```
> restart:
  with(plots):
  with(plottools):
  with(Statistics):
```

```
> # Let us take note of the initial time, just for efficiency stats

  mTime:= time():
```

```
> # let us set precision to a number of digits.

  Digits:=40:
```

```
> # let us fix the value of c and alpha. In homogeous units we have

  cv := 1:
  alv := 1:
```

```
> # The Lagrangian for material points in a Schwarzschild
  gravitational field (in polar coordinates)

  -A*c^2+dr^2/A +r^2*dte^2:
  sqrt(-%):
  L:=%;
```

$$L := \sqrt{A\,c^2 - \frac{dr^2}{A} - r^2\,dte^2} \tag{1}$$

```
> # Conjugate momenta

  diff(L, dr):
  simplify(%):
  radsimp(%) assuming r=3*al, al=1:
  pr:=%;

  diff(L, dte):
  simplify(%):
  radsimp(%) assuming r=3*al, al=1:
  pte:=%;
```

$$pr := -\frac{dr}{\sqrt{\frac{-r^2\,dte^2\,A + A^2\,c^2 - dr^2}{A}}\;A}$$

$$pte := -\frac{r^2\,dte}{\sqrt{\frac{-r^2\,dte^2\,A + A^2\,c^2 - dr^2}{A}}} \tag{2}$$

```
> # Legendre transform: solve momenta for velocities

  [Pr-pr, Pte-pte]:
  solve(%, [dr, dte]):
  allvalues(%):
  [%][1]:
  op(%):
  simplify(%):
```

```
S:=%;
```

$$S := \left[dr = -Pr\sqrt{\frac{A}{A\,Pr^2\,r^2 + Pte^2 + r^2}}\;r\,c\,A,\; dte = -\frac{Pte\sqrt{\frac{A}{A\,Pr^2\,r^2 + Pte^2 + r^2}}\;c}{r}\right] \tag{3}$$

```
> subs(S, dr):
  Dr:=%;

  subs(S, dte):
  Dte:=%;
```

$$Dr := -Pr\sqrt{\frac{A}{A\,Pr^2\,r^2 + Pte^2 + r^2}}\;r\,c\,A$$

$$Dte := -\frac{Pte\sqrt{\frac{A}{A\,Pr^2\,r^2 + Pte^2 + r^2}}\;c}{r} \tag{4}$$

```
> # Hamiltonian

  Pr*Dr+ Pte*Dte - L:
  subs(S, %):
  radsimp(%):
  simplify(%):
  %;
```

$$-\frac{\sqrt{\frac{A}{A\,Pr^2\,r^2 + Pte^2 + r^2}}\;c\left(A\,Pr^2\,r^2 + Pte^2 + r^2\right)}{r} \tag{5}$$

```
> # Again let us start from the Lagrangian

  -(1-al/r)*c^2+dr^2/(1-al/r)+r^2*dte^2:
  sqrt(-%):
  L:=%;
```

$$L := \sqrt{\left(1 - \frac{al}{r}\right)c^2 - \frac{dr^2}{1 - \frac{al}{r}} - r^2\,dte^2} \tag{6}$$

```
> diff(L, dr):
  simplify(%):
  radsimp(%) assuming r=3*al, al=1:
  pr:=%;

  diff(L, dte):
  simplify(%):
  radsimp(%) assuming r=3*al, al=1:
  pte:=%;
```

$$pr := -\frac{r^2\, dr}{\sqrt{\left(r^3\, dte^2\, al - r^4\, dte^2 + al^2\, c^2 - 2\, al\, c^2\, r + c^2\, r^2 - dr^2\, r^2\right) r\, (r - al)}}$$

$$pte := \frac{r^3\, (-r + al)\, dte}{\sqrt{\left(r^3\, dte^2\, al - r^4\, dte^2 + al^2\, c^2 - 2\, al\, c^2\, r + c^2\, r^2 - dr^2\, r^2\right) r\, (r - al)}} \tag{7}$$

```
> # Total energy first integral

  pr*dr+pte*dte-L:
  %^2:
  simplify(%):
  radsimp(%) assuming r=3*al, al=1:
  factor(%):
  subs([dr^2=dr2, dte^2=dte2], %):
  cons1:= %-c^4*ep2;
```

$$cons1 := -c^4\, ep2 - \frac{(-r + al)^3\, c^4}{\left(r^3\, dte2\, al - r^4\, dte2 + al^2\, c^2 - 2\, al\, c^2\, r + c^2\, r^2 - dr2\, r^2\right) r} \tag{8}$$

```
> # Angular momentum first integral

  pte^2:
  simplify(%):
  radsimp(%) assuming r=3*al, al=1:
  factor(%):
  subs([dr^2=dr2, dte^2=dte2], %):
  cons2:=%-k2;
```

$$cons2 := -\frac{dte2\, (-r + al)\, r^5}{r^3\, dte2\, al - r^4\, dte2 + al^2\, c^2 - 2\, al\, c^2\, r + c^2\, r^2 - dr2\, r^2} - k2 \tag{9}$$

```
> # Weierstrass equations

  [cons1, cons2]:
  solve(%, [dr2, dte2]):
  op(%):
  factor(%):
  %;

  subs(%, [dr2, dr2/dte2]):
  wPhi, wPsi := op(%):

  wPhi;
  wPsi;
```

$$\left[dr2 = \frac{(-r + al)^2\, \left(c^2\, ep2\, r^3 + al\, r^2 - r^3 + k2\, al - k2\, r\right)}{ep2\, r^5},\ dte2 = \frac{k2\, (-r + al)^2}{ep2\, r^6}\right]$$

$$\frac{(-r + al)^2\, \left(c^2\, ep2\, r^3 + al\, r^2 - r^3 + k2\, al - k2\, r\right)}{ep2\, r^5}$$

$$\frac{\left(c^2\, ep2\, r^3 + al\, r^2 - r^3 + k2\, al - k2\, r\right) r}{k2} \tag{10}$$

```
> # now we need to slow down and put Weierstrass in a smart form
  # so that later we can analytically integrate Weisrstrass
  equations.
  # essentially we want to factorize it in first order polynomials
>
  # Remember that:  -c < ep < 0
```

```
> numer(wPsi)/r:
  collect(%, r):
  P:=%;
```

$$P := \left(c^2\, ep2 - 1\right) r^3 + al\, r^2 - k2\, r + k2\, al \tag{11}$$

```
> P + (ep2*c^2-1)*(rp-r)*(r-rm)*(r-r0):
  collect(%, r):
  [subs(r=0, %), subs(r=0, diff(%, r)), subs(r=0, diff(%, r, r))]:
  simplify(%):
  solve(%, [ep2, k2, r0]):
  op(%):
  Sol1 := %;
```

$$Sol1 := \left[ep2 = -\frac{al^2\, rm + al^2\, rp - al\, rm^2 - 2\, al\, rm\, rp - al\, rp^2 + rm^2\, rp + rm\, rp^2}{c^2\left(al\, rm^2 + al\, rm\, rp + al\, rp^2 - rm^2\, rp - rm\, rp^2\right)}, k2 = -\frac{al\, rm^2\, rp^2}{al\, rm^2 + al\, rm\, rp + al\, rp^2 - rm^2\, rp - rm\, rp^2}, r0 = -\frac{al\, rm\, rp}{al\, rm + al\, rp - rp\, rm}\right] \tag{12}$$

```
> wPhi:
  subs(Sol1, %):
  simplify(%):
  mPhi := %;
```

$$mPhi := \frac{(r - rp)\, al\, (r - rm)\, (((r + rp)\, rm + r\, rp)\, al - rm\, r\, rp)\, c^2\, (r - al)^2}{r^5\, (rm + rp)\, (al - rp)\, (al - rm)} \tag{13}$$

```
> wPsi:
  subs(Sol1, %):
  simplify(%):
  mPsi := %;
```

$$mPsi := \frac{(r - rp)\, (((al - rp)\, rm + al\, rp)\, r + al\, rm\, rp)\, (r - rm)\, r}{rm^2\, rp^2} \tag{14}$$

```
> (1-al/r)*c^2/mPhi - 1/(1-al/r) - r^2/mPsi:
  sqrt(%)/c:
  simplify(%) assuming c=1, al=1, rm=10, rp=30, r=20:
  radsimp(%) assuming c=1, al=1, rm=10, rp=30, r=20:
  factor(%):
  dtau := %;
```

$$dtau := \frac{r^{3/2}\sqrt{-al\, rm^2 - al\, rm\, rp - al\, rp^2 + rm^2\, rp + rm\, rp^2}}{\sqrt{-al\, r\, rm - al\, r\, rp - al\, rm\, rp + rm\, r\, rp}\,\sqrt{al}\,\sqrt{rp - r}\,\sqrt{r - rm}\, c} \tag{15}$$

Utilities

```
> # Utility procedures
```

```
> Real:= proc(x)
  local X:
   if type(x, list) then
    map(Real, x):
   else
    X := abs(Im(x)):
    if X > 10^(-Digits+2) or X < -10^(-Digits+2) then
     print("Imaginary part neglected: ", X);
    end if:
    Re(x):
   end if:
  end:
```

```
> BranchWith := proc(S, t)
  local TSv:
   TSv:= halfT(S):
   t-tS(S, 0):
   subs(r=minS(S), %):
   evalf(%):
   %/TSv:
   evalf(%):
   floor(%):
  end:
```

```
> # When solving equations we parameterize t and theta in terms of
  r, which is not monotonic
  # thus orbits are broken in branches which need to be glue
  together to get an orbit.
  # That for satellites going along bounded orbits (and similarly
  later for light rays) can be done by the following procdure,
  # best understood by checking laler usage.
```

```
> Branch := proc(fr, branch, F, f0)
    if(floor(branch) mod 2 = 0) then
     fr:
     f0 + % + 2*F*floor(branch/2):
    else
     fr:
     f0 + 2*F*(floor(branch/2)+1) - %:
    end if:
    evalf(%):
  end:
```

```
> isOutgoing:=proc(b)
   if b mod 2 = 0 then
    return true:
   end if:
   return false:
  end:
```

```
> myRange:= proc(x1, x2)
   if x1<x2 then
    return x1..x2;
   else
    return x2..x1;
   end if:
  end:
```

```
> # Utility ciclic permutations

  Cycle:= proc(list)
   local x, n, N, r:
   x:= list[1]:
   N:= nops(list):
   r:= list:
   for n from 1 to N-1 do
    r[n] := list[n+1]:
   end do:
   r[N]:= x:
   r;
  end:
```

```
> CycleUntil:=proc(list, n)
   local r:
   if not(n in list) then
     printf("code %d is not in the list %a\n", n, list);
     return list:
   end if:
   r:= list;
   do
    r:= Cycle(r):
   until r[1]=n:
   r:
  end:
```

```
> CreateList := proc(n, v := none)
    local k:
    [seq(v,  1..n)]:
  end:
```

```
> NextEvent:=proc(Sat)
    global NextSignalAvailable:
    3*NextSignalAvailable[Sat+1] +Sat +1:
  end:
```

```
> #
  # Now let us formalize satellietes
```

```
#
> # Global variables

MaxSatellites := 3;

MaxGenerations:= 5;


MaxSignals := 2^MaxGenerations-1;
```

$$MaxSatellites := 3$$

$$MaxGenerations := 5$$

$$MaxSignals := 31 \quad \textbf{(16)}$$

```
> MaxT:= 120;
```

$$MaxT := 120 \quad \textbf{(17)}$$

```
> Ingoing := -1;
Outgoing:=  1;

Clockwise        := -1;
Counterclockwise :=  1;
```

$$Ingoing := -1$$

$$Outgoing := 1$$

$$Clockwise := -1$$

$$Counterclockwise := 1 \quad \textbf{(18)}$$

```
> #isScattering := true;
```

```
> Infalling:= 1;
SameScattering:= 2;
OtherScattering := 3;
```

$$Infalling := 1$$

$$SameScattering := 2$$

$$OtherScattering := 3 \quad \textbf{(19)}$$

```
> Side:= 1;
Front:= 2;
Behind:= 3;
```

$$Side := 1$$

$$Front := 2$$

$$Behind := 3 \quad \textbf{(20)}$$

```
>
# Verbose Flags
```

```
> ProducePlots:= true;
```

```
DebugOn := true;
DebugTimeOn := true;
DebugFlowControlOn := true;
DebugSolutionsOn := true;
```

$$ProducePlots := true$$
$$DebugOn := true$$
$$DebugTimeOn := true$$
$$DebugFlowControlOn := true$$
$$DebugSolutionsOn := true \quad (21)$$

```
> BestKnownSolution:= none:
>
```

```
ClearTypeToDo:= proc()
 global TypeToDo:
 TypeToDo := [
      true, true, true, true,
      true, true, true, true,
      true, true, true, true
       ]:
end:

ClearTypeToDo():
```

```
> TypeToDo ;
```

$$[true, true, true, true, true, true, true, true, true, true, true, true] \quad (22)$$

SearchSignal procedures

Searching for a signal is a complicated issue with many attempts and tries.
Never look for two signals at the same time.
Let us describe a search by a structure on which we can do many operations and then eventually save the result out of the structure and then

```
> TargetSat      := 1:        #  -1, 0, 1, 2
  TargetP        := 2:        #  [r, te, t]
  TargetBranch   := 3:        #  none is no sat
  TargetCol      := 4:        #  black, red, blue, green

  SourceSat      := 5:        #  0, 1, 2
  SourceBranch   := 6:        #  ..., -1, 0, 1, ...
  SourceCol      := 7:        #  black, red, blue, green
  PreviousSourceBranch:= 8:  # true | false(D)

  Gen            := 9:        # 0(D), 1, 2, ...

  IndrGuessMax   :=10:
  IndrGuessMin   :=11:
```

```
SourceCrossing:=12:

IndrmGuess    :=13:
IndK2Guess    :=14:
Indsv         :=15:
Indkv         :=16:
Indscos       :=17:

Intervalr1    :=18:
Intervalr2    :=19:
Intervalrm    :=20:
IntervalK2    :=21:

RisP          :=22:
Risr          :=23:
RisBranch     :=24:
Risrm         :=25:
RisK2         :=26:

RayType        :=27:        # Infalling | SameScattering |
OtherScattering
RayBranch      :=28:        # Ingoing | Outgoing  (at Target)
RayClock       :=29:        # Clockwise | Counterclockwise
SolveType      :=30:        # Side | Front | Behind
```

```
> SearchSignalClear := [

  none,          #x TargetSat     := 1:        #  -1, 0, 1, 2
  none,          #x TargetP       := 2:        #  [r, te, t]
  none,          #x TargetBranch  := 3:        #  none is no sat
  none,          #x TargetCol     := 4:        #  black, red, blue,
green

  none,          #x SourceSat     := 5:        #  0, 1, 2
  none,          #x SourceBranch  := 6:        #  ..., -1, 0, 1, ..
.
  none,          #x SourceCol     := 7:        #  black, red, blue,
green
  false,         # PreviousSourceBranch:= 8: # true | false(D)

  0,             #x Gen           := 9:        # 0(D), 1, 2, ...

  none,          # IndrGuessMax  :=10:
  none,          # IndrGuessMin  :=11:
  none,          # SourceCrossing:=12:

  none,          # IndrmGuess   :=13:
  none,          # IndK2Guess   :=14:
  none,          # Indsv        :=15:
  none,          # Indkv        :=16:
  none,          # Indscos      :=17:

  none,          # Intervalr1   :=18:
  none,          # Intervalr2   :=19:
  none,          # Intervalrm   :=20:
  none,          # IntervalK2   :=21:
```

```
    none,          # RisP          :=22:
    none,          # Risr          :=23:
    none,          # RisBranch     :=24:
    none,          # Risrm         :=25:
    none,          # RisK2         :=26:

    none,          # RayType        :=27:        # Infalling |
  SameScattering | OtherScattering
    none,          # RayBranch      :=28:        # Ingoing | Outgoing
  (at Target)
    none,          # RayClock       :=29:        # Clockwise |
  Counterclockwise
    none           # SolveType      :=30:        # Side | Front |
  Behind
  ]:

  SearchSignal := SearchSignalClear:



> ClearSearchSignal := proc()
    global SearchSignal;
    SearchSignal := SearchSignalClear:
  end:

> SaveSearchSignal := proc()
    return SearchSignal:
  end:

> RestoreSearchSignal := proc(saved)
    global SearchSignal;
    SearchSignal := saved:
  end:

> Get := proc(field)
  global SearchSignalFields:
    SearchSignal[field];
  end:

> Set := proc(field, val)
    global SearchSignal;
    #print("Set %d = %a", field, val);
    SearchSignal[field] := val;
  end:

> NewSearch:= proc()
    Set(RisP, none):
    Set(Risr, none):
    Set(RisBranch, none):
    Set(Risrm, none):
    Set(RisK2, none):


    #Set(RayType, none):        #check they are not set in
  LinearGuess
    #Set(RayBranch, none):
```

```
    #Set(RayClock, none):
    #Set(SolveType, none):
  end:
> SetFreeTarget := proc(P)
    Set(TargetSat, -1):
    Set(TargetP, P):
    Set(TargetCol, black):
  end:
> SetTargetPoint := proc(Sat, Pv)
    Set(TargetSat, Sat):
    Set(TargetP, Pv):
    Set(TargetBranch, BranchWith(Sat, Pv[3])):
    if Sat = 1 then
      Set(TargetCol, red):
    elif Sat = 2 then
      Set(TargetCol, blue):
    elif Sat = 3 then
      Set(TargetCol, green):
    else
      Set(TargetCol, black):
    end if:
  end:
> SetSourceSat := proc(Sat)
    Set(SourceSat, Sat):
    Set(SourceBranch, BranchWith(Sat, Get(TargetP)[3])):
    if Sat = 1 then
      Set(SourceCol, red):
    elif Sat = 2 then
      Set(SourceCol, blue):
    elif Sat = 3 then
      Set(SourceCol, green):
    else
      Set(SourceCol, black):
    end if:
  end:
> SetGeneration := proc(G)
      Set(Gen, G):
  end:
>  GetGeneration := proc()
      Get(Gen):
  end:

# if Pr<rSat.min          |     P    <--    S
#
#           Scattering                          Infalling
# sv<0        Nope                                Nope
# 0<sv<1       Out Other                            Nope
# sv>1        In Same                             In (single branch)
#
```

```
#
# if Pr>rSat.max          |     S   -->   P
#
#          Scattering                          Infalling
# sv<0       Out Same                              Out (single branch)
# 0<sv<1      Out Other                             Nope
# sv>1        Nope                                Nope
#
```

```
> LinearGuess:= proc()
   local targetR, targetTe, targetT:
   local sourceSat, sourceBranch:
   local rGuessMax, rGuessMin, d2, sv, rmGuess, kv, scos:
   local mmin, mmax, tm, tM, rguess:

   targetR, targetTe, targetT := op(Get(TargetP)):
   sourceSat := Get(SourceSat):
   sourceBranch := Get(SourceBranch):
   mmin := minS(sourceSat):
   mmax := maxS(sourceSat):


   tS(sourceSat, sourceBranch)-targetT:
   evalf(%):
   fsolve(%, r=mmin..mmax, fulldigits):
   rGuessMax:=%:
   Set(IndrGuessMax, rGuessMax):
   BranchWith(sourceSat, targetT- MaxT):
   tS(sourceSat, %)-(targetT - MaxT):
   evalf(%):
   fsolve(%, r=mmin..mmax, fulldigits):
   rGuessMin:= %:
   Set(IndrGuessMin, rGuessMin):
   # notice: it is guaranteed that rGuessMin < rGuessMax ()it
  depends on the satallite branch
   # what is guaranteed is that t at rGuessMin is less than t at
  rGuessMax
   # Therefore be careful when writing Ranges.

   tS(sourceSat, sourceBranch):
   subs(r=mmin, %):
   evalf(%):
   tm:=%:

   tS(sourceSat, sourceBranch):
   subs(r=mmax, %):
   Real(%):
   simplify(%):
   evalf(%):
   tM:=%:

  #printf("mmin= %a, mmax=%a, rGuessMax= %a, rGuessMin=%a\n", mmin,
  mmax, rGuessMax, rGuessMin);
   #    tm< targetT - MaxT (< targetT < tM ) ?
   if  targetT-MaxT < tm  and tm < targetT then
```

```
      Set(PreviousSourceBranch,false):
      Set(SourceCrossing, true):
      Set(Intervalr1, myRange(mmin, rGuessMax)):
      Set(Intervalr2, myRange(mmin, rGuessMin)):
      printf("two intervals r = %a or r = %a\n", myRange(mmin,
rGuessMax), myRange(mmin, rGuessMin)):
 elif  targetT-MaxT < tM  and tM < targetT then
      Set(PreviousSourceBranch,false):
      Set(SourceCrossing, true):
      Set(Intervalr1, myRange(rGuessMax, mmax)):
      Set(Intervalr2, myRange(rGuessMin, mmax)):
      printf("two intervals r = %a or r = %a\n", myRange
(rGuessMax, mmax), myRange(rGuessMin, mmax)):
 else
   Set(PreviousSourceBranch,false):
   Set(SourceCrossing, false):
   Set(Intervalr1, myRange(rGuessMax, rGuessMin)):
   Set(Intervalr2, none):
   printf("one interval r = %a\n", myRange(rGuessMax, rGuessMin))
:
 end if:

 (s*targetR*cos(targetTe)+(1-s)*xS(sourceSat, sourceBranch))^2
  +(s*targetR*sin(targetTe) + (1-s)*yS(sourceSat, sourceBranch))
^2:
 subs(r=rGuessMax, %):
 d2:= evalf(%):
 diff(%, s):
 solve(%, s):
 sv:=%;
 Set(Indsv, sv):

 subs(s=sv, d2):
 sqrt(%):
 evalf(%):
 rmGuess:=%;                                          # many times
the linear approximation is a good starting point to locate rm.
 Set(IndrmGuess, rmGuess):

 xS(sourceSat, sourceBranch)*targetR*sin(targetTe)-yS(sourceSat,
sourceBranch)*targetR*cos(targetTe):
 subs(r=rGuessMax, %):
 kv:=evalf(%):
 Set(Indkv, kv):

 targetR*cos(targetTe)*xS(sourceSat, sourceBranch)+targetR*sin
(targetTe)*yS(sourceSat, sourceBranch):
 subs(r=rGuessMax, %):
 scos:= %:                                            # >0 target
and source are on the same side of BH
 Set(Indscos, scos):

 if kv > 0 then
   Set(RayClock, Counterclockwise):
 else
   Set(RayClock, Clockwise):
 end if:
```

```
 Set(Intervalrm, 3/2..min(targetR, mmax)):
 Set(IntervalK2, 0..27/4):

# forse è più importante stare larghi così da sapere che in sides
non ci possono essere infalling
# e lasciare la possibilità tutta in front e behind

 if rmGuess > 4 then
    Set(SolveType, Side):
    if targetR > mmax then                    #           |
S  ---> P
      if sv<0 then
         Set(RayBranch, Outgoing):
         Set(RayType, SameScattering):
      elif sv>0 and sv<1 then
         Set(RayBranch, Outgoing):
         Set(RayType, OtherScattering):
      else       # sv>1
         printf("It cannot be s>1 and |      S  ---> P\n");
      end if:
    elif targetR < mmin then                  #           |
P  <--- S
      if sv<0 then
         printf("It cannot be s<0 and |      P  <--- S\n");
      elif sv>0 and sv<1 then
         Set(RayBranch, Outgoing):
         Set(RayType, OtherScattering):
      else       # sv>1
         Set(RayBranch, Ingoing):
         Set(RayType, SameScattering):
      end if:
    else
      printf("The user is in the satellite ring. Too messy to be
considered. At least now.\n");
    end if:
 else
    if scos > 0 then                     # same side of BH

     Set(SolveType, Front):
     if rmGuess > 1.5 then
       if targetR > mmax then                     #  |      S  --
-> P
         Set(RayBranch, Outgoing):
         Set(RayType, SameScattering ):
       elif targetR < mmin then                   #  |      P  <-
-- S
         Set(RayBranch, Ingoing):
         Set(RayType, SameScattering ):
       end if:
     else
       if targetR > mmax then                     #  |      S  --
-> P
         Set(RayBranch, Outgoing):
         Set(RayType, Infalling):
       elif targetR < mmin then                   #  |      P  <-
-- S
         Set(RayBranch, Ingoing):
```

```
          Set(RayType, Infalling):
        end if:
      end if:
     else                                # opposite sides of BH
       Set(SolveType, Behind):
       if targetR > mmax then                    #  |      S  --
-> P
          Set(RayBranch, Outgoing):
          Set(RayType, OtherScattering ):
       elif targetR < mmin then                  #  |      P  <--
- S
          Set(RayBranch, Outgoing):
          Set(RayType, OtherScattering ):
       end if:
     end if:
  end if:
end:
```

```
> SetPreviousSourceBranch := proc()
    local Crossing, Int2, sourceBranch:
    Crossing := Get(SourceCrossing):
    Set(SourceCrossing, false):

    if Crossing then
      Int2 := Get(Intervalr2):
      Set(Intervalr1, Int2):
      Set(Intervalr2, none):
      sourceBranch := Get(SourceBranch):
      Set(SourceBranch, sourceBranch-1):
    end if:
  end:
```

```
> OtherClock := proc()
   local rayClock:
   rayClock := Get(RayClock):
   Set(RayClock, -rayClock):
  end:
```

```
> CreateHint:= proc()
   local h1, h2, h3, h4, h5, h6, h7, h8:
   h1:= Get(Risr):
   h6:= Get(Intervalr1);
   printf("r=%g in [%a]\n",h1, h6):
   h2:= Get(RayType):
   if h2 = Infalling then
     h7:= Get(IntervalK2);
     h5:= Get(RisK2);
     printf("Infalling ray (K2=%g) in [%a].\n", h5, h7);
   elif h2 = SameScattering then
     h7:= Get(Intervalrm);
     h5:= Get(Risrm);
     printf("Scattering ray (rm=%g) in [%a]: target and source on
  the same branch.\n", h5, h7);
   elif h2 = OtherScattering then
     h7:= Get(Intervalrm);
     h5:= Get(Risrm);
```

```
    printf("Scattering ray (rm=%g) in [%a]: target and source on
the different branches.\n", h5, h7);
  else
    printf("Type of ray unknown.\n");
  end if:
  h3:= Get(RayClock):
  if h3 = Clockwise then
    printf("Clockwise ray.\n");
  else
    printf("Counterclockwise ray.\n");
  end if:
  h4:= Get(RayBranch):
  if h4 = Outgoing then
    printf("Ray outgoing at target.\n");
  else
    printf("Ray outgoing at target.\n");
  end if:
  h8 := Get(SolveType):
  if h8 = Side then
    printf("Solve Side.\n");
  elif h8 = Front then
    printf("Solve Front.\n");
  elif h8 = Behind then
    printf("Solve Behind.\n");
  else
    printf("Solve type unknown.\n");
  end if:
  [h1, h2, h3, h4, h5, h6, h7, h8]:
end:
```

```
> UseHint:= proc(h)
    printf("hint used Hint := %a\n", h);
    if type(h, list) and nops(h)=8 then
     Set(Risr, h[1]):
     Set(IndrGuessMax, h[1]):
     Set(RayType, h[2]):
     if h[2] = Infalling then
       Set(RisK2, h[5]):
       Set(IndK2Guess, h[5]):
       Set(IntervalK2, h[7]):
     elif h[2] = SameScattering then
       Set(Risrm, h[5]):
       Set(IndrmGuess, h[5]):
       Set(Intervalrm, h[7]):
     elif h[2] = OtherScattering then
       Set(Risrm, h[5]):
       Set(IndrmGuess, h[5]):
       Set(Intervalrm, h[7]):
     else
       printf("Type of ray unknown.\n");
     end if:
     Set(RayClock, h[3]):
     Set(RayBranch, h[4]):
     Set(Intervalr1, h[6]):
     Set(SolveType, h[8]):
    else
```

```
    printf("No hint used.\n");
    return:
  end if:
end:
```

```
> # First satellite
```

```
> rp0v := 19*alv;
  rm0v := 10*alv;

  t0in := 0:
  te0in := Pi/6:
  tau0in := 1:
```

$$rp0v := 19$$
$$rm0v := 10 \tag{23}$$

```
> Sat0 := [te0in, t0in, tau0in, rm0v, rp0v];
```

$$Sat0 := \left[\frac{\pi}{6}, 0, 1, 10, 19\right] \tag{24}$$

```
>
  # for each satellite we can analytically integrate Weierstrass
  equations
  # This is painfully detailed and sensitive.
  # Small changes end up with Maple find worse representations of the
  integrals.
```

```
> te0in:
  evalf(%);

  t0in:
  evalf(%);

  tau0in:
  evalf(%);


  rp0v:
  evalf(%);
```

```
rm0v:
evalf(%);
```

$$0.52359877559829887307710723054658381403 29$$

$$0.$$

$$1.$$

$$19.$$

$$10. \tag{25}$$

```
> ComputeSat0:= proc()
  global te0in, t0in, tau0in, rm0v, rp0v;
  global t0r, te0r, tau0r, Te0, T0, Tau0;

  mPhi:
  1/%:
  simplify(%)assuming c=cv, al=alv, rm=rm0v, rp=rp0v, r=(rm0v+rp0v)
 /2:
  sqrt(%):
  radsimp(%) assuming c=cv, al=alv, rm=rm0v, rp=rp0v, r=(rm0v+rp0v)
 /2:
  #%;
  subs(r=R, %):
  subs([c=cv, al=alv, rm=rm0v, rp=rp0v], %):
  int(%, R=rm0v..r) assuming  r>rm0v, r<rp0v:
  simplify(%) assuming r>rm0v, r<rp0v:
  radsimp(%) assuming r>rm0v, r<rp0v:
  t0r:= %:

  mPsi:
  1/%:
  simplify(%) assuming c=cv, al=alv, rm=rm0v, rp=rp0v, r=(rm0v+rp0v)
 /2:
  sqrt(%):
  radsimp(%) assuming c=cv, al=alv, rm=rm0v, rp=rp0v, r=(rm0v+rp0v)
 /2:
  #%;
  subs(r=R, %):
  subs([c=cv, al=alv, rm=rm0v, rp=rp0v], %):
  int(%, R=rm0v..r) assuming  r>rm0v, r<rp0v :
  simplify(%) assuming r>rm0v, r<rp0v:
  radsimp(%) assuming r>rm0v, r<rp0v:
  te0r := %:

  dtau :
  subs(r=R, %):
  subs([c=cv, al=alv, rm=rm0v, rp=rp0v], %):
  int(%, R=rm0v..r) assuming  r>rm0v, r<rp0v :
  simplify(%) assuming r>rm0v, r<rp0v:
  radsimp(%) assuming r>rm0v, r<rp0v:
  tau0r := %:

  te0r:
  subs(r=rp0v, %):
  simplify(%):
```

```
 Te0:=%:

 t0r:
 subs(r=rp0v, %):
 simplify(%):
 T0:=%:

 tau0r:
 subs(r=rp0v, %):
 simplify(%):
 Tau0:=%:
end:
```

```
> ComputeSat0():
```

```
>
# (half-)periods
```

```
>
evalf(2*T0);

#  578.22345916140625853458272916314050357858985318402878284242436145532288544239669
```

$$551.858605970311955031969931598092207361 7 \tag{26}$$

```
>
evalf(2*(Te0-Pi)); # Precession (in radiant)

#   0.85542692730438791614573939149815814151016744762639239692633389527823910515960 14
```

$$0.873546128865921305360799504432803492183 \tag{27}$$

```
>
evalf(2*Tau0);

#    548.26851078993541052961540893718234388592591047462285304450208564692352991626784

evalf(Tau0/T0); # Slowing time factor

#    0.94819485806592088842847416539800089835106361454635876625224347352747293450158832
```

$$522.287354766728461688751737182024559073 4$$

(28)

$$0.9464151670669527942943071936027436069656 \quad (28)$$

```
>
  # Second satellite
> rp1v := 28*alv;
  rm1v := 20*alv;

  t1in := 10:
  te1in := 7*Pi/6:
  tau1in := -1:
```

$$rp1v := 28$$
$$rm1v := 20 \quad (29)$$

```
> Sat1 := [te1in, t1in, tau1in, rm1v, rp1v];
```

$$Sat1 := \left[\frac{7\pi}{6}, 10, -1, 20, 28\right] \quad (30)$$

```
> ComputeSat1:= proc()
   global te1in, t1in, tau1in, rm1v, rp1v;
   global t1r, te1r, tau1r, Te1, T1, Tau1;

   mPhi:
   1/%:
   simplify(%)assuming c=cv, al=alv, rm=rm1v, rp=rp1v, r=(rm1v+rp1v)
  /2:
   sqrt(%):
   radsimp(%) assuming c=cv, al=alv, rm=rm1v, rp=rp1v, r=(rm1v+rp1v)
  /2:
   #%;
   subs(r=R, %):
   subs([c=cv, al=alv, rm=rm1v, rp=rp1v], %):
   int(%, R=rm1v..r) assuming  r>rm1v, r<rp1v:
   simplify(%) assuming r>rm1v, r<rp1v:
   radsimp(%) assuming r>rm1v, r<rp1v:
   t1r:= %:

   mPsi:
   1/%:
   simplify(%) assuming c=cv, al=alv, rm=rm1v, rp=rp1v, r=(rm1v+rp1v)
  /2:
   sqrt(%):
   radsimp(%) assuming c=cv, al=alv, rm=rm1v, rp=rp1v, r=(rm1v+rp1v)
  /2:
   #%;
   subs(r=R, %):
   subs([c=cv, al=alv, rm=rm1v, rp=rp1v], %):
   int(%, R=rm1v..r) assuming  r>rm1v, r<rp1v :
   simplify(%) assuming r>rm1v, r<rp1v:
   radsimp(%) assuming r>rm1v, r<rp1v:
   te1r := %:

   dtau :
```

```
  subs(r=R, %):
  subs([c=cv, al=alv, rm=rm1v, rp=rp1v], %):
  int(%, R=rm1v..r) assuming  r>rm1v, r<rp1v :
  simplify(%) assuming r>rm1v, r<rp1v:
  radsimp(%) assuming r>rm1v, r<rp1v:
  tau1r := %:

  te1r:
  subs(r=rp1v, %):
  simplify(%):
  Te1:=%:

  t1r:
  subs(r=rp1v, %):
  simplify(%):
  T1:=%:

  tau1r:
  subs(r=rp1v, %):
  simplify(%):
  Tau1:=%:
end:
```

```
> ComputeSat1():
```

```
> evalf(2*T1);

#    1184.261809516394362783565152572288103278
#
1184.2618095163943627835651525722881032778463634121270424429739336922389943742940
```

$$1117.0453880143650528563037702430694317 46 \quad (31)$$

```
> evalf(2*(Te1-Pi)); # Precession (in radiant)

#   0.43393656676173086232475433081393313687251190888468448812882215639767738953752606
```

$$0.44765740547383417354208322193206819566 5 \quad (32)$$

```
> evalf(2*Tau1);

#   1148.119711746172713886661882040516422037
#
1148.11971174617271388666188204051642203779309198998560051125124650117236797851 36

evalf(Tau1/T1); # Slowing time factor

#   0.969481327963298376788649673225728156380773250595058476841176891 44
#
0.969481327963298376788649673225728156380773250595058476841176891 44
```

```
014037883456925
```

$$1081.52885895204264646740801887285310909 0$$
$$0.968204936483864997058725659901332051383 6 \quad (33)$$

```
>
# Terzo satellite

> rp2v := 40*alv;
rm2v := 31*alv;

t2in := -10:
te2in := 3*Pi/4:
tau2in := -3:
```

$$rp2v := 40$$
$$rm2v := 31 \quad (34)$$

```
> Sat2 := [te2in, t2in, tau2in, rm2v, rp2v];
```

$$Sat2 := \left[\frac{3\pi}{4}, -10, -3, 31, 40\right] \quad (35)$$

```
> OrbitalParameters := [Sat0, Sat1, Sat2];
```

$$OrbitalParameters := \left[\left[\frac{\pi}{6}, 0, 1, 10, 19\right], \left[\frac{7\pi}{6}, 10, -1, 20, 28\right], \left[\frac{3\pi}{4}, -10, -3, 31, 40\right]\right] \quad (36)$$

```
> ComputeSat2:= proc()
 global te2in, t2in, tau2in, rm2v, rp2v;
 global t2r, te2r, tau2r, Te2, T2, Tau2;

 mPhi:
 1/%:
 simplify(%)assuming c=cv, al=alv, rm=rm2v, rp=rp2v, r=(rm2v+rp2v)
/2:
 sqrt(%):
 radsimp(%) assuming c=cv, al=alv, rm=rm2v, rp=rp2v, r=(rm2v+rp2v)
/2:
 #%;
 subs(r=R, %):
 subs([c=cv, al=alv, rm=rm2v, rp=rp2v], %):
 int(%, R=rm2v..r) assuming  r>rm2v, r<rp2v:
 simplify(%) assuming r>rm2v, r<rp2v:
 radsimp(%) assuming r>rm2v, r<rp2v:
 t2r:= %:

 mPsi:
 1/%:
 simplify(%) assuming c=cv, al=alv, rm=rm2v, rp=rp2v, r=(rm2v+rp2v)
/2:
 sqrt(%):
 radsimp(%) assuming c=cv, al=alv, rm=rm2v, rp=rp2v, r=(rm2v+rp2v)
/2:
 #%;
 subs(r=R, %):
 subs([c=cv, al=alv, rm=rm2v, rp=rp2v], %):
```

```
  int(%, R=rm2v..r) assuming  r>rm2v, r<rp2v :
  simplify(%) assuming r>rm2v, r<rp2v:
  radsimp(%) assuming r>rm2v, r<rp2v:
  te2r := %:

  dtau :
  subs(r=R, %):
  subs([c=cv, al=alv, rm=rm2v, rp=rp2v], %):
  int(%, R=rm2v..r) assuming  r>rm2v, r<rp2v :
  simplify(%) assuming r>rm2v, r<rp2v:
  radsimp(%) assuming r>rm2v, r<rp2v:
  tau2r := %:

  te2r:
  subs(r=rp2v, %):
  simplify(%):
  Te2:=%:

  t2r:
  subs(r=rp2v, %):
  simplify(%):
  T2:=%:

  tau2r:
  subs(r=rp2v, %):
  simplify(%):
  Tau2:=%:
end:
```

```
> ComputeSat2():
```

```
> evalf(2*T2);

#   1924.3157108254298524753576496804412608481353204138992565642292965042562209255413
```

$$1964.37943042340836593742930424970788409 8 \tag{37}$$

```
> evalf(2*(Te2-Pi)); # Precession (in radiant)

#    0.29437940720856936334241086222726917677259654106927873817281741201712231882042 81
```

$$0.28856769381376595952997819096930292545 7 \tag{38}$$

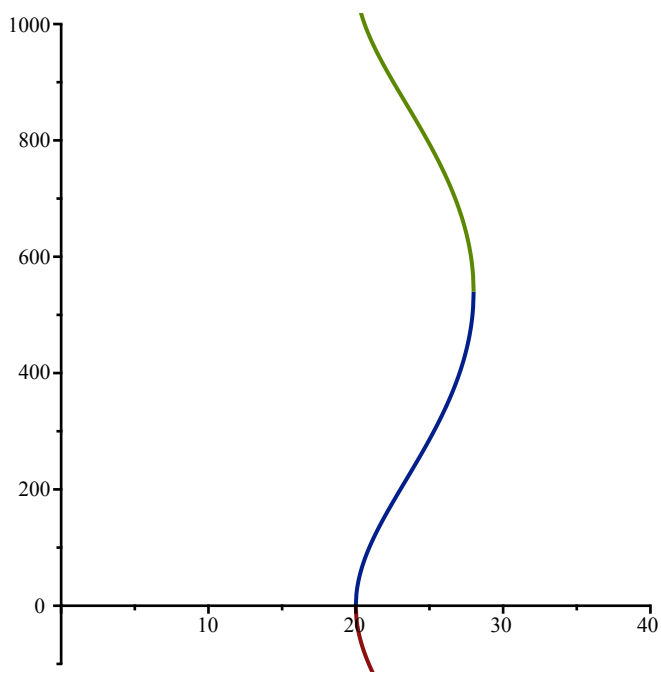
```
> evalf(2*Tau2);

#    1882.6026999684656759095351433203713923864189673264784554720763126620447505944419
```

```
evalf(Tau2/T2); # Slowing time factor

#    0.97832319789195531901590011774974859286 19
#
0.97832319789195531901590011774974859286184191659675804133111562569
483350916747710
```

$$1922.4102263793288112582801270947484102 81$$

$$0.97863487909001705248634287789275776711 86 \quad \textbf{(39)}$$

```
> # Demo
  # for plotting tau1 it takes about 4s a branch
  # Notice the usage of Branch(...)

  [
    [r, Branch(tau1r, -1, Tau1, tau1in), r=rm1v..rp1v],
    [r, Branch(tau1r,  0, Tau1, tau1in), r=rm1v..rp1v],
    [r, Branch(tau1r,  1, Tau1, tau1in), r=rm1v..rp1v]
  ]:
  plot(%, view=[0..40, -100..1000] ):
  %;
```



Satellites

Satellite 0

```
> # Satellite 0
> xS0 := proc(branchv)
    r*cos(Branch(te0r,branchv, Te0, te0in)):
```

```
end:
> yS0 := proc(branchv)
 r*sin(Branch(te0r,branchv, Te0, te0in)):
end:
> tS0 := proc(branchv)
 Branch(t0r, branchv, T0, t0in):
end:
> minS0:= proc()
  rm0v;
end:
> maxS0:= proc()
  rp0v;
end:
> halfT0:= proc()
  T0;
end:
> halfTe0:= proc()
  Te0;
end:
> halfTau0:= proc()
  Tau0;
end:
```

Satellite 1

```
> xS1 := proc(branchv)
 r*cos(Branch(te1r,branchv, Te1, te1in)):
end:
> yS1 := proc(branchv)
 r*sin(Branch(te1r,branchv, Te1, te1in)):
end:
> tS1 := proc(branchv)
 Branch(t1r, branchv, T1, t1in):
end:
> minS1:= proc()
  rm1v;
end:
> maxS1:= proc()
  rp1v;
end:
```

```
> halfT1:= proc()
    T1;
  end:
```

```
> halfTe1:= proc()
    Te1;
  end:
```

```
> halfTau1:= proc()
    Tau1;
  end:
```

Satellite 2

```
> xS2 := proc(branchv)
   r*cos(Branch(te2r,branchv, Te2, te2in)):
  end:
```

```
> yS2 := proc(branchv)
   r*sin(Branch(te2r,branchv, Te2, te2in)):
  end:
```

```
> tS2 := proc(branchv)
   Branch(t2r, branchv, T2, t2in):
  end:
```

```
> minS2:= proc()
    rm2v;
  end:
```

```
> maxS2:= proc()
    rp2v;
  end:
```

```
> halfT2:= proc()
    T2;
  end:
```

```
> halfTe2:= proc()
    Te2;
  end:
```

```
> halfTau2:= proc()
    Tau2;
  end:
```

```
> rSat:= proc(S, rv, branchv)
   rv:
```

```
  end:
> teSat:= proc(S, rv, branchv)
   if S=0 then
     Branch(te0r,branchv, Te0, te0in):
   elif S=1 then
     Branch(te1r,branchv, Te1, te1in):
   elif S=2 then
     Branch(te2r,branchv, Te2, te2in):
   end if:
   subs(r=rv, %):
   evalf(%):
  end:
> tSat:= proc(S, rv, branchv)
   if S=0 then
     Branch(t0r, branchv, T0, t0in):
   elif S=1 then
     Branch(t1r, branchv, T1, t1in):
   elif S=2 then
     Branch(t2r, branchv, T2, t2in):
   end if:
   subs(r=rv, %):
   evalf(%):
  end:
> tauSat:= proc(S, rv, branchv)
   if S=0 then
     Branch(tau0r,  branchv, Tau0, tau0in):
   elif S=1 then
     Branch(tau1r,  branchv, Tau1, tau1in):
   elif S=2 then
     Branch(tau2r,  branchv, Tau2, tau2in):
   end if:
   subs(r=rv, %):
   evalf(%):
  end:
> xS := proc(S, b)
   if S = 0 then
    xS0(b):
   elif S = 1 then
    xS1(b):
   elif S = 2 then
    xS2(b):
   end if:
  end:
> yS := proc(S, b)
   if S = 0 then
    yS0(b):
   elif S = 1 then
    yS1(b):
   elif S = 2 then
    yS2(b):
   end if:
```

```
  end:
> tS := proc(S, b)
   if S = 0 then
    tS0(b):
   elif S = 1 then
    tS1(b):
   elif S = 2 then
    tS2(b):
   end if:
  end:
> maxS := proc(S)
   if S = 0 then
    maxS0():
   elif S = 1 then
    maxS1():
   elif S = 2 then
    maxS2():
   end if:
  end:
> minS := proc(S)
   if S = 0 then
    minS0():
   elif S = 1 then
    minS1():
   elif S = 2 then
    minS2():
   end if:
  end:
> halfT:= proc(S)
   if S = 0 then
    halfT0():
   elif S = 1 then
    halfT1():
   elif S = 2 then
    halfT2():
   end if:
  end:
> halfTe:= proc(S)
   if S = 0 then
    halfTe0():
   elif S = 1 then
    halfTe1():
   elif S = 2 then
    halfTe2():
   end if:
  end:
> halfTau:= proc(S)
   if S = 0 then
    halfTau0():
   elif S = 1 then
```

```
    halfTau1():
   elif S = 2 then
    halfTau2():
   end if:
  end:
```

```
> # Plot satellites worldlines
```

```
> [xS(0, -1), yS(0, -1),tS(0, -1), r=minS(0)..maxS(0)-0.00001]:
  evalf(%):
  c0_1n := spacecurve(%, axes=box, color = red):
```

```
> [xS(0, 0), yS(0, 0),tS(0, 0), r=minS(0)..maxS(0)-0.00001]:
  evalf(%):
  c0_0 := spacecurve(%, axes=box, color = red, thickness = 4):
```

```
> [xS(0, 1), yS(0, 1),tS(0, 1), r=minS(0)..maxS(0)-0.00001]:
  evalf(%):
  c0_1 := spacecurve(%, axes=box, color = red):
```

```
> [xS(1, -1), yS(1, -1),tS(1, -1), r=rm1v..maxS(1)]:
  evalf(%):
  c1_1n := spacecurve(%, axes=box, color = blue):
```

```
> [xS(1, 0), yS(1, 0),tS(1, 0), r=minS(1)..maxS(1)]:
  evalf(%):
  c1_0 := spacecurve(%, axes=box, color = blue, thickness = 4):
```

```
> [xS(1, 1), yS(1, 1),tS(1, 1), r=minS(1)..maxS(1)]:
  evalf(%):
  c1_1 := spacecurve(%, axes=box, color = blue):
```

```
> [xS(2, -1), yS(2, -1),tS(2, -1), r=minS(2)..maxS(2)]:
  evalf(%):
  c2_1n := spacecurve(%, axes=box, color = green):
```

```
> [xS(2, 0), yS(2, 0),tS(2, 0), r=minS(2)..maxS(2)]:
  evalf(%):
  c2_0 := spacecurve(%, axes=box, color = green, thickness = 4):
```

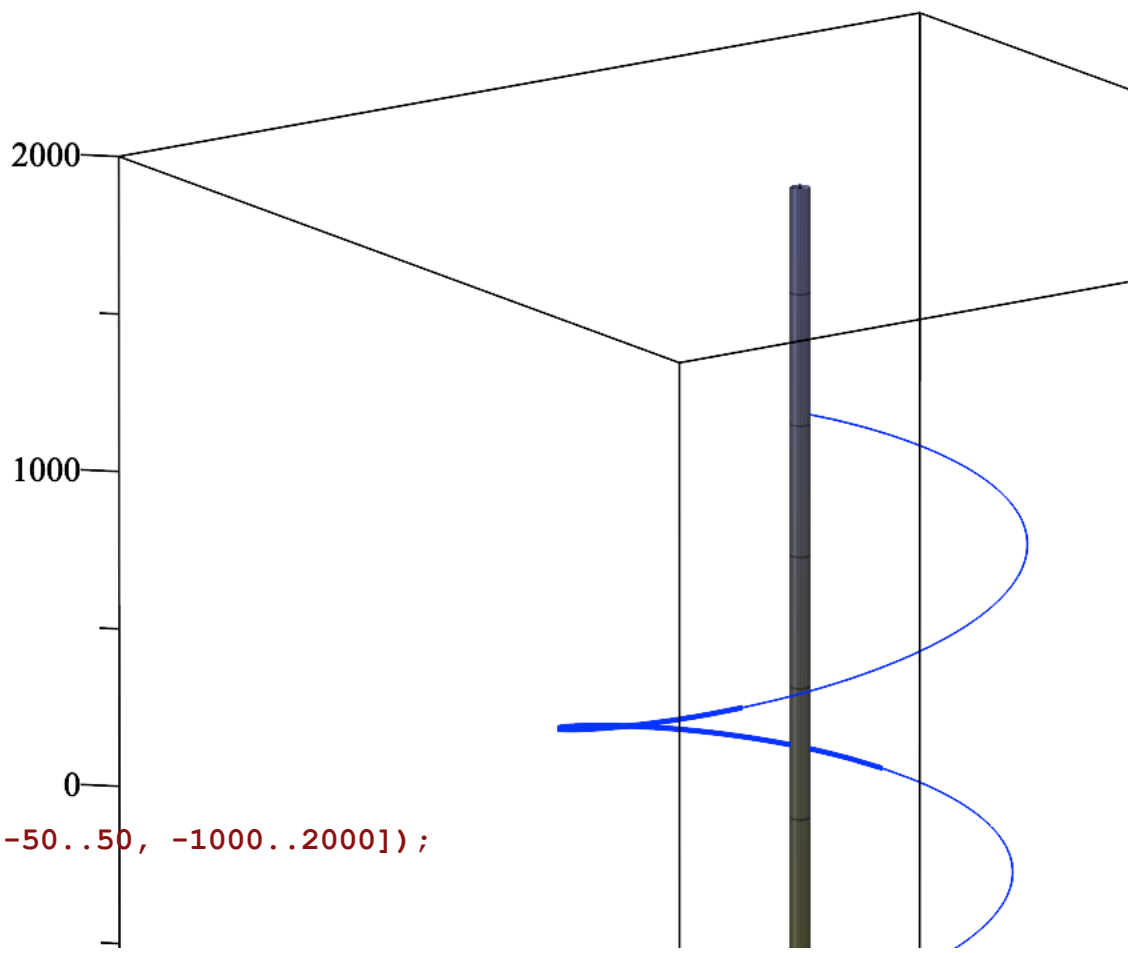
```
> [xS(2, 1), yS(2, 1),tS(2, 1), r=minS(2)..maxS(2)]:
  evalf(%):
  c2_1 := spacecurve(%, axes=box, color = green):
```

```
> BH := spacecurve([0,0,s, s=-5000..5000],
  axes=box, color = black, linestyle = dash):
```

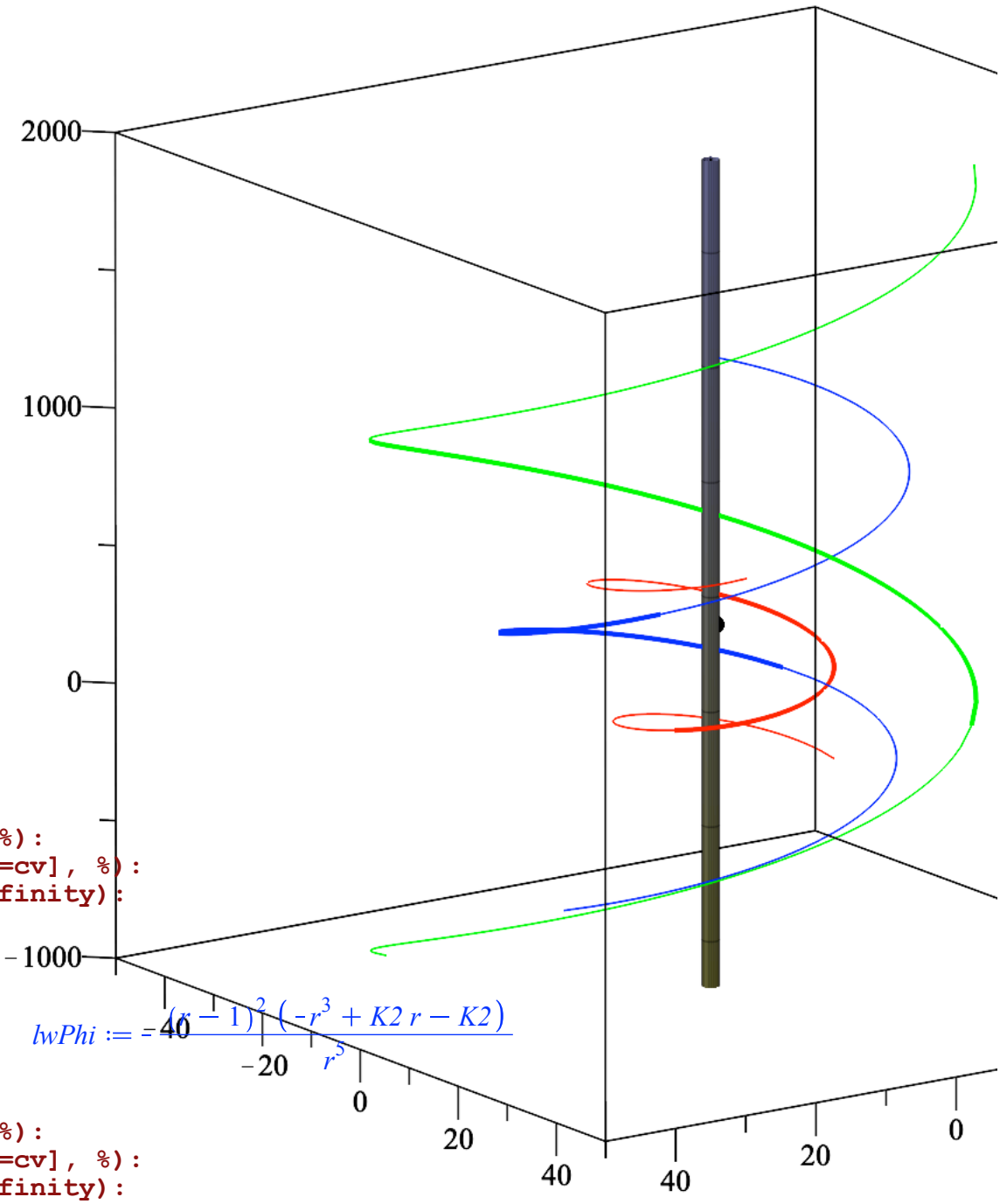
```
> [R*cos(te), R*sin(te), t]:
  subs(R=1*alv, %):
  plot3d(%, te=0..2*Pi, t=-5000..5000):
  S := %:
```

```
> ux := -alv*5:
  uy := -alv*6:
  ut := 250:

  user := pointplot3d([ux, uy, ut], color=black, symbol =
  solidcircle, symbolsize = 10):
```

```
> display(BH, S,
  c1_1n,c1_0,c1_1,
  view=[ -50..50, -50..50, -1000..2000]);
```




```
> display(BH, S,
  user,
  c0_1n,c0_0,c0_1,
  c1_1n,c1_0,c1_1,
  c2_1n,c2_0,c2_1,
  view=[ -50..50, -50..50, -1000..2000]);
```




```
> #   Light rays
```

```
> wPhi:
  subs(k2=K2*ep2,%):
  subs([al=alv, c=cv], %):
  limit(%, ep2=infinity):
  factor(%):
  lwPhi := %;
```

$$lwPhi := -\frac{(r-1)^2\left(-r^3+K2\,r-K2\right)}{r^5}$$

```
> wPsi:
  subs(k2=K2*ep2,%):
  subs([al=alv, c=cv], %):
  limit(%, ep2=infinity):
  factor(%):
  lwPsi := %;
```

$$lwPsi := -\frac{\left(-r^3+K2\,r-K2\right)r}{K2}$$

```
> # Again we need to put Weierstrass in a form suitable for later
  integration
  # even though in this case we have both scattering and infalling
  rays.

  numer(lwPsi)/r:
  lp:=%;

  subs(r=al, %):
  subs([al=alv, c=cv], %):
  %;
```

$$lp := r^3 - K2\,r + K2$$

$$1 \tag{42}$$

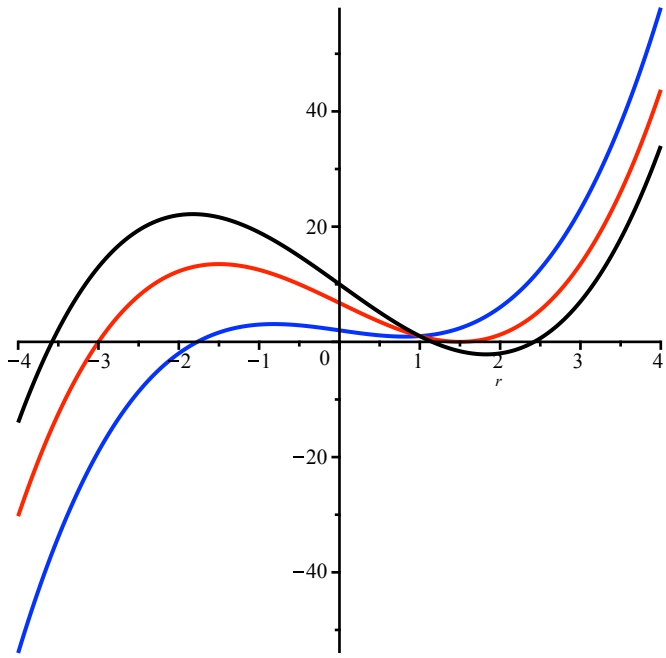
```
> lp:
  diff(%, r):
  solve(%, r):
  rcrit:=[%][1];
```

$$rcrit := \frac{\sqrt{3}\,\sqrt{K2}}{3} \tag{43}$$

```
> lp:
  subs(r=rcrit, %):
  subs(K2=K^2, %):
  radsimp(%):
  %/K^2:
  simplify(%):
  solve(%, K):
  %^2:
  K2crit:=%;
```

$$K2crit := \frac{27}{4} \tag{44}$$

```
> plot([subs(K2=2, lp),subs(K2=K2crit, lp), subs(K2=10, lp) ], r=-4.
  .4, color=[blue, red, black]);
```




```
> #
  # One real solution
  # 0 < K2 < 27/4

  #
  # To be honest we should work better for the integration of
  infalling branches
  # (to be improved, being currently numerical)
  #
```

```
> lwPhi:
  #subs([al=alv, c=cv], %):
  sqrt(%):
  radsimp(%) assuming r>0:
  simplify(%) assuming r>0:
  1/%:
  subs(r=R, %):
  Int(%, R=r0..r) assuming K2>0:
  inltr:=t0+ branch*%;

  #
  #  branch =  1 : Outgoing
  #  branch = -1 : Ingoing
```

$$inltr := \left(\int_{r0}^{r} \frac{R^{5/2}}{(R-1)\sqrt{R^3 - K2\,R + K2}}\, dR \right) branch + t0 \tag{45}$$

```
> # Test

  inltr:
  subs([t0=13, r0=5, te0=Pi/4], %):
```

```
subs([branch=Ingoing], %):
subs([K2=8/3], %):
subs([r=12], %):
evalf(%):
%;

# 4.827006692223126513866978768923036513224
#
4.827006692223126513866978768923036513238196166781075316304131624186
3216170096289
```

$$4.827006692223126513866978768923036513238 \tag{46}$$

```
> lwPsi:
sqrt(%):
radsimp(%) assuming r>0, K2>0:
simplify(%) assuming r>0:
1/%:
subs(r=R, %):
Int(%, R=r0..r):
#%;
inlter:=te0+ clock*branch*%;
```

$$inlter := \left(\int_{r0}^{r} \frac{K2}{\sqrt{R}\,\sqrt{-\left((R-1)\,K2 - R^3\right)K2}}\, dR \right) branch\ clock + te0 \tag{47}$$

```
> # Test

inlter:
subs([t0=13, r0=5, te0=Pi/4], %):
subs([branch=Ingoing, clock=Clockwise], %):
subs([K2=8/3], %):
subs([r=12], %):
evalf(%):
%;

# 0.98065682072082219966193164880092852005 02
#
0.98065682072082219966193164880092852005266434748160935140491016204
371926682759865
```

$$0.98065682072082219966193164880092852005 26 \tag{48}$$

```
>
# Three real solutions
# 27/4 < K2

> lp-(r+rh+rm)*(r-rh)*(r-rm):
expand(%):
```

```
collect(%, r):
[subs(r=0, %), subs(r=0, diff(%, r))]:
solve(%, [K2, rh]):
op(%):
allvalues(%):
[%][1]:
lSol:=%;
```

$$lSol := \left[K2 = \frac{rm^3}{rm-1}, rh = \frac{\left(-rm+1+\sqrt{rm^2+2\,rm-3}\right) rm}{2\,(rm-1)}\right] \tag{49}$$

```
> subs(lSol, rh):
radsimp(%):
rhv:=%;
```

$$rhv := -\frac{\left(rm-1-\sqrt{rm^2+2\,rm-3}\right) rm}{2\,(rm-1)} \tag{50}$$

```
> subs(lSol, K2):
radsimp(%):
K2v:=%;
```

$$K2v := \frac{rm^3}{rm-1} \tag{51}$$

```
> # verify r0 is geater than 1 nd less that 3/2

rhv:
subs([rm=2], %):
evalf(%);
```

$$1.236067977499789696409173668731276235441 \tag{52}$$

```
> lwPhi:
subs([-lp=-(r+rh+rm)*(r-rh)*(r-rm)], %):
subs([lp=(r+rh+rm)*(r-rh)*(r-rm)], %):
lPhi:=%;
```

$$lPhi := \frac{(r-1)^2\,(r+rh+rm)\,(r-rh)\,(r-rm)}{r^5} \tag{53}$$

```
>
# It is convenient to integrate from rm 'cos it removes divergences


#                      r                    r
#    ltr = t0 + b*int        = tm + b*int                    (t0,
r0, branch; r, rm)
#                      r0                   rm


# for r=r0 ltr=t0
# it is regular in r=rm


> lPhi:
```

```
simplify(%):
sqrt(%):
radsimp(%)  assuming  r>rm, rm>rh, rh<3/2, rh>1:
simplify(%) assuming  r>rm, rm>rh, rh<3/2, rh>1:
1/%:
#%;
subs(r=R, %):
int(%, R=rm..r) assuming r>rm, rm>3/2, rh<3/2, rh>1:
subs(rh=rhv, %):
radsimp(%)  assuming r>rm, rm>3/2:
simplify(%) assuming r>rm, rm>3/2:
ltr:= t0 + branch*(%- subs([r=r0], %)):
```

```
> #Test

ltr:
subs([t0=13, r0=5], %):
subs(branch = Ingoing, %):
subs([rm=3], %):
subs(r=5, %):
evalf(%):
%;

ltr:
subs([t0=13, r0=5], %):
subs(branch = Ingoing, %):
subs([rm=3], %):
subs(r=3, %):
evalf(%):
%;
```

$$13.19.156077203369455241898294305362464901 00 \tag{54}$$

```
> lwPsi:
subs([-lp=-(r+rh+rm)*(r-rh)*(r-rm)], %):
subs([lp=(r+rh+rm)*(r-rh)*(r-rm)], %):
subs(K2=K2v, %):
lPsi:=%;
```

$$lPsi := \frac{(r+rh+rm)\,(r-rh)\,(r-rm)\,r\,(rm-1)}{rm^3} \tag{55}$$

```
> # It is convenient to integrate from rm 'cos it removes divergences


#                        r                    r
#    lter = te0 + b*int        = tem + b*int
(te0, r0, branch, clock; r, rm)
#                        r0                   rm


# for r=r0 lter=te0
# it is regular in r=rm
```

```
> lPsi:
  simplify(%):
  sqrt(%):
  radsimp(%)  assuming  r>rm, rm>rh, rh<3/2, rh>1:
  simplify(%) assuming  r>rm, rm>rh, rh<3/2, rh>1:
  1/%:
  #%;
  subs(r=R, %):
  int(%, R=rm..r) assuming r>rm, rm>3/2, rh<3/2, rh>1:
  subs(rh=rhv, %):
  radsimp(%)  assuming r>rm, rm>3/2:
  simplify(%) assuming r>rm, rm>3/2:
  lter:=te0+ clock*branch*(% -subs(r=r0, %)):
```

```
> # Test

  lter:
  subs([te0=Pi/4, r0=5], %):
  subs([branch = Ingoing, clock=Clockwise], %):
  subs([rm=3], %):
  subs(r=5, %):
  evalf(%):
  %;

  lter:
  subs([te0=Pi/4, r0=5], %):
  subs([branch = Ingoing, clock=Clockwise], %):
  subs([rm=3], %):
  subs(r=3, %):
  evalf(%):
  %;
```

$$0.785398163397448309615660845819875721049 2 - 0.486413792461826073358330170920197587003 8 \tag{56}$$

Light rays

```
> # Procedures for rays
>
  #  Infalling rays are made of one branch only.

> InFallingRayThrough := proc(r0v, te0v, t0v, branchv, clockv)
    [r*cos(inlter), r*sin(inlter), inltr]:
    subs([r0=r0v, te0=te0v, t0=t0v], %):
    subs([branch=branchv, clock=clockv], %):
    evalf(%):
  end:


> ScatteringBranchThrough := proc(r0v, te0v, t0v, branchv, clockv)
```

```
    [r*cos(lter), r*sin(lter), ltr]:
    subs([r0=r0v, te0=te0v, t0=t0v], %):
    subs([branch=branchv, clock=clockv], %):
    evalf(%):
  end:
> ScatteringOtherBranchThrough := proc(r0v, te0v, t0v, branchv,
  clockv)
   local Tm, Tem, Rm:
   [r, lter, ltr]:
   subs([r0=r0v, te0=te0v, t0=t0v], %):
   subs([branch=branchv, clock=clockv], %):
   subs(r=rm, %):
   Rm, Tem, Tm := op(%):
   ScatteringBranchThrough(Rm, Tem, Tm, -branchv, clockv)
  end:
> ScatteringRayThroughEm := proc(rmv, tem, tm, clockv := Clockwise)
  :
  local crm0, crm1, eq1, eq2, dt:

   ScatteringBranchThrough(rmv, tem, tm, Ingoing, clockv):
   subs(rm=rmv, %):
   eq1:=%:
   subs(r= maxS(2), %):
   dt:= evalf(%[3]):

   ScatteringOtherBranchThrough(rmv, tem, tm, Ingoing, clockv):
   subs(rm=rmv, %):
   eq2:=%:
   subs(r= maxS(2), %)[3]- dt:
   dt:= evalf(%):
   printf("dt= %g\n", dt);

   [op(eq1), r=rmv..50]:
   crm0:= spacecurve(%, axes=box, color = black, linestyle = solid)
  :
   [op(eq2), r=rmv..50]:
   crm1:= spacecurve(%, axes=box, color = black, linestyle = solid)
  :
   [crm0, crm1], dt:
  end:
> ScatteringRayThrough := proc(rmv, clockv := Clockwise):
   ScatteringRayThroughEm(rmv, 0, 0, clockv):
  end:
> rayMin:= proc()
   local sourceR, targetR, t:
   global SearchSignal:


   sourceR := SearchSignal[Risr]:
   targetR := SearchSignal[TargetP][1]:
   t := SearchSignal[RayType]:
```

```
  if t = Infalling or t = SameScattering then
    return min(sourceR, targetR);
  elif t = OtherScattering then
    return SearchSignal[Risrm];
  else
    printf("Ray type unknown %d. \n", t);
  end if:
end:
```

Procedure for plotting

```
> # Try to get a segment from Sat 0 (r=13 on branch 1) to Sat 2
```

```
> AddPlotPointCartesian:= proc(P, PSat, G)
  local col:
  global Plots:
  if PSat = -1 then
   col := black:
  elif PSat = 0 then
   col := red:
  elif PSat = 1 then
   col := blue:
  elif PSat = 2 then
   col := green:
  end if:
  P:
  #pointplot3d(%, color=col, symbol = solidcircle, symbolsize =
 6+3*G):
  pointplot3d(%, color=col, symbol = solidcircle, symbolsize = 6):
  Plots := [op(Plots), %]:
 end:
```

```
> AddPlotPointPolar:= proc(P, PSat, G)  # Polar
  local R, Te, T, col:
  global Plots:
  R, Te, T := op(P):
  [R*cos(Te), R*sin(Te), T]:
  AddPlotPointCartesian(%, PSat, G):
 end:
```

```
> AddPlotRayScatteringSame := proc(Through, r1v, S1, G1, rmv,
 branchv, clockv)
  local eq:
  global Plots:
  ScatteringBranchThrough(Through[1], Through[2], Through[3],
 branchv, clockv):
  subs(rm=rmv, %):
  eq:= %:
  subs(r=r1v, eq):
  AddPlotPointCartesian(%, S1, G1+1):
  if r1v > Through[1] then
```

```
    eq:=[op(eq), r=Through[1]..r1v]:
   else
    eq:=[op(eq), r=r1v..Through[1]]:
   end if:
   spacecurve(eq, axes=box, color = black, linestyle = solid,
  thickness = 1):
   Plots := [op(Plots), %];
  end:
> AddPlotRayScatteringOther := proc(Through, r1v, S1, G1, rmv,
  branchv, clockv)
   local eq:
   global Plots:
    ScatteringBranchThrough(Through[1], Through[2], Through[3],
  branchv, clockv):
   subs(rm=rmv, %):
   eq:= %:
   [op(eq), r=Through[1]..rmv]:
   spacecurve(%, axes=box, color = black, linestyle = solid,
  thickness = 1):
   Plots := [op(Plots), %];

   ScatteringOtherBranchThrough(Through[1], Through[2], Through[3],
  branchv, clockv):
   subs(rm=rmv, %):
   eq:= %:
   subs(r=r1v, eq):
   AddPlotPointCartesian(%, S1, G1+1):
   [op(eq), r=rmv..r1v]:
   spacecurve(%, axes=box, color = black, linestyle = solid,
  thickness = 1):
   Plots := [op(Plots), %];
  end:
> AddPlotRayInfalling := proc(Through, r1v, S1, G1, K2v, branchv,
  clockv)
   local eq:
   global Plots:
   #printf("Step 1\n");
   InFallingRayThrough(Through[1], Through[2], Through[3], branchv,
  clockv):
   subs(K2=K2v, %):
   eq:= %:
   subs(r=r1v, eq):
   AddPlotPointCartesian(%, S1, G1):
   if r1v > Through[1] then
    eq:=[op(eq), r=Through[1]..r1v]:
   else
    eq:=[op(eq), r=r1v..Through[1]]:
   end if:
   spacecurve(eq, axes=box, color = black, linestyle = solid,
  thickness = 1):
   Plots := [op(Plots), %];
   #printf("Step out\n");
  end:
```

```
> PlotLinear := proc()
    local sourceSat, sourceBranch, rGuessMax, rGuessMin, Previous,
  targetP, St:
    global OtherPlots:
     St := time():
     sourceSat := Get(SourceSat):
     sourceBranch := Get(SourceBranch):
     rGuessMax := Get(IndrGuessMax):
     rGuessMin := Get(IndrGuessMin):
     Previous:= Get(PreviousSourceBranch):
     targetP := Get(TargetP):

     [xS(sourceSat, sourceBranch), yS(sourceSat, sourceBranch), tS
  (sourceSat, sourceBranch)]:
     subs([r=rGuessMax],%):
     pointplot3d(%, color=black, symbol = solidcircle, symbolsize =
  9):
     OtherPlots := [op(OtherPlots), %]:
     if Previous then
       [xS(sourceSat, sourceBranch-1), yS(sourceSat,
  sourceBranch-1), tS(sourceSat, sourceBranch-1)]:
       subs([r=rGuessMin],%):
       pointplot3d(%, color=black, symbol = solidcircle, symbolsize
  = 9):
       OtherPlots := [op(OtherPlots), %]:
     else
       [xS(sourceSat, sourceBranch), yS(sourceSat, sourceBranch),
  tS(sourceSat, sourceBranch)]:
       subs([r=rGuessMin],%):
       pointplot3d(%, color=black, symbol = solidcircle, symbolsize
  = 9):
       OtherPlots := [op(OtherPlots), %]:
     end if:
     [s*targetP[1]*cos(targetP[2]) + (1-s)*xS(sourceSat,
  sourceBranch),
      s*targetP[1]*sin(targetP[2]) + (1-s)*yS(sourceSat,
  sourceBranch),
      targetP[3],
      s=0..1
     ]:
     subs(r=rGuessMax, %):
     spacecurve(%, axes=box, color = black, linestyle = solid,
  thickness = 1):
     OtherPlots := [op(OtherPlots), %]:
     printf("Time Approximations %g.\n\n", time()-St);
  end:

> AddPlotRay:= proc()
    local sourceSat, targetP, risr,risrm, risK2, Gen, rayBranchv,
  rayClockv, rayTypev, St:

    St:=time():
    if Get(Risr) = none then
      printf("No solution found\n");
    else
      sourceSat:= Get(SourceSat):
```

```
      targetP:= Get(TargetP):
      risr:= Get(Risr):
      risrm:= Get(Risrm):
      risK2:= Get(RisK2):
      Gen:= GetGeneration():
      rayBranchv:= Get(RayBranch):
      rayClockv:= Get(RayClock):
      rayTypev:= Get(RayType):

      if ProducePlots then
             if rayTypev = Infalling then
                AddPlotRayInfalling(targetP, risr, sourceSat,
GetGeneration(), risK2, rayBranchv, rayClockv):
             elif rayTypev = SameScattering then
                AddPlotRayScatteringSame(targetP, risr, sourceSat,
GetGeneration(), risrm, rayBranchv, rayClockv):
             elif rayTypev = OtherScattering then
                AddPlotRayScatteringOther(targetP, risr, sourceSat,
GetGeneration(), risrm, rayBranchv, rayClockv):
             else
                  printf("Ray type unknown\n"):
             end if:
       end if:
    end if:
    printf("Time Plot %g s.\n", time()-St);
  end:
```

```
> # Test

  InFallingRayThrough(5, Pi/4, 13, Ingoing, Clockwise):
  subs([K2=8/3], %):
  subs([r=3/2], %):
  evalf(%):
  Real(%):
  %;
```

$$[1.490598274221023510802413441117938356708, -0.16768060379574733783292490090457637990 20, 19.37268235627140400435005569655112455059] \quad (57)$$

```
>
  # this are very regular and relatively fast and robust

> ScatteringBranchThrough(5, Pi/4, 13, Ingoing, Clockwise):
  subs(rm=3, %):
  subs(r=5, %):
  evalf(%):
  %;

  ScatteringBranchThrough(5, Pi/4, 13, Ingoing, Clockwise) =
  ScatteringOtherBranchThrough(5, Pi/4, 13, Ingoing, Clockwise):
  subs(rm=3, %):
  subs(r=3, %):
```

```
evalf(%):
%;

ScatteringOtherBranchThrough(5, Pi/4, 13, Ingoing, Clockwise):
subs(rm=3, %):
subs(r=5, %):
evalf(%):
%;
```

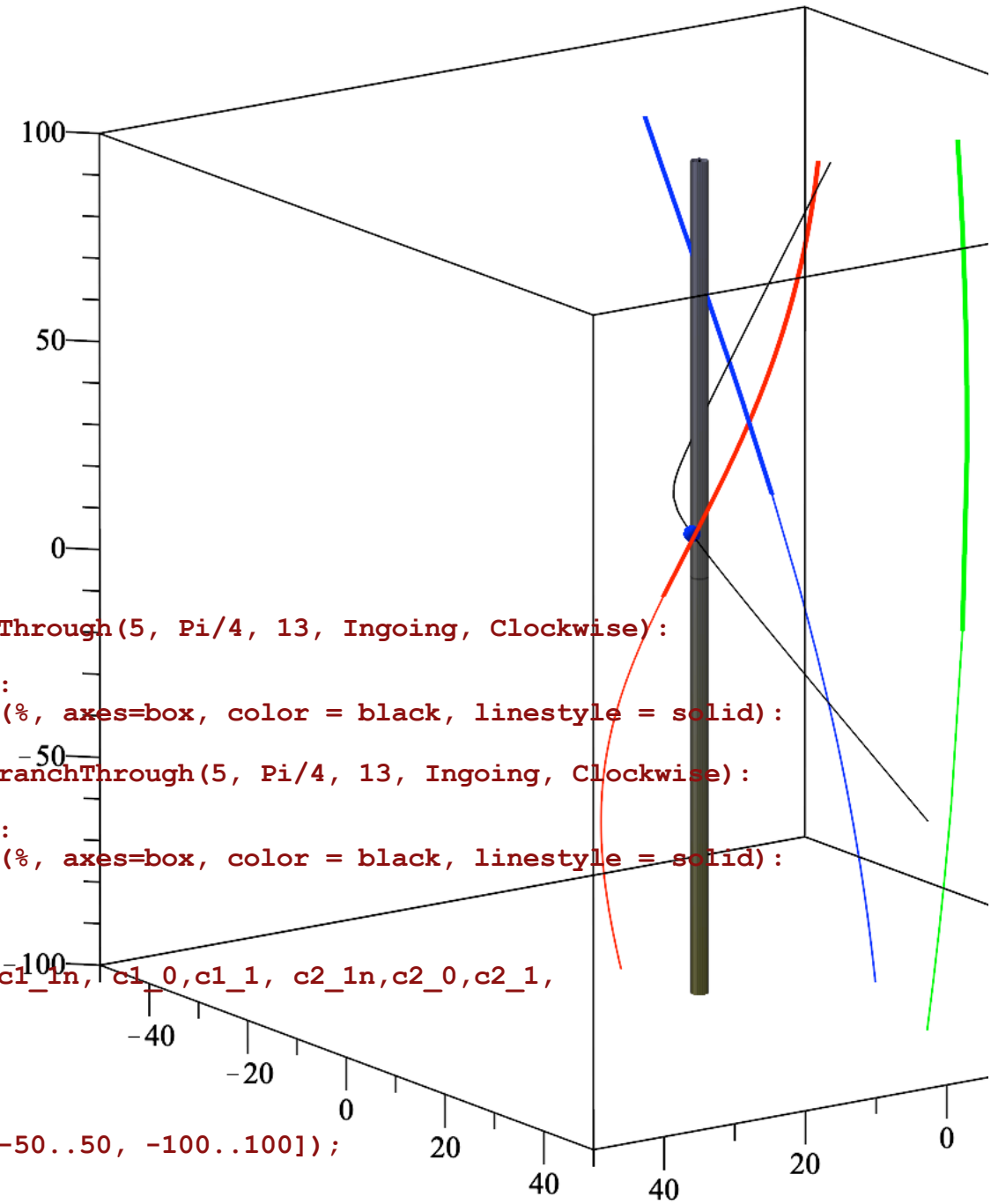
$$[3.535533905932737622004221810524245196422, 3.535533905932737622004221810524245196427, 12.999999999999999999999999999999999999998]$$

$$[2.652044829947528013947556434604041396914, -1.402375919626612534290469927664247295671, 19.156077203369455241898294305362464901 00]$$

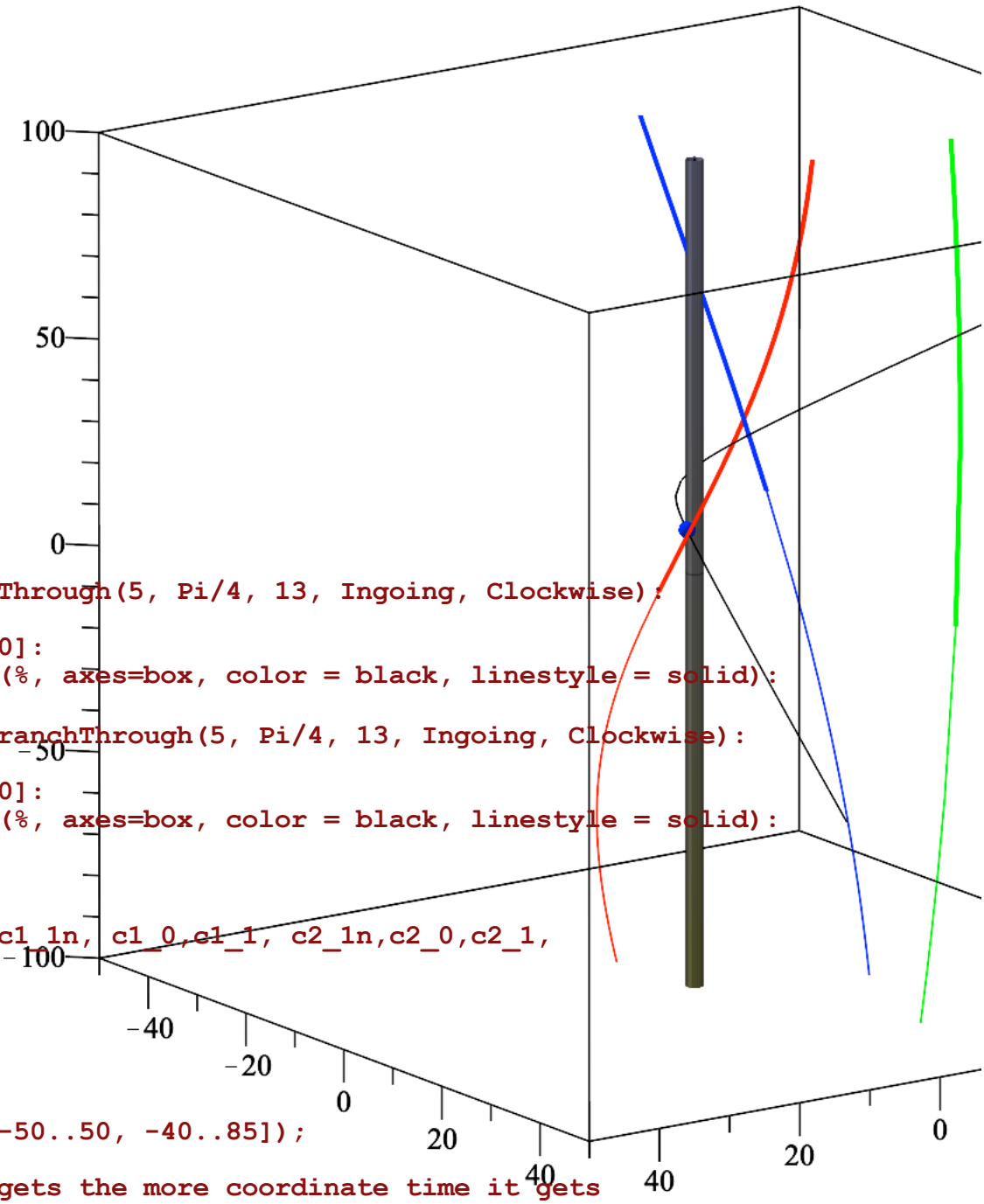
$$= [2.652044829947528013947556434604041396914, -1.402375919626612534290469927664247295671, 19.156077203369455241898294305362464901 00]$$

$$[-0.931669774155379911166785666489415733171 5, -4.912432333572164675271333260232430045110, 25.312154406738910483796588610724929802 02] \quad (58)$$

```
> #
  # Some plots to test
  #

> PM := pointplot3d([5*cos(Pi/4), 5*sin(Pi/4), 13], color=blue,
  symbol = solidcircle, symbolsize = 10):

> ScatteringBranchThrough(5, Pi/4, 13, Ingoing, Clockwise):
  subs(rm=3, %):
  [op(%), r=3..50]:
  cr1:= spacecurve(%, axes=box, color = black, linestyle = solid):

> ScatteringOtherBranchThrough(5, Pi/4, 13, Ingoing, Clockwise):
  subs(rm=3, %):
  [op(%), r=3..50]:
  cr2:= spacecurve(%, axes=box, color = black, linestyle = solid):

> display(BH, S,
  #user,
  c0_1n,c0_0,c0_1,c1_1n, c1_0,c1_1, c2_1n,c2_0,c2_1,


  PM, cr1, cr2,


  view=[ -50..50, -50..50, -100..100]);
```

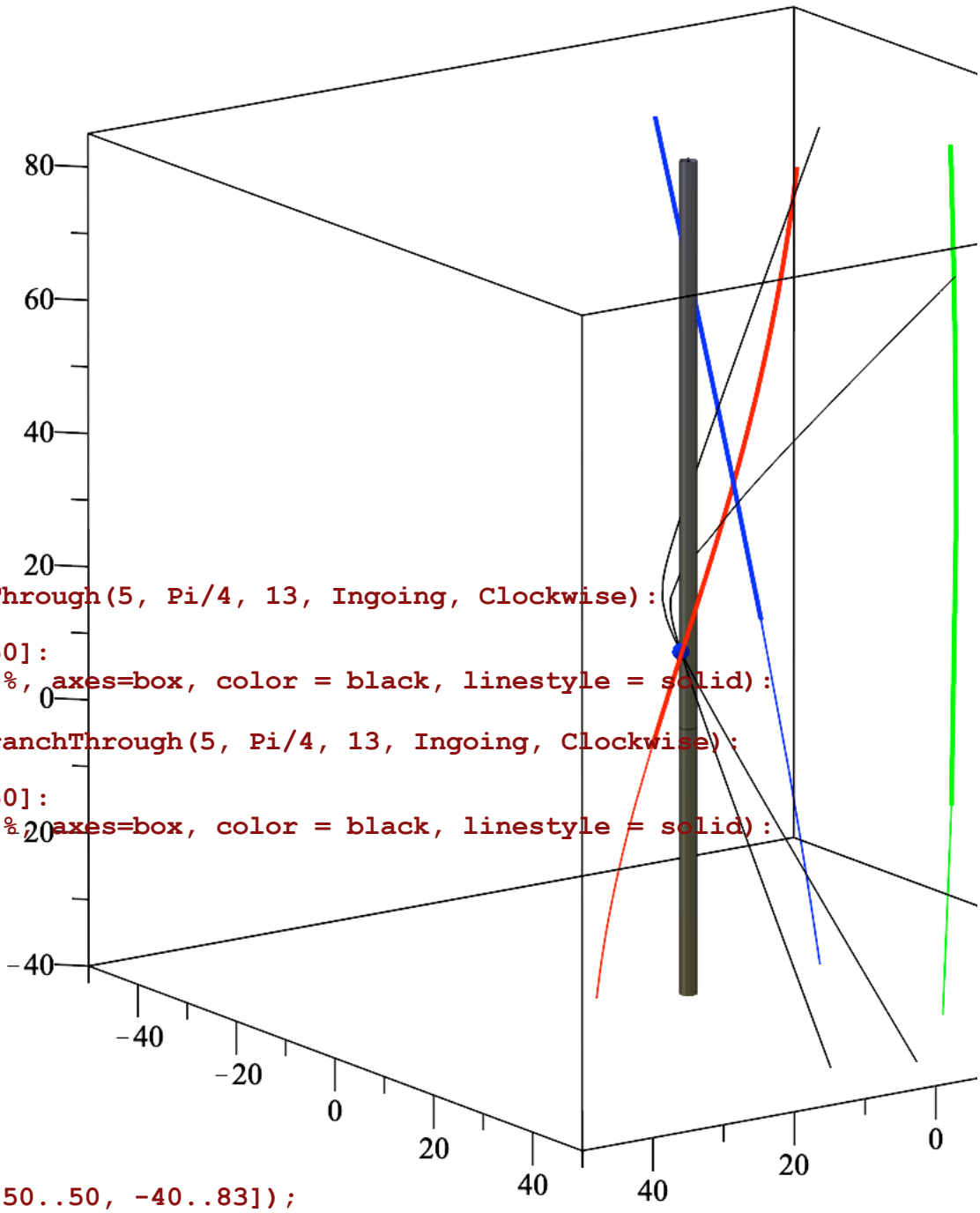


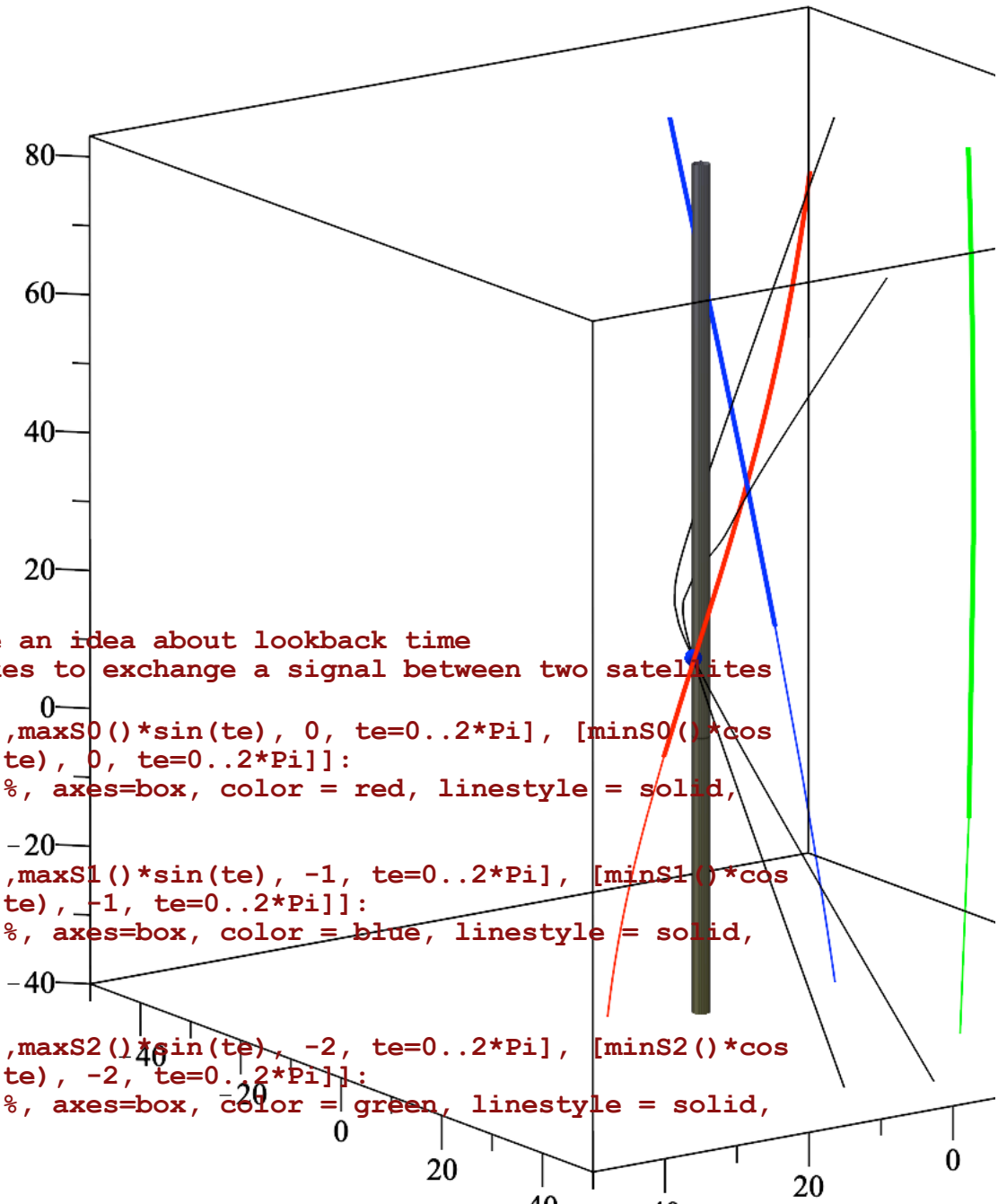

```
> ScatteringBranchThrough(5, Pi/4, 13, Ingoing, Clockwise):
  subs(rm=2, %):
  [op(%), r=2..50]:
  cr3:= spacecurve(%, axes=box, color = black, linestyle = solid):

> ScatteringOtherBranchThrough(5, Pi/4, 13, Ingoing, Clockwise):
  subs(rm=2, %):
  [op(%), r=2..50]:
  cr4:= spacecurve(%, axes=box, color = black, linestyle = solid):

> display(BH, S,
  #user,
  c0_1n,c0_0,c0_1,c1_1n, c1_0,c1_1, c2_1n,c2_0,c2_1,


  PM, #cr1, cr2,
  cr3, cr4,

  view=[ -50..50, -50..50, -100..100]);
```




```
> ScatteringBranchThrough(5, Pi/4, 13, Ingoing, Clockwise):
  subs(rm=1.8, %):
  [op(%), r=1.8..50]:
  cr5:= spacecurve(%, axes=box, color = black, linestyle = solid):

> ScatteringOtherBranchThrough(5, Pi/4, 13, Ingoing, Clockwise):
  subs(rm=1.8, %):
  [op(%), r=1.8..50]:
  cr6:= spacecurve(%, axes=box, color = black, linestyle = solid):

> display(BH, S,
  #user,
  c0_1n,c0_0,c0_1,c1_1n, c1_0,c1_1, c2_1n,c2_0,c2_1,


  PM,
  cr1, cr2,
  #cr3, cr4,
  cr5, cr6,
  view=[ -50..50, -50..50, -40..85]);

  # The closer it gets the more coordinate time it gets
  # (to be made more precise)
```

```
> ScatteringBranchThrough(5, Pi/4, 13, Ingoing, Clockwise):
  subs(rm=1.76, %):
  [op(%), r=1.76..50]:
  cr7:= spacecurve(%, axes=box, color = black, linestyle = solid):

> ScatteringOtherBranchThrough(5, Pi/4, 13, Ingoing, Clockwise):
  subs(rm=1.76, %):
  [op(%), r=1.76..50]:
  cr8:= spacecurve(%, axes=box, color = black, linestyle = solid):

> display(BH, S,
  #user,
  c0_1n,c0_0,c0_1,
  c1_1n,c1_0,c1_1,
  c2_1n,c2_0,c2_1,
  PM,
  cr1, cr2,
  #cr3, cr4,
  #cr5, cr6,
  cr7, cr8,
  #op(CC),
  view=[ -50..50, -50..50, -40..83]);
```






```
> # We need to have an idea about lookback time
  # how much it takes to exchange a signal between two satellites

> [[maxS0()*cos(te),maxS0()*sin(te), 0, te=0..2*Pi], [minS0()*cos
  (te),minS0()*sin(te), 0, te=0..2*Pi]]:
  cs0:= spacecurve(%, axes=box, color = red, linestyle = solid,
  thickness=1):

  [[maxS1()*cos(te),maxS1()*sin(te), -1, te=0..2*Pi], [minS1()*cos
  (te),minS1()*sin(te), -1, te=0..2*Pi]]:
  cs1:= spacecurve(%, axes=box, color = blue, linestyle = solid,
  thickness=1):


  [[maxS2()*cos(te),maxS2()*sin(te), -2, te=0..2*Pi], [minS2()*cos
  (te),minS2()*sin(te), -2, te=0..2*Pi]]:
  cs2:= spacecurve(%, axes=box, color = green, linestyle = solid,
  thickness=1):


  [[x,0, 1, x=-50..50], [-minS2(), 0, t, t=-100..100], [-minS1(), 0,
  t, t=-100..100]]:
  cs:= spacecurve(%, axes=box, color = black, linestyle = solid,
  thickness=1):
```

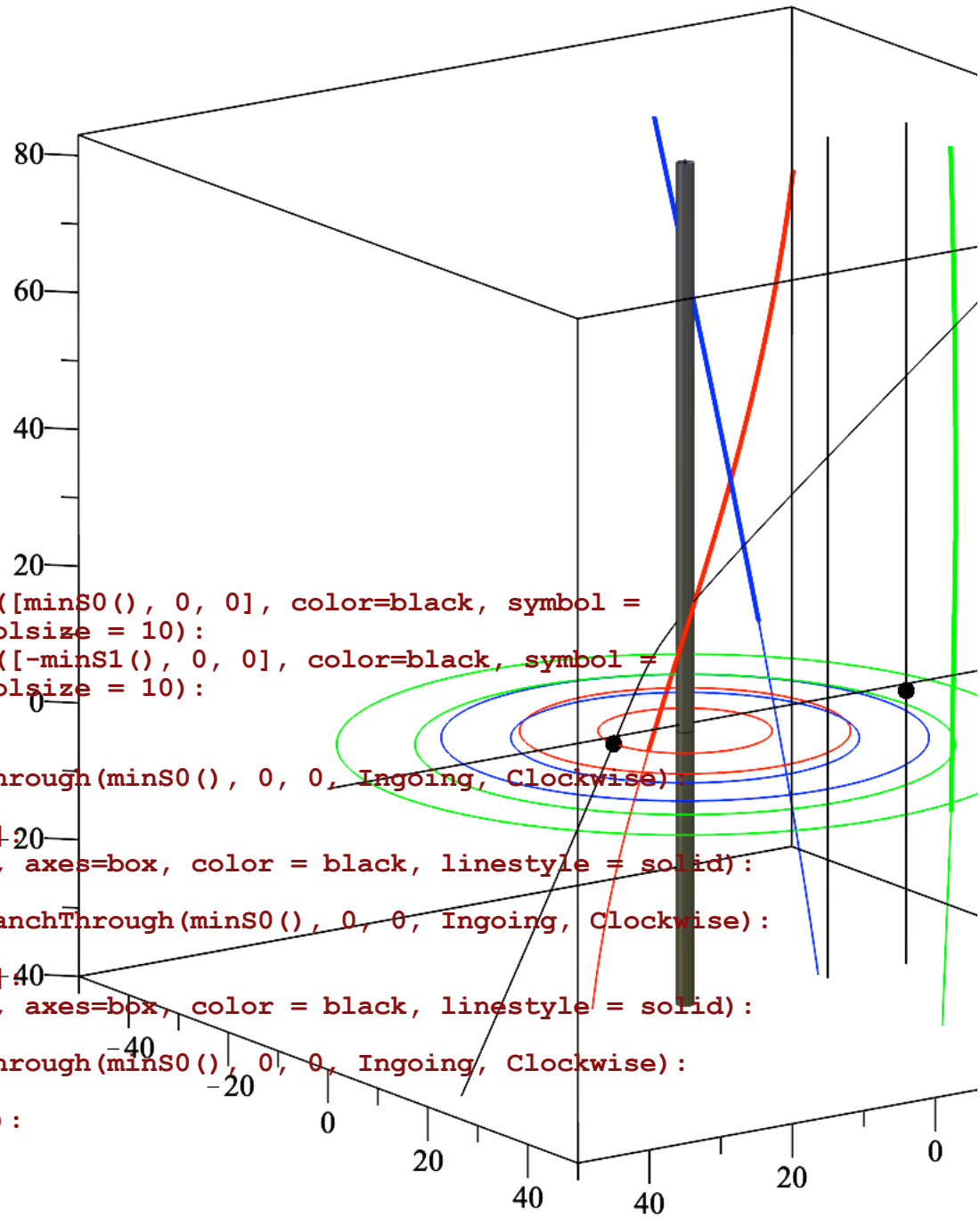
```
> P1 := pointplot3d([minS0(), 0, 0], color=black, symbol =
  solidcircle, symbolsize = 10):
  P2 := pointplot3d([-minS2(), 0, 0], color=black, symbol =
  solidcircle, symbolsize = 10):
```

```
> rmv:= 4.1;
  ScatteringBranchThrough(minS0(), 0, 0, Ingoing, Clockwise):
  subs(rm=rmv, %):
  [op(%), r=rmv..50]:
  r1:= spacecurve(%, axes=box, color = black, linestyle = solid):

  ScatteringOtherBranchThrough(minS0(), 0, 0, Ingoing, Clockwise):
  subs(rm=rmv, %):
  [op(%), r=rmv..50]:
  r2:= spacecurve(%, axes=box, color = black, linestyle = solid):

  ScatteringBranchThrough(minS0(), 0, 0, Ingoing, Clockwise):
  subs(rm=rmv, %):
  subs(r=maxS0(), %):
  %[3]:
  evalf(%):
  tt0:=%:

  ScatteringOtherBranchThrough(minS0(), 0, 0, Ingoing, Clockwise):
  subs(rm=rmv, %):
  subs(r=maxS2(), %):
  %[3]-tt0:
  evalf(%):
  MaxT02:= %; # That is the worst case scenario, with some
  overestimation.
  #It should be better than this.
```

$$rmv := 4.1$$

$$MaxT02 := 65.814471515115866361381457431832695807 71 \quad \textbf{(59)}$$

```
> display(BH, S,
  #user,
  c0_1n,c0_0,c0_1,
  c1_1n,c1_0,c1_1,
  c2_1n,c2_0,c2_1,
  P1, P2,
  cs0,cs1,cs2, cs,
  r1, r2,
  #op(CC),
  view=[ -50..50, -50..50, -40..83]);
```



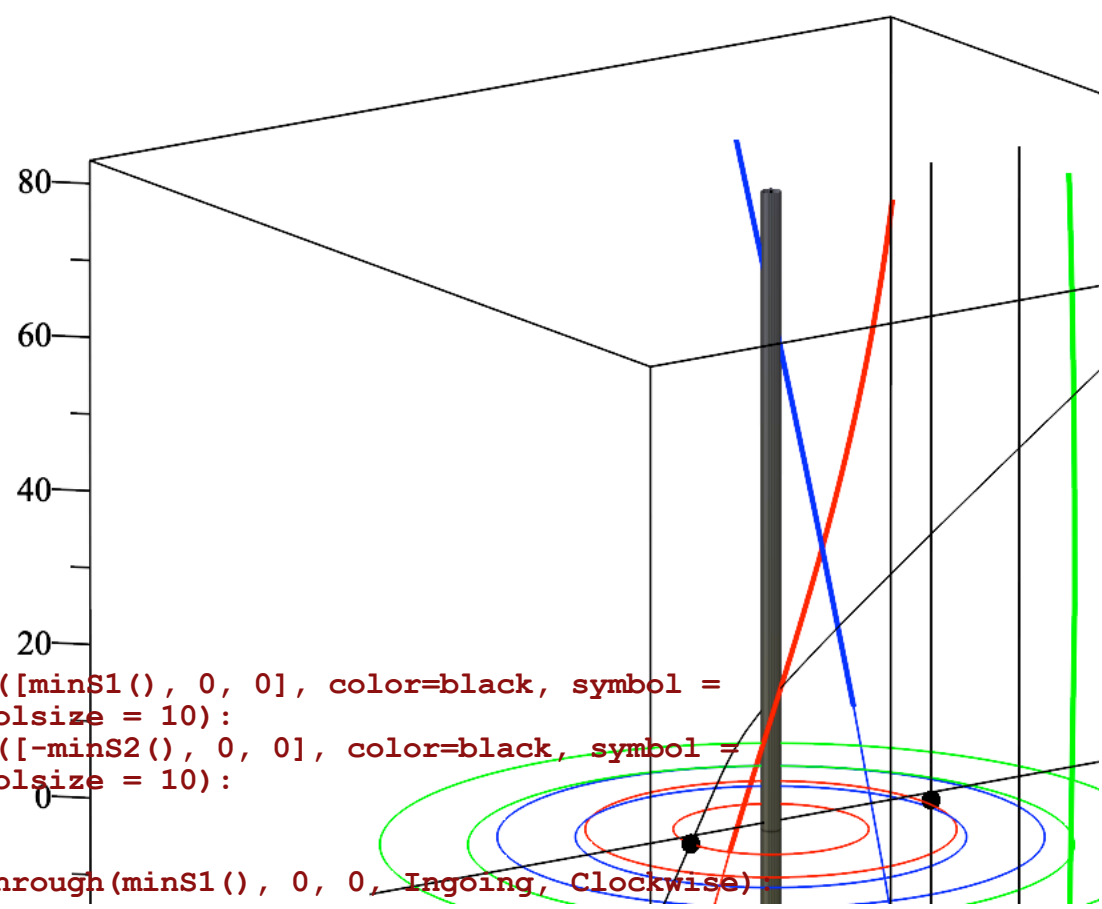

```
> P1 := pointplot3d([minS0(), 0, 0], color=black, symbol =
  solidcircle, symbolsize = 10):
  P2 := pointplot3d([-minS1(), 0, 0], color=black, symbol =
  solidcircle, symbolsize = 10):

> rmv:= 3.85;
  ScatteringBranchThrough(minS0(), 0, 0, Ingoing, Clockwise):
  subs(rm=rmv, %):
  [op(%), r=rmv..50]:
  r1:= spacecurve(%, axes=box, color = black, linestyle = solid):

  ScatteringOtherBranchThrough(minS0(), 0, 0, Ingoing, Clockwise):
  subs(rm=rmv, %):
  [op(%), r=rmv..50]:
  r2:= spacecurve(%, axes=box, color = black, linestyle = solid):

  ScatteringBranchThrough(minS0(), 0, 0, Ingoing, Clockwise):
  subs(rm=rmv, %):
  subs(r=maxS0(), %):
  %[3]:
  evalf(%):
  tt0:=%:

  ScatteringOtherBranchThrough(minS0(), 0, 0, Ingoing, Clockwise):
  subs(rm=rmv, %):
  subs(r=maxS1(), %):
  %[3]-tt0:
```

```
evalf(%):
MaxT01:= %; # That is the worst case scenario, with some
overestimation.
#It should be better than this.
```

$$rmv := 3.85$$

$$MaxT01 := 53.69078690399313438430029311307497649478 \quad \textbf{(60)}$$

```
> display(BH, S,
#user,
c0_1n,c0_0,c0_1,
c1_1n,c1_0,c1_1,
c2_1n,c2_0,c2_1,
P1, P2,
cs0,cs1,cs2, cs,
r1, r2,
#op(CC),
view=[ -50..50, -50..50, -40..83]);
```




```
> P1 := pointplot3d([minS1(), 0, 0], color=black, symbol =
solidcircle, symbolsize = 10):
P2 := pointplot3d([-minS2(), 0, 0], color=black, symbol =
solidcircle, symbolsize = 10):
```

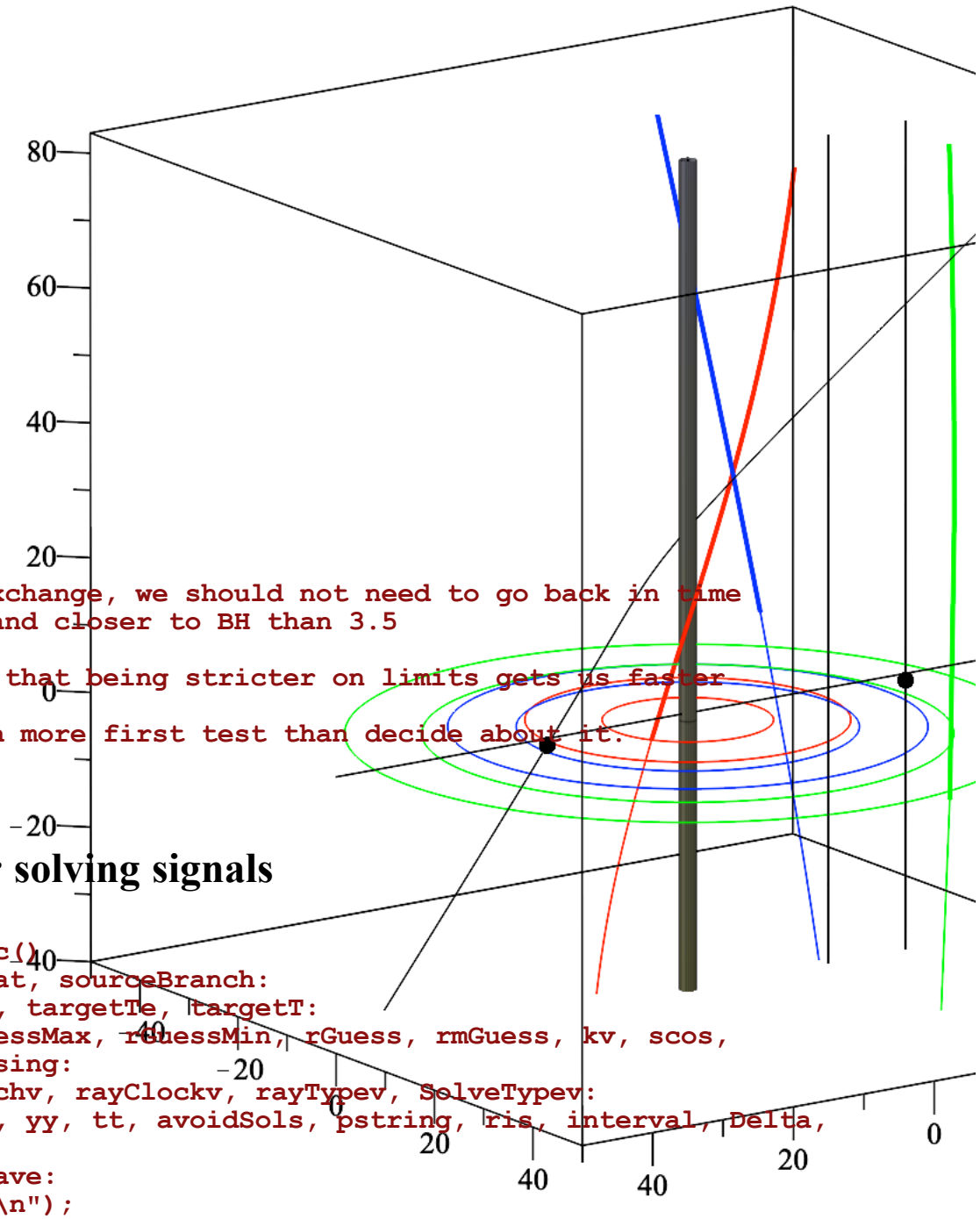
```
> rmv:= 5.1;
ScatteringBranchThrough(minS1(), 0, 0, Ingoing, Clockwise);
```

```
subs(rm=rmv, %):
[op(%), r=rmv..50]:
r1:= spacecurve(%, axes=box, color = black, linestyle = solid):

ScatteringOtherBranchThrough(minS1(), 0, 0, Ingoing, Clockwise):
subs(rm=rmv, %):
[op(%), r=rmv..50]:
r2:= spacecurve(%, axes=box, color = black, linestyle = solid):

ScatteringBranchThrough(minS1(), 0, 0, Ingoing, Clockwise):
subs(rm=rmv, %):
subs(r=maxS2(), %):
%[3]:
evalf(%):
tt0:=%:

ScatteringOtherBranchThrough(minS1(), 0, 0, Ingoing, Clockwise):
subs(rm=rmv, %):
subs(r=maxS2(), %):
%[3]-tt0:
evalf(%):
MaxT12:= %; # That is the worst case scenario, with some
overestimation.
#It should be better than this.
```

$$rmv := 5.1$$

$$MaxT12 := 86.82654937660873954821196677046734937452 \quad \textbf{(61)}$$

```
> display(BH, S,
#user,
c0_1n,c0_0,c0_1,
c1_1n,c1_0,c1_1,
c2_1n,c2_0,c2_1,
P1, P2,
cs0,cs1,cs2, cs,
r1, r2,
#op(CC),
view=[ -50..50, -50..50, -40..83]);
```




```
> # Then for any exchange, we should not need to go back in time
  further than 90 and closer to BH than 3.5
  #
  # I'm not saying that being stricter on limits gets us faster
  though.
  # I would be much more first test than decide about it.
```

Procedures for solving signals

```
> SolveOne:= proc()
   local sourceSat, sourceBranch:
   local targetR, targetTe, targetT:
   local sv, rGuessMax, rGuessMin, rGuess, rmGuess, kv, scos,
  Previous, Crossing:
   local rayBranchv, rayClockv, rayTypev, SolveTypev:
   local eqs, xx, yy, tt, avoidSols, pstring, ris, interval, Delta,
  St:
   local SignalSave:
  #printf("Step2\n");

   St:= time():
   sourceSat := Get(SourceSat):
   sourceBranch := Get(SourceBranch):
```

```
  sv := Get(Indsv):
  rGuessMax := Get(IndrGuessMax):
  rGuessMin := Get(IndrGuessMin):
  Previous := Get(PreviousSourceBranch):
  Crossing := Get(SourceCrossing):

  rmGuess := Get(IndrmGuess):
  kv := Get(Indkv):

  scos := Get(Indscos):

  rayTypev := Get(RayType):
  SolveTypev := Get(SolveType):

  targetR, targetTe, targetT := op(Get(TargetP)):

  rayBranchv := Get(RayBranch):
  rayClockv := Get(RayClock):

  if DebugFlowControlOn then
    pstring:= "I search for ":
    if rayTypev = Infalling then
      pstring := cat(pstring,"an infalling ray "):
    elif rayTypev = SameScattering then
      pstring := cat(pstring,"an scattering ray on same branch "):
    elif rayTypev = OtherScattering then
      pstring := cat(pstring,"an scattering ray on opposite
branches "):
    else
      pstring := cat(pstring,"an unknown ray "):
    end if:
    if sv<0 then
      pstring := cat(pstring,"with sv<0 (%g) "):
    elif sv>0 and sv<1 then
      pstring := cat(pstring,"with 0<sv<1 (%g) "):
    else # sv>1
      pstring := cat(pstring,"with sv>1 (%g) "):
    end if:

    if targetR < minS(sourceSat) then
      pstring := cat(pstring,"|   P  <--- S \n"):
    elif targetR > minS(sourceSat) then
      pstring := cat(pstring,"|   S  ---> P \n"):
    else
      pstring := cat(pstring,"mixing source and target.\n"):
    end if:
    printf(pstring, sv);
    printf("rGuessMin=%g   rGuessMax=%g   rmGuess=%g  k=%g   scos=
%g\n",  rGuessMin, rGuessMax, rmGuess, kv, scos);
    pstring := "branch ":
    if rayBranchv = Outgoing then
      pstring := cat(pstring," outgoing at target, "):
    else
      pstring := cat(pstring," ingoing at target, "):
    end if:
    if rayClockv = Clockwise then
      pstring := cat(pstring,"Clockwise\n"):
```

```
    else
      pstring := cat(pstring,"Counterclockwise\n"):
    end if:
    printf(pstring);
  end if:

  if rayTypev = Infalling then
    InFallingRayThrough(targetR, targetTe, targetT, rayBranchv,
rayClockv):
    xx, yy, tt := op(%):
    interval:={r= Get(Intervalr1), K2 = Get(IntervalK2)}:
  elif rayTypev = SameScattering then
    ScatteringBranchThrough(targetR, targetTe, targetT,
rayBranchv, rayClockv):
    xx, yy, tt := op(%):
    interval:={r= Get(Intervalr1), rm = Get(Intervalrm)}:
  elif rayTypev = OtherScattering then
    ScatteringOtherBranchThrough(targetR, targetTe, targetT,
rayBranchv, rayClockv):
    xx, yy, tt := op(%):
    interval:={r= Get(Intervalr1), rm = Get(Intervalrm)}:
  else
    interval:={}:
  end if:
  [xx-xS(sourceSat, sourceBranch), yy-yS(sourceSat, sourceBranch),
tt-tS(sourceSat, sourceBranch)]:
  evalf(%):
  eqs:= %:
  avoidSols := {}:
  ris := "searching":
  do
   if rayTypev = Infalling then
     printf("(infalling) fsolve({eqs1, eqs3}, {r=%g, K2=1}, %a,
avoid=%a);\n", rGuessMax, interval, avoidSols);
     fsolve({eqs[1], eqs[3]}, {r=rGuessMax, K2=1}, interval,
avoid=avoidSols, fulldigits):
     ris:= %:
   else
     # il lato alto di Intervalr1
     rGuess:= op(2, Get(Intervalr1)):
     printf("(Scattering) fsolve(eqs, {r=%g, rm=%g}, %a, avoid=%a)
;\n", rGuess, rmGuess, interval, avoidSols);
     fsolve({eqs[1], eqs[3]}, {r=rGuess, rm=rmGuess}, interval,
avoid=avoidSols, fulldigits):
     ris:= %:
   end if:
   if type(ris, 'set') then
      eqs[2]:
      subs(ris, %):
      Delta := abs(evalf(%)):
      if Delta > 10^(-Digits+7) then
        avoidSols:= {op(avoidSols), ris}:        # forse posso
restringere l'intervallo?
        if DebugSolutionsOn then
            printf("Rejected {r=%g, rm=%g} for Delta=%g\n", subs
(ris, r), subs(ris, rm), Delta);
            if DebugTimeOn then
```

```
              printf("in partial time = %g s \n", time()-St);
            end if:
        end if:
        ris:="rejected":
      else
        if DebugSolutionsOn then
          if rayTypev = Infalling then
                print(ris);
                printf("Accepted {r=%g, K2=%g} with Delta=%g\n",
subs(ris, r), subs(ris, K2), Delta);
          else
              printf("Accepted {r=%g, rm=%g} with Delta=%g\n",
subs(ris, r), subs(ris, rm), Delta);
          end if:
          subs(ris, eqs):
          printf("Equations at solution: %a", %):
        end if:
      end if:
   else
    ris:= {}:
   end if:
  until type(ris, set):
  if nops(ris)> 0 then
    [r, teSat(sourceSat, r, sourceBranch), tSat(sourceSat, r,
sourceBranch)]:
    subs(r=subs(ris, r), %):
    Set(RisP, %):
    Set(Risr, subs(ris, r)):
    Set(RisBranch, sourceBranch):

    if rayTypev = Infalling then
      Set(RisK2, subs(ris, K2)):
      ris:= [sourceSat, subs(ris, r), subs(ris, K2), sourceBranch,
rayTypev];
    else
      Set(Risrm, subs(ris, rm)):
      ris:= [sourceSat, subs(ris, r), subs(ris, rm), sourceBranch,
rayTypev];
    end if:
  else
    #  if no solution found
    #     Se crossing vando a cercare sull'altro Ramo
    #     Per -clock e infalling/scattering invece lascio fare al
chiamante
    #
    if Crossing then
      if DebugFlowControlOn and DebugTimeOn then
        printf("Turn to the other branch [nested SolveOne()]\n");
      end if:
      SignalSave := SaveSearchSignal():
      SetPreviousSourceBranch():
      ris := SolveOne():
      RestoreSearchSignal(SignalSave):
    else
      # ris:= {}:
    end if:
  end if:
```

```
  if DebugFlowControlOn and DebugTimeOn then
    printf("Solution in %gs\n\n", time()-St);
  end if:
  ris;
end:
```

```
> CompareSolution:= proc()
    local Newrm, Bestrm:
    global SearchSignal, BestKnownSolution:

    Newrm := SearchSignal[Risrm]:
    if not(Newrm = none) and BestKnownSolution = none then
      BestKnownSolution := SearchSignal:
    elif not(Newrm = none) then
       Bestrm := BestKnownSolution[Risrm]:
       if Bestrm < Newrm then
         BestKnownSolution := SearchSignal:
       end if:
    end if:
  end:
```

```
> ValidateSolution:= proc()  #  -> Bool
  local sourceR, targetR, t:
  global SearchSignal, BestKnownSolution:

  sourceR := SearchSignal[Risr]:
  targetR := SearchSignal[TargetP][1]:
  if sourceR = none then
    return false:
  end if:
  if rayMin() > 4 then
    BestKnownSolution := SearchSignal:
    return true:
  end if:
  CompareSolution():
  return false:
end:
```

```
> AcceptBestSolution := proc()
  global SearchSignal, BestKnownSolution:
  SearchSignal := BestKnownSolution:
end:
```

```
> #(Infalling0 | SameScattering1 | OtherScattering2) + (Ingoing0 +
  Outgoing1) + (Orario0 | Antitorario1)


  Sol2Code:=proc()
  local r:
  global SearchSignal:

  if Get(RayClock) = Clockwise  then
    r:= 0*1:
  else
    r:= 1*1:
```

```
  end if:
  if Get(RayBranch) = Outgoing then
    r:= r+1*2:
  else
    r:= r+0*2:
  end if:
  if Get(RayType) = Infalling  then
    r:= r+ 0*4:
  elif Get(RayType) = SameScattering  then
    r:= r+ 1*4:
  elif Get(RayType) = OtherScattering  then
    r:= r+ 2*4:
  else
    printf("RayType unknown %d\n", Get(RayType)):
  end if:
  return r+1;
end:
```

```
> DoneCode:=proc()
  local r:
  global TypeToDo:
  r:= Sol2Code():
  TypeToDo[r]:= false:
end:
```

```
> #(Infalling0 | SameScattering1 | OtherScattering2) + (Ingoing0 +
Outgoing1) + (Orario0 | Antitorario1)


Code2Sol := proc(n)
 global SearchSignal;

 if n = 1 then                  #  Infalling       Ingoing
Clockwise
   Set(RayType, Infalling);
   Set(RayClock, Clockwise);
   Set(RayBranch, Ingoing);
 elif n = 2 then                #  Infalling       Ingoing
Counterclockwise
   Set(RayType, Infalling);
   Set(RayClock, Counterclockwise);
   Set(RayBranch, Ingoing);
 elif n = 3 then                #  Infalling       Outgoing
Clockwise
   Set(RayType, Infalling);
   Set(RayClock, Clockwise);
   Set(RayBranch, Outgoing);
 elif n = 4 then                #  Infalling       Outgoing
 Counterclockwise
   Set(RayType, Infalling);
   Set(RayClock, Counterclockwise);
   Set(RayBranch, Outgoing);

 elif n = 5 then                #  SameScattering  Ingoing
 Clockwise
```

```
    Set(RayType, SameScattering);
    Set(RayClock, Clockwise);
    Set(RayBranch, Ingoing);
   elif n = 6 then                #  SameScattering  Ingoing
   Counterclockwise
    Set(RayType, SameScattering);
    Set(RayClock, Counterclockwise);
    Set(RayBranch, Ingoing);
   elif n = 7 then                #  SameScattering  Outgoing
   Clockwise
    Set(RayType, SameScattering);
    Set(RayClock, Clockwise);
    Set(RayBranch, Outgoing);
   elif n = 8 then                #  SameScattering  Outgoing
   Counterclockwise
    Set(RayType, SameScattering);
    Set(RayClock, Counterclockwise);
    Set(RayBranch, Outgoing);

   elif n = 9  then               #  OtherScattering Ingoing
   Clockwise
    Set(RayType, OtherScattering);
    Set(RayClock, Clockwise);
    Set(RayBranch, Ingoing);
   elif n = 10 then               #  OtherScattering Ingoing
   Counterclockwise
    Set(RayType, OtherScattering);
    Set(RayClock, Counterclockwise);
    Set(RayBranch, Ingoing);
   elif n =11 then                #  OtherScattering Outgoing
   Clockwise
    Set(RayType, OtherScattering);
    Set(RayClock, Clockwise);
    Set(RayBranch, Outgoing);
   elif n = 12 then               #  OtherScattering Outgoing
   Counterclockwise
    Set(RayType, OtherScattering);
    Set(RayClock, Counterclockwise);
    Set(RayBranch, Outgoing);
   else
    printf("Type Code unknown %d\n", n);
   end if:
  end:

> SolveHard:= proc(TargetSat, TargetP, SourceSat, Gen, hint:= none)
   global Plots, OtherPlots, TypeToDo, BestKnownSolution:
   local sol, OriginalSS,  loop, n, targetR, sourceMin, sourceMax,
  recCodes, St:
  #printf("Step1\n");
    St:= time():
    printf("%d --> %d  target = %a\n", SourceSat, TargetSat,
  TargetP);
    sol:=[]:

    Plots:=[];
    OtherPlots:=[];
```

```
    ClearSearchSignal():
    ClearTypeToDo();

    targetR:= TargetP[1]:
    sourceMin:= minS(SourceSat):
    sourceMax:= maxS(SourceSat):

    SetTargetPoint(TargetSat, TargetP):
    SetSourceSat(SourceSat):
    SetGeneration(Gen):
  #printf("Step1.1\n");
    LinearGuess():
  #printf("Step1.2\n");
    PlotLinear():

    OriginalSS := SaveSearchSignal():
    if type(hint, list) then
      # hint ha anche informazioni su guess e forse intervals
      UseHint(hint):
      sol := SolveOne():
      DoneCode():
      if ValidateSolution() then
        AddPlotRay():
        printf("Exiting SolveHard() after %g", time()-St);
        return sol:
      end if:
      if type(BestKnownSolution, list) then
        AcceptBestSolution():
        return sol:
      end if:
    end if:
    RestoreSearchSignal(OriginalSS):
    OriginalSS := SaveSearchSignal():
    loop:= true:
    if targetR > sourceMax then
       recCodes:= [12, 11, 7, 3, 4, 8]:
       n:= Sol2Code():
       recCodes:= CycleUntil(recCodes, n);
    elif targetR < sourceMax then
       recCodes:= [12, 11, 5, 1, 2, 6]:
       n:= Sol2Code():
       recCodes:= CycleUntil(recCodes, n);
    else
      printf("Tagent is Source ring should not happen\n"):
      printf("Exiting SolveHard() after %g", time()-St);
      return []:
    end if:
    BestKnownSolution := none:
    do
      n:= recCodes[1]:
      printf("Try code %d\n", n);
      Code2Sol(n);
      sol := SolveOne():
      DoneCode():
      if nops(sol) > 0 then
        if ValidateSolution() then
          AddPlotRay():
```

```
          printf("Exiting SolveHard() after %g", time()-St);
          return sol:
        end if:
      end if:
      recCodes := [op(2..nops(recCodes), recCodes)]:
    until nops(recCodes) = 0:

    printf("Warning: Going to unreccommended types\n");
    for n from 1 to 12 do
      if TypeToDo[n] then
        printf("Try unreccommended code %d\n", n);
        Code2Sol(n);
        sol := SolveOne():
        DoneCode():
        if ValidateSolution() then
          AddPlotRay():
          printf("Exiting SolveHard() after %g", time()-St);
          return sol:
        end if:
      end if:
    end do:
    printf("Could not find a validate solution\n");
    AcceptBestSolution():

    if SearchSignal = none then
      printf("No solution\n");
      sol:=[]:
    else   #  [sourceSat, subs(ris, r), subs(ris, rm),
  sourceBranch, rayTypev];
      if Get(RayType) = Infalling then
        sol:=[Get(SourceSat), Get(Risr), Get(RisK2), Get
  (SourceBranch), Get(RayType)]:
      else
        sol:=[Get(SourceSat), Get(Risr), Get(Risrm), Get
  (SourceBranch), Get(RayType)]:
      end if:
    end if:
    printf("Exiting SolveHard() after %g\n", time()-St);
    #  Possiamo ricostruire la BestSolution  if any se no
    sol:
  end:

> AcceptSolution:= proc()
    local sourceSat, gen, k;
    global NextSignalAvailable, ListPlots, Plots, OtherPlots,
  ListP, ListHints, ListTau:

    sourceSat := SearchSignal[SourceSat];
    gen := SearchSignal[Gen];

    ListPlots[gen+1] := [op(ListPlots[gen+1]), op(Plots)]:
    Plots:= []:
    OtherPlots:= []:

    k := 3*NextSignalAvailable[sourceSat+1] +sourceSat + 1:
    ListHints[k]:= CreateHint():
```

```
    ListP[k]:= Get(RisP):
    ListTau[k]:= tauSat(sourceSat, Get(Risr), Get(RisBranch));
    NextSignalAvailable[sourceSat+1]:= NextSignalAvailable
  [sourceSat+1] +1;
    k:
  end:
```

```
> SetBasePoint:= proc(Sat, rv, branchv)
   local n:
   global Plots, OtherPlots, NextSignalAvailable, ListHints, ListP,
  ListTau, ListPlots, MaxSignals, MaxGenerations:

   Plots:=[]:
   OtherPlots:=[]:

   NextSignalAvailable:=CreateList(3, 0);
   #ListHints:=CreateList(MaxSignals, none);
   ListP:=CreateList(MaxSignals, none);
   ListTau:=CreateList(MaxSignals, none);
   ListPlots:= CreateList(MaxGenerations, []):

   n := 3*NextSignalAvailable[Sat+1]:
   [r, teSat(Sat, r, branchv), tSat(Sat, r, branchv)]:
   subs(r=rv, %):
   evalf(%):
   ListP[n+1]:= %:
   tauSat(0, rv, 1):
   evalf(%):
   ListTau[n+1]:= %:
   NextSignalAvailable[Sat+1]:= NextSignalAvailable[Sat+1]+1:
   AddPlotPointPolar(ListP[n+1], Sat, 0):  # AddPlotPointPolar:=
  proc(P, PSat, G)
   ListPlots[1] := Plots:
   Plots:=[]:
   OtherPlots:=[]:
   n+1:
  end:
```

```
> Cascade:= proc(Sat, rv, branchv)
   local Gen, NewSignals, OldSignals, n, St, event:
   global ListHints, ListTau, ListP, MaxGenerations:

   St:= time():
   Gen := 0:
   NewSignals:= []:

   n:= SetBasePoint(Sat, rv, branchv):
   NewSignals:= [op(NewSignals), n]:

   do
     Gen := Gen+1:
     printf("Start Generation %d\n", Gen):
     OldSignals := NewSignals:
     NewSignals:= []:

     for n in OldSignals do
```

```
        event:= NextEvent(n mod 3):
    #printf("hint for event=%d\n", event);
        SolveHard((n-1) mod 3, ListP[n], n mod 3, Gen, ListHints
    [event]):
        AcceptSolution():
        NewSignals:= [op(NewSignals), %]:
        printf("\nTau %a\n\n", ListTau);

    #printf("hint for event=%d\n", event);
        event:= NextEvent((n+1) mod 3):
        SolveHard((n-1) mod 3, ListP[n], (n+1) mod 3, Gen, ListHints
    [event]):
        AcceptSolution():
        NewSignals:= [op(NewSignals), %]:

        printf("\nTau %a\n\n", ListTau);
      end do:
     until Gen = MaxGenerations-1:
     printf("Cascade time %g\n", time()-St);
     ListTau:
    end:
```

Ok, now we can go for the signals

```
> Digits:= 10;
  ListHints:=CreateList(MaxSignals, none);
  ListTau:=CreateList(31, 0);
  ListP:=CreateList(31, []);
```

$$Digits := 10$$

$$ListHints := [none, none]$$

$$ListTau := [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

$$ListP := [[\ ], [\ ]] \quad \textbf{(62)}$$

```
> BaseSat:= 0:
  BaseR:= 12:
  BaseROriginal:= BaseR:
  BaseBranch:=1:
  MaxT:=150:
```

```
> Cascade(BaseSat, BaseR, BaseBranch);

Start Generation 1
1 --> 0  target = [12., 6.217012505, 485.5490808]
```

```
one interval r = 25.56992694 .. 27.67578046
Time Approximations 0.017.

Try code 5
I search for an scattering ray on same branch with sv>1 (1.50031) |
 P  <--- S
rGuessMin=25.5699   rGuessMax=27.6758   rmGuess=8.13536  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=8.13536}, {r = 25.56992694 .
. 27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.32e-07
Equations at solution: [.4e-7, -.132e-6, .6e-6]Solution in 0.406s

Time Plot 0.816 s.
Exiting SolveHard() after 1.812r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on
the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12., 6.217012505, 485.5490808]
one interval r = 33.94922194 .. 36.10248389
Time Approximations 0.009.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=33.9492   rGuessMax=36.1025   rmGuess=9.6022  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.6022}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-08
Equations at solution: [-.3e-7, .2e-7, -.38e-5]Solution in 0.208s

Time Plot 0.843 s.
Exiting SolveHard() after 1.16r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on
the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none]
```

```
Start Generation 2
2 --> 1  target = [27.52359685, 6.583434727, 467.7873064]
one interval r = 33.70078237 .. 35.85152418
Time Approximations 0.008.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=33.7008   rGuessMax=35.8515   rmGuess=11.7215  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=11.7215}, {r = 33.70078237 .
. 35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=7e-08
Equations at solution: [-.8e-7, .7e-7, .20e-5]Solution in 0.46s

Time Plot 0.941 s.
Exiting SolveHard() after 1.508r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, none, none, 401.8817394,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none]

0 --> 1  target = [27.52359685, 6.583434727, 467.7873064]
one interval r = 12.92327158 .. 18.68550893
Time Approximations 0.012.

Try code 8
I search for an scattering ray on same branch with sv<0 (-0.315768)
|   S  ---> P
rGuessMin=18.6855   rGuessMax=12.9233   rmGuess=11.4421  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=11.4421}, {r = 12.92327158 .
. 18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=0
Equations at solution: [0., 0., .1534e-4]Solution in 9.951s

Time Plot 0.292 s.
Exiting SolveHard() after 10.521r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341, none,
401.8817394, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none]

0 --> 2  target = [35.46322963, 4.125651795, 440.6712304]
one interval r = 14.35659706 .. 18.96093397
Time Approximations 0.014.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.100126) |   S  ---> P
rGuessMin=18.9609   rGuessMax=14.3566   rmGuess=13.8899  k=-503.657
 scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=13.8899}, {r = 14.35659706 .
. 18.96093397, rm = 3/2 .. 19}, avoid={});
Rejected {r=17.0394, rm=3.67253} for Delta=33.3232
in partial time = 7.223 s
(Scattering) fsolve(eqs, {r=18.9609, rm=13.8899}, {r = 14.35659706 .
. 18.96093397, rm = 3/2 .. 19}, avoid={{r = 17.03938200, rm =
3.672532531}});
Accepted {r=17.4128, rm=1.64938} with Delta=4e-08
Equations at solution: [-.349e-6, -.4e-7, .22e-6]Solution in 25.921s

Try code 7
I search for an scattering ray on same branch with 0<sv<1 (0.100126)
|   S  ---> P
rGuessMin=18.9609   rGuessMax=14.3566   rmGuess=13.8899  k=-503.657
 scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=13.8899}, {r = 14.35659706 .
. 18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-08
Equations at solution: [.58e-7, .3e-7, -.2124e-4]Solution in 0.402s

Time Plot 0.518 s.
Exiting SolveHard() after 27.42r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341, none,
401.8817394, 389.5900147, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none]

1 --> 2  target = [35.46322963, 4.125651795, 440.6712304]
one interval r = 24.64256576 .. 27.23722351
Time Approximations 0.013.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=24.6426   rGuessMax=27.2372   rmGuess=12.0945  k=-698.357
 scos=-667.307
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=27.2372, rm=12.0945}, {r = 24.64256576 .
. 27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 0.254 s
(Scattering) fsolve(eqs, {r=27.2372, rm=12.0945}, {r = 24.64256576 .
. 27.23722351, rm = 3/2 .. 28}, avoid={{r = 26.41507064, rm =
14.37818768}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-08
Equations at solution: [0., -.26e-7, .62e-5]Solution in 2.025s

Time Plot 0.747 s.
Exiting SolveHard() after 2.936r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507890, 4.004869084, 404.8622457]
two intervals r = 16.08011004 .. 19 or r = 18.92023592 .. 19
Time Approximations 0.017.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.0257633) |   S  ---> P
rGuessMin=18.9202   rGuessMax=16.0801   rmGuess=16.0592  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.0592}, {r = 16.08011004 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=18.1979, rm=2.83177} for Delta=36.1776
in partial time = 0.984 s
(Scattering) fsolve(eqs, {r=19, rm=16.0592}, {r = 16.08011004 .. 19,
rm = 3/2 .. 19}, avoid={{r = 18.19791158, rm = 2.831771711}});
Turn to the other branch [nested SolveOne()]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0257633) |   S  ---> P
rGuessMin=18.9202   rGuessMax=16.0801   rmGuess=16.0592  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.0592}, {r = 18.92023592 .. 19,
rm = 3/2 .. 19}, avoid={});
Solution in 142.106s

Solution in 214.411s

Try code 7
I search for an scattering ray on same branch with 0<sv<1
(0.0257633) |   S  ---> P
rGuessMin=18.9202   rGuessMax=16.0801   rmGuess=16.0592  k=-511.6
```

```
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.0592}, {r = 16.08011004 .. 19,
rm = 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-08
Equations at solution: [-.34e-7, -.1e-7, .2246e-4]Solution in 0.767s

Time Plot 0.292 s.
Exiting SolveHard() after 215.817r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, none, none, 358.9736287,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

1 --> 2  target = [34.94507890, 4.004869084, 404.8622457]
one interval r = 23.84730094 .. 26.76330661
Time Approximations 0.017.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=23.8473   rGuessMax=26.7633   rmGuess=12.5587  k=-706.355
 scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=12.5587}, {r = 23.84730094 .
. 26.76330661, rm = 3/2 .. 28}, avoid={});
Rejected {r=25.7263, rm=10.625} for Delta=15.0757
in partial time = 0.664 s
(Scattering) fsolve(eqs, {r=26.7633, rm=12.5587}, {r = 23.84730094 .
. 26.76330661, rm = 3/2 .. 28}, avoid={{r = 25.72631700, rm =
10.62500428}});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-08
Equations at solution: [-.1e-7, -.26e-7, .85e-5]Solution in 2.2s

Time Plot 0.744 s.
Exiting SolveHard() after 3.188r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, none,
358.9736287, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]

1 --> 0  target = [14.19258939, 5.589637192, 443.8306593]
one interval r = 24.71083344 .. 27.27388429
```

```
Time Approximations 0.014.

Try code 5
I search for an scattering ray on same branch with sv>1 (1.09677) |
 P  <--- S
rGuessMin=24.7108   rGuessMax=27.2739   rmGuess=14.0419  k=-299.351
 scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=14.0419}, {r = 24.71083344 .
. 27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., .67e-5]Solution in 0.535s

Time Plot 0.273 s.
Exiting SolveHard() after 0.987r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and
source on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, none,
358.9736287, 398.3314715, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none]

2 --> 0  target = [14.19258939, 5.589637192, 443.8306593]
one interval r = 33.37332721 .. 35.50872230
Time Approximations 0.008.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=33.3733   rGuessMax=35.5087   rmGuess=13.6505  k=500.498
scos=58.9796
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.6505}, {r = 33.37332721 .
. 35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-08
Equations at solution: [.5e-7, -.5e-7, -.3e-6]Solution in 0.156s

Time Plot 0.708 s.
Exiting SolveHard() after 0.957r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none]
```

```
1 --> 0  target = [15.91193137, 5.187783579, 408.6577384]
one interval r = 23.93303356 .. 26.81849303
Time Approximations 0.017.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=23.933   rGuessMax=26.8185   rmGuess=15.6714  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.6714}, {r = 23.93303356 .
. 26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-08
Equations at solution: [0., -.79e-7, .305e-4]Solution in 0.304s

Time Plot 0.385 s.
Exiting SolveHard() after 1.15r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and
source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, none, none, 361.5088837, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none]

2 --> 0  target = [15.91193137, 5.187783579, 408.6577384]
one interval r = 32.91337941 .. 35.00011460
Time Approximations 0.007.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=32.9134   rGuessMax=35.0001   rmGuess=15.8731  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.8731}, {r = 32.91337941 .
. 35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., .503e-4]Solution in 0.155s

Time Plot 0.441 s.
Exiting SolveHard() after 0.906r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, none, 361.5088837, none,
```

```
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

2 --> 1  target = [26.46347110, 6.196262566, 385.4447436]
one interval r = 32.62689490 .. 34.66372796
Time Approximations 0.008.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=32.6269   rGuessMax=34.6637   rmGuess=12.8019  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=12.8019}, {r = 32.62689490 .
. 34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-08
Equations at solution: [.2e-7, -.3e-7, -.227e-4]Solution in 0.417s

Time Plot 0.929 s.
Exiting SolveHard() after 1.432r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, none, 361.5088837,
324.6714489, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110, 6.196262566, 385.4447436]
two intervals r = 16.87563409 .. 19 or r = 18.70577536 .. 19
Time Approximations 0.019.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=18.7058   rGuessMax=16.8756   rmGuess=16.0505  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=16.0505}, {r = 16.87563409 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 1.391 s
(Scattering) fsolve(eqs, {r=19, rm=16.0505}, {r = 16.87563409 .. 19,
rm = 3/2 .. 19}, avoid={{r = 18.46834175, rm = 2.336532773}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.650e-5]Solution in 6.229s

Time Plot 0.694 s.
Exiting SolveHard() after 7.323r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898872, 4.883810791, 376.6196792]
one interval r = 23.20517308 .. 26.31784245
Time Approximations 0.013.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=23.2052   rGuessMax=26.3178   rmGuess=16.114  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.114}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., .292e-4]Solution in 0.299s

Time Plot 0.665 s.
Exiting SolveHard() after 1.156r=25.872 in [23.20517308 ..
26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and
source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, none, 328.4693863, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898872, 4.883810791, 376.6196792]
one interval r = 32.52213872 .. 34.53618387
Time Approximations 0.007.

Try code 6
I search for an scattering ray on same branch with sv>1 (1.04453) |
 P  <--- S
rGuessMin=32.5221   rGuessMax=34.5362   rmGuess=17.1514  k=492.219
scos=332.478
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.1514}, {r = 32.52213872 .
. 34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Solution in 30.015s

Try code 12
I search for an scattering ray on opposite branches with sv>1
(1.04453) |   P  <--- S
```

```
rGuessMin=32.5221   rGuessMax=34.5362   rmGuess=17.1514  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.1514}, {r = 32.52213872 .
. 34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.35e-06
Equations at solution: [-.257e-5, .335e-5, -.233e-4]Solution in
0.212s

Time Plot 0.395 s.
Exiting SolveHard() after 30.707r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, none, 328.4693863, 343.8134056, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205019, 6.025813555, 351.4270303]
one interval r = 32.23723258 .. 34.17446642
Time Approximations 0.006.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=32.2372   rGuessMax=34.1745   rmGuess=13.2224  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=13.2224}, {r = 32.23723258 .
. 34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-08
Equations at solution: [-.3e-7, .6e-7, .251e-4]Solution in 0.209s

Time Plot 0.858 s.
Exiting SolveHard() after 1.355r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, none, 328.4693863, 343.8134056, none, none,
292.9996913, none, none, none, none, none, none, none, none, none,
none]

0 --> 1  target = [25.87205019, 6.025813555, 351.4270303]
two intervals r = 17.98135512 .. 19 or r = 18.00786037 .. 19
```

```
Time Approximations 0.018.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=18.0079   rGuessMax=17.9814   rmGuess=16.1349  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=16.1349}, {r = 17.98135512 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 1.086 s
(Scattering) fsolve(eqs, {r=19, rm=16.1349}, {r = 17.98135512 .. 19,
rm = 3/2 .. 19}, avoid={{r = 18.91357070, rm = 2.734500977}});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-08
Equations at solution: [-.107e-6, .3e-7, .59e-5]Solution in 3.842s

Time Plot 0.667 s.
Exiting SolveHard() after 4.76r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, 302.3138446, 328.4693863, 343.8134056, none, none,
292.9996913, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [27.02037943, 6.377943878, 423.2883285]
one interval r = 33.10127385 .. 35.21212310
Time Approximations 0.007.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=33.1013   rGuessMax=35.2121   rmGuess=12.3231  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=12.3231}, {r = 33.10127385 .
. 35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-08
Equations at solution: [.5e-7, -.5e-7, .69e-5]Solution in 0.21s

Time Plot 0.92 s.
Exiting SolveHard() after 1.21r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
```

```
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, 302.3138446, 328.4693863, 343.8134056, none, none,
292.9996913, none, none, 360.0617346, none, none, none, none, none,
none, none]

0 --> 1  target = [27.02037943, 6.377943878, 423.2883285]
two intervals r = 15.22886699 .. 19 or r = 18.99874598 .. 19
Time Approximations 0.013.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=18.9987   rGuessMax=15.2289   rmGuess=15.2012  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.2012}, {r = 15.22886699 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 1.213 s
(Scattering) fsolve(eqs, {r=19, rm=15.2012}, {r = 15.22886699 .. 19,
rm = 3/2 .. 19}, avoid={{r = 17.51537052, rm = 2.064068293}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-08
Equations at solution: [.47e-7, .1e-7, -.980e-5]Solution in 5.713s

Time Plot 0.62 s.
Exiting SolveHard() after 7.044r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533,
none, 292.9996913, none, none, 360.0617346, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953235, 4.003559814, 404.4797356]
two intervals r = 16.09683967 .. 19 or r = 18.91731436 .. 19
Time Approximations 0.016.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.0248847) |   S  ---> P
rGuessMin=18.9173   rGuessMax=16.0968   rmGuess=16.0774  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.0774}, {r = 16.09683967 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=18.2075, rm=2.82563} for Delta=36.1838
in partial time = 0.936 s
(Scattering) fsolve(eqs, {r=19, rm=16.0774}, {r = 16.09683967 .. 19,
rm = 3/2 .. 19}, avoid={{r = 18.20748052, rm = 2.825634788}});
Rejected {r=18.5743, rm=1.52462} for Delta=35.9864
```

```
in partial time = 29.265 s
(Scattering) fsolve(eqs, {r=19, rm=16.0774}, {r = 16.09683967 .. 19,
rm = 3/2 .. 19}, avoid={{r = 18.20748052, rm = 2.825634788}, {r =
18.57432205, rm = 1.524621294}});
Accepted {r=18.3927, rm=1.66564} with Delta=6e-08
Equations at solution: [-.293e-6, .6e-7, -.16e-6]Solution in 91.384s

Try code 7
I search for an scattering ray on same branch with 0<sv<1
(0.0248847) |   S  ---> P
rGuessMin=18.9173   rGuessMax=16.0968   rmGuess=16.0774  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.0774}, {r = 16.09683967 .. 19,
rm = 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-08
Equations at solution: [-.50e-7, -.1e-7, -.598e-5]Solution in 0.521s

Time Plot 0.558 s.
Exiting SolveHard() after 92.8r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533,
none, 292.9996913, 358.6434151, none, 360.0617346, none, none, none,
none, none, none, none]

1 --> 2  target = [34.93953235, 4.003559814, 404.4797356]
one interval r = 23.83864811 .. 26.75768170
Time Approximations 0.017.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=23.8386   rGuessMax=26.7577   rmGuess=12.5636  k=-706.416
 scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=12.5636}, {r = 23.83864811 .
. 26.75768170, rm = 3/2 .. 28}, avoid={});
Rejected {r=25.7186, rm=10.5867} for Delta=15.1751
in partial time = 0.691 s
(Scattering) fsolve(eqs, {r=26.7577, rm=12.5636}, {r = 23.83864811 .
. 26.75768170, rm = 3/2 .. 28}, avoid={{r = 25.71855269, rm =
10.58667550}});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., .315e-4]Solution in 2.465s

Time Plot 0.795 s.
Exiting SolveHard() after 3.503r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
```

```
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533,
328.1170924, 292.9996913, 358.6434151, none, 360.0617346, none,
none, none, none, none, none, none]

2 --> 1  target = [26.46318954, 6.196177232, 385.4273400]
one interval r = 32.62668594 .. 34.66347615
Time Approximations 0.007.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=32.6267   rGuessMax=34.6635   rmGuess=12.8021  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=12.8021}, {r = 32.62668594 .
. 34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-08
Equations at solution: [-.2e-7, .3e-7, .332e-4]Solution in 0.441s

Time Plot 0.683 s.
Exiting SolveHard() after 1.213r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533,
328.1170924, 292.9996913, 358.6434151, none, 360.0617346, none,
none, 324.6552113, none, none, none, none]

0 --> 1  target = [26.46318954, 6.196177232, 385.4273400]
two intervals r = 16.87629601 .. 19 or r = 18.70552276 .. 19
Time Approximations 0.019.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=18.7055   rGuessMax=16.8763   rmGuess=16.0506  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=16.0506}, {r = 16.87629601 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 1.37 s
```

```
(Scattering) fsolve(eqs, {r=19, rm=16.0506}, {r = 16.87629601 .. 19,
rm = 3/2 .. 19}, avoid={{r = 18.46866853, rm = 2.336690427}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.718e-7, 0., -.1616e-4]Solution in 6.196s

Time Plot 0.647 s.
Exiting SolveHard() after 7.467r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533,
328.1170924, 292.9996913, 358.6434151, none, 360.0617346,
336.5944100, none, 324.6552113, none, none, none, none]

0 --> 2  target = [34.49522661, 3.897131314, 373.7808188]
two intervals r = 17.29769086 .. 19 or r = 18.51235425 .. 19
Time Approximations 0.026.

Try code 7
I search for an scattering ray on same branch with sv<0 (-0.0522555)
|   S  ---> P
rGuessMin=18.5124   rGuessMax=17.2977   rmGuess=17.2339  k=-489.404
 scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.2339}, {r = 17.29769086 .. 19,
rm = 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-08
Equations at solution: [-.54e-7, .1e-7, -.1826e-4]Solution in 0.674s

Time Plot 0.267 s.
Exiting SolveHard() after 1.362r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533,
328.1170924, 292.9996913, 358.6434151, none, 360.0617346,
336.5944100, none, 324.6552113, 331.9380678, none, none, none]

1 --> 2  target = [34.49522661, 3.897131314, 373.7808188]
one interval r = 23.14060343 .. 26.26979834
Time Approximations 0.012.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
```

```
rGuessMin=23.1406   rGuessMax=26.2698   rmGuess=12.9464  k=-709.872
 scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=12.9464}, {r = 23.14060343 .
. 26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=4e-08
Equations at solution: [.2e-7, .4e-7, .389e-4]Solution in 0.524s

Time Plot 0.471 s.
Exiting SolveHard() after 1.155r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533,
328.1170924, 292.9996913, 358.6434151, 299.8986618, 360.0617346,
336.5944100, none, 324.6552113, 331.9380678, none, none, none]

0 --> 2  target = [33.81362495, 3.725648991, 325.8920992]
two intervals r = 18.55227050 .. 19 or r = 17.22193012 .. 19
Time Approximations 0.019.

Try code 7
I search for an scattering ray on same branch with sv<0 (-0.206409)
|   S  ---> P
rGuessMin=17.2219   rGuessMax=18.5523   rmGuess=17.891  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.891}, {r = 18.55227050 .. 19,
rm = 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=7e-08
Equations at solution: [-.173e-6, .7e-7, -.145e-4]Solution in 0.332s

Time Plot 0.491 s.
Exiting SolveHard() after 1.383r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533,
328.1170924, 292.9996913, 358.6434151, 299.8986618, 360.0617346,
336.5944100, none, 324.6552113, 331.9380678, none, none,
289.5459574]

1 --> 2  target = [33.81362495, 3.725648991, 325.8920992]
one interval r = 22.07732228 .. 25.37892164
```

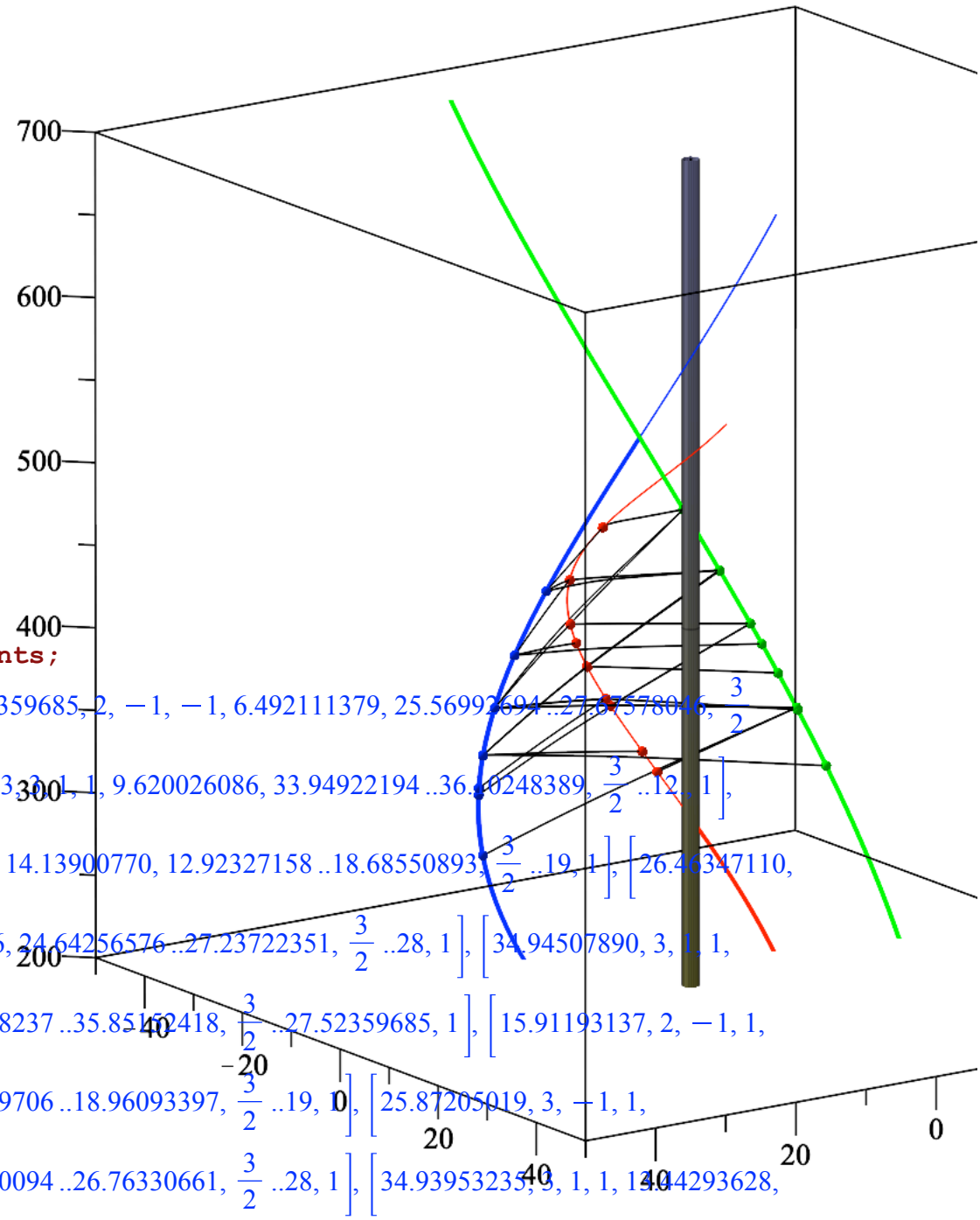
```
Time Approximations 0.01.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=22.0773   rGuessMax=25.3789   rmGuess=13.5403  k=-710.181
 scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=13.5403}, {r = 22.07732228 .
. 25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-08
Equations at solution: [-.2e-7, -.2e-7, -.410e-4]Solution in 0.206s

Time Plot 0.72 s.
Exiting SolveHard() after 1.049r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533,
328.1170924, 292.9996913, 358.6434151, 299.8986618, 360.0617346,
336.5944100, 256.1075315, 324.6552113, 331.9380678, none, none,
289.5459574]

1 --> 0  target = [17.93041370, 4.686508702, 353.3054106]
one interval r = 22.67806074 .. 25.90675353
Time Approximations 0.011.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=22.6781   rGuessMax=25.9068   rmGuess=16.1399  k=-453.132
 scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=16.1399}, {r = 22.67806074 .
. 25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-08
Equations at solution: [-.1e-7, -.23e-7, -.363e-4]Solution in 0.293s

Time Plot 0.596 s.
Exiting SolveHard() after 1.05r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and
source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
```

```
324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533,
328.1170924, 292.9996913, 358.6434151, 299.8986618, 360.0617346,
336.5944100, 256.1075315, 324.6552113, 331.9380678, 304.7995828,
none, 289.5459574]

2 --> 0  target = [17.93041370, 4.686508702, 353.3054106]
one interval r = 32.25770943 .. 34.20127520
Time Approximations 0.007.

Try code 6
I search for an scattering ray on same branch with sv>1 (1.11221) |
 P  <--- S
rGuessMin=32.2577   rGuessMax=34.2013   rmGuess=17.6855  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.6855}, {r = 32.25770943 .
. 34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=9e-08
Equations at solution: [-.5e-7, .9e-7, .15e-5]Solution in 0.171s

Time Plot 0.288 s.
Exiting SolveHard() after 0.536r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and
source on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634346, 441.6429607, 436.9174805, 422.9849341,
361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708,
358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837,
324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533,
328.1170924, 292.9996913, 358.6434151, 299.8986618, 360.0617346,
336.5944100, 256.1075315, 324.6552113, 331.9380678, 304.7995828,
323.4616909, 289.5459574]

Cascade time 435.031
```

$$[462.1634346, 441.6429607, 436.9174805, 422.9849341, 361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708, 358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837, 324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533, 328.1170924, 292.9996913, 358.6434151, 299.8986618, 360.0617346, 336.5944100, 256.1075315, 324.6552113, 331.9380678, 304.7995828, 323.4616909, 289.5459574] \quad \textbf{(63)}$$

```
> display(BH, S,
  c0_1n,c0_0,c0_1,
  c1_1n,c1_0,c1_1,
  c2_1n,c2_0,c2_1,
  op(ListPlots[1]), op(ListPlots[2]),
  op(ListPlots[3]), op(ListPlots[4]),
```

```
op(ListPlots[5]),
#op(OtherPlots),
view=[ -50..50, -50..50, 200..700]);
```




```
> Hints31:= ListHints;
```

$Hints31 := \Big[none, \Big[27.52359685, 2, -1, -1, 6.492111379, 25.56992694 ..27.67578046, \frac{3}{2} ..12., 1\Big], \Big[35.46322963, 2, 1, 1, 9.620026086, 33.94922194 ..36.40248389, \frac{3}{2} ..12., 1\Big], \Big[14.19258939, 2, 1, 1, 14.13900770, 12.92327158 ..18.68550893, \frac{3}{2} ..19, 1\Big], \Big[26.46347110, 3, -1, 1, 16.53292396, 24.64256576 ..27.23722351, \frac{3}{2} ..28, 1\Big], \Big[34.94507890, 3, 1, 1, 10.93651816, 33.70078237 ..35.85152418, \frac{3}{2} ..27.52359685, 1\Big], \Big[15.91193137, 2, -1, 1, 15.84478475, 14.35659706 ..18.96093397, \frac{3}{2} ..19, 1\Big], \Big[25.87205019, 3, -1, 1, 16.77671837, 23.84730094 ..26.76330661, \frac{3}{2} ..28, 1\Big], \Big[34.93953235, 3, 1, 1, 13.44293628,$

$33.37332721 ..35.50872230, \frac{3}{2} ..14.19258939, 1], [17.19898872, 2, -1, 1, 16.75492183,$
$16.08011004 ..19, \frac{3}{2} ..19, 1], [27.02037943, 2, -1, -1, 13.57592139, 24.71083344$
$..27.27388429, \frac{3}{2} ..14.19258939, 1], [34.49522661, 3, 1, 1, 15.76385589, 32.91337941$
$..35.00011460, \frac{3}{2} ..15.91193137, 1], [17.93041370, 3, 1, 1, 15.70097252, 16.87563409 ..19,$
$\frac{3}{2} ..19, 1], [26.46318954, 3, -1, 1, 15.90128516, 23.93303356 ..26.81849303, \frac{3}{2}$
$..15.91193137, 1], [33.81362495, 3, 1, 1, 11.78302904, 32.62689490 ..34.66372796, \frac{3}{2}$
$..26.46347110, 1], [18.68778084, 3, 1, 1, 15.36476721, 17.98135512 ..19, \frac{3}{2} ..19, 1],$
$[25.87204993, 3, -1, 1, 16.76106391, 23.20517308 ..26.31784245, \frac{3}{2} ..17.19898872, 1],$
$[34.08976800, 3, 1, 1, 17.19898432, 32.52213872 ..34.53618387, \frac{3}{2} ..17.19898872, 1],$
$[16.53336695, 3, 1, 1, 15.69073379, 15.22886699 ..19, \frac{3}{2} ..19, 1], [25.86532279, 3, -1, 1,$
$16.77921525, 23.83864811 ..26.75768170, \frac{3}{2} ..28, 1], [33.36855356, 3, 1, 1, 12.14280296,$
$32.23723258 ..34.17446642, \frac{3}{2} ..25.87205019, 1], [17.21111402, 2, -1, 1, 16.76151110,$
$16.09683967 ..19, \frac{3}{2} ..19, 1], [25.30046634, 3, -1, 1, 16.97472950, 23.14060343$
$..26.26979834, \frac{3}{2} ..28, 1], [34.32715981, 3, 1, 1, 11.39576957, 33.10127385 ..35.21212310,$
$\frac{3}{2} ..27.02037943, 1], [17.93092083, 3, 1, 1, 15.70086062, 16.87629601 ..19, \frac{3}{2} ..19, 1],$
$[24.33949902, 3, -1, 1, 17.27215877, 22.07732228 ..25.37892164, \frac{3}{2} ..28, 1],$
$[33.81339267, 3, 1, 1, 11.78320908, 32.62668594 ..34.66347615, \frac{3}{2} ..26.46318954, 1],$
$[18.05989026, 2, -1, 1, 17.06840479, 17.29769086 ..19, \frac{3}{2} ..19, 1], [25.40207153, 3, -1,$
$1, 17.00624446, 22.67806074 ..25.90675353, \frac{3}{2} ..17.93041370, 1], [33.79632952, 2, 1, -1,$
$17.86353408, 32.25770943 ..34.20127520, \frac{3}{2} ..17.93041370, 1], [18.85462971, 2, -1, 1,$

$$16.56670585, 18.55227050..19, \frac{3}{2}..19, 1\Big]\Big]$$

$$Hints31 := \Big[none, \Big[29.87826555, 2, 1, -1, 8.758183621, 28.97971405..29.97146887, \frac{3}{2}..10., 1\Big],$$
$$\Big[36.52423830, 3, 1, 1, 4.028173437, 35.46851615..37.27061300, \frac{3}{2}..10., 3\Big],$$
$$\Big[11.12399459, 2, 1, 1, 3.528747481, 10.37263699..16.44294471, \frac{3}{2}..20, 2\Big],$$
$$\Big[29.11987253, 3, -1, 1, 18.14741768, 28.13197534..29.71284135, \frac{3}{2}..30, 1\Big],$$
$$\Big[35.93679227, 3, 1, 1, 12.59355689, 35.05577445..36.91299894, \frac{3}{2}..29.87826555, 1\Big],$$
$$\Big[13.44277515, 3, -1, 1, 13.37883325, 11.41287728..17.61526950, \frac{3}{2}..20, 1\Big],$$
$$\Big[28.57854263, 3, -1, 1, 18.23665559, 27.36292487..29.35224620, \frac{3}{2}..30, 1\Big],$$
$$\Big[35.91205504, 3, 1, 1, 8.360841767, 34.72318999..36.61696587, \frac{3}{2}..11.12399459, 1\Big],$$
$$\Big[15.30161671, 2, -1, 1, 15.19622314, 13.48911124..18.91587006, \frac{3}{2}..20, 1\Big],$$
$$\Big[29.60365284, 1, -1, -1, 2.328019965, 28.24585241..29.75793176, 0..\frac{27}{4}, 2\Big],$$
$$\Big[35.31594664, 3, 1, 1, 12.75079520, 34.00163156..35.94952239, \frac{3}{2}..13.44277515, 1\Big],$$
$$\Big[16.13405333, 3, 1, 1, 15.80839013, 14.57070045..19.37856440, \frac{3}{2}..20, 1\Big],$$
$$\Big[29.10532394, 2, -1, -1, 11.51528383, 27.38039517..29.36136051, \frac{3}{2}..13.44277515, 1\Big],$$
$$\Big[34.62586393, 3, 1, 1, 13.04868683, 33.67418995..35.63445321, \frac{3}{2}..29.11987253, 1\Big],$$
$$\Big[17.91083702, 3, 1, 1, 16.23968960, 16.48814665..19.90967143, \frac{3}{2}..20, 1\Big],$$
$$\Big[28.57296034, 2, -1, -1, 15.18252650, 26.62718245..28.93700922, \frac{3}{2}..15.30161671, 1\Big],$$
$$\Big[34.84958395, 3, 1, 1, 15.20921790, 33.45600780..35.41990823, \frac{3}{2}..15.30161671, 1\Big],$$
$$\Big[13.41829467, 2, 1, 1, 12.66403723, 12.09244395..18.12235563, \frac{3}{2}..20, 1\Big],$$
$$\Big[28.55371953, 3, -1, 1, 18.23928734, 27.32888176..29.33437772, \frac{3}{2}..30, 1\Big],$$
$$\Big[34.04329235, 3, 1, 1, 13.19952918, 33.07960633..35.04036788, \frac{3}{2}..28.57854263, 1\Big],$$
$$\Big[15.37532903, 2, -1, 1, 15.25553472, 13.57904887..18.95882153, \frac{3}{2}..20, 1\Big],$$

$$\left[27.91054939, 3, -1, 1, 18.28194106, 26.47344578..28.84320324, \frac{3}{2}..30, 1\right],$$
$$\left[35.32189384, 3, 1, 1, 12.83511624, 34.40089320..36.32321658, \frac{3}{2}..29.60365284, 1\right],$$
$$\left[16.19548605, 3, 1, 1, 15.83834737, 14.63245592..19.40154367, \frac{3}{2}..20, 1\right],$$
$$\left[27.06673807, 3, -1, 1, 18.29599023, 25.41214835..28.13785667, \frac{3}{2}..30, 1\right],$$
$$\left[34.60825269, 3, 1, 1, 13.05351763, 33.65602173..35.61673250, \frac{3}{2}..29.10532394, 1\right],$$
$$\left[17.00004437, 2, -1, 1, 16.34341807, 15.64116418..19.72383047, \frac{3}{2}..20, 1\right],$$
$$\left[28.27511071, 3, -1, 1, 16.13212079, 26.23773143..28.69508492, \frac{3}{2}..16.13405333, 1\right],$$
$$\left[34.61952959, 3, 1, 1, 16.12727049, 33.19571921..35.15878287, \frac{3}{2}..16.13405333, 1\right],$$
$$\left[18.47234754, 2, -1, 1, 16.88870166, 17.60249431..20, \frac{3}{2}..20, 1\right]\Big]$$

```
> Tau10:=ListTau;
```

$T10 := [462.1634346, 441.6429607, 436.9174805, 422.9849341, 361.5258023, 401.8817394, 389.5900147, 328.4693996, 401.5075708, 358.9736287, 398.3314715, 371.4838732, 336.6121579, 361.5088837, 324.6714489, 302.3138446, 328.4693863, 343.8134056, 375.7328533, 328.1170924, 292.9996913, 358.6434151, 299.8986618, 360.0617346, 336.5944100, 256.1075315, 324.6552113, 331.9380678, 304.7995828, 323.4616909, 289.5459574]$ **(65)**

```
> # 334.332   10 digits, 30 signals without hints
  # 402.226   40 digits, 30 signals with hints
  # 478.446   40 digits, 30 signals with hints

  #+1470.84s              Digits  80, 30 signals with hints.
  #+2680.41s              Digits 120, 30 signals with hints.
```

```
> ListP;
```

$[[12., 6.217012505, 485.5490808], [27.52359685, 6.583434727, 467.7873064], [35.46322963, 4.125651795, 440.6712304], [14.19258939, 5.589637192, 443.8306593], [26.46347110, 6.196262566, 385.4447436], [34.94507890, 4.004869084, 404.8622457], [15.91193137, 5.187783579, 408.6577384], [25.87205019, 6.025813555, 351.4270303], [34.93953235, 4.003559814, 404.4797356], [17.19898872, 4.883810791, 376.6196792], [27.02037943, 6.377943878, 423.2883285], [34.49522661, 3.897131314, 373.7808188], [17.93041370, 4.686508702, 353.3054106], [26.46318954, 6.196177232, 385.4273400], [33.81362495, 3.725648991, 325.8920992], [18.68778084, 4.408641499, 317.6355763], [25.87204993, 6.025813482, 351.4270161], [34.08976800, 3.796597789, 345.4776483], [16.53336695, 5.044302184, 394.1371717], [25.86532279, 6.023953984, 351.0643079], [33.36855356,$ **(66)**

3.605714293, 293.4757844], [17.21111402, 4.880770163, 376.2749457], [25.30046634, 5.871694125, 321.9987533], [34.32715981, 3.855919918, 362.0987241], [17.93092083, 4.686358240, 353.2869287], [24.33949902, 5.620659138, 276.8366148], [33.81339267, 3.725588316, 325.8754832], [18.05989026, 4.647168297, 348.4389887], [25.40207153, 5.898629067, 327.0487018], [33.79632952, 3.721126243, 324.6541437], [18.85462971, 4.309926959, 304.3747457]]

```
> Digits:= 40;
  ListTau:=CreateList(31, 0);
  ListP:=CreateList(31, []);
```

$$Digits := 40$$

$$ListTau := [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

$$ListP := [[\ ], [\ ]] \quad \textbf{(67)}$$

```
> Cascade(BaseSat, BaseR, BaseBranch);
```

```
Start Generation 1
1 --> 0  target = [12., 6.217012502940637829614151732937593976434,
485.5490808982492308534496093965191173673]
one interval r = 25.56992693972547008995262253606783454365 ..
27.67578046428625630857456906903809480442
Time Approximations 0.051.

hint used Hint := [27.52359685, 2, -1, -1, 6.492111379, 25.56992694
.. 27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |
 P  <--- S
rGuessMin=25.5699   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 .
. 27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=0
Equations at solution: [0., 0., .5e-36]Solution in 1.155s

Time Plot 3.153 s.
Exiting SolveHard() after 6.159r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on
the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349363502477776721279324784874684,
441.6429597316308683463600766538668411865, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none]
```

```
2 --> 0  target = [12., 6.217012502940637829614151732937593976434,
485.54908089824923085344960939651911736 73]
one interval r = 33.94922193952172450684306577491961885133 ..
36.10248388941651492749251642222636168212
Time Approximations 0.025.

hint used Hint := [35.46322963, 3, 1, 1, 9.620026086, 33.94922194 ..
36.10248389, 3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=33.9492   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 .
. 36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=5e-38
Equations at solution: [.8e-37, -.5e-37, -.39e-35]Solution in 0.775s

Time Plot 3.519 s.
Exiting SolveHard() after 4.782r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on
the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none]

Start Generation 2
2 --> 1  target = [27.52359684479996584152978595423608112768,
6.583434721648324074592799441043164221029,
467.78730595863832684476535436144927539 44]
one interval r = 33.70078236348240824425290426033461490807 ..
35.85152417371059031824547448614436827730
Time Approximations 0.262.

hint used Hint := [34.94507890, 3, 1, 1, 10.93651816, 33.70078237 ..
35.85152418, 3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=33.7008   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 .
. 35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., .139e-34]Solution in 0.655s

Time Plot 4.141 s.
Exiting SolveHard() after 5.494r=34.9451 in [33.70078237 ..
35.85152418]
```

```
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84, none, none,
401.88173904228928428604361069984116175 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844799965841529785954236081127 68,
6.5834347216483240745927994410431642210 29,
467.78730595863832684476535436144927539 44]
one interval r = 12.923271608323322066574595321495782075 27 ..
18.685508940528109344439169667971551801 00
Time Approximations 0.042.

hint used Hint := [14.19258939, 2, 1, 1, 14.13900770, 12.92327158 ..
18.68550893, 3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768)
|   S  ---> P
rGuessMin=18.6855   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=8.8e-38
Equations at solution: [-.7e-37, -.88e-37, .1583e-34]Solution in
31.957s

Time Plot 1.242 s.
Exiting SolveHard() after 35.213r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08, none,
401.88173904228928428604361069984116175 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628253790402151219742545983048 78,
4.1256517968433145185385281438814667209 06,
440.67123065048064044274396492648916582 05]
one interval r = 14.356597051189930679057620672507071597 84 ..
18.960933974318739593918127349433992044 69
Time Approximations 0.042.
```

```
hint used Hint := [15.91193137, 2, -1, 1, 15.84478475, 14.35659706 .
. 18.96093397, 3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126)
|   S  ---> P
rGuessMin=18.9609   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
 scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 .
. 18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.28e-37, -.1e-37, .307e-35]Solution in
1.692s

Time Plot 1.626 s.
Exiting SolveHard() after 5.942r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08, none,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962825379040215121974254598304878,
4.12565179684331451853852814388146672090 6,
440.67123065048064044274396492648916582 05]
one interval r = 24.64256576254712802572180528368919442523 ..
27.23722351598307601530937097552762134652
Time Approximations 0.05.

hint used Hint := [26.46347110, 3, -1, 1, 16.53292396, 24.64256576 .
. 27.23722351, 3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=24.6426   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
 scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 .
. 27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.545 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 .
. 27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064385333017238330077234465321969, rm =
14.37818770404277542505590191447722338155}});
Accepted {r=26.4635, rm=16.5329} with Delta=8.0e-38
Equations at solution: [0., -.80e-37, .388e-34]Solution in 7.332s

Time Plot 2.797 s.
```

```
Exiting SolveHard() after 11.659r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888797729914637569990804457214741,
4.0048690818194448827487638012514845137 95,
404.86224501294950265590545171823193668 55]
two intervals r = 16.08011007761438284104375574889661193034 .. 19 or
r = 18.92023592144644601800070609321780284969 .. 19
Time Approximations 0.056.

hint used Hint := [17.19898872, 2, -1, 1, 16.75492183, 16.08011004 .
. 19, 3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1
(0.0257633) |   S  ---> P
rGuessMin=18.9202   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19,
rm = 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.17e-37, 0., -.38e-36]Solution in 2.183s

Time Plot 1.598 s.
Exiting SolveHard() after 6.764r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94, none, none,
358.97362823933644182158001867698955928 13, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none]
```

```
1 --> 2  target = [34.945078887977299146375699908044572147 41,
4.0048690818194448827487638012514845137 95,
404.86224501294950265590545171823193668 55]
one interval r = 23.847300927786575931659526697786917155 34 ..
26.763306600413444687053586145167169074 05
Time Approximations 0.052.

hint used Hint := [25.87205019, 3, -1, 1, 16.77671837, 23.84730094 .
. 26.76330661, 3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=23.8473   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
 scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 .
. 26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., -.88e-35]Solution in 1.197s

Time Plot 2.586 s.
Exiting SolveHard() after 5.372r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09, none,
358.97362823933644182158001867698955928 13, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none]

1 --> 0  target = [14.192589417880613743816214597088769973 24,
5.5896371829369819736442791913008774050 77,
443.83065884514772992529555109347267711 90]
one interval r = 24.710833429443324495593313941743755579 84 ..
27.273884283537815127876637074334315519 68
Time Approximations 0.048.

hint used Hint := [27.02037943, 2, -1, -1, 13.57592139, 24.71083344
.. 27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |
 P  <--- S
rGuessMin=24.7108   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
 scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 .
. 27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
```

```
Equations at solution: [-.1e-37, .81e-37, .98e-35]Solution in 1.19s

Time Plot 1.504 s.
Exiting SolveHard() after 4.301r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and
source on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793248748 74684,
441.64295973163086834636007665386684 11865,
436.91748165281983059261853942942224 33484,
422.98493397441432411106750701655786 37308,
361.52580256065215621270117170957110 58298,
401.88173904228928428604361069984116 17564,
389.59001515841913096868794166166600 70194,
328.46939893411151398613494618806651 13109, none,
358.97362823933644182158001867698955 92813,
398.33147104015507581872792756979870 53487, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

2 --> 0  target = [14.19258941788061374381621459708876997324,
5.58963718293698197364427919130087740 5077,
443.83065884514772992529555109347267 71190]
one interval r = 33.37332720438533208539377866404953276137 ..
35.50872228736422883384195797819915478947
Time Approximations 0.022.

hint used Hint := [34.93953235, 3, 1, 1, 13.44293628, 33.37332721 ..
35.50872230, 3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=33.3733   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 .
. 35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .151e-34]Solution in 0.673s

Time Plot 2.153 s.
Exiting SolveHard() after 3.227r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793248748 74684,
441.64295973163086834636007665386684 11865,
436.91748165281983059261853942942224 33484,
422.98493397441432411106750701655786 37308,
361.52580256065215621270117170957110 58298,
```

```
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [15.91193136510569595137770698583432131326,
5.18778357849831716559192757496465974486 9,
408.65773862723309892047141473126067787 13]
one interval r = 23.93303356355166970323108776934956152709 ..
26.81849303512099004369718485631702768398
Time Approximations 0.065.

hint used Hint := [26.46318954, 3, -1, 1, 15.90128516, 23.93303356 .
. 26.81849303, 3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=23.933   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 .
. 26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [.1e-37, .79e-37, .88e-35]Solution in 1.449s

Time Plot 1.79 s.
Exiting SolveHard() after 4.907r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and
source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87, none, none,
361.50888347101738332165568649713731439 20, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none]

2 --> 0  target = [15.91193136510569595137770698583432131326,
5.18778357849831716559192757496465974486 9,
408.65773862723309892047141473126067787 13]
one interval r = 32.91337941468584180453559624551954369641 ..
35.00011460045494616907519467508291514117
```

```
Time Approximations 0.022.

hint used Hint := [34.49522661, 3, 1, 1, 15.76385589, 32.91337941 ..
35.00011460, 3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=32.9134   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 .
. 35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=1.3e-37
Equations at solution: [-.12e-36, .13e-36, .80e-35]Solution in
0.661s

Time Plot 2 s.
Exiting SolveHard() after 3.013r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01, none,
361.50888347101738332165568649713731439 20, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none]

2 --> 1  target = [26.46347110539976775688849031731477848509,
6.19626256537969370216275349075650269714 2,
385.44474379327614418129435545739274961 90]
one interval r = 32.62689489860492675631649705944009283301 ..
34.66372795609603104211198035694331649846
Time Approximations 0.022.

hint used Hint := [33.81362495, 3, 1, 1, 11.78302904, 32.62689490 ..
34.66372796, 3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=32.6269   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
```

```
Equations at solution: [.3e-37, -.6e-37, -.65e-35]Solution in 0.759s

Time Plot 3.379 s.
Exiting SolveHard() after 4.466r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01, none,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]

0 --> 1  target = [26.46347110539976775688849031731477848509,
6.19626256537969370216275349075650269714 2,
385.44474379327614418129435545739274961 90]
two intervals r = 16.87563408754166121440812914403080714127 .. 19 or
r = 18.70577536964362250849095392912801103727 .. 19
Time Approximations 0.06.

hint used Hint := [17.93041370, 3, 1, 1, 15.70097252, 16.87563409 ..
19, 3/2 .. 19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=18.7058   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 5.419 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19,
rm = 3/2 .. 19}, avoid={{r =
18.46834175120789839504657743201526066543, rm =
2.33653277406135886474239591472715925008 1}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.179e-37, 0., .2178e-34]Solution in 19.903s

Time Plot 1.968 s.
Exiting SolveHard() after 24.453r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]

Start Generation 4
1 --> 0  target = [17.19898874735628255923651729245938270093,
4.88381077982614553797184240182126752788 0,
376.61967855814619245340268918025697572 30]
one interval r = 23.20517306759733802237697232416477084699 ..
26.31784243474874560818367715867804854224
Time Approximations 0.037.

hint used Hint := [25.87204993, 3, -1, 1, 16.76106391, 23.20517308 .
. 26.31784245, 3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=23.2052   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 .
. 26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.161e-34]Solution in 1.046s

Time Plot 1.816 s.
Exiting SolveHard() after 4.091r=25.872 in [23.20517308 ..
26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and
source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
```

```
389.590015158419130968687941661666007019 4,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15, none,
328.46938513483767385435600078754260890 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874735628255923651729245938270093,
4.883810779826145537971842401821267527880,
376.61967855814619245340268918025697572 30]
one interval r = 32.52213870853725673135151565769207371574 ..
34.53618386093003700718765574973058965720
Time Approximations 0.016.

hint used Hint := [34.08976800, 3, 1, 1, 17.19898432, 32.52213872 ..
34.53618387, 3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1
(1.04453) |   P  <--- S
rGuessMin=32.5221   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=6.57e-36
Equations at solution: [-.504e-35, .657e-35, -.226e-34]Solution in
0.713s

Time Plot 1.783 s.
Exiting SolveHard() after 2.796r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.590015158419130968687941661666007019 4,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15, none,
328.46938513483767385435600078754260890 80,
```

```
343.81340624896360230715542788774901323 89, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.872050175403105209417695955142983100 90,
6.025813549334585239158208828398808143178,
351.42702948382214417571102913825521608 80]
one interval r = 32.237232569114498421447246141855344390 94 ..
34.174466406139446587125927702993375044 95
Time Approximations 0.015.

hint used Hint := [33.36855356, 3, 1, 1, 12.14280296, 32.23723258 ..
34.17446642, 3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=32.2372   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 .
. 34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., -.145e-34]Solution in 0.736s

Time Plot 3.179 s.
Exiting SolveHard() after 4.189r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15, none,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542788774901323 89, none, none,
292.99969138294518055640223901684348469 92, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175403105209417695955142983100 90,
6.025813549334585239158208828398808143178,
351.42702948382214417571102913825521608 80]
two intervals r = 17.981355144431983874170804078975236281 79 .. 19 or
r = 18.007860347740994050430652411116048218 47 .. 19
Time Approximations 0.057.
```

```
hint used Hint := [18.68778084, 3, 1, 1, 15.36476721, 17.98135512 ..
19, 3/2 .. 19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=18.0079   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 4.232 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19,
rm = 3/2 .. 19}, avoid={{r =
18.913570713689207378408538835367614781 74, rm =
2.7345009931355950449955153246628959478 49}});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [-.36e-37, .1e-37, .56e-35]Solution in
12.383s

Time Plot 1.893 s.
Exiting SolveHard() after 16.457r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15,
302.31384314810880278539054351551347002 78,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542787749013238 9, none, none,
292.99969138294518055640223901684348469 92, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418720783127960221698280162527 17,
6.3779438739200435389783533607087813246 17,
423.28832783898402813927173518272824197 56]
one interval r = 33.101273843782967187971262968838331987 82 ..
35.212123086499839556580066334417117703 89
Time Approximations 0.021.

hint used Hint := [34.32715981, 3, 1, 1, 11.39576957, 33.10127385 ..
35.21212310, 3/2 .. 27.02037943, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=33.1013   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 .
. 35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.122e-34]Solution in 0.798s

Time Plot 3.479 s.
Exiting SolveHard() after 4.659r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15,
302.31384314810880278539054351551347002 78,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542788774901323 89, none, none,
292.99969138294518055640223901684348469 92, none, none,
360.06173466727435541746838478215728178 62, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418720783127960221698280162527 17,
6.3779438739200435389783533607087813246 17,
423.28832783898402813927173518272824197 56]
two intervals r = 15.228867024404914020082887630346933525 64 .. 19 or
r = 18.998745984423261094155379535643923747 00 .. 19
Time Approximations 0.047.

hint used Hint := [16.53336695, 3, 1, 1, 15.69073379, 15.22886699 ..
19, 3/2 .. 19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=18.9987   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19,
rm = 3/2 .. 19}, avoid={});
```

```
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 5.037 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19,
rm = 3/2 .. 19}, avoid={{r =
17.515370541307371830296968184275932899 43, rm =
2.0640682987050903999127283169110824270 12}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.32e-37, -.1e-37, -.1820e-34]Solution in
18.943s

Time Plot 1.877 s.
Exiting SolveHard() after 23.889r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15,
302.31384314810880278539054351551347002 78,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542788774901323 89,
375.73285290043447795783500253092730856 13, none,
292.99969138294518055640223901684348469 92, none, none,
360.06173466727435541746838478215728178 62, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343415080603666368667914992272 15,
4.0035598155192297519935817863969169914 92,
404.47973593992877766487202366228903485 84]
two intervals r = 16.096839663726623105584096975712111039 11 .. 19 or
r = 18.917314368365311838275023673243250498 06 .. 19
Time Approximations 0.053.

hint used Hint := [17.21111402, 2, -1, 1, 16.76151110, 16.09683967 .
. 19, 3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1
(0.0248847) |   S  ---> P
rGuessMin=18.9173   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19,
rm = 3/2 .. 19}, avoid={});
```

```
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.51e-37, -.1e-37, .1753e-34]Solution in
2.097s

Time Plot 1.541 s.
Exiting SolveHard() after 6.432r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15,
302.31384314810880278539054351551347002 78,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542788774901323 89,
375.73285290043447795783500253092730856 13, none,
292.99969138294518055640223901684348469 92,
358.64341560776563828789377335546371324 36, none,
360.06173466727435541746838478215728178 62, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234341508060366636866791499227215,
4.0035598155192297519935817863969169914 92,
404.47973593992877766487202366228903485 84]
one interval r = 23.83864811516925401070181528621603173110 ..
26.75768169891040670103843113853550876225
Time Approximations 0.054.

hint used Hint := [25.86532279, 3, -1, 1, 16.77921525, 23.83864811 .
. 26.75768170, 3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=23.8386   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
 scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 .
. 26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.5e-38
Equations at solution: [-.1e-37, -.25e-37, .532e-34]Solution in
1.193s

Time Plot 2.495 s.
```

```
Exiting SolveHard() after 5.186r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15,
302.31384314810880278539054351551347002 78,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542788774901323 89,
375.73285290043447795783500253092730856 13,
328.11709294326682868063061840531240315 56,
292.99969138294518055640223901684348469 92,
358.64341560776563828789377335546371324 36, none,
360.06173466727435541746838478215728178 62, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954458146650656332304152399577898,
6.19617723023314440382926937080140189027 7,
385.42734025701315342674194950890640690 63]
one interval r = 32.62668594679580861430730599464769092367 ..
34.66347615047243668966966653070068866233
Time Approximations 0.018.

hint used Hint := [33.81339267, 3, 1, 1, 11.78320908, 32.62668594 ..
34.66347615, 3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=32.6267   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 .
. 34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=8e-38
Equations at solution: [-.5e-37, .8e-37, .172e-34]Solution in 0.79s

Time Plot 3.151 s.
Exiting SolveHard() after 4.265r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and
source on the different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15,
302.31384314810880278539054351551347002 78,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542788774901323 89,
375.73285290043447795783500253092730856 13,
328.11709294326682868063061840531240315 56,
292.99969138294518055640223901684348469 92,
358.64341560776563828789377335546371324 36, none,
360.06173466727435541746838478215728178 62, none, none,
324.65521223572567440639118273934306330 08, none, none, none, none]

0 --> 1  target = [26.46318954458146650656332304152399577898,
6.19617723023314440382926937080140189027 7,
385.42734025701315342674194950890640690 63]
two intervals r = 16.87629600292858693511539432220840135730 .. 19 or
r = 18.70552276424761152487531850181966383151 .. 19
Time Approximations 0.063.

hint used Hint := [17.93092083, 3, 1, 1, 15.70086062, 16.87629601 ..
19, 3/2 .. 19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=18.7055   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 5.214 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19,
rm = 3/2 .. 19}, avoid={{r =
18.46866852532546503031552671818375815553, rm =
2.33669042812683723952335952046961564434 1}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.179e-37, 0., -.1445e-34]Solution in 19.277s

Time Plot 2.049 s.
Exiting SolveHard() after 24.024r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
```

```
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15,
302.31384314810880278539054351551347002 78,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542788774901323 89,
375.73285290043447795783500253092730856 13,
328.11709294326682868063061840531240315 56,
292.99969138294518055640223901684348469 92,
358.64341560776563828789377335546371324 36, none,
360.06173466727435541746838478215728178 62,
336.59441032236151191753282464096333626 63, none,
324.65521223572567440639118273934306330 08, none, none, none, none]

0 --> 2  target = [34.49522661165008082072914924801977531221,
3.897131315938826425809511175696600277532,
373.78081884469355781443393947213302772 14]
two intervals r = 17.29769086224237109342402341709761056719 .. 19 or
r = 18.51235425072058411055877841825442662873 .. 19
Time Approximations 0.086.

hint used Hint := [18.05989026, 2, -1, 1, 17.06840479, 17.29769086 .
. 19, 3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555)
|   S  ---> P
rGuessMin=18.5124   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
 scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19,
rm = 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., -.809e-35]Solution in 1.459s

Time Plot 1.365 s.
Exiting SolveHard() after 6.251r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15,
302.31384314810880278539054351551347002 78,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542788774901323 89,
375.73285290043447795783500253092730856 13,
328.11709294326682868063061840531240315 56,
292.99969138294518055640223901684348469 92,
358.64341560776563828789377335546371324 36, none,
360.06173466727435541746838478215728178 62,
336.59441032236151191753282464096333626 63, none,
324.65521223572567440639118273934306330 08,
331.93806791424212116529737464898354754 10, none, none, none]

1 --> 2  target = [34.49522661165008082072914924801977531221,
3.89713131593882642580951117569660027753 2,
373.78081884469355781443393947213302772 14]
one interval r = 23.14060343356794656700874621257810843342 ..
26.26979834283215497218166454938699523304
Time Approximations 0.039.

hint used Hint := [25.30046634, 3, -1, 1, 16.97472950, 23.14060343 .
. 26.26979834, 3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=23.1406   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
 scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 .
. 26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=4e-38
Equations at solution: [.2e-37, .4e-37, -.59e-35]Solution in 1.012s

Time Plot 2.85 s.
Exiting SolveHard() after 4.926r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
```

```
436.91748165281983059261853942942224334 84,
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15,
302.31384314810880278539054351551347002 78,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542788774901323 89,
375.73285290043447795783500253092730856 13,
328.11709294326682868063061840531240315 56,
292.99969138294518055640223901684348469 92,
358.64341560776563828789377335546371324 36,
299.89866204788425542623033753969888433 41,
360.06173466727435541746838478215728178 62,
336.59441032236151191753282464096333626 63, none,
324.65521223572567440639118273934306330 08,
331.93806791424212116529737464898354754 10, none, none, none]

0 --> 2  target = [33.81362495408772233776384102941444868670,
3.72564899357067124989860181185201629150 5,
325.89209972827061292564035729822378476 82]
two intervals r = 18.55227049005089555179894704717288522007 .. 19 or
r = 17.22193013587574549711947325023088000365 .. 19
Time Approximations 0.09.

hint used Hint := [18.85462971, 2, -1, 1, 16.56670585, 18.55227050 .
. 19, 3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409)
|   S  ---> P
rGuessMin=17.2219   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
 scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19,
rm = 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [-.121e-36, .5e-37, -.65e-35]Solution in
1.508s

Time Plot 1.792 s.
Exiting SolveHard() after 6.547r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
```

```
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15,
302.31384314810880278539054351551347002 78,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542788774901323 89,
375.73285290043447795783500253092730856 13,
328.11709294326682868063061840531240315 56,
292.99969138294518055640223901684348469 92,
358.64341560776563828789377335546371324 36,
299.89866204788425542623033753969888433 41,
360.06173466727435541746838478215728178 62,
336.59441032236151191753282464096333626 63, none,
324.65521223572567440639118273934306330 08,
331.93806791424212116529737464898354754 10, none, none,
289.54595772481976518148028686059313097 24]

1 --> 2  target = [33.81362495408772233776384102941444868670,
3.72564899357067124989860181185201629150 5,
325.89209972827061292564035729822378476 82]
one interval r = 22.07732229165620846710321208883760005548 ..
25.37892165298982965784819615644009584072
Time Approximations 0.031.

hint used Hint := [24.33949902, 3, -1, 1, 17.27215877, 22.07732228 .
. 25.37892164, 3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=22.0773   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
 scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 .
. 25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, -.247e-34]Solution in 0.836s

Time Plot 2.514 s.
Exiting SolveHard() after 3.97r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
441.64295973163086834636007665386684118 65,
436.91748165281983059261853942942224334 84,
```

```
422.98493397441432411106750701655786373 08,
361.52580256065215621270117170957110582 98,
401.88173904228928428604361069984116175 64,
389.59001515841913096868794166166600701 94,
328.46939893411151398613494618806651131 09,
401.50757157975460407223129891849877206 36,
358.97362823933644182158001867698955928 13,
398.33147104015507581872792756979870534 87,
371.48387394399115058524177612177482600 01,
336.61215841194302430713996040873307535 67,
361.50888347101738332165568649713731439 20,
324.67144992595116802722094998297873533 15,
302.31384314810880278539054351551347002 78,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542788774901323 89,
375.73285290043447795783500253092730856 13,
328.11709294326682868063061840531240315 56,
292.99969138294518055640223901684348469 92,
358.64341560776563828789377335546371324 36,
299.89866204788425542623033753969888433 41,
360.06173466727435541746838478215728178 62,
336.59441032236151191753282464096333626 63,
256.10753186046397444460083703407897999 18,
324.65521223572567440639118273934306330 08,
331.93806791424212116529737464898354754 10, none, none,
289.54595772481976518148028686059313097 24]

1 --> 0  target = [17.93041369709740979309933253273271318826,
4.68650870196228155456139423149427741670 9,
353.30541094994294622604476805952759249 50]
one interval r = 22.67806075122313444248268840929031612196 ..
25.90675353521168731978961475532040424362
Time Approximations 0.038.

hint used Hint := [25.40207153, 3, -1, 1, 17.00624446, 22.67806074 .
. 25.90675353, 3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=22.6781   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
 scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 .
. 25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.186e-34]Solution in
0.939s

Time Plot 1.943 s.
Exiting SolveHard() after 4.053r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and
source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493635024777767212793247848746 84,
```

```
441.642959731630868346360076653866841186 5,
436.917481652819830592618539429422243348 4,
422.984933974414324111067507016557863730 8,
361.525802560652156212701171709571105829 8,
401.881739042289284286043610699841161756 4,
389.590015158419130968687941661666007019 4,
328.469398934111513986134946188066511310 9,
401.507571579754604072231298918498772063 6,
358.973628239336441821580018676989559281 3,
398.331471040155075818727927569798705348 7,
371.483873943991150585241776121774826000 1,
336.612158411943024307139960408733075356 7,
361.508883471017383321655686497137314392 0,
324.671449925951168027220949982978735331 5,
302.313843148108802785390543515513470027 8,
328.469385134837673854356000787542608908 0,
343.813406248963602307155427887749013238 9,
375.732852900434477957835002530927308561 3,
328.117092943266828680630618405312403155 6,
292.999691382945180556402239016843484699 2,
358.643415607765638287893773355463713243 6,
299.898662047884255426230337539698884334 1,
360.061734667274355417468384782157281786 2,
336.594410322361511917532824640963336266 3,
256.107531860463974444600837034078979991 8,
324.655212235725674406391182739343063300 8,
331.938067914242121165297374648983547541 0,
304.799583254630873460020595922321782128 9, none,
289.545957724819765181480286860593130972 4]

2 --> 0  target = [17.93041369709740979309933253273271318826,
4.686508701962281554561394231494277416709,
353.305410949942946226044768059527592495 0]
one interval r = 32.25770942828860614370114686474923159331 ..
34.20127520026420397931431946702402015978
Time Approximations 0.02.

hint used Hint := [33.79632952, 2, 1, -1, 17.86353408, 32.25770943 .
. 34.20127520, 3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |
 P  <--- S
rGuessMin=32.2577   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 .
. 34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .88e-35]Solution in 0.402s

Time Plot 1.645 s.
Exiting SolveHard() after 2.384r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and
source on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493635024777767212793247848746 84,
441.6429597316308683463600766538668411865,
436.9174816528198305926185394294222433484,
422.9849339744143241110675070165578637308,
361.5258025606521562127011717095711058298,
401.8817390422892842860436106998411617564,
389.5900151584191309686879416616660070194,
328.4693989341115139861349461880665113109,
401.5075715797546040722312989184987720636,
358.9736282393364418215800186769895592813,
398.3314710401550758187279275697987053487,
371.4838739439911505852417761217748260001,
336.6121584119430243071399604087330753567,
361.5088834710173833216556864971373143920,
324.6714499259511680272209499829787353315,
302.3138431481088027853905435155134700278,
328.4693851348376738543560007875426089080,
343.8134062489636023071554278877490132389,
375.7328529004344779578350025309273085613,
328.1170929432668286806306184053124031556,
292.9996913829451805564022390168434846992,
358.6434156077656382878937733554637132436,
299.8986620478842554262303375396988843341,
360.0617346672743554174683847821572817862,
336.5944103223615119175328246409633362663,
256.1075318604639744446008370340789799918,
324.6552122357256744063911827393430633008,
331.9380679142421211652973746489835475410,
304.7995832546308734600205959223217821289,
323.4616917661055259368992235644548449218,
289.5459577248197651814802868605931309724]

Cascade time 250.544
```

[462.16343493635024777767212793247848746 84, **(68)**
441.6429597316308683463600766538668411865,
436.9174816528198305926185394294222433484,
422.9849339744143241110675070165578637308,
361.5258025606521562127011717095711058298,
401.8817390422892842860436106998411617564,
389.5900151584191309686879416616660070194,
328.4693989341115139861349461880665113109,
401.5075715797546040722312989184987720636,
358.9736282393364418215800186769895592813,
398.3314710401550758187279275697987053487,
371.4838739439911505852417761217748260001,
336.6121584119430243071399604087330753567,
361.5088834710173833216556864971373143920,
324.6714499259511680272209499829787353315,

302.31384314810880278539054351551347002 78,
328.46938513483767385435600078754260890 80,
343.81340624896360230715542788774901323 89,
375.73285290043447795783500253092730856 13,
328.11709294326682868063061840531240315 56,
292.99969138294518055640223901684348469 92,
358.64341560776563828789377335546371324 36,
299.89866204788425542623033753969888433 41,
360.06173466727435541746838478215728178 62,
336.59441032236151191753282464096333626 63,
256.10753186046397444460083703407897999 18,
324.65521223572567440639118273934306330 08,
331.93806791424212116529737464898354754 10,
304.79958325463087346002059592232178212 89,
323.46169176610552593689922356445484492 18,
289.54595772481976518148028686059313097 24]

```
> HintsOriginal:=ListHints;
```

$$HintsOriginal := \Big[none, \Big[27.52359684479996584152978595423608112768, 2, -1, -1, 6.49211140307355546658251270388993937859 4, 25.56992694..27.67578046, \frac{3}{2}..12., 1\Big], \Big[35.46322962825379040215121974254598304878, 3, 1, 1, 9.62002610443990467537032288025740016973 2, 33.94922194..36.10248389, \frac{3}{2}..12., 1\Big], \Big[14.19258941788061374381621459708876997324, 2, 1, 1, 14.13900773377946677696818354340356081958, 12.92327158..18.68550893, \frac{3}{2}..19, 1\Big], \Big[26.46347110539976775688849031731477848509, 3, -1, 1, 16.53292398122155720971187607751147172520, 24.64256576..27.23722351, \frac{3}{2}..28, 1\Big], \Big[34.94507888797729914637569990804457214741, 3, 1, 1, 10.93651821651644642196909246079097015868, 33.70078237..35.85152418, \frac{3}{2} \tag{69}$$

..27.52359685, 1], [15.91193136510569595137770698583432131326, 2, −1, 1,

15.84478473618636498341215543281814303559, 14.35659706 ..18.96093397, $\frac{3}{2}$ ..19, 1],

[25.87205017540310520941769595514298310090, 3, −1, 1,

16.77671839080472370324390883410274606898, 23.84730094 ..26.76330661, $\frac{3}{2}$ ..28, 1],

[34.93953234341508060366636866791499227215, 3, 1, 1,

13.44293633391471214310583527459503641144, 33.37332721 ..35.50872230, $\frac{3}{2}$

..14.19258939, 1], [17.19898874735628255923651729245938270093, 2, −1, 1,

16.75492183170567124243303155678476101688, 16.08011004 ..19, $\frac{3}{2}$ ..19, 1],

[27.02037941872078312796022169828016252717, 2, −1, −1,

13.57592144649376192738249951229692762748, 24.71083344 ..27.27388429, $\frac{3}{2}$

..14.19258939, 1], [34.49522661165008082072914924801977531221, 3, 1, 1,

15.76385589023872534302387520454213741716, 32.91337941 ..35.00011460, $\frac{3}{2}$

..15.91193137, 1], [17.93041369709740979309933253273271318826, 3, 1, 1,

15.70097251556007075830079142443830067567, 16.87563409 ..19, $\frac{3}{2}$ ..19, 1],

[26.46318954458146650656332304152399577898, 3, −1, 1,

15.90128515414341158765710109686370821521, 23.93303356 ..26.81849303, $\frac{3}{2}$

..15.91193137, 1], [33.81362495408772233776384102941444868670, 3, 1, 1,

11.78302906874267690462414906858869172132, 32.62689490 ..34.66372796, $\frac{3}{2}$

..26.46347110, 1], [18.68778086095717851637489883728398993543, 3, 1, 1,

15.36476719175581697177868887474571118396, 17.98135512 ..19, $\frac{3}{2}$ ..19, 1],

$$\left[25.87204991206833952374472020470551194263, 3, -1, 1, 16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245, \frac{3}{2} .. 17.19898872, 1\right], \left[34.08976799590707967844417250400164426152, 3, 1, 1, 17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387, \frac{3}{2} .. 17.19898872, 1\right], \left[16.53336698818830587377417370329609603012, 3, 1, 1, 15.69073378973079182825919789070949638294, 15.22886699 .. 19, \frac{3}{2} .. 19, 1\right],$$

$$\left[25.86532280054226848442580721202864937576, 3, -1, 1, 16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170, \frac{3}{2} .. 28, 1\right],$$

$$\left[33.36855355002008388886066412789339491074, 3, 1, 1, 12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642, \frac{3}{2} .. 25.87205019, 1\right], \left[17.21111401061511938569208469606230143811, 2, -1, 1, 16.76151108845582449866266423656100124906, 16.09683967 .. 19, \frac{3}{2} .. 19, 1\right],$$

$$\left[25.30046634035072136692300883420836359084, 3, -1, 1, 16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834, \frac{3}{2} .. 28, 1\right],$$

$$\left[34.32715979893322135156607292800685921891, 3, 1, 1, 11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310, \frac{3}{2} .. 27.02037943, 1\right], \left[17.93092083043590970169440081900921633181, 3, 1, 1, 15.70086062701339751125315740515964488012, 16.87629601 .. 19, \frac{3}{2} .. 19, 1\right],$$

$$\left[24.33949903025381421746107766068115533423, 3, -1, 1, 17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164, \frac{3}{2} .. 28, 1\right],$$

$$\left[33.81339267510015714746901816118211936913, 3, 1, 1, 11.78320912141352187071943817447189965627, 32.62668594..34.66347615, \frac{3}{2}..26.46318954, 1\right], \left[18.05989025945530489597350866091479114776, 2, -1, 1, 17.06840476846513502865031607995206863120, 17.29769086..19, \frac{3}{2}..19, 1\right], \left[25.40207153781880397105361734397047382754, 3, -1, 1, 17.00624445476621345302062451224674357053, 22.67806074..25.90675353, \frac{3}{2}..17.93041370, 1\right], \left[33.79632952269190120147214620388869542508, 2, 1, -1, 17.86353406971662964004465842689508192962, 32.25770943..34.20127520, \frac{3}{2}..17.93041370, 1\right], \left[18.85462970515005395630149798358490356795, 2, -1, 1, 16.56670584472960259009028113530215180539, 18.55227050..19, \frac{3}{2}..19, 1\right]\right]$$

```
> ListP;
```

[[12., 6.217012502940637829614151732937593976434, **(70)**
485.54908089824923085344960939651911736 73],
[27.52359684479996584152978595423608112768,
6.583434721648324074592799441043164221029,
467.78730595863832684476535436144927539 44],
[35.46322962825379040215121974254598304878,
4.125651796843314518538528143881466720906,
440.67123065048064044274396492648916582 05],
[14.19258941788061374381621459708876997324,
5.589637182936981973644279191300877405077,
443.83065884514772992529555109347267711 90],
[26.46347110539976775688849031731477848509,
6.196262565379693702162753490756502697142,
385.44474379327614418129435545739274961 90],
[34.94507888797729914637569990804457214741,
4.004869081819444882748763801251484513795,
404.86224501294950265590545171823193668 55],
[15.91193136510569595137770698583432131326,

5.18778357849831716559192757496465974486 9,
408.65773862723309892047141473126067787 13],
[25.87205017540310520941769595514298310090,
6.02581354933458523915820882839880814317 8,
351.42702948382214417571102913825521608 80],
[34.93953234341508060366636866791499227215,
4.00355981551922975199358178639691699149 2,
404.47973593992877766487202366228903485 84],
[17.19898874735628255923651729245938270093,
4.88381077982614553797184240182126752788 0,
376.61967855814619245340268918025697572 30],
[27.02037941872078312796022169828016252717,
6.37794387392004353897835336070878132461 7,
423.28832783898402813927173518272824197 56],
[34.49522661165008082072914924801977531221,
3.89713131593882642580951117569660027753 2,
373.78081884469355781443393947213302772 14],
[17.93041369709740979309933253273271318826,
4.68650870196228155456139423149427741670 9,
353.30541094994294622604476805952759249 50],
[26.46318954458146650656332304152399577898,
6.19617723023314440382926937080140189027 7,
385.42734025701315342674194950890640690 63],
[33.81362495408772233776384102941444868670,
3.72564899357067124989860181185201629150 5,
325.89209972827061292564035729822378476 82],
[18.68778086095717851637489883728398993543,
4.40864148276157707776594929632036833908 0,
317.63557439485037724263779508833691438 14],
[25.87204991206833952374472020470551194263,
6.02581347651729875413025357671595351136 3,
351.42701527668050381316113984264771801 51],
[34.08976799590707967844417250400164426152,
3.79659778949514599088697744865163398442 6,
345.47764844562532655931076908347914151 00],
[16.53336698818830587377417370329609603012,
5.04430217537899735092417810862748884883 3,
394.13717098830705173831730923850842513 87],

[25.865322800542268484425807212028649375 76,
6.023953985034046309359286105945235816303,
351.06430842091588253993965048696547755 29],
[33.36855355002008388886066412789339491074,
3.605714291664187211205152623205956224823,
293.47578374403935356430669597956582151 48],
[17.21111401061511938569208469606230143811,
4.880770162430402804896439498824244106173,
376.27494600439025577751132560418837831 85],
[25.30046634035072136692300883420836359084,
5.871694125437934501377864004692606183395,
321.99875338021932243729632790626589110 81],
[34.32715979893322135156607292800685921891,
3.855919917066455358912650990780342369879,
362.09872330873138873763289207702190937 74],
[17.93092083043590970169440081900921633181,
4.686358235794481796699313611428681982364,
353.28692860609651601176407989539002117 03],
[24.33949903025381421746107766068115533423,
5.620659139855686637113928447884253896698,
276.83661544729191672463410572896000399 82],
[33.81339267510015714746901816118211936913,
3.725588318700409459620718323791469530503,
325.87548369616797979489705282067843963 39],
[18.05989025945530489597350866091479114776,
4.647168293403685053277962322736940870496,
348.43898873486076158728936046911821970 30],
[25.40207153781880397105361734397047382754,
5.898629068111734442957332953721492409373,
327.04870231913806414377861911167973240 59],
[33.79632952269190120147214620388869542508,
3.721126245255904407032534512376987606735,
324.65414440386534784777863954837360091 0],
[18.85462970515005395630149798358490356795,
4.309926953172614028154821843449158086881,
304.37474577695551705607179409202295591 22]]

```
> Tau31Original:=ListTau;
```

Tau31Original := [462.16343493635024777767212793247848746 84, **(71)**

```
441.6429597316308683463600766538668411865,
436.9174816528198305926185394294222433484,
422.9849339744143241110675070165578637308,
361.5258025606521562127011717095711058298,
401.8817390422892842860436106998411617564,
389.5900151584191309686879416616660070194,
328.4693989341115139861349461880665113109,
401.5075715797546040722312989184987720636,
358.9736282393364418215800186769895592813,
398.3314710401550758187279275697987053487,
371.4838739439911505852417761217748260001,
336.6121584119430243071399604087330753567,
361.5088834710173833216556864971373143920,
324.6714499259511680272209499829787353315,
302.3138431481088027853905435155134700278,
328.4693851348376738543560007875426089080,
343.8134062489636023071554278877490132389,
375.7328529004344779578350025309273085613,
328.1170929432668286806306184053124031556,
292.9996913829451805564022390168434846992,
358.6434156077656382878937733554637132436,
299.8986620478842554262303375396988843341,
360.0617346672743554174683847821572817862,
336.5944103223615119175328246409633362663,
256.1075318604639744446008370340789799918,
324.6552122357256744063911827393430633008,
331.9380679142421211652973746489835475410,
304.7995832546308734600205959223217821289,
323.4616917661055259368992235644548449218,
289.5459577248197651814802868605931309724]
```

```
> PurifyTau := proc(Tau)
    local T, n, Tr:
    T:= Tau[1..3]:
    Tr:=CreateList(31, 0):

    for n from 0 to nops(Tau)-1 do
      Tr[n+1] := -Tau[n+1] +T[1+ (n mod 3)]:
    end do:
    Tr[4..MaxSignals];
  end:
```

```
> ObsSignals:=PurifyTau(Tau31Original);
```

$$ObsSignals := [39.17850096193592366660462091592062373 76, 80.11715717097871213365890494429573535 67, 35.03574261053054630657492872958108159 20, 72.57341977793111680898418627081248044 90, 113.17356079751935436022513046580032987 56, 35.40991007306522652038724051092347128 48, 103.18980669701380595609210925548892818 71, 43.31148869147579252763214908406813583 78, 65.43360770882868000737676330764741734 83, 125.55127652440722347053216752374541211 17, 80.13407626061348502470439015672952679 45, 112.24603172686866256539758944644350801 69, 159.84959178824144499228158441696501744 06, 113.17357459679319449200407586632423227 85, 93.10407540385622828546311154167323010 95, 86.43058203591576981983712540155117890 71, 113.52586678836403966572945824855443803 09, 143.91779026987465003621630041257875864 92, 103.52001932858460948977835457701477422 48, 141.74429768374661292012973911416795685 24, 76.85574698554547517515015464726496156 22, 125.56902461398873586013930329151515120 21, 185.53542787116689390175923961978786119 47, 112.26226941709415618622735669007918004 76, 130.22536702210812661237475328349493992 74, 136.84337647699999488633948073154505905 76, 113.45578988671430465571931586496739842 66, 172.61747721153048259619184107188535649 60] \quad (72)$$

```
> [xS(0, 1),yS(0, 1), tS(0, 1) ]:
  subs(r=BaseR, %):
  evalf(%):
  P0:=%;
```

$$P0 := [11.97373654559564735717785284361202977543, -0.7934942574884924253839496029588052713389, 485.54908089824923085344960939651911736 73] \quad (73)$$

```
> Q:= proc(x)
   round(x*10^36)/10^36:
  end:
```

```
> Rumor:= proc(x, si, n:= 1)
    if type(x, list) then
      return map(Rumor, x, si, n):
    else
      x+Sample(Normal(0,si), n)[1]:
      Q(%):
    end if:
  end:
```

```
> BaseROriginal;
  Sat0;
  Sat1;
  Sat2;
```

$$12$$

$$\left[\frac{\pi}{6}, 0, 1, 10, 19\right]$$

$$\left[\frac{7\pi}{6}, 10, -1, 20, 28\right]$$

$$\left[\frac{3\pi}{4}, -10, -3, 31, 40\right] \tag{74}$$

```
> Rumor(BaseROriginal, 10^(-10)):
  BaseR:=%;
```

$$BaseR := \frac{59999999998929471683147269769377237 63}{50000000000000000000000000000000000000} \tag{75}$$

```
> Rumor(Sat0, 10^(-10)):
  te0in, t0in, tau0in, rm0v, rp0v := op(%);
  ComputeSat0():
```

$$te0in, t0in, tau0in, rm0v, rp0v := \frac{52359877556869850493715832194743009}{10000000000000000000000000000000000000}, -\frac{96106200043967384190539141}{10000000000000000000000000000000000000}, \frac{25000000004066305743801690193226 2037}{25000000000000000000000000000000000000}, \frac{31249999999960686710958406604202 5197}{31250000000000000000000000000000000000}, \frac{94999999998859855305038288321420 39963}{50000000000000000000000000000000000000} \tag{76}$$

```
> Rumor(Sat1, 10^(-10)):
  te1in, t1in, tau1in, rm1v, rp1v := op(%);
  ComputeSat1():
```

$$te1in, t1in, tau1in, rm1v, rp1v := \frac{36651914289562270630175460709594656 37}{10000000000000000000000000000000000000}, \frac{50000000000655697046939727445879755 63}{50000000000000000000000000000000000000}, -\frac{10000000000071559426585504572854357411}{10000000000000000000000000000000000000}, \frac{20000000000111737560336649912355606777}{10000000000000000000000000000000000000}, \frac{27999999999874491795305514964749103397}{10000000000000000000000000000000000000} \quad (77)$$

```
> Rumor(Sat2, 10^(-10)):
  te2in, t2in, tau2in, rm2v, rp2v := op(%);
  ComputeSat2():
```

$$te2in, t2in, tau2in, rm2v, rp2v := \frac{23561944902271805124309440543464548 81}{10000000000000000000000000000000000000}, -\frac{99999999998561459769079485548459792 67}{10000000000000000000000000000000000000}, -\frac{30000000000601761596233653939950757 73}{10000000000000000000000000000000000000}, \frac{15499999999920742892181458335538303547}{50000000000000000000000000000000000000}, \frac{39999999999946202172604456699538570029}{10000000000000000000000000000000000000} \quad (78)$$

```
> MaxT:=250;
```

$$MaxT := 250 \quad (79)$$

```
> [xS(0, -1), yS(0, -1),tS(0, -1), r=minS(0)..maxS(0)-0.00001]:
  evalf(%):
  c0_1n := spacecurve(%, axes=box, color = red):
> [xS(0, 0), yS(0, 0),tS(0, 0), r=minS(0)..maxS(0)-0.00001]:
  evalf(%):
  c0_0 := spacecurve(%, axes=box, color = red, thickness = 4):
> [xS(0, 1), yS(0, 1),tS(0, 1), r=minS(0)..maxS(0)-0.00001]:
  evalf(%):
  c0_1 := spacecurve(%, axes=box, color = red):
> [xS(1, -1), yS(1, -1),tS(1, -1), r=rm1v..maxS(1)]:
  evalf(%):
  c1_1n := spacecurve(%, axes=box, color = blue):
```

```
> [xS(1, 0), yS(1, 0),tS(1, 0), r=minS(1)..maxS(1)]:
  evalf(%):
  c1_0 := spacecurve(%, axes=box, color = blue, thickness = 4):
```

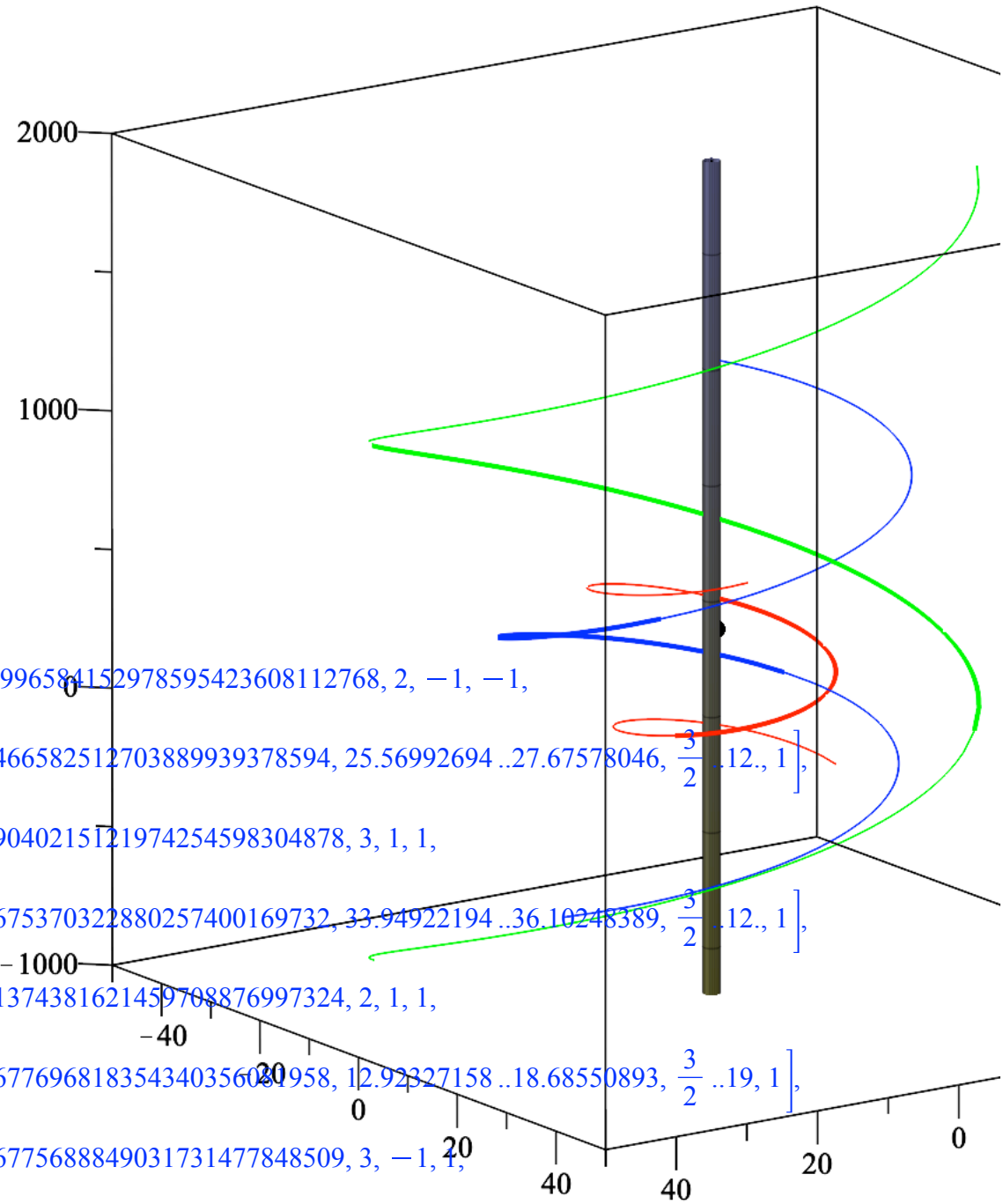
```
> [xS(1, 1), yS(1, 1),tS(1, 1), r=minS(1)..maxS(1)]:
  evalf(%):
  c1_1 := spacecurve(%, axes=box, color = blue):
```

```
> [xS(2, -1), yS(2, -1),tS(2, -1), r=minS(2)..maxS(2)]:
  evalf(%):
  c2_1n := spacecurve(%, axes=box, color = green):
```

```
> [xS(2, 0), yS(2, 0),tS(2, 0), r=minS(2)..maxS(2)]:
  evalf(%):
  c2_0 := spacecurve(%, axes=box, color = green, thickness = 4):
```

```
> [xS(2, 1), yS(2, 1),tS(2, 1), r=minS(2)..maxS(2)]:
  evalf(%):
  c2_1 := spacecurve(%, axes=box, color = green):
```

```
> display(BH, S,
  user,
  c0_1n,c0_0,c0_1,
  c1_1n,c1_0,c1_1,
  c2_1n,c2_0,c2_1,
  view=[ -50..50, -50..50, -1000..2000]);
```




```
> HintsOriginal;
```

$$\Big[\mathit{none}, \Big[27.52359684479996584152978595423608112768, 2, -1, -1,$$

$$6.49211140307355546658251270388993937859 4, 25.56992694 ..27.67578046, \frac{3}{2} ..12., 1\Big],$$

$$\Big[35.46322962825379040215121974254598304878, 3, 1, 1,$$

$$9.62002610443990467537032288025740016973 2, 33.94922194 ..36.10248389, \frac{3}{2} ..12., 1\Big],$$

$$\Big[14.19258941788061374381621459708876997324, 2, 1, 1,$$

$$14.13900773377946677696818354340356081958, 12.92327158 ..18.68550893, \frac{3}{2} ..19, 1\Big],$$

$$\Big[26.46347110539976775688849031731477848509, 3, -1, 1,$$

$$16.53292398122155720971187607751147172520, 24.64256576 ..27.23722351, \frac{3}{2} ..28, 1\Big],$$

$$\Big[34.94507888797729914637569990804457214741, 3, 1, 1,$$

10.93651821651644642196909246079097015868, 33.70078237 ..35.85152418, $\frac{3}{2}$ ..27.52359685, 1], [15.91193136510569595137770698583432131326, 2, −1, 1, 15.84478473618636498341215543281814303559, 14.35659706 ..18.96093397, $\frac{3}{2}$ ..19, 1], [25.87205017540310520941769595514298310090, 3, −1, 1, 16.77671839080472370324390883410274606898, 23.84730094 ..26.76330661, $\frac{3}{2}$ ..28, 1], [34.93953234341508060366636866791499227215, 3, 1, 1, 13.44293633391471214310583527459503641144, 33.37332721 ..35.50872230, $\frac{3}{2}$ ..14.19258939, 1], [17.19898874735628255923651729245938270093, 2, −1, 1, 16.75492183170567124243303155678476101688, 16.08011004 ..19, $\frac{3}{2}$ ..19, 1], [27.02037941872078312796022169828016252717, 2, −1, −1, 13.57592144649376192738249951229692762748, 24.71083344 ..27.27388429, $\frac{3}{2}$ ..14.19258939, 1], [34.49522661165008082072914924801977531221, 3, 1, 1, 15.76385589023872534302387520454213741716, 32.91337941 ..35.00011460, $\frac{3}{2}$ ..15.91193137, 1], [17.93041369709740979309933253273271318826, 3, 1, 1, 15.70097251556007075830079142443830067567, 16.87563409 ..19, $\frac{3}{2}$ ..19, 1], [26.46318954458146650656332304152399577898, 3, −1, 1, 15.90128515414341158765710109686370821521, 23.93303356 ..26.81849303, $\frac{3}{2}$ ..15.91193137, 1], [33.81362495408772233776384102941444868670, 3, 1, 1, 11.78302906874267690462414906858869172132, 32.62689490 ..34.66372796, $\frac{3}{2}$ ..26.46347110, 1], [18.68778086095717851637489883728398993543, 3, 1, 1,

15.364767191755816971778688874745711183 96, 17.98135512 ..19, $\frac{3}{2}$ ..19, 1],

[25.872049912068339523744720204705511942 63, 3, −1, 1,

16.761063912186698964426645072343113601 29, 23.20517308 ..26.31784245, $\frac{3}{2}$

..17.19898872, 1], [34.089767995907079678444172504001644261 52, 3, 1, 1,

17.198984348938819986702586587288643071 01, 32.52213872 ..34.53618387, $\frac{3}{2}$

..17.19898872, 1], [16.533366988188305873774173703296096030 12, 3, 1, 1,

15.690733789730791828259197890709496382 94, 15.22886699 ..19, $\frac{3}{2}$ ..19, 1],

[25.865322800542268484425807212028649375 76, 3, −1, 1,

16.779215258374334038325539120894911366 80, 23.83864811 ..26.75768170, $\frac{3}{2}$ ..28, 1],

[33.368553550020083888860664127893394910 74, 3, 1, 1,

12.142802994561336759524256526823424721 44, 32.23723258 ..34.17446642, $\frac{3}{2}$

..25.87205019, 1], [17.211114010615119385692084696062301438 11, 2, −1, 1,

16.761511088455824498662664236561001249 06, 16.09683967 ..19, $\frac{3}{2}$ ..19, 1],

[25.300466340350721366923008834208363590 84, 3, −1, 1,

16.974729527641787545085746138369074748 18, 23.14060343 ..26.26979834, $\frac{3}{2}$ ..28, 1],

[34.327159798933221351566072928006859218 91, 3, 1, 1,

11.395769599984320296717245861504673207 76, 33.10127385 ..35.21212310, $\frac{3}{2}$

..27.02037943, 1], [17.930920830435909701694400819009216331 81, 3, 1, 1,

15.700860627013397511253157405159644880 12, 16.87629601 ..19, $\frac{3}{2}$ ..19, 1],

[24.339499030253814217461077660681155334 23, 3, −1, 1,

$17.27215878251294421226707494428435581574, 22.07732228..25.37892164, \frac{3}{2}..28, 1\Big],$

$\Big[33.81339267510015714746901816118211936913, 3, 1, 1,$

$11.78320912141352187071943817447189965627, 32.62668594..34.66347615, \frac{3}{2}$

$..26.46318954, 1\Big], \Big[18.05989025945530489597350866091479114776, 2, -1, 1,$

$17.06840476846513502865031607995206863120, 17.29769086..19, \frac{3}{2}..19, 1\Big],$

$\Big[25.40207153781880397105361734397047382754, 3, -1, 1,$

$17.00624445476621345302062451224674357053, 22.67806074..25.90675353, \frac{3}{2}$

$..17.93041370, 1\Big], \Big[33.79632952269190120147214620388869542508, 2, 1, -1,$

$17.86353406971662964004465842689508192962, 32.25770943..34.20127520, \frac{3}{2}$

$..17.93041370, 1\Big], \Big[18.85462970515005395630149798358490356795, 2, -1, 1,$

$16.56670584472960259009028113530215180539, 18.55227050..19, \frac{3}{2}..19, 1\Big]\Big]$

```
> ListHint:= HintsOriginal;
  ListTau:=CreateList(31, 0);
  ListP:=CreateList(31, []);
```

$ListHint := \Big[none, \Big[27.52359684479996584152978595423608112768, 2, -1, -1,$

$6.49211140307355546658251270388993937859 4, 25.56992694..27.67578046, \frac{3}{2}..12., 1\Big],$

$\Big[35.46322962825379040215121974254598304878, 3, 1, 1,$

$9.62002610443990467537032288025740016973 2, 33.94922194..36.10248389, \frac{3}{2}..12., 1\Big],$

$\Big[14.19258941788061374381621459708876997324, 2, 1, 1,$

$14.13900773377946677696818354340356081958, 12.92327158..18.68550893, \frac{3}{2}..19, 1\Big],$

$\Big[26.46347110539976775688849031731477848509, 3, -1, 1,$

16.53292398122155720971187607751147172520, 24.64256576 ..27.23722351, $\frac{3}{2}$ ..28, 1],

[34.94507888797729914637569990804457214741, 3, 1, 1,

10.93651821651644642196909246079097015868, 33.70078237 ..35.85152418, $\frac{3}{2}$

..27.52359685, 1], [15.91193136510569595137770698583432131326, 2, −1, 1,

15.84478473618636498341215543281814303559, 14.35659706 ..18.96093397, $\frac{3}{2}$ ..19, 1],

[25.87205017540310520941769595514298310090, 3, −1, 1,

16.77671839080472370324390883410274606898, 23.84730094 ..26.76330661, $\frac{3}{2}$ ..28, 1],

[34.93953234341508060366636866791499227215, 3, 1, 1,

13.44293633391471214310583527459503641144, 33.37332721 ..35.50872230, $\frac{3}{2}$

..14.19258939, 1], [17.19898874735628255923651729245938270093, 2, −1, 1,

16.75492183170567124243303155678476101688, 16.08011004 ..19, $\frac{3}{2}$ ..19, 1],

[27.02037941872078312796022169828016252717, 2, −1, −1,

13.57592144649376192738249951229692762748, 24.71083344 ..27.27388429, $\frac{3}{2}$

..14.19258939, 1], [34.49522661165008082072914924801977531221, 3, 1, 1,

15.76385589023872534302387520454213741716, 32.91337941 ..35.00011460, $\frac{3}{2}$

..15.91193137, 1], [17.93041369709740979309933253273271318826, 3, 1, 1,

15.70097251556007075830079142443830067567, 16.87563409 ..19, $\frac{3}{2}$ ..19, 1],

[26.46318954458146650656332304152399577898, 3, −1, 1,

15.90128515414341158765710109686370821521, 23.93303356 ..26.81849303, $\frac{3}{2}$

..15.91193137, 1], [33.81362495408772233776384102941444868670, 3, 1, 1,

11.78302906874267690462414906858869172132, 32.62689490 ..34.66372796, $\frac{3}{2}$ ..26.46347110, 1], [18.68778086095717851637489883728398993543, 3, 1, 1, 15.36476719175581697177868887474571118396, 17.98135512 ..19, $\frac{3}{2}$ ..19, 1], [25.87204991206833952374472020470551194263, 3, −1, 1, 16.76106391218669896442664507234311360129, 23.20517308 ..26.31784245, $\frac{3}{2}$ ..17.19898872, 1], [34.08976799590707967844417250400164426152, 3, 1, 1, 17.19898434893881998670258658728864307101, 32.52213872 ..34.53618387, $\frac{3}{2}$ ..17.19898872, 1], [16.53336698818830587377417370329609603012, 3, 1, 1, 15.69073378973079182825919789070949638294, 15.22886699 ..19, $\frac{3}{2}$ ..19, 1], [25.86532280054226848442580721202864937576, 3, −1, 1, 16.77921525837433403832553912089491136680, 23.83864811 ..26.75768170, $\frac{3}{2}$ ..28, 1], [33.36855355002008388886066412789339491074, 3, 1, 1, 12.14280299456133675952425652682342472144, 32.23723258 ..34.17446642, $\frac{3}{2}$ ..25.87205019, 1], [17.21111401061511938569208469606230143811, 2, −1, 1, 16.76151108845582449866266423656100124906, 16.09683967 ..19, $\frac{3}{2}$ ..19, 1], [25.30046634035072136692300883420836359084, 3, −1, 1, 16.97472952764178754508574613836907474818, 23.14060343 ..26.26979834, $\frac{3}{2}$ ..28, 1], [34.32715979893322135156607292800685921891, 3, 1, 1, 11.39576959998432029671724586150467320776, 33.10127385 ..35.21212310, $\frac{3}{2}$ ..27.02037943, 1], [17.93092083043590970169440081900921633181, 3, 1, 1,

$15.7008606270133975112531574051596448801 2, 16.87629601 ..19, \frac{3}{2} ..19, 1\Big],$

$\Big[24.33949903025381421746107766068115533423, 3, -1, 1,$

$17.27215878251294421226707494428435581574, 22.07732228 ..25.37892164, \frac{3}{2} ..28, 1\Big],$

$\Big[33.81339267510015714746901816118211936913, 3, 1, 1,$

$11.78320912141352187071943817447189965627, 32.62668594 ..34.66347615, \frac{3}{2}$

$..26.46318954, 1\Big], \Big[18.05989025945530489597350866091479114776, 2, -1, 1,$

$17.06840476846513502865031607995206863120, 17.29769086 ..19, \frac{3}{2} ..19, 1\Big],$

$\Big[25.40207153781880397105361734397047382754, 3, -1, 1,$

$17.00624445476621345302062451224674357053, 22.67806074 ..25.90675353, \frac{3}{2}$

$..17.93041370, 1\Big], \Big[33.79632952269190120147214620388869542508, 2, 1, -1,$

$17.86353406971662964004465842689508192962, 32.25770943 ..34.20127520, \frac{3}{2}$

$..17.93041370, 1\Big], \Big[18.85462970515005395630149798358490356795, 2, -1, 1,$

$16.56670584472960259009028113530215180539, 18.55227050 ..19, \frac{3}{2} ..19, 1\Big]\Big]$

$ListTau := [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$

$ListP := [[\ ], [\ ]]$ **(81)**

```
> Cascade(BaseSat, BaseR, BaseBranch);
Start Generation 1
1 --> 0  target = [11.999999999785894336629453953875447526 00,
6.217012502975201913232125596883070428812,
485.54908089534509795809793505892514721 09]
one interval r = 23.40850301646789622256836121288019819880 ..
27.67578046414585476799422831967159752594
Time Approximations 0.036.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |
 P  <--- S
```

```
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 .
. 27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.29e-37
Equations at solution: [.4e-37, -.129e-36, -.13e-35]Solution in
1.199s

Time Plot 3.098 s.
Exiting SolveHard() after 5.937r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on
the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none]

2 --> 0  target = [11.999999999785894336629453953875447526 00,
6.217012502975201913232125596883070428812,
485.549080895345097958097935058925147210 9]
```

"Imaginary part neglected: ", $3.183223432214289731701923299933885596261 \times 10^{-17}$

```
one interval r = 32.628147791976204690026185311686473668 58 ..
36.102483889312443663586888136983717543 3
Time Approximations 0.022.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1,
1, 9.620026104439904675370322880257400169732, 33.94922194 ..
36.10248389, 3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 .
. 36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=1e-38
Equations at solution: [-.3e-37, .1e-37, 0.]Solution in 0.921s

Time Plot 3.234 s.
Exiting SolveHard() after 4.599r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on
the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
```

```
441.642959730441394607652802861179832682 6,
436.91748164998936082460987534285039303 09, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none]

Start Generation 2
2 --> 1  target = [27.523596844675898267292399994740153618 60,
6.583434721407766314282775060189752864155,
467.787305957607716565854917845985268848 4]
```

"Imaginary part neglected: ", $3.18322343221428973170192329993388559626 1 \times 10^{-17}$

```
one interval r = 32.419789556487257881889572178879396489 78 ..
35.851524173629117792190309955712057724 37
Time Approximations 0.02.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1,
1, 10.936518216516446421969092460790970158 68, 33.70078237 ..
35.85152418, 3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 .
. 35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, .187e-34]Solution in 0.97s

Time Plot 3.629 s.
Exiting SolveHard() after 4.964r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434933564304349033122406919760 7582,
441.642959730441394607652802861179832682 6,
436.91748164998936082460987534285039303 09, none, none,
401.881739044491732839196604202029091112 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844675898267292399994740153618 60,
6.583434721407766314282775060189752864155,
467.787305957607716565854917845985268848 4]
two intervals r = 12.923271607987371646963385634515167896 20 ..
9499999999885985530503828832142039963/50000000000000000000000000000
000000 or r = 18.394248580115380158183016013298083237 96 ..
9499999999885985530503828832142039963/50000000000000000000000000000
000000
Time Approximations 0.035.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1,
```

```
1, 14.13900773377946677696818354340356081958, 12.92327158 ..
18.68550893, 3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768)
|   S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.3e-38
Equations at solution: [-.2e-37, -.23e-37, .1187e-34]Solution in
30.24s

Time Plot 1.361 s.
Exiting SolveHard() after 33.404r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64, none,
401.88173904449173283919660420202909111 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962814319233581458741166882929558,
4.1256517968795066157644773589719011067 33,
440.67123064783892537974503535275385080 88]
two intervals r = 14.35659705092429772590778256203489820717 ..
9499999999885985530503828832142039963/50000000000000000000000000000
000000 or r = 17.70352613785965374489859532070377096395 ..
9499999999885985530503828832142039963/50000000000000000000000000000
000000
Time Approximations 0.052.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1,
1, 15.84478473618636498341215543281814303559, 14.35659706 ..
18.96093397, 3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126)
|   S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
 scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 .
. 18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [.13e-37, 0., -.3262e-34]Solution in 1.934s

Time Plot 1.299 s.
Exiting SolveHard() after 4.99r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64, none,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962814319233581458741166882929558,
4.1256517968795066157644773589719011067 33,
440.67123064783892537974503535275385080 88]
one interval r = 22.397611543616873247145176379430416959 58 ..
27.237223515844153548743217987502775738 77
Time Approximations 0.034.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1,
1, 16.532923981221557209711876077511471725 20, 24.64256576 ..
27.23722351, 3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
 scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 .
. 27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.731 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 .
. 27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643669000374546312892083907174 26, rm =
14.378187701326217733672167598333942304 35}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.31e-37
Equations at solution: [-.1e-37, -.131e-36, -.133e-34]Solution in
6.891s

Time Plot 2.501 s.
Exiting SolveHard() after 10.489r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887932896656025754890616816122 45,
4.004869081872656713173168016753856323955,
404.86224501545022370539205059267949965 52]
two intervals r = 16.08011007712401315395938046676604706063 ..
94999999998859855305038288321420399 63/500000000000000000000000000000
000000 or r = 16.41579812691202127468341845560160515269 ..
94999999998859855305038288321420399 63/500000000000000000000000000000
000000
Time Approximations 0.393.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1,
1, 16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2
.. 19, 1]
I search for an scattering ray on same branch with 0<sv<1
(0.0257633) |   S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19,
rm = 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.34e-37, .1e-37, .1832e-34]Solution in 2.4s

Time Plot 1.386 s.
Exiting SolveHard() after 5.547r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90, none, none,
358.97362824196080839567747612622097859 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none]

1 --> 2  target = [34.945078887932896656025754890616816122 45,
4.004869081872656713173168016753856323955,
404.86224501545022370539205059267949965 52]
one interval r = 21.64194399419425953238860530773691623201 ..
26.76330660035322306452723398603686583020
Time Approximations 0.051.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1,
1, 16.77671839080472370324390883410274606898, 23.84730094 ..
26.76330661, 3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
 scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 .
. 26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, .111e-34]Solution in
1.722s

Time Plot 2.396 s.
Exiting SolveHard() after 5.472r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52, none,
358.97362824196080839567747612622097859 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none]

1 --> 0  target = [14.19258941726625901580359918957455805651,
5.5896371830402204853391715391610661825 83,
443.83065884921309069076867916489068776 82]
one interval r = 22.46725374485973070637266329858101661208 ..
27.27388428347568864510213332935864647401
Time Approximations 0.034.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |
 P  <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
 scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 .
. 27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., -.18e-35]Solution in 1.258s

Time Plot 1.723 s.
Exiting SolveHard() after 4.198r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and
source on the same branch.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52, none,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

2 --> 0  target = [14.19258941726625901580359918957455805651,
5.5896371830402204853391715391610661825 83,
443.83065884921309069076867916489068776 82]
```

"Imaginary part neglected: ", $3.18322343221428973170192329993388559626 1 \times 10^{-17}$

```
one interval r = 32.15575279491742943564102576018017458757 ..
35.50872228735087961455618256253136026903
Time Approximations 0.041.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1,
1, 13.44293633391471214310583527459503641144, 33.37332721 ..
35.50872230, 3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 .
. 35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, -.86e-35]Solution in 0.782s

Time Plot 2.429 s.
Exiting SolveHard() after 3.609r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
```

```
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [15.91193136481711020848428360539751124015,
5.18778357850803989052198061095447170894 8,
408.65773862509824021973633520726000744 06]
one interval r = 21.71840114650009950324462949249388808748 ..
26.81849303499202297705269042370531336879
Time Approximations 0.058.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1,
1, 15.90128515414341158765710109686370821521, 23.93303356 ..
26.81849303, 3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
 scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 .
. 26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.37e-37
Equations at solution: [.1e-37, .237e-36, .104e-34]Solution in
1.233s

Time Plot 1.607 s.
Exiting SolveHard() after 4.602r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and
source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50, none, none,
361.50888347265249460194100068800883928 67, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none]

2 --> 0  target = [15.91193136481711020848428360539751124015,
5.18778357850803989052198061095447170894 8,
408.65773862509824021973633520726000744 06]
```

"Imaginary part neglected: ", $3.18322343221428973170192329993388559626 1 \times 10^{-17}$

```
one interval r = 31.80828598735655304187137456417366709612 ..
```

```
35.00011460034421667266017667020522387827
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1,
1, 15.76385589023872534302387520454213741716, 32.91337941 ..
35.00011460, 3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 .
. 35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .515e-34]Solution in 0.429s

Time Plot 2.229 s.
Exiting SolveHard() after 3.31r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86, none,
361.50888347265249460194100068800883928 67, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none]

2 --> 1  target = [26.46347110534272066407170631582168663425,
6.19626256515062183683141313927043786 7519,
385.44474379530512235036767581997765697 46]

"Imaginary part neglected: ", 3.1832234322142897317019232999338855962 61 × 10^-17
one interval r = 31.60836097523803625073790497494622859769 ..
34.66372795604008121370860106032171364939
Time Approximations 0.015.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1,
1, 11.78302906874267690462414906858869172132, 32.62689490 ..
34.66372796, 3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
```

```
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=9e-38
Equations at solution: [.5e-37, -.9e-37, -.47e-35]Solution in 0.526s

Time Plot 3.567 s.
Exiting SolveHard() after 4.727r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86, none,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]

0 --> 1  target = [26.46347110534272066407170631582168663425,
6.19626256515062183683141313927043786751 9,
385.44474379530512235036767581997765697 46]
two intervals r = 16.875634087103592447068281036691096276 78 ..
94999999998859855305038288321420399 63/500000000000000000000000000000
000000 or r = 15.556404938021578340854598407511182723 75 ..
94999999998859855305038288321420399 63/500000000000000000000000000000
000000
Time Approximations 0.061.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1,
1, 15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2
.. 19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 5.76 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19,
rm = 3/2 .. 19}, avoid={{r =
```

```
18.46834175085398595545335818593106353626, rm =
2.33653277420380087582807206458255715679 5}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.180e-37, 0., .2182e-34]Solution in 20.324s

Time Plot 2.003 s.
Exiting SolveHard() after 24.155r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86,
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]

Start Generation 4
1 --> 0  target = [17.19898874691299512839857856533348188462,
4.88381077986845566365827304698667527 6019,
376.61967856077056833087751034753098681 73]
one interval r = 21.11001304888343834866043110900118992921 ..
26.31784243470651232495531442529544572251
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1,
1, 16.76106391218669896442664507234311360129, 23.20517308 ..
26.31784245, 3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 .
. 26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.1e-36]Solution in
1.189s

Time Plot 1.973 s.
Exiting SolveHard() after 4.182r=25.872 in [23.20517308 ..
26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and
```

```
source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86,
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50, none,
328.46938514142074763789027786368945655 07, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874691299512839857856533348188462,
4.8838107798684556636582730469866752760 19,
376.61967856077056833087751034753098681 73]
```

"Imaginary part neglected: ", $3.18322343221428973170192329993388559626 1 \times 10^{-17}$

```
one interval r = 31.53899497698248401070949881255926683386 ..
34.53618386088045480315432324057536803274
Time Approximations 0.018.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1,
1, 17.19898434893881998670258658728864307101, 32.52213872 ..
34.53618387, 3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1
(1.04453) |   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.13e-36
Equations at solution: [-.241e-35, .313e-35, .132e-34]Solution in
0.859s

Time Plot 1.792 s.
Exiting SolveHard() after 2.963r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
```

```
441.642959730441394607652802861179832682 6,
436.917481649989360824609875342850393030 9,
422.984933978270398357247256380848702096 4,
361.525802562450128020522842394554918778 7,
401.881739044491732839196604202029091112 9,
389.590015156455425270534570841236952799 0,
328.469398940698836465565579526343595485 2,
401.507571583086982932229331852504511568 9,
358.973628241960808395677476126220978596 5,
398.331471047895047216359707013510241785 0,
371.483873942265973264012419554233048638 6,
336.612158418996004086039998559636765677 1,
361.508883472652494601941000688008839286 7,
324.671449931299513875341701830939100835 0, none,
328.469385141420747637890277863689456550 7,
343.813406251543566742224941628757098968 5, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.872050175456580397467480581860771073 43,
6.025813549130060008968205537536716218501,
351.427029490779677267946863625801527687 1]
```

"Imaginary part neglected: ", $3.183223432214289731701923299933885596261 \times 10^{-17}$

```
one interval r = 31.362302061026789036886132356685374785 07 ..
34.174466406144937509255990359184333341 68
Time Approximations 0.016.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1,
1, 12.142802994561336759524256526823424721 44, 32.23723258 ..
34.17446642, 3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 .
. 34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [-.3e-37, .6e-37, .152e-34]Solution in 0.522s

Time Plot 3.369 s.
Exiting SolveHard() after 4.489r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434933564304349033122406919760758 2,
441.642959730441394607652802861179832682 6,
436.917481649989360824609875342850393030 9,
422.984933978270398357247256380848702096 4,
361.525802562450128020522842394554918778 7,
401.881739044491732839196604202029091112 9,
389.590015156455425270534570841236952799 0,
```

```
328.46939894069883646556579526343595485 2,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86,
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50, none,
328.46938514142074763789027786368945655 07,
343.81340625154356674222494162875709896 85, none, none,
292.99969139298088863938940732218046210 22, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175456580397467480581860771073 43,
6.0258135491300600089682055375367162185 01,
351.42702949077967726794686362580152768 71]
two intervals r = 17.981355143925182072116156016025761694 94 ..
9499999999885985530503828832142039963/500000000000000000000000000000
000000 or r = 13.846080154485794068882668537871788032 94 ..
9499999999885985530503828832142039963/500000000000000000000000000000
000000
Time Approximations 0.043.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1,
1, 15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2
.. 19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 4.084 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19,
rm = 3/2 .. 19}, avoid={{r =
18.913570713374186478049719154874054530 25, rm =
2.7345009933160116104722074561100118356 93}});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [.107e-36, -.3e-37, -.36e-36]Solution in
12.324s

Time Plot 1.668 s.
Exiting SolveHard() after 15.573r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976 07582,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
```

```
389.590015156455425270534570841236952799 0,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86,
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50,
302.31384315974342317567672338606220615 06,
328.46938514142074763789027786368945655 07,
343.81340625154356674222494162875709896 85, none, none,
292.99969139298088863938940732218046210 22, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418725305808061613939759056481 16,
6.3779438737207384916569673634933894284 63,
423.28832784712375295793493746789447619 12]
```

"Imaginary part neglected: ", $3.18322343221428973170192329993388559626 1 \times 10^{-17}$

```
one interval r = 31.946618175903651742865523561524475132 86 ..
35.212123086541267363994793900815791858 11
Time Approximations 0.017.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1,
1, 11.395769599984320296717245861504673207 76, 33.10127385 ..
35.21212310, 3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 .
. 35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=8e-38
Equations at solution: [.7e-37, -.8e-37, -.271e-34]Solution in
0.931s

Time Plot 3.405 s.
Exiting SolveHard() after 4.639r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312406919760 7582,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.590015156455425270534570841236952799 0,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
```

```
358.9736282419608083956774761262209785965,
398.3314710478950472163597070135102417850,
371.4838739422659732640124195542330486386,
336.6121584189960040860399985596367656771,
361.5088834726524946019410006880088392867,
324.6714499312995138753417018309391008350,
302.3138431597434231756767233860622061506,
328.4693851414207476378902778636894565507,
343.8134062515435667422249416287570989685, none, none,
292.9996913929808886393894073221804621022, none, none,
360.0617346782310817411266825120641636291, none, none, none, none,
none, none, none]

0 --> 1  target = [27.0203794187253058080616139397590564811 6,
6.3779438737207384916569673634933894284 63,
423.2883278471237529579349374678944761912]
two intervals r = 15.2288670236153167625394293451226550807 6 ..
9499999999885985530503828832142039963/5000000000000000000000000000000
000000 or r = 17.1296577708690269932107353297174336187 8 ..
9499999999885985530503828832142039963/5000000000000000000000000000000
000000
Time Approximations 0.055.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1,
1, 15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2
.. 19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 4.966 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19,
rm = 3/2 .. 19}, avoid={{r =
17.515370540675109389914043785996023611 36, rm =
2.0640682987478326180449398656457516262 36}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.32e-37, .1e-37, -.144e-35]Solution in
19.724s

Time Plot 2.2 s.
Exiting SolveHard() after 23.844r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
```

```
389.590015156455425270534570841236952799 0,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86,
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50,
302.31384315974342317567672338606220615 06,
328.46938514142074763789027786368945655 07,
343.81340625154356674222494162875709896 85,
375.73285291419324622733963340995788615 04, none,
292.99969139298088863938940732218046210 22, none, none,
360.06173467823108174112668251206416362 91, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234338733765960299927575303480537,
4.0035598155763948327175292158903290606 88,
404.47973594358461573668946300522652604 28]
two intervals r = 16.09683966318641463804976594100069196787 ..
9499999999885985530503828832142039963/50000000000000000000000000000
000000 or r = 16.39988649115652698809677831799642952829 ..
9499999999885985530503828832142039963/50000000000000000000000000000
000000
Time Approximations 0.055.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1,
1, 16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2
.. 19, 1]
I search for an scattering ray on same branch with 0<sv<1
(0.0248847) |   S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19,
rm = 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.35e-37, -.1e-37, .3534e-34]Solution in
2.266s

Time Plot 1.765 s.
Exiting SolveHard() after 5.487r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.590015156455425270534570841236952799 0,
328.46939894069883646556557952634359548 52,
```

```
401.5075715830869829322293318525045115689,
358.9736282419608083956774761262209785965,
398.3314710478950472163597070135102417850,
371.4838739422659732640124195542330486386,
336.6121584189960040860399985596367656771,
361.5088834726524946019410006880088392867,
324.6714499312995138753417018309391008350,
302.3138431597434231756767233860622061506,
328.4693851414207476378902778636894565507,
343.8134062515435667422249416287570989685,
375.7328529141932462273396334099578861504, none,
292.9996913929808886393894073221804621022,
358.6434156113895901671092092796224300800, none,
360.0617346782310817411266825120641636291, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234338733765960299927575303480537,
4.0035598155763948327175292158903290606 88,
404.4797359435846157366894630052265260428]
one interval r = 21.63429629998864143203807303660588842602 ..
26.75768169886739596492455498281432561570
Time Approximations 0.045.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1,
1, 16.77921525837433403832553912089491136680, 23.83864811 ..
26.75768170, 3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
 scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 .
. 26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, .662e-34]Solution in 1.336s

Time Plot 2.782 s.
Exiting SolveHard() after 5.424r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349335643043490331224069197607582,
441.6429597304413946076528028611798326826,
436.9174816499893608246098753428503930309,
422.9849339782703983572472563808487020964,
361.5258025624501280205228423945549187787,
401.8817390444917328391966042020290911129,
389.5900151564554252705345708412369527990,
328.4693989406988364655655795263435954852,
401.5075715830869829322293318525045115689,
358.9736282419608083956774761262209785965,
398.3314710478950472163597070135102417850,
371.4838739422659732640124195542330486386,
```

```
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50,
302.31384315974342317567672338606220615 06,
328.46938514142074763789027786368945655 07,
343.81340625154356674222494162875709896 85,
375.73285291419324622733963340995788615 04,
328.11709295091830644461473409116098320 54,
292.99969139298088863938940732218046210 22,
358.64341561138959016710920927962243008 00, none,
360.06173467823108174112668251206416362 91, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954452171835849439694474668056555,
6.19617723000325068536376499924878781556 5,
385.42734025887460640334963441211224609 47]
```

"Imaginary part neglected: ", $3.18322343221428973170192329993388559626 1 \times 10^{-17}$

```
one interval r = 31.60822049079021188269979935689375617849 ..
34.66347615041405866468084273975961443899
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1,
1, 11.78320912141352187071943817447189965627, 32.62668594 ..
34.66347615, 3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 .
. 34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=1.1e-37
Equations at solution: [.7e-37, -.11e-36, -.175e-34]Solution in
0.839s

Time Plot 3.398 s.
Exiting SolveHard() after 4.518r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and
source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
```

```
371.48387394226597326401241955423304863 86,
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50,
302.31384315974342317567672338606220615 06,
328.46938514142074763789027786368945655 07,
343.81340625154356674222494162875709896 85,
375.73285291419324622733963340995788615 04,
328.11709295091830644461473409116098320 54,
292.99969139298088863938940732218046210 22,
358.64341561138959016710920927962243008 00, none,
360.06173467823108174112668251206416362 91, none, none,
324.65521224091777250099508043025115412 60, none, none, none, none]

0 --> 1  target = [26.46318954452171835849439694474668056555,
6.19617723000325068536376499924878781556 5,
385.42734025887460640334963441211224609 47]
two intervals r = 16.87629600249691965649438162684103928229 ..
94999999998859855305038288321420399 63/500000000000000000000000000000
000000 or r = 15.55559000649864613555413930331485144403 ..
94999999998859855305038288321420399 63/500000000000000000000000000000
000000
Time Approximations 0.054.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1,
1, 15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2
.. 19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19,
rm = 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 5.372 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19,
rm = 3/2 .. 19}, avoid={{r =
18.46866852497473692157925298048008081056, rm =
2.33669042827083765340573882902422315908 6}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.179e-37, 0., .4307e-34]Solution in 19.73s

Time Plot 2.163 s.
Exiting SolveHard() after 23.73r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312406919760 7582,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
```

```
389.5900151564554252705345708412369527990,
328.4693989406988364655657952634359548 52,
401.5075715830869829322293318525045115689,
358.9736282419608083956774761262209785965,
398.3314710478950472163597070135102417850,
371.4838739422659732640124195542330486386,
336.6121584189960040860399985596367656771,
361.5088834726524946019410006880088392867,
324.6714499312995138753417018309391008350,
302.3138431597434231756767233860622061506,
328.4693851414207476378902778636894565507,
343.8134062515435667422249416287570989685,
375.7328529141932462273396334099578861504,
328.1170929509183064446147340911609832054,
292.9996913929808886393894073221804621022,
358.6434156113895901671092092796224300800, none,
360.0617346782310817411266825120641636291,
336.5944103292434862868676437696400816707, none,
324.6552122409177725009950804302511541260, none, none, none, none]

0 --> 2  target = [34.4952266115400482782021725636013044549 4,
3.8971313159778151196990916232066837873 05,
373.7808188431769212840581545521067520266]
two intervals r = 17.297690861946892941393004673506540543 05 ..
94999999998859855305038288321420399 63/500000000000000000000000000000
000000 or r = 14.994364074156037232988123576560701707 40 ..
94999999998859855305038288321420399 63/500000000000000000000000000000
000000
Time Approximations 0.076.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1,
1, 17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2
.. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555)
|   S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
 scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19,
rm = 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [-.18e-37, .1e-37, -.2489e-34]Solution in
1.415s

Time Plot 1.67 s.
Exiting SolveHard() after 5.708r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349335643043490331224069197607582,
441.6429597304413946076528028611798326826,
436.9174816499893608246098753428503930309,
422.9849339782703983572472563808487020964,
361.5258025624501280205228423945549187787,
```

```
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86,
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50,
302.31384315974342317567672338606220615 06,
328.46938514142074763789027786368945655 07,
343.81340625154356674222494162875709896 85,
375.73285291419324622733963340995788615 04,
328.11709295091830644461473409116098320 54,
292.99969139298088863938940732218046210 22,
358.64341561138959016710920927962243008 00, none,
360.06173467823108174112668251206416362 91,
336.59441032924348628686764376964008167 07, none,
324.65521224091777250099508043025115412 60,
331.93806791351670165437004391333549016 41, none, none, none]

1 --> 2  target = [34.49522661154004827820217256360130445494,
3.89713131597781511969909162320668378730 5,
373.78018884317692128405815455210675202 66]
one interval r = 21.06068473207163600676475527714423062586 ..
26.26979834272111213429122330535996252155
Time Approximations 0.031.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1,
1, 16.97472952764178754508574613836907474818, 23.14060343 ..
26.26979834, 3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
 scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 .
. 26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [.3e-37, .7e-37, -.126e-34]Solution in 0.751s

Time Plot 2.921 s.
Exiting SolveHard() after 4.659r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312406919760 7582,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
```

```
389.5900151564554252705345708412369527990,
328.4693989406988364655655795263435954852,
401.5075715830869829322293318525045115689,
358.9736282419608083956774761262209785965,
398.3314710478950472163597070135102417850,
371.4838739422659732640124195542330486386,
336.6121584189960040860399985596367656771,
361.5088834726524946019410006880088392867,
324.6714499312995138753417018309391008350,
302.3138431597434231756767233860622061506,
328.4693851414207476378902778636894565507,
343.8134062515435667422249416287570989685,
375.7328529141932462273396334099578861504,
328.1170929509183064446147340911609832054,
292.9996913929808886393894073221804621022,
358.6434156113895901671092092796224300800,
299.8986620508042340952279834502760941683,
360.0617346782310817411266825120641636291,
336.5944103292434862868676437696400816707, none,
324.6552122409177725009950804302511541260,
331.9380679135167016543700439133354901641, none, none, none]

0 --> 2  target = [33.8136249540673777964890377556214427920 1,
3.7256489936352110555882780686178277240 48,
325.8920997339884057818185522560205054816]
two intervals r = 18.552270489660153498652957643152780514 37 ..
949999999988598553050382883214203996 3/500000000000000000000000000000
000000 or r = 12.491969358127865486537116341323360226 55 ..
949999999988598553050382883214203996 3/500000000000000000000000000000
000000
Time Approximations 0.045.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1,
1, 16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2
.. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409)
|   S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19,
rm = 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [-.69e-37, .3e-37, -.3317e-34]Solution in
1.436s

Time Plot 1.477 s.
Exiting SolveHard() after 5.027r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349335643043490331224069197607582,
441.6429597304413946076528028611798326826,
436.9174816499893608246098753428503930309,
```

```
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86,
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50,
302.31384315974342317567672338606220615 06,
328.46938514142074763789027786368945655 07,
343.81340625154356674222494162875709896 85,
375.73285291419324622733963340995788615 04,
328.11709295091830644461473409116098320 54,
292.99969139298088863938940732218046210 22,
358.64341561138959016710920927962243008 00,
299.89866205080423409522798345027609416 83,
360.06173467823108174112668251206416362 91,
336.59441032924348628686764376964008167 07, none,
324.65521224091777250099508043025115412 60,
331.93806791351670165437004391333549016 41, none, none,
289.54595773068053078726038393912294145 26]

1 --> 2  target = [33.81362495406737779648903775562144279201,
3.72564899363521105558827806861782772404 8,
325.89209973398840578181855225602050548 16]
one interval r = 20.37468935121999712219421975399398878340 ..
25.37892165304141705613648166426183308053
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1,
1, 17.27215878251294421226707494428435581574, 22.07732228 ..
25.37892164, 3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
 scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 .
. 25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .373e-34]Solution in 0.873s

Time Plot 2.44 s.
Exiting SolveHard() after 3.845r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976 07582,
441.64295973044139460765280286117983268 26,
```

```
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86,
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50,
302.31384315974342317567672338606220615 06,
328.46938514142074763789027786368945655 07,
343.81340625154356674222494162875709896 85,
375.73285291419324622733963340995788615 04,
328.11709295091830644461473409116098320 54,
292.99969139298088863938940732218046210 22,
358.64341561138959016710920927962243008 00,
299.89866205080423409522798345027609416 83,
360.06173467823108174112668251206416362 91,
336.59441032924348628686764376964008167 07,
256.10753187001502016601396388154878437 11,
324.65521224091777250099508043025115412 60,
331.93806791351670165437004391333549016 41, none, none,
289.54595773068053078726038393912294145 26]

1 --> 0  target = [17.93041369657698377152612005804669184045,
4.68650870203315993306643270676462694362 9,
353.30541095714878643556041044782357172 93]
one interval r = 20.73150479108810092606630961178221003641 ..
25.90675353526803532414595752396536766905
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1,
1, 17.00624445476621345302062451224674357053, 22.67806074 ..
25.90675353, 3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
 scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 .
. 25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.8e-38
Equations at solution: [-.2e-37, -.48e-37, -.159e-34]Solution in
0.969s

Time Plot 1.633 s.
Exiting SolveHard() after 3.726r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and
source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556579526343595485 2,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86,
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50,
302.31384315974342317567672338606220615 06,
328.46938514142074763789027786368945655 07,
343.81340625154356674222494162875709896 85,
375.73285291419324622733963340995788615 04,
328.11709295091830644461473409116098320 54,
292.99969139298088863938940732218046210 22,
358.64341561138959016710920927962243008 00,
299.89866205080423409522798345027609416 83,
360.06173467823108174112668251206416362 91,
336.59441032924348628686764376964008167 07,
256.10753187001502016601396388154878437 11,
324.65521224091777250099508043025115412 60,
331.93806791351670165437004391333549016 41,
304.79958326575195195762907544091290784 67, none,
289.54595773068053078726038393912294145 26]

2 --> 0  target = [17.93041369657698377152612005804669184045,
4.68650870203315993306643270676462694362 9,
353.30541095714878643556041044782357172 93]

"Imaginary part neglected: ", 3.183223432214289731701923299933885596261 × 10^-17
one interval r = 31.37435486982085409956186124147005693681 ..
34.20127520027382986201941761304615401237
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1,
-1, 17.86353406971662964004465842689508192962, 32.25770943 ..
34.20127520, 3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |
 P  <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 .
. 34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., -.323e-34]Solution in 0.369s

Time Plot 1.382 s.
Exiting SolveHard() after 2.384r=33.7963 in [32.25770943 ..
34.20127520]
```

```
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and
source on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493356430434903312240691976075 82,
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86,
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50,
302.31384315974342317567672338606220615 06,
328.46938514142074763789027786368945655 07,
343.81340625154356674222494162875709896 85,
375.73285291419324622733963340995788615 04,
328.11709295091830644461473409116098320 54,
292.99969139298088863938940732218046210 22,
358.64341561138959016710920927962243008 00,
299.89866205080423409522798345027609416 83,
360.06173467823108174112668251206416362 91,
336.59441032924348628686764376964008167 07,
256.10753187001502016601396388154878437 11,
324.65521224091777250099508043025115412 60,
331.93806791351670165437004391333549016 41,
304.79958326575195195762907544091290784 67,
323.46169177284247844899722495515172524 84,
289.54595773068053078726038393912294145 26]

Cascade time 241.856
```

[462.16343493356430434903312240691976075 82, **(82)**
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,

```
    371.48387394226597326401241955423304863 86,
    336.61215841899600408603999855963676567 71,
    361.50888347265249460194100068800883928 67,
    324.67144993129951387534170183093910083 50,
    302.31384315974342317567672338606220615 06,
    328.46938514142074763789027786368945655 07,
    343.81340625154356674222494162875709896 85,
    375.73285291419324622733963340995788615 04,
    328.11709295091830644461473409116098320 54,
    292.99969139298088863938940732218046210 22,
    358.64341561138959016710920927962243008 00,
    299.89866205080423409522798345027609416 83,
    360.06173467823108174112668251206416362 91,
    336.59441032924348628686764376964008167 07,
    256.10753187001502016601396388154878437 11,
    324.65521224091777250099508043025115412 60,
    331.93806791351670165437004391333549016 41,
    304.79958326575195195762907544091290784 67,
    323.46169177284247844899722495515172524 84,
    289.54595773068053078726038393912294145 26]
```

```
> ListP;
  ListTau;
```

```
[[11.99999999978589433662945395387544752600,
    6.21701250297520191323212559688307042881 2,
    485.54908089534509795809793505892514721 09],
    [27.52359684467589826729239999474015361860,
    6.58343472140776631428277506018975286415 5,
    467.78730595760771656585491784598526884 84],
    [35.46322962814319233581458741166882929558,
    4.12565179687950661576447735897190110673 3,
    440.67123064783892537974503535275385080 88],
    [14.19258941726625901580359918957455805651,
    5.58963718304022048533917153916106618258 3,
    443.83065884921309069076867916489068776 82],
    [26.46347110534272066407170631582168663425,
    6.19626256515062183683141313927043786751 9,
    385.44474379530512235036767581997765697 46],
    [34.94507888793289665602575489061681612245,
```

4.004869081872656713173168016753856323955,
404.86224501545022370539205059267949965 52],
[15.911931364817110208484283605397511240 15,
5.187783578508039890521980610954471708948,
408.65773862509824021973633520726000744 06],
[25.872050175456580397467480581860771073 43,
6.025813549130060008968205537536716218501,
351.42702949077967726794686362580152768 71],
[34.939532343387337659602999275753034805 37,
4.003559815576394832717529215890329060688,
404.47973594358461573668946300522652604 28],
[17.198988746912995128398578565333481884 62,
4.883810779868455663658273046986675276019,
376.61967856077056833087751034753098681 73],
[27.020379418725305808061613939759056481 16,
6.377943873720738491656967363493389428463,
423.28832784712375295793493746789447619 12],
[34.495226611540048278202172563601304454 94,
3.897131315977815119699091623206683787305,
373.78081884317692128405815455210675202 66],
[17.930413696576983771526120058046691840 45,
4.686508702033159933066432706764626943629,
353.30541095714878643556041044782357172 93],
[26.463189544521718358494396944746680565 55,
6.196177230003250685363764999248787815565,
385.42734025887460640334963441211224609 47],
[33.813624954067377796489037755621442792 01,
3.725648993635211055588278068617827724048,
325.89209973398840578181855225602050548 16],
[18.687780860500745570799065089207813238 96,
4.408641482855060156313509591286322988096,
317.63557440676996046723567096931971655 14],
[25.872049912121733645811621902325516848 55,
6.025813476312751104283174523460786257628,
351.42701528363366263119758949942841645 66],
[34.089767995852816260879757820007985540 32,
3.796597789549462745823636925099699811696,
345.47764844851082827989943990959803204 29],

[16.53336698723587441508917588367093200005,
5.044302175540756623401289660670894565436,
394.13717100261243167478813108842566297 16],
[25.86532280061640549029675407522495250359,
6.023953984835145976356885476090448950865,
351.06430842896906712889002186480039429 49],
[33.36855355005284846678488905015183847350,
3.605714291746724867936339722017722014127,
293.47578375455557704118282645940170461 12],
[17.21111401013589875885478803596210678899,
4.880770162481760865335368526546757682377,
376.27494600805764670822843639091819709 92],
[25.30046634035414714261067565163796370702,
5.871694125213881898681397127122916085666,
321.99875338340089644922452209415795087 51],
[34.32715979900651100987874224728811687164,
3.855919917151225728373243317560676346925,
362.09872332018534152832003348310339050 99],
[17.93092082992043870231007995312321732722,
4.686358235863902052280470950226075433983,
353.28692861312424714589227387494668782 19],
[24.33949903044041565721359638168690929643,
5.620659139671407815648558182253846721947,
276.83661545731949752782229780328539417 62],
[33.81339267507757195695479981532208302965,
3.725588318764365442423098383982999252781,
325.87548370172588431199906837989239431 49],
[18.05989025916180632769933276480679225355,
4.647168293407410910826577464860076388408,
348.43898873396235139292073114177813769 22],
[25.40207153798761325824210884953379062073,
5.898629067932587682664033182687849201886,
327.04870233076939633191924374559527026 28],
[33.79632952269098256847591903070978866390,
3.721126245325640225782102423899710476313,
324.65414441100412692649756317238014360 78],
[18.85462970482772169447801134563022915053,
4.309926953217627768139643212356678301407,

304.37474578286484191479566950706541808 52]]

[462.16343493356430434903312240691976075 82, **(83)**
441.64295973044139460765280286117983268 26,
436.91748164998936082460987534285039303 09,
422.98493397827039835724725638084870209 64,
361.52580256245012802052284239455491877 87,
401.88173904449173283919660420202909111 29,
389.59001515645542527053457084123695279 90,
328.46939894069883646556557952634359548 52,
401.50757158308698293222933185250451156 89,
358.97362824196080839567747612622097859 65,
398.33147104789504721635970701351024178 50,
371.48387394226597326401241955423304863 86,
336.61215841899600408603999855963676567 71,
361.50888347265249460194100068800883928 67,
324.67144993129951387534170183093910083 50,
302.31384315974342317567672338606220615 06,
328.46938514142074763789027786368945655 07,
343.81340625154356674222494162875709896 85,
375.73285291419324622733963340995788615 04,
328.11709295091830644461473409116098320 54,
292.99969139298088863938940732218046210 22,
358.64341561138959016710920927962243008 00,
299.89866205080423409522798345027609416 83,
360.06173467823108174112668251206416362 91,
336.59441032924348628686764376964008167 07,
256.10753187001502016601396388154878437 11,
324.65521224091777250099508043025115412 60,
331.93806791351670165437004391333549016 41,
304.79958326575195195762907544091290784 67,
323.46169177284247844899722495515172524 84,
289.54595773068053078726038393912294145 26]

```
> NewSignals:= PurifyTau(ListTau);
```

NewSignals := [39.17850095529390599178586602607105866 18, **(84)**
80.11715716799126658712996046662491390 39,
35.03574260549762798541327114082130191 80,
72.57341977710887907849855156568280795 92,

113.17356078974255814208722333483623719 74,
35.40991006690237789238054349034588146 20,
103.18980669160349595335564628069878216 17,
43.31148868254634739129309584766959089 76,
65.43360770772338756059745578861734439 23,
125.55127651456830026299312384728299508 11,
80.13407625778890000571180217317099339 59,
112.24603171868984694926817351191129219 59,
159.84959177382088117335639902085755460 76,
113.17357458902064696976252499749037613 19,
93.10407539844579408238493371409329406 24,
86.43058201937105812169348899696187460 78,
113.52586677952308816303806877001884947 72,
143.91779025700847218522046802066993092 87,
103.52001932217471418192391312729733067 82,
141.74429767963716051242481941090373851 43,
76.85574697175827908348319283078622940 18,
125.56902460432081806216547863727967908 75,
185.53542786042637444163883897963104831 15,
112.26226940907158832361479491259923890 49,
130.22536702004760269466307849358427059 41,
136.84337646468944265002372742026692483 59,
113.45578987714688237561265038769866778 25,
172.61747720288377356177273846779681930 56]

```
> ObsSignals;
```

[39.17850096193592366660462091592062373 76, **(85)**
80.11715717097871213365890494429573535 67,
35.03574261053054630657492872958108159 20,
72.57341977793111680898418627081248044 90,
113.17356079751935436022513046580032987 56,
35.40991007306522652038724051092347128 48,
103.18980669701380595609210925548892818 71,
43.31148869147579252763214908406813583 78,
65.43360770882868000737676330764741734 83,
125.55127652440722347053216752374541211 7,
80.13407626061348502470439015672952679 45,
112.24603172686866256539758944644350801 69,
159.84959178824144499228158441696501744 06,

113.17357459679319449200407586632423227 85,
93.10407540385622828546311154167323010 95,
86.43058203591576981983712540155117890 71,
113.52586678836403966572945824855443803 09,
143.91779026987465003621630041257875864 92,
103.52001932858460948977835457701477422 48,
141.74429768374661292012973911416795685 24,
76.85574698554547517515015464726496156 22,
125.56902461398873586013930329151515120 21,
185.53542787116689390175923961978786119 47,
112.26226941709415618622735669007918004 76,
130.22536702210812661237475328349493992 74,
136.84337647699999488633948073154505905 76,
113.45578988671430465571931586496739842 66,
172.61747721153048259619184107188535649 60]

```
> FitFunction:= proc(s1, s2)
    local n, k, P:
    P:= 0.0:
    n := nops(s1):
    printf("counts: %d, %d\n ", n, nops(s2));
    for k from 1 to n do
      P:= P+ (s1[n]-s2[n])^2:
    end do:
    P:
  end:
```

```
> FitFunction(NewSignals, ObsSignals);
  H:= [%];


  # 0.0003   -> Phi ~ 12.281619794122270361968983763353739043 16
  # 10^(-15) -> Phi ~ 7*10^(-26)
counts: 28, 28
```

$$2.0934361595253380316578537879613990603 20 \times 10^{-15}$$

$$H := [2.0934361595253380316578537879613990603 20 \times 10^{-15}] \quad \textbf{(86)}$$

```
>
  ProducePlots:= false:
```

```
> Stat:=proc()
   global te0in, t0in, tau0in, rm0v, rp0v:
   global te1in, t1in, tau1in, rm1v, rp1v:
   global te2in, t2in, tau2in, rm2v, rp2v:
   global H, BaseR, MaxT, ListTau, ListP, BaseSat, BaseBranch,
```

```
ObsSignals, ListHints, HintsOriginal:
 local  NewSignals:

 ListHints:= HintsOriginal:
 ListTau:=CreateList(31, 0):
 ListP:=CreateList(31, []):
 #
 Rumor(BaseROriginal, 10^(-10)):
 BaseR:=%:

 Rumor(Sat0, 10^(-10)):
 te0in, t0in, tau0in, rm0v, rp0v := op(%):
 ComputeSat0():

 Rumor(Sat1, 10^(-10)):
 te1in, t1in, tau1in, rm1v, rp1v := op(%):
 ComputeSat1():

 Rumor(Sat2, 10^(-10)):
 te2in, t2in, tau2in, rm2v, rp2v := op(%):
 ComputeSat2():

 MaxT:=250:
 Cascade(BaseSat, BaseR, BaseBranch):
 NewSignals:= PurifyTau(ListTau):
 H:= [op(H),FitFunction(NewSignals, ObsSignals)]:
end:
```

```
> H;
```

$$[2.0934361595253380316578537879613990603 20 \times 10^{-15}] \tag{87}$$

```
> evalf(BaseROriginal);
Sat0;
Sat1;
Sat2;
```

$$12.$$

$$\left[\frac{\pi}{6}, 0, 1, 10, 19\right]$$

$$\left[\frac{7\pi}{6}, 10, -1, 20, 28\right]$$

$$\left[\frac{3\pi}{4}, -10, -3, 31, 40\right] \tag{88}$$

```
> # HereHere


for n from 1 to 100 do
 printf("\n Iteration %d\n\n", n):
 Stat():
end do:
```

```
 Iteration 1

Start Generation 1
1 --> 0  target = [11.999999999798139566353807721363602408 00,
6.217012503116064474688795963244519574214,
485.549080914586325910475821703456963188 6]
one interval r = 23.408503016895994764655780150347136339 42 ..
27.675780464376604435512558951239005544 15
Time Approximations 0.044.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.8e-38
Equations at solution: [.1e-37, -.28e-37, .7e-36]Solution in 1.355s

Time Plot 0 s.
Exiting SolveHard() after 2.962r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.642959747582989218200924564825270631 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999798139566353807721363602408 00,
6.217012503116064474688795963244519574214,
485.549080914586325910475821703456963188 6]
one interval r = 32.628147792377486848678436130955087985 34 ..
36.102483889633401248495086609097915938 51
Time Approximations 0.02.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .124e-34]Solution in 0.543s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.296r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684491799982715358949036003108571,
6.58343472178259319537843471426854981 6127,
467.78730597496542011973488750609394 79599]
one interval r = 32.41978955686439600669030711783943080085 ..
35.85152417393409517357179881862153205187
Time Approximations 0.019.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=7e-38
Equations at solution: [-.8e-37, .7e-37, .19e-35]Solution in 0.602s

Time Plot 0 s.
Exiting SolveHard() after 1.381r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24, none, none,
401.88173905737220105505132989243027772 02, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684491799982715358949036003108571,
6.58343472178259319537843471426854981 6127,
467.78730597496542011973488750609394 79599]
two intervals r = 12.92327160805555702908642639963788109092 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
```

```
000 or r = 18.39424858064967320763866883677129655388 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=6.6e-38
Equations at solution: [.5e-37, .66e-37, .66e-36]Solution in 30.545s

Time Plot 0 s.
Exiting SolveHard() after 32.692r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64, none,
401.88173905737220105505132989243027772 02, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962847202810067473596225135447966,
4.12565179692740834552776141592476762062 7,
440.67123066580300563641704585419352479 91]
two intervals r = 14.356597050928045264590913422190627486 27 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000 or r = 17.70352613853436112528031484338857088243 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.14e-37, -.1e-37, .924e-35]Solution in 1.623s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.377r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64, none,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962847202810067473596225135447966,
4.12565179692740834552776141592476762062 7,
440.67123066580300563641704585419352479 91]
one interval r = 22.39761154397196971963237530890506501624 ..
27.23722351613042615795970583461388657096
Time Approximations 0.04.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.466 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064412672248231163961049298421304, rm =
14.37818770682761246876054289451400316800}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., .196e-34]Solution in 7.216s

Time Plot 0 s.
Exiting SolveHard() after 8.458r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
```

```
361.525802574510321843997570515611607757 8,
401.881739057372201055051329892430277720 2,
389.590015172557512921060339818236060557 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888206969711863829410074210463 43,
4.004869081905397783723624375606051445570,
404.862245028386759249963767805374774646 9]
two intervals r = 16.080110077352292465274844594842348347 63 ..
475000000002010042735511694797264343 9/250000000000000000000000000000000
000 or r = 16.415798127554689782758641833373780504 54 ..
475000000002010042735511694797264343 9/250000000000000000000000000000000
000
Time Approximations 0.051.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.50e-37, .1e-37, .2191e-34]Solution in 2.125s

Time Plot 0 s.
Exiting SolveHard() after 3.579r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434951927411632826889407481526755 4,
441.642959747582989218200924564825270631 8,
436.917481667796797180463854785920585832 4,
422.984933990093482926451986797095770906 4,
361.525802574510321843997570515611607757 8,
401.881739057372201055051329892430277720 2,
389.590015172557512921060339818236060557 8, none, none,
358.973628253730411981255642384562723605 5, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888206969711863829410074210463 43,
4.004869081905397783723624375606051445570,
404.862245028386759249963767805374774646 9]
one interval r = 21.641943994388896870625725040840980208 67 ..
26.763306600610673676652566925752417040 46
Time Approximations 0.044.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
```

```
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, .148e-34]Solution in 1.317s

Time Plot 0 s.
Exiting SolveHard() after 2.629r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59, none,
358.97362825373041198125564238456272360 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941756720847800592542613761213227,
5.58963718308298474965066996094947005980 4,
443.83065886151382449971118896111363926 55]
one interval r = 22.46725374509387207153273321372953943378 ..
27.27388428369270907137203753356352638856
Time Approximations 0.034.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, .10e-35]Solution in 1.256s

Time Plot 0 s.
Exiting SolveHard() after 2.58r=27.0204 in [24.71083344 .. 27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59, none,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941756720847800592542613761213227,
5.58963718308298474965066996094947005980 4,
443.83065886151382449971118896111363926 55]
one interval r = 32.15575279523690017671678689665300054858 ..
35.50872228759650370730588908142951507985
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=3e-38
Equations at solution: [.3e-37, -.3e-37, -.207e-34]Solution in 0.424s

Time Plot 0 s.
Exiting SolveHard() after 1.136r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136487368283741054092245851150401,
5.18778357860746577439673976091927111874 4,
408.65773864192301373883802792516945745 24]
one interval r = 21.71840114677964254148921267523369316246 ..
26.81849303530251393470334849494444575589
Time Approximations 0.053.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.05e-37
Equations at solution: [.1e-37, .105e-36, .150e-34]Solution in 1.305s

Time Plot 0 s.
Exiting SolveHard() after 2.704r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49, none, none,
361.50888348488458581191581849361662718 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136487368283741054092245851150401,
5.18778357860746577439673976091927111874 4,
408.65773864192301373883802792516945745 24]
one interval r = 31.80828598771406928564208700065153700107 ..
35.00011460067286990518943161440763399192
Time Approximations 0.016.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .435e-34]Solution in 0.389s

Time Plot 0 s.
Exiting SolveHard() after 1.041r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75, none,
361.50888348488458581191581849361662718 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110560139509638254699895864766760,
6.19626256550875809651504701294933734277 2,
385.44474380747138326365604918691702975 43]
one interval r = 31.60836097554850760014036853444762057713 ..
34.66372795631188984365065993232000707349
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=9e-38
Equations at solution: [.6e-37, -.9e-37, -.247e-34]Solution in 0.871s

Time Plot 0 s.
Exiting SolveHard() after 1.141r=33.8136 in [32.62689490 ..
```

```
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75, none,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110560139509638254699895864766760,
6.19626255508758096515047012949337342772,
385.44474380747138326365604918691702975 43]
two intervals r = 16.87563408735253120956768560881236631333 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000 or r = 15.55640493869460840191289168542170380041 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.358e-37, 0., .802e-35]Solution in 1.768s

Time Plot 0 s.
Exiting SolveHard() after 3.241r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
```

```
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874715370343709018351800266017791,
4.8838107799271220911964540909270066353 09,
376.61967857306958827226307249802645876 99]
one interval r = 21.11001304901636726490887530087316745333 ..
26.31784243497244402327291719749871367538
Time Approximations 0.034.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=1.50e-37
Equations at solution: [.4e-37, .150e-36, .135e-34]Solution in 1.107s

Time Plot 0 s.
Exiting SolveHard() after 1.776r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
```

```
324.671449938853145761551734113686538661 4, none,
328.46938514873133743727767335072676898 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874715370343709018351800266017791,
4.8838107799271220911964540909270066353 09,
376.61967857306958827226307249802645876 99]
one interval r = 31.53899497729175238559120297387388306147 ..
34.53618386115805194522009510027334076855
Time Approximations 0.015.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.076e-35
Equations at solution: [-.826e-35, .1076e-34, .89e-35]Solution in
0.481s

Time Plot 0 s.
Exiting SolveHard() after 1.117r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.671449938853145761551734113686538661 4, none,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017564161350099610981972724319541,
6.0258135494667924325613060757660537839 46,
351.42702949806547630264310090720725307 94]
one interval r = 31.36230206129605503478899649219893606868 ..
```

```
34.17446640636137618474725897034714344377
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, .125e-34]Solution in 0.843s

Time Plot 0 s.
Exiting SolveHard() after 1.106r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14, none,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81, none, none,
292.99969139581944761032457857947711247 39, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017564161350099610981972724319541,
6.02581354946679243256130607576605378 3946,
351.42702949806547630264310090720725307 94]
two intervals r = 17.98135514429945578944597581316100995754 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000 or r = 13.84608015499682694927867024117796256122 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000
Time Approximations 0.042.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [-.36e-37, .1e-37, -.3570e-34]Solution in 1.423s

Time Plot 0 s.
Exiting SolveHard() after 2.798r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14,
302.31384316086614426301944470878325412 38,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81, none, none,
292.99969139581944761032457857947711247 39, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941890417520192912082940229584252,
6.37794387405626903773270350355444176533 2,
423.28832785522802026813201638136787808 74]
one interval r = 31.94661817618074818288776469679542063676 ..
35.21212308673648288003541619011294924304
Time Approximations 0.016.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
```

```
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.187e-34]Solution in 0.535s

Time Plot 0 s.
Exiting SolveHard() after 1.214r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14,
302.31384316086614426301944470878325412 38,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81, none, none,
292.99969139581944761032457857947711247 39, none, none,
360.06173468217617803564849983592121289 95, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941890417520192912082940229584252,
6.37794387405626903773270350355444176533 2,
423.28832785522802026813201638136787808 74]
two intervals r = 15.22886702409288808632111558705657421017 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000 or r = 17.12965777127567967449690995043500749206 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.31e-37, .1e-37, .1705e-34]Solution in 1.521s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.552r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14,
302.31384316086614426301944470878325412 38,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81,
375.73285291568071512675825570419450523 84, none,
292.99969139581944761032457857947711247 39, none, none,
360.06173468217617803564849983592121289 95, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343644521970337118497708418342 67,
4.0035598156051237638252016087955832800 65,
404.47973595534401868344997789542845046 54]
two intervals r = 16.096839663465870771715377498679756129 25 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000 or r = 16.399886491751480137211702740768208297 07 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000
Time Approximations 0.048.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.17e-37, -.1e-37, -.2447e-34]Solution in
1.721s

Time Plot 0 s.
Exiting SolveHard() after 3.44r=17.2111 in [16.09683967 .. 19]
```

```
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14,
302.31384316086614426301944470878325412 38,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81,
375.73285291568071512675825570419450523 84, none,
292.99969139581944761032457857947711247 39,
358.64341562214226315095255440383993962 36, none,
360.06173468217617803564849983592121289 95, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234364452197033711849770841834267,
4.0035598156051237638252016087955832800 65,
404.47973595534401868344997789542845046 54]
one interval r = 21.634296300159139834902512635567572436 54 ..
26.757681699107807800000239402118996475 05
Time Approximations 0.051.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=4.9e-38
Equations at solution: [-.2e-37, -.49e-37, .67e-35]Solution in 1.31s

Time Plot 0 s.
Exiting SolveHard() after 2.967r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14,
302.31384316086614426301944470878325412 38,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81,
375.73285291568071512675825570419450523 84,
328.11709295713920515490075424989486493 60,
292.99969139581944761032457857947711247 39,
358.64341562214226315095255440383993962 36, none,
360.06173468217617803564849983592121289 95, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954478326400160650141962373770633,
6.19617723036225572176883400938619647532 5,
385.42734027121769354846961580765804334 87]
one interval r = 31.60822049110210643939297896018491404717 ..
34.66347615068843347503164825266981618248
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, .18e-35]Solution in 0.862s

Time Plot 0 s.
Exiting SolveHard() after 1.152r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
```

```
441.642959747582989218200924564825270631 8,
436.917481667796797180463854785920585832 4,
422.984933990093482926451986797095770906 4,
361.525802574510321843997570515611607757 8,
401.881739057372201055051329892430277720 2,
389.590015172557512921060339818236060557 8,
328.469398948004932005247050449207004385 9,
401.507571594815913627925223783482724446 3,
358.973628253730411981255642384562723605 5,
398.331471056016630331062243167291579994 9,
371.483873957754161843790970579126051337 5,
336.612158424822292657925119022161349996 9,
361.508883484884585811915818493616627186 8,
324.671449938853145761551734113686538661 4,
302.313843160866144263019444708783254123 8,
328.469385148731337437277673350726768989 6,
343.813406263062692446576703426332609158 1,
375.732852915680715126758255704194505238 4,
328.117092957139205154900754249894864936 0,
292.999691395819447610324578579477112473 9,
358.643415622142263150952554403839939623 6, none,
360.061734682176178035648499835921212899 5, none, none,
324.655212248636293318842338400082189563 3, none, none, none, none]

0 --> 1  target = [26.463189544783264001606501419623737706 33,
6.196177230362255721768834009386196475325,
385.427340271217693548469615807658043348 7]
two intervals r = 16.876296002739125113560297302166312422 34 ..
475000000002010042735511694797264343 9/250000000000000000000000000000000
000 or r = 15.555590007180011706092051722797630578 99 ..
475000000002010042735511694797264343 9/250000000000000000000000000000000
000
Time Approximations 0.051.

hint used Hint := [17.930920830435909701694400819009216331 81, 3, 1, 1,
15.700860627013397511253157405159644880 12, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.717e-37, 0., -.486e-35]Solution in 1.384s

Time Plot 0 s.
Exiting SolveHard() after 2.786r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434951927411632826889407481526755 4,
441.642959747582989218200924564825270631 8,
```

```
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14,
302.31384316086614426301944470878325412 38,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81,
375.73285291568071512675825570419450523 84,
328.11709295713920515490075424989486493 60,
292.99969139581944761032457857947711247 39,
358.64341562214226315095255440383993962 36, none,
360.06173468217617803564849983592121289 95,
336.59441033525017213085740643498013452 21, none,
324.65521224863629331884233840008218956 33, none, none, none, none]

0 --> 2  target = [34.49522661186661219474633198800527299637,
3.897131316021446704504564024582956861846,
373.78081885878709851886496184572868991 78]
two intervals r = 17.29769086207330666321044799845877117394 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000 or r = 14.99436407503553759747219357416324246973 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000
Time Approximations 0.088.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [-.36e-37, .1e-37, .2231e-34]Solution in 1.467s

Time Plot 0 s.
Exiting SolveHard() after 4.259r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
```

```
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14,
302.31384316086614426301944470878325412 38,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81,
375.73285291568071512675825570419450523 84,
328.11709295713920515490075424989486493 60,
292.99969139581944761032457857947711247 39,
358.64341562214226315095255440383993962 36, none,
360.06173468217617803564849983592121289 95,
336.59441033525017213085740643498013452 21, none,
324.65521224863629331884233840008218956 33,
331.93806792758885117728314058792722098 83, none, none, none]

1 --> 2  target = [34.49522661186661219474633198800527299637,
3.897131316021446704504564024582956861846,
373.78081885878709851886496184572868991 78]
one interval r = 21.06068473225621075656638606699716861771 ..
26.26979834304496438604659069088116374284
Time Approximations 0.03.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=6e-38
Equations at solution: [-.3e-37, -.6e-37, .638e-34]Solution in 0.682s

Time Plot 0 s.
Exiting SolveHard() after 1.683r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
```

```
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14,
302.31384316086614426301944470878325412 38,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81,
375.73285291568071512675825570419450523 84,
328.11709295713920515490075424989486493 60,
292.99969139581944761032457857947711247 39,
358.64341562214226315095255440383993962 36,
299.89866206048251345043788766786153246 93,
360.06173468217617803564849983592121289 95,
336.59441033525017213085740643498013452 21, none,
324.65521224863629331884233840008218956 33,
331.93806792758885117728314058792722098 83, none, none, none]

0 --> 2  target = [33.81362495429760009086869787353862936126,
3.72564899365221856430686010585371531 0312,
325.89209974149227045393102112754945398 37]
two intervals r = 18.552270489997331802002799102787423449 17 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000 or r = 12.49196935868608033238124569622389221169 ..
475000000002010042735511694797264343 9/2500000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1.0e-37
Equations at solution: [-.243e-36, .10e-36, -.1917e-34]Solution in
1.951s

Time Plot 0 s.
Exiting SolveHard() after 3.961r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14,
302.31384316086614426301944470878325412 38,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81,
375.73285291568071512675825570419450523 84,
328.11709295713920515490075424989486493 60,
292.99969139581944761032457857947711247 39,
358.64341562214226315095255440383993962 36,
299.89866206048251345043788767867853246 93,
360.06173468217617803564849983592121289 95,
336.59441033525017213085740643498013452 21, none,
324.65521224863629331884233840008218956 33,
331.93806792758885117728314058792722098 83, none, none,
289.54595773753628732876201871287122915 92]

1 --> 2  target = [33.81362495429760009086869787353862936126,
3.72564899365221856430686010585371531 0312,
325.89209974149227045393102112754945398 37]
one interval r = 20.37468935119290767319250019333086789566 ..
25.37892165323113868578587405209343471966
Time Approximations 0.023.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, .88e-35]Solution in 0.858s

Time Plot 0 s.
Exiting SolveHard() after 1.334r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14,
302.31384316086614426301944470878325412 38,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81,
375.73285291568071512675825570419450523 84,
328.11709295713920515490075424989486493 60,
292.99969139581944761032457857947711247 39,
358.64341562214226315095255440383993962 36,
299.89866206048251345043788766786153246 93,
360.06173468217617803564849983592121289 95,
336.59441033525017213085740643498013452 21,
256.10753187217842942532440608408010269 45,
324.65521224863629331884233840008218956 33,
331.93806792758885117728314058792722098 83, none, none,
289.54595773753628732876201871287122915 92]

1 --> 0  target = [17.93041369698608968930987635930075748167,
4.6865087020446861076392820612675313915 23,
353.30541096326980393747385159638127080 22]
one interval r = 20.73150479108262788600672588284050861917 ..
25.90675353543146623369879923819784401486
Time Approximations 0.028.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., -.359e-34]Solution in 0.587s

Time Plot 0 s.
Exiting SolveHard() after 1.6r=25.4021 in [22.67806074 .. 25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14,
302.31384316086614426301944470878325412 38,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81,
375.73285291568071512675825570419450523 84,
328.11709295713920515490075424989486493 60,
292.99969139581944761032457857947711247 39,
358.64341562214226315095255440383993962 36,
299.89866206048251345043788767867861532 4693,
360.06173468217617803564849983592121289 95,
336.59441033525017213085740643498013452 21,
256.10753187217842942532440608408010269 45,
324.65521224863629331884233840008218956 33,
331.93806792758885117728314058792722098 83,
304.79958326676660445332132594523307712 48, none,
289.54595773753628732876201871287122915 92]

2 --> 0  target = [17.93041369698608968930987635930075748167,
4.68650870204468610763928206126753139 1523,
353.30541096326980393747385159638127080 22]
one interval r = 31.37435487008278043449392054977690976917 ..
34.20127520047281870337891671564646498683
Time Approximations 0.014.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., -.59e-35]Solution in 0.32s

Time Plot 0 s.
Exiting SolveHard() after 0.921r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343495192741163282688940748152675 54,
441.64295974758298921820092456482527063 18,
436.91748166779679718046385478592058583 24,
422.98493399009348292645198679709577090 64,
361.52580257451032184399757051561160775 78,
401.88173905737220105505132989243027772 02,
389.59001517255751292106033981823606055 78,
328.46939894800493200524705044920700438 59,
401.50757159481591362792522378348272444 63,
358.97362825373041198125564238456272360 55,
398.33147105601663033106224316729157999 49,
371.48387395775416184379097057912605133 75,
336.61215842482229265792511902216134999 69,
361.50888348488458581191581849361662718 68,
324.67144993885314576155173411368653866 14,
302.31384316086614426301944470878325412 38,
328.46938514873133743727767335072676898 96,
343.81340626306269244657670342633260915 81,
375.73285291568071512675825570419450523 84,
328.11709295713920515490075424989486493 60,
292.99969139581944761032457857947711247 39,
358.64341562214226315095255440383993962 36,
299.89866206048251345043788766786153246 93,
360.06173468217617803564849983592121289 95,
336.59441033525017213085740643498013452 21,
256.10753187217842942532440608408010269 45,
324.65521224863629331884233840008218956 33,
331.93806792758885117728314058792722098 83,
304.79958326676660445332132594523307712 48,
323.46169177890151665945720860058486146 89,
289.54595773753628732876201871287122915 92]

Cascade time 103.944
counts: 28, 28

 Iteration 2

Start Generation 1
1 --> 0  target = [12.00000000004179370945989573038912286100,
6.21701250277489436921668817280061043373 9,
485.54908089419685639874821807519449487 05]
one interval r = 23.40850301651265435643256832863991459592 ..
27.67578046431783846861322220938100553801
Time Approximations 0.038.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
```

```
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=0
Equations at solution: [0., 0., -.2e-36]Solution in 1.306s

Time Plot 0 s.
Exiting SolveHard() after 2.817r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000004179370945989573038912286100,
6.217012502774894369216688172800610433739,
485.549080894196856398748218075194494870 5]
one interval r = 32.628147792100581028942939627477262444 42 ..
36.102483889313322457822066141332655936 50
Time Approximations 0.02.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, .14e-35]Solution in 0.52s

Time Plot 0 s.
Exiting SolveHard() after 1.29r=35.4632 in [33.94922194 .. 36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844811832512316397450611502720 09,
6.583434721643320798284175480025174816764,
467.787305953477134257691872683842550154 6]
```

```
one interval r = 32.41978955658648221382910061047113775422 ..
35.85152417359505904224236360232165033002
Time Approximations 0.019.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=1.2e-37
Equations at solution: [-.14e-36, .12e-36, .314e-34]Solution in 0.587s

Time Plot 0 s.
Exiting SolveHard() after 1.324r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93, none, none,
401.88173903475914048269085362589336636 43, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684481183251231639745061150272009,
6.58343472164332079828417548002517481676 4,
467.78730595347713425769187268384255015 46]
two intervals r = 12.92327160844172610946538132513290584656 ..
47499999999957827345070130865702780 51/2500000000000000000000000000000000
000 or r = 18.39424858019491322654407217823212842044 ..
47499999999957827345070130865702780 51/2500000000000000000000000000000000
000
Time Approximations 0.037.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.2e-38
Equations at solution: [.2e-37, .32e-37, -.1621e-34]Solution in 30.441s

Time Plot 0 s.
```

```
Exiting SolveHard() after 32.452r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72, none,
401.88173903475914048269085362589336636 43, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962815470454674849732658237642282,
4.12565179663632193952376970112880159785 2,
440.67123064608018613874094398896931888 27]
two intervals r = 14.35659705128100245587940053618577463792 ..
474999999999578273450701308657027805 1/2500000000000000000000000000000000
000 or r = 17.70352613794797760339335339430667153250 ..
474999999999578273450701308657027805 1/2500000000000000000000000000000000
000
Time Approximations 0.043.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [-.57e-37, -.3e-37, -.2254e-34]Solution in
1.586s

Time Plot 0 s.
Exiting SolveHard() after 2.918r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72, none,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
1 --> 2  target = [35.46322962815470454674849732658237642282,
4.12565179663632193952376970112880159785 2,
440.67123064608018613874094398896931888 27]
one interval r = 22.39761154362579325188331960773356192611 ..
27.23722351598807505117363992620715419561
Time Approximations 0.035.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.897 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064378107083464753719672018298572, rm =
14.37818770143557449394935602639311148729}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.7e-38
Equations at solution: [-.1e-37, -.27e-37, .321e-34]Solution in 7.061s

Time Plot 0 s.
Exiting SolveHard() after 8.316r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888784183226803645217409095896211,
4.00486908160127176413201045098620214007 7,
404.86224500546319254948634119105239209 14]
two intervals r = 16.08011007783727411255577345624149102909 ..
47499999999957827345070130865702780 51/2500000000000000000000000000000000
000 or r = 16.41579812660318287614539529983597370864 ..
47499999999957827345070130865702780 51/2500000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
```

```
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.50e-37, .1e-37, -.1732e-34]Solution in 1.843s

Time Plot 0 s.
Exiting SolveHard() after 3.697r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35, none, none,
358.97362823173210818648967998849571193 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887841832268036452174090958962 11,
4.0048690816012717641320104509862021400 77,
404.86224500546319254948634119105239209 14]
one interval r = 21.641943994023902619784872988882780110 22 ..
26.763306600356931951003905448495283675 18
Time Approximations 0.046.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=1.01e-37
Equations at solution: [.2e-37, .101e-36, .230e-34]Solution in 1.283s

Time Plot 0 s.
Exiting SolveHard() after 3.01r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92, none,
358.97362823173210818648967998849571193 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941816776327515440104087632393361,
5.5896371827253540677211870548269317408 05,
443.83065883697174397020714593279052676 87]
one interval r = 22.46725374463842965980296822120390962922 ..
27.27388428350107420308483495547605135859
Time Approximations 0.04.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.35e-37
Equations at solution: [-.1e-37, .135e-36, .55e-35]Solution in 1.23s

Time Plot 0 s.
Exiting SolveHard() after 2.586r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92, none,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941816776327515440104087632393361,
```

```
5.589637182725354067721187054826931740805,
443.83065883697174397020714593279052676 87]
one interval r = 32.155752794945081308716308662674806356 15 ..
35.508722287210124571259244668611757213 74
Time Approximations 0.018.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [-.8e-37, .7e-37, -.112e-34]Solution in 0.419s

Time Plot 0 s.
Exiting SolveHard() after 1.096r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136522168413226248287385410294329,
5.18778357833440682456070704012217939774 1,
408.65773862229357925062529105481697935 1]
one interval r = 21.71840114647325358558896874355455557657 ..
26.81849303510270831811380455462587447264
Time Approximations 0.053.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.6e-38
Equations at solution: [0., .26e-37, -.254e-34]Solution in 1.24s

Time Plot 0 s.
Exiting SolveHard() after 2.676r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58, none, none,
361.50888346410488383349304760203845078 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136522168413226248287385410294329,
5.1877835783344068245607070401221793977 41,
408.65773862222935792506252910548169793 51]
one interval r = 31.80828598750196547156541742226520736363 ..
35.00011460035454154662046567157558495325
Time Approximations 0.016.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, .9e-36]Solution in 0.372s

Time Plot 0 s.
Exiting SolveHard() after 1.031r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73, none,
361.50888346410488383349304760203845078 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110533108615644209348336711942018,
6.19626256536225613953246146130493838140 3,
385.44474378578755873512455493319403481 78]
one interval r = 31.60836097534129490036414611991495655097 ..
34.66372795596560377844086194468815130784
Time Approximations 0.015.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., .147e-34]Solution in 0.505s

Time Plot 0 s.
Exiting SolveHard() after 1.131r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73, none,
```

```
361.508883464104883833493047602038450787 4,
324.67144991608020648176565652531552531 67, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110533108615644209348336711942018,
6.19626256536225613953246146130493838140 3,
385.44474378578755873512455493319403481 78]
two intervals r = 16.87563408773844516209530947924921590157 ..
47499999999957827345070130865702780 51/25000000000000000000000000000000000
000 or r = 15.55640493766061845863539494500090580606 ..
47499999999957827345070130865702780 51/25000000000000000000000000000000000
000
Time Approximations 0.053.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 5.162 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175129631384934778477049360398303, rm
= 2.33653277419862753731011081309387473115 9}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.538e-37, 0., .693e-35]Solution in 19.597s

Time Plot 0 s.
Exiting SolveHard() after 21.425r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.508883464104883833493047602038450787 4,
324.67144991608020648176565652531552531 67, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]
```

```
Start Generation 4
1 --> 0  target = [17.19898874755363792162917486536119868714,
4.88381077964122634989891699459669127758 9,
376.61967855025366286727592207986570418 50]
one interval r = 21.11001304871528850783604669544191516431 ..
26.31784243466816447650113996648934472430
Time Approximations 0.03.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.78e-35]Solution in 1.108s

Time Plot 0 s.
Exiting SolveHard() after 2.132r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67, none,
328.46938512494865011877096605082012317 06, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874755363792162917486536119868714,
4.88381077964122634989891699459669127758 9,
376.61967855025366286727592207986570418 50]
one interval r = 31.53899497708512809639295335015270924395 ..
34.53618386079624691919471146356638097809
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
```

```
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=7.44e-36
Equations at solution: [-.571e-35, .744e-35, -.180e-34]Solution in
0.832s

Time Plot 0 s.
Exiting SolveHard() after 1.121r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275387110166145 8,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67, none,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017526156491892038370570113290363,
6.0258135493007029740062016094555052141 46,
351.42702947338045846189106186456663806 68]
one interval r = 31.362302061106667597508489210275213456 89 ..
34.174466405976967372718733420270744665 28
Time Approximations 0.015.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=1.2e-37
```

```
Equations at solution: [-.6e-37, .12e-36, -.23e-35]Solution in 0.855s

Time Plot 0 s.
Exiting SolveHard() after 1.101r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67, none,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96, none, none,
292.99969137026440704222511439408501255 43, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017526156491892038370570113290363,
6.02581354930070297400620160945550521 4146,
351.42702947338045846189106186456663806 68]
two intervals r = 17.98135514465072937447760720308079595791 ..
47499999999957827345070130865702780 51/25000000000000000000000000000000000
000 or r = 13.84608015360931011842583473813050201377 ..
47499999999957827345070130865702780 51/25000000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 3.935 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.91357071373738154430980757073732022015, rm
= 2.7345009934115125824219866974377227522 34}});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
```

```
Equations at solution: [-.71e-37, .2e-37, -.3234e-34]Solution in
12.023s

Time Plot 0 s.
Exiting SolveHard() after 13.351r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67,
302.31384313523530048749320456038334881 61,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96, none, none,
292.99969137026440704222511439408501255 43, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941863623050253330065614468864787,
6.37794387388922441034547438390672250057 8,
423.28832782842145564331725497012294988 68]
one interval r = 31.94661817588686097649317033745236487913 ..
35.21212308631547740463669127389788770013
Time Approximations 0.016.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.80e-35]Solution in 0.533s

Time Plot 0 s.
Exiting SolveHard() after 1.209r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67,
302.31384313523530048749320456038334881 61,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96, none, none,
292.99969137026440704222511439408501255 43, none, none,
360.06173465459086303113161763591769095 35, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941863623050253330065614468864787,
6.37794387388922441034547438390672250057 8,
423.28832782842145564331725497012294988 68]
two intervals r = 15.22886702479522802691680214935837355833 ..
474999999999578273450701308657027805 1/2500000000000000000000000000000000
000 or r = 17.12965777032318652081488432837654260979 ..
474999999999578273450701308657027805 1/2500000000000000000000000000000000
000
Time Approximations 0.059.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 4.867 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541557962426173954681991922592 14, rm
= 2.0640682988056813128725245705185010950 02}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.31e-37, -.1e-37, .3799e-34]Solution in
19.495s

Time Plot 0 s.
Exiting SolveHard() after 21.505r=16.5334 in [15.22886699 .. 19]
```

```
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67,
302.31384313523530048749320456038334881 61,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96,
375.73285288682514473310785166213215296 51, none,
292.99969137026440704222511439408501255 43, none, none,
360.06173465459086303113161763591769095 35, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343268443418640669618853139742 90,
4.0035598152983872242827536541287744530 83,
404.47973593166555762919109291774593846 83]
two intervals r = 16.096839663982996156031843113060656428 61 ..
474999999999578273450701308657027805 1/2500000000000000000000000000000000
000 or r = 16.399886490765687508437726299830111304 06 ..
474999999999578273450701308657027805 1/2500000000000000000000000000000000
000
Time Approximations 0.41.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.49e-37, -.1e-37, .322e-35]Solution in 1.955s

Time Plot 0 s.
Exiting SolveHard() after 3.706r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67,
302.31384313523530048749320456038334881 61,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96,
375.73285288682514473310785166213215296 51, none,
292.99969137026440704222511439408501255 43,
358.64341559948893394222004229808612295 98, none,
360.06173465459086303113161763591769095 35, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234326844341864066961885313974290,
4.0035598152983872242827536541287744530 83,
404.47973593166555762919109291774593846 83]
one interval r = 21.63429629977977683055764531945613341454 ..
26.75768169884220716062207038881792872404
Time Approximations 0.045.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., -.200e-34]Solution in 1.277s

Time Plot 0 s.
Exiting SolveHard() after 3.116r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
```

```
441.642959726860242431244176751434314601 2,
436.917481648305723135128295865577926939 3,
422.984933966613395423943310526363814117 2,
361.525802553623189063557645209296489882 0,
401.881739034759140482690853625893366364 3,
389.590015153596173979175042989912217343 5,
328.469398924219384534984572099762741299 2,
401.507571571464534913204747883695232883 2,
358.973628231732108186489679988495711938 8,
398.331471030130859324252753871101661458,
371.483873938187988279368340057080910897 3,
336.612158401777563189355111149820681861 0,
361.508883464104883833493047602038450787 4,
324.671449916080206481765656525315525316 7,
302.313843135235300487493204560383348816 1,
328.469385124948650118770966050820123170 6,
343.813406240494794704567777076506582569 6,
375.732852886825144733107851662132152965 1,
328.117092932659096308812808595059693078 2,
292.999691370264407042225114394085012554 3,
358.643415599488933942220042298086122959 8, none,
360.061734654590863031131617635917690953 5, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954451471282595274700326442836079,
6.196177230216293676046592153623015901061,
385.42734024964436807614691484706748417 95]
one interval r = 31.60822049089580364512118119782786828979 ..
34.66347615034374753459602410588275676389
Time Approximations 0.016.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.199e-34]Solution in 0.888s

Time Plot 0 s.
Exiting SolveHard() after 1.167r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.642959726860242431244176751434314601 2,
436.917481648305723135128295865577926939 3,
422.984933966613395423943310526363814117 2,
```

```
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67,
302.31384313523530048749320456038334881 61,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96,
375.73285288682514473310785166213215296 51,
328.11709293265909630881280859505969307 82,
292.99969137026440704222511439408501255 43,
358.64341559948893394222004229808612295 98, none,
360.06173465459086303113161763591769095 35, none, none,
324.65521222596645499597262271281807057 64, none, none, none, none]

0 --> 1  target = [26.46318954451471282595274700326442836079,
6.19617723021629367604659215362301590106 1,
385.42734024964436807614691484706748417 95]
two intervals r = 16.87629600312078882878624727515605852088 ..
474999999999578273450701308657027805 1/2500000000000000000000000000000000
000 or r = 15.55559000615108026027389573011324997083 ..
474999999999578273450701308657027805 1/2500000000000000000000000000000000
000
Time Approximations 0.06.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 5.677 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852541160013254676154635613362020, rm
= 2.33669042826305467334984697068131642961 9}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.718e-37, 0., -.1651e-34]Solution in 19.952s

Time Plot 0 s.
Exiting SolveHard() after 21.409r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67,
302.31384313523530048749320456038334881 61,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96,
375.73285288682514473310785166213215296 51,
328.11709293265909630881280859505969307 82,
292.99969137026440704222511439408501255 43,
358.64341559948893394222004229808612295 98, none,
360.06173465459086303113161763591769095 35,
336.59441031231834006912266985424835169 51, none,
324.65521222596645499597262271281807057 64, none, none, none, none]

0 --> 2  target = [34.49522661154839080832919308531303021116,
3.8971313157260590669822292020653938315 77,
373.78081883897012184245747519639421291 64]
two intervals r = 17.297690862360351633052208158436499674 23 ..
474999999999578273450701308657027805 1/2500000000000000000000000000000000000
000 or r = 14.994364074018880379235878063248674613 23 ..
474999999999578273450701308657027805 1/2500000000000000000000000000000000000
000
Time Approximations 0.073.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.72e-37, 0., -.510e-35]Solution in 1.457s

Time Plot 0 s.
Exiting SolveHard() after 3.698r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67,
302.31384313523530048749320456038334881 61,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96,
375.73285288682514473310785166213215296 51,
328.11709293265909630881280859505969307 82,
292.99969137026440704222511439408501255 43,
358.64341559948893394222004229808612295 98, none,
360.06173465459086303113161763591769095 35,
336.59441031231834006912266985424835169 51, none,
324.65521222596645499597262271281807057 64,
331.93806790806639992847968520305099393 55, none, none, none]

1 --> 2  target = [34.49522661154839080832919308531303021116,
3.8971313157260590669822292020653938315 77,
373.78081883897012184245747519639421291 64]
one interval r = 21.06068473201403551467390265401570973444 ..
26.26979834278683626989685159752758031516
Time Approximations 0.031.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=4e-38
Equations at solution: [-.2e-37, -.4e-37, .106e-34]Solution in 1.124s

Time Plot 0 s.
Exiting SolveHard() after 2.172r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
```

```
441.642959726860242431244176751434314601 2,
436.917481648305723135128295865577926939 3,
422.984933966613395423943310526363814117 2,
361.525802553623189063557645209296489882 0,
401.881739034759140482690853625893366364 3,
389.590015153596173979175042989912217343 5,
328.469398924219384534984572099762741299 2,
401.507571571464534913204747883695232883 2,
358.973628231732108186489679988495711938 8,
398.331471030130859324252753871101661458,
371.483873938187988279368340057080910897 3,
336.612158401777563189355111149820681861 0,
361.508883464104883833493047602038450787 4,
324.671449916080206481765656525315525316 7,
302.313843135235300487493204560383348816 1,
328.469385124948650118770966050820123170 6,
343.813406240494794704567777076506582569 6,
375.732852886825144733107851662132152965 1,
328.117092932659096308812808595059693078 2,
292.999691370264407042225114394085012554 3,
358.643415599488933942220042298086122959 8,
299.898662039606479424643745364385208182 7,
360.061734654590863031131617635917690953 5,
336.594410312318340069122669854248351695 1, none,
324.655212225966454995972622712818070576 4,
331.938067908066399928479685203050993935 5, none, none, none]

0 --> 2  target = [33.813624953942346045127206147425333607 20,
3.725648993341787137989004231135685108253,
325.892099718380914042177904918701978783 8]
two intervals r = 18.552270490184111462110549232039249175 64 ..
474999999999578273450701308657027805 1/250000000000000000000000000000000
000 or r = 12.491969357319693859048488710112408414 95 ..
474999999999578273450701308657027805 1/250000000000000000000000000000000
000
Time Approximations 0.036.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [.140e-36, -.6e-37, .512e-35]Solution in 1.38s

Time Plot 0 s.
Exiting SolveHard() after 3.457r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67,
302.31384313523530048749320456038334881 61,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96,
375.73285288682514473310785166213215296 51,
328.11709293265909630881280859505969307 82,
292.99969137026440704222511439408501255 43,
358.64341559948893394222004229808612295 98,
299.89866203960647942464374536438520818 27,
360.06173465459086303113161763591769095 35,
336.59441031231834006912266985424835169 51, none,
324.65521222596645499597262271281807057 64,
331.93806790806639992847968520305099393 55, none, none,
289.54595771483984650469261939148541617 54]

1 --> 2  target = [33.81362495394234604512720614742533360720,
3.72564899334178713798900423113568510825 3,
325.89209971838091404217790491870197878 38]
one interval r = 20.37468935104372120535297618105252705256 ..
25.37892165284430339404814636569025012896
Time Approximations 0.023.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., -.132e-34]Solution in 0.501s

Time Plot 0 s.
Exiting SolveHard() after 1.367r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67,
302.31384313523530048749320456038334881 61,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96,
375.73285288682514473310785166213215296 51,
328.11709293265909630881280859505969307 82,
292.99969137026440704222511439408501255 43,
358.64341559948893394222004229808612295 98,
299.89866203960647942464374536438520818 27,
360.06173465459086303113161763591769095 35,
336.59441031231834006912266985424835169 51,
256.10753184837789861519227112929099661 57,
324.65521222596645499597262271281807057 64,
331.93806790806639992847968520305099393 55, none, none,
289.54595771483984650469261939148541617 54]

1 --> 0  target = [17.93041369732431541079549913961234537077,
4.68650870176006348428911566723487080736 1,
353.30541093939867454997514605660721637 17]
one interval r = 20.73150479083064744957970167783538798229 ..
25.90675353506950096650594272334163157641
Time Approximations 0.029.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.2e-38
Equations at solution: [-.1e-37, -.22e-37, .287e-34]Solution in 0.96s

Time Plot 0 s.
Exiting SolveHard() after 2.025r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67,
302.31384313523530048749320456038334881 61,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96,
375.73285288682514473310785166213215296 51,
328.11709293265909630881280859505969307 82,
292.99969137026440704222511439408501255 43,
358.64341559948893394222004229808612295 98,
299.89866203960647942464374536438520818 27,
360.06173465459086303113161763591769095 35,
336.59441031231834006912266985424835169 51,
256.10753184837789861519227112929099661 57,
324.65521222596645499597262271281807057 64,
331.93806790806639992847968520305099393 55,
304.79958324213644084490772403937313693 05, none,
289.54595771483984650469261939148541617 54]

2 --> 0  target = [17.93041369732431541079549913961234537077,
4.68650870176006348428911566723487080736 1,
353.30541093939867454997514605660721637 17]
one interval r = 31.37435486989623342738375219544792187237 ..
34.20127520009961589076241014855823059591
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.61e-35]Solution in 0.356s

Time Plot 0 s.
Exiting SolveHard() after 0.626r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
```

```
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493246439589830347928652522941 65,
441.64295972686024243124417675143431460 12,
436.91748164830572313512829586557792693 93,
422.98493396661339542394331052636381411 72,
361.52580255362318906355764520929648988 20,
401.88173903475914048269085362589336636 43,
389.59001515359617397917504298991221734 35,
328.46939892421938453498457209976274129 92,
401.50757157146453491320474788369523288 32,
358.97362823173210818648967998849571193 88,
398.33147103013085932425275378711016614 58,
371.48387393818798827936834005708091089 73,
336.61215840177756318935511114982068186 10,
361.50888346410488383349304760203845078 74,
324.67144991608020648176565652531552531 67,
302.31384313523530048749320456038334881 61,
328.46938512494865011877096605082012317 06,
343.81340624049479470456777707650658256 96,
375.73285288682514473310785166213215296 51,
328.11709293265909630881280859505969307 82,
292.99969137026440704222511439408501255 43,
358.64341559948893394222004229808612295 98,
299.89866203960647942464374536438520818 27,
360.06173465459086303113161763591769095 35,
336.59441031231834006912266985424835169 51,
256.10753184837789861519227112929099661 57,
324.65521222596645499597262271281807057 64,
331.93806790806639992847968520305099393 55,
304.79958324213644084490772403937313693 05,
323.46169175520156084263011682900537155 61,
289.54595771483984650469261939148541617 54]

Cascade time 168.987
counts: 28, 28

 Iteration 3

Start Generation 1
1 --> 0  target = [11.999999999870866270265172518936348018 00,
6.2170125028925632225856037197773577944 23,
485.54908089927159126078967022015472099 12]
one interval r = 23.408503016604213129640492752858362055 14 ..
27.675780464334568449126510994769679638 47
Time Approximations 0.036.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.4921114030735554665825127038899393785 94, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
```

```
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.5e-38
Equations at solution: [-.1e-37, .25e-37, .361e-36]Solution in 1.265s

Time Plot 0 s.
Exiting SolveHard() after 2.701r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999870866270265172518936348018 00,
6.2170125028925632225856037197773577944 23,
485.54908089927159126078967022015472099 12]
one interval r = 32.628147792161266442075830687657379513 96 ..
36.102483889447682842910971607925486818 76
Time Approximations 0.02.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=1.0e-37
Equations at solution: [.15e-36, -.10e-36, -.11e-35]Solution in 0.518s

Time Plot 0 s.
Exiting SolveHard() after 1.285r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
```

```
2 --> 1  target = [27.52359684484541332768095192050950224988,
6.583434721614183390193729507839409327961,
467.78730595940028335064862154919187578 23]
one interval r = 32.41978955664991980302143921186448001829 ..
35.85152417373799106240102023276877146558
Time Approximations 0.02.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .151e-34]Solution in 0.946s

Time Plot 0 s.
Exiting SolveHard() after 1.295r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194 14981,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65, none, none,
401.88173904273531129113884480231586875 83, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684484541332768095192050950224988,
6.583434721614183390193729507839409327961,
467.78730595940028335064862154919187578 23]
two intervals r = 12.92327160822611402521583212059543739155 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
or r = 18.39424858039182780881701320637658322794 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
Time Approximations 0.037.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.2e-38
Equations at solution: [-.3e-37, -.32e-37, .12245e-34]Solution in
```

```
28.726s

Time Plot 0 s.
Exiting SolveHard() after 30.907r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91, none,
401.88173904273531129113884480231586875 83, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962828425153335648894154319137125,
4.12565179678677309315476858811136695 2399,
440.67123065146933545316008887603999018 21]
two intervals r = 14.35659705111367577772346230775920503577 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
or r = 17.70352613817552387097915437346382155383 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
Time Approximations 0.041.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [.57e-37, .3e-37, -.44562e-34]Solution in 1.627s

Time Plot 0 s.
Exiting SolveHard() after 3.457r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91, none,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962828425153335648894154319137125,
4.12565179678677309315476858811136695239 9,
440.67123065146933545316008887603999018 21]
one interval r = 22.39761154371764622230349442820869635573 ..
27.23722351603037002196815844909488295926
Time Approximations 0.033.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.47 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064387758303038845994217814116757, rm =
14.37818770295731945219916684203322309324}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., .26e-37, -.8107e-35]Solution in 6.818s

Time Plot 0 s.
Exiting SolveHard() after 8.068r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888799919116411562585141736260048,
4.00486908176113106511503793144348699120 0,
404.86224501336974956144548576386562917 30]
two intervals r = 16.08011007760635466249043270293410820104 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
or r = 16.41579812698393799684863199648021731214 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
Time Approximations 0.052.
```

```
hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.17e-37, .1e-37, .10600e-34]Solution in 2.077s

Time Plot 0 s.
Exiting SolveHard() after 3.501r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19, none, none,
358.97362823995622178823568288776157605 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888799919116411562585141736260048,
4.0048690817611310651150379314434869912 00,
404.86224501336974956144548576386562917 30]
one interval r = 21.641943994153295795651878576226357582 55 ..
26.763306600452860465492660121107516394 50
Time Approximations 0.047.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=1.00e-37
Equations at solution: [-.3e-37, -.100e-36, -.37024e-34]Solution in
1.32s

Time Plot 0 s.
Exiting SolveHard() after 2.734r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
```

```
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91, none,
358.97362823995622178823568288776157605 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941783946425432787228181866125219,
5.58963718287021436970291008784471194607 8,
443.83065884538717812214348935281662877 84]
one interval r = 22.46725374479796795436269478158936793818 ..
27.27388428357652083957009661289617669379
Time Approximations 0.033.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .4091e-35]Solution in 1.24s

Time Plot 0 s.
Exiting SolveHard() after 2.565r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91, none,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none]

2 --> 0  target = [14.19258941783946425432787228181866125219,
5.589637182870214369702910087844711946078,
443.83065884538717812214348935281662877 84]
one interval r = 32.15575279502458298160247310863671305788 ..
35.50872228738393058104342583517639375953
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, -.95e-35]Solution in 0.833s

Time Plot 0 s.
Exiting SolveHard() after 1.163r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136506782129198931930588435395616,
5.187783578439070446947238698956579072928,
408.65773862823452089756202786563639039 30]
one interval r = 21.71840114656645627425032633965524152596 ..
26.81849303516884475807653777588195025412
Time Approximations 0.056.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
```

```
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=5.2e-38
Equations at solution: [-.1e-37, -.52e-37, -.6090e-35]Solution in
1.331s

Time Plot 0 s.
Exiting SolveHard() after 2.795r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33, none, none,
361.50888347161210753433986838336213963 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.911931365067821291989319305884353956 16,
5.1877835784390704469472386989565790729 28,
408.65773862823452089756202786563639039 30]
one interval r = 31.808285987540269718047817720034754366 31 ..
35.000114600485306563935241694391110170 80
Time Approximations 0.024.

hint used Hint := [34.495226611650080820729149248019775312 21, 3, 1, 1,
15.763855890238725343023875204542137417 16, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=7e-38
Equations at solution: [-.7e-37, .7e-37, -.310e-34]Solution in 0.795s

Time Plot 0 s.
Exiting SolveHard() after 1.109r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
```

```
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29, none,
361.50888347161210753433986838336213963 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110544108546723694386505254908669,
6.19626256534530167846570898125200274995 8,
385.44474379382617455870682103930284731 21]
one interval r = 31.60836097538549923813218424350782479740 ..
34.66372795611958295900122555315475409892
Time Approximations 0.016.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.161e-34]Solution in 0.507s

Time Plot 0 s.
Exiting SolveHard() after 0.775r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
```

```
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29, none,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110544108546723694386505254908669,
6.1962625653453016784657089812520027499 58,
385.44474379382617455870682103930284731 21]
two intervals r = 16.875634087546802701793448476937922358 63 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
or r = 15.556404938081378608314621457518192605 81 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
Time Approximations 0.051.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.717e-37, 0., -.4672e-35]Solution in 1.426s

Time Plot 0 s.
Exiting SolveHard() after 3.279r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29,
336.61215841204347251440655318889525382 21,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
```

```
1 --> 0  target = [17.19898874737112226403325230455917459467,
4.88381077976239161676151264780002985358 2,
376.61967855864425878606176606169631906 75]
one interval r = 21.11001304883194372974837824990360774479 ..
26.31784243478928956704462134833602641360
Time Approximations 0.032.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=5.1e-38
Equations at solution: [-.1e-37, -.51e-37, -.10592e-34]Solution in
0.774s

Time Plot 0 s.
Exiting SolveHard() after 1.839r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29,
336.61215841204347251440655318889525382 21,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51, none,
328.46938513489206776398157296507304231 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874737112226403325230455917459467,
4.88381077976239161676151264780002985358 2,
376.61967855864425878606176606169631906 75]
one interval r = 31.53899497712714825627911223476653174295 ..
34.53618386095273198954616476116036590320
Time Approximations 0.015.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
```

```
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=9.17e-36
Equations at solution: [.704e-35, -.917e-35, -.39e-35]Solution in
0.439s

Time Plot 0 s.
Exiting SolveHard() after 1.141r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29,
336.61215841204347251440655318889525382 21,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51, none,
328.46938513489206776398157296507304231 96,
343.81340624957260829431384566897971443 01, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017543504535247975554813863783891,
6.0258135492977223197447515593127643706 81,
351.42702948382652632220966385663051230 99]
one interval r = 31.362302061147324767472494058063588238 64 ..
34.174466406154781041244401382903645538 79
Time Approximations 0.015.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
```

```
Equations at solution: [.3e-37, -.6e-37, -.168e-34]Solution in 0.85s

Time Plot 0 s.
Exiting SolveHard() after 1.071r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29,
336.61215841204347251440655318889525382 21,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51, none,
328.46938513489206776398157296507304231 96,
343.81340624957260829431384566897971443 01, none, none,
292.99969138264927181839866981495059543 38, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175435045352479755548138637838 91,
6.0258135492977223197447515593127643706 81,
351.42702948382652632220966385663051230 99]
two intervals r = 17.981355144479706258785029410484165639 65 ..
37109375000144373420254467145821143/195312500000000000000000000000000000
or r = 13.846080154189311535218144144960793991 27 ..
37109375000144373420254467145821143/195312500000000000000000000000000000
Time Approximations 0.039.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [-.90e-37, .3e-37, -.273e-36]Solution in 1.418s

Time Plot 0 s.
Exiting SolveHard() after 2.772r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29,
336.61215841204347251440655318889525382 21,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51,
302.31384314768898313574009339214694822 57,
328.46938513489206776398157296507304231 96,
343.81340624957260829431384566897971443 01, none, none,
292.99969138264927181839866981495059543 38, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941875245037178022289581293552938,
6.37794387388177299621994917876552362 2625,
423.28832783878060836563284888435467590 68]
one interval r = 31.94661817597425914093834722998997869869 ..
35.21212308651290152319764331572018709201
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.181e-34]Solution in 0.927s

Time Plot 0 s.
Exiting SolveHard() after 1.237r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
```

```
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29,
336.61215841204347251440655318889525382 21,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51,
302.31384314768898313574009339214694822 57,
328.46938513489206776398157296507304231 96,
343.81340624957260829431384566897971443 01, none, none,
292.99969138264927181839866981495059543 38, none, none,
360.06173466680263707095358248165253403 39, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941875245037178022289581293552938,
6.37794387388177299621994917876552362262 5,
423.28832783878060836563284888435467590 68]
two intervals r = 15.22886702440791743123547299531108970672 ..
37109375000144373420254467145821143/195312500000000000000000000000000000
or r = 17.12965777075381905311314654460457679891 ..
37109375000144373420254467145821143/195312500000000000000000000000000000
Time Approximations 0.46.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [.46e-37, .2e-37, -.964e-36]Solution in 1.559s

Time Plot 0 s.
Exiting SolveHard() after 3.613r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
```

```
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29,
336.61215841204347251440655318889525382 21,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51,
302.31384314768898313574009339214694822 57,
328.46938513489206776398157296507304231 96,
343.81340624957260829431384566897971443 01,
375.73285289973575894797523012498439065 78, none,
292.99969138264927181839866981495059543 38, none, none,
360.06173466680263707095358248165253403 39, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234343592601266667873178568494014,
4.0035598154606711042548407237785856088 11,
404.47973594027713632825300319230439174 60]
two intervals r = 16.09683966372211313778786563371904572313 ..
37109375000144373420254467145821143/195312500000000000000000000000000000
or r = 16.39988649117654768483408837021815806153 ..
37109375000144373420254467145821143/195312500000000000000000000000000000
Time Approximations 0.046.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.52e-37, -.1e-37, -.4781e-35]Solution in 1.75s

Time Plot 0 s.
Exiting SolveHard() after 3.112r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29,
```

```
336.61215841204347251440655318889525382 21,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51,
302.31384314768898313574009339214694822 57,
328.46938513489206776398157296507304231 96,
343.81340624957260829431384566897971443 01,
375.73285289973575894797523012498439065 78, none,
292.99969138264927181839866981495059543 38,
358.64341560832328847749662421923200620 32, none,
360.06173466680263707095358248165253403 39, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234343592601266667873178568494014,
4.0035598154606711042548407237785856088 11,
404.47973594027713632825300319230439174 60]
one interval r = 21.634296299923011031815588325467317959 33 ..
26.757681698948757255467275237962042268 12
Time Approximations 0.044.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.1e-38
Equations at solution: [.2e-37, .51e-37, -.28215e-34]Solution in 1.357s

Time Plot 0 s.
Exiting SolveHard() after 3.146r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29,
336.61215841204347251440655318889525382 21,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51,
302.31384314768898313574009339214694822 57,
```

```
328.46938513489206776398157296507304231 96,
343.81340624957260829431384566897971443 01,
375.73285289973575894797523012498439065 78,
328.11709294325468733736651497327608655 41,
292.99969138264927181839866981495059543 38,
358.64341560832328847749662421923200620 32, none,
360.06173466680263707095358248165253403 39, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954462293982013336367176643020460,
6.19617723019879965622356849047797541429 8,
385.42734025757279305140141384591475407 98]
one interval r = 31.60822049093910895760357679062630147372 ..
34.66347615049612743212255625702548947852
Time Approximations 0.015.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=9e-38
Equations at solution: [.6e-37, -.9e-37, -.156e-34]Solution in 0.508s

Time Plot 0 s.
Exiting SolveHard() after 0.776r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29,
336.61215841204347251440655318889525382 21,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51,
302.31384314768898313574009339214694822 57,
328.46938513489206776398157296507304231 96,
343.81340624957260829431384566897971443 01,
375.73285289973575894797523012498439065 78,
```

```
328.1170929432546873373665149732760865541,
292.9996913826492718183986698149505954338,
358.6434156083232884774966242192320062032, none,
360.0617346668026370709535824816525340339, none, none,
324.6552122359576544204750235758743219839, none, none, none, none]

0 --> 1  target = [26.46318954462293982013336367176643020460,
6.196177230198799656223568490477975414298,
385.42734025757279305140141384591475407 98]
two intervals r = 16.87629600293337892282046741930697873789 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
or r = 15.55559000656670741917836970765653487193 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
Time Approximations 0.051.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.179e-37, 0., -.4038e-35]Solution in 1.441s

Time Plot 0 s.
Exiting SolveHard() after 3.304r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349374346535195717959077719414981,
441.6429597324253811668962847427717158763,
436.9174816538226480537049007358636171965,
422.9849339747850008407179879610260914191,
361.5258025612375392834290772563573710799,
401.8817390427353112911388448023158687583,
389.5900151595190915567115206228755081319,
328.4693989341656427540139874398168025291,
401.5075715801303053464289948193385200249,
358.9736282399562217882356828877615760547,
398.3314710400086897166541810382010451933,
371.4838739450339347828241739440595995129,
336.6121584120434725144065531888952538221,
361.5088834716121075343398683833621396325,
324.6714499261741882018843976840345558451,
302.3138431476889831357400933921469482257,
328.4693851348920677639815729650730423196,
343.8134062495726082943138456689797144301,
375.7328528997357589479752301249843906578,
328.1170929432546873373665149732760865541,
292.9996913826492718183986698149505954338,
```

```
358.6434156083232884774966242192320062032, none,
360.0617346668026370709535824816525340339,
336.5944103224717691656906843736990720169, none,
324.6552122359576544204750235758743219839, none, none, none, none]

0 --> 2  target = [34.4952266116804262398331560144008774219 5,
3.8971313158827611004917261246078871226 77,
373.7808188457245475171617554229454721423]
two intervals r = 17.297690862241318375997893991679084814 15 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
or r = 14.994364074392195295688567916077914277 31 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
Time Approximations 0.073.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [.90e-37, -.1e-37, .12053e-34]Solution in 1.48s

Time Plot 0 s.
Exiting SolveHard() after 3.845r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349374346535195717959077719414981,
441.6429597324253811668962847427717158763,
436.9174816538226480537049007358636171965,
422.9849339747850008407179879610260914191,
361.5258025612375392834290772563573710799,
401.8817390427353112911388448023158687583,
389.5900151595190915567115206228755081319,
328.4693989341656427540139874398168025291,
401.5075715801303053464289948193385200249,
358.9736282399562217882356828877615760547,
398.3314710400086897166541810382010451933,
371.4838739450339347828241739440595995129,
336.6121584120434725144065531888952538221,
361.5088834716121075343398683833621396325,
324.6714499261741882018843976840345558451,
302.3138431476889831357400933921469482257,
328.4693851348920677639815729650730423196,
343.8134062495726082943138456689797144301,
375.7328528997357589479752301249843906578,
328.1170929432546873373665149732760865541,
292.9996913826492718183986698149505954338,
358.6434156083232884774966242192320062032, none,
```

```
360.06173466680263707095358248165253403 39,
336.59441032247176916569068437369907201 69, none,
324.65521223595765442047502357587432198 39,
331.93806791538760773319020273243805519 62, none, none, none]

1 --> 2  target = [34.49522661168042623983315601440087742195,
3.89713131588276110049172612460788712267 7,
373.78081884572454751716175542294547214 23]
one interval r = 21.06068473209987956335763018881701184748 ..
26.26979834288180722384585987097778486936
Time Approximations 0.029.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, .33418e-34]Solution in 0.689s

Time Plot 0 s.
Exiting SolveHard() after 1.733r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194 14981,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29,
336.61215841204347251440655318889525382 21,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51,
302.31384314768898313574009339214694822 57,
328.46938513489206776398157296507304231 96,
343.81340624957260829431384566897971443 01,
375.73285289973575894797523012498439065 78,
328.11709294325468733736651497327608655 41,
292.99969138264927181839866981495059543 38,
358.64341560832328847749662421923200620 32,
299.89866204849350075262114337074588255 97,
```

```
360.06173466680263707095358248165253403 39,
336.59441032247176916569068437369907201 69, none,
324.65521223595765442047502357587432198 39,
331.93806791538760773319020273243805519 62, none, none, none]

0 --> 2  target = [33.81362495410548296124824621798351686296,
3.72564899351179740246828957596738752396 6,
325.89209972846380716609952222648466956 88]
two intervals r = 18.552270490107316698483405558447431316 93 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
or r = 12.491969357860410032134096797932105984 83 ..
371093750001443734202544671458211 43/195312500000000000000000000000000000
Time Approximations 0.035.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=9e-38
Equations at solution: [-.227e-36, .9e-37, -.2120e-35]Solution in
1.476s

Time Plot 0 s.
Exiting SolveHard() after 3.543r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.64295973242538116689628474277171587 63,
436.91748165382264805370490073586361719 65,
422.98493397478500084071798796102609141 91,
361.52580256123753928342907725635737107 99,
401.88173904273531129113884480231586875 83,
389.59001515951909155671152062287550813 19,
328.46939893416564275401398743981680252 91,
401.50757158013030534642899481933852002 49,
358.97362823995622178823568288776157605 47,
398.33147104000868971665418103820104519 33,
371.48387394503393478282417394405959951 29,
336.61215841204347251440655318889525382 21,
361.50888347161210753433986838336213963 25,
324.67144992617418820188439768403455584 51,
302.31384314768898313574009339214694822 57,
328.46938513489206776398157296507304231 96,
343.81340624957260829431384566897971443 01,
375.73285289973575894797523012498439065 78,
328.11709294325468733736651497327608655 41,
292.99969138264927181839866981495059543 38,
```

```
358.6434156083232884774966242192320062032,
299.8986620484935007526211433707458825597,
360.0617346668026370709535824816525340339,
336.5944103224717691656906843736990720169, none,
324.6552122359576544204750235758743219839,
331.9380679153876077331902027324380551962, none, none,
289.5459577252000157405472315121780249076]

1 --> 2  target = [33.81362495410548296124824621798351686296,
3.7256489935117974024682895759673875239 66,
325.8920997284638071660995222264846695688]
one interval r = 20.37468935112293850141242585447779575490 ..
25.37892165302589114615328461887606529148
Time Approximations 0.024.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .7016e-35]Solution in 0.518s

Time Plot 0 s.
Exiting SolveHard() after 1.453r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493743465351957179590777194149 81,
441.6429597324253811668962847427717158763,
436.9174816538226480537049007358636171965,
422.9849339747850008407179879610260914191,
361.5258025612375392834290772563573710799,
401.8817390427353112911388448023158687583,
389.5900151595190915567115206228755081319,
328.4693989341656427540139874398168025291,
401.5075715801303053464289948193385200249,
358.9736282399562217882356828877615760547,
398.3314710400086897166541810382010451933,
371.4838739450339347828241739440595995129,
336.6121584120434725144065531888952538221,
361.5088834716121075343398683833621396325,
324.6714499261741882018843976840345558451,
302.3138431476889831357400933921469482257,
328.4693851348920677639815729650730423196,
343.8134062495726082943138456689797144301,
375.7328528997357589479752301249843906578,
328.1170929432546873373665149732760865541,
```

```
292.99969138264927181839866981495059543 38,
358.6434156083232884774966242192320062032,
299.8986620484935007526211433707458825597,
360.0617346668026370709535824816525340339,
336.5944103224717691656906843736990720169,
256.1075318602963506283943741076396488476,
324.6552122359576544204750235758743219839,
331.9380679153876077331902027324380551962, none, none,
289.5459577252000157405472315121780249076]

1 --> 0  target = [17.9304136971453351040384459429314451753 1,
4.6865087018951011890586344590336258076 14,
353.3054109499024730545513627462007238425]
one interval r = 20.7315047909552352821656126683569586582 4 ..
25.9067535352428022253401112757718064665 5
Time Approximations 0.03.

hint used Hint := [25.4020715378188039710536173439704738275 4, 3, -1, 1,
17.0062444547662134530206245122467435705 3, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=6.9e-38
Equations at solution: [-.3e-37, -.69e-37, .27663e-34]Solution in
1.001s

Time Plot 0 s.
Exiting SolveHard() after 2.181r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349374346535195717959077719414981,
441.6429597324253811668962847427717158763,
436.9174816538226480537049007358636171965,
422.9849339747850008407179879610260914191,
361.5258025612375392834290772563573710799,
401.8817390427353112911388448023158687583,
389.5900151595190915567115206228755081319,
328.4693989341656427540139874398168025291,
401.5075715801303053464289948193385200249,
358.9736282399562217882356828877615760547,
398.3314710400086897166541810382010451933,
371.4838739450339347828241739440595995129,
336.6121584120434725144065531888952538221,
361.5088834716121075343398683833621396325,
324.6714499261741882018843976840345558451,
302.3138431476889831357400933921469482257,
328.4693851348920677639815729650730423196,
```

```
343.8134062495726082943138456689797144301,
375.7328528997357589479752301249843906578,
328.1170929432546873373665149732760865541,
292.9996913826492718183986698149505954338,
358.6434156083232884774966242192320062032,
299.8986620484935007526211433707458825597,
360.0617346668026370709535824816525340339,
336.5944103224717691656906843736990720169,
256.1075318602963506283943741076396488476,
324.6552122359576544204750235758743219839,
331.9380679153876077331902027324380551962,
304.7995832541418151721282483237452713288, none,
289.5459577252000157405472315121780249076]

2 --> 0  target = [17.93041369714533510403844594293144517531,
4.686508701895101189058634459033625807614,
353.3054109499024730545513627462007238425]
one interval r = 31.37435486993854605891710399317629370195 ..
34.20127520027891453096120731696676262026
Time Approximations 0.014.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., -.131e-34]Solution in 0.356s

Time Plot 0 s.
Exiting SolveHard() after 0.619r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349374346535195717959077719414981,
441.6429597324253811668962847427717158763,
436.9174816538226480537049007358636171965,
422.9849339747850008407179879610260914191,
361.5258025612375392834290772563573710799,
401.8817390427353112911388448023158687583,
389.5900151595190915567115206228755081319,
328.4693989341656427540139874398168025291,
401.5075715801303053464289948193385200249,
358.9736282399562217882356828877615760547,
398.3314710400086897166541810382010451933,
371.4838739450339347828241739440595995129,
336.6121584120434725144065531888952538221,
361.5088834716121075343398683833621396325,
```

```
324.67144992617418820188439768403455584 51,
302.31384314768898313574009339214694822 57,
328.46938513489206776398157296507304231 96,
343.81340624957260829431384566897971443 01,
375.73285289973575894797523012498439065 78,
328.11709294325468733736651497327608655 41,
292.99969138264927181839866981495059543 38,
358.64341560832328847749662421923200620 32,
299.89866204849350075262114337074588255 97,
360.06173466680263707095358248165253403 39,
336.59441032247176916569068437369907201 69,
256.10753186029635062839437410763964884 76,
324.65521223595765442047502357587432198 39,
331.93806791538760773319020273243805519 62,
304.79958325414181517212824832374527132 88,
323.46169176623337981006371294097809460 18,
289.54595772520001574054723151217802490 76]

Cascade time 101.108
counts: 28, 28

 Iteration 4

Start Generation 1
1 --> 0  target = [11.99999999999289707130714472331375316000,
6.21701250303714293129469587586713604013 4,
485.54908089577638494882648452091443495 41]
one interval r = 23.40850301645322825404987990887279387005 ..
27.67578046424296178293599811806286916510
Time Approximations 0.035.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.32e-37
Equations at solution: [-.4e-37, .132e-36, .6e-36]Solution in 1.312s

Time Plot 0 s.
Exiting SolveHard() after 2.755r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none]

2 --> 0  target = [11.999999999992897071307144723313753160 00,
6.217012503037142931294695875867136040134,
485.549080895776384948826484520914434954 1]
one interval r = 32.628147792144496469330857054637072364 94 ..
36.102483889368179964642072178975821182 71
Time Approximations 0.02.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .89e-35]Solution in 0.955s

Time Plot 0 s.
Exiting SolveHard() after 1.341r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844765786204301424582213801376 71,
6.583434721566907678007065255803128054130,
467.787305957449253626094446831816681791 5]
one interval r = 32.419789556656396369108770676691081254 26 ..
35.851524173683601829986556286865266426 05
Time Approximations 0.024.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .233e-34]Solution in 1.082s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.462r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67, none, none,
401.88173904128609629470767434922099318 54, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844765786204301424582213801376 71,
6.5834347215669076780070652558031280541 30,
467.78730595744925362609444683181668179 15]
two intervals r = 12.923271608269719646876959465845041548 78 ..
19000000000031433327629899685968433957/10000000000000000000000000000000
00000 or r = 18.394248580319033445959478945823839031 10 ..
19000000000031433327629899685968433957/10000000000000000000000000000000
00000
Time Approximations 0.067.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.3e-38
Equations at solution: [-.2e-37, -.33e-37, -.914e-35]Solution in 30.13s

Time Plot 0 s.
Exiting SolveHard() after 32.539r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88, none,
401.88173904128609629470767434922099318 54, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628189288889535951864317806078 86,
```

```
4.125651796766772406495042188087737468465,
440.671230646317368231930402797951543319 9]
two intervals r = 14.35659705131598348566925200740586761816 ..
190000000000314333276298996859684339 57/100000000000000000000000000000000
00000 or r = 17.70352613799276265552230677163974994122 ..
190000000000314333276298996859684339 57/100000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.14e-37, -.1e-37, -.4891e-34]Solution in 1.67s

Time Plot 0 s.
Exiting SolveHard() after 3.547r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593 03314,
441.64295973034525717581149308815731 92676,
436.91748164869099135744896889203946 69567,
422.98493397677806915142819813955704 36388, none,
401.88173904128609629470767434922099 31854,
389.59001515247415517003042692434016 59842, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962818928888953595186431780607886,
4.125651796766772406495042188087737468465,
440.671230646317368231930402797951543319 9]
one interval r = 22.39761154352103721987010984896405219807 ..
27.23722351591228890345080921558215041402
Time Approximations 0.033.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
```

```
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.49 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643720933921867483431087042481 40, rm =
14.37818770186861900146580244340682164003}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.05e-37
Equations at solution: [-.1e-37, -.105e-36, .83e-35]Solution in 7.056s

Time Plot 0 s.
Exiting SolveHard() after 8.305r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887966320797475503217012421 22349,
4.0048690817533891489733942559240247748 98,
404.86224501197972840949241432167876480 47]
two intervals r = 16.08011007760428314196541995429987663886 ..
19000000000031433327629899685968433957/10000000000000000000000000000000
00000 or r = 16.41579812690494659442098770030035175783 ..
19000000000031433327629899685968433957/10000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.17e-37, 0., .244e-35]Solution in 2.122s

Time Plot 0 s.
Exiting SolveHard() after 3.615r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42, none, none,
358.97362823590527556950360868479645835 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888796632079747550321701242122349,
4.0048690817533891489733942559240247748 98,
404.86224501197972840949241432167876480 47]
one interval r = 21.641943994031607459300787798712578209 20 ..
26.763306600377860461012514981091235554 67
Time Approximations 0.045.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.5e-38
Equations at solution: [-.1e-37, -.25e-37, .346e-34]Solution in 1.404s

Time Plot 0 s.
Exiting SolveHard() after 2.83r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40, none,
358.97362823590527556950360868479645835 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589417663840510119630351175122766 72,
5.5896371830985188157777634489626682833 94,
443.83065884753246933722292764887893119 87]
```

```
one interval r = 22.46725374476293646322795436267535621320 ..
27.27388428354314768421228861358099912299
Time Approximations 0.033.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, .81e-35]Solution in 1.264s

Time Plot 0 s.
Exiting SolveHard() after 2.632r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40, none,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941766384051011963035117512276672,
5.589637183098518815777763448962668283394,
443.83065884753246933722292764887893119 87]
one interval r = 32.15575279508068648637939808996159617472 ..
35.50872228739336886827914259748004607811
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
```

```
Equations at solution: [.3e-37, -.2e-37, .125e-34]Solution in 0.883s

Time Plot 0 s.
Exiting SolveHard() after 1.242r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136533468747358191779491052362922,
5.18778357856114299948929240428928801170 6,
408.65773862086955447314598937735216473 0]
one interval r = 21.71840114632435436727222566974194954375 ..
26.81849303500807440941110953568628415429
Time Approximations 0.055.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [.1e-37, .79e-37, .176e-34]Solution in 1.379s

Time Plot 0 s.
Exiting SolveHard() after 2.953r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
```

```
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18, none, none,
361.50888346727330213657218686067294451 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136533468747358191779491052362922,
5.1877835785611429994892924042892880117 06,
408.65773862086955447314598937735216473 0]
one interval r = 31.80828598751248871885151222434781619204 ..
35.00011460036494303047620713592631212534
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=1.0e-37
Equations at solution: [-.9e-37, .10e-36, .28e-35]Solution in 0.812s

Time Plot 0 s.
Exiting SolveHard() after 1.117r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03, none,
361.50888346727330213657218686067294451 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 1  target = [26.46347110531792567669972319439955003724,
6.196262565285140588541917439709511582133,
385.44474378954441910303933812768340514 94]
one interval r = 31.60836097539401665382693671066678482724 ..
34.66372795604933624305376184703693536769
Time Approximations 0.016.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, -.197e-34]Solution in 0.579s

Time Plot 0 s.
Exiting SolveHard() after 0.873r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593 03314,
441.64295973034525717581149308815731 92676,
436.91748164869099135744896889203946 69567,
422.98493397677806915142819813955704 36388,
361.52580255689739744351335434484921 44830,
401.88173904128609629470767434922099 31854,
389.59001515247415517003042692434016 59842,
328.46939893331261127522809075353885 36540,
401.50757157901282740194193243157871 36230,
358.97362823590527556950360868479645 83530,
398.33147104516225764991511992273346 80818,
371.48387393529288285414677208723315 21903, none,
361.50888346727330213657218686067294 45175,
324.67144992258382012501248618909532 62432, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110531792567669972319439955003724,
6.196262565285140588541917439709511582133,
385.44474378954441910303933812768340514 94]
two intervals r = 16.87563408764890603905981561296588433356 ..
19000000000031433327629899685968433957/10000000000000000000000000000000
00000 or r = 15.55640493786318419418499448926127062742 ..
19000000000031433327629899685968433957/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [.896e-37, -.1e-37, .2205e-34]Solution in 1.534s

Time Plot 0 s.
Exiting SolveHard() after 3.572r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874745986242981045774678114017961,
4.8838107799207468508687948924481429178 80,
376.61967855443482505258975316090320937 91]
one interval r = 21.11001304865612285733142988963965651082 ..
26.31784243466467545338323692460569825246
Time Approximations 0.031.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.59e-35]Solution in 0.778s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.887r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32, none,
328.46938513403961541367606784568248186 93, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874745986242981045774678114017961,
4.8838107799207468508687948924481429178 80,
376.61967855443482505258975316090320937 91]
one interval r = 31.53899497713928557685059970360308153958 ..
34.53618386088569577346673152745364055915
Time Approximations 0.015.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.0e-37
Equations at solution: [-.23e-36, .30e-36, .137e-34]Solution in 0.436s

Time Plot 0 s.
Exiting SolveHard() after 1.129r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
```

```
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32, none,
328.46938513403961541367606784568248186 93,
343.81340624219947384688251793425754269 34, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017536779916419384404442010403672,
6.0258135492546667081648432011569188946 77,
351.42702948312979544170065877402607855 11]
one interval r = 31.36230206119037310898221260015956299598 ..
34.17446640614434656846596145161871442290
Time Approximations 0.018.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, -.42e-35]Solution in 0.958s

Time Plot 0 s.
Exiting SolveHard() after 1.199r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
```

```
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32, none,
328.46938513403961541367606784568248186 93,
343.81340624219947384688251793425754269 34, none, none,
292.99969138241825305105679769971835809 52, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175367799164193844044420104036 72,
6.0258135492546667081648432011569188946 77,
351.42702948312979544170065877402607855 11]
two intervals r = 17.981355144442721489939168335832080545 78 ..
19000000000031433327629899685968433957/10000000000000000000000000000000
00000 or r = 13.846080154141314218826343974062265460 46 ..
19000000000031433327629899685968433957/10000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=4e-38
Equations at solution: [-.125e-36, .4e-37, .1007e-34]Solution in 1.521s

Time Plot 0 s.
Exiting SolveHard() after 2.969r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32,
302.31384315114209946075173834989614727 00,
328.46938513403961541367606784568248186 93,
```

```
343.8134062421994738468825179342575426934, none, none,
292.9996913824182530510567976997183580952, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941876894843785107001728366323534,
6.3779438738687374933517785836370446301 97,
423.2883278442613183716581289971469125810]
one interval r = 31.9466181760648567342935435594989936578 2 ..
35.21212308657537577761147904577677581567
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, .94e-35]Solution in 1.067s

Time Plot 0 s.
Exiting SolveHard() after 1.411r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349341272598686131749498959303314,
441.6429597303452571758114930881573192676,
436.9174816486909913574489688920394669567,
422.9849339767780691514281981395570436388,
361.5258025568973974435133543448492144830,
401.8817390412860962947076743492209931854,
389.5900151524741551700304269243401659842,
328.4693989333126112752280907535388536540,
401.5075715790128274019419324315787136230,
358.9736282359052755695036086847964583530,
398.3314710451622576499151199227334680818,
371.4838739352928828541467720872331521903,
336.6121584120031942455480999627852256265,
361.5088834672733021365721868606729445175,
324.6714499225838201250124861890953262432,
302.3138431511420994607517383498961472700,
328.4693851340396154136760678456824818693,
343.8134062421994738468825179342575426934, none, none,
292.9996913824182530510567976997183580952, none, none,
360.0617346723355112775708118974688347591, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941876894843785107001728366323534,
6.3779438738687374933517785836370446301 97,
```

```
423.28832784426131837165812899714691258 10]
two intervals r = 15.22886702407830403240065320994763364769 ..
190000000000314333276298996859684339 57/100000000000000000000000000000000000
00000 or r = 17.12965777092581889425236460753967949860 ..
190000000000314333276298996859684339 57/100000000000000000000000000000000000
00000
Time Approximations 0.071.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.31e-37, .1e-37, -.814e-35]Solution in 2.166s

Time Plot 0 s.
Exiting SolveHard() after 3.997r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32,
302.31384315114209946075173834989614727 00,
328.46938513403961541367606784568248186 93,
343.81340624219947384688251793425754269 34,
375.73285290991815552373054940627958600 32, none,
292.99969138241825305105679769971835809 52, none, none,
360.06173467233551127757081189746883475 91, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343408059826937101416101843305 38,
4.0035598154540877280552135704970659116 93,
404.47973593922623952562130771135686307 91]
two intervals r = 16.09683966370511499638415696823627026395 ..
190000000000314333276298996859684339 57/100000000000000000000000000000000000
```

```
00000 or r = 16.3998864911115950272633042270501848529 3 ..
190000000000314333276298996859684339 57/1000000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.7615110884558244986626642365610012490 6, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.17e-37, 0., -.2101e-34]Solution in 1.931s

Time Plot 0 s.
Exiting SolveHard() after 3.461r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32,
302.31384315114209946075173834989614727 00,
328.46938513403961541367606784568248186 93,
343.81340624219947384688251793425754269 34,
375.73285290991815552373054940627958600 32, none,
292.99969138241825305105679769971835809 52,
358.64341560456358980011842295343205431 94, none,
360.06173467233551127757081189746883475 91, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234340805982693710141610184330538,
4.0035598154540877280552135704970659116 93,
404.47973593922623952562130771135686307 91]
one interval r = 21.63429629980802404021190194600669547962 ..
26.75768169887872747085097384323358193878
Time Approximations 0.051.
```

```
hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [0., .26e-37, .288e-34]Solution in 1.464s

Time Plot 0 s.
Exiting SolveHard() after 3.512r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593 03314,
441.64295973034525717581149308815731 92676,
436.91748164869099135744896889203946 69567,
422.98493397677806915142819813955704 36388,
361.52580255689739744351335434484921 44830,
401.88173904128609629470767434922099 31854,
389.59001515247415517003042692434016 59842,
328.46939893331261127522809075353885 36540,
401.50757157901282740194193243157871 36230,
358.97362823590527556950360868479645 83530,
398.33147104516225764991511992273346 80818,
371.48387393529288285414677208723315 21903,
336.61215841200319424554809996278522 56265,
361.50888346727330213657218686067294 45175,
324.67144992258382012501248618909532 62432,
302.31384315114209946075173834989614 72700,
328.46938513403961541367606784568248 18693,
343.81340624219947384688251793425754 26934,
375.73285290991815552373054940627958 60032,
328.11709294271407817607701264425903 56918,
292.99969138241825305105679769971835 80952,
358.64341560456358980011842295343205 43194, none,
360.06173467233551127757081189746883 47591, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954449979701565215547273764266283,
6.19617723013864470190321727916466596 1266,
385.42734025329240987983321617663538 56165]
one interval r = 31.608220490947643513573843259940125353 81 ..
34.663476150425904785036962357213139 38972
Time Approximations 0.016.

hint used Hint := [33.813392675100157147469018161182119 36913, 3, 1, 1,
11.783209121413521870719438174471899 65627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=5e-38
Equations at solution: [-.3e-37, .5e-37, -.28e-35]Solution in 0.545s

Time Plot 0 s.
Exiting SolveHard() after 0.83r=33.8134 in [32.62668594 .. 34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32,
302.31384315114209946075173834989614727 00,
328.46938513403961541367606784568248186 93,
343.81340624219947384688251793425754269 34,
375.73285290991815552373054940627958600 32,
328.11709294271407817607701264425903569 18,
292.99969138241825305105679769971835809 52,
358.64341560456358980011842295343205431 94, none,
360.06173467233551127757081189746883475 91, none, none,
324.65521223236858025888147373836831257 88, none, none, none, none]

0 --> 1  target = [26.46318954449979701565215547273764266283,
6.19617723013864470190321727916466596126 6,
385.42734025329240987983321617663538561 65]
two intervals r = 16.87629600303540905703663555109769003533 ..
19000000000031433327629899685968433957/10000000000000000000000000000000
00000 or r = 15.55559000634856212131876516790833097061 ..
19000000000031433327629899685968433957/10000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.538e-37, 0., .1160e-34]Solution in 1.648s

Time Plot 0 s.
Exiting SolveHard() after 3.693r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32,
302.31384315114209946075173834989614727 00,
328.46938513403961541367606784568248186 93,
343.81340624219947384688251793425754269 34,
375.73285290991815552373054940627958600 32,
328.11709294271407817607701264425903569 18,
292.99969138241825305105679769971835809 52,
358.64341560456358980011842295343205431 94, none,
360.06173467233551127757081189746883475 91,
336.59441032243286289428553039928962404 45, none,
324.65521223236858025888147373836831257 88, none, none, none, none]

0 --> 2  target = [34.49522661153246250443181009329461872051,
3.89713131584506778618377411759658275715 5,
373.78018183585393252079486528869541450 91]
two intervals r = 17.29769086252079889290680872359254835123 ..
19000000000031433327629899685968433957/10000000000000000000000000000000
00000 or r = 14.99436407388838662331549502101408512602 ..
19000000000031433327629899685968433957/10000000000000000000000000000000
00000
Time Approximations 0.085.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
```

```
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.54e-37, 0., .187e-35]Solution in 1.68s

Time Plot 0.001 s.
Exiting SolveHard() after 4.35r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32,
302.31384315114209946075173834989614727 00,
328.46938513403961541367606784568248186 93,
343.81340624219947384688251793425754269 34,
375.73285290991815552373054940627958600 32,
328.11709294271407817607701264425903569 18,
292.99969138241825305105679769971835809 52,
358.64341560456358980011842295343205431 94, none,
360.06173467233551127757081189746883475 91,
336.59441032243286289428553039928962404 45, none,
324.65521223236858025888147373836831257 88,
331.93806790381657033421106777143438119 18, none, none, none]

1 --> 2  target = [34.49522661153246250443181009329461872051,
3.8971313158450677861837741175965827571 55,
373.78081883585393252079486528869541450 91]
one interval r = 21.06068473182698416392293621945469300515 ..
26.26979834265995707122726586332291012642
Time Approximations 0.03.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
```

```
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., .105e-34]Solution in 0.716s

Time Plot 0 s.
Exiting SolveHard() after 1.917r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593 03314,
441.64295973034525717581149308815731 92676,
436.91748164869099135744896889203946 69567,
422.98493397677806915142819813955704 36388,
361.52580255689739744351335434484921 44830,
401.88173904128609629470767434922099 31854,
389.59001515247415517003042692434016 59842,
328.46939893331261127522809075353885 36540,
401.50757157901282740194193243157871 36230,
358.97362823590527556950360868479645 83530,
398.33147104516225764991511992273346 80818,
371.48387393529288285414677208723315 21903,
336.61215841200319424554809996278522 56265,
361.50888346727330213657218686067294 45175,
324.67144992258382012501248618909532 62432,
302.31384315114209946075173834989614 72700,
328.46938513403961541367606784568248 18693,
343.81340624219947384688251793425754 26934,
375.73285290991815552373054940627958 60032,
328.11709294271407817607701264425903 56918,
292.99969138241825305105679769971835 80952,
358.64341560456358980011842295343205 43194,
299.89866203986847463488235714918917 07227,
360.06173467233551127757081189746883 47591,
336.59441032243286289428553039928962 40445, none,
324.65521223236858025888147373836831 25788,
331.93806790381657033421106777143438 11918, none, none, none]

0 --> 2  target = [33.81362495406075758165402751709221196102,
3.72564899349567762662759130965391754 9042,
325.89209972488141715037912734395443 15179]
two intervals r = 18.552270490120041803915837037108771370 25 ..
190000000000314333276298996859684339 57/100000000000000000000000000000000
00000 or r = 12.491969357667201625172283084595024102 20 ..
190000000000314333276298996859684339 57/100000000000000000000000000000000
00000
Time Approximations 0.038.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [-.155e-36, .6e-37, -.776e-35]Solution in 1.566s

Time Plot 0 s.
Exiting SolveHard() after 3.754r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32,
302.31384315114209946075173834989614727 00,
328.46938513403961541367606784568248186 93,
343.81340624219947384688251793425754269 34,
375.73285290991815552373054940627958600 32,
328.11709294271407817607701264425903569 18,
292.99969138241825305105679769971835809 52,
358.64341560456358980011842295343205431 94,
299.89866203986847463488235714918917072 27,
360.06173467233551127757081189746883475 91,
336.59441032243286289428553039928962404 45, none,
324.65521223236858025888147373836831257 88,
331.93806790381657033421106777143438119 18, none, none,
289.54595771912348989904403833129894879 47]

1 --> 2  target = [33.81362495406075758165402751709221196102,
3.72564899349567762662759130965391754904 2,
325.89209972488141715037912734395443151 79]
one interval r = 20.37468935096903544193135953861873060028 ..
25.37892165289801777830859899638041577125
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=5e-38
Equations at solution: [.5e-37, .5e-37, -.311e-34]Solution in 0.569s

Time Plot 0 s.
Exiting SolveHard() after 1.524r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32,
302.31384315114209946075173834989614727 00,
328.46938513403961541367606784568248186 93,
343.81340624219947384688251793425754269 34,
375.73285290991815552373054940627958600 32,
328.11709294271407817607701264425903569 18,
292.99969138241825305105679769971835809 52,
358.64341560456358980011842295343205431 94,
299.89866203986847463488235714918917072 27,
360.06173467233551127757081189746883475 91,
336.59441032243286289428553039928962404 45,
256.10753185747012579495343567856377026 3,
324.65521223236858025888147373836831257 88,
331.93806790381657033421106777143438119 18, none, none,
289.54595771912348989904403833129894879 47]

1 --> 0  target = [17.93041369709009620285040268901716263100,
4.68650870208917433663587346155229976146 3,
353.30541094987608622010242485071533598 40]
one interval r = 20.73150479084661100055165599233324731491 ..
25.90675353518806395516056900276940935442
Time Approximations 0.028.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
```

```
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.2e-38
Equations at solution: [-.1e-37, -.22e-37, .161e-34]Solution in 1.046s

Time Plot 0 s.
Exiting SolveHard() after 2.201r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32,
302.31384315114209946075173834989614727 00,
328.46938513403961541367606784568248186 93,
343.81340624219947384688251793425754269 34,
375.73285290991815552373054940627958600 32,
328.11709294271407817607701264425903569 18,
292.99969138241825305105679769971835809 52,
358.64341560456358980011842295343205431 94,
299.89866203986847463488235714918917072 27,
360.06173467233551127757081189746883475 91,
336.59441032243286289428553039928962404 45,
256.10753185747012579495343357678563770 263,
324.65521223236858025888147373836831257 88,
331.93806790381657033421106777143438119 18,
304.79958325758582516328585583569061188 01, none,
289.54595771912348989904403833129894879 47]

2 --> 0  target = [17.93041369709009620285040268901716263100,
4.68650870208917433663587346155229976146 3,
353.30541094987608622010242485071533598 40]
one interval r = 31.37435486998564429144581920392423100503 ..
34.20127520027761017598082049910026154689
```

```
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=9e-38
Equations at solution: [-.5e-37, .9e-37, .328e-34]Solution in 0.353s

Time Plot 0 s.
Exiting SolveHard() after 0.604r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493412725986861317494989593033 14,
441.64295973034525717581149308815731926 76,
436.91748164869099135744896889203946695 67,
422.98493397677806915142819813955704363 88,
361.52580255689739744351335434484921448 30,
401.88173904128609629470767434922099318 54,
389.59001515247415517003042692434016598 42,
328.46939893331261127522809075353885365 40,
401.50757157901282740194193243157871362 30,
358.97362823590527556950360868479645835 30,
398.33147104516225764991511992273346808 18,
371.48387393529288285414677208723315219 03,
336.61215841200319424554809996278522562 65,
361.50888346727330213657218686067294451 75,
324.67144992258382012501248618909532624 32,
302.31384315114209946075173834989614727 00,
328.46938513403961541367606784568248186 93,
343.81340624219947384688251793425754269 34,
375.73285290991815552373054940627958600 32,
328.11709294271407817607701264425903569 18,
292.99969138241825305105679769971835809 52,
358.64341560456358980011842295343205431 94,
299.89866203986847463488235714918917072 27,
360.06173467233551127757081189746883475 91,
336.59441032243286289428553039928962404 45,
256.10753185747012579495343576785637702 63,
324.65521223236858025888147373836831257 88,
331.93806790381657033421106777143438119 18,
304.79958325758582516328585583569061188 01,
323.46169176235857670140902471392969856 36,
289.54595771912348989904403833129894879 47]

Cascade time 107.279
```

```
counts: 28, 28

 Iteration 5

Start Generation 1
1 --> 0  target = [11.999999999886600159065172248396562972 00,
6.217012503119007126079417910381576456098,
485.549080905674176673198901469146968279 3]
one interval r = 23.408503016709811115638878775148356029 76 ..
27.675780464415595409649730518044569502 82
Time Approximations 0.036.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.7e-38
Equations at solution: [-.1e-37, .27e-37, .4e-36]Solution in 1.444s

Time Plot 0 s.
Exiting SolveHard() after 3.013r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999886600159065172248396562972 00,
6.217012503119007126079417910381576456098,
485.549080905674176673198901469146968279 3]
one interval r = 32.628147792174355860346873335081169009 53 ..
36.102483889415221027458480184974370280 21
Time Approximations 0.024.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
```

```
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., .61e-35]Solution in 1.1s

Time Plot 0 s.
Exiting SolveHard() after 1.537r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684494999861942674611300475472730,
6.58343472162810554991900212449655030519 8,
467.78730596721209326739573203115624134 08]
one interval r = 32.41978955667882190992697638369137711406 ..
35.85152417373125920699499123201239346076
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.91e-35]Solution in 1.188s

Time Plot 0 s.
Exiting SolveHard() after 1.6r=34.9451 in [33.70078237 .. 35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55, none, none,
401.88173905234464932615876244908869786 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684494999861942674611300475472730,
6.58343472162810554991900212449655030519 8,
467.78730596721209326739573203115624134 08]
```

```
two intervals r = 12.92327160809515299876481974859684825542 ..
189999999999211381706392479216515469 63/10000000000000000000000000000000
00000 or r = 18.39424858037947515023838278256924460047 ..
189999999999211381706392479216515469 63/10000000000000000000000000000000
00000
Time Approximations 0.043.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.2e-38
Equations at solution: [.1e-37, .12e-37, -.804e-35]Solution in 29.678s

Time Plot 0 s.
Exiting SolveHard() after 31.638r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50, none,
401.88173905234464932615876244908869786 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962825967586504325619482459395519,
4.1256517969712465139444944451079195589 79,
440.67123065745959944822672337523979783 95]
two intervals r = 14.35659705099715990122288067713769357303 ..
189999999999211381706392479216515469 63/10000000000000000000000000000000
00000 or r = 17.70352613819061711467147221378692511763 ..
189999999999211381706392479216515469 63/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
```

```
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [-.56e-37, -.3e-37, .3593e-34]Solution in 1.646s

Time Plot 0 s.
Exiting SolveHard() after 3.084r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50, none,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962825967586504325619482459395519,
4.1256517969712465139444944451079195589 79,
440.67123065745959944822672337523979783 95]
one interval r = 22.397611543781273597403983279812888623 50 ..
27.237223516130794486282794837877931186 23
Time Approximations 0.033.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.605 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070644045881203064954001623562372 28, rm =
14.378187705386714925319579324841487714 09}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .625e-34]Solution in 7.507s

Time Plot 0 s.
Exiting SolveHard() after 8.803r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
```

```
441.642959739992109455200319575258847798 2,
436.917481659725053606336406349913012075 5,
422.984933985365801795246061443376267535 0,
361.525802569599823693627683872877210119 5,
401.881739052344649326158762449088697863 2,
389.590015164572059389729915161923141722 6, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888039102196123820987559489360 94,
4.004869081958631664132931418956127706060,
404.862245023149972696073964502650994903 8]
two intervals r = 16.080110077259199450307920583184368892 19 ..
189999999999211381706392479216515469 63/100000000000000000000000000000000
00000 or r = 16.415798127260037776612534285820149403 60 ..
189999999999211381706392479216515469 63/100000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.17e-37, 0., -.2268e-34]Solution in 2.016s

Time Plot 0 s.
Exiting SolveHard() after 3.564r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434943048835304914529540297330753 2,
441.642959739992109455200319575258847798 2,
436.917481659725053606336406349913012075 5,
422.984933985365801795246061443376267535 0,
361.525802569599823693627683872877210119 5,
401.881739052344649326158762449088697863 2,
389.590015164572059389729915161923141722 6, none, none,
358.973628248386507909440462922055749257 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888039102196123820987559489360 94,
4.004869081958631664132931418956127706060,
404.862245023149972696073964502650994903 8]
one interval r = 21.641943994257715608241570177828632151 64 ..
26.763306600622111289523036246292679276 46
```

```
Time Approximations 0.045.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .503e-34]Solution in 1.443s

Time Plot 0 s.
Exiting SolveHard() after 2.917r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41, none,
358.97362824838650790944046292205574925 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941743662239591152124504495074333,
5.58963718315376724208148114968376891442 8,
443.83065885702431686511381095015930853 72]
one interval r = 22.46725374498910197195854934507898296889 ..
27.27388428374032456064744342100345140873
Time Approximations 0.035.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [0., -.81e-37, .25e-35]Solution in 1.412s

Time Plot 0 s.
```

```
Exiting SolveHard() after 2.863r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41, none,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941743662239591152124504495074333,
5.5896371831537672420814811496837689144 28,
443.83065885702431686511381095015930853 72]
one interval r = 32.15575279509145989708427674887607180723 ..
35.50872228743971241267317277922380468565
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, -.180e-34]Solution in 0.889s

Time Plot 0 s.
Exiting SolveHard() after 1.217r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
```

```
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136490155949822823278890458020246,
5.18778357863774613986630763599626900780 3,
408.65773863399880933651848250679929563 37]
one interval r = 21.71840114659453202569196812877335146162 ..
26.81849303527799952588552809002274425896
Time Approximations 0.049.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.37e-37
Equations at solution: [-.2e-37, -.237e-36, -.254e-34]Solution in
1.373s

Time Plot 0 s.
Exiting SolveHard() after 2.902r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84, none, none,
361.50888347987312586310539336183529731 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136490155949822823278890458020246,
5.18778357863774613986630763599626900780 3,
408.65773863399880933651848250679929563 37]
one interval r = 31.80828598754214885789055767193765857496 ..
35.00011460046603840484652307002506009566
```

```
Time Approximations 0.016.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, .1e-36]Solution in 0.418s

Time Plot 0 s.
Exiting SolveHard() after 0.691r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23, none,
361.50888347987312586310539336183529731 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110560273935317933873005058065712,
6.19626256536766307905000607353090155773 3,
385.44474380247365574887764735963533181 02]
one interval r = 31.60836097540764416794594830610177169754 ..
34.66372795614757481329657264395519770586
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
```

```
Accepted {r=33.8136, rm=11.783} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, .265e-34]Solution in 0.954s

Time Plot 0 s.
Exiting SolveHard() after 1.225r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23, none,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110560273935317933873005058065712,
6.19626256536766307905000607353090155773 3,
385.44474380247365574887764735963533181 02]
two intervals r = 16.87563408724644299824439106606012084780 ..
18999999999921138170639247921651546963/10000000000000000000000000000000
00000 or r = 15.55640493838829226264374715032265453460 ..
18999999999921138170639247921651546963/10000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 5.485 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175100833822901040534275153716320, rm
= 2.33653277396953981010824881462530930386 3}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.5141e-34]Solution in 19.625s

Time Plot 0 s.
```

```
Exiting SolveHard() after 21.15r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874705151959842195814487508386601,
4.8838107799832818270091925298938142867 98,
376.61967856790903387486297258483991875 32]
one interval r = 21.11001304888538850122904368342839287193 ..
26.31784243496484160681869709880721561714
Time Approximations 0.03.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .88e-35]Solution in 1.227s

Time Plot 0 s.
Exiting SolveHard() after 2.389r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
```

```
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64, none,
328.46938514671866566578007385175496676 83, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874705151959842195814487508386601,
4.8838107799832818270091925298938142867 98,
376.61967856790903387486297258483991875 32]
one interval r = 31.53899497715140433468942830069920283288 ..
34.53618386099148444455869388406921341968
Time Approximations 0.015.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=4.19e-36
Equations at solution: [-.322e-35, .419e-35, -.455e-34]Solution in
0.442s

Time Plot 0 s.
Exiting SolveHard() after 0.703r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
```

```
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64, none,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017567436730880798726602093901208,
6.0258135493408865129359813570807351268 66,
351.42702949605297049913109405483288079 63]
one interval r = 31.362302061181337796585049651432597598 84 ..
34.174466406240241629043367960872440958 93
Time Approximations 0.015.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.56e-35]Solution in 0.942s

Time Plot 0 s.
Exiting SolveHard() after 1.164r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64, none,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59, none, none,
292.99969139645371225454057246796342944 08, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017567436730880798726602093901208,
6.0258135493408865129359813570807351268 66,
```

```
351.42702949605297049913109405483288079 63]
two intervals r = 17.98135514410391992210857093824430574516 ..
189999999999211381706392479216515469 63/100000000000000000000000000000000
00000 or r = 13.84608015482040438325917218681827913064 ..
189999999999211381706392479216515469 63/100000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 4.217 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.91357071353605772902792893814250554695, rm
= 2.7345009929383988494106280428649843387 34}});
Accepted {r=18.6878, rm=15.3648} with Delta=7e-38
Equations at solution: [-.213e-36, .7e-37, .3036e-34]Solution in
12.445s

Time Plot 0 s.
Exiting SolveHard() after 13.897r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59, none, none,
292.99969139645371225454057246796342944 08, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941897397279698618537876662451824,
```

```
6.37794387392995037392061596334067576310 9,
423.28832785341625531096749503133854031 25]
one interval r = 31.946618176064205421725310922228119467 25 ..
35.212123086618271406215335619656000360 96
Time Approximations 0.016.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=8e-38
Equations at solution: [.7e-37, -.8e-37, .154e-34]Solution in 0.53s

Time Plot 0 s.
Exiting SolveHard() after 1.369r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59, none, none,
292.99969139645371225454057246796342944 08, none, none,
360.06173468283898543084659477400487944 17, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941897397279698618537876662451824,
6.37794387392995037392061596334067576310 9,
423.28832785341625531096749503133854031 25]
two intervals r = 15.22886702383557675268572853077647642658 ..
189999999999211381706392479216515469 63/10000000000000000000000000000000
00000 or r = 17.12965777112674367404227936066584551097 ..
189999999999211381706392479216515469 63/10000000000000000000000000000000
```

```
00000
Time Approximations 0.536.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 4.803 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370540879597110303070650165175842 18, rm
= 2.0640682986177908180499104393450465535 28}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [-.15e-37, 0., -.1330e-34]Solution in 19.6s

Time Plot 0 s.
Exiting SolveHard() after 21.801r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59,
375.73285291757039591592379139551159422 11, none,
292.99969139645371225454057246796342944 08, none, none,
360.06173468283898543084659477400487944 17, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343487237941648960166024202996 40,
4.0035598156608460602339742586582189857 36,
404.47973595083710209966029811126575679 18]
two intervals r = 16.096839663340838976277874876572094397 88 ..
189999999999211381706392479216515469 63/10000000000000000000000000000000
```

```
00000 or r = 16.39988649148676177447837278309874027359 ..
189999999999211381706392479216515469 63/10000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.33e-37, -.1e-37, -.229e-35]Solution in 2.016s

Time Plot 0 s.
Exiting SolveHard() after 4.007r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733 07532,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59,
375.73285291757039591592379139551159422 11, none,
292.99969139645371225454057246796342944 08,
358.64341561742861795469549950762768973 20, none,
360.06173468283898543084659477400487944 17, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234348723794164896016602420299640,
4.0035598156608460602339742586582189857 36,
404.47973595083710209966029811126575679 18]
one interval r = 21.63429630004248897041137634481972353751 ..
26.75768169912971519109781272380711352659
Time Approximations 0.045.
```

```
hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, .107e-34]Solution in 1.473s

Time Plot 0 s.
Exiting SolveHard() after 2.934r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59,
375.73285291757039591592379139551159422 11,
328.11709295580158624558380836482532779 05,
292.99969139645371225454057246796342944 08,
358.64341561742861795469549950762768973 20, none,
360.06173468283898543084659477400487944 17, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954478291579013665537321902382355,
6.19617723022065149664102463792716627800 7,
385.42734026611611322267719783928947336 69]
one interval r = 31.608220490960408138776122195027483081 23 ..
34.663476150522615993336466166650868094 21
Time Approximations 0.016.

hint used Hint := [33.813392675100157147469018161182119369 13, 3, 1, 1,
11.783209121413521870719438174471899656 27, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., .53e-35]Solution in 0.991s

Time Plot 0 s.
Exiting SolveHard() after 1.273r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59,
375.73285291757039591592379139551159422 11,
328.11709295580158624558380836482532779 05,
292.99969139645371225454057246796342944 08,
358.64341561742861795469549950762768973 20, none,
360.06173468283898543084659477400487944 17, none, none,
324.65521224631026385733061341818349921 26, none, none, none, none]

0 --> 1  target = [26.46318954478291579013665537321902382355,
6.19617723022065149664102463792716627 8007,
385.42734026611611322267719783928947336 69]
two intervals r = 16.87629600263698330635196707390322815704 ..
189999999999211381706392479216515469 63/10000000000000000000000000000000
00000 or r = 15.55559000686881408432981224848717097916 ..
189999999999211381706392479216515469 63/10000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 5.156 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852512771275656473203327470084597, rm
= 2.33669042803585316116524508741520400806 7}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.179e-37, 0., -.2794e-34]Solution in 19.685s

Time Plot 0 s.
Exiting SolveHard() after 21.192r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59,
375.73285291757039591592379139551159422 11,
328.11709295580158624558380836482532779 05,
292.99969139645371225454057246796342944 08,
358.64341561742861795469549950762768973 20, none,
360.06173468283898543084659477400487944 17,
336.59441033361860495021850029412129635 86, none,
324.65521224631026385733061341818349921 26, none, none, none, none]

0 --> 2  target = [34.49522661166612569683916537384458641579,
3.89713131606583464366680957838239662943 7,
373.78081885127146930175049418641581618 90]
two intervals r = 17.297690862051085140011676479403253522 36 ..
18999999999921138170639247921651546963/10000000000000000000000000000000
00000 or r = 14.994364074593089764269309595014332222 83 ..
18999999999921138170639247921651546963/10000000000000000000000000000000
00000
```

```
Time Approximations 0.072.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.54e-37, 0., .2440e-34]Solution in 1.537s

Time Plot 0 s.
Exiting SolveHard() after 4.081r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59,
375.73285291757039591592379139551159422 11,
328.11709295580158624558380836482532779 05,
292.99969139645371225454057246796342944 08,
358.64341561742861795469549950762768973 20, none,
360.06173468283898543084659477400487944 17,
336.59441033361860495021850029412129635 86, none,
324.65521224631026385733061341818349921 26,
331.93806792025704883888668289652868644 33, none, none, none]

1 --> 2  target = [34.49522661166612569683916537384458641579,
3.8971313160658346436668095783823966294 37,
373.78018885127146930175049418641581618 90]
one interval r = 21.06068473208484186541903971769690682372 ..
26.26979834299523587702737316099008031981
Time Approximations 0.03.
```

```
hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., .68e-35]Solution in 0.664s

Time Plot 0 s.
Exiting SolveHard() after 1.783r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59,
375.73285291757039591592379139551159422 11,
328.11709295580158624558380836482532779 05,
292.99969139645371225454057246796342944 08,
358.64341561742861795469549950762768973 20,
299.89866205640384316856222637722497767 12,
360.06173468283898543084659477400487944 17,
336.59441033361860495021850029412129635 86, none,
324.65521224631026385733061341818349921 26,
331.93806792025704883888668289652868644 33, none, none, none]

0 --> 2  target = [33.81362495417131813628927713540658918148,
3.72564899371366399639812931906371003927 9,
325.89209973910951215965414464688336232 17]
two intervals r = 18.55227048982372199584591609739629275086 ..
18999999999921138170639247921651546963/10000000000000000000000000000000
00000 or r = 12.49196935848828291271562105326697256255 ..
18999999999921138170639247921651546963/10000000000000000000000000000000
```

```
00000
Time Approximations 0.036.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=2e-38
Equations at solution: [-.51e-37, .2e-37, -.2085e-34]Solution in 1.478s

Time Plot 0 s.
Exiting SolveHard() after 3.725r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59,
375.73285291757039591592379139551159422 11,
328.11709295580158624558380836482532779 05,
292.99969139645371225454057246796342944 08,
358.64341561742861795469549950762768973 20,
299.89866205640384316856222637722497767 12,
360.06173468283898543084659477400487944 17,
336.59441033361860495021850029412129635 86, none,
324.65521224631026385733061341818349921 26,
331.93806792025704883888668289652868644 33, none, none,
289.54595773476468258800781127494247249 15]

1 --> 2  target = [33.81362495417131813628927713540658918148,
3.72564899371366399639812931906371003927 9,
325.89209973910951215965414464688336232 17]
one interval r = 20.37468935110539353141120015648127102209 ..
```

```
25.37892165324321105372265939274139066996
Time Approximations 0.023.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.2e-37, -.2e-37, .121e-34]Solution in 0.997s

Time Plot 0 s.
Exiting SolveHard() after 1.472r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59,
375.73285291757039591592379139551159422 11,
328.11709295580158624558380836482532779 05,
292.99969139645371225454057246796342944 08,
358.64341561742861795469549950762768973 20,
299.89866205640384316856222637722497767 12,
360.06173468283898543084659477400487944 17,
336.59441033361860495021850029412129635 86,
256.10753187282728147780144175335330231 58,
324.65521224631026385733061341818349921 26,
331.93806792025704883888668289652868644 33, none, none,
289.54595773476468258800781127494247249 15]

1 --> 0  target = [17.93041369676673848483189793013607655062,
4.68650870213153154021088830563499480714 8,
```

```
353.30541096208018363730120065587886937 46]
one interval r = 20.73150479101396500039745868832845114350 ..
25.90675353548033050926376809333802503497
Time Approximations 0.029.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, -.239e-34]Solution in 1.066s

Time Plot 0 s.
Exiting SolveHard() after 1.735r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59,
375.73285291757039591592379139551159422 11,
328.11709295580158624558380836482532779 05,
292.99969139645371225454057246796342944 08,
358.64341561742861795469549950762768973 20,
299.89866205640384316856222637722497767 12,
360.06173468283898543084659477400487944 17,
336.59441033361860495021850029412129635 86,
256.10753187282728147780144175335330231 58,
324.65521224631026385733061341818349921 26,
331.93806792025704883888668289652868644 33,
304.79958326878754272059845057870453114 17, none,
289.54595773476468258800781127494247249 15]
```

```
2 --> 0  target = [17.93041369676673848483189793013607655062,
4.68650870213153154021088830563499480714 8,
353.30541096208018363730120065587886937 46]
one interval r = 31.37435486997325825736105166430789346968 ..
34.20127520036343497094425764391397579455
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .3e-36]Solution in 0.363s

Time Plot 0 s.
Exiting SolveHard() after 0.629r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494304883530491452954029733075 32,
441.64295973999210945520031957525884779 82,
436.91748165972505360633640634991301207 55,
422.98493398536580179524606144337626753 50,
361.52580256959982369362768387287721011 95,
401.88173905234464932615876244908869786 32,
389.59001516457205938972991516192314172 26,
328.46939894599481866768313303603394342 41,
401.50757159050235707156327422125192946 43,
358.97362824838650790944046292205574925 78,
398.33147105420632746776765669187265172 84,
371.48387395050191066899948825640059773 23,
336.61215842329669232761633080220914696 13,
361.50888347987312586310539336183529731 19,
324.67144993662396748950817004965147571 64,
302.31384316218680757144488096440373383 19,
328.46938514671866566578007385175496676 83,
343.81340625818945974936162167220149296 59,
375.73285291757039591592379139551159422 11,
328.11709295580158624558380836482532779 05,
292.99969139645371225454057246796342944 08,
358.64341561742861795469549950762768973 20,
299.89866205640384316856222637722497767 12,
360.06173468283898543084659477400487944 17,
336.59441033361860495021850029412129635 86,
256.10753187282728147780144175335330231 58,
324.65521224631026385733061341818349921 26,
331.93806792025704883888668289652868644 33,
```

```
304.79958326878754272059845057870453114 17,
323.46169177751609073325217488361766979 95,
289.54595773476468258800781127494247249 15]

Cascade time 170.413
counts: 28, 28

 Iteration 6

Start Generation 1
1 --> 0  target = [11.999999999893863210040320286490949108 00,
6.217012503015603041650705530674212806349,
485.54908090112065171809965740982135274 37]
one interval r = 23.408503016637874669112321368008347881 57 ..
27.675780464304611616180127726687913180 06
Time Approximations 0.036.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.04e-37
Equations at solution: [.3e-37, -.104e-36, .1e-36]Solution in 1.429s

Time Plot 0 s.
Exiting SolveHard() after 3.445r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999893863210040320286490949108 00,
6.217012503015603041650705530674212806349,
485.54908090112065171809965740982135274 37]
one interval r = 32.628147792032100942718390374059248683 64 ..
36.102483889354331103769909815297001717 55
Time Approximations 0.02.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
```

```
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., .103e-34]Solution in 0.526s

Time Plot 0 s.
Exiting SolveHard() after 0.9r=35.4632 in [33.94922194 .. 36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684482816921732417636261003278852,
6.58343472161949695074557684287098456522 1,
467.78730596211835753277725271722917348 71]
one interval r = 32.41978955652692930048299951185583696322 ..
35.85152417365597004280567610119363272374
Time Approximations 0.018.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=6e-38
Equations at solution: [-.8e-37, .6e-37, .107e-34]Solution in 0.59s

Time Plot 0 s.
Exiting SolveHard() after 1.452r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82, none, none,
401.88173904570677438264941530855550710 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684482816921732417636261003278852,
6.58343472161949695074557684287098456522 1,
467.78730596211835753277725271722917348 71]
two intervals r = 12.92327160818404405857874491895204661542 ..
237500000000570913546149993718876393 9/1250000000000000000000000000000000
000 or r = 18.39424858041182833673688150793175385510 ..
237500000000570913546149993718876393 9/1250000000000000000000000000000000
000
Time Approximations 0.037.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.1e-38
Equations at solution: [-.1e-37, -.11e-37, -.828e-35]Solution in
31.035s

Time Plot 0 s.
Exiting SolveHard() after 33.391r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71, none,
401.88173904570677438264941530855550710 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962817317826515097001919397270367,
4.12565179680061589948481914818391381 6586,
440.67123065230361451346765645864209654 64]
two intervals r = 14.35659705114767123197927663343991018060 ..
237500000000570913546149993718876393 9/1250000000000000000000000000000000
000 or r = 17.70352613816612100432605404333214265791 ..
237500000000570913546149993718876393 9/1250000000000000000000000000000000
000
Time Approximations 0.043.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
```

```
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=4e-38
Equations at solution: [-.70e-37, -.4e-37, .3346e-34]Solution in 1.727s

Time Plot 0 s.
Exiting SolveHard() after 3.274r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71, none,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962817317826515097001919397270367,
4.12565179680061589948481914818391381658 6,
440.67123065230361451346765645864209654 64]
one interval r = 22.39761154373625298562778567111670366931 ..
27.23722351600020319866933609796225815465
Time Approximations 0.035.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.616 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064386300583718035287912341575922, rm =
14.37818770306478423448410099435527282352}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.32e-37
Equations at solution: [-.2e-37, -.132e-36, -.202e-34]Solution in
7.502s

Time Plot 0 s.
Exiting SolveHard() after 8.913r=26.4635 in [24.64256576 ..
27.23722351]
```

```
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887915649554624349757547006005 83,
4.0048690817818801533772793192700297622 48,
404.86224501636262805688261643039309530 53]
two intervals r = 16.080110077519664148294005386019463295 71 ..
23750000000057091354614999371887639 39/125000000000000000000000000000000
000 or r = 16.415798127082925209813355063567725086 87 ..
23750000000057091354614999371887639 39/125000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.16e-37, 0., .558e-35]Solution in 1.84s

Time Plot 0 s.
Exiting SolveHard() after 3.431r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53, none, none,
358.97362824139713870109854895382240732 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none]

1 --> 2  target = [34.94507888791564955462434975754700600583,
4.00486908178188015337727931927002976224 8,
404.86224501636262805688261643039309530 53]
one interval r = 21.641943994220250753147110063205571932 93 ..
26.763306600461013829681101501433648259 71
Time Approximations 0.052.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., .392e-34]Solution in 1.487s

Time Plot 0 s.
Exiting SolveHard() after 3.023r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388 80858,
441.64295973498952956243284442401538 13491,
436.91748165468045543897753628393409 86182,
422.98493397959989768038813777494539 69671,
361.52580256234324428037775840319406 83385,
401.88173904570677438264941530855550 71061,
389.59001515918412760473774334154890 44853,
328.46939893725444269761651595690545 45805, none,
358.97362824139713870109854895382240 73227, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941765777168407476886985104473697,
5.58963718303279622205873855239182545877 7,
443.83065885041054309558685200209077601 89]
one interval r = 22.46725374490863024074182055497862369368 ..
27.27388428359383075102662517102088707802
Time Approximations 0.035.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
```

```
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [.1e-37, -.81e-37, .71e-35]Solution in 1.385s

Time Plot 0 s.
Exiting SolveHard() after 2.862r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05, none,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941765777168407476886985104473697,
5.589637183032796222058738552391825458777,
443.83065885041054309558685200209077601 89]
one interval r = 32.15575279492010077315061869127583810865 ..
35.50872228733343093360731586741963410060
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .312e-34]Solution in 0.951s

Time Plot 0 s.
Exiting SolveHard() after 1.284r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365133906318488319985127227869 80,
5.1877835785356147359362275683446972643 59,
408.65773862782313519955023524476365205 94]
one interval r = 21.718401146564487569339540721071978323 32 ..
26.818493035126644489426859540288298872 28
Time Approximations 0.051.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.58e-37
Equations at solution: [-.2e-37, -.158e-36, .182e-34]Solution in 1.504s

Time Plot 0 s.
Exiting SolveHard() after 3.174r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63, none, none,
361.50888347271766673981004779293459184 52, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0  target = [15.91193136513390631848831998512722786980,
5.1877835785356147359362275683446972643 59,
408.65773862782313519955023524476365205 94]
one interval r = 31.80828598737400071951288338431328209163 ..
35.00011460035284137109124994625836286966
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.61e-35]Solution in 0.393s

Time Plot 0 s.
Exiting SolveHard() after 0.672r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01, none,
361.50888347271766673981004779293459184 52, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110542747778074844434609003248039,
6.19626256534319079530047605141277834973 6,
385.44474379504416146631622028407733563 87]
one interval r = 31.60836097522993852987518917751985200596 ..
34.66372795600790892184446801516651325650
Time Approximations 0.015.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
```

```
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, -.111e-34]Solution in 0.514s

Time Plot 0 s.
Exiting SolveHard() after 1.445r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01, none,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110542747778074844434609003248039,
6.196262565343190795300476051412778349736,
385.44474379504416146631622028407733563 87]
two intervals r = 16.87563408752902892236020341796453611786 ..
23750000000057091354614999371887639 39/12500000000000000000000000000000000
000 or r = 15.55640493812179756215747242986082752623 ..
23750000000057091354614999371887639 39/12500000000000000000000000000000000
000
Time Approximations 0.052.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.359e-37, 0., -.3040e-34]Solution in 1.649s

Time Plot 0 s.
```

```
Exiting SolveHard() after 3.304r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874734204500896318907134551176277,
4.88381077987650620766627246203347931281 1,
376.61967856009216016922863586687565737 18]
one interval r = 21.11001304886934726240883554054061575021 ..
26.31784243478152943753977374468964873553
Time Approximations 0.03.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.379e-34]Solution in 1.303s

Time Plot 0 s.
Exiting SolveHard() after 2.503r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
```

```
361.525802562343244280377758403194068338 5,
401.881739045706774382649415308555507106 1,
389.590015159184127604737743341548904485 3,
328.469398937254442697616515956905454580 5,
401.507571583293450256909148983382474794 5,
358.973628241397138701098548953822407322 7,
398.331471046608297828235330860563336686 3,
371.483873943329183683912505399803350590 1,
336.612158415360750666077035287612749102 8,
361.508883472717666739810047792934591845 2,
324.671449927738156775981418883749069200 2, none,
328.469385137981169163676821741537764398 4, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747342045008963189071345511762 77,
4.883810779876506207666272462033479312811,
376.619678560092160169228635866875657371 8]
one interval r = 31.538994976971812207616490349881852527 91 ..
34.536183860843206545265743150824495372 84
Time Approximations 0.015.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=6.62e-36
Equations at solution: [-.508e-35, .662e-35, .29e-35]Solution in 0.459s

Time Plot 0 s.
Exiting SolveHard() after 0.727r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434939248187343959685903033888085 8,
441.642959734989529562432844424015381349 1,
436.917481654680455438977536283934098618 2,
422.984933979599897680388137774945396967 1,
361.525802562343244280377758403194068338 5,
401.881739045706774382649415308555507106 1,
389.590015159184127604737743341548904485 3,
328.469398937254442697616515956905454580 5,
401.507571583293450256909148983382474794 5,
358.973628241397138701098548953822407322 7,
398.331471046608297828235330860563336686 3,
371.483873943329183683912505399803350590 1,
336.612158415360750666077035287612749102 8,
361.508883472717666739810047792934591845 2,
```

```
324.67144992773815677598141888374906920 02, none,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.872050175466953305842812197011682487 77,
6.0258135493059569319291756307875279694 44,
351.42702948708591664580397738798722022 81]
one interval r = 31.362302060999067732315772981065045391 37 ..
34.174466406067453846980471860560248335 46
Time Approximations 0.014.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=9e-38
Equations at solution: [-.5e-37, .9e-37, .282e-34]Solution in 0.487s

Time Plot 0 s.
Exiting SolveHard() after 1.25r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02, none,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86, none, none,
292.99969138611027135924553390925912629 96, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175466953305842812197011682487 77,
6.0258135493059569319291756307875279694 44,
351.42702948708591664580397738798722022 81]
two intervals r = 17.981355144397392802603896050406329542 02 ..
```

```
2375000000005709135461499937188763939/125000000000000000000000000000000
000 or r = 13.846080154367633313154337648613776451 90 ..
2375000000005709135461499937188763939/125000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [-.72e-37, .2e-37, .494e-34]Solution in 1.629s

Time Plot 0 s.
Exiting SolveHard() after 3.215r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388 80858,
441.64295973498952956243284442401538 13491,
436.91748165468045543897753628393409 86182,
422.98493397959989768038813777494539 69671,
361.52580256234324428037775840319406 83385,
401.88173904570677438264941530855550 71061,
389.59001515918412760473774334154890 44853,
328.46939893725444269761651595690545 45805,
401.50757158329345025690914898338247 47945,
358.97362824139713870109854895382240 73227,
398.33147104660829782823533086056333 66863,
371.48387394332918368391250539980335 05901,
336.61215841536075066607703528761274 91028,
361.50888347271766673981004779293459 18452,
324.67144992773815677598141888374906 92002,
302.31384315294112302481953304972780 58700,
328.46938513798116916367682174153776 43984,
343.81340624932444042664734994415220 37686, none, none,
292.99969138611027135924553390925912 62996, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418805061671876577309185912 15960,
6.3779438739064739353800120697521808 01632,
423.28832784564599092476045966127447 66054]
one interval r = 31.946618175880752823247341240651177 38979 ..
35.212123086487412620443845634363935 90491
Time Approximations 0.027.

hint used Hint := [34.327159798933221351566072928006859 21891, 3, 1, 1,
11.395769599984320296717245861504673 20776, 33.10127385 .. 35.21212310,
```

```
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.359e-34]Solution in 1.074s

Time Plot 0 s.
Exiting SolveHard() after 1.431r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02,
302.31384315294112302481953304972780587 00,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86, none, none,
292.99969138611027135924553390925912629 96, none, none,
360.06173467365802977252923544270222855 73, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418805061671876577309185912159 60,
6.3779438739064739353800120697521808016 32,
423.28832784564599092476045966127447660 54]
two intervals r = 15.22886702413612732913894052862705473269 ..
23750000000057091354614999371887639 39/12500000000000000000000000000000000
000 or r = 17.12965777096888783066319993304297446875 ..
23750000000057091354614999371887639 39/12500000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
```

```
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.32e-37, -.1e-37, .2040e-34]Solution in 1.717s

Time Plot 0 s.
Exiting SolveHard() after 3.583r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02,
302.31384315294112302481953304972780587 00,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86,
375.73285290903429444582221827361104267 52, none,
292.99969138611027135924553390925912629 96, none, none,
360.06173467365802977252923544270222855 73, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234335518078706253079323982146004,
4.0035598154820878630079126826717476932 14,
404.47973594346595612635786704126370386 19]
two intervals r = 16.09683966362685401818780020755102834551 ..
23750000000057091354614999371887639 39/12500000000000000000000000000000000
000 or r = 16.39988649128417706279394579247158459607 ..
23750000000057091354614999371887639 39/12500000000000000000000000000000000
000
Time Approximations 0.048.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.34e-37, 0., .2337e-34]Solution in 2.464s

Time Plot 0 s.
Exiting SolveHard() after 3.983r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02,
302.31384315294112302481953304972780587 00,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86,
375.73285290903429444582221827361104267 52, none,
292.99969138611027135924553390925912629 96,
358.64341560993307952660232108195848016 16, none,
360.06173467365802977252923544270222855 73, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234335518078706253079323982146004,
4.0035598154820878630079126826717476932 14,
404.47973594346595612635786704126370386 19]
one interval r = 21.634296299993866933841808249132955379 24 ..
26.757681698959932743980337556561082966 75
Time Approximations 0.045.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.333e-34]Solution in 1.522s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.984r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02,
302.31384315294112302481953304972780587 00,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86,
375.73285290903429444582221827361104267 52,
328.11709294652378976569753298291533521 38,
292.99969138611027135924553390925912629 96,
358.64341560993307952660232108195848016 16, none,
360.06173467365802977252923544270222855 73, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954460933359688206216735986339178,
6.19617723019668795386218797509677067661 3,
385.42734025879062974256310517322674902 13]
one interval r = 31.608220490783543979369783545971261712 25 ..
34.663476150384448963052443283901572398 57
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, .96e-35]Solution in 0.553s

Time Plot 0 s.
Exiting SolveHard() after 0.848r=33.8134 in [32.62668594 ..
34.66347615]
```

```
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02,
302.31384315294112302481953304972780587 00,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86,
375.73285290903429444582221827361104267 52,
328.11709294652378976569753298291533521 38,
292.99969138611027135924553390925912629 96,
358.64341560993307952660232108195848016 16, none,
360.06173467365802977252923544270222855 73, none, none,
324.65521223752148341378119644827304497 69, none, none, none, none]

0 --> 1  target = [26.46318954460933359688206216735986339178,
6.19617723019668795386218797509677067661 3,
385.42734025879062974256310517322674902 13]
two intervals r = 16.87629600291560406262748419538130754596 ..
23750000000057091354614999371887639 39/12500000000000000000000000000000000
000 or r = 15.55559000660713370820013498143101530209 ..
23750000000057091354614999371887639 39/12500000000000000000000000000000000
000
Time Approximations 0.054.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.359e-37, 0., -.1194e-34]Solution in 1.633s

Time Plot 0 s.
Exiting SolveHard() after 3.821r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
```

```
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02,
302.31384315294112302481953304972780587 00,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86,
375.73285290903429444582221827361104267 52,
328.11709294652378976569753298291533521 38,
292.99969138611027135924553390925912629 96,
358.64341560993307952660232108195848016 16, none,
360.06173467365802977252923544270222855 73,
336.59441032578884929388661852851859455 77, none,
324.65521223752148341378119644827304497 69, none, none, none, none]

0 --> 2  target = [34.49522661152380977438643594095598340030,
3.8971313158864450791585955854858999741 22,
373.78081884393389328664950272769690228 88]
two intervals r = 17.297690862322626149656263276831154081 04 ..
23750000000057091354614999371887639 39/1250000000000000000000000000000000
000 or r = 14.994364074293691271238290502102067903 00 ..
23750000000057091354614999371887639 39/1250000000000000000000000000000000
000
Time Approximations 0.087.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.54e-37, 0., .927e-35]Solution in 1.73s

Time Plot 0 s.
Exiting SolveHard() after 3.867r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02,
302.31384315294112302481953304972780587 00,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86,
375.73285290903429444582221827361104267 52,
328.11709294652378976569753298291533521 38,
292.99969138611027135924553390925912629 96,
358.64341560993307952660232108195848016 16, none,
360.06173467365802977252923544270222855 73,
336.59441032578884929388661852851859455 77, none,
324.65521223752148341378119644827304497 69,
331.93806791263086137494978625772942183 58, none, none, none]

1 --> 2  target = [34.49522661152380977438643594095598340030,
3.89713131588644507915859558548589997412 2,
373.78081884393389328664950272769690228 88]
one interval r = 21.06068473208152893046056056311534202530 ..
26.26979834281959667580172617462394019547
Time Approximations 0.032.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [-.1e-37, -.3e-37, -.734e-34]Solution in 1.28s

Time Plot 0 s.
Exiting SolveHard() after 2.516r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02,
302.31384315294112302481953304972780587 00,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86,
375.73285290903429444582221827361104267 52,
328.11709294652378976569753298291533521 38,
292.99969138611027135924553390925912629 96,
358.64341560993307952660232108195848016 16,
299.89866204717987106732044246943670579 15,
360.06173467365802977252923544270222855 73,
336.59441032578884929388661852851859455 77, none,
324.65521223752148341378119644827304497 69,
331.93806791263086137494978625772942183 58, none, none, none]

0 --> 2  target = [33.81362495398965322834906562692027243634,
3.72564899352649878679248429106582085159 6,
325.89209973001286771008091454770326526 83]
two intervals r = 18.552270490070536566734847606899339918 45 ..
23750000000057091354614999371887639 39/1250000000000000000000000000000000
000 or r = 12.491969357970998979376716120893771245 52 ..
23750000000057091354614999371887639 39/1250000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=2e-38
Equations at solution: [.34e-37, -.2e-37, .257e-34]Solution in 1.776s

Time Plot 0 s.
Exiting SolveHard() after 3.759r=18.8546 in [18.55227050 .. 19]
```

```
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02,
302.31384315294112302481953304972780587 00,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86,
375.73285290903429444582221827361104267 52,
328.11709294652378976569753298291533521 38,
292.99969138611027135924553390925912629 96,
358.64341560993307952660232108195848016 16,
299.89866204717987106732044246943670579 15,
360.06173467365802977252923544270222855 73,
336.59441032578884929388661852851859455 77, none,
324.65521223752148341378119644827304497 69,
331.93806791263086137494978625772942183 58, none, none,
289.54595772541273274372786660577351695 22]

1 --> 2  target = [33.81362495398965322834906562692027243634,
3.72564899352649878679248429106582085159 6,
325.89209973001286771008091454770326526 83]
one interval r = 20.37468935115915796766999424451409288622 ..
25.37892165303239553471379913922794352763
Time Approximations 0.023.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .87e-35]Solution in 0.508s

Time Plot 0 s.
Exiting SolveHard() after 1.555r=24.3395 in [22.07732228 ..
```

```
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02,
302.31384315294112302481953304972780587 00,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86,
375.73285290903429444582221827361104267 52,
328.11709294652378976569753298291533521 38,
292.99969138611027135924553390925912629 96,
358.64341560993307952660232108195848016 16,
299.89866204717987106732044246943670579 15,
360.06173467365802977252923544270222855 73,
336.59441032578884929388661852851859455 77,
256.10753186202913552351061229855213731 99,
324.65521223752148341378119644827304497 69,
331.93806791263086137494978625772942183 58, none, none,
289.54595772541273274372786660577351695 22]

1 --> 0  target = [17.93041369705825863352260852565467293494,
4.68650870202397507984851437766820922301 5,
353.30541095330105477877728139075788067 62]
one interval r = 20.73150479102136376047859290777762290009 ..
25.90675353527660063184877233129510613753
Time Approximations 0.029.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.5e-38
Equations at solution: [-.1e-37, -.25e-37, -.146e-34]Solution in 1.133s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.372r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02,
302.31384315294112302481953304972780587 00,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86,
375.73285290903429444582221827361104267 52,
328.11709294652378976569753298291533521 38,
292.99969138611027135924553390925912629 96,
358.64341560993307952660232108195848016 16,
299.89866204717987106732044246943670579 15,
360.06173467365802977252923544270222855 73,
336.59441032578884929388661852851859455 77,
256.10753186202913552351061229855213731 99,
324.65521223752148341378119644827304497 69,
331.93806791263086137494978625772942183 58,
304.79958325919528489794308459388105774 35, none,
289.54595772541273274372786660577351695 22]

2 --> 0  target = [17.93041369705825863352260852565467293494,
4.68650870202397507984851437766820922301 5,
353.30541095330105477877728139075788067 62]
one interval r = 31.37435486979168748858473337878440218400 ..
34.20127520019388990338678054715051682 37
Time Approximations 0.014.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
```

```
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [.1e-37, -.3e-37, .294e-34]Solution in 0.342s

Time Plot 0 s.
Exiting SolveHard() after 0.591r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493924818734395968590303388808 58,
441.64295973498952956243284442401538134 91,
436.91748165468045543897753628393409861 82,
422.98493397959989768038813777494539696 71,
361.52580256234324428037775840319406833 85,
401.88173904570677438264941530855550710 61,
389.59001515918412760473774334154890448 53,
328.46939893725444269761651595690545458 05,
401.50757158329345025690914898338247479 45,
358.97362824139713870109854895382240732 27,
398.33147104660829782823533086056333668 63,
371.48387394332918368391250539980335059 01,
336.61215841536075066607703528761274910 28,
361.50888347271766673981004779293459184 52,
324.67144992773815677598141888374906920 02,
302.31384315294112302481953304972780587 00,
328.46938513798116916367682174153776439 84,
343.81340624932444042664734994415220376 86,
375.73285290903429444582221827361104267 52,
328.11709294652378976569753298291533521 38,
292.99969138611027135924553390925912629 96,
358.64341560993307952660232108195848016 16,
299.89866204717987106732044246943670579 15,
360.06173467365802977252923544270222855 73,
336.59441032578884929388661852851859455 77,
256.10753186202913552351061229852137319 9,
324.65521223752148341378119644827304497 69,
331.93806791263086137494978625772942183 58,
304.79958325919528489794308459388105774 35,
323.46169176763522335260926849849918463 21,
289.54595772541273274372786660577351695 22]

Cascade time 109.631
counts: 28, 28

 Iteration 7

Start Generation 1
1 --> 0  target = [11.99999999995214925680753607604031340300,
6.21701250283896808090758035551224494667 8,
485.54908090007137333605716590468938597 64]
one interval r = 23.40850301661635516621170792250492840495 ..
27.67578046431598879358074924302891953141
Time Approximations 0.048.
```

```
hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.2e-38
Equations at solution: [.1e-37, -.52e-37, .11e-35]Solution in 1.671s

Time Plot 0 s.
Exiting SolveHard() after 3.487r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999995214925680753607604031340300,
6.217012502838968080907580355512244946678,
485.54908090007137333605716590468938597 64]
one interval r = 32.62814779222698817576412704562308692646 ..
36.10248388946443817509057396836801366082
Time Approximations 0.024.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=6e-38
Equations at solution: [-.9e-37, .6e-37, .3e-36]Solution in 1.216s

Time Plot 0 s.
Exiting SolveHard() after 1.673r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684482036406211699793556812949387,
6.58343472170542613762217478944407221 8021,
467.78730595927616512805237679756401070 07]
one interval r = 32.41978955670807380835253633319362997546 ..
35.85152417374497095106470905813589372466
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, -.182e-34]Solution in 0.69s

Time Plot 0 s.
Exiting SolveHard() after 1.1r=34.9451 in [33.70078237 .. 35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83, none, none,
401.88173904058341362198129590430698830 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684482036406211699793556812949387,
6.58343472170542613762217478944407221 8021,
467.78730595927616512805237679756401070 07]
two intervals r = 12.92327160831550066713738363702635354754 ..
9499999999968675391524278694688968363/50000000000000000000000000000000
000 or r = 18.39424858025612595323031072931553826754 ..
9499999999968675391524278694688968363/50000000000000000000000000000000
000
Time Approximations 0.046.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
```

```
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=4.3e-38
Equations at solution: [-.4e-37, -.43e-37, -.1428e-34]Solution in
33.618s

Time Plot 0 s.
Exiting SolveHard() after 36.364r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79, none,
401.88173904058341362198129590430698830 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962830426874081516074999853157940,
4.12565179670362265488316763793634366740 5,
440.67123065189709900804325949034500835 04]
two intervals r = 14.35659705111804237636506703397697303075 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000000
000 or r = 17.70352613806785583560568982968651073530 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [-.56e-37, -.3e-37, .179e-35]Solution in 1.938s

Time Plot 0 s.
Exiting SolveHard() after 3.599r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79, none,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962830426874081516074999853157940,
4.1256517967036226548831676379363436674 05,
440.67123065189709900804325949034500835 04]
one interval r = 22.397611543725352779702822736136569721 58 ..
27.237223516013012938835662386525495261 49
Time Approximations 0.039.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.891 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643868475135677535790666180415 85, rm =
14.378187703082278573916196480814432617 56}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.32e-37
Equations at solution: [-.1e-37, -.132e-36, .467e-34]Solution in 8.117s

Time Plot 0 s.
Exiting SolveHard() after 9.593r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888798832417184851246226843044269,
4.0048690816691031146767226421325779109 04,
```

```
404.86224501117715066215599301046684383 86]
two intervals r = 16.08011007766747291372059353289234681352 ..
949999999996867539152427869468896836 3/5000000000000000000000000000000000
000 or r = 16.41579812680201672046931937380828093982 ..
949999999996867539152427869468896836 3/5000000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., -.726e-35]Solution in 2.19s

Time Plot 0 s.
Exiting SolveHard() after 3.857r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16, none, none,
358.97362823773707670268603796024029910 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888798832417184851246226843044269,
4.00486908166910311467672264213257791090 4,
404.86224501117715066215599301046684383 86]
one interval r = 21.64194399411113378184134412245155789396 ..
26.76330660040050470338499882302138397705
Time Approximations 0.05.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
```

```
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, -.216e-34]Solution in 1.583s

Time Plot 0 s.
Exiting SolveHard() after 3.178r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26, none,
358.97362823773707670268603796024029910 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941802250001348554083075890904801,
5.58963718277056836584928294116935360992 1,
443.83065884250217801677025085434755055 46]
one interval r = 22.46725374473220708570675471651754281461 ..
27.27388428352087347327431820682713853731
Time Approximations 0.037.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.4e-38
Equations at solution: [0., .54e-37, -.65e-35]Solution in 1.417s

Time Plot 0 s.
Exiting SolveHard() after 3.008r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
```

```
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26, none,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941802250001348554083075890904801,
5.5896371827705683658492829411693536099 21,
443.83065884250217801677025085434755055 46]
one interval r = 32.155752795058076828190132096769338648 54 ..
35.50872228735566161679452217251505157068
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.262e-34]Solution in 1.045s

Time Plot 0 s.
Exiting SolveHard() after 1.408r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136503496814275230162280738888958,
5.1877835783757233345001053164269339597 57,
408.65773862831660379324221788594222605 05]
one interval r = 21.718401146569477007203664759690957302 81 ..
26.818493035149630174419776551553896139 93
```

```
Time Approximations 0.053.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=5.2e-38
Equations at solution: [.1e-37, .52e-37, .88e-35]Solution in 1.535s

Time Plot 0 s.
Exiting SolveHard() after 3.234r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56, none, none,
361.50888346936330531515878473151593038 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136503496814275230162280738888958,
5.18778357837572333450010531642693395975 7,
408.65773862831660379324221788594222605 05]
one interval r = 31.80828598761103011604539015374517367553 ..
35.00011460050665851120408726445570943760
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
```

```
Equations at solution: [.3e-37, -.2e-37, -.322e-34]Solution in 0.412s

Time Plot 0 s.
Exiting SolveHard() after 0.716r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867702084 20, none,
361.50888346936330531515878473151593038 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110538465387996865114196582764687,
6.19626256542513154112873159956092754437 0,
385.44474379150131641529611894542982476 08]
one interval r = 31.60836097544024740831659708915988382063 ..
34.66372795611083321680535439658777323609
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, -.61e-35]Solution in 0.569s

Time Plot 0 s.
Exiting SolveHard() after 1.4r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
```

```
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867702084 20, none,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110538465387996865114196582764687,
6.19626256542513154112873159956092754437 0,
385.44474379150131641529611894542982476 08]
two intervals r = 16.87563408757388869532122151260667719520 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000000
000 or r = 15.55640493790526863840420405419654809711 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000000
000
Time Approximations 0.06.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 6.124 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175118003965460680770210890244151, rm
= 2.33653277413648030251470562861198991310 7}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.179e-37, 0., .2011e-34]Solution in 22.382s

Time Plot 0 s.
Exiting SolveHard() after 24.124r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
```

```
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867704204 20,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874737904874826293597941858316499,
4.8838107796752277918076771449836141677 79,
376.61967855637925488324965196895391986 23]
one interval r = 21.11001304879643582216478648489908005294 ..
26.31784243473299558029955620182169571437
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=5.0e-38
Equations at solution: [-.1e-37, -.50e-37, .129e-34]Solution in 0.827s

Time Plot 0 s.
Exiting SolveHard() after 2.126r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867704204 20,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99, none,
328.46938513011649440039333779883756146 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
2 --> 0  target = [17.19898874737904874826293597941858316499,
4.883810779675227791807677144983614167779,
376.61967855637925488324965196895391986 23]
one interval r = 31.53899497718457335650551911389347472803 ..
34.53618386094675019357019640099641233584
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=7.3e-37
Equations at solution: [.56e-36, -.73e-36, -.15e-35]Solution in 0.491s

Time Plot 0 s.
Exiting SolveHard() after 1.321r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867702084 20,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99, none,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017532907745661980709242901262407,
6.025813549363944839738306772124973077938,
351.42702947898915207607008636592717301 40]
one interval r = 31.36230206119441478439371905862234996011 ..
34.17446640611775994291883053474211246879
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
```

```
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .217e-34]Solution in 1.131s

Time Plot 0 s.
Exiting SolveHard() after 1.409r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825258885757208677020842 0,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99, none,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68, none, none,
292.99969137587285752846795266130416356 59, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175329077456619807092429012624 07,
6.0258135493639448397383067721249730779 38,
351.42702947898915207607008636592717301 40]
two intervals r = 17.981355144512174535981691934468439114 17 ..
9499999999968675391524278694688968363/50000000000000000000000000000000
000 or r = 13.846080153922963473842721492897555409 77 ..
9499999999968675391524278694688968363/50000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 4.786 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713661576821238286221738813326 61, rm
= 2.7345009933112624776187923406661886921 81}});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [.36e-37, -.1e-37, -.1574e-34]Solution in
13.514s

Time Plot 0 s.
Exiting SolveHard() after 15.161r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867702084 20,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68, none, none,
292.99969137587285752846795266130416356 59, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941866370842733948081349792137039,
6.37794387394922179437674503150978780550 2,
423.28832783367415404425632362938183094 39]
one interval r = 31.94661817599200390504166828184901487214 ..
35.21212308645622400006985913387320605176
Time Approximations 0.017.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
```

```
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, .190e-34]Solution in 0.582s

Time Plot 0 s.
Exiting SolveHard() after 1.54r=34.3272 in [33.10127385 .. 35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825258885757208677020842 0,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68, none, none,
292.99969137587285752846795266130416356 59, none, none,
360.06173465988799888703555531205554200 04, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941866370842733948081349792137039,
6.37794387394922179437674503150978780550 2,
423.28832783367415404425632362938183094 39]
two intervals r = 15.22886702464990208100413213111883817228 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000
000 or r = 17.12965777046254184321157123091521318402 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000
000
Time Approximations 0.069.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 5.307 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
```

```
= 3/2 .. 19}, avoid={{r = 17.51537054142280166488755824670789091844, rm
= 2.0640682987693040733937789631284581614 63}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.46e-37, .1e-37, -.1796e-34]Solution in 21.712s

Time Plot 0 s.
Exiting SolveHard() after 24.303r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825258885757208677020842 0,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68,
375.73285289144835839546930314802699685 28, none,
292.99969137587285752846795266130416356 59, none, none,
360.06173465988799888703555531205554200 04, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343415562315139372504099369 90257,
4.0035598153663812745667562347558011457 76,
404.47973593742427367335836883932851483 06]
two intervals r = 16.09683966381127462577443173053449293699 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000
000 or r = 16.39988649096728743950009963848625369944 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
```

```
Equations at solution: [-.35e-37, 0., .4322e-34]Solution in 2.176s

Time Plot 0 s.
Exiting SolveHard() after 3.825r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867702084 20,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68,
375.73285289144835839546930314802699685 28, none,
292.99969137587285752846795266130416356 59,
358.64341560553399996638601828091385921 89, none,
360.06173465988799888703555531205554200 04, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234341556231513937250409936990257,
4.0035598153663812745667562347558011457 76,
404.47973593742427367335836883932851483 06]
one interval r = 21.634296299867754089810445833933639113 32 ..
26.757681698886641393921405246868549167 25
Time Approximations 0.05.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.2e-38
Equations at solution: [-.2e-37, -.52e-37, .110e-34]Solution in 1.453s

Time Plot 0 s.
Exiting SolveHard() after 2.992r=25.8653 in [23.83864811 ..
26.75768170]
```

```
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825258885757208677020842 0,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68,
375.73285289144835839546930314802699685 28,
328.11709293786861704120139698851546175 44,
292.99969137587285752846795266130416356 59,
358.64341560553399996638601828091385921 89, none,
360.06173465988799888703555531205554200 04, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954456802812293473169427485302008,
6.19617723027909033962312895367835064769 1,
385.42734025534198689606778803420381666 01]
one interval r = 31.60822049099462070320368926976405676248 ..
34.66347615048874220480192196533924639721
Time Approximations 0.531.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=8e-38
Equations at solution: [-.5e-37, .8e-37, .330e-34]Solution in 0.491s

Time Plot 0 s.
Exiting SolveHard() after 1.289r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867702084 20,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68,
375.73285289144835839546930314802699685 28,
328.11709293786861704120139698851546175 44,
292.99969137587285752846795266130416356 59,
358.64341560553399996638601828091385921 89, none,
360.06173465988799888703555531205554200 04, none, none,
324.65521223166831424243131913751121255 26, none, none, none, none]

0 --> 1  target = [26.46318954456802812293473169427485302008,
6.19617723027909033962312895367835064769 1,
385.42734025534198689606778803420381666 01]
two intervals r = 16.87629600295685376665647129332906356340 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000000
000 or r = 15.55559000639501551371858949812812534647 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000000
000
Time Approximations 0.062.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 6.185 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852529564723607731693797989521507, rm
= 2.33669042820103920371969690021431324 1747}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.538e-37, 0., .1349e-34]Solution in 21.817s

Time Plot 0 s.
```

```
Exiting SolveHard() after 23.524r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867702084 20,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68,
375.73285289144835839546930314802699685 28,
328.11709293786861704120139698851546175 44,
292.99969137587285752846795266130416356 59,
358.64341560553399996638601828091385921 89, none,
360.06173465988799888703555531205554200 04,
336.59441031728874786013620884766265681 79, none,
324.65521223166831424243131913751121255 26, none, none, none, none]

0 --> 2  target = [34.49522661170347833140042420739438489809,
3.89713131579728812034144142903469903698 7,
373.78081884542990013908459599608268824 67]
two intervals r = 17.29769086217636440198707551385823578659 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000000
000 or r = 14.99436407432744449246244037730253192648 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000000
000
Time Approximations 0.074.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.108e-36, 0., -.1414e-34]Solution in 1.705s

Time Plot 0 s.
```

```
Exiting SolveHard() after 3.718r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867702084 20,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68,
375.73285289144835839546930314802699685 28,
328.11709293786861704120139698851546175 44,
292.99969137587285752846795266130416356 59,
358.64341560553399996638601828091385921 89, none,
360.06173465988799888703555531205554200 04,
336.59441031728874786013620884766265681 79, none,
324.65521223166831424243131913751121255 26,
331.93806791483243052346404707199962616 66, none, none, none]

1 --> 2  target = [34.49522661170347833140042420739438489809,
3.89713131579728812034144142903469903698 7,
373.78081884542990013908459599608268824 67]
one interval r = 21.06068473209921851985657073171606161840 ..
26.26979834285880504669515743591570880901
Time Approximations 0.03.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, -.413e-34]Solution in 1.187s

Time Plot 0 s.
Exiting SolveHard() after 2.339r=25.3005 in [23.14060343 ..
26.26979834]
```

```
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825258885757208677020842 0,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68,
375.73285289144835839546930314802699685 28,
328.11709293786861704120139698851546175 44,
292.99969137587285752846795266130416356 59,
358.64341560553399996638601828091385921 89,
299.89866204544909991416207479207211882 39,
360.06173465988799888703555531205554200 04,
336.59441031728874786013620884766265681 79, none,
324.65521223166831424243131913751121255 26,
331.93806791483243052346404707199962616 66, none, none, none]

0 --> 2  target = [33.81362495408036838484295575124659842231,
3.7256489934112575610774370188277954243 04,
325.89209972399361833113135580547074362 35]
two intervals r = 18.552270490070990391382090132106812570 47 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000000
000 or r = 12.491969357671371670171249702941308557 98 ..
94999999999686753915242786946889683 63/50000000000000000000000000000000000
000
Time Approximations 0.035.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=7e-38
Equations at solution: [.157e-36, -.7e-37, -.2147e-34]Solution in
1.487s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.316r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867702084 20,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68,
375.73285289144835839546930314802699685 28,
328.11709293786861704120139698851546175 44,
292.99969137587285752846795266130416356 59,
358.64341560553399996638601828091385921 89,
299.89866204544909991416207479207211882 39,
360.06173465988799888703555531205554200 04,
336.59441031728874786013620884766265681 79, none,
324.65521223166831424243131913751121255 26,
331.93806791483243052346404707199962616 66, none, none,
289.54595772097338533817562148776356213 92]

1 --> 2  target = [33.81362495408036838484295575124659842231,
3.7256489934112575610774370188277954243 04,
325.89209972399361833113135580547074362 35]
one interval r = 20.37468935108118214354208093393759120628 ..
25.37892165292201946909733950439332233888
Time Approximations 0.023.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .469e-34]Solution in 0.497s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.518r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867702084 20,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68,
375.73285289144835839546930314802699685 28,
328.11709293786861704120139698851546175 44,
292.99969137587285752846795266130416356 59,
358.64341560553399996638601828091385921 89,
299.89866204544909991416207479207211882 39,
360.06173465988799888703555531205554200 04,
336.59441031728874786013620884766265681 79,
256.10753185342756191606093839105803171 60,
324.65521223166831424243131913751121255 26,
331.93806791483243052346404707199962616 66, none, none,
289.54595772097338533817562148776356213 92]

1 --> 0  target = [17.93041369719947123653286369592226361721,
4.68650870178094076219527588011808758083 8,
353.30541094444944361509009544720227939 10]
one interval r = 20.73150479088020718295768811037869508386 ..
25.90675353512593575312090297379353081156
Time Approximations 0.028.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
```

```
Equations at solution: [-.1e-37, -.23e-37, -.38e-35]Solution in 0.598s

Time Plot 0 s.
Exiting SolveHard() after 1.79r=25.4021 in [22.67806074 .. 25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825255888575720867702084 20,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68,
375.73285289144835839546930314802699685 28,
328.11709293786861704120139698851546175 44,
292.99969137587285752846795266130416356 59,
358.64341560553399996638601828091385921 89,
299.89866204544909991416207479207211882 39,
360.06173465988799888703555531205554200 04,
336.59441031728874786013620884766265681 79,
256.10753185342756191606093839105803171 60,
324.65521223166831424243131913751121255 26,
331.93806791483243052346404707199962616 66,
304.79958324613514315679047722617035370 06, none,
289.54595772097338533817562148776356213 92]

2 --> 0  target = [17.93041369719947123653286369592226361721,
4.68650870178094076219527588011808758083 8,
353.30541094444944361509009544720227939 10]
one interval r = 31.37435486998096470624329780357377693320 ..
34.20127520023262689664802931575500548441
Time Approximations 0.014.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
```

```
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, .27e-35]Solution in 0.317s

Time Plot 0 s.
Exiting SolveHard() after 1.105r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493814206260105942181950087357 25,
441.64295973222231663974658296670613766 12,
436.91748165423447092299425383739843086 83,
422.98493397200078752623471890345367161 79,
361.52580255889730178584592368701554205 90,
401.88173904058341362198129590430698830 04,
389.59001515954094526333207164507268823 16,
328.46939892938780059072710340106859861 26,
401.50757157733254965344888234525236477 07,
358.97362823773707670268603796024029910 92,
398.33147103495933749881083741802083349 56,
371.48387394473825258885757208677020842 0,
336.61215840676447735995302672427736182 54,
361.50888346936330531515878473151593038 15,
324.67144992179712908376490093503107599 99,
302.31384314003285095991827660458290191 22,
328.46938513011649440039333779883756146 50,
343.81340624725281368291021389350382531 68,
375.73285289144835839546930314802699685 28,
328.11709293786861704120139698851546175 44,
292.99969137587285752846795266130416356 59,
358.64341560553399996638601828091385921 89,
299.89866204544909991416207479207211882 39,
360.06173465988799888703555531205554200 04,
336.59441031728874786013620884766265681 79,
256.10753185342756191606093839105803171 60,
324.65521223166831424243131913751121255 26,
331.93806791483243052346404707199962616 66,
304.79958324613514315679047722617035370 06,
323.46169176112608201735874757087451246 93,
289.54595772097338533817562148776356213 92]

Cascade time 188.078
counts: 28, 28

 Iteration 8

Start Generation 1
1 --> 0  target = [12.00000000010797123561347176700843957900,
6.21701250281740364311167164207372138 4228,
485.54908089053941019204144325883499841 79]
one interval r = 23.40850301635388359537349446618714446869 ..
27.67578046420287200469284321253558744851
Time Approximations 0.036.
```

```
hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.2e-38
Equations at solution: [.2e-37, -.52e-37, -.3e-36]Solution in 0.88s

Time Plot 0 s.
Exiting SolveHard() after 3.02r=27.5236 in [25.56992694 .. 27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000010797123561347176700843957900,
6.217012502817403643111671642073721384228,
485.549080890539410192041443258834998417 9]
one interval r = 32.62814779199635620318059817101265075104 ..
36.10248389341055178106159339301217774 87
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, .15750e-34]Solution in 0.557s

Time Plot 0 s.
Exiting SolveHard() after 1.523r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
```

```
441.6429597247505819289575049355954572786,
436.9174816460632776527859866739029450603, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684471093779115539083589867960821,
6.58343472146539896797099198941906038626 7,
467.78730595153657330507334694702393904 78]
one interval r = 32.41978955649475870708597892246625742723 ..
35.85152417363722603886101373270991738828
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, .1870e-35]Solution in 1.107s

Time Plot 0 s.
Exiting SolveHard() after 1.477r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349288782829836384814747254234729,
441.6429597247505819289575049355954572786,
436.9174816460632776527859866739029450603, none, none,
401.8817390370987245475805607997298055845, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684471093779115539083589867960821,
6.58343472146539896797099198941906038626 7,
467.78730595153657330507334694702393904 78]
two intervals r = 12.92327160840965487308701253866064489005 ..
18999999999876063709742907046705568107/10000000000000000000000000000000
00000 or r = 18.39424858007229242857228769456364856001 ..
18999999999876063709742907046705568107/10000000000000000000000000000000
00000
Time Approximations 0.036.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
```

```
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.1e-38
Equations at solution: [.1e-37, .21e-37, .289e-35]Solution in 29.469s

Time Plot 0 s.
Exiting SolveHard() after 31.255r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60, none,
401.88173903709872454758056079972980558 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962817435103398990979108459321498,
4.1256517968023602280453222269314763730 02,
440.67123064361648525981771294608207836 42]
two intervals r = 14.35659705126025571228416300750324958318 ..
18999999999876063709742907046705568107/10000000000000000000000000000000
00000 or r = 17.70352613780361138420608273065148556955 ..
18999999999876063709742907046705568107/10000000000000000000000000000000
00000
Time Approximations 0.043.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.29e-37, .1e-37, -.123e-35]Solution in 1.782s

Time Plot 0 s.
Exiting SolveHard() after 3.315r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60, none,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962817435103398990979108459321498,
4.1256517968023602280453222269314763730 02,
440.67123064361648525981771294608207836 42]
one interval r = 22.397611543488848496757977840347262191 85 ..
27.237223515881468141629774585129544608 21
Time Approximations 0.033.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.607 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643667219996976819895703578957 36, rm =
14.378187701206169418000751866261801139 87}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., .183e-34]Solution in 6.672s

Time Plot 0 s.
Exiting SolveHard() after 8.044r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888790928186918723771945623460443,
4.0048690817834083472427236588587953960 73,
```

```
404.86224500768364513635888061932053370 41]
two intervals r = 16.08011007759061751035940717715014900737 ..
189999999998760637097429070467055681 07/10000000000000000000000000000000000
00000 or r = 16.41579812664241093147862326558591204190 ..
189999999998760637097429070467055681 07/10000000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.35e-37, 0., -.3619e-34]Solution in 2.604s

Time Plot 0 s.
Exiting SolveHard() after 4.21r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84, none, none,
358.97362823480866353269890824799614503 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888790928186918723771945623460443,
4.00486908178340834724272365885879539607 3,
404.86224500768364513635888061932053370 41]
one interval r = 21.64194399397870695316910690368702486358 ..
26.76330660031688159615297066324600693188
Time Approximations 0.044.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
```

```
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .299e-34]Solution in 0.92s

Time Plot 0 s.
Exiting SolveHard() after 2.394r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79, none,
358.97362823480866353269890824799614503 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941788438279082489946091089442924,
5.5896371828534907070187677486350122297 71,
443.83065883958347267677398933828659524 30]
one interval r = 22.46725374461393877859393160619715938812 ..
27.27388428345280718807729582394385488289
Time Approximations 0.034.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [-.1e-37, .81e-37, .40e-35]Solution in 1.42s

Time Plot 0 s.
Exiting SolveHard() after 2.898r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
```

```
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79, none,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941788438279082489946091089442924,
5.5896371828534907070187677486350122297 71,
443.83065883958347267677398933828659524 30]
one interval r = 32.15575279489134020973638285637435650315 ..
35.50872228730457475495709317174701229329
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=6e-38
Equations at solution: [.8e-37, -.6e-37, .2594e-35]Solution in 0.976s

Time Plot 0 s.
Exiting SolveHard() after 1.334r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136511786921888386380225911014375,
5.1877835784109578956243919097887540572 07,
408.65773862120254538399820743259645747 03]
one interval r = 21.71840114636355213322586783686944391296 ..
26.81849303501482260748419913909486227398
```

```
Time Approximations 0.051.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=5.3e-38
Equations at solution: [-.1e-37, -.53e-37, .8e-36]Solution in 0.887s

Time Plot 0 s.
Exiting SolveHard() after 2.541r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47, none, none,
361.50888346663754871224954322541647004 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136511786921888386380225911014375,
5.1877835784109578956243919097887540572 07,
408.65773862120254538399820743259645747 03]
one interval r = 31.80828598739525490986488099797708667104 ..
35.00011460037704019959614916259213421789
Time Approximations 0.022.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=1.0e-37
```

```
Equations at solution: [-.9e-37, .10e-36, -.65566e-34]Solution in
0.408s

Time Plot 0 s.
Exiting SolveHard() after 1.248r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46, none,
361.50888346663754871224954322541647004 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110530922199772188292619303711726,
6.19626256520667024155950591655069899725 8,
385.44474378880954679674626921514683091 43]
one interval r = 31.60836097525899347029358683172185209811 ..
34.66372795603357767103620728075304885418
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., .13808e-34]Solution in 1.041s

Time Plot 0 s.
Exiting SolveHard() after 1.313r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46, none,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110530922199772188292619303711726,
6.19626256520667024155950591655069899725 8,
385.44474378880954679674626921514683091 43]
two intervals r = 16.875634087477589942805591457626896733 72 ..
189999999998760637097429070467055681 07/100000000000000000000000000000000
00000 or r = 15.556404937759114550622289401146714685 79 ..
189999999998760637097429070467055681 07/100000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 5.31 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175110133501609358203315055888461, rm
= 2.33653277423590179637641830117230986336 7}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.358e-37, 0., .2178e-34]Solution in 19.906s

Time Plot 0 s.
Exiting SolveHard() after 21.561r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
```

```
401.881739037098724547580560799729805584 5,
389.590015152494856354866989940486244188 4,
328.469398931332722354176106700096151327 9,
401.507571574997310817682523033565388532 6,
358.973628234808663532698908247996145033 1,
398.331471036044626313925727698244288034 7,
371.483873938938911358040348985371718944 6,
336.612158409454082654529108107122375520 0,
361.508883466637548712249543225416470048 1,
324.671449923432502188024807196791562356 4, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747292852465349376204704431846 42,
4.883810779752465920631597095657692761654,
376.619678553576923172122931602261505357 0]
one interval r = 21.110013048678447260732818833533206562 84 ..
26.317842434653995780149772846613952203 88
Time Approximations 0.03.

hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.761063912186698964426645072343113601 29, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, -.214e-34]Solution in 0.724s

Time Plot 0 s.
Exiting SolveHard() after 1.876r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434928878282983638481474725423472 9,
441.642959724750581928957504935595457278 6,
436.917481646063277652785986673902945060 3,
422.984933968937625052603578687189619326 0,
361.525802556345752906493282543212246489 9,
401.881739037098724547580560799729805584 5,
389.590015152494856354866989940486244188 4,
328.469398931332722354176106700096151327 9,
401.507571574997310817682523033565388532 6,
358.973628234808663532698908247996145033 1,
398.331471036044626313925727698244288034 7,
371.483873938938911358040348985371718944 6,
336.612158409454082654529108107122375520 0,
361.508883466637548712249543225416470048 1,
324.671449923432502188024807196791562356 4, none,
328.469385132056852329029194306341880825 4, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874729285246534937620470443184642,
4.883810779752465920631597095657692761654,
376.61967855357692317212293160226150535 70]
one interval r = 31.53899497700055065353154085069445853794 ..
34.53618386086342819271334235934400472431
Time Approximations 0.015.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=4.21e-36
Equations at solution: [.325e-35, -.421e-35, .8052e-35]Solution in
0.959s

Time Plot 0 s.
Exiting SolveHard() after 1.229r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542 34729,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64, none,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017533258347730797598237923815615,
6.025813549168441760356281339492909710803,
351.42702948092367864336063784469297473 08]
one interval r = 31.36230206103622699326715416306192804705 ..
34.17446640608931779852937831943768762570
Time Approximations 0.014.
```

```
hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.1e-37, -.3e-37, .5385e-35]Solution in 0.492s

Time Plot 0 s.
Exiting SolveHard() after 0.737r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64, none,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11, none, none,
292.99969138196371682968552893920154133 66, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017533258347730797598237923815615,
6.0258135491684417603562813394929097108 03,
351.42702948092367864336063784469297473 08]
two intervals r = 17.98135514431255282437208178418699519774 ..
189999999998760637097429070467055681 07/100000000000000000000000000000000
00000 or r = 13.84608015398386387158161908376261279043 ..
189999999998760637097429070467055681 07/100000000000000000000000000000000
00000
Time Approximations 0.043.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
```

```
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 5.113 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713561027308331889145082166211 52, rm
= 2.7345009934132086928874601459165359339 28}});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [.54e-37, -.1e-37, -.161e-35]Solution in 13.778s

Time Plot 0 s.
Exiting SolveHard() after 15.32r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11, none, none,
292.99969138196371682968552893920154133 66, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941864440881087131999361387058117,
6.37794387374905814790646618125808133130 2,
423.28832783472317941328832213789490506 08]
one interval r = 31.94661817585746944590263631892964919927 ..
35.21212308645226518306045919999882716385
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.26510e-34]Solution in 0.561s

Time Plot 0 s.
Exiting SolveHard() after 1.469r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11, none, none,
292.99969138196371682968552893920154133 66, none, none,
360.06173466486152751666627467000080330 33, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941864440881087131999361387058117,
6.37794387374905814790646618125808133 1302,
423.28832783472317941328832213789490506 08]
two intervals r = 15.22886702433959454909396383844279822193 ..
189999999998760637097429070467055681 07/10000000000000000000000000000000
00000 or r = 17.12965777048713450499502168351745111159 ..
189999999998760637097429070467055681 07/10000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
```

```
in partial time = 5.569 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.51537054120237406849691057596533531065, rm
= 2.06406829880539834166481800105547181462 8}});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [-.32e-37, -.2e-37, .789e-35]Solution in 21.569s

Time Plot 0 s.
Exiting SolveHard() after 23.656r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11,
375.73285289817943819850088285435975383 93, none,
292.99969138196371682968552893920154133 66, none, none,
360.06173466486152751666627467000080330 33, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234335336520909718245375598804368,
4.00355981548470469311962481705204499367 7,
404.47973593510582419065470857181749130 10]
two intervals r = 16.09683966368333760678915133932190263667 ..
189999999998760637097429070467055681 07/100000000000000000000000000000000
00000 or r = 16.39988649085616658391299540385921336755 ..
189999999998760637097429070467055681 07/100000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.69e-37, .1e-37, -.1559e-34]Solution in 2.279s

Time Plot 0 s.
Exiting SolveHard() after 3.96r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11,
375.73285298179438198500882854359753839 3, none,
292.99969138196371682968552893920154133 66,
358.64341560362061476896300732140944515 18, none,
360.06173466486152751666627467000080330 33, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234335336520909718245375598804368,
4.0035598154847046931196248170520449936 77,
404.47973593510582419065470857181749130 10]
one interval r = 21.63429629975881508687577545973875802041 ..
26.75768169882022312972830580712317294286
Time Approximations 0.052.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, -.206e-34]Solution in 1.615s

Time Plot 0 s.
```

```
Exiting SolveHard() after 3.405r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11,
375.73285289817943819850088285435975383 93,
328.11709294089646214565317288870498862 24,
292.99969138196371682968552893920154133 66,
358.64341560362061476896300732140944515 18, none,
360.06173466486152751666627467000080330 33, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954448969368213747076863093902350,
6.19617723005975001841485212657327419724 0,
385.42734025247101996265924214959037073 38]
one interval r = 31.60822049081191635734423666481135131040 ..
34.66347615040888510110628734749695081700
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, .15741e-34]Solution in 1.145s

Time Plot 0 s.
Exiting SolveHard() after 1.455r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
```

```
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11,
375.73285298179438198500882854359753839 3,
328.11709294089646214565317288870498862 24,
292.99969138196371682968552893920154133 66,
358.64341560362061476896300732140944515 18, none,
360.06173466486152751666627467000080330 33, none, none,
324.65521223313657308571010103345473302 17, none, none, none, none]

0 --> 1  target = [26.46318954448969368213747076863093902350,
6.19617723005975001841485212657327419724 0,
385.42734025247101996265924214959037073 38]
two intervals r = 16.87629600286738282454849159002039840921 ..
189999999998760637097429070467055681 07/100000000000000000000000000000000
00000 or r = 15.55559000624045140407860511898777415319 ..
189999999998760637097429070467055681 07/100000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 5.505 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852522031519621055176563283852962, rm
= 2.33669042830211041567671491777986206838 8}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.358e-37, 0., -.873e-35]Solution in 20.779s
```

```
Time Plot 0 s.
Exiting SolveHard() after 22.537r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269898024799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11,
375.73285289817943819850088285435975383 93,
328.11709294089646214565317288870498862 24,
292.99969138196371682968552893920154133 66,
358.64341560362061476896300732140944515 18, none,
360.06173466486152751666627467000080330 33,
336.59441031979551868026820210087246170 64, none,
324.65521223313657308571010103345473302 17, none, none, none, none]

0 --> 2  target = [34.49522661157458908678939142802654383141,
3.89713131590285699492958385495992883444 8,
373.78081883956717002496428514925293727 12]
two intervals r = 17.29769086219692841262338428077058172134 ..
189999999998760637097429070467055681 07/100000000000000000000000000000000
00000 or r = 14.99436407401172802959499623251599954092 ..
189999999998760637097429070467055681 07/100000000000000000000000000000000
00000
Time Approximations 0.087.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., .255e-35]Solution in 1.793s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.69r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11,
375.73285289817943819850088285435975383 93,
328.11709294089646214565317288870498862 24,
292.99969138196371682968552893920154133 66,
358.64341560362061476896300732140944515 18, none,
360.06173466486152751666627467000080330 33,
336.59441031979551868026820210087246170 64, none,
324.65521223313657308571010103345473302 17,
331.93806790985928511676168060153127553 37, none, none, none]

1 --> 2  target = [34.49522661157458908678939142802654383141,
3.89713131590285699492958385495992883444 8,
373.78018839567170024964285149252937271 2]
one interval r = 21.06068473192871601816026954139334702011 ..
26.26979834272712089691752999151488545900
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [-.2e-37, -.5e-37, -.558e-34]Solution in 1.348s

Time Plot 0 s.
Exiting SolveHard() after 2.062r=25.3005 in [23.14060343 ..
```

```
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11,
375.73285298179438198500882854359753839 3,
328.11709294089646214565317288870498862 24,
292.99969138196371682968552893920154133 66,
358.64341560362061476896300732140944515 18,
299.89866204527313270381193562205160539 80,
360.06173466486152751666627467000080330 33,
336.59441031979551868026820210087246170 64, none,
324.65521223313657308571010103345473302 17,
331.93806790985928511676168060153127553 37, none, none, none]

0 --> 2  target = [33.81362495403518838199329729549257445755,
3.7256489935433778606032545174981898151 5,
325.89209972573326433410969810177299764 38]
two intervals r = 18.552270489925784244024699816581012121 76 ..
189999999998760637097429070467055681 07/10000000000000000000000000000000
00000 or r = 12.491969357695493356390300619788105870 79 ..
189999999998760637097429070467055681 07/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=0
Equations at solution: [0., 0., .1470e-34]Solution in 1.668s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.734r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11,
375.73285289817943819850088285435975383 93,
328.11709294089646214565317288870498862 24,
292.99969138196371682968552893920154133 66,
358.64341560362061476896300732140944515 18,
299.89866204527313270381193562205160539 80,
360.06173466486152751666627467000080330 33,
336.59441031979551868026820210087246170 64, none,
324.65521223313657308571010103345473302 17,
331.93806790985928511676168060153127553 37, none, none,
289.54595772275560058216239428331803768 81]

1 --> 2  target = [33.81362495403518838199329729549257445755,
3.72564899354337778606032545174981898151 5,
325.89209972573326433410969810177299764 38]
one interval r = 20.37468935102178059332389083400200821830 ..
25.37892165292317541569550486060637437029
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=6e-38
Equations at solution: [.4e-37, .6e-37, -.195e-34]Solution in 1.145s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.693r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11,
375.73285289817943819850088285435975383 93,
328.11709294089646214565317288870498862 24,
292.99969138196371682968552893920154133 66,
358.64341560362061476896300732140944515 18,
299.89866204527313270381193562205160539 80,
360.06173466486152751666627467000080330 33,
336.59441031979551868026820210087246170 64,
256.10753186026948757596520660392244720 61,
324.65521223313657308571010103345473302 17,
331.93806790985928511676168060153127553 37, none, none,
289.54595772275560058216239428331803768 81]

1 --> 0  target = [17.93041369696550505173492905128085903492,
4.68650870190485545076751271006566879225 7,
353.30541094749375078963341726976637653 43]
one interval r = 20.73150479085167629130470572103137337291 ..
25.90675353514971057963165757579558043895
Time Approximations 0.633.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
```

```
Equations at solution: [0., 0., .364e-34]Solution in 0.65s

Time Plot 0 s.
Exiting SolveHard() after 2.009r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11,
375.73285289817943819850088285435975383 93,
328.11709294089646214565317288870498862 24,
292.99969138196371682968552893920154133 66,
358.64341560362061476896300732140944515 18,
299.89866204527313270381193562205160539 80,
360.06173466486152751666627467000080330 33,
336.59441031979551868026820210087246170 64,
256.10753186026948757596520660392244720 61,
324.65521223313657308571010103345473302 17,
331.93806790985928511676168060153127553 37,
304.79958325401447899585232121699292353 11, none,
289.54595772275560058216239428331803768 81]

2 --> 0  target = [17.93041369696550505173492905128085903492,
4.68650870190485545076751271006566879225 7,
353.30541094749375078963341726976637653 43]
one interval r = 31.37435486983068579157470620730948655358 ..
34.20127520022105162311519401241087833483
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, .3158e-35]Solution in 0.349s

Time Plot 0 s.
Exiting SolveHard() after 0.649r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492887828298363848147472542347 29,
441.64295972475058192895750493559545727 86,
436.91748164606327765278598667390294506 03,
422.98493396893762505260357868718961932 60,
361.52580255634575290649328254321224648 99,
401.88173903709872454758056079972980558 45,
389.59001515249485635486698994048624418 84,
328.46939893133272235417610670009615132 79,
401.50757157499731081768252303356538853 26,
358.97362823480866353269890824799614503 31,
398.33147103604462631392572769824428803 47,
371.48387393893891135804034898537171894 46,
336.61215840945408265452910810712237552 00,
361.50888346663754871224954322541647004 81,
324.67144992343250218802480719679156235 64,
302.31384314713427323681008108799362479 76,
328.46938513205685232902919430634188082 54,
343.81340624524386856661471629946588721 11,
375.73285298179438198500882854359753839 3,
328.11709294089646214565317288870498862 24,
292.99969138196371682968552893920154133 66,
358.64341560362061476896300732140944515 18,
299.89866204527313270381193562205160539 80,
360.06173466486152751666627467000080330 33,
336.59441031979551868026820210087246170 64,
256.10753186026948757596520660392244720 61,
324.65521223313657308571010103345473302 17,
331.93806790985928511676168060153127553 37,
304.79958325401447899585232121699292353 11,
323.46169176428607138812407682720237983 72,
289.54595772275560058216239428331803768 81]

Cascade time 176.676
counts: 28, 28

 Iteration 9

Start Generation 1
1 --> 0  target = [12.00000000011580551198727967935119766800,
6.21701250286000502149339562877690498171 2,
485.54908089557533060086785545221329519 48]
one interval r = 23.40850301653119372619044932300386422247 ..
27.67578046427839440147810668682417770472
```

```
Time Approximations 0.038.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.2e-38
Equations at solution: [.1e-37, -.52e-37, .1e-36]Solution in 0.991s

Time Plot 0 s.
Exiting SolveHard() after 2.71r=27.5236 in [25.56992694 .. 27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000011580551198727967935119766800,
6.217012502860005021493395628776904981712,
485.549080895575330600867855452213295 1948]
one interval r = 32.628147792057277418019315036841804406 51 ..
36.10248388938385371096915902233275686533
Time Approximations 0.021.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, .41e-35]Solution in 0.583s

Time Plot 0 s.
Exiting SolveHard() after 1.646r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684478036670220250387155492721727,
6.58343472167677567388525019522541812167 1,
467.78730595531853793333167794508341072 44]
one interval r = 32.41978955654030393606492859127244746082 ..
35.85152417366505475288774620323613158631
Time Approximations 0.02.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., -.308e-34]Solution in 0.611s

Time Plot 0 s.
Exiting SolveHard() after 1.646r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32, none, none,
401.88173903841449066283374265924050924 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684478036670220250387155492721727,
6.58343472167677567388525019522541812167 1,
467.78730595531853793333167794508341072 44]
two intervals r = 12.92327160850513409344492518728410525758 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000 or r = 18.39424858032123784295816808314367246877 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000
Time Approximations 0.038.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
```

```
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.3e-38
Equations at solution: [-.3e-37, -.33e-37, .100e-34]Solution in 30.59s

Time Plot 0 s.
Exiting SolveHard() after 32.47r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56, none,
401.88173903841449066283374265924050924 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962821679024829033141673854556056,
4.12565179680023767376458309841252451026 8,
440.67123064815099493012582794568271041 23]
two intervals r = 14.35659705134676799874951047371984888396 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000 or r = 17.70352613808530277189874180463447250729 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000
Time Approximations 0.559.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., .84e-35]Solution in 1.773s

Time Plot 0 s.
Exiting SolveHard() after 3.841r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56, none,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962821679024829033141673854556056,
4.1256517968002376737645830984125245102 68,
440.67123064815099493012582794568271041 23]
one interval r = 22.397611543668583629787478273724028627 72 ..
27.237223515966208424647382843809230274 46
Time Approximations 0.036.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.629 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643826234423345106529482686399 87, rm =
14.378187703892046782880044143020080913 77}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, -.520e-34]Solution in 6.777s

Time Plot 0 s.
Exiting SolveHard() after 7.608r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888790700773554535194484849410441,
4.0048690817699920492485614288971648899 88,
```

```
404.8622450088228914850824274759643706624]
two intervals r = 16.08011007786085905350085576405774468100 ..
190000000000992690509773741954101700 17/1000000000000000000000000000000000
00000 or r = 16.41579812679990559845985232148779584463 ..
190000000000992690509773741954101700 17/1000000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.17e-37, 0., .99e-35]Solution in 2.447s

Time Plot 0 s.
Exiting SolveHard() after 4.103r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05, none, none,
358.97362823560364757471884158675601970 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888790700773554535194484849410441,
4.0048690817699920492485614288971648899 88,
404.8622450088228914850824274759643706624]
one interval r = 21.64194399409658169249710465796676577916 ..
26.76330660036411898183503796067503165692
Time Approximations 0.044.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
```

```
Accepted {r=25.8721, rm=16.7767} with Delta=5.0e-38
Equations at solution: [.1e-37, .50e-37, -.63e-35]Solution in 0.915s

Time Plot 0 s.
Exiting SolveHard() after 2.462r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71, none,
358.97362823560364757471884158675601970 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941818056773013604774744181312169,
5.58963718283979848467537855589549475338 3,
443.83065884003003815654203062914775704 84]
one interval r = 22.46725374470255318931720578656202638260 ..
27.27388428348965302715835377786129954717
Time Approximations 0.036.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.3e-38
Equations at solution: [0., .53e-37, .14e-35]Solution in 1.425s

Time Plot 0 s.
Exiting SolveHard() after 2.937r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
```

```
361.525802557281369101081754017157108532 4,
401.881739038414490662833742659240509240 4,
389.590015156319662940904782073581247990 5,
328.469398929091355033130528871110264867 1, none,
358.973628235603647574718841586756019707 5,
398.331471033800088235535379877567881371 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589418180567730136047747441813121 69,
5.589637182839798484675378555895494753383,
443.830658840030038156542030629147757048 4]
one interval r = 32.155752794900692180262133084193678289 75 ..
35.508722287287751815816582601410801245 27
Time Approximations 0.018.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.44e-35]Solution in 0.965s

Time Plot 0 s.
Exiting SolveHard() after 1.297r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434933822719163802717661434590328 1,
441.642959728374646400909671978004589390 3,
436.917481650701738171865788274701012133 2,
422.984933969539926954868405072723551365 6,
361.525802557281369101081754017157108532 4,
401.881739038414490662833742659240509240 4,
389.590015156319662940904782073581247990 5,
328.469398929091355033130528871110264867 1,
401.507571575516058337086771484306619686 1,
358.973628235603647574718841586756019707 5,
398.331471033800088235535379877567881371 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365265586520853623567654050200 54,
5.187783578447683451297266857228580179975,
408.657738625076454256812356846067417166 6]
one interval r = 21.718401146535589681529291518272481270 64 ..
26.818493035101047453597751077149602425 99
```

```
Time Approximations 0.051.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.11e-37
Equations at solution: [.2e-37, .211e-36, .39e-35]Solution in 1.488s

Time Plot 0 s.
Exiting SolveHard() after 3.105r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10, none, none,
361.50888346769438027785311341206227077 11, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136526558652085362356765405020054,
5.18778357844768345129726685722858017997 5,
408.65773862507645425681235684606741716 66]
one interval r = 31.80828598743844370139878315396953290622 ..
35.00011460041397931380267258747102413416
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=6e-38
```

```
Equations at solution: [.4e-37, -.6e-37, -.260e-34]Solution in 0.39s

Time Plot 0 s.
Exiting SolveHard() after 0.681r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10, none,
361.50888346769438027785311341206227077 11, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110535684453049443501733665358662,
6.19626256540593842302115543843383929741 1,
385.44474378983217717204351947046269078 23]
one interval r = 31.60836097527689478862248162544321690462 ..
34.66372795603182896173377873582637092893
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.152e-34]Solution in 1.05s

Time Plot 0 s.
Exiting SolveHard() after 1.305r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
```

```
441.642959728374646400909671978004589390 3,
436.917481650701738171865788274701012133 2,
422.984933969539926954868405072723551365 6,
361.525802557281369101081754017157108532 4,
401.881739038414490662833742659240509240 4,
389.590015156319662940904782073581247990 5,
328.469398929091355033130528871110264867 1,
401.507571575516058337086771484306619686 1,
358.973628235603647574718841586756019707 5,
398.331471033800088235535379877567881371 0,
371.483873942193872830966695580620763931 0, none,
361.508883467694380277853113412062270771 1,
324.671449921937244383527315239790073328 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.463471105356844530494435017336653586 62,
6.196262565405938423021155438433839297411,
385.444743789832177172043519470462690782 3]
two intervals r = 16.875634087749705939996799808850011491 73 ..
190000000000992690509773741954101700 17/100000000000000000000000000000000
00000 or r = 15.556404937897118902087595569432730388 07 ..
190000000000992690509773741954101700 17/100000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.179e-37, 0., .121e-34]Solution in 1.578s

Time Plot 0 s.
Exiting SolveHard() after 3.268r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434933822719163802717661434590328 1,
441.642959728374646400909671978004589390 3,
436.917481650701738171865788274701012133 2,
422.984933969539926954868405072723551365 6,
361.525802557281369101081754017157108532 4,
401.881739038414490662833742659240509240 4,
389.590015156319662940904782073581247990 5,
328.469398929091355033130528871110264867 1,
401.507571575516058337086771484306619686 1,
358.973628235603647574718841586756019707 5,
398.331471033800088235535379877567881371 0,
```

```
371.48387394219387283096669558062076393 10,
336.61215840709931582540922989355322076 54,
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874757290284043279272141707391399,
4.8838107779766847309836200951693775629474,
376.61967855430401015101251065192395931 39]
one interval r = 21.11001304879790612595175029229257320798 ..
26.31784243469783439837211089306393279507
Time Approximations 0.578.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.3e-38
Equations at solution: [0., .23e-37, .57e-35]Solution in 0.73s

Time Plot 0 s.
Exiting SolveHard() after 1.903r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10,
336.61215840709931582540922989355322076 54,
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89, none,
328.46938512981873845458459478520542535 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874757290284043279272141707391399,
4.8838107779766847309836200951693775629474,
376.61967855430401015101251065192395931 39]
one interval r = 31.53899497701640932131045830249244502586 ..
```

```
34.53618386085840404771822452322650769373
Time Approximations 0.015.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.13e-36
Equations at solution: [.86e-36, -.113e-35, .307e-34]Solution in 0.977s

Time Plot 0 s.
Exiting SolveHard() after 1.252r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10,
336.61215840709931582540922989355322076 54,
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89, none,
328.46938512981873845458459478520542535 15,
343.81340624562410430125912282529954057 92, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017532497802253639006757854459251,
6.0258135493512245454655447658440846334 90,
351.42702947867665044986455612134659271 05]
one interval r = 31.36230206103390833088066760631853330066 ..
34.17446640604467972826822312426193137711
Time Approximations 0.015.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
```

```
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .264e-34]Solution in 0.513s

Time Plot 0 s.
Exiting SolveHard() after 0.771r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10,
336.61215840709931582540922989355322076 54,
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89, none,
328.46938512981873845458459478520542535 15,
343.81340624562410430125912282529954057 92, none, none,
292.99969137733547828927204839037327835 43, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017532497802253639006757854459251,
6.0258135493512245454655447658440846334 90,
351.42702947867665044986455612134659271 05]
two intervals r = 17.98135514466045412422242343896237088809 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000 or r = 13.84608015391761550573709644821984335675 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=0
```

```
Equations at solution: [0., 0., -.50e-35]Solution in 1.571s

Time Plot 0 s.
Exiting SolveHard() after 3.073r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10,
336.61215840709931582540922989355322076 54,
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89,
302.31384314172473094600040645230093752 17,
328.46938512981873845458459478520542535 15,
343.81340624562410430125912282529954057 92, none, none,
292.99969137733547828927204839037327835 43, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418645435085184216708396019295 43,
6.3779438739327193487092273033338485595 94,
423.28832783247624960350546892976947766 34]
one interval r = 31.946618175842045874871898809831707100 99 ..
35.212123086398825511709714025267015875 92
Time Approximations 0.017.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, .69e-35]Solution in 0.525s

Time Plot 0 s.
Exiting SolveHard() after 1.38r=34.3272 in [33.10127385 .. 35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10,
336.61215840709931582540922989355322076 54,
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89,
302.31384314172473094600040645230093752 17,
328.46938512981873845458459478520542535 15,
343.81340624562410430125912282529954057 92, none, none,
292.99969137733547828927204839037327835 43, none, none,
360.06173466040054594201183706948382867 62, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941864543508518421670839601929543,
6.37794387393271934870922730333848559594,
423.28832783247624960350546892976947766 34]
two intervals r = 15.22886702477234597463468207007460771660 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000 or r = 17.12965777053497950621670002079418534259 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000
Time Approximations 0.054.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [-.32e-37, -.2e-37, -.223e-34]Solution in 1.82s

Time Plot 0 s.
Exiting SolveHard() after 3.596r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
```

```
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10,
336.61215840709931582540922989355322076 54,
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89,
302.31384314172473094600040645230093752 17,
328.46938512981873845458459478520542535 15,
343.81340624562410430125912282529954057 92,
375.73285289221840295569044291739753325 03, none,
292.99969137733547828927204839037327835 43, none, none,
360.06173466040054594201183706948382867 62, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234333932042595838277734968047593,
4.0035598154684991559819610945135301046 20,
404.47973593543028252761259208359601460 20]
two intervals r = 16.09683966398915682740178544765535919085 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000 or r = 16.39988649097946366059203190242286838495 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.17e-37, 0., .39e-35]Solution in 1.935s

Time Plot 0 s.
Exiting SolveHard() after 3.54r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
```

```
401.881739038414490662833742659240509240 4,
389.590015156319662940904782073581247990 5,
328.469398929091355033130528871110264867 1,
401.507571575516058337086771484306619686 1,
358.973628235603647574718841586756019707 5,
398.331471033800088235535379877567881371 0,
371.483873942193872830966695580620763931 0,
336.612158407099315825409229893553220765 4,
361.508883467694380277853113412062270771 1,
324.671449921937244383527315239790073328 9,
302.313843141724730946000406452300937521 7,
328.469385129818738454584594785205425351 5,
343.813406245624104301259122825299540579 2,
375.732852892218402955690442917397533250 3, none,
292.999691377335478289272048390373278354 3,
358.643415603710606180247868382542036899 4, none,
360.061734660400545942011837069483828676 2, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343339320425958382777349680475 93,
4.003559815468499155981961094513530104620,
404.479735935430282527612592083596014602 0]
one interval r = 21.634296299860539848317120947634817044 77 ..
26.757681698855504548620065813252236221 12
Time Approximations 0.045.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, .3e-36]Solution in 1.481s

Time Plot 0 s.
Exiting SolveHard() after 3.006r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434933822719163802717661434590328 1,
441.642959728374646400909671978004589390 3,
436.917481650701738171865788274701012133 2,
422.984933969539926954868405072723551365 6,
361.525802557281369101081754017157108532 4,
401.881739038414490662833742659240509240 4,
389.590015156319662940904782073581247990 5,
328.469398929091355033130528871110264867 1,
401.507571575516058337086771484306619686 1,
```

```
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10,
336.61215840709931582540922989355322076 54,
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89,
302.31384314172473094600040645230093752 17,
328.46938512981873845458459478520542535 15,
343.81340624562410430125912282529954057 92,
375.73285892218402955690442917397533250 3,
328.11709293790429559527534926270428281 71,
292.99969137733547828927204839037327835 43,
358.64341560371060618024786838254203689 94, none,
360.06173466040054594201183706948382867 62, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954453933572150676586606958911612,
6.19617723025962970247782734955525570 5289,
385.42734025361833704445927499980577004 79]
one interval r = 31.60822049083082458668277802965960929947 ..
34.66347615040894240063857670675960417093
Time Approximations 0.651.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., -.300e-34]Solution in 0.506s

Time Plot 0 s.
Exiting SolveHard() after 1.423r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10,
```

```
336.61215840709931582540922989355322076 54,
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89,
302.31384314172473094600040645230093752 17,
328.46938512981873845458459478520542535 15,
343.81340624562410430125912282529954057 92,
375.73285289221840295569044291739753325 03,
328.11709293790429559527534926270428281 71,
292.99969137733547828927204839037327835 43,
358.64341560371060618024786838254203689 94, none,
360.06173466040054594201183706948382867 62, none, none,
324.65521223175760232032757235847051868 91, none, none, none, none]

0 --> 1  target = [26.46318954453933572150676586606958911612,
6.19617723025962970247782734955525570 5289,
385.42734025361833704445927499980577004 79]
two intervals r = 16.87629600313475254690204253591899939341 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000 or r = 15.55559000638428053621578513140970127883 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., .43e-35]Solution in 1.585s

Time Plot 0 s.
Exiting SolveHard() after 3.317r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10,
336.61215840709931582540922989355322076 54,
```

```
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89,
302.31384314172473094600040645230093752 17,
328.46938512981873845458459478520542535 15,
343.81340624562410430125912282529954057 92,
375.73285289221840295569044291739753325 03,
328.11709293790429559527534926270428281 71,
292.99969137733547828927204839037327835 43,
358.64341560371060618024786838254203689 94, none,
360.06173466040054594201183706948382867 62,
336.59441031756800734462345382795092998 81, none,
324.65521223175760232032757235847051868 91, none, none, none, none]

0 --> 2  target = [34.49522661160442872555164498574901320729,
3.89713131589642127469301715032869171060 5,
373.78081884268973248056203726018692388 86]
two intervals r = 17.29769086239362511531127202274204028140 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000 or r = 14.99436407424378739197916045023016802087 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000
Time Approximations 0.073.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.37e-37, 0., -.135e-34]Solution in 1.648s

Time Plot 0 s.
Exiting SolveHard() after 4.385r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10,
336.61215840709931582540922989355322076 54,
```

```
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89,
302.31384314172473094600040645230093752 17,
328.46938512981873845458459478520542535 15,
343.81340624562410430125912282529954057 92,
375.73285289221840295569044291739753325 03,
328.11709293790429559527534926270428281 71,
292.99969137733547828927204839037327835 43,
358.64341560371060618024786838254203689 94, none,
360.06173466040054594201183706948382867 62,
336.59441031756800734462345382795092998 81, none,
324.65521223175760232032757235847051868 91,
331.93806791227839517704231993438839384 56, none, none, none]

1 --> 2  target = [34.49522661160442872555164498574901320729,
3.8971313158964212746930171503286917106 05,
373.78081884268973248056203726018692388 86]
one interval r = 21.06068473209035319920110022529099370402 ..
26.26979834281209540099523607063774708807
Time Approximations 0.037.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [.2e-37, .5e-37, .289e-34]Solution in 1.3s

Time Plot 0 s.
Exiting SolveHard() after 2.001r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10,
336.61215840709931582540922989355322076 54,
361.50888346769438027785311341206227077 11,
```

```
324.67144992193724438352731523979007332 89,
302.31384314172473094600040645230093752 17,
328.46938512981873845458459478520542535 15,
343.81340624562410430125912282529954057 92,
375.73285289221840295569044291739753325 03,
328.11709293790429559527534926270428281 71,
292.99969137733547828927204839037327835 43,
358.64341560371060618024786838254203689 94,
299.89866204482205413424703599438815551 24,
360.06173466040054594201183706948382867 62,
336.59441031756800734462345382795092998 81, none,
324.65521223175760232032757235847051868 91,
331.93806791227839517704231993438839384 56, none, none, none]

0 --> 2  target = [33.81362495400128691062584651420470201933,
3.72564899351954785268310791672110320 7215,
325.89209972399650607249190838999575998 22]
two intervals r = 18.552270490223765074815605328724096015 75 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000 or r = 12.491969357626955651294199988712229363 54 ..
19000000000099269050977374195410170017/10000000000000000000000000000000
00000
Time Approximations 0.036.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=4e-38
Equations at solution: [-.103e-36, .4e-37, .69e-35]Solution in 1.642s

Time Plot 0 s.
Exiting SolveHard() after 3.517r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
371.48387394219387283096669558062076393 10,
```

```
336.61215840709931582540922989355322076 54,
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89,
302.31384314172473094600040645230093752 17,
328.46938512981873845458459478520542535 15,
343.81340624562410430125912282529954057 92,
375.73285289221840295569044291739753325 03,
328.11709293790429559527534926270428281 71,
292.99969137733547828927204839037327835 43,
358.64341560371060618024786838254203689 94,
299.89866204482205413424703599438815551 24,
360.06173466040054594201183706948382867 62,
336.59441031756800734462345382795092998 81, none,
324.65521223175760232032757235847051868 91,
331.93806791227839517704231993438839384 56, none, none,
289.54595772074461896772130123050630399 20]

1 --> 2  target = [33.81362495400128691062584651420470201933,
3.72564899351954785268310791672110320 7215,
325.89209972399650607249190838999575998 22]
one interval r = 20.37468935112688871063901953217968579129 ..
25.37892165292628348699258236555078635119
Time Approximations 0.023.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, .184e-34]Solution in 0.487s

Time Plot 0 s.
Exiting SolveHard() after 1.576r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
358.97362823560364757471884158675601970 75,
398.33147103380008823553537987756788137 10,
```

```
371.48387394219387283096669558062076393 10,
336.61215840709931582540922989355322076 54,
361.50888346769438027785311341206227077 11,
324.67144992193724438352731523979007332 89,
302.31384314172473094600040645230093752 17,
328.46938512981873845458459478520542535 15,
343.81340624562410430125912282529954057 92,
375.73285289221840295569044291739753325 03,
328.11709293790429559527534926270428281 71,
292.99969137733547828927204839037327835 43,
358.64341560371060618024786838254203689 94,
299.89866204482205413424703599438815551 24,
360.06173466040054594201183706948382867 62,
336.59441031756800734462345382795092998 81,
256.10753185533215438830386051899320879 55,
324.65521223175760232032757235847051868 91,
331.93806791227839517704231993438839384 56, none, none,
289.54595772074461896772130123050630399 20]

1 --> 0  target = [17.93041369732416046688134642740193959821,
4.68650870189855087182249789980602247339 7,
353.30541094496031097175151904237716228 83]
one interval r = 20.73150479092958962160239569028423241294 ..
25.90675353513687281535427371532530627510
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.2e-38
Equations at solution: [-.1e-37, -.22e-37, -.138e-34]Solution in 0.643s

Time Plot 0 s.
Exiting SolveHard() after 1.878r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.64295972837464640090967197800458939 03,
436.91748165070173817186578827470101213 32,
422.98493396953992695486840507272355136 56,
361.52580255728136910108175401715710853 24,
401.88173903841449066283374265924050924 04,
389.59001515631966294090478207358124799 05,
328.46939892909135503313052887111026486 71,
401.50757157551605833708677148430661968 61,
```

```
358.9736282356036475747188415867560197075,
398.3314710338000882355353798775678813710,
371.4838739421938728309666955806207639310,
336.6121584070993158254092298935532207654,
361.5088834676943802778531134120622707711,
324.6714499219372443835273152397900733289,
302.3138431417247309460004064523009375217,
328.4693851298187384545845947852054253515,
343.8134062456241043012591228252995405792,
375.7328528922184029556904429173975332503,
328.1170929379042955952753492627042828171,
292.9996913773354782892720483903732783543,
358.6434156037106061802478683825420368994,
299.8986620448220541342470359943881555124,
360.0617346604005459420118370694838286762,
336.5944103175680073446234538279509299881,
256.1075318553215438830386051899320879 55,
324.6552122317576023203275723584705186891,
331.9380679122783951770423199343883938456,
304.7995832485275724476269538904877206723, none,
289.5459577207446189677213012305063039920]

2 --> 0  target = [17.93041369732416046688134642740193959821,
4.6865087018985508718224978998060224733 97,
353.3054109449603109717515190423771622883]
one interval r = 31.37435486982617810212640990680321950805 ..
34.20127520017206795880188119796808330424
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, .514e-34]Solution in 0.865s

Time Plot 0 s.
Exiting SolveHard() after 1.145r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493382271916380271766143459032 81,
441.6429597283746464009096719780045893903,
436.9174816507017381718657882747010121332,
422.9849339695399269548684050727235513656,
361.5258025572813691010817540171571085324,
401.8817390384144906628337426592405092404,
```

```
389.5900151563196629409047820735812479905,
328.4693989290913550331305288711102648671,
401.5075715755160583370867714843066196861,
358.9736282356036475747188415867560197075,
398.3314710338000882355353798775678813710,
371.4838739421938728309666955806207639310,
336.6121584070993158254092298935532207654,
361.5088834676943802778531134120622707711,
324.6714499219372443835273152397900733289,
302.3138431417247309460004064523009375217,
328.4693851298187384545845947852054253515,
343.8134062456241043012591228252995405792,
375.7328528922184029556904429173975332503,
328.1170929379042955952753492627042828171,
292.9996913773354782892720483903732783543,
358.6434156037106061802478683825420368994,
299.8986620448220541342470359943881555124,
360.0617346604005459420118370694838286762,
336.5944103175680073446234538279509299881,
256.1075318553321543883038605189932087955,
324.6552122317576023203275723584705186891,
331.9380679122783951770423199343883938456,
304.7995832485275724476269538904877206723,
323.4616917616973249742800202196138611031,
289.5459577207446189677213012305063039920]

Cascade time 107.464
counts: 28, 28

 Iteration 10

Start Generation 1
1 --> 0  target = [12.0000000001293830367647557592147054270 0,
6.2170125029003047120812078385521890749 81,
485.5490809033578244374944065212477204588]
one interval r = 23.40850301675806124463511807614955895294 ..
27.67578046452006591885157583253148042073
Time Approximations 0.036.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.09e-37
Equations at solution: [-.7e-37, .209e-36, .19e-35]Solution in 1.437s

Time Plot 0.001 s.
Exiting SolveHard() after 3.052r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000012938303676475575921470542700,
6.2170125029003047120812078385521890749 81,
485.54908090335782443749440652124772045 88]
one interval r = 32.628147792135483864637823068785405 81775 ..
36.102483889487508648604876183323620382 14
Time Approximations 0.021.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=6e-38
Equations at solution: [.9e-37, -.6e-37, .27e-35]Solution in 1.222s

Time Plot 0 s.
Exiting SolveHard() after 1.621r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684503150523803664024491676206996,
6.5834347216313869989241910481229651664 91,
467.78730596346588488552751675593694183 61]
one interval r = 32.419789556617585992331314009170404720 26 ..
35.851524173774697423521261019988491633 83
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.318e-34]Solution in 1.187s

Time Plot 0 s.
Exiting SolveHard() after 1.565r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25, none, none,
401.88173904562607961290546963458724648 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596845031505238036640244916762069 96,
6.5834347216313869989241910481229651664 91,
467.78730596346588488552751675593694183 61]
two intervals r = 12.923271608476029671236579016068793253 45 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000 or r = 18.394248580543542841932798228904663988 16 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000
Time Approximations 0.037.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=9.8e-38
Equations at solution: [-.8e-37, -.98e-37, -.2082e-34]Solution in
29.86s

Time Plot 0 s.
Exiting SolveHard() after 31.737r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84, none,
401.88173904562607961290546963458724648 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962830367462009588874495796226262,
4.12565179672607332274053097118878405357 4,
440.67123065467578982283420900796588920 32]
two intervals r = 14.35659705138246707543417328635575550010 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000 or r = 17.70352613832124385933859618024400735501 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., -.1077e-34]Solution in 1.788s

Time Plot 0 s.
Exiting SolveHard() after 3.89r=15.9119 in [14.35659706 .. 18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84, none,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962830367462009588874495796226262,
4.12565179672607332274053097118878405357 4,
440.67123065467578982283420900796588920 32]
one interval r = 22.39761154383087226735188086238679021470 ..
27.23722351620694257492073180211939563927
Time Approximations 0.035.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
```

```
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.64 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070644034487278838428611808544318 37, rm =
14.378187702469663089029213644644313268 09}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., -.650e-34]Solution in 6.733s

Time Plot 0 s.
Exiting SolveHard() after 7.524r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888007710981775514292758520846 78,
4.0048690816998680564219594090679325351 76,
404.86224501635055551625185239520930531 40]
two intervals r = 16.080110077841419219847261383585057005 14 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000 or r = 16.415798127145537718901560312358409128 86 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.17e-37, 0., -.2486e-34]Solution in 2.502s

Time Plot 0 s.
Exiting SolveHard() after 4.163r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70, none, none,
358.97362824153741836835587646780628271 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888007710981775514292758520846 78,
4.0048690816998680564219594090679325351 76,
404.86224501635055551625185239520930531 40]
one interval r = 21.641943994242006832331286481035386621 71 ..
26.763306600623080243633196449132052175 29
Time Approximations 0.049.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=5.2e-38
Equations at solution: [.2e-37, .52e-37, -.87e-35]Solution in 0.904s

Time Plot 0 s.
Exiting SolveHard() after 2.45r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
```

```
328.46939893558034854291028591840110114 77, none,
358.97362824153741836835587646780628271 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941806707140612549092034214642185,
5.5896371828972886433151223619231439696 14,
443.83065884945382803605626299613610397 04]
one interval r = 22.46725374493181767985492925734774182937 ..
27.27388428376316409965606389445002006650
Time Approximations 0.033.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, -.19e-35]Solution in 1.435s

Time Plot 0 s.
Exiting SolveHard() after 2.912r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77, none,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

2 --> 0  target = [14.19258941806707140612549092034214642185,
5.5896371828972886433151223619231439696 14,
443.83065884945382803605626299613610397 04]
one interval r = 32.15575279498361309766375231726141867888 ..
35.50872228741635472266797025716283010760
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.45e-35]Solution in 1s

Time Plot 0 s.
Exiting SolveHard() after 1.33r=34.9395 in [33.37332721 .. 35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136535233709642926299552024618915,
5.1877835784535765567243296458794094736 13,
408.65773863021946598486870359360050940 42]
one interval r = 21.71840114663717469813313342111489459205 ..
26.81849303532518951842576108197571477682
Time Approximations 0.056.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, -.86e-35]Solution in 1.499s

Time Plot 0 s.
Exiting SolveHard() after 3.155r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53, none, none,
361.50888347331946636599477211300386617 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136535233709642926299552024618915,
5.1877835784535765567243296458794094736 13,
408.65773863021946598486870359360050940 42]
one interval r = 31.80828598746744693519988634065554120458 ..
35.00011460048024502672652854087933451762
Time Approximations 0.016.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.21e-35]Solution in 0.371s

Time Plot 0 s.
Exiting SolveHard() after 0.643r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
```

```
358.9736282415374183683558764678062827163,
398.3314710438359235122837530836416404253,
371.4838739456026952408714511050866960190, none,
361.5088834733194663659947721130038661775, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110559033924665853055490543939634,
6.196262565356391386179662773174135185785,
385.44474379567035374447307704709015231 94]
one interval r = 31.60836097530533553584866960690209671077 ..
34.66372795610709615803058574127391201317
Time Approximations 0.016.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., .78e-35]Solution in 1.052s

Time Plot 0 s.
Exiting SolveHard() after 1.309r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349414564394476874938349326955864,
441.6429597362546431196946565440061735520,
436.9174816569349230624063316843083095125,
422.9849339787331361060377135254679715984,
361.5258025628889912710518995321863310474,
401.8817390456260796129054696345872464881,
389.5900151614553361408820751197838457470,
328.4693989355803485429102859184011011477,
401.5075715827846148133774783144930846145,
358.9736282415374183683558764678062827163,
398.3314710438359235122837530836416404253,
371.4838739456026952408714511050866960190, none,
361.5088834733194663659947721130038661775,
324.6714499268929411957027525282929861859, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110559033924665853055490543939634,
6.196262565356391386179662773174135185785,
385.44474379567035374447307704709015231 94]
two intervals r = 16.87563408780143515265748638677052969051 ..
190000000002042715168609742082774333 99/100000000000000000000000000000000
```

```
00000 or r = 15.5564049382062190529923688212021371356 3 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.179e-37, 0., .1497e-34]Solution in 1.64s

Time Plot 0 s.
Exiting SolveHard() after 3.286r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874761965306966156797706685477296,
4.88381077978159160858963741924840763526 1,
376.61967856028578641032588993022457988 62]
one interval r = 21.11001304887997265856519906463451343483 ..
26.31784243493308326706998692210086282421
Time Approximations 0.034.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
```

```
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=1.27e-37
Equations at solution: [.4e-37, .127e-36, .193e-34]Solution in 1.295s

Time Plot 0 s.
Exiting SolveHard() after 1.927r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59, none,
328.46938513630849679063575658293389797 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874761965306966156797706685477296,
4.8838107797815916085896374192484076352 61,
376.61967856028578641032588993022457988 62]
one interval r = 31.53899497704321371872253030959055665254 ..
34.53618386093519460342059618915093028654
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.86e-36
Equations at solution: [.144e-35, -.186e-35, .15e-35]Solution in 1.042s

Time Plot 0 s.
Exiting SolveHard() after 1.316r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
```

```
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59, none,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017557158205441577591383956623597,
6.0258135493092468768865439083203188790 33,
351.42702948543319290729297600126463710 30]
one interval r = 31.36230206105737889718561237375674301360 ..
34.17446640613049403195342939734484717594
Time Approximations 0.015.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=1.2e-37
Equations at solution: [.6e-37, -.12e-36, .223e-34]Solution in 0.484s

Time Plot 0 s.
Exiting SolveHard() after 0.73r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
```

```
401.881739045626079612905469634587246488 1,
389.590015161455336140882075119783845747 0,
328.469398935580348542910285918401101147 7,
401.507571582784614813377478314493084614 5,
358.973628241537418368355876467806282716 3,
398.331471043835923512283753083641640425 3,
371.483873945602695240871451105086696019 0,
336.612158413799035644238441176375702636 7,
361.508883473319466365994772113003866177 5,
324.671449926892941195702752528292986185 9, none,
328.469385136308496790635756582933897975 8,
343.813406249623288172034101077712369909 6, none, none,
292.999691383112447301621809855571854753 9, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175571582054415775913839566235 97,
6.025813549309246876886543908320318879033,
351.427029485433192907292976001264637103 0]
two intervals r = 17.981355144692259720417780877817638548 07 ..
19000000000204271516860974208277433399/100000000000000000000000000000000
00000 or r = 13.846080154327444688580941837611172086 91 ..
19000000000204271516860974208277433399/100000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [-.71e-37, .2e-37, .187e-34]Solution in 1.596s

Time Plot 0 s.
Exiting SolveHard() after 3.108r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434941456439447687493834932695586 4,
441.642959736254643119694656544006173552 0,
436.917481656934923062406331684308309512 5,
422.984933978733136106037713525467971598 4,
361.525802562888991271051899532186331047 4,
401.881739045626079612905469634587246488 1,
389.590015161455336140882075119783845747 0,
328.469398935580348542910285918401101147 7,
401.507571582784614813377478314493084614 5,
358.973628241537418368355876467806282716 3,
398.331471043835923512283753083641640425 3,
```

```
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96, none, none,
292.99969138311244730162180985557185475 39, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418939160316186230580803087053 60,
6.3779438739015275861925742059280412758 07,
423.28832784285404545184515291602778206 52]
one interval r = 31.946618175925866846833188661679403374 69 ..
35.212123086541301858105651364418500984 09
Time Approximations 0.017.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=6e-38
Equations at solution: [-.5e-37, .6e-37, .152e-34]Solution in 0.565s

Time Plot 0 s.
Exiting SolveHard() after 1.459r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96, none, none,
```

```
292.99969138311244730162180985557185475 39, none, none,
360.06173466964755141922601917431738281 00, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941893916031618623058080308705360,
6.37794387390152758619257420592804127580 7,
423.28832784285404545184515291602778206 52]
two intervals r = 15.22886702461364881272367352728336918021 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000 or r = 17.12965777094157738728889032548776298798 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., .2630e-34]Solution in 1.696s

Time Plot 0 s.
Exiting SolveHard() after 3.62r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96,
375.73285290375350942685942161309871246 63, none,
292.99969138311244730162180985557185475 39, none, none,
360.06173466964755141922601917431738281 00, none, none, none, none,
none, none, none]
```

```
0 --> 2  target = [34.93953234344085361006678078956818950425,
4.00355981539858232474703003365579735314 1,
404.47973594301621755356094447470318577 57]
two intervals r = 16.09683966396725461455208659821250094226 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000 or r = 16.39988649132837742424552864539242464175 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000
Time Approximations 0.047.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.17e-37, 0., -.1399e-34]Solution in 1.942s

Time Plot 0 s.
Exiting SolveHard() after 3.518r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96,
375.73285290375350942685942161309871246 63, none,
292.99969138311244730162180985557185475 39,
358.64341560969438316283604871411597087 89, none,
360.06173466964755141922601917431738281 00, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234344085361006678078956818950425,
4.00355981539858232474703003365579735314 1,
```

```
404.47973594301621755356094447470318577 57]
one interval r = 21.634296300006669057075094240116373387 60 ..
26.757681699115372072817966315595893997 93
Time Approximations 0.05.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, -.604e-34]Solution in 1.475s

Time Plot 0 s.
Exiting SolveHard() after 3.051r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96,
375.73285290375350942685942161309871246 63,
328.11709294444647242497554180275062110 47,
292.99969138311244730162180985557185475 39,
358.64341560969438316283604871411597087 89, none,
360.06173466964755141922601917431738281 00, none, none, none, none,
none, none, none]

2 --> 1  target = [26.463189544773120093576996115741920732 91,
6.196177230210172204233545821325319008 944,
385.42734025947448308835174348581468090 78]
one interval r = 31.608220490859405041327756459630391364 17 ..
34.663476150484468583501981333013380898 84
```

```
Time Approximations 0.601.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .21e-35]Solution in 0.526s

Time Plot 0 s.
Exiting SolveHard() after 1.398r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96,
375.73285290375350942685942161309871246 63,
328.11709294444647242497554180275062110 47,
292.99969138311244730162180985557185475 39,
358.64341560969438316283604871411597087 89, none,
360.06173466964755141922601917431738281 00, none, none,
324.65521223673004834624161243601407072 22, none, none, none, none]

0 --> 1  target = [26.46318954477312009357699611574192073291,
6.19617723021017220423354582132531900894 4,
385.42734025947448308835174348581468090 78]
two intervals r = 16.87629600318579771014658358368262721025 ..
190000000002042715168609742082774333 99/100000000000000000000000000000000
00000 or r = 15.55559000669425191105784850070010656093 ..
190000000002042715168609742082774333 99/100000000000000000000000000000000
00000
```

```
Time Approximations 0.056.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., -.105e-35]Solution in 1.681s

Time Plot 0 s.
Exiting SolveHard() after 3.409r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96,
375.73285290375350942685942161309871246 63,
328.11709294444647242497554180275062110 47,
292.99969138311244730162180985557185475 39,
358.64341560969438316283604871411597087 89, none,
360.06173466964755141922601917431738281 00,
336.59441032428600706661861853363320825 16, none,
324.65521223673004834624161243601407072 22, none, none, none, none]

0 --> 2  target = [34.49522661164727428275547165962199549666,
3.89713131581328404225215338199372119490 4,
373.78081884633108731938248016478473328 26]
two intervals r = 17.29769086252099843930794036888166328947 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000 or r = 14.99436407446373047487882463936314438164 ..
19000000000204271516860974208277433399/10000000000000000000000000000000
00000
```

```
Time Approximations 0.075.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [0., 0., -.853e-35]Solution in 1.686s

Time Plot 0 s.
Exiting SolveHard() after 4.477r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96,
375.73285290375350942685942161309871246 63,
328.11709294444647242497554180275062110 47,
292.99969138311244730162180985557185475 39,
358.64341560969438316283604871411597087 89, none,
360.06173466964755141922601917431738281 00,
336.59441032428600706661861853363320825 16, none,
324.65521223673004834624161243601407072 22,
331.93806791479871522466210126483835540 15, none, none, none]

1 --> 2  target = [34.49522661164727428275547165962199549666,
3.8971313158132840422521533819937211949 04,
373.78081884633108731938248016478473328 26]
one interval r = 21.06068473212867266023920594981322512052 ..
26.26979834300726416416936085101521445786
Time Approximations 0.031.
```

```
hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=1.2e-37
Equations at solution: [.5e-37, .12e-36, -.386e-34]Solution in 1.253s

Time Plot 0 s.
Exiting SolveHard() after 1.891r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96,
375.73285290375350942685942161309871246 63,
328.11709294444647242497554180275062110 47,
292.99969138311244730162180985557185475 39,
358.64341560969438316283604871411597087 89,
299.89866204770731059311693701123217521 85,
360.06173466964755141922601917431738281 00,
336.59441032428600706661861853363320825 16, none,
324.65521223673004834624161243601407072 22,
331.93806791479871522466210126483835540 15, none, none, none]

0 --> 2  target = [33.81362495406274209409390149563038095888,
3.72564899344263671969747223632433795817 6,
325.89209972922262491159706032069407924 88]
two intervals r = 18.55227049030197814646713289452612161951 ..
190000000002042715168609742082774333 99/100000000000000000000000000000000
00000 or r = 12.49196935797637669795612347559033467953 ..
190000000002042715168609742082774333 99/100000000000000000000000000000000
```

```
00000
Time Approximations 0.037.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=0
Equations at solution: [-.17e-37, 0., -.110e-34]Solution in 1.666s

Time Plot 0 s.
Exiting SolveHard() after 3.553r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96,
375.73285290375350942685942161309871246 63,
328.11709294444647242497554180275062110 47,
292.99969138311244730162180985557185475 39,
358.64341560969438316283604871411597087 89,
299.89866204770731059311693701123217521 85,
360.06173466964755141922601917431738281 00,
336.59441032428600706661861853363320825 16, none,
324.65521223673004834624161243601407072 22,
331.93806791479871522466210126483835540 15, none, none,
289.54595772467217730701700699926131398 48]

1 --> 2  target = [33.81362495406274209409390149563038095888,
3.72564899344263671969747223632433795817 6,
325.89209972922262491159706032069407924 88]
one interval r = 20.37468935113661430831603300173611882026 ..
```

```
25.37892165313805663766095612579618573856
Time Approximations 0.023.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=6e-38
Equations at solution: [.5e-37, .6e-37, .649e-34]Solution in 0.487s

Time Plot 0 s.
Exiting SolveHard() after 1.631r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96,
375.73285290375350942685942161309871246 63,
328.11709294444647242497554180275062110 47,
292.99969138311244730162180985557185475 39,
358.64341560969438316283604871411597087 89,
299.89866204770731059311693701123217521 85,
360.06173466964755141922601917431738281 00,
336.59441032428600706661861853363320825 16,
256.10753185962929727566766966208923899 03,
324.65521223673004834624161243601407072 22,
331.93806791479871522466210126483835540 15, none, none,
289.54595772467217730701700699926131398 48]

1 --> 0  target = [17.93041369735416970357404068259493193253,
4.68650870191993181576877588953335461240 3,
```

```
353.30541095174038004184115291236123451 40]
one interval r = 20.73150479099468341696562854180224233334 ..
25.90675353538411459575287669670435995618
Time Approximations 0.028.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., .130e-34]Solution in 0.588s

Time Plot 0 s.
Exiting SolveHard() after 1.844r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96,
375.73285290375350942685942161309871246 63,
328.11709294444647242497554180275062110 47,
292.99969138311244730162180985557185475 39,
358.64341560969438316283604871411597087 89,
299.89866204770731059311693701123217521 85,
360.06173466964755141922601917431738281 00,
336.59441032428600706661861853363320825 16,
256.10753185962929727566766966208923899 03,
324.65521223673004834624161243601407072 22,
331.93806791479871522466210126483835540 15,
304.79958325579414767789430423440824446 65, none,
289.54595772467217730701700699926131398 48]
```

```
2 --> 0  target = [17.93041369735416970357404068259493193253,
4.686508701919931815768775889533354612403,
353.30541095174038004184115291236123451 40]
one interval r = 31.37435486985050659570899289286735249661 ..
34.20127520025840519569716073054588021775
Time Approximations 0.6.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, .101e-34]Solution in 0.316s

Time Plot 0 s.
Exiting SolveHard() after 1.166r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494145643944768749383493269558 64,
441.64295973625464311969465654400617355 20,
436.91748165693492306240633168430830951 25,
422.98493397873313610603771352546797159 84,
361.52580256288899127105189953218633104 74,
401.88173904562607961290546963458724648 81,
389.59001516145533614088207511978384574 70,
328.46939893558034854291028591840110114 77,
401.50757158278461481337747831449308461 45,
358.97362824153741836835587646780628271 63,
398.33147104383592351228375308364164042 53,
371.48387394560269524087145110508669601 90,
336.61215841379903564423844117637570263 67,
361.50888347331946636599477211300386617 75,
324.67144992689294119570275252829298618 59,
302.31384314924540438875090046213403494 64,
328.46938513630849679063575658293389797 58,
343.81340624962328817203410107771236990 96,
375.73285290375350942685942161309871246 63,
328.11709294444647242497554180275062110 47,
292.99969138311244730162180985557185475 39,
358.64341560969438316283604871411597087 89,
299.89866204770731059311693701123217521 85,
360.06173466964755141922601917431738281 00,
336.59441032428600706661861853363320825 16,
256.10753185962929727566766966208923899 03,
324.65521223673004834624161243601407072 22,
331.93806791479871522466210126483835540 15,
```

```
304.79958325579414767789430423440824446 65,
323.46169176633904916745991361579473088 61,
289.54595772467217730701700699926131398 48]

Cascade time 106.797
counts: 28, 28

 Iteration 11

Start Generation 1
1 --> 0  target = [12.00000000006897930966475616401563849600,
6.21701250299296567041652786092264181982 8,
485.54908089935250250474448151220211141 43]
one interval r = 23.40850301657314134875602973462334035158 ..
27.67578046426759369717054686365578742181
Time Approximations 0.039.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.4e-38
Equations at solution: [-.1e-37, .24e-37, .9e-36]Solution in 1.573s

Time Plot 0 s.
Exiting SolveHard() after 3.322r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000006897930966475616401563849600,
6.21701250299296567041652786092264181982 8,
485.54908089935250250474448151220211141 43]
```

"Imaginary part neglected: ", $3.18322343222580196599031280552569102413 6 \times 10^{-17}$

```
one interval r = 32.62814779207569415036979136072084380604 ..
36.10248388942994405724791345062158796004
Time Approximations 0.044.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
```

```
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, .118e-34]Solution in 0.56s

Time Plot 0 s.
Exiting SolveHard() after 1.011r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684478254625845400544849520878320,
6.58343472172960707187499653027041629585 7,
467.78730595961689682849408407424409391 08]
```

"Imaginary part neglected: ", $3.18322343222580196599031280552569102413 6 \times 10^{-17}$

```
one interval r = 32.41978955656119121313057071271078122267 ..
35.85152417371795167800614187673990616256
Time Approximations 0.042.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, -.126e-34]Solution in 0.579s

Time Plot 0 s.
Exiting SolveHard() after 1.754r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64, none, none,
401.88173904360757626022369699925483057 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684478254625845400544849520878320,
6.58343472172960707187499653027041629585 7,
467.78730595961689682849408407424409391 08]
two intervals r = 12.92327160837005424169792259368939864497 ..
189999999999165239181310049453042333 43/100000000000000000000000000000000
00000 or r = 18.39424858023230808590911534264962034278 ..
189999999999165239181310049453042333 43/100000000000000000000000000000000
00000
Time Approximations 0.036.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.09e-37
Equations at solution: [-.9e-37, -.109e-36, -.771e-35]Solution in
31.439s

Time Plot 0 s.
Exiting SolveHard() after 33.448r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58, none,
401.88173904360757626022369699925483057 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962826430229860791443452246837095,
4.12565179695760549746939473171629075424 3,
440.67123065215920837354429427451931336 80]
two intervals r = 14.35659705116103094513088038988376245284 ..
189999999999165239181310049453042333 43/100000000000000000000000000000000
00000 or r = 17.70352613804374618359453718497656855967 ..
189999999999165239181310049453042333 43/100000000000000000000000000000000
```

```
00000
Time Approximations 0.041.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.43e-37, -.2e-37, -.970e-35]Solution in 2.047s

Time Plot 0 s.
Exiting SolveHard() after 3.738r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58, none,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962826430229860791443452246837095,
4.12565179695760549746939473171629075 4243,
440.67123065215920837354429427451931336 80]
one interval r = 22.39761154370489169870039691835203437826 ..
27.23722351597765972329657229823585969606
Time Approximations 0.033.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.801 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064391218781382061578479436022941, rm =
14.37818770650535206680031856565727102393}});
```

```
Accepted {r=26.4635, rm=16.5329} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, -.161e-34]Solution in 7.762s

Time Plot 0 s.
Exiting SolveHard() after 9.29r=26.4635 in [24.64256576 .. 27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888797361334212860833389652768440,
4.0048690819327901173151992297308674899 62,
404.86224501431602714820943594538024070 57]
two intervals r = 16.08011007756436690674298729498321871625 ..
189999999999165239181310049453042333 43/10000000000000000000000000000000
00000 or r = 16.41579812690330357042116710120045989946 ..
189999999999165239181310049453042333 43/10000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.34e-37, 0., -.4433e-34]Solution in 2.257s

Time Plot 0 s.
Exiting SolveHard() after 3.936r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
```

```
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13, none, none,
358.97362824128814384926141533672030726 00, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888797361334212860833389652768440,
4.0048690819327901173151992297308674899 62,
404.86224501431602714820943594538024070 57]
one interval r = 21.641943994147815059144603254581635714 78 ..
26.763306600411245215766152569566778858 77
Time Approximations 0.05.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.568e-34]Solution in 1.752s

Time Plot 0 s.
Exiting SolveHard() after 3.445r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10, none,
358.97362824128814384926141533672030726 00, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941791809563757058470124006160264,
5.5896371829830068005847090250152441557 19,
443.83065884562800073436358316873229910 34]
one interval r = 22.467253744775036975058734201755879678 05 ..
27.273884283518000814884465780847041237 66
Time Approximations 0.038.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
```

```
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.08e-37
Equations at solution: [-.1e-37, .108e-36, .51e-35]Solution in 1.62s

Time Plot 0 s.
Exiting SolveHard() after 2.54r=27.0204 in [24.71083344 .. 27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10, none,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941791809563757058470124006160264,
5.589637182983006800584709025015244155719,
443.83065884562800073436358316873229910 34]
```

"Imaginary part neglected: ", $3.18322343222580196599031280552569102 4136 \times 10^{-17}$

```
one interval r = 32.15575279492991726105893218949996761372 ..
35.50872228735831330682524109678052466223
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., .33e-35]Solution in 1.153s

Time Plot 0 s.
Exiting SolveHard() after 1.502r=34.9395 in [33.37332721 ..
```

```
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136501021198779526035531838231195,
5.18778357856416974458647293932009539039 2,
408.65773862968576764718100380315231879 85]
one interval r = 21.71840114657112368192027228259858505275 ..
26.81849303513365630727236251433926438325
Time Approximations 0.051.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.7e-38
Equations at solution: [0., .27e-37, 0.]Solution in 0.942s

Time Plot 0 s.
Exiting SolveHard() after 2.7r=26.4632 in [23.93303356 .. 26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
```

```
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49, none, none,
361.50888347305200869958169730188266545 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136501021198779526035531838231195,
5.1877835785641697445864729393200953903 92,
408.65773862968576764718100380315231879 85]
```

"Imaginary part neglected: ", $3.1832234322258019659903128055256910241 36 \times 10^{-17}$

```
one interval r = 31.80828598744824676468178920037755547201 ..
35.00011460046806975592063767778525626950
Time Approximations 0.015.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .89e-35]Solution in 0.411s

Time Plot 0 s.
Exiting SolveHard() after 1.423r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29, none,
361.50888347305200869958169730188266545 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110541397823542443421322310482477,
6.1962625654661526219257383758027815128 44,
```

385.4447437954084724236658743350977023671]

"Imaginary part neglected: ", $3.18322343222580196599031280552569102413 6 \times 10^{-17}$

one interval r = 31.60836097528826147003313584963540090178 ..
34.66372795609788969027330393943075565434
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) | P <--- S
rGuessMin=31.6084 rGuessMax=33.8136 rmGuess=11.783 k=708.892
scos=-582.197
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, .112e-34]Solution in 1.207s

Time Plot 0 s.
Exiting SolveHard() after 1.483r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29, none,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1 target = [26.46347110541397823542443421322310482477,
6.19626255466152621925738375802781512844,
385.4447437954084724236658743350977023671]
two intervals r = 16.87563408744849268217759097321773943196 ..
189999999999165239181310049453042333 43/10000000000000000000000000000000
00000 or r = 15.55640493806323855769368937765355563161 ..
189999999999165239181310049453042333 43/10000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,

```
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 6.372 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175111472902192582379789057141786, rm
= 2.33653277396080970341169822114570766953 7}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.717e-37, 0., -.1578e-34]Solution in 22.644s

Time Plot 0 s.
Exiting SolveHard() after 24.379r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874725520665768915388626061339457,
4.88381077988585931883594748315751574 9147,
376.61967856041884405147505595142576080 53]
one interval r = 21.11001304884101299499561461326709118662 ..
26.31784243476769919266637807085166628117
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.518e-34]Solution in 1.452s

Time Plot 0 s.
Exiting SolveHard() after 2.107r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43, none,
328.46938513661336096219943226655065774 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874725520665768915388626061339457,
4.88381077988585931883594748315751574 9147,
376.61967856041884405147505595142576080 53]
```

"Imaginary part neglected: ", $3.18322343222580196599031280552569102 4136 \times 10^{-17}$

```
one interval r = 31.53899497702905242682299694818731119420 ..
34.53618386093132158252957612160960064416
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=6.14e-36
Equations at solution: [-.471e-35, .614e-35, .69e-35]Solution in 1.159s

Time Plot 0 s.
Exiting SolveHard() after 1.434r=34.0898 in [32.52213872 ..
```

```
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43, none,
328.46938513661336096219943226655065774 33,
343.81340625221394746386515516469391576 75, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017542123813327657074811775845133,
6.02581354941973094404961656795694967037 0,
351.42702948566559713771473404301839491 25]
```

"Imaginary part neglected: ", $3.18322343222580196599031280552569102413 6 \times 10^{-17}$

```
one interval r = 31.36230206104313368639198935771818595197 ..
34.17446640612693559992049024417914614749
Time Approximations 0.015.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=1.5e-37
Equations at solution: [-.7e-37, .15e-36, -.248e-34]Solution in 0.536s

Time Plot 0 s.
Exiting SolveHard() after 0.8r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43, none,
328.46938513661336096219943226655065774 33,
343.81340625221394746386515516469391576 75, none, none,
292.99969138512594226060237310798403429 63, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017542123813327657074811775845133,
6.02581354941973094404961656795694967037 0,
351.42702948566559713771473404301839491 25]
two intervals r = 17.98135514433931875415332197536649057192 ..
189999999999165239181310049453042333 43/10000000000000000000000000000000
00000 or r = 13.84608015426340457815596914520120634667 ..
189999999999165239181310049453042333 43/10000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 4.297 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.91357071360199514625947727303205245025, rm
= 2.73450099297219142313972802205183114 1163}});
Accepted {r=18.6878, rm=15.3648} with Delta=4e-38
Equations at solution: [.107e-36, -.4e-37, -.362e-35]Solution in
13.495s

Time Plot 0 s.
Exiting SolveHard() after 15.168r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43,
302.31384314909219009071316605622137060 71,
328.46938513661336096219943226655065774 33,
343.81340625221394746386515516469391576 75, none, none,
292.99969138512594226060237310798403429 63, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941869795297007157676992706524698,
6.37794387399679170058700269006788988473 4,
423.28832783906283215456166347468374272 02]
```

"Imaginary part neglected: ", $3.18322343222580196599031280552569102413 6 \times 10^{-17}$

```
one interval r = 31.94661817587496520934861883785971042084 ..
35.21212308648259120549349589346276218739
Time Approximations 0.017.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.122e-34]Solution in 0.526s

Time Plot 0 s.
Exiting SolveHard() after 1.603r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
```

```
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43,
302.31384314909219009071316605622137060 71,
328.46938513661336096219943226655065774 33,
343.81340625221394746386515516469391576 75, none, none,
292.99969138512594226060237310798403429 63, none, none,
360.06173466777376232985351882061518997 03, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941869795297007157676992706524698,
6.37794387399679170058700269006788988473 4,
423.28832783906283215456166347468374272 02]
two intervals r = 15.22886702443447415973845169565984619 73 ..
189999999999165239181310049453042333 43/100000000000000000000000000000000
00000 or r = 17.12965777062447817813753267651897347383 ..
189999999999165239181310049453042333 43/100000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 5.532 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.51537054128397979368263261294804089844, rm
= 2.06406829865594977923792421205637139160 7}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.31e-37, .1e-37, -.1147e-34]Solution in 22.208s

Time Plot 0 s.
Exiting SolveHard() after 24.177r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
```

```
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43,
302.31384314909219009071316605622137060 71,
328.46938513661336096219943226655065774 33,
343.81340625221394746386515516469391576 75,
375.73285289964392967693276410885879666 52, none,
292.99969138512594226060237310798403429 63, none, none,
360.06173466777376232985351882061518997 03, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234341196758698927589327917651332,
4.0035598156327361744712499014838445404 99,
404.47973594134220263584151679283720765 19]
two intervals r = 16.09683966367424306481799794371045757440 ..
189999999999165239181310049453042333 43/100000000000000000000000000000000
00000 or r = 16.39988649110143943192027811701917193165 ..
189999999999165239181310049453042333 43/100000000000000000000000000000000
00000
Time Approximations 0.054.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.15e-37, 0., -.3479e-34]Solution in 2.683s

Time Plot 0 s.
Exiting SolveHard() after 4.678r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
```

```
389.590015160530265000467305084276575251 3,
328.469398935888030130290664317194734561 0,
401.507571581118772575832434783461742266 6,
358.973628241288143849261415336720307260 0,
398.331471040215350777461946415477472474 9,
371.483873947237886443081349361190867682 9,
336.612158413251553076847662186959190919 6,
361.508883473052008699581697301882665451 7,
324.671449928389359796639513620133923174 3,
302.313843149092190090713166056221370607 1,
328.469385136613360962199432266550657743 3,
343.813406252213947463865155164693915767 5,
375.732852899643929676932764108858796665 2, none,
292.999691385125942260602373107984034296 3,
358.643415609758835938348112441192890670 2, none,
360.061734667773762329853518820615189970 3, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343411967586989275893279176513 32,
4.003559815632736174471249901483844540499,
404.479735941342202635841516792837207651 9]
one interval r = 21.634296299919911760145615375698992882 14 ..
26.757681698908974950801212737649946859 32
Time Approximations 0.044.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., .155e-34]Solution in 1.698s

Time Plot 0 s.
Exiting SolveHard() after 3.425r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434937180240821317797068516523404 8,
441.642959732565532844291538906058834177 0,
436.917481654443737267705138434377068696 4,
422.984933974647072478094846716944152545 8,
361.525802562710066127119510793666991563 7,
401.881739043607576260223696999254830577 5,
389.590015160530265000467305084276575251 3,
328.469398935888030130290664317194734561 0,
401.507571581118772575832434783461742266 6,
358.973628241288143849261415336720307260 0,
```

```
398.3314710402153507774619464154774724749,
371.4838739472378864430813493611908676829,
336.6121584132515530768476621869591909196,
361.5088834730520086995816973018826654517,
324.6714499283893597966395136201339231743,
302.3138431490921900907131660562213706071,
328.4693851366133609621994322665506577433,
343.8134062522139474638651551646939157675,
375.7328528996439296769327641088587966652,
328.1170929450866009277668368191649144718,
292.9996913851259422606023731079840342963,
358.6434156097588359383481124411928906702, none,
360.0617346677737623298535188206151899703, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954459529415998730683396797662142,
6.196177230319485936101530305994848479260,
385.4273402591215307143886050883020765862]
```

"Imaginary part neglected: ", $3.183223432225801965990312805525691024136 \times 10^{-17}$

```
one interval r = 31.608220490841595644683496607823005055 30 ..
34.66347615047394371753667156593255248852
Time Approximations 0.015.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.158e-34]Solution in 0.578s

Time Plot 0 s.
Exiting SolveHard() after 0.854r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349371802408213177970685165234048,
441.6429597325655328442915389060588341770,
436.9174816544437372677051384343770686964,
422.9849339746470724780948467169441525458,
361.5258025627100661271195107936669915637,
401.8817390436075762602236969992548305775,
389.5900151605302650004673050842765752513,
328.4693989358880301302906643171947345610,
401.5075715811187725758324347834617422666,
358.9736282412881438492614153367203072600,
```

```
398.331471040215350777461946415477472 4749,
371.483873947237886443081349361190867 6829,
336.612158413251553076847662186959190 9196,
361.508883473052008699581697301882665 4517,
324.671449928389359796639513620133923 1743,
302.313843149092190090713166056221370 6071,
328.469385136613360962199432266550657 7433,
343.813406252213947463865155164693915 7675,
375.732852899643929676932764108858796 6652,
328.117092945086600927766836819164914 4718,
292.999691385125942260602373107984034 2963,
358.643415609758835938348112441192890 6702, none,
360.061734667773762329853518820615189 9703, none, none,
324.655212238141525696273344859302995 3811, none, none, none, none]

0 --> 1  target = [26.463189544595294159987306833967976 62142,
6.196177230319485936101530305994848479260,
385.427340259121530714388605088302076 5862]
two intervals r = 16.876296002836322610125647229108687 01934 ..
189999999999165239181310049453042333 43/100000000000000000000000000000000
00000 or r = 15.555590006547031039356218279375709962 44 ..
189999999999165239181310049453042333 43/100000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.930920830435909701694400819009216 33181, 3, 1, 1,
15.700860627013397511253157405159644880 12, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 5.978 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852523274547517787479615982446845, rm
= 2.33669042802648042524268795529875647 1297}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.359e-37, 0., -.931e-35]Solution in 21.836s

Time Plot 0 s.
Exiting SolveHard() after 23.603r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434937180240821317797068516523 4048,
441.642959732565532844291538906058834 1770,
436.917481654443737267705138434377068 6964,
422.984933974647072478094846716944152 5458,
361.525802562710066127119510793666991 5637,
401.881739043607576260223696999254830 5775,
```

```
389.5900151605302650004673050842765752513,
328.4693989358880301302906643171947345610,
401.5075715811187725758324347834617422666,
358.9736282412881438492614153367203072600,
398.3314710402153507774619464154774724749,
371.4838739472378864430813493611908676829,
336.6121584132515530768476621869591909196,
361.5088834730520086995816973018826654517,
324.6714499283893597966395136201339231743,
302.3138431490921900907131660562213706071,
328.4693851366133609621994322665506577433,
343.8134062522139474638651551646939157675,
375.7328528996439296769327641088587966652,
328.1170929450866009277668368191649144718,
292.9996913851259422606023731079840342963,
358.6434156097588359383481124411928906702, none,
360.0617346677737623298535188206151899703,
336.5944103236455949046228032575153619216, none,
324.6552122381415256962733448593029953811, none, none, none, none]

0 --> 2  target = [34.4952266116658905752300226886296709216 1,
3.8971313160591202489882204576952941136 88,
373.7808188480322410449202274647473003095]
two intervals r = 17.297690862104070833412883503974467325 24 ..
189999999999165239181310049453042333 43/1000000000000000000000000000000000
00000 or r = 14.994364074434721323599207547602060551 66 ..
189999999999165239181310049453042333 43/1000000000000000000000000000000000
00000
Time Approximations 0.076.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [.17e-37, -.1e-37, .832e-35]Solution in 1.848s

Time Plot 0 s.
Exiting SolveHard() after 4.087r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349371802408213177970685165234048,
441.6429597325655328442915389060588341770,
436.9174816544437372677051384343770686964,
422.9849339746470724780948467169441525458,
361.5258025627100661271195107936669915637,
401.8817390436075762602236969992548305775,
```

```
389.590015160530265000467305084276575251 3,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43,
302.31384314909219009071316605622137060 71,
328.46938513661336096219943226655065774 33,
343.81340625221394746386515516469391576 75,
375.73285289964392967693276410885879666 52,
328.11709294508660092776683681916491447 18,
292.99969138512594226060237310798403429 63,
358.64341560975883593834811244119289067 02, none,
360.06173466777376232985351882061518997 03,
336.59441032364559490462280325751536192 16, none,
324.65521223814152569627334485930299538 11,
331.93806791799334131716754743796172013 88, none, none, none]

1 --> 2  target = [34.49522661166589057523002268862967092161,
3.897131316059120248988220457695294113688,
373.78081884803224104492022746474730030 95]
one interval r = 21.06068473211782728284312303363636614568 ..
26.26979834287005671389016945288262957488
Time Approximations 0.029.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, .370e-34]Solution in 0.681s

Time Plot 0 s.
Exiting SolveHard() after 2.023r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.590015160530265000467305084276575251 3,
```

```
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43,
302.31384314909219009071316605622137060 71,
328.46938513661336096219943226655065774 33,
343.81340625221394746386515516469391576 75,
375.73285289964392967693276410885879666 52,
328.11709294508660092776683681916491447 18,
292.99969138512594226060237310798403429 63,
358.64341560975883593834811244119289067 02,
299.89866205147354288337872931344367798 09,
360.06173466777376232985351882061518997 03,
336.59441032364559490462280325751536192 16, none,
324.65521223814152569627334485930299538 11,
331.93806791799334131716754743796172013 88, none, none, none]

0 --> 2  target = [33.81362495407657943032228386819645855900,
3.72564899368814560417594952365971245020 4,
325.89209973078254546006513902124512378 71]
two intervals r = 18.55227048994487380616585919624006049909 ..
189999999999165239181310049453042333 43/1000000000000000000000000000000000
00000 or r = 12.49196935802880486907578639990276759286 ..
189999999999165239181310049453042333 43/1000000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=0
Equations at solution: [0., 0., .1201e-34]Solution in 1.72s

Time Plot 0 s.
Exiting SolveHard() after 3.754r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
```

```
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43,
302.31384314909219009071316605622137060 71,
328.46938513661336096219943226655065774 33,
343.81340625221394746386515516469391576 75,
375.73285289964392967693276410885879666 52,
328.11709294508660092776683681916491447 18,
292.99969138512594226060237310798403429 63,
358.64341560975883593834811244119289067 02,
299.89866205147354288337872931344367798 09,
360.06173466777376232985351882061518997 03,
336.59441032364559490462280325751536192 16, none,
324.65521223814152569627334485930299538 11,
331.93806791799334131716754743796172013 88, none, none,
289.54595772792446590977907304855813255 60]

1 --> 2  target = [33.81362495407657943032228386819645855900,
3.72564899368814560417594952365971245020 4,
325.89209973078254546006513902124512378 71]
one interval r = 20.37468935112978585480917726914479577704 ..
25.37892165302798306789663286058620451831
Time Approximations 0.025.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .87e-35]Solution in 0.554s

Time Plot 0 s.
Exiting SolveHard() after 1.925r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
```

```
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43,
302.31384314909219009071316605622137060 71,
328.46938513661336096219943226655065774 33,
343.81340625221394746386515516469391576 75,
375.73285289964392967693276410885879666 52,
328.11709294508660092776683681916491447 18,
292.99969138512594226060237310798403429 63,
358.64341560975883593834811244119289067 02,
299.89866205147354288337872931344367798 09,
360.06173466777376232985351882061518997 03,
336.59441032364559490462280325751536192 16,
256.10753186328828858330236967513937163 42,
324.65521223814152569627334485930299538 11,
331.93806791799334131716754743796172013 88, none, none,
289.54595772792446590977907304855813255 60]

1 --> 0  target = [17.93041369701189807479573192823668603736,
4.68650870201363274674794891190209897968 0,
353.30541095153693338649638805060905348 46]
one interval r = 20.73150479095915623356525267143616965155 ..
25.90675353522474370860281699816810941707
Time Approximations 0.029.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.5e-38
Equations at solution: [.1e-37, .25e-37, .48e-35]Solution in 0.598s

Time Plot 0 s.
Exiting SolveHard() after 2.041r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
```

```
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43,
302.31384314909219009071316605622137060 71,
328.46938513661336096219943226655065774 33,
343.81340625221394746386515516469391576 75,
375.73285289964392967693276410885879666 52,
328.11709294508660092776683681916491447 18,
292.99969138512594226060237310798403429 63,
358.64341560975883593834811244119289067 02,
299.89866205147354288337872931344367798 09,
360.06173466777376232985351882061518997 03,
336.59441032364559490462280325751536192 16,
256.10753186328828858330236967513937163 42,
324.65521223814152569627334485930299538 11,
331.93806791799334131716754743796172013 88,
304.79958325567555877052064949109618780 62, none,
289.54595772792446590977907304855813255 60]

2 --> 0  target = [17.93041369701189807479573192823668603736,
4.68650870201363274674794891190209897968 0,
353.30541095153693338649638805060905348 46]

"Imaginary part neglected: ", 3.183223432225801965990312805525691024136 × 10^-17
one interval r = 31.37435486983350275489968337884415119421 ..
34.20127520024870711814310850372965556582
Time Approximations 0.023.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=1.2e-37
Equations at solution: [.7e-37, -.12e-36, -.323e-34]Solution in 0.99s

Time Plot 0 s.
Exiting SolveHard() after 1.257r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493718024082131779706851652340 48,
441.64295973256553284429153890605883417 70,
436.91748165444373726770513843437706869 64,
422.98493397464707247809484671694415254 58,
361.52580256271006612711951079366699156 37,
401.88173904360757626022369699925483057 75,
389.59001516053026500046730508427657525 13,
328.46939893588803013029066431719473456 10,
401.50757158111877257583243478346174226 66,
358.97362824128814384926141533672030726 00,
398.33147104021535077746194641547747247 49,
371.48387394723788644308134936119086768 29,
336.61215841325155307684766218695919091 96,
361.50888347305200869958169730188266545 17,
324.67144992838935979663951362013392317 43,
302.31384314909219009071316605622137060 71,
328.46938513661336096219943226655065774 33,
343.81340625221394746386515516469391576 75,
375.73285289964392967693276410885879666 52,
328.11709294508660092776683681916491447 18,
292.99969138512594226060237310798403429 63,
358.64341560975883593834811244119289067 02,
299.89866205147354288337872931344367798 09,
360.06173466777376232985351882061518997 03,
336.59441032364559490462280325751536192 16,
256.10753186328828858330236967513937163 42,
324.65521223814152569627334485930299538 11,
331.93806791799334131716754743796172013 88,
304.79958325567555877052064949109618780 62,
323.46169176885408794331483202803972139 23,
289.54595772792446590977907304855813255 60]

Cascade time 186.964
counts: 28, 28

 Iteration 12

Start Generation 1
1 --> 0  target = [11.99999999997507507736205330014818841100,
6.21701250280918751358380036663501295466 4,
485.54908089510185557177022295327536562 69]
one interval r = 23.40850301645281798795469214182819795917 ..
27.67578046434258209415862491116750048067
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.5e-38
Equations at solution: [-.2e-37, .55e-37, .2e-36]Solution in 0.943s

Time Plot 0 s.
Exiting SolveHard() after 2.737r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999975075077362053300148188411 00,
6.2170125028091875135838003666350129546 64,
485.54908089510185557177022295327536562 69]
one interval r = 32.628147792081875467396609898389513066 32 ..
36.102483889358208524701553424268921725 78
Time Approximations 0.021.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .139e-34]Solution in 0.579s

Time Plot 0 s.
Exiting SolveHard() after 1.691r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844852592878099084793882627293 18,
```

```
6.583434721453129607771409954220651122213,
467.787305955865565052831234439041868144 7]
one interval r = 32.41978955658012151110091532907666629641 ..
35.85152417365709482646038156400474763512
Time Approximations 0.023.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, -.178e-34]Solution in 1.321s

Time Plot 0 s.
Exiting SolveHard() after 1.689r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51, none, none,
401.88173904085856126320320910967927270 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844852592878099084793882627293 18,
6.583434721453129607771409954220651122213,
467.787305955865565052831234439041868144 7]
two intervals r = 12.92327160826430733843440653737296361105 ..
4749999999962030546427879073801068029/2500000000000000000000000000000000
000 or r = 18.39424858012649467153868535017494536184 ..
4749999999962030546427879073801068029/2500000000000000000000000000000000
000
Time Approximations 0.038.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.5e-38
Equations at solution: [-.5e-37, -.55e-37, .58e-36]Solution in 33.223s
```

```
Time Plot 0 s.
Exiting SolveHard() after 35.244r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76, none,
401.88173904085856126320320910967927270 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962820229736268050353724722781755,
4.1256517967592070660432955875049614958 29,
440.67123064788114037516407682921613246 38]
two intervals r = 14.35659705110273041669244793287336801180 ..
4749999999962030546427879073801068029/2500000000000000000000000000000000
000 or r = 17.70352613789761083358786044104164120599 ..
4749999999962030546427879073801068029/2500000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [.57e-37, .3e-37, .2494e-34]Solution in 1.837s

Time Plot 0 s.
Exiting SolveHard() after 3.616r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76, none,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
1 --> 2  target = [35.46322962820229736268050353724722781755,
4.12565179675920706604329558750496149582 9,
440.67123064788114037516407682921613246 38]
one interval r = 22.39761154354564073481102189356654361715 ..
27.23722351602645234694288355631393530162
Time Approximations 0.034.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.824 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064379728501570223426547891497049, rm =
14.37818770060993212795126285769169938509}});
Accepted {r=26.4635, rm=16.5329} with Delta=8.0e-38
Equations at solution: [-.1e-37, -.80e-37, -.284e-34]Solution in 7.833s

Time Plot 0 s.
Exiting SolveHard() after 9.528r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888794189099820775610449332782896,
4.00486908173887404455547036498221787732 4,
404.86224501147655277273716831964621839 69]
two intervals r = 16.08011007745938264489345881448285089804 ..
47499999999620305464278790738010680 29/2500000000000000000000000000000000
000 or r = 16.41579812677280773849671282297839813476 ..
47499999999620305464278790738010680 29/2500000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
```

```
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.51e-37, -.1e-37, .170e-35]Solution in 2.39s

Time Plot 0 s.
Exiting SolveHard() after 4.218r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69, none, none,
358.97362823843537509653844572456668647 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887941890998207756104493327828 96,
4.0048690817388740445554703649822178773 24,
404.86224501147655277273716831964621839 69]
one interval r = 21.641943993991914370457807933622731951 24 ..
26.763306600456834958713655227977530145 84
Time Approximations 0.049.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, .701e-34]Solution in 1.842s

Time Plot 0 s.
Exiting SolveHard() after 3.523r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26, none,
358.97362823843537509653844572456668647 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941774678760416092224930264422400,
5.5896371828211426105231484437523977223 54,
443.83065884348008266721887363910423347 08]
one interval r = 22.46725374466537425903675863584508878851 ..
27.27388428359321906861549074936504850854
Time Approximations 0.038.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.0e-38
Equations at solution: [.1e-37, -.80e-37, -.51e-35]Solution in 1.652s

Time Plot 0 s.
Exiting SolveHard() after 2.575r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26, none,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]
```

```
2 --> 0  target = [14.19258941774678760416092224930264422400,
5.589637182821142610523148443752397722354,
443.83065884348008266721887363910423347 08]
one interval r = 32.15575279497497410701516057249174140855 ..
35.50872228732618762848229530644817964684
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=9e-38
Equations at solution: [-.10e-36, .9e-37, -.17e-35]Solution in 1.057s

Time Plot 0 s.
Exiting SolveHard() after 1.401r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136497009011373949300724385083951,
5.187783578373166206246747845403041052344,
408.65773862533959988332339797777635638 18]
one interval r = 21.71840114638744148163727239528367330733 ..
26.81849303515985664566739980518227595351
Time Approximations 0.05.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.37e-37
Equations at solution: [.2e-37, .237e-36, .157e-34]Solution in 0.986s

Time Plot 0 s.
Exiting SolveHard() after 2.811r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41, none, none,
361.50888347002577823542895674101017432 86, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136497009011373949300724385083951,
5.18778357837316620624674784540304105 2344,
408.65773862533959988332339797777635638 18]
one interval r = 31.80828598748426522590846940629717436576 ..
35.00011460041344532529773002394228485200
Time Approximations 0.016.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=1.0e-37
Equations at solution: [-.9e-37, .10e-36, .249e-34]Solution in 0.384s

Time Plot 0 s.
Exiting SolveHard() after 1.496r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51, none,
361.50888347002577823542895674101017432 86, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110544451378023774616098164079367,
6.1962625651952314142874821135194857791 48,
385.44474379239900105786249607494164549 80]
one interval r = 31.608360975344342249463461981699892873 66 ..
34.663727956069258090342057001631925409 85
Time Approximations 0.016.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., .242e-34]Solution in 1.221s

Time Plot 0 s.
Exiting SolveHard() after 1.47r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
```

```
371.48387394292345131378881727285283083 51, none,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110544451378023774616098164079367,
6.19626256519523141428748211351948577914 8,
385.44474379239900105786249607494164549 80]
two intervals r = 16.875634087362358029667593725074028545 54 ..
47499999999620305464278790738010680 29/2500000000000000000000000000000000
000 or r = 15.556404937907207989978818233143420616 42 ..
47499999999620305464278790738010680 29/2500000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 5.898 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175101955635043185320690722660525, rm
= 2.33653277428186556908585832828104978717 3}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.896e-37, 0., -.58e-36]Solution in 21.435s

Time Plot 0 s.
Exiting SolveHard() after 23.224r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]
```

```
Start Generation 4
1 --> 0  target = [17.19898874717561768924342132133367647238,
4.883810779705904297821850498995395692827,
376.61967855734787593703391957493671292 19]
one interval r = 21.11001304866274880164126666654487621706 ..
26.31784243479118132500075100379200114403
Time Approximations 0.03.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, .231e-34]Solution in 0.74s

Time Plot 0 s.
Exiting SolveHard() after 2.104r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58, none,
328.46938513496232007899534361788049983 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874717561768924342132133367647238,
4.883810779705904297821850498995395692827,
376.61967855734787593703391957493671292 19]
one interval r = 31.53899497708756776639973621464886320 70 ..
34.53618386090441154210044261841604504495
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
```

```
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.14e-36
Equations at solution: [-.87e-36, .114e-35, -.203e-34]Solution in
1.126s

Time Plot 0 s.
Exiting SolveHard() after 1.402r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58, none,
328.46938513496232007899534361788049983 29,
343.81340624897905629468283133866259251 26, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017545302905159551212461789668211,
6.0258135491563780674340349551486844294 36,
351.42702948403093205775455357356180389 44]
one interval r = 31.362302061118868567515013154378226970 90 ..
34.174466406128244257547416949226858743 15
Time Approximations 0.016.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
```

```
Equations at solution: [0., 0., -.287e-34]Solution in 0.538s

Time Plot 0 s.
Exiting SolveHard() after 0.784r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58, none,
328.46938513496232007899534361788049983 29,
343.81340624897905629468283133866259251 26, none, none,
292.99969138444952852576658742372197076 25, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175453029051595512124617896682 11,
6.0258135491563780674340349551486844294 36,
351.42702948403093205775455357356180389 44]
two intervals r = 17.981355144230355609843173767501546481 95 ..
474999999996203054642787907380106802 9/2500000000000000000000000000000000
000 or r = 13.846080154147203033314356714959040819 64 ..
474999999996203054642787907380106802 9/2500000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 5.008 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713513555825049282150910367572 49, rm
= 2.7345009934998099872051266307877349512 42}});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
```

```
Equations at solution: [-.71e-37, .2e-37, -.1171e-34]Solution in
13.423s

Time Plot 0 s.
Exiting SolveHard() after 15.12r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58,
302.31384314927601310606012839734596756 88,
328.46938513496232007899534361788049983 29,
343.81340624897905629468283133866259251 26, none, none,
292.99969138444952852576658742372197076 25, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941878036241904807500576480040839,
6.37794387373508984037799631397104237040 7,
423.28832783819693955268771191424563477 83]
one interval r = 31.94661817593906377153325975299672084620 ..
35.21212308647440863872654263029177048469
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=7e-38
Equations at solution: [-.7e-37, .7e-37, .251e-34]Solution in 0.592s

Time Plot 0 s.
Exiting SolveHard() after 1.72r=34.3272 in [33.10127385 .. 35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58,
302.31384314927601310606012839734596756 88,
328.46938513496232007899534361788049983 29,
343.81340624897905629468283133866259251 26, none, none,
292.99969138444952852576658742372197076 25, none, none,
360.06173466776321950954242512163288073 34, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418780362419048075005764800408 39,
6.3779438737350898403779963139710423704 07,
423.28832783819693955268771191424563477 83]
two intervals r = 15.22886702422023600318884529908563096903 ..
474999999996203054642787907380106802 9/2500000000000000000000000000000000
000 or r = 17.12965777057988656087343416683799141988 ..
474999999996203054642787907380106802 9/2500000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 5.591 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541095706096891836606459793785 33, rm
= 2.0640682988322076581504050632028815332 44}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [-.15e-37, 0., -.2657e-34]Solution in 23.353s

Time Plot 0 s.
Exiting SolveHard() after 25.461r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58,
302.31384314927601310606012839734596756 88,
328.46938513496232007899534361788049983 29,
343.81340624897905629468283133866259251 26,
375.73285290081407862854202597590207941 40, none,
292.99969138444952852576658742372197076 25, none, none,
360.06173466776321950954242512163288073 34, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343385329949231534586192988 22650,
4.0035598154399923865610224932492164142 09,
404.47973593884601507686453942099640387 41]
two intervals r = 16.09683966355452814258056660584220374050 ..
474999999996203054642787907380106802 9/2500000000000000000000000000000000
000 or r = 16.39988649098491090962844270724511814654 ..
474999999996203054642787907380106802 9/2500000000000000000000000000000000
000
Time Approximations 0.058.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.33e-37, .1e-37, .4323e-34]Solution in 2.422s

Time Plot 0 s.
Exiting SolveHard() after 4.473r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58,
302.31384314927601310606012839734596756 88,
328.46938513496232007899534361788049983 29,
343.81340624897905629468283133866259251 26,
375.73285290081407862854202597590207941 40, none,
292.99969138444952852576658742372197076 25,
358.64341560720248141034851863999591361 54, none,
360.06173466776321950954242512163288073 34, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343385329949231534586192988226 50,
4.0035598154399923865610224932492164142 09,
404.47973593884601507686453942099640387 41]
one interval r = 21.634296299770593996594325964631226926 82 ..
26.757681698959385037439631773267061452 27
Time Approximations 0.049.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, .254e-34]Solution in 1.823s

Time Plot 0 s.
Exiting SolveHard() after 3.623r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
```

```
361.525802559726127041664021276754064004 4,
401.881739040858561263203209109679272709 9,
389.590015156454810456610287918786066156 9,
328.469398934237987947751015470352961662 6,
401.507571578705568163595904220881772862 1,
358.973628238435375096538445724566686473 0,
398.331471039319849424770531226083244224 1,
371.483873942923451313788817272852830835 1,
336.612158412125943961637943445704237656 1,
361.508883470025778235428956741010174328 6,
324.671449926400929802925892718079605405 8,
302.313843149276013106060128397345967568 8,
328.469385134962320078995343617880499832 9,
343.813406248979056294682831338662592512 6,
375.732852900814078628542025975902079414 0,
328.117092943752787819672830744027800283 8,
292.999691384449528525766587423721970762 5,
358.643415607202481410348518639995913615 4, none,
360.061734667763219509542425121632880733 4, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954462511421680580808328387114921,
6.196177230048351761475418106536456016090,
385.42734025606855808257849652582008599 08]
one interval r = 31.60822049089733106735135590172884198071 ..
34.66347615044468780912657125499136742429
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., -.69e-35]Solution in 1.285s

Time Plot 0 s.
Exiting SolveHard() after 1.59r=33.8134 in [32.62668594 .. 34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.642959728866782176381889740575900289 1,
436.917481650285750710847481819023071535 1,
422.984933972645569140343960468688206397 6,
361.525802559726127041664021276754064004 4,
401.881739040858561263203209109679272709 9,
389.590015156454810456610287918786066156 9,
328.469398934237987947751015470352961662 6,
```

```
401.5075715787055681635959042208817728621,
358.9736282384353750965384457245666864730,
398.3314710393198494247705312260832442241,
371.4838739429234513137888172728528308351,
336.6121584121259439616379434457042376561,
361.5088834700257782354289567410101743286,
324.6714499264009298029258927180796054058,
302.3138431492760131060601283973459675688,
328.4693851349623200789953436178804998329,
343.8134062489790562946828313386625925126,
375.7328529008140786285420259759020794140,
328.1170929437527878196728307440278002838,
292.9996913844495285257665874237219707625,
358.6434156072024814103485186399959136154, none,
360.0617346677632195095424251216328807334, none, none,
324.6552122361125245865305016768673308237, none, none, none, none]

0 --> 1  target = [26.46318954462511421680580808328387114921,
6.196177230048351761475418106536456016090,
385.4273402560685580825784965258200859908]
two intervals r = 16.87629600275185148158209906970928850584 ..
474999999996203054642787907380106802 9/2500000000000000000000000000000000
000 or r = 15.55559000638894638618783318601785083182 ..
474999999996203054642787907380106802 9/2500000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 5.82 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852513839942886681225277078460884, rm
= 2.33669042834801323970072663392587497195 5}});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [-.717e-37, .1e-37, .3846e-34]Solution in
21.353s

Time Plot 0 s.
Exiting SolveHard() after 23.162r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
```

```
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58,
302.31384314927601310606012839734596756 88,
328.46938513496232007899534361788049983 29,
343.81340624897905629468283133866259251 26,
375.73285290081407862854202597590207941 40,
328.11709294375278781967283074402780028 38,
292.99969138444952852576658742372197076 25,
358.64341560720248141034851863999591361 54, none,
360.06173466776321950954242512163288073 34,
336.59441032247559303884749389751284879 78, none,
324.65521223611252458653050167686733082 37, none, none, none, none]

0 --> 2  target = [34.49522661162007237322985363131372773652,
3.8971313158593669064222997082527299845 97,
373.78081884359104346844099030891291471 32]
two intervals r = 17.297690862072731290196278006188212496 85 ..
474999999996203054642787907380106802 9/2500000000000000000000000000000000
000 or r = 14.994364074196388816163531536949244331 50 ..
474999999996203054642787907380106802 9/2500000000000000000000000000000000
000
Time Approximations 0.107.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.36e-37, 0., .2144e-34]Solution in 1.135s

Time Plot 0 s.
Exiting SolveHard() after 4.23r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
```

```
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58,
302.31384314927601310606012839734596756 88,
328.46938513496232007899534361788049983 29,
343.81340624897905629468283133866259251 26,
375.73285290081407862854202597590207941 40,
328.11709294375278781967283074402780028 38,
292.99969138444952852576658742372197076 25,
358.64341560720248141034851863999591361 54, none,
360.06173466776321950954242512163288073 34,
336.59441032247559303884749389751284879 78, none,
324.65521223611252458653050167686733082 37,
331.93806791373912858305702696608113217 34, none, none, none]

1 --> 2  target = [34.49522661162007237322985363131372773652,
3.8971313158593669064222997082527299845 97,
373.78081884359104346844099030891291471 32]
one interval r = 21.06068473191444654237121229378317373985 ..
26.26979834286822169881449386797665235052
Time Approximations 0.033.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [-.3e-37, -.7e-37, .197e-34]Solution in 1.562s

Time Plot 0 s.
Exiting SolveHard() after 3.011r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
```

```
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58,
302.31384314927601310606012839734596756 88,
328.46938513496232007899534361788049983 29,
343.81340624897905629468283133866259251 26,
375.73285290081407862854202597590207941 40,
328.11709294375278781967283074402780028 38,
292.99969138444952852576658742372197076 25,
358.64341560720248141034851863999591361 54,
299.89866204836081677045911763432633880 74,
360.06173466776321950954242512163288073 34,
336.59441032247559303884749389751284879 78, none,
324.65521223611252458653050167686733082 37,
331.93806791373912858305702696608113217 34, none, none, none]

0 --> 2  target = [33.81362495407971575010494381982231911644,
3.72564899349662571627868953933343566007 6,
325.89209972871983217168596460365603819 04]
two intervals r = 18.552270489861254562544279051017128753 64 ..
474999999996203054642787907380106802 9/2500000000000000000000000000000000
000 or r = 12.491969357871000405520937270690915235 81 ..
474999999996203054642787907380106802 9/2500000000000000000000000000000000
000
Time Approximations 0.035.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [-.156e-36, .6e-37, .248e-35]Solution in 1.737s

Time Plot 0 s.
Exiting SolveHard() after 3.852r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
```

```
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58,
302.31384314927601310606012839734596756 88,
328.46938513496232007899534361788049983 29,
343.81340624897905629468283133866259251 26,
375.73285290081407862854202597590207941 40,
328.11709294375278781967283074402780028 38,
292.99969138444952852576658742372197076 25,
358.64341560720248141034851863999591361 54,
299.89866204836081677045911763432633880 74,
360.06173466776321950954242512163288073 34,
336.59441032247559303884749389751284879 78, none,
324.65521223611252458653050167686733082 37,
331.93806791373912858305702696608113217 34, none, none,
289.54595772577235143294593780511490087 72]

1 --> 2  target = [33.81362495407971575010494381982231911644,
3.7256489934966257162786895393334356600 76,
325.89209972871983217168596460365603819 04]
one interval r = 20.37468935094525012648668676542795928133 ..
25.37892165303337016924135303417564076081
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=1e-38
Equations at solution: [-.1e-37, -.1e-37, -.71e-35]Solution in 0.59s

Time Plot 0 s.
Exiting SolveHard() after 1.112r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
```

```
441.6429597288667821763818897405759002891,
436.9174816502857507108474818190230715351,
422.9849339726455691403439604686882063976,
361.5258025597261270416640212767540640044,
401.8817390408585612632032091096792727099,
389.5900151564548104566102879187860661569,
328.4693989342379879477510154703529616626,
401.5075715787055681635959042208817728621,
358.9736282384353750965384457245666864730,
398.3314710393198494247705312260832442241,
371.4838739429234513137888172728528308351,
336.6121584121259439616379434457042376561,
361.5088834700257782354289567410101743286,
324.6714499264009298029258927180796054058,
302.3138431492760131060601283973459675688,
328.4693851349623200789953436178804998329,
343.8134062489790562946828313386625925126,
375.7328529008140786285420259759020794140,
328.1170929437527878196728307440278002838,
292.9996913844495285257665874237219707625,
358.6434156072024814103485186399959136154,
299.8986620483608167704591176343263388074,
360.0617346677632195095424251216328807334,
336.5944103224755930388474938975128487978,
256.1075318623737222378846682251120114465,
324.6552122361125245865305016768673308237,
331.9380679137391285830570269660811321734, none, none,
289.5459577257723514329459378051149008772]

1 --> 0  target = [17.93041369689163434795776339274477500287,
4.686508701847370082575580126274398233495,
353.3054109502609669332968745473774289113]
one interval r = 20.73150479079689935157824493174707060109 ..
25.90675353526438237511463869064518281572
Time Approximations 0.03.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.141e-34]Solution in 1.324s

Time Plot 0 s.
Exiting SolveHard() after 2.747r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58,
302.31384314927601310606012839734596756 88,
328.46938513496232007899534361788049983 29,
343.81340624897905629468283133866259251 26,
375.73285290081407862854202597590207941 40,
328.11709294375278781967283074402780028 38,
292.99969138444952852576658742372197076 25,
358.64341560720248141034851863999591361 54,
299.89866204836081677045911763432633880 74,
360.06173466776321950954242512163288073 34,
336.59441032247559303884749389751284879 78,
256.10753186237372223788466822511201144 65,
324.65521223611252458653050167686733082 37,
331.93806791373912858305702696608113217 34,
304.79958325584670962095963417709179372 25, none,
289.54595772577235143294593780511490087 72]

2 --> 0  target = [17.93041369689163434795776339274477500287,
4.68650870184737008257558012627439823349 5,
353.30541095026096693329687454737742891 13]
one interval r = 31.37435486991111253212715766802570249915 ..
34.20127520025457173520802907066719587648
Time Approximations 0.014.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .660e-34]Solution in 0.37s

Time Plot 0 s.
Exiting SolveHard() after 0.617r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493317550950967950445321468928 92,
441.64295972886678217638188974057590028 91,
436.91748165028575071084748181902307153 51,
422.98493397264556914034396046868820639 76,
361.52580255972612704166402127675406400 44,
401.88173904085856126320320910967927270 99,
389.59001515645481045661028791878606615 69,
328.46939893423798794775101547035296166 26,
401.50757157870556816359590422088177286 21,
358.97362823843537509653844572456668647 30,
398.33147103931984942477053122608324422 41,
371.48387394292345131378881727285283083 51,
336.61215841212594396163794344570423765 61,
361.50888347002577823542895674101017432 86,
324.67144992640092980292589271807960540 58,
302.31384314927601310606012839734596756 88,
328.46938513496232007899534361788049983 29,
343.81340624897905629468283133866259251 26,
375.73285290081407862854202597590207941 40,
328.11709294375278781967283074402780028 38,
292.99969138444952852576658742372197076 25,
358.64341560720248141034851863999591361 54,
299.89866204836081677045911763432633880 74,
360.06173466776321950954242512163288073 34,
336.59441032247559303884749389751284879 78,
256.10753186237372223788466822511201144 65,
324.65521223611252458653050167686733082 37,
331.93806791373912858305702696608113217 34,
304.79958325584670962095963417709179372 25,
323.46169176718465390254807122678744002 76,
289.54595772577235143294593780511490087 72]

Cascade time 190.287
counts: 28, 28

 Iteration 13

Start Generation 1
1 --> 0  target = [12.00000000006677327533079704243437539700,
6.21701250293503810172601673315045941648 3,
485.54908090032497043668945001338986953 92]
one interval r = 23.40850301653040690451276935419005745069 ..
27.67578046417459593822636503525739849631
Time Approximations 0.037.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
```

```
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.05e-37
Equations at solution: [-.4e-37, .105e-36, -.4e-36]Solution in 1.663s

Time Plot 0 s.
Exiting SolveHard() after 3.543r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000006677327533079704243437539700,
6.217012502935038101726016733150459416483,
485.54908090032497043668945001338986953 92]
one interval r = 32.628147792168713026201815795205604720 56 ..
36.10248388941214174548796230486370506981
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, .129e-34]Solution in 1.373s

Time Plot 0 s.
Exiting SolveHard() after 1.79r=35.4632 in [33.94922194 .. 36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684468825103432906656570553259581,
6.583434721799584673260643528033982148489,
467.78730595977926283939555852454244422 41]
```

```
one interval r = 32.41978955665198233914417646392841819340 ..
35.85152417369616862823243655986955655418
Time Approximations 0.02.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=6e-38
Equations at solution: [-.8e-37, .6e-37, -.164e-34]Solution in 1.353s

Time Plot 0 s.
Exiting SolveHard() after 1.725r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60, none, none,
401.88173904025865621579706268231592408 14, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684468825103432906656570553259581,
6.58343472179958467326064352803398214 8489,
467.78730595977926283939555852454244422 41]
two intervals r = 12.92327160841256686953946914093629912449 ..
474999999998721355291449552789024752 9/2500000000000000000000000000000000
000 or r = 18.39424858025976484881941394448387783367 ..
474999999998721355291449552789024752 9/2500000000000000000000000000000000
000
Time Approximations 0.037.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.0e-38
Equations at solution: [-.1e-37, -.10e-37, .1044e-35]Solution in
32.444s
```

```
Time Plot 0 s.
Exiting SolveHard() after 34.519r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07, none,
401.88173904025865621579706268231592408 14, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962824257236330462006627811447487,
4.12565179673854519101903365206110573678 8,
440.67123065151698708584726372880339083 34]
two intervals r = 14.35659705124750583410611953992221319470 ..
47499999999872135529144955278902475 29/2500000000000000000000000000000000
000 or r = 17.70352613804490138386709863011073729265 ..
47499999999872135529144955278902475 29/2500000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [.28e-37, .2e-37, -.16194e-34]Solution in 1.986s

Time Plot 0 s.
Exiting SolveHard() after 2.961r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07, none,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
1 --> 2  target = [35.46322962824257236330462006627811447487,
4.12565179673854519101903365206110573678 8,
440.67123065151698708584726372880339083 34]
one interval r = 22.39761154362479304723666136410474049885 ..
27.23722351588095602993004409739157082491
Time Approximations 0.034.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.782 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064378573266158186659622145987088, rm =
14.37818770456551928911943992558665338220}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., -.481e-34]Solution in 7.606s

Time Plot 0 s.
Exiting SolveHard() after 9.208r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888793178138468397573655286876663,
4.00486908170507705151523874643523070431 7,
404.86224501119282457131078218489841411 36]
two intervals r = 16.08011007775228121323774233144650546670 ..
47499999999872135529144955278902475 29/2500000000000000000000000000000000
000 or r = 16.41579812678952047405263980789752607171 ..
47499999999872135529144955278902475 29/2500000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
```

```
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.34e-37, 0., -.16688e-34]Solution in 2.355s

Time Plot 0 s.
Exiting SolveHard() after 4.139r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91, none, none,
358.97362823669100363831482653625073731 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888793178138468397573655286876663,
4.00486908170507705151523874643523070431 7,
404.86224501119282457131078218489841411 36]
one interval r = 21.641943994017178429414764027391749834 40 ..
26.763306600283855767087618493193184981 67
Time Approximations 0.045.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .256e-34]Solution in 1.695s

Time Plot 0 s.
Exiting SolveHard() after 3.501r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31, none,
358.97362823669100363831482653625073731 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941808681515159548917370505715119,
5.5896371828841232063785802776542516809 90,
443.83065884341249614636257391350439085 71]
one interval r = 22.46725374466023896904849902423192478978 ..
27.27388428340269082055054452663461658264
Time Approximations 0.033.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.4e-38
Equations at solution: [.1e-37, -.54e-37, .38e-35]Solution in 1.671s

Time Plot 0 s.
Exiting SolveHard() after 3.39r=27.0204 in [24.71083344 .. 27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31, none,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941808681515159548917370505715119,
```

```
5.589637182884123206378580277654251680990,
443.830658843412496146362573913504390857 1]
one interval r = 32.155752795005184310017862968263457427 25 ..
35.508722287312579417646743596355046856 57
Time Approximations 0.019.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=9e-38
Equations at solution: [-.10e-36, .9e-37, .46e-35]Solution in 0.491s

Time Plot 0 s.
Exiting SolveHard() after 0.828r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434938353966237486508544424387907 5,
441.642959732725868096702468207052513115 5,
436.917481653521247214699695299438918216 0,
422.984933972807857504292103010040051790 7,
361.525802557888651082975722784730195209 1,
401.881739040258656215797062682315924081 4,
389.590015158264285271788756216810058229 1,
328.469398928744979006674446838098468493 1,
401.507571576848114746924721660666895666 3,
358.973628236691003638314826536250737314 2,
398.331471036209387495788392762285913942 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365179959861953250336515078850 25,
5.187783578469192453040863195548448563754,
408.657738627048589427952052990199976407 7]
one interval r = 21.718401146449769661731346694341907148 25 ..
26.818493035013452144605505608060245419 36
Time Approximations 0.051.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.11e-37
Equations at solution: [-.2e-37, -.211e-36, .71e-35]Solution in 1.659s

Time Plot 0 s.
Exiting SolveHard() after 3.556r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20, none, none,
361.50888346839545592891797339228782962 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136517995986195325033651507885025,
5.18778357846919245304086319554844856 3754,
408.65773862704858942795205299019997640 77]
one interval r = 31.80828598753602281229385462991786538903 ..
35.00011460043157085512658125603525479806
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=8e-38
Equations at solution: [-.7e-37, .8e-37, -.94e-35]Solution in 0.397s

Time Plot 0 s.
Exiting SolveHard() after 0.678r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07, none,
361.50888346839545592891797339228782962 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110525567785043218412584112070243,
6.19626256550934724892199697341657462987 4,
385.44474379044016743643958471310064717 39]
one interval r = 31.60836097536776642301041182803045521492 ..
34.66372795603832642896721144884714187090
Time Approximations 0.015.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, -.64e-35]Solution in 0.585s

Time Plot 0 s.
Exiting SolveHard() after 1.612r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07, none,
```

```
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110525567785043218412584112070243,
6.19626256550934724892199697341657462987 4,
385.44474379044016743643958471310064717 39]
two intervals r = 16.875634087686287908630621891845173260 33 ..
474999999998721355291449552789024752 9/2500000000000000000000000000000000
000 or r = 15.556404937844164793723191657430628872 16 ..
474999999998721355291449552789024752 9/2500000000000000000000000000000000
000
Time Approximations 0.058.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 6.255 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.468341751248135908155059147467610594 07, rm
= 2.336532774003436958688986606535261367386}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.538e-37, 0., .11458e-34]Solution in 21.79s

Time Plot 0 s.
Exiting SolveHard() after 23.783r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]
```

```
Start Generation 4
1 --> 0  target = [17.19898874748080438698912554946196162774,
4.88381077977504292047761593241433416379 9,
376.61967855536669573586718635802099936 40]
one interval r = 21.11001304868208536948464342734182800934 ..
26.31784243460627892164320032282389318684
Time Approximations 0.031.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, -.271e-34]Solution in 1.571s

Time Plot 0 s.
Exiting SolveHard() after 2.967r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68, none,
328.46938512947490922842681857824185799 14, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874748080438698912554946196162774,
4.88381077977504292047761593241433416379 9,
376.61967855536669573586718635802099936 40]
one interval r = 31.53899497711275228813189115305500379780 ..
34.53618386087478944882150692675429811013
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
```

```
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.114e-35
Equations at solution: [.856e-35, -.1114e-34, -.170e-34]Solution in
0.516s

Time Plot 0 s.
Exiting SolveHard() after 0.79r=34.0898 in [32.52213872 .. 34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68, none,
328.46938512947490922842681857824185799 14,
343.81340624474935057701042360812590173 58, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017521206222490705290813283415790,
6.02581354944889973530030306528542697473 7,
351.42702947830016067899466509691954427 26]
one interval r = 31.36230206112556025698311173941860271881 ..
34.17446640604998460403622305068082499915
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .132e-34]Solution in 1.224s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.464r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68, none,
328.46938512947490922842681857824185799 14,
343.81340624474935057701042360812590173 58, none, none,
292.99969137436407882440277361250329262 12, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175212062224907052908132834157 90,
6.0258135494488997353003030652854269747 37,
351.42702947830016067899466509691954427 26]
two intervals r = 17.981355144580599706629166822618474156 37 ..
474999999998721355291449552789024752 9/2500000000000000000000000000000000
000 or r = 13.846080153883747310261374010667852125 79 ..
474999999998721355291449552789024752 9/2500000000000000000000000000000000
000
Time Approximations 0.039.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 4.319 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713692389903004517045980453662 80, rm
= 2.7345009930784292703183888126726624362 08}});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [.52e-37, -.1e-37, -.14328e-34]Solution in
13.684s
```

```
Time Plot 0 s.
Exiting SolveHard() after 15.421r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68,
302.31384313972413345549985771371774836 53,
328.46938512947490922842681857824185799 14,
343.81340624474935057701042360812590173 58, none, none,
292.99969137436407882440277361250329262 12, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941855823591592101856007067566226,
6.37794387404567861330605945072480550721 6,
423.28832783493950410949253542724867573 67]
one interval r = 31.94661817594129419724642460013753059954 ..
35.21212308641808155968575865965891758470
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=1.0e-37
Equations at solution: [.9e-37, -.10e-36, .64e-35]Solution in 1.311s

Time Plot 0 s.
Exiting SolveHard() after 1.652r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68,
302.31384313972413345549985771371774836 53,
328.46938512947490922842681857824185799 14,
343.81340624474935057701042360812590173 58, none, none,
292.99969137436407882440277361250329262 12, none, none,
360.06173466024724189006905282571390959 30, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941855823591592101856007067566226,
6.37794387404567861330605945072480550721 6,
423.28832783493950410949253542724867573 67]
two intervals r = 15.22886702468693737370979041716907995436 ..
474999999998721355291449552789024752 9/2500000000000000000000000000000000
000 or r = 17.12965777049490022828180244516972607895 ..
474999999998721355291449552789024752 9/2500000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 5.606 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.51537054145934210083494521708265957281, rm
= 2.06406829867837169975498214593946949580 3}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.16e-37, .1e-37, .32286e-34]Solution in 22.22s

Time Plot 0 s.
Exiting SolveHard() after 24.276r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68,
302.31384313972413345549985771371774836 53,
328.46938512947490922842681857824185799 14,
343.81340624474935057701042360812590173 58,
375.73285289311007543888829369288967494 68, none,
292.99969137436407882440277361250329262 12, none, none,
360.06173466024724189006905282571390959 30, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234335664532330035795445823897659,
4.0035598154017934262266708263226416850 50,
404.47973593727669702622601900106801127 62]
two intervals r = 16.09683966390294810413701174029218612145 ..
4749999999987213552914495527890247529/2500000000000000000000000000000000
000 or r = 16.39988649094800754465923568995140111148 ..
4749999999987213552914495527890247529/2500000000000000000000000000000000
000
Time Approximations 0.822.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.17e-37, 0., .3174e-35]Solution in 2.366s

Time Plot 0 s.
Exiting SolveHard() after 4.096r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
```

```
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68,
302.31384313972413345549985771371774836 53,
328.46938512947490922842681857824185799 14,
343.81340624474935057701042360812590173 58,
375.73285289311007543888829369288967494 68, none,
292.99969137436407882440277361250329262 12,
358.64341560434610302308522294924775872 97, none,
360.06173466024724189006905282571390959 30, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234335664532330035795445823897659,
4.0035598154017934262266708263226416850 50,
404.47973593727669702622601900106801127 62]
one interval r = 21.634296299770508323827353556387135890 55 ..
26.757681698767694838540040427963789151 07
Time Approximations 0.049.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, .431e-34]Solution in 1.02s

Time Plot 0 s.
Exiting SolveHard() after 2.783r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
```

```
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68,
302.31384313972413345549985771371774836 53,
328.46938512947490922842681857824185799 14,
343.81340624474935057701042360812590173 58,
375.73285289311007543888829369288967494 68,
328.11709293707544683368258575304352660 53,
292.99969137436407882440277361250329262 12,
358.64341560434610302308522294924775872 97, none,
360.06173466024724189006905282571390959 30, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954443973401581154549359572138786,
6.19617723036351123575313011174748700812 2,
385.42734025432280522473937470151292010 17]
one interval r = 31.60822049092247901119942674854804770850 ..
34.66347615041684231688500246174838077893
Time Approximations 0.015.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, .280e-34]Solution in 0.509s

Time Plot 0 s.
Exiting SolveHard() after 1.555r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
```

```
398.331471036209387495788392762285913942 0,
371.483873942171619532263789780017879050 7,
336.612158406048207366533644110313915233 9,
361.508883468395455928917973392287829626 2,
324.671449919957933538885803077188182266 8,
302.313843139724133455499857713717748365 3,
328.469385129474909228426818578241857991 4,
343.813406244749350577010423608125901735 8,
375.732852893110075438888293692889674946 8,
328.117092937075446833682585753043526605 3,
292.999691374364078824402773612503292621 2,
358.643415604346103023085222949247758729 7, none,
360.061734660247241890069052825713909593 0, none, none,
324.655212229868266660298489783654027752 7, none, none, none, none]

0 --> 1  target = [26.463189544439734015811545493595721387 86,
6.196177230363511235753130111747487008122,
385.427340254322805224739374701512920101 7]
two intervals r = 16.876296003067639299904974146420997923 19 ..
474999999998721355291449552789024752 9/2500000000000000000000000000000000
000 or r = 15.555590006335874851845982146756275550 85 ..
474999999998721355291449552789024752 9/2500000000000000000000000000000000
000
Time Approximations 0.052.

hint used Hint := [17.930920830435909701694400819009216331 81, 3, 1, 1,
15.700860627013397511253157405159644880 12, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 5.822 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852536293718187584942861529259213, rm
= 2.33669042806758115346680842598348217012 2}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.538e-37, 0., -.46503e-34]Solution in 21.645s

Time Plot 0 s.
Exiting SolveHard() after 23.486r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.642959732725868096702468207052513115 5,
436.917481653521247214699695299438918216 0,
422.984933972807857504292103010040051790 7,
361.525802557888651082975722784730195209 1,
401.881739040258656215797062682315924081 4,
```

```
389.5900151582642852717887562168100582291,
328.4693989287449790066744468380984684931,
401.5075715768481147469247216606668956663,
358.9736282366910036383148265362507373142,
398.3314710362093874957883927622859139420,
371.4838739421716195322637897800178790507,
336.6121584060482073665336441103139152339,
361.5088834683954559289179733922878296262,
324.6714499199579335388858030771881822668,
302.3138431397241334554998577137177483653,
328.4693851294749092284268185782418579914,
343.8134062447493505770104236081259017358,
375.7328528931100754388882936928896749468,
328.1170929370754468336825857530435266053,
292.9996913743640788244027736125032926212,
358.6434156043461030230852229492477587297, none,
360.0617346602472418900690528257139095930,
336.5944103166152983074501347685359239069, none,
324.6552122298682666602984897836540277527, none, none, none, none]

0 --> 2  target = [34.4952266116132176881424981420956471197 5,
3.8971313158249648316338215772261629108 80,
373.7808188431517705600237331360603783729]
two intervals r = 17.2976908623187180671453769276349902155 8 ..
4749999999987213552914495527890247529/2500000000000000000000000000000000
000 or r = 14.9943640742054193729232290389037330725 5 ..
4749999999987213552914495527890247529/2500000000000000000000000000000000
000
Time Approximations 0.083.

hint used Hint := [18.0598902594553048959735086609147911477 6, 2, -1, 1,
17.0684047684651350286503160799520686312 0, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.36e-37, 0., .33081e-34]Solution in 1.862s

Time Plot 0 s.
Exiting SolveHard() after 5.011r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349383539662374865085444243879075,
441.6429597327258680967024682070525131155,
436.9174816535212472146996952994389182160,
422.9849339728078575042921030100400517907,
361.5258025578886510829757227847301952091,
401.8817390402586562157970626823159240814,
```

```
389.590015158264285271788756216810058229 1,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68,
302.31384313972413345549985771371774836 53,
328.46938512947490922842681857824185799 14,
343.81340624474935057701042360812590173 58,
375.73285289311007543888829369288967494 68,
328.11709293707544683368258575304352660 53,
292.99969137436407882440277361250329262 12,
358.64341560434610302308522294924775872 97, none,
360.06173466024724189006905282571390959 30,
336.59441031661529830745013476853592390 69, none,
324.65521222986826666029848978365402775 27,
331.93806791172197853131078308946850933 12, none, none, none]

1 --> 2  target = [34.49522661161321768814249814209564711975,
3.897131315824964831633821577226162910880,
373.78018184315177056002373313606037837 29]
one interval r = 21.06068473196310816069886386277560455084 ..
26.26979834271099666639748731159425196137
Time Approximations 0.03.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [.2e-37, .5e-37, -.155e-34]Solution in 0.742s

Time Plot 0 s.
Exiting SolveHard() after 2.123r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.590015158264285271788756216810058229 1,
```

```
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68,
302.31384313972413345549985771371774836 53,
328.46938512947490922842681857824185799 14,
343.81340624474935057701042360812590173 58,
375.73285289311007543888829369288967494 68,
328.11709293707544683368258575304352660 53,
292.99969137436407882440277361250329262 12,
358.64341560434610302308522294924775872 97,
299.89866204270341624263128670804178539 44,
360.06173466024724189006905282571390959 30,
336.59441031661529830745013476853592390 69, none,
324.65521222986826666029848978365402775 27,
331.93806791172197853131078308946850933 12, none, none, none]

0 --> 2  target = [33.81362495400025585580167512525375463302,
3.72564899344091118499994945333371908670 43,
325.89209972245672327075655886116678682 75]
two intervals r = 18.552270490133669064717500746804727019 92 ..
474999999998721355291449552789024752 9/2500000000000000000000000000000000
000 or r = 12.491969357600124251710461534577432478 34 ..
474999999998721355291449552789024752 9/2500000000000000000000000000000000
000
Time Approximations 0.037.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [-.140e-36, .6e-37, .15063e-34]Solution in
1.801s

Time Plot 0 s.
Exiting SolveHard() after 3.917r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
```

```
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68,
302.31384313972413345549985771371774836 53,
328.46938512947490922842681857824185799 14,
343.81340624474935057701042360812590173 58,
375.73285289311007543888829369288967494 68,
328.11709293707544683368258575304352660 53,
292.99969137436407882440277361250329262 12,
358.64341560434610302308522294924775872 97,
299.89866204270341624263128670804178539 44,
360.06173466024724189006905282571390959 30,
336.59441031661529830745013476853592390 69, none,
324.65521222986826666029848978365402775 27,
331.93806791172197853131078308946850933 12, none, none,
289.54595771847483227560520536457776523 31]

1 --> 2  target = [33.81362495400025585580167512525375463302,
3.72564899344091118499949453333719086704 3,
325.89209972245672327075655886116678682 75]
one interval r = 20.37468935096291864016289635049138963159 ..
25.37892165279150250131145536785446384162
Time Approximations 0.023.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.2e-37, -.2e-37, -.388e-34]Solution in 0.528s

Time Plot 0 s.
Exiting SolveHard() after 1.852r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
```

```
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68,
302.31384313972413345549985771371774836 53,
328.46938512947490922842681857824185799 14,
343.81340624474935057701042360812590173 58,
375.73285289311007543888829369288967494 68,
328.11709293707544683368258575304352660 53,
292.99969137436407882440277361250329262 12,
358.64341560434610302308522294924775872 97,
299.89866204270341624263128670804178539 44,
360.06173466024724189006905282571390959 30,
336.59441031661529830745013476853592390 69,
256.10753185137587626143234288320690692 43,
324.65521222986826666029848978365402775 27,
331.93806791172197853131078308946850933 12, none, none,
289.54595771847483227560520536457776523 31]

1 --> 0  target = [17.93041369726877195495559409271116483707,
4.68650870188526352873886152384305533696 4,
353.30541094378669430818557188072623047 31]
one interval r = 20.73150479077159996292201910346349672542 ..
25.90675353500911403713198687890667181912
Time Approximations 0.028.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=7.1e-38
Equations at solution: [-.3e-37, -.71e-37, .33e-35]Solution in 0.597s

Time Plot 0 s.
Exiting SolveHard() after 2.094r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
```

```
441.6429597327258680967024682070525131155,
436.9174816535212472146996952994389182160,
422.9849339728078575042921030100400517907,
361.5258025578886510829757227847301952091,
401.8817390402586562157970626823159240814,
389.5900151582642852717887562168100582291,
328.4693989287449790066744468380984684931,
401.5075715768481147469247216606668956663,
358.9736282366910036383148265362507373142,
398.3314710362093874957883927622859139420,
371.4838739421716195322637897800178790507,
336.6121584060482073665336441103139152339,
361.5088834683954559289179733922878296262,
324.6714499199579335388858030771881822668,
302.3138431397241334554998577137177483653,
328.4693851294749092284268185782418579914,
343.8134062447493505770104236081259017358,
375.7328528931100754388882936928896749468,
328.1170929370754468336825857530435266053,
292.9996913743640788244027736125032926212,
358.6434156043461030230852229492477587297,
299.8986620427034162426312867080417853944,
360.0617346602472418900690528257139095930,
336.5944103166152983074501347685359239069,
256.1075318513758762614323428832069069243,
324.6552122298682666602984897836540277527,
331.9380679117219785313107830894685093312,
304.7995832458838209301232116851592652983, none,
289.5459577184748322756052053645777652331]

2 --> 0  target = [17.93041369726877195495559409271116483707,
4.6865087018852635287388615238430553369 64,
353.3054109437866943081855718807262304731]
one interval r = 31.37435486991223671467183548961721452897 ..
34.20127520016526245581612610964385058437
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=9e-38
Equations at solution: [.5e-37, -.9e-37, .188e-34]Solution in 1.076s

Time Plot 0 s.
Exiting SolveHard() after 1.342r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493835396623748650854442438790 75,
441.64295973272586809670246820705251311 55,
436.91748165352124721469969529943891821 60,
422.98493397280785750429210301004005179 07,
361.52580255788865108297572278473019520 91,
401.88173904025865621579706268231592408 14,
389.59001515826428527178875621681005822 91,
328.46939892874497900667444683809846849 31,
401.50757157684811474692472166066689566 63,
358.97362823669100363831482653625073731 42,
398.33147103620938749578839276228591394 20,
371.48387394217161953226378978001787905 07,
336.61215840604820736653364411031391523 39,
361.50888346839545592891797339228782962 62,
324.67144991995793353888580307718818226 68,
302.31384313972413345549985771371774836 53,
328.46938512947490922842681857824185799 14,
343.81340624474935057701042360812590173 58,
375.73285289311007543888829369288967494 68,
328.11709293707544683368258575304352660 53,
292.99969137436407882440277361250329262 12,
358.64341560434610302308522294924775872 97,
299.89866204270341624263128670804178539 44,
360.06173466024724189006905282571390959 30,
336.59441031661529830745013476853592390 69,
256.10753185137587626143234288320690692 43,
324.65521222986826666029848978365402775 27,
331.93806791172197853131078308946850933 12,
304.79958324588382093012321168515926529 83,
323.46169175884945938734951206297250924 87,
289.54595771847483227560520536457776523 31]

Cascade time 190.126
counts: 28, 28

 Iteration 14

Start Generation 1
1 --> 0  target = [12.00000000007598087466023998351314330700,
6.21701250290094798550839259761132817 7818,
485.54908089449270455534992840653006889 15]
```

"Imaginary part neglected: ", $1.8899423791487321694159180727752604522 14 \times 10^{-17}$

```
one interval r = 23.40850301645078052325554312114727793572 ..
27.67578046423610038887324443933103624721
Time Approximations 0.039.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
```

```
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=8.0e-38
Equations at solution: [.2e-37, -.80e-37, -.1132e-35]Solution in 0.941s

Time Plot 0 s.
Exiting SolveHard() after 2.75r=27.5236 in [25.56992694 .. 27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000007598087466023998351314330700,
6.21701250290094798550839259761132817 8818,
485.54908089449270455534992840653006889 15]
one interval r = 32.62814779219079294752802437516238082018 ..
36.10248389947173557520360692063725442459
Time Approximations 0.02.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .77e-35]Solution in 0.541s

Time Plot 0 s.
Exiting SolveHard() after 1.753r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684473869700265731297891106403104,
```

```
6.583434721716321484260332415301465099698,
467.787305954251721036945061885956916478 0]
one interval r = 32.41978955667807774748012559584417865450 ..
35.85152417375511007843536972198761643273
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .171e-34]Solution in 1.351s

Time Plot 0 s.
Exiting SolveHard() after 1.685r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23, none, none,
401.88173903705924043939667089367925805 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684473869700265731297891106403104,
6.583434721716321484260332415301465099698,
467.787305954251721036945061885956916478 0]
two intervals r = 12.92327160844267613592492473293976425247 ..
18999999999965850282856547380650829161/100000000000000000000000000000000
00000 or r = 18.39424858019080770079727309543865198665 ..
18999999999965850282856547380650829161/100000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=6.5e-38
Equations at solution: [.6e-37, .65e-37, .207e-35]Solution in 31.034s
```

```
Time Plot 0 s.
Exiting SolveHard() after 33.166r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18, none,
401.88173903705924043939667089367925805 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962830572258332089240625380063184,
4.1256517968176227201678727151261774276 31,
440.67123064676659813016706398382086220 25]
two intervals r = 14.35659705127548576129218946221074122805 ..
189999999999658502828565473806508291 61/100000000000000000000000000000000
00000 or r = 17.70352613794977330278971715341472302899 ..
189999999999658502828565473806508291 61/100000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [-.57e-37, -.3e-37, -.4470e-34]Solution in
2.042s

Time Plot 0 s.
Exiting SolveHard() after 3.834r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18, none,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962830572258332089240625380063184,
4.12565179681762272016787271512617742763 1,
440.67123064676659813016706398382086220 25]
```

"Imaginary part neglected: ", $1.8899423791487321694159180727752604522 14 \times 10^{-17}$

```
one interval r = 22.39761154356723253237267806414927493859 ..
27.23722351592060525077131156983921573138
Time Approximations 0.039.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 2.036 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064378828414779427467300440621999, rm =
14.37818770456767172509704223912215828725}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., .2261e-35]Solution in 7.919s

Time Plot 0 s.
Exiting SolveHard() after 8.737r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888800152417040105180106586272616,
4.00486908178778434278126552323075328920 5,
404.86224500745387134791721526076285340 30]
two intervals r = 16.08011007776486717191457252777992336564 ..
189999999999658502828565473806508291 61/10000000000000000000000000000000
00000 or r = 16.41579812667185443774265627509148636556 ..
```

```
189999999999658502828565473806508291 61/1000000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.33e-37, .1e-37, .2740e-34]Solution in 2.455s

Time Plot 0.001 s.
Exiting SolveHard() after 4.419r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66, none, none,
358.97362823420571110066298466530622063 34, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888800152417040105180106586272616,
4.0048690817877843427812655232307532892 05,
404.86224500745387134791721526076285340 30]

"Imaginary part neglected: ", 1.8899423791487321694159180727752604522 14 × 10^-17
one interval r = 21.641943993985169462155096450677977794 79 ..
26.763306600315714987660652369319163834 44
Time Approximations 0.044.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
```

```
Equations at solution: [0., 0., .27357e-34]Solution in 0.946s

Time Plot 0 s.
Exiting SolveHard() after 2.783r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19, none,
358.97362823420571110066298466530622063 34, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941809770784480724346747897083927,
5.5896371828742700135372667018731118162 04,
443.83065883891805725426941274975403712 00]
```

"Imaginary part neglected: ", $1.8899423791487321694159180727752604522 14 \times 10^{-17}$

```
one interval r = 22.46725374460819531361603769210627101591 ..
27.27388428344734476157594438294253904250
Time Approximations 0.054.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [.1e-37, -.81e-37, -.4619e-35]Solution in 1.697s

Time Plot 0 s.
Exiting SolveHard() after 3.532r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
```

```
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19, none,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941809770784480724346747897083927,
5.589637182874270013537266701873111816204,
443.83065883918057254269412749754037120 0]
one interval r = 32.15575279504342345090942999306980553329 ..
35.50872228738017875367738992753247613121
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, .66e-35]Solution in 0.493s

Time Plot 0 s.
Exiting SolveHard() after 0.862r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136518160237838425273501184772992,
5.187783578472159902217165561751542200961,
408.65773862348639805410799583928686034 87]
```

"Imaginary part neglected: ", $1.8899423791487321694159180727752604522 14 \times 10^{-17}$

```
one interval r = 21.71840114642072037571820982589363856344 ..
26.81849303504987377103491704909720513882
Time Approximations 0.066.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.11e-37
Equations at solution: [.2e-37, .211e-36, .8409e-35]Solution in 1.809s

Time Plot 0 s.
Exiting SolveHard() after 3.695r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93, none, none,
361.50888346598525261643717934200409504 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136518160237838425273501184772992,
5.1877835784721599022171655617515422009 61,
408.65773862348639805410799583928686034 87]
one interval r = 31.80828598758458491454678605344846351731 ..
35.00011460050467694263540345966717575492
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=7e-38
Equations at solution: [-.7e-37, .7e-37, .151e-34]Solution in 0.401s

Time Plot 0 s.
Exiting SolveHard() after 0.693r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09, none,
361.50888346598525261643717934200409504 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110530009227373825585822922392068,
6.19626256544174961312575007573418187007 4,
385.44474378809861332212880652445983184 48]
one interval r = 31.60836097542723913830176500781736137036 ..
34.66372795612437148461401187126485291335
Time Approximations 0.015.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.53e-35]Solution in 0.504s

Time Plot 0 s.
Exiting SolveHard() after 1.574r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09, none,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110530009227373825585822922392068,
6.19626256544174961312575007573418187007 4,
385.44474378809861332212880652445983184 48]
two intervals r = 16.87563408765812148765317960971169502805 ..
189999999999658502828565473806508291 61/100000000000000000000000000000000
00000 or r = 15.55640493775919968291629569563963127615 ..
189999999999658502828565473806508291 61/100000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 5.928 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175125477580436157304565937476413, rm
= 2.33653277403990070862142311032285130084 0}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.180e-37, 0., -.4207e-34]Solution in 21.203s

Time Plot 0 s.
Exiting SolveHard() after 23.105r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
```

```
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874746984843340981349191580479263,
4.8838107797882912336984199169377257598 66,
376.61967855276260697254898872650469559 96]
```

"Imaginary part neglected: ", $1.8899423791487321694159180727752604522 14 \times 10^{-17}$

```
one interval r = 21.11001304867671546338713542647731484807 ..
26.31784243464277470459378180929488291749
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., .13711e-34]Solution in 0.802s

Time Plot 0 s.
Exiting SolveHard() after 2.403r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
```

```
398.331471032640404073985547599089024559 3,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21, none,
328.46938512812719572137792457836528115 38, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874746984843340981349191580479263,
4.8838107797882912336984199169377257598 66,
376.61967855276260697254898872650469559 96]
one interval r = 31.53899497717025477267337156379308144388 ..
34.53618386095531837373457103631013448321
Time Approximations 0.015.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.081e-35
Equations at solution: [.830e-35, -.1081e-34, .389e-34]Solution in
0.463s

Time Plot 0 s.
Exiting SolveHard() after 1.61r=34.0898 in [32.52213872 .. 34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.331471032640404073985547599089024559 3,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21, none,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017526211561478124247074875711939,
```

```
6.025813549386714041961912559132892774631,
351.42702947696590690874109710854261250 49]
one interval r = 31.362302061191982395291961995596509306 87 ..
34.174466406144008082858149556714473893 34
Time Approximations 0.015.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .192e-34]Solution in 1.274s

Time Plot 0 s.
Exiting SolveHard() after 1.498r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21, none,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81, none, none,
292.99969137536576811133457852508075218 25, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175262115614781242470748757119 39,
6.025813549386714041961912559132892774631,
351.42702947696590690874109710854261250 49]
two intervals r = 17.981355144551710536008394889702276168 87 ..
189999999999965850282856547380650829161/100000000000000000000000000000000
00000 or r = 13.846080153800292647737128419485804496 96 ..
189999999999965850282856547380650829161/100000000000000000000000000000000
00000
Time Approximations 0.041.
```

```
hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 5.209 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.91357071370073775471794091418301782886, rm
= 2.7345009931235017043197379645493455306 79}});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [.54e-37, -.1e-37, -.290e-35]Solution in 13.998s

Time Plot 0 s.
Exiting SolveHard() after 14.953r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21,
302.31384313994391584528306893066591273 21,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81, none, none,
292.99969137536576811133457852508075218 25, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941860156792186030650657796086702,
6.37794387397159851764468463003781049671 8,
423.28832783131571650733771776418134045 65]
one interval r = 31.94661817598889704120606914159632502279 ..
35.21212308649347944155014070212021709897
Time Approximations 0.017.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., .36e-35]Solution in 1.34s

Time Plot 0 s.
Exiting SolveHard() after 1.661r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21,
302.31384313994391584528306893066591273 21,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81, none, none,
292.99969137536576811133457852508075218 25, none, none,
360.06173465894760240276827313477302063 64, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418601567921860306506577960867 02,
6.3779438739715985176446846300378104967 18,
423.28832783131571650733771776418134045 65]
two intervals r = 15.228867024677346573380805225956238853 50 ..
189999999999658502828565473806508291 61/10000000000000000000000000000000
00000 or r = 17.129657770409958406720076454969542479 04 ..
189999999999658502828565473806508291 61/10000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
```

```
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 4.99 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541483902536302171441943600877 45, rm
= 2.0640682987077893257623676341256464018 12}});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [-.48e-37, -.2e-37, -.1299e-34]Solution in 21.8s

Time Plot 0 s.
Exiting SolveHard() after 23.847r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21,
302.31384313994391584528306893066591273 21,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81,
375.73285289104131885480401182187905736 23, none,
292.99969137536576811133457852508075218 25, none, none,
360.06173465894760240276827313477302063 64, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343433380262095141857210716 2028,
4.0035598154861630532854749423626876311 80,
404.47973593402255473001942469144616508 48]
two intervals r = 16.096839663894649124748229314258848166 72 ..
189999999999658502828565473806508291 61/100000000000000000000000000000
00000 or r = 16.399886490849914755187370781443383527 81 ..
189999999999658502828565473806508291 61/100000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
```

```
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.67e-37, -.1e-37, -.238e-35]Solution in 3.145s

Time Plot 0 s.
Exiting SolveHard() after 4.107r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21,
302.31384313994391584528306893066591273 21,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81,
375.73285289104131885480401182187905736 23, none,
292.99969137536576811133457852508075218 25,
358.64341560227974426255301042706200379 72, none,
360.06173465894760240276827313477302063 64, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234343333802620951418572107162028,
4.0035598154861630532854749423626876311 80,
404.47973593402255473001942469144616508 48]

"Imaginary part neglected: ", 1.8899423791487321694159180727752604522 14 × 10^-17
one interval r = 21.63429629974825224778337469622732856158 ..
26.75768169880648811597404198290217631824
Time Approximations 0.051.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
```

```
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=1.01e-37
Equations at solution: [-.3e-37, -.101e-36, .11299e-34]Solution in
1.751s

Time Plot 0 s.
Exiting SolveHard() after 2.737r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21,
302.31384313994391584528306893066591273 21,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81,
375.73285289104131885480401182187905736 23,
328.11709293617711043570395920433047037 62,
292.99969137536576811133457852508075218 25,
358.64341560227974426255301042706200379 72, none,
360.06173465894760240276827313477302063 64, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954448268905021493126181464306753,
6.19617723029547358419085743217480190016 9,
385.42734025189148022034767704781339797 61]
one interval r = 31.60822049098122711974645086883846441132 ..
34.66347615050158506059688334626677813881
Time Approximations 0.025.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, -.101e-34]Solution in 1.34s

Time Plot 0 s.
Exiting SolveHard() after 1.631r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21,
302.31384313994391584528306893066591273 21,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81,
375.73285289104131885480401182187905736 23,
328.11709293617711043570395920433047037 62,
292.99969137536576811133457852508075218 25,
358.64341560227974426255301042706200379 72, none,
360.06173465894760240276827313477302063 64, none, none,
324.65521222977167615972849865893390750 01, none, none, none, none]

0 --> 1  target = [26.46318954448268905021493126181464306753,
6.19617723029547358419085743217480190016 9,
385.42734025189148022034767704781339797 61]
two intervals r = 16.87629600304290343664585319394306701297 ..
189999999999658502828565473806508291 61/1000000000000000000000000000000000
00000 or r = 15.55559000624668167669613179556123257092 ..
189999999999658502828565473806508291 61/1000000000000000000000000000000000
00000
Time Approximations 0.054.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 6.01 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852537126961489027078106050311807, rm
= 2.33669042810486701866881262378465952394 9}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.179e-37, 0., .1309e-34]Solution in 22.347s

Time Plot 0 s.
Exiting SolveHard() after 23.381r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21,
302.31384313994391584528306893066591273 21,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81,
375.73285289104131885480401182187905736 23,
328.11709293617711043570395920433047037 62,
292.99969137536576811133457852508075218 25,
358.64341560227974426255301042706200379 72, none,
360.06173465894760240276827313477302063 64,
336.59441031577803353929559615367587217 10, none,
324.65521222977167615972849865893390750 01, none, none, none, none]

0 --> 2  target = [34.49522661169827048641870536598063589823,
3.89713131591309123028101272778430009736 4,
373.78081884089896504836386740504774861 17]
two intervals r = 17.29769086229785615666317574577701080919 ..
189999999999965850282856547380650829161/100000000000000000000000000000000
00000 or r = 14.99436407410798751454451974782731901512 ..
189999999999965850282856547380650829161/100000000000000000000000000000000
00000
Time Approximations 0.075.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.54e-37, 0., -.669e-35]Solution in 1.91s

Time Plot 0 s.
Exiting SolveHard() after 4.269r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21,
302.31384313994391584528306893066591273 21,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81,
375.73285289104131885480401182187905736 23,
328.11709293617711043570395920433047037 62,
292.99969137536576811133457852508075218 25,
358.64341560227974426255301042706200379 72, none,
360.06173465894760240276827313477302063 64,
336.59441031577803353929559615367587217 10, none,
324.65521222977167615972849865893390750 01,
331.93806791055197445096801110327014204 41, none, none, none]

1 --> 2  target = [34.49522661169827048641870536598063589823,
3.89713131591309123028101272778430009736 4,
373.78081884089896504836386740504774861 17]
```

"Imaginary part neglected: ", $1.8899423791487321694159180727752604522 14 \times 10^{-17}$

```
one interval r = 21.06068473196433944755703129346972334863 ..
26.26979834275231042016013727112527427421
Time Approximations 0.032.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
```

```
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [-.3e-37, -.7e-37, -.25470e-34]Solution in
0.714s

Time Plot 0 s.
Exiting SolveHard() after 2.24r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21,
302.31384313994391584528306893066591273 21,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81,
375.73285289104131885480401182187905736 23,
328.11709293617711043570395920433047037 62,
292.99969137536576811133457852508075218 25,
358.64341560227974426255301042706200379 72,
299.89866204275517374833024673039677727 14,
360.06173465894760240276827313477302063 64,
336.59441031577803353929559615367587217 10, none,
324.65521222977167615972849865893390750 01,
331.93806791055197445096801110327014204 41, none, none, none]

0 --> 2  target = [33.81362495410168359180744917091964261585,
3.72564899353599480574978961112270962662 5,
325.89209972198033629346668611469404611 30]
two intervals r = 18.552270490110632779752894511888292277 27 ..
18999999999965850282856547380650829161/10000000000000000000000000000000
00000 or r = 12.491969357520797318183385852150347894 72 ..
18999999999965850282856547380650829161/10000000000000000000000000000000
00000
Time Approximations 0.036.
```

```
hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1e-38
Equations at solution: [.17e-37, -.1e-37, .17e-36]Solution in 1.897s

Time Plot 0 s.
Exiting SolveHard() after 4.034r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21,
302.31384313994391584528306893066591273 21,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81,
375.73285289104131885480401182187905736 23,
328.11709293617711043570395920433047037 62,
292.99969137536576811133457852508075218 25,
358.64341560227974426255301042706200379 72,
299.89866204275517374833024673039677727 14,
360.06173465894760240276827313477302063 64,
336.59441031577803353929559615367587217 10, none,
324.65521222977167615972849865893390750 01,
331.93806791055197445096801110327014204 41, none, none,
289.54595771887049104330296790336947368 79]

1 --> 2  target = [33.81362495410168359180744917091964261585,
3.72564899353599480574978961112270962 6625,
325.89209972198033629346668611469404611 30]

"Imaginary part neglected: ", 1.8899423791487321694159180727752604522 14 × 10^-17
one interval r = 20.37468935099499494189828888861987146046 ..
```

```
25.37892165285141966146441309076113948918
Time Approximations 0.025.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.2e-37, .2e-37, .30954e-34]Solution in 1.29s

Time Plot 0 s.
Exiting SolveHard() after 1.783r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21,
302.31384313994391584528306893066591273 21,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81,
375.73285289104131885480401182187905736 23,
328.11709293617711043570395920433047037 62,
292.99969137536576811133457852508075218 25,
358.64341560227974426255301042706200379 72,
299.89866204275517374833024673039677727 14,
360.06173465894760240276827313477302063 64,
336.59441031577803353929559615367587217 10,
256.10753185308704389708729493987588234 42,
324.65521222977167615972849865893390750 01,
331.93806791055197445096801110327014204 41, none, none,
289.54595771887049104330296790336947368 79]

1 --> 0  target = [17.93041369722317432521471347426606114288,
4.68650870191409385164049126007788410710 2,
```

```
353.30541094299591742850296602265034987 22]
```

"Imaginary part neglected: ", $1.8899423791487321694159180727752604522 14 \times 10^{-17}$

```
one interval r = 20.7315047907985042776452687038365179084 5 ..
25.906753535069738753276312102458576733 91
Time Approximations 0.03.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, .52052e-34]Solution in
1.438s

Time Plot 0 s.
Exiting SolveHard() after 2.13r=25.4021 in [22.67806074 .. 25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493282442908735923741860940779 75,
441.64295972730516227571050968744702797 55,
436.91748164933305332183177329977042985 23,
422.98493396852783843437310469076268236 18,
361.52580255556572029254230960342789653 02,
401.88173903705924043939667089367925805 65,
389.59001515486542984339337232519873304 66,
328.46939892739969554412082440608201203 19,
401.50757157412292612368738352424378238 06,
358.97362823420571110066298466530622063 34,
398.33147103264040407398554759908902455 93,
371.48387394042462398248207079209072097 09,
336.61215840530250786745352487405330011 19,
361.50888346598525261643717934200409504 49,
324.67144991994506066523057259799103955 21,
302.31384313994391584528306893066591273 21,
328.46938512812719572137792457836528115 38,
343.81340624393871652034525441055610084 81,
375.73285289104131885480401182187905736 23,
328.11709293617711043570395920433047037 62,
292.99969137536576811133457852508075218 25,
358.64341560227974426255301042706200379 72,
299.89866204275517374833024673039677727 14,
360.06173465894760240276827313477302063 64,
336.59441031577803353929559615367587217 10,
256.10753185308704389708729493987588234 42,
324.65521222977167615972849865893390750 01,
```

```
331.9380679105519744509680111032701420441,
304.7995832463905304034322572215614735621, none,
289.5459577188704910433029679033694736879]

2 --> 0  target = [17.93041369722317432521471347426606114288,
4.686508701914093851640491260077884107102,
353.30541094299591742850296602265034987 22]
one interval r = 31.37435486998221452430415391831986747825 ..
34.20127520026739723683058775381044114420
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., -.161e-34]Solution in 0.364s

Time Plot 0 s.
Exiting SolveHard() after 0.641r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349328244290873592374186094077975,
441.6429597273051622757105096874470279755,
436.9174816493330533218317732997704298523,
422.9849339685278384343731046907626823618,
361.5258025555657202925423096034278965302,
401.8817390370592404393966708936792580565,
389.5900151548654298433933723251987330466,
328.4693989273996955441208244060820120319,
401.5075715741229261236873835242437823806,
358.9736282342057111006629846653062206334,
398.3314710326404040739855475990890245593,
371.4838739404246239824820707920907209709,
336.6121584053025078674535248740533001119,
361.5088834659852526164371793420040950449,
324.6714499199450606652305725979910395521,
302.3138431399439158452830689306659127321,
328.4693851281271957213779245783652811538,
343.8134062439387165203452544105561008481,
375.7328528910413188548040118218790573623,
328.1170929361771104357039592043304703762,
292.9996913753657681113345785250807521825,
358.6434156022797442625530104270620037972,
299.8986620427551737483302467303967772714,
360.0617346589476024027682731347730206364,
```

```
336.594410315778033539295596153675872171 0,
256.107531853087043897087294939875882344 2,
324.655212229771676159728498658933907500 1,
331.938067910551974450968011103270142044 1,
304.799583246390530403432257221561473562 1,
323.461691759676136723466681658191963989 6,
289.545957718870491043302967903369473687 9]

Cascade time 185.563
counts: 28, 28

 Iteration 15

Start Generation 1
1 --> 0  target = [12.00000000010406979252323204364583142900,
6.217012502931143479735002041013928814313,
485.549080892631981708023392054841763622 0]
one interval r = 23.40850301646466129367656505147514646193 ..
27.67578046436779335344001998207762448833
Time Approximations 0.035.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.08e-37
Equations at solution: [-.3e-37, .108e-36, .21e-35]Solution in 1.687s

Time Plot 0 s.
Exiting SolveHard() after 3.556r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434931249978116896592788385173203 2,
441.642959725939868974906558088758474791 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000010406979252323204364583142900,
6.217012502931143479735002041013928814313,
485.549080892631981708023392054841763622 0]
one interval r = 32.62814779200156371492511857089759953781 ..
36.10248388936160171264402707300041405222
Time Approximations 0.021.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
```

```
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=8e-38
Equations at solution: [-.12e-36, .8e-37, -.2061e-35]Solution in 0.596s

Time Plot 0 s.
Exiting SolveHard() after 0.989r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684486367696377370590972029644480,
6.58343472170566691036002779048019837159 2,
467.78730595257914264027132714837665283 27]
one interval r = 32.41978955648603911006515722210473625215 ..
35.85152417364250617193428996785698916811
Time Approximations 0.019.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .374e-36]Solution in 0.599s

Time Plot 0 s.
Exiting SolveHard() after 1.814r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42, none, none,
401.88173903307730037865636016521279619 83, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844863676963773705909720296444 80,
6.5834347217056669103600277904801983715 92,
467.78730595257914264027132714837665283 27]
two intervals r = 12.923271608446948160530731687181915837 07 ..
949999999993486596640095605637213980 1/500000000000000000000000000000000
000 or r = 18.394248580080634761017710561800556054 52 ..
949999999993486596640095605637213980 1/500000000000000000000000000000000
000
Time Approximations 0.039.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.4e-38
Equations at solution: [-.5e-37, -.54e-37, .129e-35]Solution in 32.423s

Time Plot 0 s.
Exiting SolveHard() after 34.584r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67, none,
401.88173903307730037865636016521279619 83, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628149937527941246391198727350 15,
4.1256517966137599100241640291608716806 67,
440.67123064266304713703959375068343943 67]
two intervals r = 14.356597051384520595204547717817712575 03 ..
949999999993486596640095605637213980 1/500000000000000000000000000000000
000 or r = 17.703526137762696528195327920138778903 44 ..
949999999993486596640095605637213980 1/500000000000000000000000000000000
000
Time Approximations 0.047.
```

```
hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [-.57e-37, -.3e-37, -.334e-35]Solution in 2.183s

Time Plot 0 s.
Exiting SolveHard() after 4.178r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67, none,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.463229628149937527941246391198727350 15,
4.1256517966137599100241640291608716806 67,
440.67123064266304713703959375068343943 67]
one interval r = 22.397611543515655604558965439338196332 40 ..
27.237223516004392440682347251002758433 08
Time Approximations 0.04.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 2.119 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643776858691325666430688057723 25, rm =
14.378187701790786246967279657453472705 91}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., -.569e-34]Solution in 8.82s
```

```
Time Plot 0 s.
Exiting SolveHard() after 10.662r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888783790650399535375015020249250,
4.00486908158397285182442822115190614972 6,
404.86224500360611779730225199775500359 74]
two intervals r = 16.08011007781972641671602115272909595315 ..
9499999999934865966400956056372139801/500000000000000000000000000000000
000 or r = 16.41579812646726284895616856818278967245 ..
9499999999934865966400956056372139801/500000000000000000000000000000000
000
Time Approximations 0.055.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.17e-37, 0., -.4501e-34]Solution in 3.431s

Time Plot 0 s.
Exiting SolveHard() after 4.522r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
```

```
389.590015148466778087652089899531320 1392, none, none,
358.97362822779410084261705834862502761 37, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888783790650399535375015020249250,
4.00486908158397285182442822115190614972 6,
404.86224500360611779730225199775500359 74]
one interval r = 21.641943993933474536669348623569382707 70 ..
26.763306600379144593144689657266808241 63
Time Approximations 0.053.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., .208e-34]Solution in 1.92s

Time Plot 0 s.
Exiting SolveHard() after 2.949r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533695591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.590015148466778087652089899531320 1392,
328.46939892119538593020714720547970802 36, none,
358.97362822779410084261705834862502761 37, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941804926545647829066215124694097,
5.58963718291610796763469246053819215574 2,
443.83065883790042856851589299192271724 51]
one interval r = 22.467253744625564944992290785173686860 92 ..
27.273884283568664027474099900099775761 71
Time Approximations 0.04.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
```

```
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [-.1e-37, .27e-37, .80e-35]Solution in 1.031s

Time Plot 0 s.
Exiting SolveHard() after 3.012r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36, none,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589418049265456478290662151246940 97,
5.5896371829161079676346924605381921557 42,
443.83065883790042856851589299192271724 51]
one interval r = 32.155752794851960360343519518001852479 92 ..
35.508722287269813453832632274718039108 52
Time Approximations 0.021.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .6573e-35]Solution in 0.492s

Time Plot 0 s.
Exiting SolveHard() after 1.787r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136537872050881871879212745356893,
5.18778357845021626508009334028537079540 2,
408.65773861656659625854026423059352570 09]
one interval r = 21.71840114630704990447510457916259440429 ..
26.81849303507155728726957021010270432566
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.11e-37
Equations at solution: [.2e-37, .211e-36, -.52e-35]Solution in 1.905s

Time Plot 0 s.
Exiting SolveHard() after 3.973r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19, none, none,
```

```
361.50888345967567690905288488411679098 54, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136537872050881871879212745356893,
5.1877835784502162650800933402853707954 02,
408.65773861656659625854026423059352570 09]
one interval r = 31.80828598733012356498273331218365081715 ..
35.00011460029774071214495037968704011563
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.502e-36]Solution in 0.426s

Time Plot 0 s.
Exiting SolveHard() after 0.726r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37, none,
361.50888345967567690905288488411679098 54, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110530217649058345970403737489925,
6.1962625654081582943191549062491290868 74,
385.44474378123023571133368651074660015 22]
one interval r = 31.60836097517436985008643906637916343971 ..
34.66372795591078665680245140698901760013
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
```

```
11.783029068742676904624149068588691721 32, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.12783e-34]Solution in 1.409s

Time Plot 0 s.
Exiting SolveHard() after 1.685r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37, none,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110530217649058345970403737489925,
6.19626256540815829431915490624912908687 4,
385.44474378123023571133368651074660015 22]
two intervals r = 16.875634087804318852431475587314689829 95 ..
94999999999348659664009560563721398 01/50000000000000000000000000000000
000 or r = 15.556404937404664304265734003949378225 32 ..
94999999999348659664009560563721398 01/50000000000000000000000000000000
000
Time Approximations 0.054.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 5.46 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175127657807902161366080027458013, rm
= 2.33653277407326545555769770999836763856 1}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.538e-37, 0., -.3915e-34]Solution in 22.626s

Time Plot 0 s.
Exiting SolveHard() after 24.592r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874759836312838905483290712598774,
4.88381077977209739067805716110578134829 3,
376.61967854585052203605923982499176520 88]
one interval r = 21.11001304857130461411226272507060668733 ..
26.31784243463657184644344790792593243736
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=1.00e-37
Equations at solution: [.3e-37, .100e-36, .415e-34]Solution in 0.872s

Time Plot 0 s.
Exiting SolveHard() after 2.519r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
```

```
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93, none,
328.46938512192649745019164357177150108 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874759836312838905483290712598774,
4.8838107797720973906780571611057813482 93,
376.61967854585052203605923982499176520 88]
one interval r = 31.53899497691780933357145933516432923584 ..
34.53618386073837158131380906089771805579
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=7.76e-36
Equations at solution: [-.596e-35, .776e-35, -.11770e-34]Solution in
1.295s

Time Plot 0 s.
Exiting SolveHard() after 1.606r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
```

```
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93, none,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017523996452590011716695266087109,
6.0258135493539384547873012398714903028 24,
351.42702947032361504002288488674948914 30]
one interval r = 31.36230206094450264882027336145250023277 ..
34.17446640592342178627748761093000093932
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [-.3e-37, .6e-37, -.21376e-34]Solution in 0.564s

Time Plot 0 s.
Exiting SolveHard() after 0.833r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
```

```
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93, none,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28, none, none,
292.99969136654646317740843849206810497 25, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175239964525900117166952660871 09,
6.0258135493539384547873012398714903028 24,
351.42702947032361504002288488674948914 30]
two intervals r = 17.981355144616742902883072403375518585 45 ..
949999999993486596640095605637213980 1/500000000000000000000000000000000
000 or r = 13.846080153440417897414005636752350856 13 ..
949999999993486596640095605637213980 1/500000000000000000000000000000000
000
Time Approximations 0.043.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 4.796 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713649021710708740778648459073 31, rm
= 2.7345009931760200330910069019355207293 60}});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [-.17e-37, 0., -.1068e-34]Solution in 15.773s

Time Plot 0 s.
Exiting SolveHard() after 17.714r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533695591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
```

```
324.67144991096299101218795297009452158 93,
302.31384313418431010993482255724859465 14,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28, none, none,
292.99969136654646317740843849206810497 25, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418721205068837810265689343417 08,
6.3779438739679446557935909498728095361 09,
423.28832783081816080021358627426294341 67]
one interval r = 31.946618175797043929402918868740537479 39 ..
35.212123086384303314656935846658906786 17
Time Approximations 0.02.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=6e-38
Equations at solution: [-.5e-37, .6e-37, .12479e-34]Solution in 0.634s

Time Plot 0 s.
Exiting SolveHard() after 0.97r=34.3272 in [33.10127385 .. 35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533695591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93,
302.31384313418431010993482255724859465 14,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28, none, none,
292.99969136654646317740843849206810497 25, none, none,
360.06173465598534820900137354888540390 67, none, none, none, none,
none, none, none]
```

```
0 --> 1  target = [27.020379418721205068837810265689343417 08,
6.3779438739679446557935909498728095361 09,
423.28832783081816080021358627426294341 67]
two intervals r = 15.22886702459168752229499522763532897057 ..
949999999993486596640095605637213980 1/500000000000000000000000000000000000
000 or r = 17.12965777033728072956900186243787699929 ..
949999999993486596640095605637213980 1/500000000000000000000000000000000000
000
Time Approximations 0.067.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 6.554 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541389373312588040617563773579 74, rm
= 2.0640682986926880538775361939213550804 76}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.15e-37, .1e-37, -.1433e-34]Solution in 24.691s

Time Plot 0 s.
Exiting SolveHard() after 26.823r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517 32032,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533695591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93,
302.31384313418431010993482255724859465 14,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28,
375.73285289154430071399315335955392167 49, none,
292.99969136654646317740843849206810497 25, none, none,
360.06173465598534820900137354888540390 67, none, none, none, none,
```

```
none, none, none]

0 --> 2  target = [34.93953234326312549558102638580260783338,
4.0035598152808145745039014493504780708 10,
404.47973592972851212826097368519365748 54]
two intervals r = 16.09683966396854334688243823225963843455 ..
949999999993486596640095605637213980 1/500000000000000000000000000000000
000 or r = 16.39988649062652228390634478833337346371 ..
949999999993486596640095605637213980 1/500000000000000000000000000000000
000
Time Approximations 0.052.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.34e-37, -.1e-37, -.1262e-34]Solution in
2.408s

Time Plot 0 s.
Exiting SolveHard() after 4.436r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93,
302.31384313418431010993482255724859465 14,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28,
375.73285289154430071399315335955392167 49, none,
292.99969136654646317740843849206810497 25,
358.64341559548097844503014562477659817 12, none,
360.06173465598534820900137354888540390 67, none, none, none, none,
none, none, none]
```

```
1 --> 2  target = [34.93953234326312549558102638580260783338,
4.0035598152808145745039014493504780708 10,
404.47973592972851212826097368519365748 54]
one interval r = 21.63429629968761243777941534977638622698 ..
26.75768169886310073815263722517730994156
Time Approximations 0.048.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=4.9e-38
Equations at solution: [.1e-37, .49e-37, -.329e-34]Solution in 1.941s

Time Plot 0 s.
Exiting SolveHard() after 3.952r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93,
302.31384313418431010993482255724859465 14,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28,
375.73285289154430071399315335955392167 49,
328.11709292956162873405351946777510187 69,
292.99969136654646317740843849206810497 25,
358.64341559548097844503014562477659817 12, none,
360.06173465598534820900137354888540390 67, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954448669042071261407189874270445,
6.19617723026246776405794704283621836270 0,
385.42734024514252605511339519835403396 08]
```

```
one interval r = 31.6082204907293231024145068720356664698 9 ..
34.663476150289722691750728096963678927 59
Time Approximations 0.018.

hint used Hint := [33.813392675100157147469018161182119369 13, 3, 1, 1,
11.783209121413521870719438174471899656 27, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .12536e-34]Solution in 0.579s

Time Plot 0 s.
Exiting SolveHard() after 0.901r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93,
302.31384313418431010993482255724859465 14,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28,
375.73285289154430071399315335955392167 49,
328.11709292956162873405351946777510187 69,
292.99969136654646317740843849206810497 25,
358.64341559548097844503014562477659817 12, none,
360.06173465598534820900137354888540390 67, none, none,
324.65521222090100678484718912017552107 51, none, none, none, none]

0 --> 1  target = [26.46318954448669042071261407189874270445,
6.19617723026246776405794704283621836270 0,
385.42734024514252605511339519835403396 08]
two intervals r = 16.87629600318452143085851549085585975726 ..
9499999999934865966400956056372139801/50000000000000000000000000000000
000 or r = 15.55559000589772034168535521605853283583 ..
```

```
9499999999934865966400956056372139801/500000000000000000000000000000000
000
Time Approximations 0.06.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 6.676 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852539079193529964816135172520813, rm
= 2.33669042813715767741413344619516822915 1}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.717e-37, 0., -.5482e-34]Solution in 25.206s

Time Plot 0 s.
Exiting SolveHard() after 27.276r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93,
302.31384313418431010993482255724859465 14,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28,
375.73285289154430071399315335955392167 49,
328.11709292956162873405351946777510187 69,
292.99969136654646317740843849206810497 25,
358.64341559548097844503014562477659817 12, none,
360.06173465598534820900137354888540390 67,
336.59441030983988719711380878183557783 82, none,
324.65521222090100678484718912017552107 51, none, none, none, none]

0 --> 2  target = [34.49522661143538510968828343844846134563,
```

```
3.897131315686756664740981730753564515922,
373.780818830861336705737487472036947209 2]
two intervals r = 17.29769086252731950788304517877768471169 ..
94999999999348659664009560563721398 01/50000000000000000000000000000000000
000 or r = 14.99436407358623938064195880494797952158 ..
94999999999348659664009560563721398 01/50000000000000000000000000000000000
000
Time Approximations 0.085.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.108e-36, 0., -.774e-35]Solution in 2.241s

Time Plot 0 s.
Exiting SolveHard() after 4.884r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93,
302.31384313418431010993482255724859465 14,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28,
375.73285289154430071399315335955392167 49,
328.11709292956162873405351946777510187 69,
292.99969136654646317740843849206810497 25,
358.64341559548097844503014562477659817 12, none,
360.06173465598534820900137354888540390 67,
336.59441030983988719711380878183557783 82, none,
324.65521222090100678484718912017552107 51,
331.93806789861035604772423249032421012 18, none, none, none]
```

```
1 --> 2  target = [34.49522661143538510968828343844846134563,
3.897131315686756664740981730753564515922,
373.78081883086133670573748747203694720 92]
one interval r = 21.06068473180622646638805997367355031383 ..
26.26979834269048161114946083250506073180
Time Approximations 0.913.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [.2e-37, .5e-37, .111e-34]Solution in 0.739s

Time Plot 0 s.
Exiting SolveHard() after 2.344r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533695591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93,
302.31384313418431010993482255724859465 14,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28,
375.73285289154430071399315335955392167 49,
328.11709292956162873405351946777510187 69,
292.99969136654646317740843849206810497 25,
358.64341559548097844503014562477659817 12,
299.89866203086340135618326916655073490 46,
360.06173465598534820900137354888540390 67,
336.59441030983988719711380878183557783 82, none,
324.65521222090100678484718912017552107 51,
331.93806789861035604772423249032421012 18, none, none, none]
```

```
0 --> 2  target = [33.81362495384138746477804564849583019840,
3.72564899331125995957505048282826824344 5,
325.89209971300602610567721954436149906 76]
two intervals r = 18.552270490162298993878660438604787250 47 ..
949999999993486596640095605637213980 1/500000000000000000000000000000000
000 or r = 12.491969357063157327847846638348745230 01 ..
949999999993486596640095605637213980 1/500000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [.155e-36, -.6e-37, .223e-35]Solution in 1.928s

Time Plot 0 s.
Exiting SolveHard() after 4.382r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93,
302.31384313418431010993482255724859465 14,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28,
375.73285289154430071399315335955392167 49,
328.11709292956162873405351946777510187 69,
292.99969136654646317740843849206810497 25,
358.64341559548097844503014562477659817 12,
299.89866203086340135618326916655073490 46,
360.06173465598534820900137354888540390 67,
336.59441030983988719711380878183557783 82, none,
324.65521222090100678484718912017552107 51,
```

```
331.9380678986103560477242324903242101218, none, none,
289.5459577077989597757093892290187741606]

1 --> 2  target = [33.8136249538413874647780456484958301984 0,
3.7256489933112599595750504828282682434 45,
325.8920997130060261056772195443614990676]
one interval r = 20.37468935089778217819974649353309324138 ..
25.37892165276298240928742269834689073202
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=3e-38
Equations at solution: [.3e-37, .3e-37, .367e-34]Solution in 1.386s

Time Plot 0 s.
Exiting SolveHard() after 1.961r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349312499781168965927883851732032,
441.6429597259398689749065580887584747913,
436.9174816450971793446800553369359171142,
422.9849339677461375018192214760028537567,
361.5258025491400449113624833789346799282,
401.8817390330773003786563601652127961983,
389.5900151484667780876520898995313201392,
328.4693989211953859302071472054797080236,
401.5075715697044716262522886355245373198,
358.9736282277941008426170583486250276137,
398.3314710324129391100605258741185163819,
371.4838739303936300247270443468693417537,
336.6121583992425302532232940946225077880,
361.5088834596756769090528848841167909854,
324.6714499109629910121879529700945215893,
302.3138431341843101099348225572485946514,
328.4693851219264974501916435717715010884,
343.8134062335192691595793553405660191028,
375.7328528915443007139931533595539216749,
328.1170929295616287340535194677751018769,
292.9996913665464631774084384920681049725,
358.6434155954809784450301456247765981712,
299.8986620308634013561832691665507349046,
360.0617346559853482090013735488854039067,
336.5944103098398871971138087818355778382,
```

```
256.10753184219099451589440152766492961 66,
324.65521222090100678484718912017552107 51,
331.93806789861035604772423249032421012 18, none, none,
289.54595770779895977570938922901877416 06]

1 --> 0  target = [17.93041369728936715885257154243465857679,
4.68650870190308664219259817717233761460 4,
353.30541093645888332313321838087781514 75]
one interval r = 20.73150479071015470125502392255272545367 ..
25.90675353505101583136846600959254409182
Time Approximations 0.033.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., .134e-34]Solution in 1.5s

Time Plot 0 s.
Exiting SolveHard() after 2.263r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533695591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93,
302.31384313418431010993482255724859465 14,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28,
375.73285289154430071399315335955392167 49,
328.11709292956162873405351946777510187 69,
292.99969136654646317740843849206810497 25,
358.64341559548097844503014562477659817 12,
299.89866203086340135618326916655073490 46,
```

```
360.06173465598534820900137354888540390 67,
336.59441030983988719711380878183557783 82,
256.10753184219099451589440152766492961 66,
324.65521222090100678484718912017552107 51,
331.93806789861035604772423249032421012 18,
304.79958324062959551927750864077495868 54, none,
289.54595770779895977570938922901877416 06]

2 --> 0  target = [17.93041369728936715885257154243465857679,
4.68650870190308664219259817717233761460 4,
353.30541093645888332313321838087781514 75]
one interval r = 31.37435486973514656645154843237538967197 ..
34.20127520004885951126838254137795899262
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [.5e-37, -.8e-37, -.5247e-35]Solution in 0.385s

Time Plot 0 s.
Exiting SolveHard() after 0.645r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493124997811689659278838517320 32,
441.64295972593986897490655808875847479 13,
436.91748164509717934468005533693591711 42,
422.98493396774613750181922147600285375 67,
361.52580254914004491136248337893467992 82,
401.88173903307730037865636016521279619 83,
389.59001514846677808765208989953132013 92,
328.46939892119538593020714720547970802 36,
401.50757156970447162625228863552453731 98,
358.97362822779410084261705834862502761 37,
398.33147103241293911006052587411851638 19,
371.48387393039363002472704434686934175 37,
336.61215839924253025322329409462250778 80,
361.50888345967567690905288488411679098 54,
324.67144991096299101218795297009452158 93,
302.31384313418431010993482255724859465 14,
328.46938512192649745019164357177150108 84,
343.81340623351926915957935534056601910 28,
375.73285289154430071399315335955392167 49,
328.11709292956162873405351946777510187 69,
```

```
292.99969136654646317740843849206810497 25,
358.6434155954809784450301456247765981712,
299.8986620308634013561832691665507349046,
360.0617346559853482090013735488854039067,
336.5944103098398871971138087818355778382,
256.1075318421909945158944015276649296166,
324.6552122209010067848471891201755210751,
331.9380678986103560477242324903242101218,
304.7995832406295955192775086407749586854,
323.4616917493906380833303063802550394699,
289.5459577077989597757093892290187741606]

Cascade time 202.6
counts: 28, 28

 Iteration 16

Start Generation 1
1 --> 0  target = [11.999999999947096253585473996040826952 00,
6.217012502848829039408890393125530256274,
485.5490808986008424261783434801109877052]
one interval r = 23.40850301651065037649404683823480092238 ..
27.67578046433731482518567893500020226228
Time Approximations 0.042.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.04e-37
Equations at solution: [-.3e-37, .104e-36, .7e-36]Solution in 1.861s

Time Plot 0 s.
Exiting SolveHard() after 3.892r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.6429597313790338495374198507773989473, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999947096253585473996040826952 00,
6.217012502848829039408890393125530256274,
485.5490808986008424261783434801109877052]
one interval r = 32.62814779219035684730882754886709509666 ..
```

```
36.10248388950747811563726135294517910756
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .8266e-35]Solution in 0.628s

Time Plot 0 s.
Exiting SolveHard() after 1.052r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684484629241667898422149078045568,
6.58343472160855707591684373267241105 6213,
467.78730595852838814897246694810910 11895]
one interval r = 32.41978955667525184390182691645881452990 ..
35.85152417379232079914044207704027341543
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., -.24851e-34]Solution in 0.662s

Time Plot 0 s.
Exiting SolveHard() after 1.91r=34.9451 in [33.70078237 .. 35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80, none, none,
401.88173904151352456226720614204554278 43, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684484629241667898422149078045568,
6.58343472160855707591684373267241105621 3,
467.78730595852838814897246694810910118 95]
two intervals r = 12.92327160831198797277519615587186293243 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000 or r = 18.39424858036501798968017611270582495719 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.4e-38
Equations at solution: [.5e-37, .54e-37, .9e-37]Solution in 34.31s

Time Plot 0 s.
Exiting SolveHard() after 36.547r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90, none,
401.88173904151352456226720614204554278 43, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962833642566460269312420058798988,
4.12565179674525782796161010856883207004 7,
440.67123065072096471331640292824669826 88]
two intervals r = 14.35659705118512902781550662666087402805 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000 or r = 17.70352613814353077479056557324498195590 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
```

```
00000
Time Approximations 0.047.

hint used Hint := [15.9119313651056959513777069858343213132 6, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., .1173e-34]Solution in 2.171s

Time Plot 0 s.
Exiting SolveHard() after 4.118r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90, none,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962833642566460269312420058798988,
4.12565179674525782796161010856883207004 7,
440.67123065072096471331640292824669826 88]
one interval r = 22.39761154358677212018628070603014183731 ..
27.23722351603024676888810877487053060378
Time Approximations 0.039.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 2.123 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064385201522170157969375227744384, rm =
14.37818770239511915065451614100547098760}});
```

```
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, .35e-35]Solution in 8.968s

Time Plot 0 s.
Exiting SolveHard() after 10.767r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888803777235369097685914732559842,
4.00486908171781014970963312969334861392 8,
404.86224501202694675255915403881200765 98]
two intervals r = 16.08011007768789112027189116990637332140 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000 or r = 16.41579812691968658622218384458695936797 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.34e-37, 0., .1241e-34]Solution in 2.396s

Time Plot 0 s.
Exiting SolveHard() after 3.497r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
```

```
422.984933973369761353611888426878940389O,
361.525802559707598240366763956943032774 3,
401.881739041513524562267206142045542784 3,
389.590015158664597650412589598585053078 9, none, none,
358.973628238565309170811626253545847318 2, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888037772353690976859147325598 42,
4.004869081717810149709633129693348613928,
404.862245012026946752559154038812007659 8]
one interval r = 21.641943993981906467598639531788642382 43 ..
26.763306600437385248320136199105904387 82
Time Approximations 0.054.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., .103e-34]Solution in 1.965s

Time Plot 0 s.
Exiting SolveHard() after 3.948r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936800788310499984815593579825 2,
441.642959731379033849537419850777398947 3,
436.917481653183238475801835015994839208 0,
422.984933973369761353611888426878940389O,
361.525802559707598240366763956943032774 3,
401.881739041513524562267206142045542784 3,
389.590015158664597650412589598585053078 9,
328.469398932095651426085336330658755303 1, none,
358.973628238565309170811626253545847318 2, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589417950147022097919946418582773 79,
5.589637182822608956183536469648170154881,
443.830658843908929284458366409202569619 9]
one interval r = 22.467253744653519301952226022457274967 24 ..
27.273884283568404450725894794778122551 42
Time Approximations 0.034.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
```

```
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.34e-37
Equations at solution: [-.1e-37, .134e-36, -.59e-35]Solution in 0.946s

Time Plot 0 s.
Exiting SolveHard() after 2.869r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31, none,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941795014702209791994641858277379,
5.589637182822608956183536469648170154881,
443.83065884390892928445836640920256961 99]
one interval r = 32.15575279504192050699904751779252165691 ..
35.50872228742605131908802846932187715978
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.2807e-35]Solution in 0.472s

Time Plot 0 s.
Exiting SolveHard() after 1.797r=34.9395 in [33.37332721 ..
35.50872230]
```

```
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136513075974953438604573924084399,
5.1877835784026492978654234359432718308 38,
408.65773862736101237851197312320309609 14]
one interval r = 21.71840114640738694334287705142814045343 ..
26.81849303516120853626302760426083472001
Time Approximations 0.059.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.37e-37
Equations at solution: [.2e-37, .237e-36, .306e-34]Solution in 1.881s

Time Plot 0 s.
Exiting SolveHard() after 3.873r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
```

```
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22, none, none,
361.50888347010488809636692416610040314 79, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.911931365130759749534386045739240843 99,
5.1877835784026492978654234359432718308 38,
408.65773862736101237851197312320309609 14]
one interval r = 31.808285987562212075832410848669930216 00 ..
35.000114600531208414451364142501345928 10
Time Approximations 0.018.

hint used Hint := [34.495226611650080820729149248019775312 21, 3, 1, 1,
15.763855890238725343023875204542137417 16, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., .16782e-34]Solution in 0.41s

Time Plot 0 s.
Exiting SolveHard() after 0.703r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39, none,
361.50888347010488809636692416610040314 79, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.463471105418941326131304322811062302 65,
6.1962625653382657200363824178698937209 21,
385.44474379246025750216453397865526406 07]
one interval r = 31.608360975402638335983676106805608814 13 ..
34.663727956155258127650922308823741656 74
```

```
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .3423e-35]Solution in 1.385s

Time Plot 0 s.
Exiting SolveHard() after 1.655r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39, none,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110541894132613130432281106230265,
6.196262565338265720036382417869893720921,
385.44474379246025750216453397865526406 07]
two intervals r = 16.875634087616312873677123938376847809 24 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000 or r = 15.556404938010609657103230905192914137 94 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000
Time Approximations 0.954.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.1203e-34]Solution in 1.156s

Time Plot 0 s.
Exiting SolveHard() after 3.201r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874743672049659647480086041291195,
4.8838107797228390739352623651962128326 54,
376.61967855721714438557013456954843337 14]
one interval r = 21.11001304863961558705728460864639373166 ..
26.31784243476283259916316156803143048108
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.152e-34]Solution in 0.864s

Time Plot 0 s.
Exiting SolveHard() after 2.471r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82, none,
328.46938513282266215661075999611943847 28, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874743672049659647480086041291195,
4.8838107797228390739352623651962128326 54,
376.61967855721714438557013456954843337 14]
one interval r = 31.53899497714379188501744770627279817075 ..
34.53618386098639540938125127542610621738
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=5.59e-36
Equations at solution: [.429e-35, -.559e-35, -.13910e-34]Solution in
0.5s

Time Plot 0 s.
Exiting SolveHard() after 1.679r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
```

```
389.590015158664597650412589598585053078 9,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82, none,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017538855528592212473181510470436,
6.0258135492879743532023626554834843859 67,
351.42702948190474953132762721235341867 28]
one interval r = 31.36230206116108655232252301113147706578 ..
34.17446640617832575429503763204217455750
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=9e-38
Equations at solution: [-.4e-37, .9e-37, .9108e-35]Solution in 0.521s

Time Plot 0 s.
Exiting SolveHard() after 0.758r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.590015158664597650412589598585053078 9,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82, none,
```

```
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51, none, none,
292.99969138043251386002160678214157805 93, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017538855528592212473181510470436,
6.0258135492879743532023626554834843859 67,
351.42702948190474953132762721235341867 28]
two intervals r = 17.981355144540637416560334512369657655 57 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000 or r = 13.846080154086246087225709729251888392 17 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000
Time Approximations 0.043.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [-.73e-37, .2e-37, .332e-35]Solution in 1.942s

Time Plot 0 s.
Exiting SolveHard() after 3.933r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903849537419850777398947 3,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51, none, none,
292.99969138043251386002160678214157805 93, none, none, none, none,
none, none, none, none, none, none]
```

```
2 --> 1  target = [27.02037941873371232175138329320654551342,
6.37794387387163584625828743997579495518 2,
423.28832783682463355869537844550445909 61]
one interval r = 31.946618175986300257498115551920645523 88 ..
35.21212308654514997564319346937790632998
Time Approximations 0.021.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.10018e-34]Solution in 1.413s

Time Plot 0 s.
Exiting SolveHard() after 1.761r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51, none, none,
292.99969138043251386002160678214157805 93, none, none,
360.06173466455228075250356919470729997 47, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941873371232175138329320654551342,
6.37794387387163584625828743997579495518 2,
423.28832783682463355869537844550445909 61]
two intervals r = 15.22886702453055607549010362525888896541 ..
190000000000703266265536353127422527 31/100000000000000000000000000000000
00000 or r = 17.12965777067440306813675319209582169193 ..
```

```
190000000000703266265536353127422527 31/1000000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [-.46e-37, -.2e-37, -.1304e-34]Solution in
2.101s

Time Plot 0 s.
Exiting SolveHard() after 4.247r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51,
375.73285289715986962112563018558648177 37, none,
292.99969138043251386002160678214157805 93, none, none,
360.06173466455228075250356919470729997 47, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343472211306851558968852001980 24,
4.0035598154168220634014235321070806836 03,
404.47973593877957040611691140455184847 79]
two intervals r = 16.096839663810162452878013858546610004 73 ..
190000000000703266265536353127422527 31/1000000000000000000000000000000000
00000 or r = 16.399886491105723430953817789092306401 24 ..
190000000000703266265536353127422527 31/1000000000000000000000000000000000
00000
```

```
Time Approximations 0.944.

hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.187e-35]Solution in 2.333s

Time Plot 0 s.
Exiting SolveHard() after 4.261r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51,
375.73285289715986962112563018558648177 37, none,
292.99969138043251386002160678214157805 93,
358.64341560679846531812697288687397231 34, none,
360.06173466455228075250356919470729997 47, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234347221130685155896885200198024,
4.0035598154168220634014235321070806836 03,
404.47973593877957040611691140455184847 79]
one interval r = 21.63429629974824907279814789983580609116 ..
26.75768169893089178687384307575592224318
Time Approximations 0.049.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .152e-34]Solution in 1.863s

Time Plot 0 s.
Exiting SolveHard() after 2.852r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51,
375.73285289715986962112563018558648177 37,
328.11709294104229339469648681748854846 07,
292.99969138043251386002160678214157805 93,
358.64341560679846531812697288687397231 34, none,
360.06173466455228075250356919470729997 47, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954460116759726726075307946932519,
6.19617723019187841522835143292633220746 2,
385.42734025623024846547336196891249698 89]
one interval r = 31.608220490956436610310160194848344764 02 ..
34.663476150532138517572165763601612481 28
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
```

```
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .9724e-35]Solution in 1.51s

Time Plot 0 s.
Exiting SolveHard() after 1.819r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51,
375.73285289715986962112563018558648177 37,
328.11709294104229339469648681748854846 07,
292.99969138043251386002160678214157805 93,
358.64341560679846531812697288687397231 34, none,
360.06173466455228075250356919470729997 47, none, none,
324.65521223427848005301731462913153403 71, none, none, none, none]

0 --> 1  target = [26.46318954460116759726726075307946932519,
6.19617723019187841522835143292633220746 2,
385.42734025623024846547336196891249698 89]
two intervals r = 16.87629600300198831268603479366439964948 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000 or r = 15.55559000649702897918209835746076745894 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000
Time Approximations 1.033.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
```

```
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.359e-37, 0., .1180e-34]Solution in 1.126s

Time Plot 0 s.
Exiting SolveHard() after 3.238r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51,
375.73285289715986962112563018558648177 37,
328.11709294104229339469648681748854846 07,
292.99969138043251386002160678214157805 93,
358.64341560679846531812697288687397231 34, none,
360.06173466455228075250356919470729997 47,
336.59441032058954876587646698565692174 489, none,
324.65521223427848005301731462913153403 71, none, none, none, none]

0 --> 2  target = [34.49522661171975377189133794574653430713,
3.89713131584073776448206607449884709902 8,
373.78081884471494831241359262712556361 19]
two intervals r = 17.29769086229057639933539824792827903624 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000 or r = 14.99436407433688174276244518226586488805 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000
Time Approximations 0.081.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
```

```
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [0., 0., .103e-35]Solution in 2.029s

Time Plot 0 s.
Exiting SolveHard() after 5.393r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51,
375.73285289715986962112563018558648177 37,
328.11709294104229339469648681748854846 07,
292.99969138043251386002160678214157805 93,
358.64341560679846531812697288687397231 34, none,
360.06173466455228075250356919470729997 47,
336.59441032058954876587646698565921744 89, none,
324.65521223427848005301731462913153403 71,
331.93806791426897102355038219111565227 24, none, none, none]

1 --> 2  target = [34.49522661171975377189133794574653430713,
3.89713131584073776448206607449884709902 8,
373.78018184471494831241359262712556361 19]
one interval r = 21.06068473191298415153697032292276496429 ..
26.26979834286133596892480703378754621524
Time Approximations 0.034.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, .745e-34]Solution in 1.67s

Time Plot 0 s.
Exiting SolveHard() after 2.388r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51,
375.73285289715986962112563018558648177 37,
328.11709294104229339469648681748854846 07,
292.99969138043251386002160678214157805 93,
358.64341560679846531812697288687397231 34,
299.89866204674257314293979052512272953 31,
360.06173466455228075250356919470729997 47,
336.59441032058954876587646698565921744 89, none,
324.65521223427848005301731462913153403 71,
331.93806791426897102355038219111565227 24, none, none, none]

0 --> 2  target = [33.81362495412756680998200161890725004133,
3.72564899346697851806582905486975850775 5,
325.89209972663174091344336827379226373 10]
two intervals r = 18.552270490144308123877748479449031093 28 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000 or r = 12.491969357769299947546592381561204672 51 ..
19000000000070326626553635312742252731/10000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
```

```
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=0
Equations at solution: [0., 0., -.1490e-34]Solution in 2.093s

Time Plot 0 s.
Exiting SolveHard() after 4.498r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51,
375.73285289715986962112563018558648177 37,
328.11709294104229339469648681748854846 07,
292.99969138043251386002160678214157805 93,
358.64341560679846531812697288687397231 34,
299.89866204674257314293979052512272953 31,
360.06173466455228075250356919470729997 47,
336.59441032058954876587646698565921744 89, none,
324.65521223427848005301731462913153403 71,
331.93806791426897102355038219111565227 24, none, none,
289.54595772333757429667172832384436087 57]

1 --> 2  target = [33.81362495412756680998200161890725004133,
3.72564899346697851806582905486975850775 5,
325.89209972663174091344336827379226373 10]
one interval r = 20.37468935090167810881599813231570299353 ..
25.37892165296740070450387120158165020317
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
```

```
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .385e-34]Solution in 0.598s

Time Plot 0 s.
Exiting SolveHard() after 1.137r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51,
375.73285289715986962112563018558648177 37,
328.11709294104229339469648681748854846 07,
292.99969138043251386002160678214157805 93,
358.64341560679846531812697288687397231 34,
299.89866204674257314293979052512272953 31,
360.06173466455228075250356919470729997 47,
336.59441032058954876587646698565921744 89,
256.10753185782413639435612493981004209 89,
324.65521223427848005301731462913153403 71,
331.93806791426897102355038219111565227 24, none, none,
289.54595772333757429667172832384436087 57]

1 --> 0  target = [17.93041369720894744677420221853539237169,
4.6865087018523407636854721752702385087 55,
353.30541094794319054705430470191195366 42]
one interval r = 20.73150479074177612937474539153653099574 ..
25.90675353519656106295402000020838370147
Time Approximations 0.033.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=7.1e-38
Equations at solution: [-.3e-37, -.71e-37, -.23e-35]Solution in 0.688s

Time Plot 0 s.
Exiting SolveHard() after 2.364r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51,
375.73285289715986962112563018558648177 37,
328.11709294104229339469648681748854846 07,
292.99969138043251386002160678214157805 93,
358.64341560679846531812697288687397231 34,
299.89866204674257314293979052512272953 31,
360.06173466455228075250356919470729997 47,
336.59441032058954876587646698565692174 489,
256.10753185782413639435612493981004209 89,
324.65521223427848005301731462913153403 71,
331.93806791426897102355038219111565227 24,
304.79958325155682999942744229597473158 10, none,
289.54595772333757429667172832384436087 57]

2 --> 0  target = [17.93041369720894744677420221853539237169,
4.68650870185234076368547217527023850875 5,
353.30541094794319054705430470191195366 42]
one interval r = 31.37435486995198289352332895013274067002 ..
34.20127520030213577034860440946991032193
Time Approximations 0.014.
```

```
hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.45190e-34]Solution in 0.369s

Time Plot 0 s.
Exiting SolveHard() after 1.627r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680078831049998481559357982 52,
441.64295973137903384953741985077739894 73,
436.91748165318323847580183501599483920 80,
422.98493397336976135361188842687894038 90,
361.52580255970759824036676395694303277 43,
401.88173904151352456226720614204554278 43,
389.59001515866459765041258959858505307 89,
328.46939893209565142608533633065875530 31,
401.50757157875712445460193121172582648 78,
358.97362823856530917081162625354584731 82,
398.33147103790556628215854066684208640 22,
371.48387394413747939386332019184095453 39,
336.61215841013739005422492029346858282 00,
361.50888347010488809636692416610040314 79,
324.67144992447320773647103313158806423 82,
302.31384314528363603520065238506062027 18,
328.46938513282266215661075999611943847 28,
343.81340624817008005249382677012248711 51,
375.73285289715986962112563018558648177 37,
328.11709294104229339469648681748854846 07,
292.99969138043251386002160678214157805 93,
358.64341560679846531812697288687397231 34,
299.89866204674257314293979052512272953 31,
360.06173466455228075250356919470729997 47,
336.59441032058954876587646698565921744 89,
256.10753185782413639435612493981004209 89,
324.65521223427848005301731462913153403 71,
331.93806791426897102355038219111565227 24,
304.79958325155682999942744229597473158 10,
323.46169176434789406666981650401007926 91,
289.54595772333757429667172832384436087 57]

Cascade time 124.321
counts: 28, 28
```

```
 Iteration 17

Start Generation 1
1 --> 0  target = [11.999999999997088568751374579374473866 00,
6.217012503006056412599121066178317629814,
485.549080905713549187419082965899889606 1]
one interval r = 23.408503016673432797017936937721233630 56 ..
27.675780464353607909385115251506411695 90
Time Approximations 0.043.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.5e-38
Equations at solution: [-.1e-37, .25e-37, .8e-36]Solution in 1.998s

Time Plot 0 s.
Exiting SolveHard() after 3.156r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434943497780177587855267051359491 4,
441.642959738243702102064337992014239624 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999997088568751374579374473866 00,
6.217012503006056412599121066178317629814,
485.549080905713549187419082965899889606 1]
one interval r = 32.628147792191791918045082312145951278 47 ..
36.102483889432978964496580320026101484 46
Time Approximations 0.023.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=5e-38
Equations at solution: [.6e-37, -.5e-37, -.10139e-34]Solution in 1.622s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.061r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684487360170627916939968251617507,
6.58343472179636536475397039171046760030 4,
467.78730596544288138956920727510308846 47]
one interval r = 32.41978955667570160316772276287622980111 ..
35.85152417372311991391588444967964799655
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., -.14551e-34]Solution in 0.682s

Time Plot 0 s.
Exiting SolveHard() after 1.072r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12, none, none,
401.88173904686117347385238586250919441 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684487360170627916939968251617507,
6.58343472179636536475397039171046760030 4,
467.78730596544288138956920727510308846 47]
two intervals r = 12.92327160836408127592542470962337602006 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000000
```

```
000 or r = 18.39424858060992778735741681837610701776 ..
950000000011559072362979964737952031 9/500000000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=6.5e-38
Equations at solution: [.5e-37, .65e-37, -.100e-35]Solution in 35.169s

Time Plot 0 s.
Exiting SolveHard() after 37.572r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46, none,
401.88173904686117347385238586250919441 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962827061520069799610200937096498,
4.12565179682261741623950580290266104851 0,
440.67123065705504002395341933465115164 05]
two intervals r = 14.35659705126208592041952809693208219233 ..
950000000011559072362979964737952031 9/500000000000000000000000000000000000
000 or r = 17.70352613841575029783480020169318142026 ..
950000000011559072362979964737952031 9/500000000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=4e-38
Equations at solution: [-.86e-37, -.4e-37, -.2136e-34]Solution in
```

```
2.377s

Time Plot 0 s.
Exiting SolveHard() after 4.496r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46, none,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962827061520069799610200937096498,
4.12565179682261741623950580290266104851 0,
440.67123065705504002395341933465115164 05]
one interval r = 22.39761154373725112734400788692211728703 ..
27.23722351606980698614475477874099554524
Time Approximations 0.043.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 2.301 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064399693122108350214749222423920, rm =
14.37818770567745821806300298194612804711}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., -.836e-34]Solution in 8.615s

Time Plot 0 s.
Exiting SolveHard() after 10.591r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
```

```
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887976332210027689616673094977 86,
4.0048690817926992089472009452693858765 16,
404.86224501767983400762524433709977007 97]
two intervals r = 16.080110077795681165504738040253820202 95 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000
000 or r = 16.415798127218149798153472782365697613 51 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000
000
Time Approximations 0.051.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=2e-38
Equations at solution: [.85e-37, .2e-37, -.433e-35]Solution in 2.58s

Time Plot 0 s.
Exiting SolveHard() after 4.762r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49, none, none,
358.97362824289815210055939637120336249 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887976332210027689616673094977 86,
4.0048690817926992089472009452693858765 16,
404.86224501767983400762524433709977007 97]
one interval r = 21.641943994114416477839380268385386608 93 ..
26.763306600489896911879200711044499322 01
Time Approximations 0.052.
```

```
hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, .722e-34]Solution in 1.075s

Time Plot 0 s.
Exiting SolveHard() after 3.25r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84, none,
358.97362824289815210055939637120336249 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941802814249101542930368892923928,
5.58963718297118646168020257481999480207 5,
443.83065885023100998274619345623469518 07]
one interval r = 22.46725374480371850843579226208414754260 ..
27.27388428360577473929551220120044834281
Time Approximations 0.035.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., -.70e-35]Solution in 2.073s

Time Plot 0 s.
Exiting SolveHard() after 4.082r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
```

```
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84, none,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941802814249101542930368892923928,
5.5896371829711864616802025748199948020 75,
443.83065885023100998274619345623469518 07]
one interval r = 32.15575279503692591418202806930332568173 ..
35.50872228735874748941212486898759124956
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=6e-38
Equations at solution: [-.7e-37, .6e-37, -.19625e-34]Solution in 0.479s

Time Plot 0 s.
Exiting SolveHard() after 0.836r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
```

```
398.33147104370065807466595439955795375 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136525170012614545042930373519074,
5.1877835785437993655752180574241959200 84,
408.65773863272279813635544024060078707 01]
one interval r = 21.71840114653468280620255434711414768328 ..
26.81849303520733733161399591975785232748
Time Approximations 0.059.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.59e-37
Equations at solution: [.1e-37, .159e-36, .31e-35]Solution in 2.058s

Time Plot 0 s.
Exiting SolveHard() after 4.182r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78, none, none,
361.50888347481421038660135419259620401 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136525170012614545042930373519074,
5.1877835785437993655752180574241959200 84,
408.65773863272279813635544024060078707 01]
one interval r = 31.80828598754824625185385520576827455653 ..
35.00011460046410379174409290268468037134
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
```

```
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=4e-38
Equations at solution: [.5e-37, -.4e-37, .19292e-34]Solution in 0.44s

Time Plot 0 s.
Exiting SolveHard() after 0.755r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21, none,
361.50888347481421038660135419259620401 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110546609400068708273141706906414,
6.19626255516416209290182024841208251575,
385.44474379709681819936500204092798292 37]
one interval r = 31.60836097538227833144030933982277981641 ..
34.66372795608625786081926639045184391899
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=1.4e-37
Equations at solution: [-.9e-37, .14e-36, .19428e-34]Solution in 1.543s

Time Plot 0 s.
```

```
Exiting SolveHard() after 1.829r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21, none,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110546609400068708273141706906414,
6.19626256551641620929018202484120825 1575,
385.44474379709681819936500204092798292 37]
two intervals r = 16.87563408776372098693120102280640949506 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000000
000 or r = 15.55640493828622150531377631272895996494 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000000
000
Time Approximations 0.054.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.716e-37, 0., .681e-35]Solution in 1.146s

Time Plot 0 s.
Exiting SolveHard() after 3.393r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
```

```
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874758199318737053906038954859756,
4.8838107798608208447367688762598912790 41,
376.61967856186940462932047175398392235 23]
one interval r = 21.11001304874647978770355699703316271502 ..
26.31784243481453672090890774924654823405
Time Approximations 0.034.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=4.9e-38
Equations at solution: [-.2e-37, -.49e-37, .197e-34]Solution in 0.856s

Time Plot 0.001 s.
Exiting SolveHard() after 2.521r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
```

```
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17, none,
328.46938513678188542963832203622810090 14, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874758199318737053906038954859756,
4.8838107798608208447367688762598912790 41,
376.61967856186940462932047175398392235 23]
one interval r = 31.53899497712337584429055199015174762003 ..
34.53618386092076500431601992369357616771
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.89e-36
Equations at solution: [-.299e-35, .389e-35, -.29888e-34]Solution in
0.499s

Time Plot 0.001 s.
Exiting SolveHard() after 1.816r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17, none,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017543889132898206367893297367901,
6.0258135494631918285259595075830022517 30,
351.42702948583760487632910177064187598 64]
```

```
one interval r = 31.3623020611347589113286953225783729376 7 ..
34.174466406110963271682845040972554654 33
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.31499e-34]Solution in 0.531s

Time Plot 0 s.
Exiting SolveHard() after 0.793r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17, none,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31, none, none,
292.99969138273059857185799814201344698 29, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017543889132898206367893297367901,
6.0258135494631918285259595075830022517 30,
351.42702948583760487632910177064187598 64]
two intervals r = 17.98135514470326157993806684375936565841 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000000
000 or r = 13.84608015435219064127187964814587233475 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
```

```
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.71e-37, -.2e-37, .780e-35]Solution in 2.071s

Time Plot 0 s.
Exiting SolveHard() after 4.089r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17,
302.31384314810541728913202467578768841 15,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31, none, none,
292.99969138273059857185799814201344698 29, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418777684561434912455985582169 88,
6.3779438740548782118298652954827823389 68,
423.28832784264254401832848571181427058 51]
one interval r = 31.946618175976583854557560853433558454 96 ..
35.212123086478954321556329519604891585 43
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
```

```
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.7114e-35]Solution in 0.613s

Time Plot 0 s.
Exiting SolveHard() after 0.964r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17,
302.31384314810541728913202467578768841 15,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31, none, none,
292.99969138273059857185799814201344698 29, none, none,
360.06173466876744106723761985210207067 91, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941877684561434912455985582169 88,
6.37794387405487821182986529548278233896 8,
423.28832784264254401832848571181427058 51]
two intervals r = 15.22886702463168987126145989742727559119 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000000
000 or r = 17.12965777095452205001881512738292616567 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000000
000
Time Approximations 0.063.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.32e-37, .1e-37, -.2510e-34]Solution in 2.266s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.627r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17,
302.31384314810541728913202467578768841 15,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31,
375.73285290185757327364039510495986994 64, none,
292.99969138273059857185799814201344698 29, none, none,
360.06173466876744106723761985210207067 91, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343405978651277068689573803824 79,
4.0035598154905437644646028344971121204 59,
404.47973594409197279753486179626971848 44]
two intervals r = 16.096839663932820907743740355564003864 94 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000000
000 or r = 16.399886491390505024268448086055230353 13 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000000
000
Time Approximations 0.048.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.33e-37, .1e-37, -.2132e-34]Solution in 2.477s

Time Plot 0 s.
Exiting SolveHard() after 4.619r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17,
302.31384314810541728913202467578768841 15,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31,
375.73285290185757327364039510495986994 64, none,
292.99969138273059857185799814201344698 29,
358.64341561083634964594150465369768185 13, none,
360.06173466876744106723761985210207067 91, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234340597865127706868957380382479,
4.0035598154905437644646028344971121204 59,
404.47973594409197279753486179626971848 44]
one interval r = 21.63429629987388183183756714903330336031 ..
26.75768169897861207249572592375464507159
Time Approximations 0.047.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.3e-38
Equations at solution: [0., -.23e-37, -.120e-34]Solution in 1.014s

Time Plot 0 s.
Exiting SolveHard() after 3.038r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17,
302.31384314810541728913202467578768841 15,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31,
375.73285290185757327364039510495986994 64,
328.11709294468540948187750881952991830 78,
292.99969138273059857185799814201344698 29,
358.64341561083634964594150465369768185 13, none,
360.06173466876744106723761985210207067 91, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954464942214449693279751276707054,
6.19617723037036055455759095695421398905 7,
385.42734026093438893855490556867489305 12]
one interval r = 31.60822049093662189063992677648322088889 ..
34.66347615046412231053811122680940019656
Time Approximations 0.015.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=9e-38
Equations at solution: [.6e-37, -.9e-37, -.14345e-34]Solution in 0.575s

Time Plot 0 s.
Exiting SolveHard() after 1.959r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
```

```
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17,
302.31384314810541728913202467578768841 15,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31,
375.73285290185757327364039510495986994 64,
328.11709294468540948187750881952991830 78,
292.99969138273059857185799814201344698 29,
358.64341561083634964594150465369768185 13, none,
360.06173466876744106723761985210207067 91, none, none,
324.65521223736859636287703911942616947 40, none, none, none, none]

0 --> 1  target = [26.46318954464942214449693279751276707054,
6.19617723037036055455759095695421398905 7,
385.42734026093438893855490556867489305 12]
two intervals r = 16.87629600314682278614852209607821556152 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000000
000 or r = 15.55559000677582228313352373640792631714 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000000
000
Time Approximations 0.062.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [-.538e-37, .1e-37, .1088e-34]Solution in 2.256s

Time Plot 0 s.
Exiting SolveHard() after 4.47r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
```

```
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17,
302.31384314810541728913202467578768841 15,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31,
375.73285290185757327364039510495986994 64,
328.11709294468540948187750881952991830 78,
292.99969138273059857185799814201344698 29,
358.64341561083634964594150465369768185 13, none,
360.06173466876744106723761985210207067 91,
336.59441032415105921788198844517388181 60, none,
324.65521223736859636287703911942616947 40, none, none, none, none]

0 --> 2  target = [34.49522661164927710897758601746346676682,
3.8971313159105800119178180926559482004 16,
373.78081884896217819320432894198993092 89]
two intervals r = 17.29769086244934157577440517325313701445 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000
000 or r = 14.99436407460410457929009199967238970757 ..
95000000001155907236297996473795203 19/50000000000000000000000000000000
000
Time Approximations 0.086.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [-.54e-37, .1e-37, -.879e-35]Solution in 2.31s

Time Plot 0 s.
Exiting SolveHard() after 5.187r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
```

```
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17,
302.31384314810541728913202467578768841 15,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31,
375.73285290185757327364039510495986994 64,
328.11709294468540948187750881952991830 78,
292.99969138273059857185799814201344698 29,
358.64341561083634964594150465369768185 13, none,
360.06173466876744106723761985210207067 91,
336.59441032415105921788198844517388181 60, none,
324.65521223736859636287703911942616947 40,
331.93806791730445619763038740082956813 72, none, none, none]

1 --> 2  target = [34.49522661164927710897758601746346676682,
3.89713131591058001191781809265594820041 6,
373.78081884896217819320432894198993092 89]
one interval r = 21.06068473201205176991289530945642829478 ..
26.26979834290751670032293602408746763197
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, .404e-34]Solution in 1.806s

Time Plot 0 s.
Exiting SolveHard() after 2.551r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
```

```
401.881739046861173473852385862509194412 4,
389.590015163686873086722700292974746174 9,
328.469398936052950942234091651884822808 4,
401.507571583771719725536097923131568030 1,
358.973628242898152100559396371203362494 4,
398.331471043700658074665954399557953757 8,
371.483873948110952727971944763032338492 1,
336.612158413629966393210062166505853914 2,
361.508883474814210386601354192596204019 7,
324.671449927500309160035565866802826581 7,
302.313843148105417289132024675787688411 5,
328.469385136781885429638322036228100901 4,
343.813406251369810375281466379207544813 1,
375.732852901857573273640395104959869946 4,
328.117092944685409481877508819529918307 8,
292.999691382730598571857998142013446982 9,
358.643415610836349645941504653697681851 3,
299.898662049362727002722996742501944325 5,
360.061734668767441067237619852102070679 1,
336.594410324151059217881988445173881816 0, none,
324.655212237368596362877039119426169474 0,
331.938067917304456197630387400829568137 2, none, none, none]

0 --> 2  target = [33.813624954059175118118217698118681371 11,
3.725648993533028446291775039493235058373,
325.892099729909710255075369840553321284 3]
two intervals r = 18.552270490314191594888023546336851394 96 ..
95000000001155907236297996473795203 19/500000000000000000000000000000000
000 or r = 12.491969358006116229460064024073694949 65 ..
95000000001155907236297996473795203 19/500000000000000000000000000000000
000
Time Approximations 0.037.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [.121e-36, -.5e-37, -.494e-35]Solution in 1.214s

Time Plot 0 s.
Exiting SolveHard() after 3.786r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434943497780177587855267051359491 4,
441.642959738243702102064337992014239624 9,
436.917481659196836918661980559506962811 2,
```

```
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17,
302.31384314810541728913202467578768841 15,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31,
375.73285290185757327364039510495986994 64,
328.11709294468540948187750881952991830 78,
292.99969138273059857185799814201344698 29,
358.64341561083634964594150465369768185 13,
299.89866204936272700272299674250194432 55,
360.06173466876744106723761985210207067 91,
336.59441032415105921788198844517388181 60, none,
324.65521223736859636287703911942616947 40,
331.93806791730445619763038740082956813 72, none, none,
289.54595772545003886628176511392103287 06]

1 --> 2  target = [33.81362495405917511811821769811868137111,
3.72564899353302844629177503949323505837 3,
325.89209972990971025507536984055332128 43]
one interval r = 20.37468935097156108013938488934123506609 ..
25.37892165301502962558618418581413210098
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .565e-34]Solution in 0.596s

Time Plot 0 s.
Exiting SolveHard() after 2.138r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
```

```
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17,
302.31384314810541728913202467578768841 15,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31,
375.73285290185757327364039510495986994 64,
328.11709294468540948187750881952991830 78,
292.99969138273059857185799814201344698 29,
358.64341561083634964594150465369768185 13,
299.89866204936272700272299674250194432 55,
360.06173466876744106723761985210207067 91,
336.59441032415105921788198844517388181 60,
256.10753185949633751613023004139358674 63,
324.65521223736859636287703911942616947 40,
331.93806791730445619763038740082956813 72, none, none,
289.54595772545003886628176511392103287 06]

1 --> 0  target = [17.93041369737609585223585591592392478100,
4.68650870198564050605813116860829984738 5,
353.30541095172855326230807729882537999 76]
one interval r = 20.73150479082762809465118620674423452340 ..
25.90675353524339393074481184255970676463
Time Approximations 0.033.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., .319e-34]Solution in 0.686s

Time Plot 0 s.
Exiting SolveHard() after 2.521r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17,
302.31384314810541728913202467578768841 15,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31,
375.73285290185757327364039510495986994 64,
328.11709294468540948187750881952991830 78,
292.99969138273059857185799814201344698 29,
358.64341561083634964594150465369768185 13,
299.89866204936272700272299674250194432 55,
360.06173466876744106723761985210207067 91,
336.59441032415105921788198844517388181 60,
256.10753185949633751613023004139358674 63,
324.65521223736859636287703911942616947 40,
331.93806791730445619763038740082956813 72,
304.79958325464272063980349032782577703 31, none,
289.54595772545003886628176511392103287 06]

2 --> 0  target = [17.93041369737609585223585591592392478100,
4.68650870198564050605813116860829984 7385,
353.30541095172855326230807729882537999 76]
one interval r = 31.37435486992477158050423933506068951321 ..
34.20127520023204285307307211999035117167
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [-.5e-37, .8e-37, -.4913e-35]Solution in 1.363s

Time Plot 0 s.
Exiting SolveHard() after 1.649r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494349778017758785526705135949 14,
441.64295973824370210206433799201423962 49,
436.91748165919683691866198055950696281 12,
422.98493397931459997488746139586397789 46,
361.52580256435122313206243366328071976 93,
401.88173904686117347385238586250919441 24,
389.59001516368687308672270029297474617 49,
328.46939893605295094223409165188482280 84,
401.50757158377171972553609792313156803 01,
358.97362824289815210055939637120336249 44,
398.33147104370065807466595439955795375 78,
371.48387394811095272797194476303233849 21,
336.61215841362996639321006216650585391 42,
361.50888347481421038660135419259620401 97,
324.67144992750030916003556586680282658 17,
302.31384314810541728913202467578768841 15,
328.46938513678188542963832203622810090 14,
343.81340625136981037528146637920754481 31,
375.73285290185757327364039510495986994 64,
328.11709294468540948187750881952991830 78,
292.99969138273059857185799814201344698 29,
358.64341561083634964594150465369768185 13,
299.89866204936272700272299674250194432 55,
360.06173466876744106723761985210207067 91,
336.59441032415105921788198844517388181 60,
256.10753185949633751613023004139358674 63,
324.65521223736859636287703911942616947 40,
331.93806791730445619763038740082956813 72,
304.79958325464272063980349032782577703 31,
323.46169176669768428333540192463951188 52,
289.54595772545003886628176511392103287 06]

Cascade time 128.823
counts: 28, 28

 Iteration 18

Start Generation 1
1 --> 0  target = [12.00000000020153686035796893770227857200,
6.21701250295520747873246541307430203006 9,
485.54908089891952681092962386731000416 27]
one interval r = 23.40850301651627662488781700055763451619 ..
27.67578046429541394870210209731641985610
Time Approximations 0.042.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
```

```
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.05e-37
Equations at solution: [-.3e-37, .105e-36, .2e-36]Solution in 2.006s

Time Plot 0 s.
Exiting SolveHard() after 4.201r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000020153686035796893770227857200,
6.21701250295520747873246541307430203006 9,
485.54908089891952681092962386731000416 27]
one interval r = 32.62814779201791759372860567760405450729 ..
36.10248389374639730882022459314506088 6
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=6e-38
Equations at solution: [-.9e-37, .6e-37, .2485e-35]Solution in 0.618s

Time Plot 0 s.
Exiting SolveHard() after 1.046r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684479560368616119770405425685017,
6.58343472192123945176667949267153001146 3,
```

```
467.787305957671162640979268860194676103 2]
one interval r = 32.41978955648590642835464287477369395726 ..
35.85152417364113728556418465159033940459
Time Approximations 0.02.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=9e-38
Equations at solution: [-.10e-36, .9e-37, .6655e-35]Solution in 0.648s

Time Plot 0 s.
Exiting SolveHard() after 2.152r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87, none, none,
401.88173903854850920306354872560622798 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684479560368616119770405425685017,
6.58343472192123945176667949267153001146 3,
467.787305957671162640979268860194676103 2]
```

"Imaginary part neglected: ", $1.10311211490393897687000784504334616314 8 \times 10^{-17}$

```
two intervals r = 12.92327160862169217145638904899107250930 ..
19000000000111832725362225033004650119/10000000000000000000000000000000
00000 or r = 18.39424858035466642513517827526821214164 ..
19000000000111832725362225033004650119/10000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
```

```
Accepted {r=14.1926, rm=14.139} with Delta=2.3e-38
Equations at solution: [-.2e-37, -.23e-37, .2847e-35]Solution in 36.5s

Time Plot 0 s.
Exiting SolveHard() after 38.889r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64, none,
401.88173903854850920306354872560622798 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962820273730253498311607996110835,
4.12565179688007680306491298686971676015 4,
440.67123065129819218910898295948134266 84]
```

"Imaginary part neglected: ", $1.10311211490393897687000784504334616314 8 \times 10^{-17}$

```
two intervals r = 14.35659705139203474095531356354701626110 ..
19000000000111832725362225033004650119/10000000000000000000000000000000
00000 or r = 17.70352613816051077861524014653854941761 ..
19000000000111832725362225033004650119/10000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=5e-38
Equations at solution: [.84e-37, .5e-37, .25730e-34]Solution in 2.41s

Time Plot 0 s.
Exiting SolveHard() after 4.454r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
```

```
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64, none,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848660813503084 7, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962820273730253498311607996110835,
4.1256517968800768030649129868697167601 54,
440.67123065129819218910898295948134266 84]
one interval r = 22.397611543608689223682713591949051479 57 ..
27.237223515997382569285396915125736103 55
Time Approximations 0.043.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.28 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643950181222882781478530039128 00, rm =
14.378187707992147336139651786711292086 73}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, .92e-35]Solution in 7.788s

Time Plot 0 s.
Exiting SolveHard() after 9.765r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848660813503084 7, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888785035152178837085128794375727,
4.0048690818404550703867011794158434555 58,
404.86224500921985290788525827698666617 98]
```

"Imaginary part neglected: ", $1.10311211490393897687000784504334616314 8 \times 10^{-17}$

```
two intervals r = 16.08011007799886948997734020301487688441 ..
190000000001118327253622250330046501 19/100000000000000000000000000000000
00000 or r = 16.41579812680154481781829640165908775034 ..
190000000001118327253622250330046501 19/100000000000000000000000000000000
00000
Time Approximations 0.058.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.67e-37, -.1e-37, .16147e-34]Solution in
2.499s

Time Plot 0 s.
Exiting SolveHard() after 3.518r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848660813503084 7, none, none,
358.97362823602618843686261525819796303 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888785035152178837085128794375727,
4.0048690818404550703867011794158434555 58,
404.86224500921985290788525827698666617 98]
one interval r = 21.64194399394342970108775322396096845162 ..
26.76330660036124046444235317794485845798
Time Approximations 0.05.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [.2e-37, .49e-37, -.501e-34]Solution in 2.141s

Time Plot 0 s.
Exiting SolveHard() after 4.26r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37, none,
358.97362823602618843686261525819796303 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941841886438345003416240715204049,
5.58963718288936978162126238559563800462 0,
443.83065883953249120682770150447927214 73]
one interval r = 22.46725374456526636629164932245274025582 ..
27.27388428347819876661092696669377831235
Time Approximations 0.034.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., -.28e-35]Solution in 1.011s

Time Plot 0 s.
Exiting SolveHard() after 3.089r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
```

```
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37, none,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941841886438345003416240715204049,
5.5896371828893697816212623855956380046 20,
443.83065883953249120682770150447927214 73]
one interval r = 32.155752794811787824163499439728841321 63 ..
35.508722287221444566738333353909706308 62
Time Approximations 0.021.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=9e-38
Equations at solution: [-.11e-36, .9e-37, -.21199e-34]Solution in
1.468s

Time Plot 0 s.
Exiting SolveHard() after 1.847r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136529532858279955575517326844963,
5.1877835785419029582462566610528473894 25,
408.65773862800398822018115092993456253 74]
```

```
one interval r = 21.71840114643724312630894934830012681883 ..
26.81849303513501057159814542790439957090
Time Approximations 1.143.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.85e-37
Equations at solution: [.1e-37, .185e-36, -.78e-35]Solution in 1.03s

Time Plot 0 s.
Exiting SolveHard() after 3.276r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08, none, none,
361.50888346827017647548879866842231311 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136529532858279955575517326844963,
5.18778357854190295824625666105284738 9425,
408.65773862800398822018115092993456253 74]
one interval r = 31.80828598737842832307316938776538764125 ..
35.00011460039435133957421909972060563905
Time Approximations 0.02.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
```

```
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [.3e-37, -.3e-37, -.5281e-35]Solution in 1.428s

Time Plot 0 s.
Exiting SolveHard() after 1.773r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00, none,
361.50888346827017647548879866842231311 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110535218307847783066304008612163,
6.19626256564319150288365250507577332308 3,
385.44474379031087935430636163888249764 86]
one interval r = 31.60836097519216186856467076166704759173 ..
34.66372795597457020171615522289558892376
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, .17556e-34]Solution in 0.622s

Time Plot 0 s.
Exiting SolveHard() after 0.904r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848660813503084 7,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00, none,
361.50888346827017647548879866842231311 9,
324.67144992014255305899559147585031142 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110535218307847783066304008612163,
6.19626256564319150288365250507577332308 3,
385.44474379031087935430636163888249764 86]

"Imaginary part neglected: ", 1.10311211490393897687000784504334616314 8 × 10^-17
two intervals r = 16.875634087859137222505818419107590916 04 ..
19000000000111832725362225033004650119/100000000000000000000000000000000
00000 or r = 15.556404937912961374905058235238131941 22 ..
19000000000111832725362225033004650119/100000000000000000000000000000000
00000
Time Approximations 0.063.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.16988e-34]Solution in 2.21s

Time Plot 0 s.
Exiting SolveHard() after 4.682r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848660813503084 7,
```

```
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666647,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946939144025047777 99,
361.50888346827017647548879866842231311 9,
324.67144992014255305899559147585031142 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874767278553123865519872093669593,
4.8838107798387869263943722634386743763 52,
376.61967855474996372792937277128344538 59]
one interval r = 21.11001304862017733929018478345821875070 ..
26.31784243469075353112463167536931127935
Time Approximations 0.038.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, .154e-34]Solution in 1.81s

Time Plot 0 s.
Exiting SolveHard() after 2.499r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666647,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946939144025047777 99,
361.50888346827017647548879866842231311 9,
324.67144992014255305899559147585031142 65, none,
328.46938512773933129390421139736994909 94, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
2 --> 0  target = [17.19898874767278553123865519872093669593,
4.883810779838786926394372263438674376352,
376.61967855474996372792937277128344538 59]
one interval r = 31.53899497693058505141950469801474933565 ..
34.53618386079982254859020892548585553868
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=4.92e-36
Equations at solution: [.378e-35, -.492e-35, .28471e-34]Solution in
0.524s

Time Plot 0 s.
Exiting SolveHard() after 0.84r=34.0898 in [32.52213872 .. 34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946391440250477779 9,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65, none,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017526237134706106662238806380082,
6.025813549575414748784958396003673369774,
351.42702947651417895222823603216753093 86]
one interval r = 31.36230206092928669541875830311435376122 ..
34.17446640594635345254398942303891589799
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
```

```
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=9e-38
Equations at solution: [.4e-37, -.9e-37, .1067e-35]Solution in 1.657s

Time Plot 0 s.
Exiting SolveHard() after 1.907r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946939144025047777 99,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65, none,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14, none, none,
292.99969137302325766038718316537896984 01, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017526237134706106662238806380082,
6.02581354957541474878495839600367336977 4,
351.42702947651417895222823603216753093 86]

"Imaginary part neglected: ", 1.10311211490393897687000784504334616314 8 × 10^-17
two intervals r = 17.98135514480205470209025725322136625004 ..
19000000000111832725362225033004650119/10000000000000000000000000000000
00000 or r = 13.84608015382271824933094412112534712794 ..
19000000000111832725362225033004650119/10000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [.17e-37, -.1e-37, -.1608e-35]Solution in 2.163s

Time Plot 0 s.
Exiting SolveHard() after 3.263r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848660813503084 7,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946939144025047777 99,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14, none, none,
292.99969137302325766038718316537896984 01, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418601813412844841275971069295 87,
6.3779438741539121907983434833391069218 77,
423.28832782967016594609279228147604113 66]
one interval r = 31.946618175729129348307999996380827702 17 ..
35.212123086297386931876405253682885503 18
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=1.0e-37
Equations at solution: [-.9e-37, .10e-36, -.9801e-35]Solution in 1.605s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.964r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946391440250477779 9,
361.50888346827017647548879866842231311 9,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14, none, none,
292.99969137302325766038718316537896984 01, none, none,
360.06173465559956215627580256677582945 42, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941860181341284484127597106929587,
6.37794387415391219079834348333910692187 7,
423.28832782967016594609279228147604113 66]
```

"Imaginary part neglected: ", $1.10311211490393897687000784504334616314 8 \times 10^{-17}$

```
two intervals r = 15.22886702509092287861057520047890162167 ..
19000000000111832725362225033004650119/10000000000000000000000000000000
00000 or r = 17.12965777041549004961592702266981246629 ..
19000000000111832725362225033004650119/10000000000000000000000000000000
00000
Time Approximations 0.065.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [-.62e-37, -.2e-37, .28561e-34]Solution in
2.336s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.906r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946939144025047777 99,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14,
375.73285288610164936910331768929045730 94, none,
292.99969137302325766038718316537896984 01, none, none,
360.06173465559956215627580256677582945 42, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343273177121704155185941814907 23,
4.0035598155367277620116487911453060934 78,
404.47973593517532952438471701856772335 15]
```

"Imaginary part neglected: ", $1.10311211490393897687000784504334616314 8 \times 10^{-17}$

```
two intervals r = 16.09683966415512737164979648673557551484 ..
19000000000111832725362225033004650119/10000000000000000000000000000000
00000 or r = 16.39988649095411399904378171896295108077 ..
19000000000111832725362225033004650119/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [-.17e-37, 0., .45347e-34]Solution in 2.581s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.598r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848660813503084 7,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946939144025047777 99,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14,
375.73285288610164936910331768929045730 94, none,
292.99969137302325766038718316537896984 01,
358.64341560356989025902217935353724053 51, none,
360.06173465559956215627580256677582945 42, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234327317712170415518594181490723,
4.0035598155367277620116487911453060934 78,
404.47973593517532952438471701856772335 15]
one interval r = 21.634296299694011723122928546144448112 65 ..
26.757681698842983216134397240068401859 42
Time Approximations 0.053.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, .727e-34]Solution in 2.097s

Time Plot 0 s.
Exiting SolveHard() after 4.224r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946939144025047777 99,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14,
375.73285288610164936910331768929045730 94,
328.11709293522196086567584555251126401 16,
292.99969137302325766038718316537896984 01,
358.64341560356989025902217935353724053 51, none,
360.06173465559956215627580256677582945 42, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954453618008162614995098052350798,
6.19617723049734043365552970319657294 3629,
385.42734025419031931930848579513920314 17]
one interval r = 31.60822049074684291505715840948028923138 ..
34.66347615035303162077210296145858908276
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .21748e-34]Solution in 0.584s

Time Plot 0 s.
Exiting SolveHard() after 0.891r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848660813503084 7,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946939144025047777 99,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14,
375.73285288610164936910331768929045730 94,
328.11709293522196086567584555251126401 16,
292.99969137302325766038718316537896984 01,
358.64341560356989025902217935353724053 51, none,
360.06173465559956215627580256677582945 42, none, none,
324.65521223004990942613083605460337931 88, none, none, none, none]

0 --> 1  target = [26.46318954453618008162614995098052350798,
6.19617723049734043365552970319657294362 9,
385.42734025419031931930848579513920314 17]
```

"Imaginary part neglected: ", $1.10311211490393897687000784504334616314 8 \times 10^{-17}$

```
two intervals r = 16.87629600324061478046228465125933068005 ..
19000000000111832725362225033004650119/10000000000000000000000000000000000
00000 or r = 15.55559000640450258684830308361778902983 ..
19000000000111832725362225033004650119/10000000000000000000000000000000000
00000
Time Approximations 0.07.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.538e-37, 0., -.31500e-34]Solution in 2.223s

Time Plot 0 s.
Exiting SolveHard() after 4.52r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848660813503084 7,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946391440250477779 9,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14,
375.73285288610164936910331768929045730 94,
328.11709293522196086567584555251126401 16,
292.99969137302325766038718316537896984 01,
358.64341560356989025902217935353724053 51, none,
360.06173465559956215627580256677582945 42,
336.59441031508560636875630787656259707 41, none,
324.65521223004990942613083605460337931 88, none, none, none, none]

0 --> 2  target = [34.49522661157789700476817963872315169999,
3.8971313159746962658608091970636232092 56,
373.78081884537748531276916317761342482 71]
```

"Imaginary part neglected: ", $1.10311211490393897687000784504334616314 8 \times 10^{-17}$

```
two intervals r = 17.29769086241322507881172781579929433573 ..
19000000000111832725362225033004650119/10000000000000000000000000000000
00000 or r = 14.99436407437734314678635514451909174044 ..
19000000000111832725362225033004650119/10000000000000000000000000000000
00000
Time Approximations 0.087.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., -.2222e-35]Solution in 2.264s

Time Plot 0 s.
Exiting SolveHard() after 5.055r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946391440250477779 9,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14,
375.73285288610164936910331768929045730 94,
328.11709293522196086567584555251126401 16,
292.99969137302325766038718316537896984 01,
358.64341560356989025902217935353724053 51, none,
360.06173465559956215627580256677582945 42,
336.59441031508560636875630787656259707 41, none,
324.65521223004990942613083605460337931 88,
331.93806791468096395711741479590084235 77, none, none, none]

1 --> 2  target = [34.49522661157789700476817963872315169999,
3.8971313159746962658608091970636232092 56,
373.78081884537748531276916317761342482 71]
one interval r = 21.06068473194876350743108248585162264738 ..
26.26979834284250452959538035323270978581
Time Approximations 1.113.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [-.2e-37, -.5e-37, .320e-34]Solution in 0.716s

Time Plot 0 s.
Exiting SolveHard() after 2.537r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946939144025047777 99,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14,
375.73285288610164936910331768929045730 94,
328.11709293522196086567584555251126401 16,
292.99969137302325766038718316537896984 01,
358.64341560356989025902217935353724053 51,
299.89866204482960414841573629471136480 04,
360.06173465559956215627580256677582945 42,
336.59441031508560636875630787656259707 41, none,
324.65521223004990942613083605460337931 88,
331.93806791468096395711741479590084235 77, none, none, none]

0 --> 2  target = [33.81362495390912794326169929136519058562,
3.72564899358214772043172497483729934385 1,
325.89209972241690724612577990554866072 62]
```

"Imaginary part neglected: ", $1.10311211490393897687000784504334616314 8 \times 10^{-17}$

```
two intervals r = 18.552270490307363777834600239847357718 43 ..
19000000000111832725362225033004650119/10000000000000000000000000000000000
00000 or r = 12.491969357593201434073574500573191609 46 ..
19000000000111832725362225033004650119/10000000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=2e-38
Equations at solution: [-.53e-37, .2e-37, .20396e-34]Solution in 2.292s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.799r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946391440250477779 9,
361.50888346827017647548879866842231311 9,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14,
375.73285288610164936910331768929045730 94,
328.11709293522196086567584555251126401 16,
292.99969137302325766038718316537896984 01,
358.64341560356989025902217935353724053 51,
299.89866204482960414841573629471136480 04,
360.06173465559956215627580256677582945 42,
336.59441031508560636875630787656259707 41, none,
324.65521223004990942613083605460337931 88,
331.93806791468096395711741479590084235 77, none, none,
289.54595771935484699357328553227650911 42]

1 --> 2  target = [33.81362495390912794326169929136519058562,
3.7256489935821477204317249748372993438 51,
325.89209972241690724612577990554866072 62]
one interval r = 20.37468935088730138431128481498724411790 ..
25.37892165286142302757415795970165152755
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, .804e-34]Solution in 1.604s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.157r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946391440250477779 9,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14,
375.73285288610164936910331768929045730 94,
328.11709293522196086567584555251126401 16,
292.99969137302325766038718316537896984 01,
358.64341560356989025902217935353724053 51,
299.89866204482960414841573629471136480 04,
360.06173465559956215627580256677582945 42,
336.59441031508560636875630787656259707 41,
256.10753185143005978264274759972709671 12,
324.65521223004990942613083605460337931 88,
331.93806791468096395711741479590084235 77, none, none,
289.54595771935484699357328553227650911 42]

1 --> 0  target = [17.93041369748341894872972169017174888076,
4.68650870194626467179708557065613766881 3,
353.30541094228706921580339984962397559 43]
one interval r = 20.73150479068592612113104051067519887746 ..
25.90675353506580214478466334614021855378
Time Approximations 0.033.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.9e-38
```

```
Equations at solution: [-.2e-37, -.49e-37, .109e-34]Solution in 0.703s

Time Plot 0 s.
Exiting SolveHard() after 1.412r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946939144025047777 99,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14,
375.73285288610164936910331768929045730 94,
328.11709293522196086567584555251126401 16,
292.99969137302325766038718316537896984 01,
358.64341560356989025902217935353724053 51,
299.89866204482960414841573629471136480 04,
360.06173465559956215627580256677582945 42,
336.59441031508560636875630787656259707 41,
256.10753185143005978264274759972709671 12,
324.65521223004990942613083605460337931 88,
331.93806791468096395711741479590084235 77,
304.79958324329952794436476960910525704 17, none,
289.54595771935484699357328553227650911 42]

2 --> 0  target = [17.93041369748341894872972169017174888076,
4.68650870194626467179708557065613766881 3,
353.30541094228706921580339984962397559 43]
one interval r = 31.37435486971815161786290780718345631883 ..
34.20127520006659953854062283714992307330
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .45968e-34]Solution in 0.369s

Time Plot 0 s.
Exiting SolveHard() after 1.652r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493703281475510275693482248519 83,
441.64295973068879710148365399279614943 17,
436.91748165352613356300895294761871041 87,
422.98493396910593368578782355472410035 64,
361.52580255776648264775590603797543302 72,
401.88173903854850920306354872560622798 61,
389.59001515911918875702848606081350308 47,
328.46939892700963485925547835832905274 37,
401.50757157501239256945537358176555666 47,
358.97362823602618843686261525819796303 47,
398.33147103109130355924140625103222138 08,
371.48387394456889540993272274127241837 00,
336.61215840452172527946939144025047777 99,
361.50888346827017647548879866842231311 19,
324.67144992014255305899559147585031142 65,
302.31384313665031799277220470211833039 84,
328.46938512773933129390421139736994909 94,
343.81340624583201688866481902094085894 14,
375.73285288610164936910331768929045730 94,
328.11709293522196086567584555251126401 16,
292.99969137302325766038718316537896984 01,
358.64341560356989025902217935353724053 51,
299.89866204482960414841573629471136480 04,
360.06173465559956215627580256677582945 42,
336.59441031508560636875630787656259707 41,
256.10753185143005978264274759972709671 12,
324.65521223004990942613083605460337931 88,
331.93806791468096395711741479590084235 77,
304.79958324329952794436476960910525704 17,
323.46169175915223336560272925286342670 21,
289.54595771935484699357328553227650911 42]

Cascade time 130.148
counts: 28, 28

 Iteration 19

Start Generation 1
1 --> 0  target = [11.99999999971534535415352251318687332500,
6.21701250311436662717963473736573806 3667,
485.54908090608185650374939582894505146 78]
```

"Imaginary part neglected: ", $1.8899423791419816359681003477207760107 12 \times 10^{-17}$

```
one interval r = 23.40850301672876964542820125779281466694 ..
27.67578046424074200369398459916583510243
Time Approximations 0.044.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.7e-38
Equations at solution: [-.1e-37, .27e-37, .7e-36]Solution in 2.025s

Time Plot 0 s.
Exiting SolveHard() after 4.248r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999715345354153522513186873325 00,
6.2170125031143666271796347373657380636 67,
485.54908090608185650374939582894505146 78]
one interval r = 32.62814779226778145232582924711371193945 ..
36.10248388954345242733715814605909636460
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [.6e-37, -.4e-37, .79e-35]Solution in 0.63s

Time Plot 0 s.
Exiting SolveHard() after 1.065r=35.4632 in [33.94922194 ..
36.10248389]
```

```
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844774008960814211290513351724 61,
6.583434721726077880629418432491352476822,
467.787305966917847855402058823528373909 8]
one interval r = 32.419789556762388016311098606219253661 60 ..
35.851524173846558856269905543075862676 10
Time Approximations 0.021.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.215e-34]Solution in 0.661s

Time Plot 0 s.
Exiting SolveHard() after 2.125r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90, none, none,
401.88173904985236069349019639818770399 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844774008960814211290513351724 61,
6.583434721726077880629418432491352476822,
467.787305966917847855402058823528373909 8]
two intervals r = 12.923271608017889684856031397136772448 80 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000 or r = 18.394248580586582247023540442334148370 44 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000
```

```
Time Approximations 0.046.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=4.3e-38
Equations at solution: [-.4e-37, -.43e-37, -.776e-35]Solution in
34.266s

Time Plot 0 s.
Exiting SolveHard() after 36.789r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49, none,
401.88173904985236069349019639818770399 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962836822068311474748874627984542,
4.12565179688205020152919254866277379290 4,
440.67123065701015927681108848964467068 30]
two intervals r = 14.35659705100988156702527488056974670447 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000 or r = 17.70352613837193723569727058628826547355 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.15e-37, -.1e-37, -.210e-35]Solution in 2.473s

Time Plot 0 s.
```

```
Exiting SolveHard() after 3.556r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49, none,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962836822068311474748874627984542,
4.12565179688205020152919254866277379290 4,
440.67123065701015927681108848964467068 30]
```

"Imaginary part neglected: ", $1.8899423791419816359681003477207760107 12 \times 10^{-17}$

```
one interval r = 22.39761154383850252424579252088372304551 ..
27.23722351596205108927003349408973417465
Time Approximations 0.042.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 2.396 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064390734512665078116557481028465, rm =
14.37818770539616220841606608971013053515}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.58e-37
Equations at solution: [-.2e-37, -.158e-36, .86e-35]Solution in 8.697s

Time Plot 0 s.
Exiting SolveHard() after 10.764r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
```

```
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888113458136334728439911214922 71,
4.0048690818631871541441767788408170950 68,
404.86224502073073613410508792967387865 69]
two intervals r = 16.080110077442606097654178415840305370 73 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000 or r = 16.415798127311340531742088333722593346 25 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.86e-37, -.1e-37, -.841e-35]Solution in 2.592s

Time Plot 0 s.
Exiting SolveHard() after 4.863r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48, none, none,
358.97362824562344397814327124789779887 67, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888113458136334728439911214922 71,
4.0048690818631871541441767788408170950 68,
404.86224502073073613410508792967387865 69]
```

"Imaginary part neglected: ", $1.8899423791419816359681003477207760107 12 \times 10^{-17}$

```
one interval r = 21.641943994323076079717978581271013504 22 ..
26.763306600440657480718385270666071011 40
Time Approximations 0.058.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=5.2e-38
Equations at solution: [-.1e-37, -.52e-37, -.428e-34]Solution in 2.204s

Time Plot 0 s.
Exiting SolveHard() after 3.257r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95, none,
358.97362824562344397814327124789779887 67, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941753338356110014643392339090624,
5.58963718310853085822658693787527369552 4,
443.83065885479410838508361516004181419 60]
```

"Imaginary part neglected: ", $1.8899423791419816359681003477207760107 12 \times 10^{-17}$

```
one interval r = 22.467253745002793409313905861154954997 13 ..
27.273884283549988625716211663512363398 69
Time Approximations 0.04.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
```

```
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.34e-37
Equations at solution: [.1e-37, -.134e-36, .149e-34]Solution in 0.982s

Time Plot 0 s.
Exiting SolveHard() after 3.165r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95, none,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941753338356110014643392339090624,
5.589637183108530858226586937875273695524,
443.83065885479410838508361516004181419 60]
one interval r = 32.15575279515283493736611902121002567357 ..
35.50872228752314896450713818155712292112
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., .234e-34]Solution in 0.459s

Time Plot 0 s.
Exiting SolveHard() after 1.829r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
```

```
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136503914129435657074447610557186,
5.18778357860883411711793712716763754541 3,
408.65773863247631422393498015920730476 47]
```

"Imaginary part neglected: ", $1.8899423791419816359681003477207760107 12 \times 10^{-17}$

```
one interval r = 21.71840114667254524483797698313098412957 ..
26.81849303510806187701162496300440332901
Time Approximations 0.056.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.90e-37
Equations at solution: [.3e-37, .290e-36, .13e-35]Solution in 0.983s

Time Plot 0 s.
Exiting SolveHard() after 3.254r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55, none, none,
361.50888347679640073792556995887356865 57, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none]

2 --> 0  target = [15.91193136503914129435657074447610557186,
5.1877835786088341171179371271676375454 13,
408.65773863247631422393498015920730476 47]
one interval r = 31.80828598761077249086549142773616344823 ..
35.00011460055397319696253113722591817152
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.223e-34]Solution in 1.494s

Time Plot 0 s.
Exiting SolveHard() after 1.828r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36, none,
361.50888347679640073792556995887356865 57, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110541596061585289267408550547187,
6.19626256544594542588564496472065151052 0,
385.44474379921961529753513727803921017 98]
one interval r = 31.60836097546258849634018588224811920967 ..
34.66372795620705577436667672596901364739
Time Approximations 0.016.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [-.3e-37, .6e-37, .21e-35]Solution in 0.58s

Time Plot 0 s.
Exiting SolveHard() after 0.871r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36, none,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110541596061585289267408550547187,
6.19626256544594542588564496472065151052 0,
385.44474379921961529753513727803921017 98]
two intervals r = 16.875634087485110850730317185703562220 24 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000 or r = 15.556404938353536966141385559458004019 77 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000
Time Approximations 0.07.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.538e-37, 0., .2153e-34]Solution in 2.217s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.461r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874730154284166897977836523789391,
4.88381077994502687303978065919756859089 7,
376.61967856449381769926006432432109906 36]
```

"Imaginary part neglected: ", $1.8899423791419816359681003477207760107 12 \times 10^{-17}$

```
one interval r = 21.11001304897428141180330376965702318181 ..
26.31784243477914015248531375122429573004
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., .106e-34]Solution in 1.938s

Time Plot 0 s.
Exiting SolveHard() after 2.642r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94, none,
328.46938514173500503785005795459508104 01, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874730154284166897977836523789391,
4.8838107799450268730397806591975685908 97,
376.61967856449381769926006432432109906 36]
one interval r = 31.53899497720606312398183478817976311104 ..
34.53618386104748276648827527542868197405
Time Approximations 0.018.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.30e-36
Equations at solution: [-.100e-35, .130e-35, -.40e-35]Solution in
0.521s

Time Plot 0 s.
Exiting SolveHard() after 0.815r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
```

```
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94, none,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017546984710410419768003865518545,
6.0258135494070128102279900777072618569 17,
351.42702949093234171535722587966378731 24]
one interval r = 31.362302061228428265084062450621428943 61 ..
34.174466406269006648050944282591425493 63
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .99e-35]Solution in 0.536s

Time Plot 0 s.
Exiting SolveHard() after 0.784r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94, none,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14, none, none,
292.99969138929149579101753218618631210 72, none, none, none, none,
```

```
none, none, none, none, none, none]

0 --> 1  target = [25.872050175469847104104197680038655185 45,
6.025813549407012810227990077707261856917,
351.42702949093234171535722587966378731 24]
two intervals r = 17.981355144406247747470039011203839327 88 ..
190000000001403860935761575259485587 93/100000000000000000000000000000000
00000 or r = 13.846080154585306004968981664550624312 24 ..
190000000001403860935761575259485587 93/100000000000000000000000000000000
00000
Time Approximations 0.047.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=7e-38
Equations at solution: [-.231e-36, .7e-37, -.1277e-34]Solution in 2.41s

Time Plot 0 s.
Exiting SolveHard() after 4.474r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14, none, none,
292.99969138929149579101753218618631210 72, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418768633619605191794549599871 32,
6.377943874008721300463322398923900037718,
423.28832784964978782456873836441104799 18]
```

```
one interval r = 31.946618176110913846183065669379206511 42 ..
35.21212308667601074624283530255125732205
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.233e-34]Solution in 0.624s

Time Plot 0 s.
Exiting SolveHard() after 0.99r=34.3272 in [33.10127385 .. 35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14, none, none,
292.99969138929149579101753218618631210 72, none, none,
360.06173467702869027205037787970549868 72, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941876863361960519179454959987132,
6.37794387400872130046332239892390003771 8,
423.28832784964978782456873836441104799 18]
two intervals r = 15.22886702405339103484959267433667967981 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000 or r = 17.12965777116859597747997440301093188000 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000
Time Approximations 0.061.
```

```
hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [.17e-37, 0., -.1903e-34]Solution in 2.292s

Time Plot 0 s.
Exiting SolveHard() after 4.619r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14,
375.73285291224825128689543757342165999 27, none,
292.99969138929149579101753218618631210 72, none, none,
360.06173467702869027205037787970549868 72, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343552295711915319875504290850 59,
4.0035598155632230362760063666037952580 00,
404.47973594778059901565618163372922134 25]
two intervals r = 16.096839663552630965074867853886090383 69 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000 or r = 16.399886491510654913221285526192410045 03 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000
Time Approximations 0.054.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.424e-35]Solution in 2.782s

Time Plot 0 s.
Exiting SolveHard() after 5.143r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14,
375.73285291224825128689543757342165999 27, none,
292.99969138929149579101753218618631210 72,
358.64341561411370171786435801159456360 52, none,
360.06173467702869027205037787970549868 72, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234355229571191531987550429085059,
4.0035598155632230362760006366603795258000,
404.47973594778059901565618163372922134 25]

"Imaginary part neglected: ", 1.8899423791419816359681003477207760107 12 × 10^-17
one interval r = 21.63429630009567503759059469760290007486 ..
26.75768169893902420832250815456481347200
Time Approximations 0.062.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
```

```
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=9.8e-38
Equations at solution: [-.2e-37, -.98e-37, -.5e-36]Solution in 2.317s

Time Plot 0 s.
Exiting SolveHard() after 3.401r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14,
375.73285291224825128689543757342165999 27,
328.11709295022822027191606826713365735 60,
292.99969138929149579101753218618631210 72,
358.64341561411370171786435801159456360 52, none,
360.06173467702869027205037787970549868 72, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954459791118794973000188239830690,
6.19617723029946805150433264850289000 1424,
385.42734026297129196992452064125838638 23]
one interval r = 31.60822049101623585552882393999230803676 ..
34.66347615058367486129957383088733784681
Time Approximations 0.019.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .135e-34]Solution in 1.856s

Time Plot 0 s.
Exiting SolveHard() after 2.192r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14,
375.73285291224825128689543757342165999 27,
328.11709295022822027191606826713365735 60,
292.99969138929149579101753218618631210 72,
358.64341561411370171786435801159456360 52, none,
360.06173467702869027205037787970549868 72, none, none,
324.65521224103482392663353082715269082 56, none, none, none, none]

0 --> 1  target = [26.46318954459791118794973000188239830690,
6.19617723029946805150433264850289000 1424,
385.42734026297129196992452064125838638 23]
two intervals r = 16.87629600287151030572036447303342895277 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000 or r = 15.55559000683912478451664827229715877608 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000
Time Approximations 0.063.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., .770e-35]Solution in 2.366s

Time Plot 0 s.
Exiting SolveHard() after 3.581r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14,
375.73285291224825128689543757342165999 27,
328.11709295022822027191606826713365735 60,
292.99969138929149579101753218618631210 72,
358.64341561411370171786435801159456360 52, none,
360.06173467702869027205037787970549868 72,
336.59441032915130042924620772429188917 17, none,
324.65521224103482392663353082715269082 56, none, none, none, none]

0 --> 2  target = [34.49522661173211052790521147842309927127,
3.8971313159696547212443070687153173977 43,
373.78081884857345387563629037502504894 21]
two intervals r = 17.29769086227784192810389054254759310167 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000 or r = 14.99436407455241758334098691871486608720 ..
19000000000140386093576157525948558793/10000000000000000000000000000000
00000
Time Approximations 0.086.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.18e-37, 0., .3774e-34]Solution in 2.345s

Time Plot 0 s.
Exiting SolveHard() after 5.123r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14,
375.73285291224825128689543757342165999 27,
328.11709295022822027191606826713365735 60,
292.99969138929149579101753218618631210 72,
358.64341561411370171786435801159456360 52, none,
360.06173467702869027205037787970549868 72,
336.59441032915130042924620772429188917 17, none,
324.65521224103482392663353082715269082 56,
331.93806791712600114186588497446144688 84, none, none, none]

1 --> 2  target = [34.495226611732110527905211478423099271 27,
3.8971313159696547212443070687153173977 43,
373.78081884857345387563629037502504894 21]
```

"Imaginary part neglected: ", $1.8899423791419816359681003477207760107 12 \times 10^{-17}$

```
one interval r = 21.060684732190439945213118735071682612 23 ..
26.269798342823016687606895480873509496 04
Time Approximations 0.038.

hint used Hint := [25.300466340350721366923008834208363590 84, 3, -1, 1,
16.974729527641787545085746138369074748 18, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=4e-38
Equations at solution: [-.2e-37, -.4e-37, .13e-35]Solution in 1.823s

Time Plot 0 s.
Exiting SolveHard() after 2.566r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14,
375.73285291224825128689543757342165999 27,
328.11709295022822027191606826713365735 60,
292.99969138929149579101753218618631210 72,
358.64341561411370171786435801159456360 52,
299.89866205117549951484654228331876839 07,
360.06173467702869027205037787970549868 72,
336.59441032915130042924620772429188917 17, none,
324.65521224103482392663353082715269082 56,
331.93806791712600114186588497446144688 84, none, none, none]

0 --> 2  target = [33.81362495419474020645393711596344007851,
3.72564899360804113665387803328324428 4238,
325.89209973374043136955447721325588656 32]
two intervals r = 18.552270490111269825858854820194989793 30 ..
190000000001403860935761575259485587 93/10000000000000000000000000000000
00000 or r = 12.491969358161672863191087292540252541 86 ..
190000000001403860935761575259485587 93/10000000000000000000000000000000
00000
Time Approximations 0.038.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
```

```
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [.139e-36, -.5e-37, .1822e-34]Solution in 1.2s

Time Plot 0 s.
Exiting SolveHard() after 3.813r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14,
375.73285291224825128689543757342165999 27,
328.11709295022822027191606826713365735 60,
292.99969138929149579101753218618631210 72,
358.64341561411370171786435801159456360 52,
299.89866205117549951484654228331876839 07,
360.06173467702869027205037787970549868 72,
336.59441032915130042924620772429188917 17, none,
324.65521224103482392663353082715269082 56,
331.93806791712600114186588497446144688 84, none, none,
289.54595772911432181611958373994789104 18]

1 --> 2  target = [33.81362495419474020645393711596344007851,
3.72564899360804113665387803328324428 4238,
325.89209973374043136955447721325588656 32]
```

"Imaginary part neglected: ", $1.8899423791419816359681003477207760107 12 \times 10^{-17}$

```
one interval r = 20.37468935124646768764264620753151901675 ..
25.37892165304764612286966029471808739656
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
```

```
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=6e-38
Equations at solution: [-.4e-37, -.6e-37, .243e-34]Solution in 0.58s

Time Plot 0 s.
Exiting SolveHard() after 2.171r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14,
375.73285291224825128689543757342165999 27,
328.11709295022822027191606826713365735 60,
292.99969138929149579101753218618631210 72,
358.64341561411370171786435801159456360 52,
299.89866205117549951484654228331876839 07,
360.06173467702869027205037787970549868 72,
336.59441032915130042924620772429188917 17,
256.10753186517853455526500242657285364 50,
324.65521224103482392663353082715269082 56,
331.93806791712600114186588497446144688 84, none, none,
289.54595772911432181611958373994789104 18]

1 --> 0  target = [17.93041369707457832025543401509399766267,
4.68650870208464124395680840725869959705 4,
353.30541095679490541137251836457524412 75]

"Imaginary part neglected: ", 1.8899423791419816359681003477207760107 12 × 10^-17
```

```
one interval r = 20.7315047911102726468061225687073716502 6 ..
25.906753535271912104710948692867639212 79
Time Approximations 0.032.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.8e-38
Equations at solution: [.3e-37, .48e-37, -.99e-35]Solution in 0.656s

Time Plot 0 s.
Exiting SolveHard() after 2.572r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14,
375.73285291224825128689543757342165999 27,
328.11709295022822027191606826713365735 60,
292.99969138929149579101753218618631210 72,
358.64341561411370171786435801159456360 52,
299.89866205117549951484654228331876839 07,
360.06173467702869027205037787970549868 72,
336.59441032915130042924620772429188917 17,
256.10753186517853455526500242657285364 50,
324.65521224103482392663353082715269082 56,
331.93806791712600114186588497446144688 84,
304.79958326200209104359814911204670080 21, none,
289.54595772911432181611958373994789104 18]
```

```
2 --> 0  target = [17.93041369707457832025543401509399766267,
4.68650870208464124395680840725869959705 4,
353.30541095679490541137251836457524412 75]
one interval r = 31.37435487001885192818638253579166376231 ..
34.20127520039001780612035397691377059411
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [.1e-37, -.3e-37, .153e-34]Solution in 0.351s

Time Plot 0 s.
Exiting SolveHard() after 0.635r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494393334396432696407853150228 18,
441.64295973966767468878139199985524153 79,
436.91748165916164192259353479740796807 90,
422.98493398378295341088431446577263172 49,
361.52580256641691477712036532150491467 62,
401.88173904985236069349019639818770399 61,
389.59001516364177063374187276874253376 48,
328.46939894100819633064018749321503477 95,
401.50757158738668832127009887484328380 52,
358.97362824562344397814327124789779887 67,
398.33147105051417447240146699800904408 55,
371.48387394773605872923467265977555393 36,
336.61215841871790015658311435149251207 38,
361.50888347679640073792556995887356865 57,
324.67144993124664863211015852190738984 94,
302.31384315595836041463402940543026128 08,
328.46938514173500503785005795459508104 01,
343.81340625357269128182520611106617298 14,
375.73285291224825128689543757342165999 27,
328.11709295022822027191606826713365735 60,
292.99969138929149579101753218618631210 72,
358.64341561411370171786435801159456360 52,
299.89866205117549951484654228331876839 07,
360.06173467702869027205037787970549868 72,
336.59441032915130042924620772429188917 17,
256.10753186517853455526500242657285364 50,
324.65521224103482392663353082715269082 56,
331.93806791712600114186588497446144688 84,
```

```
304.79958326200209104359814911204670080 21,
323.46169177111634464890404062891168772 63,
289.54595772911432181611958373994789104 18]

Cascade time 127.662
counts: 28, 28

 Iteration 20

Start Generation 1
1 --> 0  target = [12.00000000001935157533490479842395160100,
6.21701250291917074145193389190414397301 1,
485.54908090267280058066823656823159487 96]
one interval r = 23.40850301662597525939460722867002567439 ..
27.67578046434880662728292392310882855911
Time Approximations 0.042.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.7e-38
Equations at solution: [.1e-37, -.27e-37, .457e-36]Solution in 2.071s

Time Plot 0 s.
Exiting SolveHard() after 4.259r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000001935157533490479842395160100,
6.21701250291917074145193389190414397301 1,
485.54908090267280058066823656823159487 96]
one interval r = 32.62814779214875520391797879481526064437 ..
36.10248388950241175409901662596157940745
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
```

```
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.123e-34]Solution in 0.626s

Time Plot 0 s.
Exiting SolveHard() after 1.058r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684487392802594375336717729164993,
6.58343472154521174871793131164854251 3747,
467.78730596358135136500002670522790045 68]
one interval r = 32.41978955664035917969350343091572538637 ..
35.85152417380048087607775273774986153865
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, -.83e-35]Solution in 0.659s

Time Plot 0 s.
Exiting SolveHard() after 2.123r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58, none, none,
401.88173904784321516867009534396967241 92, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684487392802594375336717729164993,
6.58343472154521174871793131164854251374 7,
467.78730596358135136500002670522790045 68]
two intervals r = 12.92327160829308351426085394428211868501 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000 or r = 18.39424858039919349373626438133419945489 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000
Time Approximations 0.043.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.5e-38
Equations at solution: [.4e-37, .55e-37, .791e-35]Solution in 34.864s

Time Plot 0 s.
Exiting SolveHard() after 37.463r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23, none,
401.88173904784321516867009534396967241 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962832728679967676921183208853354,
4.12565179680769348471407960289567907854 6,
440.67123065467546363448993699721518601 15]
two intervals r = 14.35659705117628296334533248196819405521 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000 or r = 17.70352613819360038105524923719643126956 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
```

```
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.14e-37, .1e-37, -.1047e-34]Solution in 1.348s

Time Plot 0 s.
Exiting SolveHard() after 3.526r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23, none,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962832728679967676921183208853354,
4.1256517968076934847140796028956790785 46,
440.67123065467546363448993699721518601 15]
one interval r = 22.397611543711103153503553352223445243 28 ..
27.237223516058386478824356324797637903 06
Time Approximations 0.038.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 2.261 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643902746983378676727449969026 01, rm =
14.378187702341888596862322181035810720 01}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, .6787e-35]Solution in 8.721s

Time Plot 0 s.
Exiting SolveHard() after 10.662r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
```

```
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888806298414363042665081434041168,
4.0048690817892107753923895581677875500 45,
404.86224501857521318038197678403080781 38]
two intervals r = 16.08011007752385517144506915111960229860 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000 or r = 16.41579812713958945555187518519384316563 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.52e-37, -.1e-37, -.2413e-34]Solution in
2.602s

Time Plot 0 s.
Exiting SolveHard() after 4.791r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88, none, none,
358.97362824432840411699894995003889524 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none]

1 --> 2  target = [34.945078888062984143630426650814340411 68,
4.0048690817892107753923895581677875500 45,
404.86224501857521318038197678403080781 38]
one interval r = 21.641943994162374935229252270249863649 75 ..
26.763306600518583703350946911025925554 28
Time Approximations 0.048.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.61418e-34]Solution in
1.061s

Time Plot 0 s.
Exiting SolveHard() after 3.22r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10, none,
358.97362824432840411699894995003889524 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589417754158081302901451898445441 94,
5.5896371829384292837391108525394047035 73,
443.83065885154735546412609349925328645 26]
one interval r = 22.467253744858643950044018374069204373 78 ..
27.273884283637507343408732647719654308 72
Time Approximations 0.035.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
```

```
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., .5688e-35]Solution in 0.978s

Time Plot 0 s.
Exiting SolveHard() after 3.166r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10, none,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941775415808130290145189844544194,
5.589637182938429283739110852539404703573,
443.83065885154735546412609349925328645 26]
one interval r = 32.15575279502718724782201889323482767517 ..
35.50872228747015622306861185505916749479
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, .141e-34]Solution in 0.448s

Time Plot 0 s.
Exiting SolveHard() after 1.866r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136508528870201203862883212263710,
5.1877835784676306357465577736277169513 63,
408.65773863127783546702213142269424063 21]
one interval r = 21.71840114653500901916283827775268921475 ..
26.81849303520214572974285858780794510919
Time Approximations 0.059.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.06e-37
Equations at solution: [.2e-37, .106e-36, -.13331e-34]Solution in 0.99s

Time Plot 0 s.
Exiting SolveHard() after 3.377r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10, none, none,
361.50888347573506481372427922332514274 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0  target = [15.91193136508528870201203862883212263710,
5.1877835784676306357465577736277169513 63,
408.65773863127783546702213142269424063 21]
one interval r = 31.808285987498536268634955295402051116 64 ..
35.00011460051866466000711662651588884821
Time Approximations 0.019.

hint used Hint := [34.495226611650080820729149248019775312 21, 3, 1, 1,
15.763855890238725343023875204542137417 16, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., .303e-34]Solution in 1.49s

Time Plot 0 s.
Exiting SolveHard() after 1.817r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94, none,
361.50888347573506481372427922332514274 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110550265169947648860305622517317,
6.19626256528032525209534025911209104002 5,
385.44474379840886619950482132234212817 77]
one interval r = 31.608360975348888636325416049853025066 03 ..
34.66372795616944997794833359020702844915
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
```

```
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.218e-34]Solution in 0.571s

Time Plot 0 s.
Exiting SolveHard() after 0.869r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94, none,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110550265169947648860305622517317,
6.19626256528032525209534025911209104002 5,
385.44474379840886619950482132234212817 77]
two intervals r = 16.875634087476839693329045890329752858 08 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000 or r = 15.556404938254454699565578631595642647 51 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.897e-37, 0., .2270e-34]Solution in 2.236s

Time Plot 0 s.
```

```
Exiting SolveHard() after 4.69r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874728644003630138457522311654250,
4.88381077980533179356724156199995189 1524,
376.61967856351562275016768623428497560 11]
one interval r = 21.11001304880842217987431536492531225803 ..
26.31784243485754812195381135470515858481
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., .11385e-34]Solution in 1.869s

Time Plot 0 s.
Exiting SolveHard() after 2.536r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
```

```
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46, none,
328.46938514090856637024490143362720921 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874728644003630138457522311654250,
4.8838107798053317935672415619999518915 24,
376.61967856351562275016768623428497560 11]
one interval r = 31.53899497708937051822969018949377859179 ..
34.53618386100442684586840961746860384621
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=4.3e-37
Equations at solution: [-.33e-36, .43e-36, -.393e-34]Solution in 0.524s

Time Plot 0 s.
Exiting SolveHard() after 0.806r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
```

```
324.67144993172085361865601615220569858 46, none,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017553743173980386938298191278588,
6.0258135492436898253485712253354968379 39,
351.42702949031480888116277804276404805 11]
one interval r = 31.362302061109984602236529665581485195 57 ..
34.174466406222431931144766711223783983 85
Time Approximations 0.017.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.1e-37, -.3e-37, .171e-34]Solution in 1.627s

Time Plot 0 s.
Exiting SolveHard() after 1.878r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46, none,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61, none, none,
292.99969138999584160833778146327961082 77, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017553743173980386938298191278588,
6.0258135492436898253485712253354968379 39,
351.42702949031480888116277804276404805 11]
```

```
two intervals r = 17.98135514434773906890309835756584510025 ..
380000000000458438402634901383721511 3/2000000000000000000000000000000000
000 or r = 13.84608015454289627059517815334467799455 ..
380000000000458438402634901383721511 3/2000000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.54e-37, -.2e-37, -.558e-35]Solution in 2.193s

Time Plot 0 s.
Exiting SolveHard() after 3.231r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61, none, none,
292.99969138999584160833778146327961082 77, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941884477907838505159705764434070,
6.37794387383051531776782694102772608604 3,
423.28832784651739230638289174065312713 97]
one interval r = 31.94661817598305158491183152994841717396 ..
35.21212308661764554539085618209277336577
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
```

```
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.117e-34]Solution in 0.606s

Time Plot 0 s.
Exiting SolveHard() after 0.968r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61, none, none,
292.99969138999584160833778146327961082 77, none, none,
360.06173467527670288310244610322996632 46, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941884477907838505159705764434070,
6.37794387383051531776782694102772608 6043,
423.28832784651739230638289174065312713 97]
two intervals r = 15.22886702422052650305279961830875056013 ..
380000000000458438402634901383721511 3/2000000000000000000000000000000000
000 or r = 17.12965777095885095110818673378647105905 ..
380000000000458438402634901383721511 3/2000000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.15e-37, .1e-37, .598e-35]Solution in 2.368s

Time Plot 0 s.
Exiting SolveHard() after 4.758r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61,
375.73285290912987729834628382111201951 96, none,
292.99969138999584160833778146327961082 77, none, none,
360.06173467527670288310244610322996632 46, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343505069503999495388775273838 68,
4.0035598154900412747976843855233720692 04,
404.47973594585801150591311230600736225 01]
two intervals r = 16.096839663622923715860318936188956654 70 ..
380000000000458438402634901383721511 3/2000000000000000000000000000000000
000 or r = 16.399886491348727985716259190112358986 07 ..
380000000000458438402634901383721511 3/2000000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.17e-37, -.1e-37, -.2400e-34]Solution in
2.619s

Time Plot 0 s.
Exiting SolveHard() after 4.743r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61,
375.73285290912987729834628382111201951 96, none,
292.99969138999584160833778146327961082 77,
358.64341561302032670817934524659514320 38, none,
360.06173467527670288310244610322996632 46, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234350506950399949538877527383868,
4.0035598154900412747976843855233720692 04,
404.47973594585801150591311230600736225 01]
one interval r = 21.63429629993925234661354053233603766451 ..
26.75768169902013603355579684284545522136
Time Approximations 0.044.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=1.24e-37
```

```
Equations at solution: [-.3e-37, -.124e-36, .20130e-34]Solution in
1.032s

Time Plot 0 s.
Exiting SolveHard() after 3.126r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61,
375.73285290912987729834628382111201951 96,
328.11709294961814290317698531631416021 97,
292.99969138999584160833778146327961082 77,
358.64341561302032670817934524659514320 38, none,
360.06173467527670288310244610322996632 46, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954468370583052115111308768664754,
6.19617723013358009177480701996463110498 4,
385.42734026210578359301313124701386789 94]
one interval r = 31.60822049090209050725885108215146272166 ..
34.66347615054527080185195891487372385126
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., -.263e-34]Solution in 1.599s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.92r=33.8134 in [32.62668594 .. 34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61,
375.73285290912987729834628382111201951 96,
328.11709294961814290317698531631416021 97,
292.99969138999584160833778146327961082 77,
358.64341561302032670817934524659514320 38, none,
360.06173467527670288310244610322996632 46, none, none,
324.65521224145796234070889875690775251 98, none, none, none, none]

0 --> 1  target = [26.46318954468370583052115111308768664754,
6.19617723013358009177480701996463110498 4,
385.42734026210578359301313124701386789 94]
two intervals r = 16.87629600286529584298766104128566762312 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000 or r = 15.55559000673749620761678633570631015774 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000
Time Approximations 0.064.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.179e-37, 0., -.706e-35]Solution in 2.284s

Time Plot 0 s.
```

```
Exiting SolveHard() after 3.465r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61,
375.73285290912987729834628382111201951 96,
328.11709294961814290317698531631416021 97,
292.99969138999584160833778146327961082 77,
358.64341561302032670817934524659514320 38, none,
360.06173467527670288310244610322996632 46,
336.59441032838441283020651410861989087 52, none,
324.65521224145796234070889875690775251 98, none, none, none, none]

0 --> 2  target = [34.49522661170273168702617907105365013984,
3.8971313159031139504157171271841150017 45,
373.78081884861134577228446339743906573 81]
two intervals r = 17.29769086222378858731624934404826990351 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000 or r = 14.99436407450860642855471583912395350253 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000
Time Approximations 0.091.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [.90e-37, -.1e-37, .866e-35]Solution in 2.337s

Time Plot 0 s.
```

```
Exiting SolveHard() after 5.182r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61,
375.73285290912987729834628382111201951 96,
328.11709294961814290317698531631416021 97,
292.99969138999584160833778146327961082 77,
358.64341561302032670817934524659514320 38, none,
360.06173467527670288310244610322996632 46,
336.59441032838441283020651410861989087 52, none,
324.65521224145796234070889875690775251 98,
331.93806791779155740149247726639457511 04, none, none, none]

1 --> 2  target = [34.49522661170273168702617907105365013984,
3.8971313159031139504157171271841150017 45,
373.78081884861134577228446339743906573 81]
one interval r = 21.06068473203940018475420669313391301276 ..
26.26979834291686846590155128074278591767
Time Approximations 0.03.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, -.4740e-35]Solution in 0.779s

Time Plot 0 s.
Exiting SolveHard() after 2.598r=25.3005 in [23.14060343 ..
26.26979834]
```

```
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61,
375.73285290912987729834628382111201951 96,
328.11709294961814290317698531631416021 97,
292.99969138999584160833778146327961082 77,
358.64341561302032670817934524659514320 38,
299.89866205238061668066919218461365003 99,
360.06173467527670288310244610322996632 46,
336.59441032838441283020651410861989087 52, none,
324.65521224145796234070889875690775251 98,
331.93806791779155740149247726639457511 04, none, none, none]

0 --> 2  target = [33.81362495415328005528721334941453376661,
3.72564899354250210842276924191725708407 0,
325.89209973412213144348053382963912060 90]
two intervals r = 18.552270490009639896989229506910104923 68 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000 or r = 12.491969358222843042586305103473438001 58 ..
38000000000045843840263490138372151 13/20000000000000000000000000000000000
000
Time Approximations 0.038.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [-.67e-37, .3e-37, .180e-35]Solution in 2.208s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.746r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61,
375.73285290912987729834628382111201951 96,
328.11709294961814290317698531631416021 97,
292.99969138999584160833778146327961082 77,
358.64341561302032670817934524659514320 38,
299.89866205238061668066919218461365003 99,
360.06173467527670288310244610322996632 46,
336.59441032838441283020651410861989087 52, none,
324.65521224145796234070889875690775251 98,
331.93806791779155740149247726639457511 04, none, none,
289.54595773014431181924078642941898194 10]

1 --> 2  target = [33.81362495415328005528721334941453376661,
3.7256489935425021084227692419172570840 70,
325.89209973412213144348053382963912060 90]
one interval r = 20.37468935105596455159085559555796617890 ..
25.37892165311568450014206104588939759002
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, -.17988e-34]Solution in
0.606s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.141r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61,
375.73285290912987729834628382111201951 96,
328.11709294961814290317698531631416021 97,
292.99969138999584160833778146327961082 77,
358.64341561302032670817934524659514320 38,
299.89866205238061668066919218461365003 99,
360.06173467527670288310244610322996632 46,
336.59441032838441283020651410861989087 52,
256.10753186671893191981721200212465831 82,
324.65521224145796234070889875690775251 98,
331.93806791779155740149247726639457511 04, none, none,
289.54595773014431181924078642941898194 10]

1 --> 0  target = [17.93041369701144957668699205414100772172,
4.68650870194846248310383688513828008 4245,
353.30541095642173584449239943578678472 44]
one interval r = 20.73150479093278981599330546384656419570 ..
25.90675353534533960370999229634329792453
Time Approximations 0.029.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=9.4e-38
```

```
Equations at solution: [.4e-37, .94e-37, -.2496e-35]Solution in 0.659s

Time Plot 0 s.
Exiting SolveHard() after 2.489r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256408816315765997157680735779 4,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61,
375.73285290912987729834628382111201951 96,
328.11709294961814290317698531631416021 97,
292.99969138999584160833778146327961082 77,
358.64341561302032670817934524659514320 38,
299.89866205238061668066919218461365003 99,
360.06173467527670288310244610322996632 46,
336.59441032838441283020651410861989087 52,
256.10753186671893191981721200212465831 82,
324.65521224145796234070889875690775251 98,
331.93806791779155740149247726639457511 04,
304.79958326177805809576535209328706962 10, none,
289.54595773014431181924078642941898194 10]

2 --> 0  target = [17.93041369701144957668699205414100772172,
4.68650870194846248310383688513828008 4245,
353.30541095642173584449239943578678472 44]
one interval r = 31.37435486990229162365741718057072515168 ..
34.20127520034758455650832407097782375617
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=1.1e-37
Equations at solution: [-.8e-37, .11e-36, -.49e-35]Solution in 1.391s

Time Plot 0 s.
Exiting SolveHard() after 1.688r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494041779762288521076949053497 98,
441.64295973621403781678391474883874161 24,
436.91748165697604606509218301083841537 58,
422.98493398035381990688053598863454975 23,
361.52580256540881631576599715768073577 94,
401.88173904784321516867009534396967241 92,
389.59001516213714435402064058411300967 88,
328.46939894018337983597072465297977827 10,
401.50757158560541192231959076694178001 92,
358.97362824432840411699894995003889524 76,
398.33147104725307770709000144749775773 10,
371.48387394787296422928770874364366630 94,
336.61215841800687233679086792930018191 50,
361.50888347573506481372427922332514274 51,
324.67144993172085361865601615220569858 46,
302.31384315540405882179375567272974072 44,
328.46938514090856637024490143362720921 23,
343.81340625403260557485611931708047404 61,
375.73285290912987729834628382111201951 96,
328.11709294961814290317698531631416021 97,
292.99969138999584160833778146327961082 77,
358.64341561302032670817934524659514320 38,
299.89866205238061668066919218461365003 99,
360.06173467527670288310244610322996632 46,
336.59441032838441283020651410861989087 52,
256.10753186671893191981721200212465831 82,
324.65521224145796234070889875690775251 98,
331.93806791779155740149247726639457511 04,
304.79958326177805809576535209328706962 10,
323.46169177218472188855401228529662677 83,
289.54595773014431181924078642941898194 10]

Cascade time 128.227
counts: 28, 28

 Iteration 21

Start Generation 1
1 --> 0  target = [11.99999999995140887848010519049235634600,
6.21701250295895748478807852573990396660 9,
485.54908089497241600928287101430927617 16]
one interval r = 23.40850301648372004686218736470459213220 ..
27.67578046417225540933686399295939355480
```

```
Time Approximations 0.042.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.12e-37
Equations at solution: [-.7e-37, .212e-36, .16e-35]Solution in 2.055s

Time Plot 0 s.
Exiting SolveHard() after 4.338r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999995140887848010519049235634600,
6.217012502958957484788078525739903966609,
485.549080894972416009282871014309276171 6]
one interval r = 32.628147792011093513681820587994142889 59 ..
36.10248389331186539743681957797443938 35
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, .108e-34]Solution in 0.621s

Time Plot 0 s.
Exiting SolveHard() after 1.058r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844683448842878048517126521520 45,
6.583434721674965068551813653597514121650,
467.787305955375988644567184780502929231 1]
one interval r = 32.419789556502187748606537138710901564 93 ..
35.851524173622255554239117905779669728 61
Time Approximations 0.021.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=7e-38
Equations at solution: [-.8e-37, .7e-37, -.319e-34]Solution in 0.666s

Time Plot 0 s.
Exiting SolveHard() after 2.13r=34.9451 in [33.70078237 .. 35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31, none, none,
401.88173903890704431630911244329240560 98, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844683448842878048517126521520 45,
6.583434721674965068551813653597514121650,
467.787305955375988644567184780502929231 1]
two intervals r = 12.923271608267285058306705405672085433 58 ..
189999999998720571787529010009030242 63/100000000000000000000000000000000
00000 or r = 18.394248580141004070497786079393194446 35 ..
189999999998720571787529010009030242 63/100000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
```

```
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.3e-38
Equations at solution: [-.2e-37, -.33e-37, .8596e-35]Solution in
35.367s

Time Plot 0 s.
Exiting SolveHard() after 37.958r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09, none,
401.88173903890704431630911244329240560 98, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962815799315377899159623452661254,
4.12565179683313107029374458800737566682 2,
440.67123064703974230565296228011401265 80]
two intervals r = 14.35659705113421461418365977998173202479 ..
18999999999872057178752901000903024263/10000000000000000000000000000000
00000 or r = 17.70352613789507185463885274402157495201 ..
18999999999872057178752901000903024263/10000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., .15747e-34]Solution in 2.471s

Time Plot 0 s.
Exiting SolveHard() after 4.698r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09, none,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962815799315377899159623452661254,
4.1256517968331310702937445880073756668 22,
440.67123064703974230565296228011401265 80]
one interval r = 22.397611543623896744303581231548537221 79 ..
27.237223515862088899679368796906535088 85
Time Approximations 0.037.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 2.3 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643740079072729372718879225390 94, rm =
14.378187704150434628475523864270869270 23}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., -.60e-35]Solution in 8.758s

Time Plot 0 s.
Exiting SolveHard() after 9.635r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]
```

```
Start Generation 3
0 --> 2  target = [34.945078887874534420827543125883397774 27,
4.004869081808583429968821461683770045780,
404.862245009467296798802732317863487703 2]
two intervals r = 16.080110077547982071768506292228161042 72 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000 or r = 16.415798126707029507503471362992779265 03 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., -.37384e-34]Solution in 2.833s

Time Plot 0 s.
Exiting SolveHard() after 5.189r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74, none, none,
358.97362823574755991774391102651268103 36, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887874534420827543125883397774 27,
4.004869081808583429968821461683770045780,
404.862245009467296798802732317863487703 2]
one interval r = 21.641943994099357095389699766094165949 38 ..
26.763306600291424919012835289003785383 94
Time Approximations 0.052.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., .233e-34]Solution in 1.094s

Time Plot 0 s.
Exiting SolveHard() after 3.521r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42, none,
358.97362823574755991774391102651268103 36, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941782205948089804465261331442977,
5.58963718294989612835150204702118013133 1,
443.83065884178173961021381335988127587 15]
one interval r = 22.46725374472007711786386069334784017144 ..
27.27388428341790676925197715160420710517
Time Approximations 0.042.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., .22e-35]Solution in 2.095s

Time Plot 0 s.
Exiting SolveHard() after 4.55r=27.0204 in [24.71083344 .. 27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
```

```
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42, none,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941782205948089804465261331442977,
5.589637182949896128351502047021180131331,
443.83065884178173961021381335988127587 15]
one interval r = 32.15575279488065823863828779622112019300 ..
35.50872228727008026899995901141957881137
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .198e-34]Solution in 0.527s

Time Plot 0 s.
Exiting SolveHard() after 0.907r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136504801487812469917237093571514,
5.187783578505628827508628559158066575310,
408.65773862358803646218111018649065266 86]
```

```
one interval r = 21.7184011464942598390225264163009276680 1 ..
26.81849303499649926522971376260462916492
Time Approximations 0.059.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.90e-37
Equations at solution: [-.3e-37, -.290e-36, -.30e-35]Solution in 2.296s

Time Plot 0 s.
Exiting SolveHard() after 4.542r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68, none, none,
361.50888346742277843125974901815689249 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136504801487812469917237093571514,
5.18778357850562882750862855915806657531 0,
408.65773862358803646218111018649065266 86]
one interval r = 31.80828598738651374346907430314187251612 ..
35.00011460035048408109914742569834183981
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
```

```
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .398e-34]Solution in 0.461s

Time Plot 0 s.
Exiting SolveHard() after 0.767r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05, none,
361.50888346742277843125974901815689249 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110527440208925975948542427784194,
6.19626256540004720566608373199462797498 0,
385.44474378950211853398275123254963869 64]
one interval r = 31.60836097523603878485709466153359978406 ..
34.66372795598570347048299023860243329154
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., -.91e-35]Solution in 0.568s

Time Plot 0 s.
Exiting SolveHard() after 0.854r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05, none,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110527440208925975948542427784194,
6.1962625654000472056660837319946279749 80,
385.44474378950211853398275123254963869 64]
two intervals r = 16.875634087478807224662063169775124477 74 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000 or r = 15.556404937784966351495506287301139448 68 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000
Time Approximations 0.064.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.791 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175111199562587543774754978837335, rm
= 2.33653277404342695812502530340760572737 6}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.897e-37, 0., .21158e-34]Solution in 25.368s

Time Plot 0 s.
Exiting SolveHard() after 27.686r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
```

```
401.881739038907044316309112443292405609 8,
389.590015154861030868420844483967486437 4,
328.469398930478488829847836651439241574 2,
401.507571576272167279740122735987821413 6,
358.973628235747559917743911026512681033 6,
398.331471036869996256168056735944328466 8,
371.483873940242273506642238830345230890 5,
336.612158408115847548582524200931012453 0,
361.508883467422778431259749018156892491 6,
324.671449922103943345990998431368924666 4, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874728577980446605604118792083469,
4.883810779831264819520189441317540940191,
376.619678554463603744771005503887413724 4]
one interval r = 21.11001304879776898504829852289958982421 ..
26.31784243462576547467567699541844433210
Time Approximations 0.031.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.357e-34]Solution in 0.835s

Time Plot 0 s.
Exiting SolveHard() after 2.588r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434933124276349463777763447603812 1,
441.642959728501440790160578940905333165 1,
436.917481649475597115332349617350791003 1,
422.984933971114784672298793341783372170 9,
361.525802557039492725085371622350175644 5,
401.881739038907044316309112443292405609 8,
389.590015154861030868420844483967486437 4,
328.469398930478488829847836651439241574 2,
401.507571576272167279740122735987821413 6,
358.973628235747559917743911026512681033 6,
398.331471036869996256168056735944328466 8,
371.483873940242273506642238830345230890 5,
336.612158408115847548582524200931012453 0,
361.508883467422778431259749018156892491 6,
324.671449922103943345990998431368924666 4, none,
328.469385131205126813414351781076352150 5, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874728577980446605604118792083469,
4.8838107798312648195201894413175409401 91,
376.61967855446360374477100550388741372 44]
one interval r = 31.53899497697948559976087311371602351442 ..
34.53618386081955704156365342472619332508
Time Approximations 0.015.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=2.2e-37
Equations at solution: [-.17e-36, .22e-36, -.489e-34]Solution in 0.491s

Time Plot 0 s.
Exiting SolveHard() after 1.935r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64, none,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017528200708236450335156979494622,
6.0258135493530250226130159759344934855 28,
351.42702948002047862587794054974713631 29]
one interval r = 31.36230206100444697337576276084236157296 ..
34.17446640602322299816482650874979942588
Time Approximations 0.015.
```

```
hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [-.3e-37, .6e-37, -.394e-34]Solution in 0.511s

Time Plot 0 s.
Exiting SolveHard() after 0.743r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64, none,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26, none, none,
292.99969137907344620245319906861607021 86, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017528200708236450335156979494622,
6.025813549353025022613015975934493485528,
351.42702948002047862587794054974713631 29]
two intervals r = 17.98135514435201179959637829656218527120 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000 or r = 13.84608015393815467435228380956912560078 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
```

```
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 5.101 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713574852757517378789824205549 77, rm
= 2.7345009931098929452127460115349839877 44}});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [-.36e-37, .1e-37, .14186e-34]Solution in
15.341s

Time Plot 0 s.
Exiting SolveHard() after 17.511r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26, none, none,
292.99969137907344620245319906861607021 86, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941859925015494413466667225321254,
6.37794387394390685302418890927610303321 1,
423.28832783555396044932258324666162401 90]
one interval r = 31.94661817583367748253871785368220028650 ..
35.21212308640108110359798771531769997185
Time Approximations 0.017.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.214e-34]Solution in 0.626s

Time Plot 0 s.
Exiting SolveHard() after 0.947r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26, none, none,
292.99969137907344620245319906861607021 86, none, none,
360.06173466374359832569373146093329716 00, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418599250154944134666672253212 54,
6.3779438739439068530241889092761030332 11,
423.28832783555396044932258324666162401 90]
two intervals r = 15.22886702434273048185634493344134015774 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000 or r = 17.12965777050585271656303425311022050625 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000
Time Approximations 0.064.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 6.827 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541220118151545877660510677898 58, rm
= 2.0640682986909440902194471587504810965 89}});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [.46e-37, .2e-37, .24461e-34]Solution in 24.776s

Time Plot 0 s.
Exiting SolveHard() after 27.307r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26,
375.73285289694307831963449453734330620 00, none,
292.99969137907344620245319906861607021 86, none, none,
360.06173466374359832569373146093329716 00, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343310763448702599984104344651 72,
4.0035598155080113957114592668020578606 38,
404.47973593634411307160656350745999665 90]
two intervals r = 16.096839663664554036072986807895421536 93 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000 or r = 16.399886490898313188888310985108315075 90 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000
Time Approximations 0.047.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.17e-37, 0., .14580e-34]Solution in 2.693s

Time Plot 0 s.
Exiting SolveHard() after 4.772r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26,
375.73285289694307831963449453734330620 00, none,
292.99969137907344620245319906861607021 86,
358.64341560408838924338125026314931862 61, none,
360.06173466374359832569373146093329716 00, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234331076344870259998410434465172,
4.0035598155080113957114592668020578606 38,
404.47973593634411307160656350745999665 90]
one interval r = 21.63429629986878312756141136841992706229 ..
26.75768169878689408512268059692583675851
Time Approximations 0.044.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.400e-34]Solution in 0.962s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.089r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26,
375.73285289694307831963449453734330620 00,
328.11709293953961231385224138365564743 18,
292.99969137907344620245319906861607021 86,
358.64341560408838924338125026314931862 61, none,
360.06173466374359832569373146093329716 00, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954445640272853875541086598225036,
6.19617723025358808239555663202125962071 4,
385.42734025325770005995869027256625945 11]
one interval r = 31.60822049078972213190986436185018188567 ..
34.66347615036237490241802573298047849704
Time Approximations 0.016.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, -.103e-34]Solution in 1.596s

Time Plot 0 s.
Exiting SolveHard() after 1.867r=33.8134 in [32.62668594 ..
34.66347615]
```

```
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26,
375.73285289694307831963449453734330620 00,
328.11709293953961231385224138365564743 18,
292.99969137907344620245319906861607021 86,
358.64341560408838924338125026314931862 61, none,
360.06173466374359832569373146093329716 00, none, none,
324.65521223189577956546624400720889310 32, none, none, none, none]

0 --> 1  target = [26.46318954445640272853875541086598225036,
6.19617723025358808239555663202125962071 4,
385.42734025325770005995869027256625945 11]
two intervals r = 16.87629600286501803973697949620160714947 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000 or r = 15.55559000627071627078735593949617047730 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 5.77 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852522920432286548045230823439593, rm
= 2.33669042810873376685549361801375491917 2}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
```

```
Equations at solution: [-.896e-37, 0., -.30585e-34]Solution in 23.509s

Time Plot 0 s.
Exiting SolveHard() after 25.571r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26,
375.73285289694307831963449453734330620 00,
328.11709293953961231385224138365564743 18,
292.99969137907344620245319906861607021 86,
358.64341560408838924338125026314931862 61, none,
360.06173466374359832569373146093329716 00,
336.59441031855328786699592155315977623 41, none,
324.65521223189577956546624400720889310 32, none, none, none, none]

0 --> 2  target = [34.49522661153658061223990916450992949668,
3.8971313159261140640282671092216516360 02,
373.78081884083402429795974188509540024 35]
two intervals r = 17.29769086217656811160587607854474891322 ..
18999999999872057178752901000903024263/10000000000000000000000000000000
00000 or r = 14.99436407407023271603462380526715206072 ..
18999999999872057178752901000903024263/10000000000000000000000000000000
00000
Time Approximations 0.076.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
```

```
Equations at solution: [-.72e-37, 0., -.29836e-34]Solution in 2.206s

Time Plot 0 s.
Exiting SolveHard() after 4.979r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26,
375.73285289694307831963449453734330620 00,
328.11709293953961231385224138365564743 18,
292.99969137907344620245319906861607021 86,
358.64341560408838924338125026314931862 61, none,
360.06173466374359832569373146093329716 00,
336.59441031855328786699592155315977623 41, none,
324.65521223189577956546624400720889310 32,
331.93806791033330309124460817706098697 30, none, none, none]

1 --> 2  target = [34.49522661153658061223990916450992949668,
3.8971313159261140640282671092216516360 02,
373.78081884083402429795974188509540024 35]
one interval r = 21.06068473205611724730050836431014567372 ..
26.26979834270651081756356502645430451994
Time Approximations 0.032.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [-.2e-37, -.5e-37, .32e-35]Solution in 1.832s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.47r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26,
375.73285289694307831963449453734330620 00,
328.11709293953961231385224138365564743 18,
292.99969137907344620245319906861607021 86,
358.64341560408838924338125026314931862 61,
299.89866204391790818899533846239272408 60,
360.06173466374359832569373146093329716 00,
336.59441031855328786699592155315977623 41, none,
324.65521223189577956546624400720889310 32,
331.93806791033330309124460817706098697 30, none, none, none]

0 --> 2  target = [33.81362495396545949586983595483978481015,
3.72564899355660432575736574105928631042 4,
325.89209972430613549418788988314269021 28]
two intervals r = 18.552270489958776964016823063709545175 97 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000 or r = 12.491969357632502927122028058312843357 53 ..
189999999998720571787529010009030242 63/10000000000000000000000000000000
00000
Time Approximations 0.036.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1.6e-37
```

```
Equations at solution: [.383e-36, -.16e-36, -.2071e-35]Solution in
1.053s

Time Plot 0 s.
Exiting SolveHard() after 3.468r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26,
375.73285289694307831963449453734330620 00,
328.11709293953961231385224138365564743 18,
292.99969137907344620245319906861607021 86,
358.64341560408838924338125026314931862 61,
299.89866204391790818899533846239272408 60,
360.06173466374359832569373146093329716 00,
336.59441031855328786699592155315977623 41, none,
324.65521223189577956546624400720889310 32,
331.93806791033330309124460817706098697 30, none, none,
289.54595772083983278855262257290080541 69]

1 --> 2  target = [33.81362495396545949586983595483978481015,
3.72564899355660432575736574105928631042 4,
325.89209972430613549418788988314269021 28]
one interval r = 20.37468935113727033363771617054577852932 ..
25.37892165287181974738953030433240121964
Time Approximations 0.023.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
```

```
Accepted {r=24.3395, rm=17.2722} with Delta=8e-38
Equations at solution: [-.6e-37, -.8e-37, .247e-34]Solution in 0.507s

Time Plot 0 s.
Exiting SolveHard() after 2.181r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26,
375.73285289694307831963449453734330620 00,
328.11709293953961231385224138365564743 18,
292.99969137907344620245319906861607021 86,
358.64341560408838924338125026314931862 61,
299.89866204391790818899533846239272408 60,
360.06173466374359832569373146093329716 00,
336.59441031855328786699592155315977623 41,
256.10753185641680002696197542073960766 74,
324.65521223189577956546624400720889310 32,
331.93806791033330309124460817706098697 30, none, none,
289.54595772083983278855262257290080541 69]

1 --> 0  target = [17.93041369702215479134100412292046432749,
4.68650870196396432791972251097888173470 8,
353.30541094600800427027874175021306600 25]
one interval r = 20.73150479094733759684906324057132383103 ..
25.90675353508776553431489032525132319574
Time Approximations 0.031.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.277e-34]Solution in 1.868s

Time Plot 0 s.
Exiting SolveHard() after 2.547r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26,
375.73285289694307831963449453734330620 00,
328.11709293953961231385224138365564743 18,
292.99969137907344620245319906861607021 86,
358.64341560408838924338125026314931862 61,
299.89866204391790818899533846239272408 60,
360.06173466374359832569373146093329716 00,
336.59441031855328786699592155315977623 41,
256.10753185641680002696197542073960766 74,
324.65521223189577956546624400720889310 32,
331.93806791033330309124460817706098697 30,
304.79958325074756687888436180687084592 88, none,
289.54595772083983278855262257290080541 69]

2 --> 0  target = [17.93041369702215479134100412292046432749,
4.68650870196396432791972251097888173470 8,
353.30541094600800427027874175021306600 25]
one interval r = 31.37435486979492504325988642316522425320 ..
34.20127520014636736310524956691986086247
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
```

```
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .161e-34]Solution in 0.367s

Time Plot 0 s.
Exiting SolveHard() after 0.64r=33.7963 in [32.25770943 .. 34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493312427634946377776344760381 21,
441.64295972850144079016057894090533316 51,
436.91748164947559711533234961735079100 31,
422.98493397111478467229879334178337217 09,
361.52580255703949272508537162235017564 45,
401.88173903890704431630911244329240560 98,
389.59001515486103086842084448396748643 74,
328.46939893047848882984783665143924157 42,
401.50757157627216727974012273598782141 36,
358.97362823574755991774391102651268103 36,
398.33147103686999625616805673594432846 68,
371.48387394024227350664223883034523089 05,
336.61215840811584754858252420093101245 30,
361.50888346742277843125974901815689249 16,
324.67144992210394334599099843136892466 64,
302.31384314427894984572107511005013288 55,
328.46938513120512681341435178107635215 05,
343.81340624517127787617174584634254240 26,
375.73285289694307831963449453734330620 00,
328.11709293953961231385224138365564743 18,
292.99969137907344620245319906861607021 86,
358.64341560408838924338125026314931862 61,
299.89866204391790818899533846239272408 60,
360.06173466374359832569373146093329716 00,
336.59441031855328786699592155315977623 41,
256.10753185641680002696197542073960766 74,
324.65521223189577956546624400720889310 32,
331.93806791033330309124460817706098697 30,
304.79958325074756687888436180687084592 88,
323.46169176209213898087953803366814541 91,
289.54595772083983278855262257290080541 69]

Cascade time 210.513
counts: 28, 28

 Iteration 22

Start Generation 1
1 --> 0  target = [12.00000000000486537165179220898006559300,
6.21701250312722647695641300218508762450 0,
485.54908089865293801941969984126596526 31]
```

```
one interval r = 23.40850301654083100657559686398340310804 ..
27.67578046433195496097535725593381751923
Time Approximations 0.036.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.32e-37
Equations at solution: [-.4e-37, .132e-36, .4e-36]Solution in 0.989s

Time Plot 0 s.
Exiting SolveHard() after 3.142r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000000486537165179220898006559300,
6.217012503127226476956413002185087624500,
485.549080898652938019419699841265965263 1]
one interval r = 32.62814779208404982875821980169953281890 ..
36.10248388938558636844400767013978181134
Time Approximations 0.019.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=1e-38
Equations at solution: [-.3e-37, .1e-37, -.123e-34]Solution in 0.526s

Time Plot 0 s.
Exiting SolveHard() after 2.364r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684485374368929904948900582742482,
6.58343472169637360266998729562640081550 4,
467.78730595997028324064729692995637968 01]
one interval r = 32.41978955658527979526193535158901515721 ..
35.85152417369196252920352265233616483368
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., .27e-35]Solution in 0.583s

Time Plot 0 s.
Exiting SolveHard() after 0.937r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09, none, none,
401.88173904276130302135855700799079172 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684485374368929904948900582742482,
6.58343472169637360266998729562640081550 4,
467.78730595997028324064729692995637968 01]
two intervals r = 12.92327160827377148247274559932442824387 ..
189999999999925439061886507849467327 31/10000000000000000000000000000000
00000 or r = 18.39424858032148331298529358828144240075 ..
189999999999925439061886507849467327 31/10000000000000000000000000000000
00000
Time Approximations 0.039.
```

```
hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=6.5e-38
Equations at solution: [-.5e-37, -.65e-37, .242e-35]Solution in 35.634s

Time Plot 0 s.
Exiting SolveHard() after 38.119r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70, none,
401.88173904276130302135855700799079172 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962818989046453852770982262841194,
4.12565179681550610502777286127587280451 6,
440.67123064876545897790506562800651217 63]
two intervals r = 14.35659705129321760666578682618942120212 ..
189999999999925439061886507849467327 31/100000000000000000000000000000000
00000 or r = 17.70352613801922993740299894936161714574 ..
189999999999925439061886507849467327 31/100000000000000000000000000000000
00000
Time Approximations 0.043.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.13e-37, .1e-37, .3793e-34]Solution in 1.323s

Time Plot 0 s.
Exiting SolveHard() after 3.833r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70, none,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962818989046453852770982262841194,
4.12565179681550610502777286127587280451 6,
440.67123064876545897790506562800651217 63]
one interval r = 22.39761154358686541681245597810424798468 ..
27.23722351600157998222666118183846442469
Time Approximations 0.035.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 2.611 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064385790302071582648378700257923, rm =
14.37818770412520815756693322826810339649}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., .26e-37, -.688e-34]Solution in 9.313s

Time Plot 0 s.
Exiting SolveHard() after 11.459r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887943360875879220379259228894 34,
4.004869081798969947505976065045031657131,
404.86224501347737291761525938746302767 38]
two intervals r = 16.08011007759807138869164326114653123712 ..
189999999999925439061886507849467327 31/100000000000000000000000000000000
00000 or r = 16.41579812693240790935087117025653758828 ..
189999999999925439061886507849467327 31/100000000000000000000000000000000
00000
Time Approximations 0.047.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.18e-37, 0., -.3648e-34]Solution in 2.875s

Time Plot 0 s.
Exiting SolveHard() after 5.213r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25, none, none,
358.97362823696842200466536320450325882 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887943360875879220379259228894 34,
4.004869081798969947505976065045031657131,
404.86224501347737291761525938746302767 38]
one interval r = 21.64194399406570647635082878201680033823 ..
26.76330660045492905106904413033036193678
Time Approximations 0.051.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
```

```
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.3e-38
Equations at solution: [0., .23e-37, -.192e-34]Solution in 2.354s

Time Plot 0 s.
Exiting SolveHard() after 3.338r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16, none,
358.97362823696842200466536320450325882 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941768958825917348524128028154264,
5.5896371831686568405557255871842956000 41,
443.83065884910789516497342625223313148 94]
one interval r = 22.46725374481046820389706831465615705970 ..
27.27388428362222519644576531940612831574
Time Approximations 0.037.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, -.87e-35]Solution in 0.918s

Time Plot 0 s.
Exiting SolveHard() after 3.086r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16, none,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941768958825917348524128028154264,
5.5896371831686568405557255871842956000 41,
443.83065884910789516497342625223313148 94]
one interval r = 32.15575279499099766389749084515057022860 ..
35.50872228738222688949552560527508943995
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.145e-34]Solution in 0.422s

Time Plot 0 s.
Exiting SolveHard() after 0.766r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136530750837561751318804614228305,
```

```
5.18778357863522834938733312415376472327 3,
408.65773862289947666781127791124341367 80]
one interval r = 21.71840114637046582702146281267440188009 ..
26.81849303509295854629944854254947772923
Time Approximations 0.062.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.85e-37
Equations at solution: [-.2e-37, -.185e-36, -.181e-34]Solution in
2.428s

Time Plot 0 s.
Exiting SolveHard() after 4.75r=26.4632 in [23.93303356 .. 26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27, none, none,
361.50888346844320945266663145747196879 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136530750837561751318804614228305,
5.18778357863522834938733312415376472327 3,
408.65773862289947666781127791124341367 80]
one interval r = 31.80828598741608049806712470959637028591 ..
35.00011460035078473099189000845062840477
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .301e-34]Solution in 0.463s

Time Plot 0 s.
Exiting SolveHard() after 0.784r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13, none,
361.50888346844320945266663145747196879 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110538746607550304210855369693034,
6.19626256541003344256538965734793337479 5,
385.44474379063693882954510465658521068 69]
one interval r = 31.60836097528284633611613207947991875269 ..
34.66372795601484080659039276346111370742
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, -.176e-34]Solution in 1.839s

Time Plot 0 s.
Exiting SolveHard() after 2.13r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13, none,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110538746607550304210855369693034,
6.1962625654100334425653896573479333747 95,
385.44474379063693882954510465658521068 69]
two intervals r = 16.87563408764529984908795313839934071514 ..
189999999999925439061886507849467327 31/100000000000000000000000000000000
00000 or r = 15.55640493789146043277024905864004455225 ..
189999999999925439061886507849467327 31/100000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.805 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175126741453053594152276657967107, rm
= 2.33653277392583299378828785926324846218 5}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.179e-37, 0., -.2379e-34]Solution in 26.554s

Time Plot 0 s.
Exiting SolveHard() after 27.655r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
```

```
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874744697439842852752438624418142,
4.8838107799852401595818680726104449522 44,
376.61967855562328808076793888713428615 49]
one interval r = 21.11001304867530239181478408071128102236 ..
26.31784243473502567738515624467700179114
Time Approximations 0.033.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=5.2e-38
Equations at solution: [-.2e-37, -.52e-37, .171e-34]Solution in 2.037s

Time Plot 0 s.
Exiting SolveHard() after 2.696r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
```

```
324.67144992264359446439796827872753581 72, none,
328.46938513427499486223072586531809091 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874744697439842852752438624418142,
4.8838107799852401595818680726104449522 44,
376.61967855562328808076793888713428615 49]
one interval r = 31.538994977026623608945618227692891415 65 ..
34.536183860849952669811553060249970828 50
Time Approximations 0.017.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=6.22e-36
Equations at solution: [-.478e-35, .622e-35, -.140e-34]Solution in
0.519s

Time Plot 0 s.
Exiting SolveHard() after 0.834r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72, none,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017541691151159452458395286556262,
6.0258135493756695940033860978678024447 44,
351.42702948330467657369728227295499067 26]
one interval r = 31.362302061065231347928623199518486032 88 ..
```

```
34.17446640608643493640782269795538385427
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, .203e-34]Solution in 1.907s

Time Plot 0 s.
Exiting SolveHard() after 2.159r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72, none,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49, none, none,
292.99969138159424477521733160797689569 46, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017541691151159452458395286556262,
6.02581354937566959400338609786780244474 4,
351.42702948330467657369728227295499067 26]
two intervals r = 17.98135514444413725795425034771615982401 ..
189999999999925439061886507849467327 31/10000000000000000000000000000000
00000 or r = 13.84608015415861758849543609157288065964 ..
189999999999925439061886507849467327 31/10000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 5.844 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713692677226080968495426385747 56, rm
= 2.7345009928707238610960448779511558424 08}});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [-.35e-37, .1e-37, -.3020e-34]Solution in
17.637s

Time Plot 0 s.
Exiting SolveHard() after 18.627r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49, none, none,
292.99969138159424477521733160797689569 46, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941883724587173974797462578119015,
6.37794387399128508700226907397123078510 4,
423.28832784504450375621337809108255129 63]
one interval r = 31.94661817596099161642140751118097356192 ..
35.21212308654725228051925311337126547140
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., .33e-35]Solution in 0.624s

Time Plot 0 s.
Exiting SolveHard() after 0.96r=34.3272 in [33.10127385 .. 35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49, none, none,
292.99969138159424477521733160797689569 46, none, none,
360.06173467213768185026284321356838860 05, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941883724587173974797462578119015,
6.37794387399128508700226907397123078510 4,
423.28832784504450375621337809108255129 63]
two intervals r = 15.22886702412197948295298360013706078201 ..
189999999999925439061886507849467327 31/100000000000000000000000000000000
00000 or r = 17.12965777091130236353884303498940843724 ..
189999999999925439061886507849467327 31/100000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 6.364 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541138613412410619283431579728 59, rm
= 2.0640682985769905834818486311323070231 32}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.15e-37, .1e-37, .1329e-34]Solution in 26.058s

Time Plot 0 s.
Exiting SolveHard() after 28.697r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49,
375.73285290937962496541956114598804935 27, none,
292.99969138159424477521733160797689569 46, none, none,
360.06173467213768185026284321356838860 05, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343381203392893687851383337464 00,
4.0035598154987741966135673077692869973 98,
404.47973594046273118715888794161430571 11]
two intervals r = 16.096839663710027033316743427480975070 12 ..
189999999999925439061886507849467327 31/10000000000000000000000000000000
00000 or r = 16.399886491128543059185732004771488866 85 ..
189999999999925439061886507849467327 31/10000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
```

```
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.4248e-34]Solution in 3.099s

Time Plot 0 s.
Exiting SolveHard() after 4.225r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49,
375.73285290937962496541956114598804935 27, none,
292.99969138159424477521733160797689569 46,
358.64341560540140584789126924746164047 60, none,
360.06173467213768185026284321356838860 05, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234338120339289368785138333746400,
4.00355981549877419661356730776928699739 8,
404.47973594046273118715888794161430571 11]
one interval r = 21.63429629983678170248606754730469408236 ..
26.75768169895193543072025895208937907218
Time Approximations 0.053.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.3e-38
```

```
Equations at solution: [-.1e-37, -.23e-37, -.261e-34]Solution in 2.569s

Time Plot 0 s.
Exiting SolveHard() after 4.958r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49,
375.73285290937962496541956114598804935 27,
328.11709294270767475198023498492415523 30,
292.99969138159424477521733160797689569 46,
358.64341560540140584789126924746164047 60, none,
360.06173467213768185026284321356838860 05, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954457002417821143392665615159907,
6.19617723026374657030572179115784264257 7,
385.42734025442747203804404084558983958 30]
one interval r = 31.60822049083681212543543121899070417453 ..
34.66347615039201972243130414742724885928
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., .407e-34]Solution in 0.593s

Time Plot 0 s.
```

```
Exiting SolveHard() after 0.907r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49,
375.73285290937962496541956114598804935 27,
328.11709294270767475198023498492415523 30,
292.99969138159424477521733160797689569 46,
358.64341560540140584789126924746164047 60, none,
360.06173467213768185026284321356838860 05, none, none,
324.65521223246803178002196305037182358 83, none, none, none, none]

0 --> 1  target = [26.46318954457002417821143392665615159907,
6.19617723026374657030572179115784264257 7,
385.42734025442747203804404084558983958 30]
two intervals r = 16.87629600303017272080656798001874542521 ..
189999999999925439061886507849467327 31/100000000000000000000000000000000
00000 or r = 15.55559000637884764646132886972159644966 ..
189999999999925439061886507849467327 31/100000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 6.701 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852538395535767863769127779961676, rm
```

```
= 2.336690427990790816399177299855815949633}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., .203e-35]Solution in 25.223s

Time Plot 0 s.
Exiting SolveHard() after 27.658r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49,
375.73285290937962496541956114598804935 27,
328.11709294270767475198023498492415523 30,
292.99969138159424477521733160797689569 46,
358.64341560540140584789126924746164047 60, none,
360.06173467213768185026284321356838860 05,
336.59441032226295381212917116111859287 63, none,
324.65521223246803178002196305037182358 83, none, none, none, none]

0 --> 2  target = [34.49522661150183492969570672914382279883,
3.897131315890963086927158866158665844764,
373.78081883745646781226759133677565643 15]
two intervals r = 17.29769086249202155136700343253688613356 ..
189999999999925439061886507849467327 31/10000000000000000000000000000000
00000 or r = 14.99436407395387428039084715784947007104 ..
189999999999925439061886507849467327 31/10000000000000000000000000000000
00000
Time Approximations 0.093.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.36e-37, 0., -.2400e-34]Solution in 2.669s

Time Plot 0 s.
Exiting SolveHard() after 5.806r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49,
375.73285290937962496541956114598804935 27,
328.11709294270767475198023498492415523 30,
292.99969138159424477521733160797689569 46,
358.64341560540140584789126924746164047 60, none,
360.06173467213768185026284321356838860 05,
336.59441032226295381212917116111859287 63, none,
324.65521223246803178002196305037182358 83,
331.93806790498863953631932352132638683 34, none, none, none]

1 --> 2  target = [34.49522661150183492969570672914382279883,
3.897131315890963086927158866158665844764,
373.78081883745646781226759133677565643 15]
one interval r = 21.06068473185239784905991402657004149535 ..
26.26979834273710556729080423431208834573
Time Approximations 0.034.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
```

```
Equations at solution: [0., 0., -.196e-34]Solution in 0.755s

Time Plot 0 s.
Exiting SolveHard() after 2.774r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49,
375.73285290937962496541956114598804935 27,
328.11709294270767475198023498492415523 30,
292.99969138159424477521733160797689569 46,
358.64341560540140584789126924746164047 60,
299.89866204018711903487435457567493436 99,
360.06173467213768185026284321356838860 05,
336.59441032226295381212917116111859287 63, none,
324.65521223246803178002196305037182358 83,
331.93806790498863953631932352132638683 34, none, none, none]

0 --> 2  target = [33.81362495399330833226770225610909285133,
3.72564899353570037495551168109820125418 3,
325.89209972492914400623753074812462136 38]
two intervals r = 18.55227049010618939745433396774010962013 ..
189999999999925439061886507849467327 31/100000000000000000000000000000000
00000 or r = 12.49196935770969725397232940218546464962 ..
189999999999925439061886507849467327 31/100000000000000000000000000000000
00000
Time Approximations 0.038.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [.139e-36, -.6e-37, -.960e-35]Solution in 1.087s

Time Plot 0 s.
Exiting SolveHard() after 3.924r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49,
375.73285290937962496541956114598804935 27,
328.11709294270767475198023498492415523 30,
292.99969138159424477521733160797689569 46,
358.64341560540140584789126924746164047 60,
299.89866204018711903487435457567493436 99,
360.06173467213768185026284321356838860 05,
336.59441032226295381212917116111859287 63, none,
324.65521223246803178002196305037182358 83,
331.93806790498863953631932352132638683 34, none, none,
289.54595771895393272426342798474086969 21]

1 --> 2  target = [33.81362495399330833226770225610909285133,
3.72564899353570037495551168109820125418 3,
325.89209972492914400623753074812462136 38]
one interval r = 20.37468935095945710532813257636933523750 ..
25.37892165294002730821837583499297500462
Time Approximations 0.023.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.2e-37, .4e-37, .365e-34]Solution in 0.532s

Time Plot 0 s.
Exiting SolveHard() after 2.419r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49,
375.73285290937962496541956114598804935 27,
328.11709294270767475198023498492415523 30,
292.99969138159424477521733160797689569 46,
358.64341560540140584789126924746164047 60,
299.89866204018711903487435457567493436 99,
360.06173467213768185026284321356838860 05,
336.59441032226295381212917116111859287 63,
256.10753185635334857397517110457601482 15,
324.65521223246803178002196305037182358 83,
331.93806790498863953631932352132638683 34, none, none,
289.54595771895393272426342798474086969 21]

1 --> 0  target = [17.93041369710169957056025093820300730785,
4.68650870214181884604943275167026529 8724,
353.30541094973270056661383121376179065 21]
one interval r = 20.73150479083856461576148933108224286216 ..
25.90675353523163816981528899059811958898
Time Approximations 0.028.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
```

```
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., -.150e-34]Solution in 0.629s

Time Plot 0 s.
Exiting SolveHard() after 2.586r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49,
375.73285290937962496541956114598804935 27,
328.11709294270767475198023498492415523 30,
292.99969138159424477521733160797689569 46,
358.64341560540140584789126924746164047 60,
299.89866204018711903487435457567493436 99,
360.06173467213768185026284321356838860 05,
336.59441032226295381212917116111859287 63,
256.10753185635334857397517110457601482 15,
324.65521223246803178002196305037182358 83,
331.93806790498863953631932352132638683 34,
304.79958325645093094045701050055310081 07, none,
289.54595771895393272426342798474086969 21]

2 --> 0  target = [17.93041369710169957056025093820300730785,
4.68650870214181884604943275167026529872 4,
353.30541094973270056661383121376179065 21]
one interval r = 31.37435486985877034446028386139506778124 ..
34.20127520021572050242413241052187540651
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
```

```
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, -.36e-35]Solution in 0.344s

Time Plot 0 s.
Exiting SolveHard() after 0.616r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493678174912396790767324665520 11,
441.64295973287186586117045309348159611 09,
436.91748165109619679548846025446438653 09,
422.98493397820835807713713660618841171 70,
361.52580255802594808119792787904480361 99,
401.88173904276130302135855700799079172 53,
389.59001515433769685564593854518290975 25,
328.46939893354688491512692516225524087 16,
401.50757158023257998537558126235236768 55,
358.97362823696842200466536320450325882 30,
398.33147104599316478156404913116681485 27,
371.48387393687102115541186947806969050 13,
336.61215841178989373699480794554266076 73,
361.50888346844320945266663145747196879 45,
324.67144992264359446439796827872753581 72,
302.31384315003228937677999683893714376 12,
328.46938513427499486223072586531809091 45,
343.81340624304628579129645660929300195 49,
375.73285290937962496541956114598804935 27,
328.11709294270767475198023498492415523 30,
292.99969138159424477521733160797689569 46,
358.64341560540140584789126924746164047 60,
299.89866204018711903487435457567493436 99,
360.06173467213768185026284321356838860 05,
336.59441032226295381212917116111859287 63,
256.10753185635334857397517110457601482 15,
324.65521223246803178002196305037182358 83,
331.93806790498863953631932352132638683 34,
304.79958325645093094045701050055310081 07,
323.46169176204264620186221916666184343 70,
289.54595771895393272426342798474086969 21]

Cascade time 217.479
counts: 28, 28

 Iteration 23

Start Generation 1
```

```
1 --> 0  target = [11.999999999969482727164209446463677186 00,
6.217012502899260346715822128433526705860,
485.549080899593170945267475946251572653 5]
```

"Imaginary part neglected: ", $1.889942379135098434201053520278850669975 \times 10^{-17}$

```
one interval r = 23.40850301654958862664484955797116798160 ..
27.67578046413217151565126731590853043013
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=0
Equations at solution: [0., 0., .1516e-35]Solution in 0.986s

Time Plot 0 s.
Exiting SolveHard() after 3.303r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885 80326,
441.64295973389916860670404861726901725 91, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999969482727164209446463677186 00,
6.217012502899260346715822128433526705860,
485.549080899593170945267475946251572653 5]
one interval r = 32.62814779242411666797520295398356248697 ..
36.10248388954779355309847936302165519009
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=1e-38
Equations at solution: [.3e-37, -.1e-37, .75e-35]Solution in 2.05s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.479r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684466012539333110449147718092665,
6.58343472152615084484589058005936328 8917,
467.78730596066548827031249790437421156 64]
one interval r = 32.41978955693502176999069554125760287434 ..
35.85152417386140508015775912287631180332
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=1.1e-37
Equations at solution: [-.13e-36, .11e-36, .99e-35]Solution in 0.653s

Time Plot 0 s.
Exiting SolveHard() after 1.038r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84, none, none,
401.88173904458585368281858858052117026 06, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684466012539333110449147718092665,
6.58343472152615084484589058005936328 8917,
467.78730596066548827031249790437421156 64]
two intervals r = 12.92327160821305894420536808228672941658 ..
9499999999917400031813837744259769 49/50000000000000000000000000000000000
```

```
0 or r = 18.39424858019202261596998197516490525065 ..
949999999991740003181383774425976949/500000000000000000000000000000000000
0
Time Approximations 0.037.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=4.4e-38
Equations at solution: [-.3e-37, -.44e-37, .1287e-34]Solution in
35.585s

Time Plot 0 s.
Exiting SolveHard() after 38.413r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885 80326,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51, none,
401.88173904458585368281858858052117026 06, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962840066392119120651120784222448,
4.12565179674611259896172639017085080358 9,
440.67123065113175773760937714501920531 97]
two intervals r = 14.35659705110217382682173631654749590295 ..
949999999991740003181383774425976949/500000000000000000000000000000000000
0 or r = 17.70352613796059887405312104303335191304 ..
949999999991740003181383774425976949/500000000000000000000000000000000000
0
Time Approximations 0.041.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
```

```
Equations at solution: [.27e-37, .1e-37, -.2612e-34]Solution in 1.266s

Time Plot 0 s.
Exiting SolveHard() after 3.62r=15.9119 in [14.35659706 .. 18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51, none,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962840066392119120651120784222448,
4.12565179674611259896172639017085080358 9,
440.67123065113175773760937714501920531 97]
```

"Imaginary part neglected: ", $1.8899423791350984342010535202788506699 75 \times 10^{-17}$

```
one interval r = 22.39761154367901704598324951181891386853 ..
27.23722351584186732362224230399656088132
Time Approximations 0.035.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 2.336 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064368151991247781943805890775839, rm =
14.37818770118467269333268279915357775432}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, -.39283e-34]Solution in 9.024s

Time Plot 0 s.
Exiting SolveHard() after 11.338r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888817048673466117894562102728885,
4.0048690817286381027154411911097284414 88,
404.86224501528077976921641619210508564 34]
two intervals r = 16.08011007740841723139171922293535079526 ..
9499999999917400031813837744259769 49/50000000000000000000000000000000000
0 or r = 16.41579812690518837575894394213265324983 ..
9499999999917400031813837744259769 49/50000000000000000000000000000000000
0
Time Approximations 0.047.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., -.726e-35]Solution in 2.912s

Time Plot 0 s.
Exiting SolveHard() after 5.263r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50, none, none,
358.97362824068774676650736054759938566 59, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888817048673466117894562102728885,
4.0048690817286381027154411911097284414 88,
404.86224501528077976921641619210508564 34]
```

"Imaginary part neglected: ", $1.889942379135098434201053520278850669975 \times 10^{-17}$

```
one interval r = 21.641943994183865164505721284377870064 99 ..
26.763306600314417520539432421170536231 29
Time Approximations 0.05.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., .60282e-34]Solution in 2.342s

Time Plot 0 s.
Exiting SolveHard() after 3.309r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26, none,
358.97362824068774676650736054759938566 59, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941764370526357498196396989797772,
5.589637182915801453919085696923035730839,
443.83065884875814157144526469486813841 32]
```

"Imaginary part neglected: ", $1.889942379135098434201053520278850669975 \times 10^{-17}$

```
one interval r = 22.467253744839113132806731994811381130 91 ..
27.273884283429124170945172816329166252 87
Time Approximations 0.033.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
```

```
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [.1e-37, -.81e-37, .9502e-35]Solution in 0.969s

Time Plot 0 s.
Exiting SolveHard() after 3.243r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26, none,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941764370526357498196396989797772,
5.5896371829158014539190856969230357308 39,
443.83065884875814157144526469486813841 32]
one interval r = 32.155752795345942862797300226496546116 09 ..
35.508722287551820033179842406605228694 07
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.196e-34]Solution in 1.681s

Time Plot 0 s.
Exiting SolveHard() after 2.058r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365002341076606321679185529 20311,
5.1877835784268131104121287076461741171 17,
408.65773862731901654610428526851090065 34]
```

"Imaginary part neglected: ", $1.8899423791350984342010535202788506699 75 \times 10^{-17}$

```
one interval r = 21.718401146537817090253442910918692022 28 ..
26.818493034987170483907737515147246709 30
Time Approximations 0.059.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.38e-37
Equations at solution: [-.2e-37, -.238e-36, -.25272e-34]Solution in
2.221s

Time Plot 0 s.
Exiting SolveHard() after 3.268r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
```

```
398.33147104519582278542611759383945978 70, none, none,
361.50888347243085695290099271168068761 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136500234107660632167918552920311,
5.18778357842681311041212870764617411711 7,
408.65773862731901654610428526851090065 34]
one interval r = 31.80828598783794440601960854403717512349 ..
35.00011460061318536422333778851991638783
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, -.125e-34]Solution in 0.427s

Time Plot 0 s.
Exiting SolveHard() after 0.736r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57, none,
361.50888347243085695290099271168068761 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110529483011987378971035382051473,
6.19626256524992546690959902577255672764 6,
385.44474379443416738370962731042649397 56]
one interval r = 31.60836097570845228625829153622243732291 ..
34.66372795628462105906577091070717635675
Time Approximations 0.018.
```

```
hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., -.232e-34]Solution in 1.764s

Time Plot 0 s.
Exiting SolveHard() after 2.044r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57, none,
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110529483011987378971035382051473,
6.19626256524992546690959902577255672764 6,
385.44474379443416738370962731042649397 56]
two intervals r = 16.87563408737426438097966683793590168006 ..
94999999999174000318138377442597694 9/50000000000000000000000000000000000
0 or r = 15.55640493798764286165664214835167722148 ..
94999999999174000318138377442597694 9/50000000000000000000000000000000000
0
Time Approximations 0.056.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.748 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175101065205837660809346130924636, rm
= 2.33653277420423853007300972074083020221 8}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.358e-37, 0., -.2416e-34]Solution in 25.922s

Time Plot 0 s.
Exiting SolveHard() after 26.992r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885 80326,
441.64295973389916860670404861726901 72591,
436.91748165347598406111589873366296 62584,
422.98493397758992929186256036247185 54151,
361.52580256208746121140484158835948 46267,
401.88173904458585368281858858052117 02606,
389.59001515827515748523495787607864 26450,
328.46939893710109618148241502531549 65026,
401.50757158225732671522536868598297 45167,
358.97362824068774676650736054759938 56659,
398.33147104519582278542611759383945 97870,
371.48387394349623894262843937334484 27757,
336.61215841411394313517884876194196 96534,
361.50888347243085695290099271168068 76123,
324.67144992742878905445145490741396 00240, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874716806018182619991612187958112,
4.88381077976316315530292039589153852 4931,
376.61967855977861619303351227766662 96148]
```

"Imaginary part neglected: ", $1.8899423791350984342010535202788506699 75 \times 10^{-17}$

```
one interval r = 21.11001304886108921274684145485159320644 ..
26.31784243465716840465816997269985569748
Time Approximations 0.03.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, -.656e-36]Solution in 0.761s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.785r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57,
336.61215841411394313517884876194196965 34,
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40, none,
328.46938513782666290122183650625050384 39, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874716806018182619991612187958112,
4.8838107797631631553029203958915385249 31,
376.61967855977861619303351227766662961 48]
one interval r = 31.53899497745808298870679463030973699881 ..
34.53618386113120854582712704904704970581
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=9.24e-36
Equations at solution: [.710e-35, -.924e-35, .64e-35]Solution in 0.449s

Time Plot 0 s.
Exiting SolveHard() after 0.73r=34.0898 in [32.52213872 .. 34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
```

```
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57,
336.61215841411394313517884876194196965 34,
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40, none,
328.46938513782666290122183650625050384 39,
343.81340624984601735877622062372794912 58, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017534926441898790406276191106809,
6.0258135492117594317972929220879815021 18,
351.42702948657604942527010939517965687 22]
one interval r = 31.362302061497963356141909092197028274 13 ..
34.17446640637314654587258323585791920547
Time Approximations 0.013.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=1.5e-37
Equations at solution: [.7e-37, -.15e-36, .193e-34]Solution in 0.523s

Time Plot 0 s.
Exiting SolveHard() after 2.145r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57,
```

```
336.61215841411394313517884876194196965 34,
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40, none,
328.46938513782666290122183650625050384 39,
343.81340624984601735877622062372794912 58, none, none,
292.99969138590554130056656785198134907 50, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175349264418987904062761911068 09,
6.0258135492117594317972929220879815021 18,
351.42702948657604942527010939517965687 22]
two intervals r = 17.981355144210902904126079807079192699 52 ..
9499999999917400031813837744259769 49/5000000000000000000000000000000000
0 or r = 13.846080154297180715933509652504745520 93 ..
9499999999917400031813837744259769 49/5000000000000000000000000000000000
0
Time Approximations 0.039.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 4.198 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713496148650344559716467912730 35, rm
= 2.7345009933638875027148620980693966377 24}});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [.18e-37, 0., -.3425e-34]Solution in 15.06s

Time Plot 0 s.
Exiting SolveHard() after 17.371r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57,
336.61215841411394313517884876194196965 34,
```

```
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40,
302.31384315175879260215494634106514968 54,
328.46938513782666290122183650625050384 39,
343.81340624984601735877622062372794912 58, none, none,
292.99969138590554130056656785198134907 50, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941864343357327374218600586698291,
6.3779438738093234645664259904470286340 08,
423.28832784385349339246610973753037621 32]
one interval r = 31.946618176321659203549943898628750706 36 ..
35.212123086718262580563545403358699272 30
Time Approximations 0.018.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=7e-38
Equations at solution: [-.6e-37, .7e-37, .223e-34]Solution in 1.985s

Time Plot 0 s.
Exiting SolveHard() after 2.345r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57,
336.61215841411394313517884876194196965 34,
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40,
302.31384315175879260215494634106514968 54,
328.46938513782666290122183650625050384 39,
343.81340624984601735877622062372794912 58, none, none,
292.99969138590554130056656785198134907 50, none, none,
360.06173467223116920463739997181024068 52, none, none, none, none,
```

```
none, none, none]

0 --> 1  target = [27.020379418643433573273742186005866982 91,
6.3779438738093234645664259904470286340 08,
423.28832784385349339246610973753037621 32]
two intervals r = 15.22886702408831740601427600796621110079 ..
9499999999917400031813837744259769 49/50000000000000000000000000000000000
0 or r = 17.12965777075069513992515254768301619074 ..
9499999999917400031813837744259769 49/50000000000000000000000000000000000
0
Time Approximations 0.06.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.097 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370540986809723373177926607199574 57, rm
= 2.0640682987641935389019047964218607774 16}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., .2326e-34]Solution in 24.577s

Time Plot 0 s.
Exiting SolveHard() after 25.905r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57,
336.61215841411394313517884876194196965 34,
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40,
302.31384315175879260215494634106514968 54,
328.46938513782666290122183650625050384 39,
343.81340624984601735877622062372794912 58,
375.73285290659619070446048813940004557 19, none,
```

```
292.99969138590554130056656785198134907 50, none, none,
360.06173467223116920463739997181024068 52, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234361156636288814720684349610915,
4.0035598154291543032446980355281798946 03,
404.47973594247085059967258878690711003 55]
two intervals r = 16.09683966351126670231061480189977624524 ..
9499999999917400031813837744259769 49/50000000000000000000000000000000000
0 or r = 16.39988649111041656291607845348573431745 ..
9499999999917400031813837744259769 49/50000000000000000000000000000000000
0
Time Approximations 0.046.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.892e-35]Solution in 2.821s

Time Plot 0 s.
Exiting SolveHard() after 5.063r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885 80326,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57,
336.61215841411394313517884876194196965 34,
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40,
302.31384315175879260215494634106514968 54,
328.46938513782666290122183650625050384 39,
343.81340624984601735877622062372794912 58,
375.73285290659619070446048813940004557 19, none,
292.99969138590554130056656785198134907 50,
358.64341560929980903905337376834868585 23, none,
360.06173467223116920463739997181024068 52, none, none, none, none,
```

```
none, none, none]

1 --> 2  target = [34.93953234361156636288814720684349610915,
4.0035598154291543032446980355281798946 03,
404.47973594247085059967258878690711003 55]
```

"Imaginary part neglected: ", $1.8899423791350984342010535202788506699 75 \times 10^{-17}$

```
one interval r = 21.63429629995941580241111738885753977267 ..
26.75768169881473411024915534399378969623
Time Approximations 0.048.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, .36890e-34]Solution in
0.998s

Time Plot 0 s.
Exiting SolveHard() after 3.395r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885 80326,
441.64295973389916860670404861726901 72591,
436.91748165347598406111589873366296 62584,
422.98493397758992929186256036247185 54151,
361.52580256208746121140484158835948 46267,
401.88173904458585368281858858052117 02606,
389.59001515827515748523495787607864 26450,
328.46939893710109618148241502531549 65026,
401.50757158225732671522536868598297 45167,
358.97362824068774676650736054759938 56659,
398.33147104519582278542611759383945 97870,
371.48387394349623894262843937334484 27757,
336.61215841411394313517884876194196 96534,
361.50888347243085695290099271168068 76123,
324.67144992742878905445145490741396 00240,
302.31384315175879260215494634106514 96854,
328.46938513782666290122183650625050 38439,
343.81340624984601735877622062372794 91258,
375.73285290659619070446048813940004 55719,
328.11709294645054014967075559687655 40575,
292.99969138590554130056656785198134 90750,
358.64341560929980903905337376834868 58523, none,
```

```
360.06173467223116920463739997181024068 52, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954447617597414481787623875538804,
6.196177230103265363672052904037952923430,
385.42734025814871771969453911161725962 09]
one interval r = 31.608220491261811166743241245354539014 67 ..
34.663476150660713013061127727689807575 72
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, -.206e-34]Solution in 1.897s

Time Plot 0 s.
Exiting SolveHard() after 2.195r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885 80326,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57,
336.61215841411394313517884876194196965 34,
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40,
302.31384315175879260215494634106514968 54,
328.46938513782666290122183650625050384 39,
343.81340624984601735877622062372794912 58,
375.73285290659619070446048813940004557 19,
328.11709294645054014967075559687655405 75,
292.99969138590554130056656785198134907 50,
358.64341560929980903905337376834868585 23, none,
360.06173467223116920463739997181024068 52, none, none,
324.65521223718234089638008616182117304 22, none, none, none, none]
```

```
0 --> 1  target = [26.46318954447617597414481787623875538804,
6.196177230103265363672052904037952923430,
385.42734025814871771969453911161725962 09]
two intervals r = 16.876296002762035136180414901202624532 31 ..
949999999991740003181383774425976949/500000000000000000000000000000000000
0 or r = 15.555590006471508626446087785223378530 00 ..
949999999991740003181383774425976949/500000000000000000000000000000000000
0
Time Approximations 0.055.

hint used Hint := [17.930920830435909701694400819009216331 81, 3, 1, 1,
15.700860627013397511253157405159644880 12, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.624 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852512864625244353041447444577134, rm
= 2.33669042826995987793902980710299166 3493}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.717e-37, 0., -.132e-35]Solution in 25.756s

Time Plot 0 s.
Exiting SolveHard() after 26.813r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885 80326,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57,
336.61215841411394313517884876194196965 34,
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40,
302.31384315175879260215494634106514968 54,
328.46938513782666290122183650625050384 39,
343.81340624984601735877622062372794912 58,
375.73285290659619070446048813940004557 19,
328.11709294645054014967075559687655405 75,
292.99969138590554130056656785198134907 50,
```

```
358.6434156092998090390533737683486858523, none,
360.0617346722311692046373999718102406852,
336.5944103245094727942762590818348482498, none,
324.6552122371823408963800861618211730422, none, none, none, none]

0 --> 2  target = [34.4952266118220701856313171686625177395 6,
3.8971313158388771218861460382089745573 16,
373.7808188441741088689747012808939562024]
two intervals r = 17.297690862127290134263070134070623269 81 ..
9499999999917400031813837744259769 49/5000000000000000000000000000000000000
0 or r = 14.994364074219075656347967427785733975 43 ..
9499999999917400031813837744259769 49/5000000000000000000000000000000000000
0
Time Approximations 0.097.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [.18e-37, -.1e-37, .106e-35]Solution in 2.648s

Time Plot 0 s.
Exiting SolveHard() after 5.585r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349373645556106697573907288580326,
441.6429597338991686067040486172690172591,
436.9174816534759840611158987336629662584,
422.9849339775899292918625603624718554151,
361.5258025620874612114048415883594846267,
401.8817390445858536828185885805211702606,
389.5900151582751574852349578760786426450,
328.4693989371010961814824150253154965026,
401.5075715822573267152253686859829745167,
358.9736282406877467665073605475993856659,
398.3314710451958227854261175938394597870,
371.4838739434962389426284393733448427757,
336.6121584141139431351788487619419696534,
361.5088834724308569529009927116806876123,
324.6714499274287890544514549074139600240,
302.3138431517587926021549463410651496854,
328.4693851378266629012218365062505038439,
343.8134062498460173587762206237279491258,
375.7328529065961907044604881394000455719,
328.1170929464505401496707555968765540575,
292.9996913859055413005665678519813490750,
```

```
358.6434156092998090390533737683486858523, none,
360.0617346722311692046373999718102406852,
336.5944103245094727942762590818348482498, none,
324.6552122371823408963800861618211730422,
331.9380679131706569163762918971871747935, none, none, none]

1 --> 2  target = [34.4952266118220701856313171686625177 3956,
3.8971313158388771218861460382089745573 16,
373.7808188441741088689747012808939562024]
```

"Imaginary part neglected: ", $1.8899423791350984342010535202788506699 75 \times 10^{-17}$

```
one interval r = 21.06068473208402284012065850084789528198 ..
26.26979834270579966422649919294044564409
Time Approximations 0.037.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [.1e-37, .3e-37, -.39066e-34]Solution in 0.851s

Time Plot 0 s.
Exiting SolveHard() after 1.6r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349373645556106697573907288580326,
441.6429597338991686067040486172690172591,
436.9174816534759840611158987336629662584,
422.9849339775899292918625603624718554151,
361.5258025620874612114048415883594846267,
401.8817390445858536828185885805211702606,
389.5900151582751574852349578760786426450,
328.4693989371010961814824150253154965026,
401.5075715822573267152253686859829745167,
358.9736282406877467665073605475993856659,
398.3314710451958227854261175938394597870,
371.4838739434962389426284393733448427757,
336.6121584141139431351788487619419696534,
361.5088834724308569529009927116806876123,
324.6714499274287890544514549074139600240,
302.3138431517587926021549463410651496854,
328.4693851378266629012218365062505038439,
343.8134062498460173587762206237279491258,
375.7328529065961907044604881394000455719,
328.1170929464505401496707555968765540575,
```

```
292.99969138590554130056656785198134907 50,
358.6434156092998090390533737683486858523,
299.8986620482563192887388326422908678566,
360.0617346722311692046373999718102406852,
336.5944103245094727942762590818348482498, none,
324.6552122371823408963800861618211730422,
331.9380679131706569163762918971871747935, none, none, none]

0 --> 2  target = [33.8136249543205840957588500274930898852 8,
3.7256489934793778782113925908274391929 12,
325.8920997297755939231614400744131060447]
two intervals r = 18.552270489865783031293534283216656230 71 ..
9499999999917400031813837744259769 49/50000000000000000000000000000000000
0 or r = 12.491969357974471516160454129679716434 56 ..
9499999999917400031813837744259769 49/50000000000000000000000000000000000
0
Time Approximations 0.043.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=8e-38
Equations at solution: [-.191e-36, .8e-37, .1420e-34]Solution in 2.3s

Time Plot 0 s.
Exiting SolveHard() after 5.009r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57,
336.61215841411394313517884876194196965 34,
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40,
302.31384315175879260215494634106514968 54,
328.46938513782666290122183650625050384 39,
343.81340624984601735877622062372794912 58,
```

```
375.7328529065961907044604881394000455719,
328.1170929464505401496707555968765540575,
292.9996913859055413005665678519813490750,
358.6434156092998090390533737683486858523,
299.8986620482563192887388326422908678566,
360.0617346722311692046373999718102406852,
336.5944103245094727942762590818348482498, none,
324.6552122371823408963800861618211730422,
331.9380679131706569163762918971871747935, none, none,
289.5459577256317440496897471097635567444]

1 --> 2  target = [33.8136249543205840957588500274930898852 8,
3.7256489934793778782113925908274391929 12,
325.8920997297755939231614400744131060447]
```

"Imaginary part neglected: ", $1.889942379135098434201053520278850669975 \times 10^{-17}$

```
one interval r = 20.37468935117055707424141534292232680640 ..
25.37892165293084144048928036348627550266
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [-.3e-37, -.4e-37, .21106e-34]Solution in 1.73s

Time Plot 0 s.
Exiting SolveHard() after 2.23r=24.3395 in [22.07732228 .. 25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349373645556106697573907288580326,
441.6429597338991686067040486172690172591,
436.9174816534759840611158987336629662584,
422.9849339775899292918625603624718554151,
361.5258025620874612114048415883594846267,
401.8817390445858536828185885805211702606,
389.5900151582751574852349578760786426450,
328.4693989371010961814824150253154965026,
401.5075715822573267152253686859829745167,
358.9736282406877467665073605475993856659,
398.3314710451958227854261175938394597870,
371.4838739434962389426284393733448427757,
336.6121584141139431351788487619419696534,
361.5088834724308569529009927116806876123,
324.6714499274287890544514549074139600240,
```

```
302.31384315175879260215494634106514968 54,
328.46938513782666290122183650625050384 39,
343.81340624984601735877622062372794912 58,
375.73285290659619070446048813940004557 19,
328.11709294645054014967075559687655405 75,
292.99969138590554130056656785198134907 50,
358.64341560929980903905337376834868585 23,
299.89866204825631928873883264229086785 66,
360.06173467223116920463739997181024068 52,
336.59441032450947279427625908183484824 98,
256.10753186269115111591789617603431576 32,
324.65521223718234089638008616182117304 22,
331.93806791317065691637629189718717479 35, none, none,
289.54595772563174404968974710976355674 44]

1 --> 0  target = [17.930413696883344955562545833502358666 21,
4.6865087019014022941130907608592023070 78,
353.30541095241385351605497783492871826 12]

"Imaginary part neglected: ", 1.8899423791350984342010535202788506699 75 × 10^-17
one interval r = 20.731504791014041216776947665229346247 88 ..
25.906753535151200553390770634094508098 84
Time Approximations 0.031.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., .7072e-35]Solution in 0.635s

Time Plot 0 s.
Exiting SolveHard() after 1.329r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
```

```
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57,
336.61215841411394313517884876194196965 34,
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40,
302.31384315175879260215494634106514968 54,
328.46938513782666290122183650625050384 39,
343.81340624984601735877622062372794912 58,
375.73285290659619070446048813940004557 19,
328.11709294645054014967075559687655405 75,
292.99969138590554130056656785198134907 50,
358.64341560929980903905337376834868585 23,
299.89866204825631928873883264229086785 66,
360.06173467223116920463739997181024068 52,
336.59441032450947279427625908183484824 98,
256.10753186269115111591789617603431576 32,
324.65521223718234089638008616182117304 22,
331.93806791317065691637629189718717479 35,
304.79958325841716483973722798308482383 28, none,
289.54595772563174404968974710976355674 44]

2 --> 0  target = [17.93041369688334495556254583350235866621,
4.68650870190140229411309076085920230707 8,
353.30541095241385351605497783492871826 12]
one interval r = 31.37435487028716727083785628049474802230 ..
34.20127520049264004090398230151358407962
Time Approximations 0.014.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=1.4e-37
Equations at solution: [-.9e-37, .14e-36, -.201e-34]Solution in 0.339s

Time Plot 0 s.
Exiting SolveHard() after 1.943r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493736455561066975739072885803 26,
441.64295973389916860670404861726901725 91,
436.91748165347598406111589873366296625 84,
422.98493397758992929186256036247185541 51,
361.52580256208746121140484158835948462 67,
401.88173904458585368281858858052117026 06,
389.59001515827515748523495787607864264 50,
```

```
328.46939893710109618148241502531549650 26,
401.50757158225732671522536868598297451 67,
358.97362824068774676650736054759938566 59,
398.33147104519582278542611759383945978 70,
371.48387394349623894262843937334484277 57,
336.61215841411394313517884876194196965 34,
361.50888347243085695290099271168068761 23,
324.67144992742878905445145490741396002 40,
302.31384315175879260215494634106514968 54,
328.46938513782666290122183650625050384 39,
343.81340624984601735877622062372794912 58,
375.73285290659619070446048813940004557 19,
328.11709294645054014967075559687655405 75,
292.99969138590554130056656785198134907 50,
358.64341560929980903905337376834868585 23,
299.89866204825631928873883264229086785 66,
360.06173467223116920463739997181024068 52,
336.59441032450947279427625908183484824 98,
256.10753186269115111591789617603431576 32,
324.65521223718234089638008616182117304 22,
331.93806791317065691637629189718717479 35,
304.79958325841716483973722798308482383 28,
323.46169176777426216354075295950893706 49,
289.54595772563174404968974710976355674 44]

Cascade time 213.61
counts: 28, 28

 Iteration 24

Start Generation 1
1 --> 0  target = [11.99999999995115429095242765677131361400,
6.21701250302537866820898504941178658927 0,
485.54908090433668480357967771674918450 13]
one interval r = 23.40850301674211395757168596471767553253 ..
27.67578046445216002173477129131282244811
Time Approximations 0.039.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.29e-37
Equations at solution: [-.4e-37, .129e-36, .1107e-35]Solution in 2.333s

Time Plot 0 s.
Exiting SolveHard() after 4.712r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999951154290952427656771313614 00,
6.2170125030253786682089850494117865892 70,
485.54908090433668480357967771674918450 13]
one interval r = 32.628147792171449535088768665511418577 02 ..
36.102483889463417213456928629652432559 56
Time Approximations 0.021.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .53e-35]Solution in 0.565s

Time Plot 0 s.
Exiting SolveHard() after 0.954r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844980222430224471652962655057 70,
6.5834347215627973852915043796658729924 18,
467.78730596570254438762199467858296976 93]
one interval r = 32.419789556671402023026199085938669667 82 ..
35.851524173772885879437364272431040727 74
Time Approximations 0.021.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
```

```
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, .117e-34]Solution in 1.971s

Time Plot 0 s.
Exiting SolveHard() after 2.322r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49, none, none,
401.88173905165131328401025919626409025 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844980222430224471652962655057 70,
6.5834347215627973852915043796658729924 18,
467.78730596570254438762199467858296976 93]
two intervals r = 12.923271608213268480627739754657627029 93 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000
0 or r = 18.394248580514771386845659663380754784 72 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000
0
Time Approximations 0.043.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=7.6e-38
Equations at solution: [-.6e-37, -.76e-37, -.1205e-34]Solution in
35.886s

Time Plot 0 s.
Exiting SolveHard() after 38.639r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37, none,
401.88173905165131328401025919626409025 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962831061214520664199770136762075,
4.12565179697612963860577029799828606505 3,
440.67123065703175797572392088987906484 31]
two intervals r = 14.35659705110766001506695438027057529513 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000
0 or r = 17.70352613832825704534390334148498375235 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000
0
Time Approximations 0.055.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.15e-37, .1e-37, -.12e-36]Solution in 2.874s

Time Plot 0 s.
Exiting SolveHard() after 4.057r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37, none,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962831061214520664199770136762075,
4.12565179697612963860577029799828606505 3,
440.67123065703175797572392088987906484 31]
one interval r = 22.39761154384851467567534375395090866201 ..
27.23722351616722160980318919085984532146
Time Approximations 0.035.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
```

```
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.27 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070644062787249915497632250891868 30, rm =
14.378187704713699158803996377705099056 68}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.05e-37
Equations at solution: [-.1e-37, -.105e-36, .56468e-34]Solution in
7.244s

Time Plot 0 s.
Exiting SolveHard() after 9.535r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888076918087087827672317453829 69,
4.0048690819625455180610088138220164596 23,
404.86224502236920065336434052863509773 60]
two intervals r = 16.080110077428009932974212028065340100 01 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000
0 or r = 16.415798127346559076191266056537170375 91 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000
0
Time Approximations 0.052.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., -.296e-35]Solution in 2.804s

Time Plot 0 s.
Exiting SolveHard() after 5.236r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47, none, none,
358.97362824913331388317819560406879598 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888076918087087827672317453829 69,
4.0048690819625455180610088138220164596 23,
404.86224502236920065336434052863509773 60]
one interval r = 21.641943994330807647736689027977724343 94 ..
26.763306600650229977994795873708372552 21
Time Approximations 0.046.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, .29054e-34]Solution in
0.957s

Time Plot 0 s.
Exiting SolveHard() after 3.436r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
```

```
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64, none,
358.97362824913331388317819560406879598 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941761686022031506644083256281544,
5.5896371830651980102995129018885891533 22,
443.83065885516337514869987465719363174 74]
one interval r = 22.46725374502358431953373718573650406533 ..
27.27388428376070682131762970570001353304
Time Approximations 0.035.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.0e-38
Equations at solution: [-.1e-37, .80e-37, -.1950e-35]Solution in 0.953s

Time Plot 0 s.
Exiting SolveHard() after 3.184r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64, none,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941761686022031506644083256281544,
5.5896371830651980102995129018885891533 22,
443.83065885516337514869987465719363174 74]
one interval r = 32.15575279507740753970802413516905153258 ..
35.50872228747073332455320928454745964051
Time Approximations 0.018.
```

```
hint used Hint := [34.9395323434150806036663686679149922721 5, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., -.158e-34]Solution in 0.433s

Time Plot 0 s.
Exiting SolveHard() after 1.999r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372581486543484 7,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136501040734227987908129325232115,
5.18778357858454244902125690239738657037 1,
408.65773863448564144956918476884039822 29]
one interval r = 21.71840114669183160036451047928940810652 ..
26.81849303532468070637350702435541704717
Time Approximations 0.06.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=0
Equations at solution: [0., 0., .16107e-34]Solution in 1.047s

Time Plot 0 s.
```

```
Exiting SolveHard() after 3.54r=26.4632 in [23.93303356 .. 26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11, none, none,
361.50888348050983538265681829648285296 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136501040734227987908129325232115,
5.18778357858454244902125690239738657037 1,
408.65773863448564144956918476884039822 29]
one interval r = 31.80828598754666420313658405708016042362 ..
35.00011460052308018627206846436157372545
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., -.233e-34]Solution in 1.706s

Time Plot 0 s.
Exiting SolveHard() after 2.037r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
```

```
389.590015165639855407169997115002274914 7,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82, none,
361.50888348050983538265681829648285296 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110565067762918150023072075646220,
6.19626256531211140999278455363149867514 4,
385.44474380299189160450740588277933848 77]
one interval r = 31.60836097540866759506490665662753263853 ..
34.66372795619988728311448203513146111567
Time Approximations 0.016.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.153e-34]Solution in 0.549s

Time Plot 0 s.
Exiting SolveHard() after 0.833r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.590015165639855407169997115002274914 7,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82, none,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110565067762918150023072075646220,
6.19626256531211140999278455363149867514 4,
```

```
385.44474380299189160450740588277933848 77]
two intervals r = 16.875634087378593814543579660563077241 51 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000000
0 or r = 15.556404938508325117452605401514013478 33 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000000
0
Time Approximations 0.059.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [-.717e-37, .1e-37, .5051e-34]Solution in 2.391s

Time Plot 0 s.
Exiting SolveHard() after 4.991r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747203865391905430371913398 03209,
4.883810779932482115160518634161938438335,
376.61967856807559141131721279368011588 05]
one interval r = 21.110013048986371804973773375669274210 62 ..
26.317842435007413665302476702031451435 84
Time Approximations 0.036.

hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.761063912186698964426645072343113601 29, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=5.2e-38
Equations at solution: [-.1e-37, -.52e-37, -.21033e-34]Solution in
0.851s

Time Plot 0 s.
Exiting SolveHard() after 1.55r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84, none,
328.46938514707846270181950654383708228 00, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874720386539190543037191339803209,
4.8838107799324821151605186341619384383 35,
376.61967856807559141131721279368011588 05]
one interval r = 31.53899497714839571493383432449646113825 ..
34.53618386103674153078395947166289553591
Time Approximations 0.015.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=5.74e-36
Equations at solution: [-.440e-35, .574e-35, -.331e-34]Solution in
0.474s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.226r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84, none,
328.46938514707846270181950654383708228 00,
343.81340625945153128510803599421727409 53, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017571930019885244643018019089932,
6.02581354928399546740194310861618542410 3,
351.42702949625486483323421479197807463 81]
one interval r = 31.36230206117547957153782543839788033294 ..
34.17446640628040279249254139232074826188
Time Approximations 0.015.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=4e-38
Equations at solution: [.1e-37, -.4e-37, -.262e-34]Solution in 0.525s

Time Plot 0 s.
Exiting SolveHard() after 0.761r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84, none,
328.46938514707846270181950654383708228 00,
343.81340625945153128510803599421727409 53, none, none,
292.99969139748255076038095756604805607 50, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017571930019885244643018019089932,
6.02581354928399546740194310861618542410 3,
351.42702949625486483323421479197807463 81]
two intervals r = 17.98135514427050804787182690059083927993 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000000
0 or r = 13.84608015487342737827368776937499296493 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000000
0
Time Approximations 0.04.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=5e-38
Equations at solution: [-.178e-36, .5e-37, -.45e-35]Solution in 2.324s

Time Plot 0 s.
Exiting SolveHard() after 4.702r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
```

```
389.590015165639855407169997115002274914 7,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84,
302.31384316306297298071177520967594776 57,
328.46938514707846270181950654383708228 00,
343.81340625945153128510803599421727409 53, none, none,
292.99969139748255076038095756604805607 50, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941898644891708627579549042214993,
6.3779438738617922367149512271800443861 33,
423.28832785131175522713391078176199041 44]
one interval r = 31.94661817604594551301566224260115627032 ..
35.21212308664104563902872742730994839415
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.75e-35]Solution in 0.622s

Time Plot 0 s.
Exiting SolveHard() after 0.935r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.590015165639855407169997115002274914 7,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
```

```
361.508883480509835382656818296482852969 6,
324.671449937918151661422773521152788488 4,
302.313843163062972980711775209675947765 7,
328.469385147078462701819506543837082280 0,
343.813406259451531285108035994217274095 3, none, none,
292.999691397482550760380957566048056075 0, none, none,
360.061734681641057425720052157010279604 6, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418986448917086275795490422149 93,
6.377943873861792236714951227180044386133,
423.288327851311755227133910781761990414 4]
two intervals r = 15.22886702405096295688416687886633595510 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000
0 or r = 17.12965777118811715418552974486152023235 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000
0
Time Approximations 0.055.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [-.16e-37, 0., -.2003e-34]Solution in 2.477s

Time Plot 0 s.
Exiting SolveHard() after 5.066r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434942431304960431284649981032260 1,
441.642959738689252374423725881486543484 7,
436.917481659377855116297809279905286734 9,
422.984933984226671947405954974526377523 7,
361.525802570269004108287752401214643342 9,
401.881739051651313284010259196264090252 4,
389.590015165639855407169997115002274914 7,
328.469398946355563961151170977024467316 4,
401.507571589925857869947163294458985652 0,
358.973628249133313883178195604068795980 4,
398.331471052324802172076703886065312361 1,
371.483873952013900933863710728085816208 2,
336.612158424447300618325737975942310993 4,
361.508883480509835382656818296482852969 6,
324.671449937918151661422773521152788488 4,
302.313843163062972980711775209675947765 7,
328.469385147078462701819506543837082280 0,
```

```
343.8134062594515312851080359942172740953,
375.7328529159194335211094166373933093592, none,
292.9996913974825507603809575660480560750, none, none,
360.0617346816410574257200521570102796046, none, none, none, none,
none, none, none]

0 --> 2  target = [34.9395323435267007285660361092795975721 4,
4.0035598156651707697355748509839606842 92,
404.4797359501757790482542148552227886499]
two intervals r = 16.09683966350473557786630859820599029919 ..
9500000000052274584915161450345223 77/50000000000000000000000000000000000
0 or r = 16.39988649157775947017647572105642988679 ..
9500000000052274584915161450345223 77/50000000000000000000000000000000000
0
Time Approximations 0.05.

hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.7615110884558244986626642365610012490 6, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.50e-37, -.1e-37, .136e-35]Solution in 2.669s

Time Plot 0 s.
Exiting SolveHard() after 3.647r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349424313049604312846499810322601,
441.6429597386892523744237258814865434847,
436.9174816593778551162978092799052867349,
422.9849339842266719474059549745263775237,
361.5258025702690041082877524012146433429,
401.8817390516513132840102591962640902524,
389.5900151656398554071699971150022749147,
328.4693989463555639611511709770244673164,
401.5075715899258578699471632944589856520,
358.9736282491333138831781956040687959804,
398.3314710523248021720767038860653123611,
371.4838739520139009338637107280858162082,
336.6121584244473006183257379759423109934,
361.5088834805098353826568182964828529696,
324.6714499379181516614227735211527884884,
302.3138431630629729807117752096759477657,
328.4693851470784627018195065438370822800,
343.8134062594515312851080359942172740953,
375.7328529159194335211094166373933093592, none,
292.9996913974825507603809575660480560750,
```

```
358.6434156182781349882074141705521437827, none,
360.0617346816410574257200521570102796046, none, none, none, none,
none, none, none]

1 --> 2  target = [34.9395323435267007285660361092795975721 4,
4.0035598156651707697355748509839606842 92,
404.4797359501757790482542148552227886499]
one interval r = 21.6342963001181189197506042217000674241 6 ..
26.7576816991595612951076631652475730748 0
Time Approximations 0.045.

hint used Hint := [25.8653228005422684844258072120286493757 6, 3, -1, 1,
16.7792152583743340383255391208949113668 0, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, -.40237e-34]Solution in
1.057s

Time Plot 0 s.
Exiting SolveHard() after 3.383r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349424313049604312846499810322601,
441.6429597386892523744237258814865434847,
436.9174816593778551162978092799052867349,
422.9849339842266719474059549745263775237,
361.5258025702690041082877524012146433429,
401.8817390516513132840102591962640902524,
389.5900151656398554071699971150022749147,
328.4693989463555639611511709770244673164,
401.5075715899258578699471632944589856520,
358.9736282491333138831781956040687959804,
398.3314710523248021720767038860653123611,
371.4838739520139009338637107280858162082,
336.6121584244473006183257379759423109934,
361.5088834805098353826568182964828529696,
324.6714499379181516614227735211527884884,
302.3138431630629729807117752096759477657,
328.4693851470784627018195065438370822800,
343.8134062594515312851080359942172740953,
375.7328529159194335211094166373933093592,
328.1170929562725149894263466942194757467,
292.9996913974825507603809575660480560750,
358.6434156182781349882074141705521437827, none,
360.0617346816410574257200521570102796046, none, none, none, none,
```

```
none, none, none]

2 --> 1  target = [26.46318954483031463753744186304140069392,
6.19617723016493618417780909435965457479 0,
385.42734026660094898956578543758099442 04]
one interval r = 31.60822049096115926674234607211868005398 ..
34.66347615057444131026566970627005884201
Time Approximations 0.016.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .97e-35]Solution in 0.515s

Time Plot 0 s.
Exiting SolveHard() after 2.081r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84,
302.31384316306297298071177520967594776 57,
328.46938514707846270181950654383708228 00,
343.81340625945153128510803599421727409 53,
375.73285291591943352110941663739330935 92,
328.11709295627251498942634669421947574 67,
292.99969139748255076038095756604805607 50,
358.64341561827813498820741417055214378 27, none,
360.06173468164105742572005215701027960 46, none, none,
324.65521224757330042664699951382387263 19, none, none, none, none]

0 --> 1  target = [26.46318954483031463753744186304140069392,
```

```
6.196177230164936184177809094359654574790,
385.427340266600948989565785437580994420 4]
two intervals r = 16.876296002770420550792581135824978975 81 ..
95000000000522745849151614503452237 7/500000000000000000000000000000000000
0 or r = 15.555590006987262119399541505064287439 55 ..
95000000000522745849151614503452237 7/500000000000000000000000000000000000
0
Time Approximations 0.059.

hint used Hint := [17.930920830435909701694400819009216331 81, 3, 1, 1,
15.700860627013397511253157405159644880 12, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.895e-37, 0., .945e-35]Solution in 1.146s

Time Plot 0 s.
Exiting SolveHard() after 3.576r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84,
302.31384316306297298071177520967594776 57,
328.46938514707846270181950654383708228 00,
343.81340625945153128510803599421727409 53,
375.73285291591943352110941663739330935 92,
328.11709295627251498942634669421947574 67,
292.99969139748255076038095756604805607 50,
358.64341561827813498820741417055214378 27, none,
360.06173468164105742572005215701027960 46,
336.59441033473517476178105298692183197 39, none,
324.65521224757330042664699951382387263 19, none, none, none, none]

0 --> 2  target = [34.495226611729619731713902448818610371 19,
```

```
3.897131316077808309024465369658143719224,
373.780818852746552147426240247068187850 4]
two intervals r = 17.29769086216055793230015459507564055697 ..
95000000000052274584915161450345223 77/50000000000000000000000000000000000000
0 or r = 14.99436407474546379804147701191238746423 ..
95000000000052274584915161450345223 77/50000000000000000000000000000000000000
0
Time Approximations 0.079.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.54e-37, 0., -.2095e-34]Solution in 2.482s

Time Plot 0 s.
Exiting SolveHard() after 5.359r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84,
302.31384316306297298071177520967594776 57,
328.46938514707846270181950654383708228 00,
343.81340625945153128510803599421727409 53,
375.73285291591943352110941663739330935 92,
328.11709295627251498942634669421947574 67,
292.99969139748255076038095756604805607 50,
358.64341561827813498820741417055214378 27, none,
360.06173468164105742572005215701027960 46,
336.59441033473517476178105298692183197 39, none,
324.65521224757330042664699951382387263 19,
331.93806792293561028584944018675880480 85, none, none, none]
```

```
1 --> 2  target = [34.49522661172961973171390244881861037119,
3.8971313160778083090244653696581437192 24,
373.78081885274655214742624024706818785 04]
one interval r = 21.06068473220922177553379300661371673056 ..
26.26979834306021958765228040749733677359
Time Approximations 0.03.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [.3e-37, .7e-37, .10744e-34]Solution in 0.717s

Time Plot 0 s.
Exiting SolveHard() after 2.781r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372581486543484 7,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84,
302.31384316306297298071177520967594776 57,
328.46938514707846270181950654383708228 00,
343.81340625945153128510803599421727409 53,
375.73285291591943352110941663739330935 92,
328.11709295627251498942634669421947574 67,
292.99969139748255076038095756604805607 50,
358.64341561827813498820741417055214378 27,
299.89866205884605933393903372723763596 50,
360.06173468164105742572005215701027960 46,
336.59441033473517476178105298692183197 39, none,
324.65521224757330042664699951382387263 19,
331.93806792293561028584944018675880480 85, none, none, none]
```

```
0 --> 2  target = [33.81362495422055182903056209519003620659,
3.72564899372529141431217094321442218311 7,
325.89209974036353182507374275087635613 63]
two intervals r = 18.552270489981677177513014420340072627 29 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000
0 or r = 12.491969358544533676249049264645700163 77 ..
95000000000522745849151614503452237 7/50000000000000000000000000000000000
0
Time Approximations 0.037.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [.139e-36, -.5e-37, -.27e-35]Solution in 2.475s

Time Plot 0 s.
Exiting SolveHard() after 5.125r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84,
302.31384316306297298071177520967594776 57,
328.46938514707846270181950654383708228 00,
343.81340625945153128510803599421727409 53,
375.73285291591943352110941663739330935 92,
328.11709295627251498942634669421947574 67,
292.99969139748255076038095756604805607 50,
358.64341561827813498820741417055214378 27,
299.89866205884605933393903372723763596 50,
360.06173468164105742572005215701027960 46,
336.59441033473517476178105298692183197 39, none,
324.65521224757330042664699951382387263 19,
```

```
331.938067922935610285849440186758804 8085, none, none,
289.5459577371755155769450474310105401162]

1 --> 2  target = [33.81362495422055182903056209519003620659,
3.7256489937252914143121709432144221831 17,
325.8920997403635318250737427508763561363]
one interval r = 20.37468935123140421709518373907505894484 ..
25.37892165331220400303914348054488947588
Time Approximations 0.025.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, .19371e-34]Solution in 0.526s

Time Plot 0 s.
Exiting SolveHard() after 1.021r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84,
302.31384316306297298071177520967594776 57,
328.46938514707846270181950654383708228 00,
343.81340625945153128510803599421727409 53,
375.73285291591943352110941663739330935 92,
328.11709295627251498942634669421947574 67,
292.99969139748255076038095756604805607 50,
358.64341561827813498820741417055214378 27,
299.89866205884605933393903372723763596 50,
360.06173468164105742572005215701027960 46,
336.59441033473517476178105298692183197 39,
```

```
256.10753187506643404880011252179576919 73,
324.65521224757330042664699951382387263 19,
331.93806792293561028584944018675880480 85, none, none,
289.54595773717551557694504743101054011 62]

1 --> 0  target = [17.93041369692146507095793589906017729342,
4.6865087020882384356593070497123275371 55,
353.30541096267950046961181482839387297 81]
one interval r = 20.731504791128015136849740114049439536 63 ..
25.906753535532410229077249548615251195 75
Time Approximations 0.03.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.5e-38
Equations at solution: [-.1e-37, -.45e-37, .31360e-34]Solution in
0.681s

Time Plot 0 s.
Exiting SolveHard() after 2.746r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84,
302.31384316306297298071177520967594776 57,
328.46938514707846270181950654383708228 00,
343.81340625945153128510803599421727409 53,
375.73285291591943352110941663739330935 92,
328.11709295627251498942634669421947574 67,
292.99969139748255076038095756604805607 50,
358.64341561827813498820741417055214378 27,
```

```
299.89866205884605933393903372723763596 50,
360.06173468164105742572005215701027960 46,
336.59441033473517476178105298692183197 39,
256.10753187506643404880011252179576919 73,
324.65521224757330042664699951382387263 19,
331.93806792293561028584944018675880480 85,
304.79958326962438324911849396531183926 14, none,
289.54595773717551557694504743101054011 62]

2 --> 0  target = [17.93041369692146507095793589906017729342,
4.68650870208823843565930704971232753715 5,
353.30541096267950046961181482839387297 81]
one interval r = 31.37435486997018472893606185036475791866 ..
34.20127520040968835319456272291606435089
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, .60e-35]Solution in 0.334s

Time Plot 0 s.
Exiting SolveHard() after 0.615r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494243130496043128464998103226 01,
441.64295973868925237442372588148654348 47,
436.91748165937785511629780927990528673 49,
422.98493398422667194740595497452637752 37,
361.52580257026900410828775240121464334 29,
401.88173905165131328401025919626409025 24,
389.59001516563985540716999711500227491 47,
328.46939894635556396115117097702446731 64,
401.50757158992585786994716329445898565 20,
358.97362824913331388317819560406879598 04,
398.33147105232480217207670388606531236 11,
371.48387395201390093386371072808581620 82,
336.61215842444730061832573797594231099 34,
361.50888348050983538265681829648285296 96,
324.67144993791815166142277352115278848 84,
302.31384316306297298071177520967594776 57,
328.46938514707846270181950654383708228 00,
343.81340625945153128510803599421727409 53,
375.73285291591943352110941663739330935 92,
```

```
328.11709295627251498942634669421947574 67,
292.99969139748255076038095756604805607 50,
358.64341561827813498820741417055214378 27,
299.89866205884605933393903372723763596 50,
360.06173468164105742572005215701027960 46,
336.59441033473517476178105298692183197 39,
256.10753187506643404880011252179576919 73,
324.65521224757330042664699951382387263 19,
331.93806792293561028584944018675880480 85,
304.79958326962438324911849396531183926 14,
323.46169177914796236843066321532531926 34,
289.54595773717551557694504743101054011 62]

Cascade time 131.128
counts: 28, 28

 Iteration 25

Start Generation 1
1 --> 0  target = [12.0000000000168440751140274154795740660 0,
6.2170125030742238136470297298375803205 79,
485.54908089460805111568224596624433069 61]
one interval r = 23.408503016462321211072992744485162079 74 ..
27.675780464299439647940196969830830822 15
Time Approximations 0.035.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.4921114030735554665825127038899393785 94, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=0
Equations at solution: [0., 0., -.3e-36]Solution in 0.924s

Time Plot 0 s.
Exiting SolveHard() after 3.332r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.0000000000168440751140274154795740660 0,
6.2170125030742238136470297298375803205 79,
485.54908089460805111568224596624433069 61]
```

```
one interval r = 32.628147792226417083478045879535314708 41 ..
36.10248388941724501541122044770484688852
Time Approximations 0.02.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, -.196e-34]Solution in 0.558s

Time Plot 0 s.
Exiting SolveHard() after 2.333r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684480555979816146692490149800460,
6.58343472178968374601794162725124642255 5,
467.78730595495184343340124686635965510 52]
one interval r = 32.41978955672620243743271004426552496246 ..
35.85152417371508520322949334691302698327
Time Approximations 0.023.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .72e-35]Solution in 0.648s

Time Plot 0 s.
Exiting SolveHard() after 1.023r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
```

```
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26, none, none,
401.88173903609962734599857344851114302 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684480555979816146692490149800460,
6.58343472178968374601794162725124642255 5,
467.78730595495184343340124686635965510 52]
two intervals r = 12.92327160833660495911103020655108567047 ..
9499999999940753339754126140213503279/500000000000000000000000000000000000
000 or r = 18.39424858013478419501401940574763209630 ..
9499999999940753339754126140213503279/500000000000000000000000000000000000
000
Time Approximations 0.043.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.3e-38
Equations at solution: [-.3e-37, -.33e-37, -.3107e-35]Solution in
38.718s

Time Plot 0 s.
Exiting SolveHard() after 41.517r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08, none,
401.88173903609962734599857344851114302 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962823449693436528471649090418982,
4.12565179675833215930713116187267087925 1,
440.67123064462650126693526432092273838 49]
two intervals r = 14.35659705130185462774869223856551559606 ..
```

```
9499999999940753339754126140213503279/500000000000000000000000000000000
000 or r = 17.70352613782299191799097959546758658921 ..
9499999999940753339754126140213503279/500000000000000000000000000000000
000
Time Approximations 1.54.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.15e-37, .1e-37, -.28498e-34]Solution in 1.303s

Time Plot 0 s.
Exiting SolveHard() after 3.973r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08, none,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962823449693436528471649090418982,
4.1256517967583321593071311618726708792 51,
440.67123064462650126693526432092273838 49]
one interval r = 22.39761154351521196218201382916116934420 ..
27.23722351595202908755261484639299852991
Time Approximations 0.033.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 2.684 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
```

```
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064380113210581569839602014217317, rm =
14.37818770458842175605596997722785403946}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, -.727e-34]Solution in 9.418s

Time Plot 0 s.
Exiting SolveHard() after 11.619r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888796253340299552968872279644384,
4.0048690817326554962940124396663215496 38,
404.86224500665440364579028059329446018 65]
two intervals r = 16.08011007770626834490926387114507789393 ..
94999999999407533397541261402135032 79/50000000000000000000000000000000000
000 or r = 16.41579812658961556316140130862169161028 ..
94999999999407533397541261402135032 79/50000000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.51e-37, -.1e-37, -.24119e-34]Solution in
2.905s

Time Plot 0 s.
Exiting SolveHard() after 5.466r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89, none, none,
358.97362823028569237834982536244116076 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888796253340299552968872279644384,
4.0048690817326554962940124396663215496 38,
404.86224500665440364579028059329446018 65]
one interval r = 21.641943993953223375626538363242482401 89 ..
26.763306600354629485610178519807592966 24
Time Approximations 0.049.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.245e-34]Solution in 2.234s

Time Plot 0 s.
Exiting SolveHard() after 3.226r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67, none,
358.97362823028569237834982536244116076 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589417889014756672880623360580644 36,
5.5896371830697775452217706173863127314 98,
443.83065884133145755720511681573132649 90]
one interval r = 22.467253744657542716072357935592345893 62 ..
27.273884283532036781041326262995736572 93
```

```
Time Approximations 0.039.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .21e-35]Solution in 2.306s

Time Plot 0 s.
Exiting SolveHard() after 3.272r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67, none,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941788901475667288062336058064436,
5.589637183069777545221770617386312731498,
443.83065884133145755720511681573132649 90]
one interval r = 32.15575279511661468405438234928323501884 ..
35.50872228737338667504842973611571499801
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.56e-35]Solution in 0.472s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.843r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67,
401.50757157297918691946966788906724059 68,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136530545080089995988926587475501,
5.18778357858243489886968071668619664 1068,
408.65773861864773993617077892844054171 98]
one interval r = 21.71840114630778586331848053050556002537 ..
26.81849303503158215616653158712407615962
Time Approximations 0.062.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, -.113e-34]Solution in 2.337s

Time Plot 0 s.
Exiting SolveHard() after 4.735r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
```

```
401.881739036099627345998573448511143026 5,
389.590015149989286050927465118421865208 9,
328.469398924629528800985740687961280086 7,
401.507571572979186919469667889067240596 8,
358.973628230285692378349825362441160768 4,
398.331471036177117557816692510862822301 2, none, none,
361.508883462082340498388016871406061346 7, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.911931365305450800899959889265874755 01,
5.187783578582434898869680716686196641068,
408.657738618647739936170778928440541719 8]
one interval r = 31.808285987598091275766811811217881838 13 ..
35.000114600405727576311167664677994848 46
Time Approximations 0.017.

hint used Hint := [34.495226611650080820729149248019775312 21, 3, 1, 1,
15.763855890238725343023875204542137417 16, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=7e-38
Equations at solution: [.7e-37, -.7e-37, -.99e-35]Solution in 0.433s

Time Plot 0 s.
Exiting SolveHard() after 0.738r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434932638110146380136719100296389 3,
441.642959727865887879874330702965614894 9,
436.917481647060409891509603169553984582 6,
422.984933970531681484972898053125496970 8,
361.525802551584442605766061642018597137 5,
401.881739036099627345998573448511143026 5,
389.590015149989286050927465118421865208 9,
328.469398924629528800985740687961280086 7,
401.507571572979186919469667889067240596 8,
358.973628230285692378349825362441160768 4,
398.331471036177117557816692510862822301 2,
371.483873932582514376559013081007335791 2, none,
361.508883462082340498388016871406061346 7, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.463471105282969240125720521127892332 30,
6.196262565494041748193031471336878021352,
```

```
385.4447437841335887543379099125541852668]
one interval r = 31.60836097545883962303767561105755699169 ..
34.66372795604676707339982782672495747363
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.90e-35]Solution in 1.799s

Time Plot 0 s.
Exiting SolveHard() after 2.067r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67,
401.50757157297918691946966788906724059 68,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12,
371.48387393258251437655901308100733579 12, none,
361.50888346208234049838801687140606134 67,
324.67144991447775233942930981362569705 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110528296924012572052112789233230,
6.19626256549404174819303147133687802135 2,
385.4447437841335887543379099125541852668]
two intervals r = 16.87563408771361693274889371857735823129 ..
9499999999940753339754126140213503279/50000000000000000000000000000000
000 or r = 15.55640493753320315462714785524110759178 ..
9499999999940753339754126140213503279/50000000000000000000000000000000
000
Time Approximations 0.054.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.898 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175124883528809249399209162156703, rm
= 2.33653277391556707972052774236305240384 8}});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [.715e-37, -.1e-37, .28222e-34]Solution in
26.343s

Time Plot 0 s.
Exiting SolveHard() after 27.386r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67,
401.50757157297918691946966788906724059 68,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12,
371.48387393258251437655901308100733579 12,
336.61215840252775773587918532186973089 77,
361.50888346208234049838801687140606134 67,
324.67144991447775233942930981362569705 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874750912413119346954383365449953,
4.88381077991095758229069985781166145251 2,
376.61967854893542950055825504559587475 85]
one interval r = 21.11001304858562965283892774931187591472 ..
26.31784243462365866523356446403019882747
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
```

```
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.175e-34]Solution in 0.876s

Time Plot 0 s.
Exiting SolveHard() after 3.205r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67,
401.50757157297918691946966788906724059 68,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12,
371.48387393258251437655901308100733579 12,
336.61215840252775773587918532186973089 77,
361.50888346208234049838801687140606134 67,
324.67144991447775233942930981362569705 27, none,
328.46938512535962403749786783798931419 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874750912413119346954383365449953,
4.8838107799109575822906998578116614525 12,
376.61967854893542950055825504559587475 85]
one interval r = 31.53899497720665867970981846844878838573 ..
34.53618386088311372022672708207390305160
Time Approximations 0.018.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=4.1e-37
Equations at solution: [-.31e-36, .41e-36, -.218e-34]Solution in 0.516s

Time Plot 0 s.
Exiting SolveHard() after 0.844r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67,
401.50757157297918691946966788906724059 68,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12,
371.48387393258251437655901308100733579 12,
336.61215840252775773587918532186973089 77,
361.50888346208234049838801687140606134 67,
324.67144991447775233942930981362569705 27, none,
328.46938512535962403749786783798931419 18,
343.81340623662494808595344099543173327 97, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017525160092231755104160693364573,
6.02581354944507825400447211149805044095 3,
351.42702947424627732358803363793942340 50]
one interval r = 31.36230206124560812760277394340236468014 ..
34.17446640609762908854469397058727110653
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=1.2e-37
Equations at solution: [.6e-37, -.12e-36, -.225e-34]Solution in 0.554s

Time Plot 0 s.
Exiting SolveHard() after 2.134r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
```

```
328.46939892462952880098574068796128008 67,
401.50757157297918691946966788906724059 68,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12,
371.48387393258251437655901308100733579 12,
336.61215840252775773587918532186973089 77,
361.50888346208234049838801687140606134 67,
324.67144991447775233942930981362569705 27, none,
328.46938512535962403749786783798931419 18,
343.81340623662494808595344099543173327 97, none, none,
292.99969137101034793755301844387970922 37, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175251600922317551041606933645 73,
6.0258135494450782540044721114980504409 53,
351.42702947424627732358803363793942340 50]
two intervals r = 17.981355144529803213308774393357032534 56 ..
949999999994075333975412614021350327 9/500000000000000000000000000000000
000 or r = 13.846080153634743983792749875970965603 53 ..
949999999994075333975412614021350327 9/500000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 4.371 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713638225104543896210228979540 39, rm
= 2.7345009928928985744987779051572672595 24}});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [0., 0., -.9581e-35]Solution in 15.669s

Time Plot 0 s.
Exiting SolveHard() after 18.106r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67,
```

```
401.50757157297918691946966788906724059 68,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12,
371.48387393258251437655901308100733579 12,
336.61215840252775773587918532186973089 77,
361.50888346208234049838801687140606134 67,
324.67144991447775233942930981362569705 27,
302.31384313840540936143726642292333668 52,
328.46938512535962403749786783798931419 18,
343.81340623662494808595344099543173327 97, none, none,
292.99969137101034793755301844387970922 37, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418705060793885877808593008641 29,
6.3779438740603553165387156618432018231 12,
423.28832783507834428468417801224381789 53]
one interval r = 31.946618176079331846185248462853622442 07 ..
35.212123086513823186461747103350686849 33
Time Approximations 0.02.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, -.99e-35]Solution in 1.888s

Time Plot 0 s.
Exiting SolveHard() after 2.241r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67,
401.50757157297918691946966788906724059 68,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12,
371.48387393258251437655901308100733579 12,
336.61215840252775773587918532186973089 77,
361.50888346208234049838801687140606134 67,
324.67144991447775233942930981362569705 27,
```

```
302.31384313840540936143726642292333366852,
328.46938512535962403749786783798931419 18,
343.81340623662494808595344099543173327 97, none, none,
292.99969137101034793755301844387970922 37, none, none,
360.06173466077396416983933152259206806 94, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418705060793885877808593008641 29,
6.3779438740603553165387156618432018231 12,
423.28832783507834428468417801224381789 53]
two intervals r = 15.228867024405065596364032516022132286 10 ..
9499999999940753339754126140213503279/50000000000000000000000000000000000
000 or r = 17.129657770488620255720400068503861345 04 ..
9499999999940753339754126140213503279/50000000000000000000000000000000000
000
Time Approximations 0.077.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 6.282 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541289344800262065856147067336 08, rm
= 2.0640682985910780819847933724538726871 95}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.31e-37, .1e-37, .38511e-34]Solution in 25.605s

Time Plot 0 s.
Exiting SolveHard() after 28.406r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67,
401.50757157297918691946966788906724059 68,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12,
371.48387393258251437655901308100733579 12,
336.61215840252775773587918532186973089 77,
361.50888346208234049838801687140606134 67,
```

```
324.671449914477752339429309813625697052 7,
302.313843138405409361437266422923336685 2,
328.469385125359624037497867837989314191 8,
343.813406236624948085953440995431733279 7,
375.732852896332819563703317060114843974 3, none,
292.999691371010347937553018443879709223 7, none, none,
360.061734660773964169839331522592068069 4, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343391758174235141382124319328 42,
4.0035598154303870448540470041139535419 51,
404.479735933034845819837680530213215060 6]
two intervals r = 16.096839663844136187710053071498857008 53 ..
949999999994075333975412614021350327 9/500000000000000000000000000000000
000 or r = 16.399886490759905079045248146170649435 11 ..
949999999994075333975412614021350327 9/500000000000000000000000000000000
000
Time Approximations 0.055.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=2e-38
Equations at solution: [.85e-37, .2e-37, -.14443e-34]Solution in 3.065s

Time Plot 0 s.
Exiting SolveHard() after 4.133r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434932638110146380136719100296389 3,
441.642959727865887879874330702965614894 9,
436.917481647060409891509603169553984582 6,
422.984933970531681484972898053125496970 8,
361.525802551584442605766061642018597137 5,
401.881739036099627345998573448511143026 5,
389.590015149989286050927465118421865208 9,
328.469398924629528800985740687961280086 7,
401.507571572979186919469667889067240596 8,
358.973628230285692378349825362441160768 4,
398.331471036177117557816692510862822301 2,
371.483873932582514376559013081007335791 2,
336.612158402527757735879185321869730897 7,
361.508883462082340498388016871406061346 7,
324.671449914477752339429309813625697052 7,
302.313843138405409361437266422923336685 2,
328.469385125359624037497867837989314191 8,
```

```
343.8134062366249480859534409954317332797,
375.7328528963328195637033170601148439743, none,
292.9996913710103479375530184438797092237,
358.6434155981958208837176540711446741356, none,
360.0617346607739641698393315225920680694, none, none, none, none,
none, none, none]

1 --> 2  target = [34.9395323433917581742351413821243193284 2,
4.0035598154303870448540470041139535419 51,
404.4797359330348458198376805302132150606]
one interval r = 21.63429629971245370822187302293319072907 ..
26.75768169884253891463164100074793391874
Time Approximations 0.052.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=4.9e-38
Equations at solution: [.2e-37, .49e-37, .831e-34]Solution in 1.077s

Time Plot 0 s.
Exiting SolveHard() after 3.623r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349326381101463801367191002963893,
441.6429597278658878798743307029656148949,
436.9174816470604098915096031695539845826,
422.9849339705316814849728980531254969708,
361.5258025515844426057660616420185971375,
401.8817390360996273459985734485111430265,
389.5900151499892860509274651184218652089,
328.4693989246295288009857406879612800867,
401.5075715729791869194696678890672405968,
358.9736282302856923783498253624411607684,
398.3314710361771175578166925108628223012,
371.4838739325825143765590130810073357912,
336.6121584025277577358791853218697308977,
361.5088834620823404983880168714060613467,
324.6714499144777523394293098136256970527,
302.3138431384054093614372664229233366852,
328.4693851253596240374978678379893141918,
343.8134062366249480859534409954317332797,
375.7328528963328195637033170601148439743,
328.1170929332332912847823436550634556548,
292.9996913710103479375530184438797092237,
```

```
358.6434155981958208837176540711446741356, none,
360.0617346607739641698393315225920680694, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954446686166810628055066101398985,
6.196177230348160873647502587948747484273,
385.42734024800706454671082121379850436 86]
one interval r = 31.60822049101348571088520148064413182458 ..
34.66347615042515361472572127261068527115
Time Approximations 0.016.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .216e-34]Solution in 0.563s

Time Plot 0 s.
Exiting SolveHard() after 2.261r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349326381101463801367191002963893,
441.6429597278658878798743307029656148949,
436.9174816470604098915096031695539845826,
422.9849339705316814849728980531254969708,
361.5258025515844426057660616420185971375,
401.8817390360996273459985734485111430265,
389.5900151499892860509274651184218652089,
328.4693989246295288009857406879612800867,
401.5075715729791869194696678890672405968,
358.9736282302856923783498253624411607684,
398.3314710361771175578166925108628223012,
371.4838739325825143765590130810073357912,
336.6121584025277577358791853218697308977,
361.5088834620823404983880168714060613467,
324.6714499144777523394293098136256970527,
302.3138431384054093614372664229233366852,
328.4693851253596240374978678379893141918,
343.8134062366249480859534409954317332797,
375.7328528963328195637033170601148439743,
328.1170929332332912847823436550634556548,
292.9996913710103479375530184438797092237,
358.6434155981958208837176540711446741356, none,
360.0617346607739641698393315225920680694, none, none,
324.6552122243795537069104246717413298328, none, none, none, none]
```

```
0 --> 1  target = [26.46318954446686166810628055066101398985,
6.196177230348160873647502587948747484273,
385.42734024800706454671082121379850436 86]
two intervals r = 16.87629600309531095333767523031188274491 ..
949999999994075333975412614021350327 9/500000000000000000000000000000000000
000 or r = 15.555590006024451212045016697139904789 82 ..
949999999994075333975412614021350327 9/500000000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 6.634 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852536379098229020008688128280313, rm
= 2.33669042797977092884652565813628976 6442}});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [.538e-37, -.1e-37, .21709e-34]Solution in
25.228s

Time Plot 0 s.
Exiting SolveHard() after 27.754r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67,
401.50757157297918691946966788906724059 68,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12,
371.48387393258251437655901308100733579 12,
336.61215840252775773587918532186973089 77,
361.50888346208234049838801687140606134 67,
324.67144991447775233942930981362569705 27,
302.31384313840540936143726642292333668 52,
328.46938512535962403749786783798931419 18,
343.81340623662494808595344099543173327 97,
375.73285289633281956370331706011484397 43,
328.11709293323329128478234365506345565 48,
```

```
292.99969137101034793755301844387970922 37,
358.6434155981958208837176540711446741356, none,
360.0617346607739641698393315225920680694,
336.5944103130854907351462464256054773546, none,
324.6552122243795537069104246717413298328, none, none, none, none]

0 --> 2  target = [34.4952266115693252776021392913830786069 8,
3.8971313158330567552780329171336536837 01,
373.7808188330602190147513975924865081304]
two intervals r = 17.2976908624712103066697166474439691038 1 ..
9499999999940753339754126140213503279/500000000000000000000000000000000
000 or r = 14.9943640736857051079096915832170775120 7 ..
9499999999940753339754126140213503279/500000000000000000000000000000000
000
Time Approximations 0.09.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.18e-37, 0., -.5951e-35]Solution in 2.711s

Time Plot 0 s.
Exiting SolveHard() after 5.816r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349326381101463801367191002963893,
441.6429597278658878798743307029656148949,
436.9174816470604098915096031695539845826,
422.9849339705316814849728980531254969708,
361.5258025515844426057660616420185971375,
401.8817390360996273459985734485111430265,
389.5900151499892860509274651184218652089,
328.4693989246295288009857406879612800867,
401.5075715729791869194696678890672405968,
358.9736282302856923783498253624411607684,
398.3314710361771175578166925108628223012,
371.4838739325825143765590130810073357912,
336.6121584025277577358791853218697308977,
361.5088834620823404983880168714060613467,
324.6714499144777523394293098136256970527,
302.3138431384054093614372664229233366852,
328.4693851253596240374978678379893141918,
343.8134062366249480859534409954317332797,
375.7328528963328195637033170601148439743,
328.1170929332332912847823436550634556548,
```

```
292.99969137101034793755301844387970922 37,
358.6434155981958208837176540711446741356, none,
360.0617346607739641698393315225920680694,
336.5944103130854907351462464256054773546, none,
324.6552122243795537069104246717413298328,
331.9380679003892595924414587433737176703, none, none, none]

1 --> 2  target = [34.49522661156932527760213929138307860698,
3.8971313158330567552780329171336536837 01,
373.7808188330602190147513975924865081304]
one interval r = 21.06068473180451078943041536190810087546 ..
26.26979834266354311990493759574009649024
Time Approximations 0.035.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, .336e-34]Solution in 0.79s

Time Plot 0 s.
Exiting SolveHard() after 2.988r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349326381101463801367191002963893,
441.6429597278658878798743307029656148949,
436.9174816470604098915096031695539845826,
422.9849339705316814849728980531254969708,
361.5258025515844426057660616420185971375,
401.8817390360996273459985734485111430265,
389.5900151499892860509274651184218652089,
328.4693989246295288009857406879612800867,
401.5075715729791869194696678890672405968,
358.9736282302856923783498253624411607684,
398.3314710361771175578166925108628223012,
371.4838739325825143765590130810073357912,
336.6121584025277577358791853218697308977,
361.5088834620823404983880168714060613467,
324.6714499144777523394293098136256970527,
302.3138431384054093614372664229233366852,
328.4693851253596240374978678379893141918,
343.8134062366249480859534409954317332797,
375.7328528963328195637033170601148439743,
328.1170929332332912847823436550634556548,
292.99969137101034793755301844387970922 37,
```

```
358.6434155981958208837176540711446741356,
299.8986620335068789532254073216428286753,
360.0617346607739641698393315225920680694,
336.5944103130854907351462464256054773546, none,
324.6552122243795537069104246717413298328,
331.9380679003892595924414587433737176703, none, none, none]

0 --> 2  target = [33.8136249540279217732064489320825327933 5,
3.7256489934636625457652891802549966803 70,
325.8920997165670571480963726616172264092]
two intervals r = 18.552270490115029009924534560208127488 21 ..
9499999999940753339754126140213503279/500000000000000000000000000000000
000 or r = 12.491969357232862669201174096961349574 59 ..
9499999999940753339754126140213503279/500000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1e-38
Equations at solution: [-.35e-37, .1e-37, .12024e-34]Solution in 2.399s

Time Plot 0 s.
Exiting SolveHard() after 5.292r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349326381101463801367191002963893,
441.6429597278658878798743307029656148949,
436.9174816470604098915096031695539845826,
422.9849339705316814849728980531254969708,
361.5258025515844426057660616420185971375,
401.8817390360996273459985734485111430265,
389.5900151499892860509274651184218652089,
328.4693989246295288009857406879612800867,
401.5075715729791869194696678890672405968,
358.9736282302856923783498253624411607684,
398.3314710361771175578166925108628223012,
371.4838739325825143765590130810073357912,
336.6121584025277577358791853218697308977,
361.5088834620823404983880168714060613467,
324.6714499144777523394293098136256970527,
302.3138431384054093614372664229233366852,
328.4693851253596240374978678379893141918,
343.8134062366249480859534409954317332797,
375.7328528963328195637033170601148439743,
```

```
328.11709293323329128478234365506345565 48,
292.99969137101034793755301844387970922 37,
358.64341559819582088371765407114467413 56,
299.89866203350687895322540732164282867 53,
360.06173466077396416983933152259206806 94,
336.59441031308549073514624642560547735 46, none,
324.65521222437955370691042467174132983 28,
331.93806790038925959244145874337371767 03, none, none,
289.54595771080349020927322799582795794 70]

1 --> 2  target = [33.81362495402792177320644893208253279335,
3.72564899346366254576528918025499668037 0,
325.89209971656705714809637266161722640 92]
one interval r = 20.37468935089943552757195530125153337102 ..
25.37892165277681893958626603423170577846
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, -.264e-34]Solution in 0.6s

Time Plot 0 s.
Exiting SolveHard() after 1.144r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67,
401.50757157297918691946966788906724059 68,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12,
371.48387393258251437655901308100733579 12,
336.61215840252775773587918532186973089 77,
361.50888346208234049838801687140606134 67,
324.67144991447775233942930981362569705 27,
302.31384313840540936143726642292333668 52,
328.46938512535962403749786783798931419 18,
343.81340623662494808595344099543173327 97,
```

```
375.7328528963328195637033170601148439743,
328.1170929332332912847823436550634556548,
292.9996913710103479375530184438797092237,
358.6434155981958208837176540711446741356,
299.8986620335068789532254073216428286753,
360.0617346607739641698393315225920680694,
336.5944103130854907351462464256054773546,
256.1075318460670190260383423106100295241,
324.6552122243795537069104246717413298328,
331.9380679003892595924414587433737176703, none, none,
289.5459577108034902092732279958279579470]

1 --> 0  target = [17.93041369720049348354950332179930628685,
4.6865087020469631134492587617261300078 16,
353.3054109403640079258595662910928836154]
one interval r = 20.73150479072945945498579807769381319255 ..
25.90675353506158618920658696901942189875
Time Approximations 0.031.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.9e-38
Equations at solution: [.2e-37, .49e-37, -.61e-35]Solution in 0.68s

Time Plot 0 s.
Exiting SolveHard() after 2.821r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349326381101463801367191002963893,
441.6429597278658878798743307029656148949,
436.9174816470604098915096031695539845826,
422.9849339705316814849728980531254969708,
361.5258025515844426057660616420185971375,
401.8817390360996273459985734485111430265,
389.5900151499892860509274651184218652089,
328.4693989246295288009857406879612800867,
401.5075715729791869194696678890672405968,
358.9736282302856923783498253624411607684,
398.3314710361771175578166925108628223012,
371.4838739325825143765590130810073357912,
336.6121584025277577358791853218697308977,
361.5088834620823404983880168714060613467,
324.6714499144777523394293098136256970527,
302.3138431384054093614372664229233366852,
```

```
328.46938512535962403749786783798931419 18,
343.81340623662494808595344099543173327 97,
375.73285289633281956370331706011484397 43,
328.11709293323329128478234365506345565 48,
292.99969137101034793755301844387970922 37,
358.64341559819582088371765407114467413 56,
299.89866203350687895322540732164282867 53,
360.06173466077396416983933152259206806 94,
336.59441031308549073514624642560547735 46,
256.10753184606701902603834231061002952 41,
324.65521222437955370691042467174132983 28,
331.93806790038925959244145874337371767 03,
304.79958324486368113225638833984789564 73, none,
289.54595771080349020927322799582795794 70]

2 --> 0  target = [17.93041369720049348354950332179930628685,
4.6865087020469631134492587617261300078 16,
353.30541094036400792585956629109288361 54]
one interval r = 31.37435487003577864931334973863084294090 ..
34.20127520022152645551848219687364171895
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=9e-38
Equations at solution: [.5e-37, -.9e-37, -.311e-34]Solution in 0.359s

Time Plot 0 s.
Exiting SolveHard() after 0.651r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493263811014638013671910029638 93,
441.64295972786588787987433070296561489 49,
436.91748164706040989150960316955398458 26,
422.98493397053168148497289805312549697 08,
361.52580255158444260576606164201859713 75,
401.88173903609962734599857344851114302 65,
389.59001514998928605092746511842186520 89,
328.46939892462952880098574068796128008 67,
401.50757157297918691946966788906724059 68,
358.97362823028569237834982536244116076 84,
398.33147103617711755781669251086282230 12,
371.48387393258251437655901308100733579 12,
336.61215840252775773587918532186973089 77,
```

```
361.50888346208234049838801687140606134 67,
324.67144991447775233942930981362569705 27,
302.31384313840540936143726642292333668 52,
328.46938512535962403749786783798931419 18,
343.81340623662494808595344099543173327 97,
375.73285289633281956370331706011484397 43,
328.11709293323329128478234365506345565 48,
292.99969137101034793755301844387970922 37,
358.64341559819582088371765407114467413 56,
299.89866203350687895322540732164282867 53,
360.06173466077396416983933152259206806 94,
336.59441031308549073514624642560547735 46,
256.10753184606701902603834231061002952 41,
324.65521222437955370691042467174132983 28,
331.93806790038925959244145874337371767 03,
304.79958324486368113225638833984789564 73,
323.46169175324706890311240815589627102 10,
289.54595771080349020927322799582795794 70]

Cascade time 223.01
counts: 28, 28

 Iteration 26

Start Generation 1
1 --> 0  target = [11.99999999994578199004672709374859611700,
6.21701250295778318006757151533392406569 9,
485.54908089989375981753949728996117668 88]
```

"Imaginary part neglected: ", $1.8899423791569025751853528309229169021 66 \times 10^{-17}$

```
one interval r = 23.40850301661202417095348552293033513960 ..
27.67578046435270384107800789132264954934
Time Approximations 0.043.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.33e-37
Equations at solution: [.4e-37, -.133e-36, -.9e-36]Solution in 0.991s

Time Plot 0 s.
Exiting SolveHard() after 3.589r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999994578199004672709374859611700,
6.2170125029577831800675715153339240656 99,
485.54908089989375981753949728996117668 88]
one interval r = 32.628147792164048044344584149724844733 36 ..
36.102483889419847561914392557207438838 10
Time Approximations 0.021.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=8e-38
Equations at solution: [-.12e-36, .8e-37, .199e-34]Solution in 0.582s

Time Plot 0 s.
Exiting SolveHard() after 2.303r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684485690016312015853987167727248,
6.5834347218031399314119262923918986486 97,
467.78730595920506170945412298835401853 11]
one interval r = 32.419789556645009559517741252373007221 71 ..
35.851524173700896832916639736263901666 17
Time Approximations 0.022.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .323e-34]Solution in 2.041s

Time Plot 0 s.
Exiting SolveHard() after 2.417r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35, none, none,
401.88173904148632814746174817402560614 01, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844856900163120158539871677272 48,
6.583434721803139931411926292391898648697,
467.78730595920506170945412298835401853 11]
two intervals r = 12.923271608319392331526235609782991263 65 ..
190000000000026057488798764161952976 33/100000000000000000000000000000000
00000 or r = 18.394248580315161966022980858346809610 51 ..
190000000000026057488798764161952976 33/100000000000000000000000000000000
00000
Time Approximations 0.064.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=4.3e-38
Equations at solution: [-.4e-37, -.43e-37, -.72e-35]Solution in 37.532s

Time Plot 0 s.
Exiting SolveHard() after 40.177r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28, none,
```

```
401.881739041486328147461748174025606140 1, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962826544991842047648560258647718,
4.12565179687585716256181169972136561263 4,
440.67123065230475559592959456021148074 89]
two intervals r = 14.35659705111722752305305550046851948980 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000 or r = 17.70352613813492624518328532739929140140 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [.27e-37, .2e-37, .221e-34]Solution in 2.733s

Time Plot 0 s.
Exiting SolveHard() after 3.82r=15.9119 in [14.35659706 .. 18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28, none,
401.881739041486328147461748174025606140 1,
389.59001516062999574560172842407142191 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962826544991842047648560258647718,
4.12565179687585716256181169972136561263 4,
440.67123065230475559592959456021148074 89]
```

"Imaginary part neglected: ", $1.8899423791569025751853528309229169021 66 \times 10^{-17}$

```
one interval r = 22.39761154372189798106510374975631917641 ..
27.23722351605311653952363434451998828975
Time Approximations 0.041.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.262 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643975356409162167547793502405 71, rm =
14.37818770647502425119214231810432601259}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.06e-37
Equations at solution: [.1e-37, .106e-36, .16e-35]Solution in 9.088s

Time Plot 0 s.
Exiting SolveHard() after 11.406r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887952782956522489150630543707 32,
4.0048690818425081491611962156533681200 37,
404.86224501197452773743843877521285535 52]
two intervals r = 16.080110077672471283713773801088743015 12 ..
190000000000026057488798764161952976 33/100000000000000000000000000000000
00000 or r = 16.415798126878069002673814501779571327 67 ..
190000000000026057488798764161952976 33/100000000000000000000000000000000
00000
Time Approximations 0.054.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
```

```
Equations at solution: [.17e-37, 0., -.149e-34]Solution in 2.874s

Time Plot 0 s.
Exiting SolveHard() after 3.926r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29, none, none,
358.97362823918607051029803147719807371 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888795278295652248915063054370732,
4.0048690818425081491611962156533681200 37,
404.86224501197452773743843877521285535 52]
```

"Imaginary part neglected: ", $1.8899423791569025751853528309229169021 66 \times 10^{-17}$

```
one interval r = 21.641943994104718954584547834475082038 96 ..
26.763306600444057452932223514781285402 66
Time Approximations 0.053.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=1.24e-37
Equations at solution: [-.4e-37, -.124e-36, .116e-34]Solution in 1.069s

Time Plot 0 s.
Exiting SolveHard() after 3.581r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
```

```
361.525802560614202023280813916773019057 1,
401.881739041486328147461748174025606140 1,
389.590015160629995745601728424071421912 9,
328.469398931464656271269737108152505472 2, none,
358.973628239186070510298031477198073716 2, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589418023721887059392121629764751 00,
5.589637182897468497875106950456088735257,
443.830658842826956208216956806146189887 9]
```

"Imaginary part neglected: ", $1.889942379156902575185352830922916902166 \times 10^{-17}$

```
one interval r = 22.467253744727844437060009113555230617 95 ..
27.273884283560168510480885467944472485 53
Time Approximations 0.042.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.88e-37
Equations at solution: [.2e-37, -.188e-36, -.1e-36]Solution in 2.421s

Time Plot 0 s.
Exiting SolveHard() after 4.807r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434938073596476092504775191007024 4,
441.642959732397111584160492939052453983 8,
436.917481654755425113021458630573193383 5,
422.984933972404729126188957724580523452 8,
361.525802560614202023280813916773019057 1,
401.881739041486328147461748174025606140 1,
389.590015160629995745601728424071421912 9,
328.469398931464656271269737108152505472 2, none,
358.973628239186070510298031477198073716 2,
398.331471035827083721080104414932432698 6, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589418023721887059392121629764751 00,
5.589637182897468497875106950456088735257,
443.830658842826956208216956806146189887 9]
one interval r = 32.155752794997389428947361716288064130 24 ..
```

```
35.50872228731584210996120862774586048816
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .96e-35]Solution in 0.488s

Time Plot 0 s.
Exiting SolveHard() after 0.853r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136502627738879962820365314698473,
5.18778357851111057031485007722077835068 5,
408.65773862936217501417576185680349869 39]
```

"Imaginary part neglected: ", $1.8899423791569025751853528309229169021 66 \times 10^{-17}$

```
one interval r = 21.71840114656935597398216938533109800006 ..
26.81849303519693743464383336999709984952
Time Approximations 0.058.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [.1e-37, .79e-37, -.50e-35]Solution in 1.028s

Time Plot 0 s.
Exiting SolveHard() after 3.579r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86, none, none,
361.50888347104053975458182443547751059 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136502627738879962820365314698473,
5.18778357851111057031485007722077835068 5,
408.65773862936217501417576185680349869 39]
one interval r = 31.80828598755392416974632017562244778561 ..
35.00011460047498396729971227334589880438
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=8e-38
Equations at solution: [.8e-37, -.8e-37, -.182e-34]Solution in 0.424s

Time Plot 0 s.
Exiting SolveHard() after 2.111r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02, none,
361.50888347104053975458182443547751059 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110543870011333758484982729916561,
6.19626256553171139507296145536798808426 6,
385.44474379302048416870166881116769301 1]
one interval r = 31.60836097538435990398899521026508271057 ..
34.66372795608429313988851090704255175224
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.45e-35]Solution in 0.557s

Time Plot 0 s.
Exiting SolveHard() after 0.832r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02, none,
```

```
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110543870011333758484982729916561,
6.19626256553171139507296145536798808426 6,
385.44474379302048416870166881116769301 1]
two intervals r = 16.875634087563789551861942861530786537 14 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000 or r = 15.556404938010385267397882295966700440 45 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000
Time Approximations 0.058.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.180e-37, 0., .97e-35]Solution in 2.565s

Time Plot 0 s.
Exiting SolveHard() after 5.1r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874737961167046282814296417893056,
4.88381077981436481088878890051936358809 1,
376.61967855780081366079736030597183094 51]
```

"Imaginary part neglected: ", $1.8899423791569025751853528309229169021 66 \times 10^{-17}$

```
one interval r = 21.11001304879144387696259142247058037102 ..
26.31784243478518781452877180773146235313
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=1.01e-37
Equations at solution: [-.3e-37, -.101e-36, .267e-34]Solution in 2.259s

Time Plot 0 s.
Exiting SolveHard() after 2.969r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67, none,
328.46938513219226575627609579811645901 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874737961167046282814296417893056,
4.88381077981436481088878890051936358809 1,
376.61967855780081366079736030597183094 51]
one interval r = 31.53899497712677000689855794948810891107 ..
34.53618386091808250897768471895062911594
Time Approximations 0.015.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
```

```
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=7.38e-36
Equations at solution: [.567e-35, -.738e-35, -.24e-35]Solution in
0.515s

Time Plot 0 s.
Exiting SolveHard() after 0.787r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67, none,
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017538863998330639274885620214159,
6.0258135494731854454304324560933689761 53,
351.42702948088174384853244034850954634 74]
one interval r = 31.36230206113646116132969158093939875612 ..
34.17446640609360062271680863224306897513
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [-.3e-37, .6e-37, -.1e-36]Solution in 0.541s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.786r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67, none,
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91, none, none,
292.99969137869100569547619542797838798 51, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017538863998330639274885620214159,
6.02581354947318544543043245609336897615 3,
351.42702948088174384853244034850954634 74]
two intervals r = 17.98135514451882730290139267518192722644 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000 or r = 13.84608015403602122606924617866401758104 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [-.17e-37, 0., .190e-34]Solution in 2.455s

Time Plot 0 s.
Exiting SolveHard() after 4.763r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67,
302.31384314246924913563135725270145775 72,
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91, none, none,
292.99969137869100569547619542797838798 51, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941870597381832735279647721322975,
6.37794387405089033734159047984521197139 9,
423.28832783431751545651178889147983907 10]
one interval r = 31.94661817593264431134878445941374258488 ..
35.21212308641970966445876225960267119635
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.153e-34]Solution in 0.619s

Time Plot 0 s.
Exiting SolveHard() after 0.972r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
```

```
361.525802560614202023280813916773019057 1,
401.881739041486328147461748174025606140 1,
389.590015160629995745601728424071421912 9,
328.469398931464656271269737108152505472 2,
401.507571578448568876680075959613832127 3,
358.973628239186070510298031477198073716 2,
398.331471035827083721080104414932432698 6,
371.483873946563519415715004917780066180 2,
336.612158408863688993232594210497413341 4,
361.508883471040539754581824435477510591 5,
324.671449924246380502198045231009221886 7,
302.313843142469249135631357252701457757 2,
328.469385132192265756276095798116459017 8,
343.813406249457386379288998172739522739 1, none, none,
292.999691378691005695476195427978387985 1, none, none,
360.061734661493878158762617909865298445 5, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418705973818327352796477213229 75,
6.377943874050890337341590479845211971399,
423.288327834317515456511788891479839071 0]
two intervals r = 15.22886702464947518482045955432715953289 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000 or r = 17.12965777053532260694788343381902075037 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000
Time Approximations 0.065.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.46e-37, .1e-37, -.81e-35]Solution in 2.844s

Time Plot 0 s.
Exiting SolveHard() after 5.495r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434938073596476092504775191007024 4,
441.642959732397111584160492939052453983 8,
436.917481654755425113021458630573193383 5,
422.984933972404729126188957724580523452 8,
361.525802560614202023280813916773019057 1,
401.881739041486328147461748174025606140 1,
389.590015160629995745601728424071421912 9,
328.469398931464656271269737108152505472 2,
```

```
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67,
302.31384314246924913563135725270145775 72,
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91,
375.73285289266415992242151327513933828 54, none,
292.99969137869100569547619542797838798 51, none, none,
360.06173466149387815876261790986529844 55, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343383152742171265626517143463 36,
4.0035598155405306247190337386682372861 70,
404.47973593843950004592569750265271433 88]
two intervals r = 16.096839663807012329452761987266164603 17 ..
19000000000002605748879876416195297633/1000000000000000000000000000000
00000 or r = 16.399886491052345947624806898271520531 58 ..
19000000000002605748879876416195297633/1000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.33e-37, 0., .120e-34]Solution in 1.41s

Time Plot 0 s.
Exiting SolveHard() after 3.82r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
```

```
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67,
302.31384314246924913563135725270145775 72,
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91,
375.73285289266415992242151327513933828 54, none,
292.99969137869100569547619542797838798 51,
358.64341560717125878395635676208910138 28, none,
360.06173466149387815876261790986529844 55, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343383152742171265626517143463 36,
4.0035598155405306247190337386682372861 70,
404.47973593843950004592569750265271433 88]
```

"Imaginary part neglected: ", $1.8899423791569025751853528309229169021 66 \times 10^{-17}$

```
one interval r = 21.634296299865564333013756430655361721 01 ..
26.757681698933381639014632824212225603 35
Time Approximations 0.053.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.505e-34]Solution in 2.665s

Time Plot 0 s.
Exiting SolveHard() after 5.152r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
```

```
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67,
302.31384314246924913563135725270145775 72,
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91,
375.73285289266415992242151327513933828 54,
328.11709294014610388799668056780191540 71,
292.99969137869100569547619542797838798 51,
358.64341560717125878395635676208910138 28, none,
360.06173466149387815876261790986529844 55, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954462141393396129733033115576576,
6.19617723038547050912224719410101252026 4,
385.42734025682035451686407102039866001 91]
one interval r = 31.60822049093840155471542631122746137059 ..
34.66347615046161042109004696372969568090
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, .34e-35]Solution in 0.581s

Time Plot 0 s.
Exiting SolveHard() after 0.876r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67,
```

```
302.31384314246924913563135725270145775 72,
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91,
375.73285289266415992242151327513933828 54,
328.11709294014610388799668056780191540 71,
292.99969137869100569547619542797838798 51,
358.64341560717125878395635676208910138 28, none,
360.06173466149387815876261790986529844 55, none, none,
324.65521223407950944734782311523697588 06, none, none, none, none]

0 --> 1  target = [26.46318954462141393396129733033115576576,
6.19617723038547050912224719410101252026 4,
385.42734025682035451686407102039866001 91]
two intervals r = 16.87629600294832070834814764227934593625 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000 or r = 15.55559000649821915724288279501391495481 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.538e-37, 0., -.158e-34]Solution in 2.497s

Time Plot 0 s.
Exiting SolveHard() after 5.074r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67,
302.31384314246924913563135725270145775 72,
```

```
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91,
375.73285289266415992242151327513933828 54,
328.11709294014610388799668056780191540 71,
292.99969137869100569547619542797838798 51,
358.64341560717125878395635676208910138 28, none,
360.06173466149387815876261790986529844 55,
336.59441031934633373275739215318686474 90, none,
324.65521223407950944734782311523697588 06, none, none, none, none]

0 --> 2  target = [34.49522661167916674226620813322182908081,
3.8971313159738738535350371714919614528 22,
373.78081884716995539366050617568672943 85]
two intervals r = 17.297690862168193758981967772950267448 53 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000 or r = 14.994364074441135534877865287820663533 54 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000
Time Approximations 0.084.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.19e-37, 0., .102e-34]Solution in 1.192s

Time Plot 0 s.
Exiting SolveHard() after 4.163r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67,
302.31384314246924913563135725270145775 72,
```

```
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91,
375.73285289266415992242151327513933828 54,
328.11709294014610388799668056780191540 71,
292.99969137869100569547619542797838798 51,
358.64341560717125878395635676208910138 28, none,
360.06173466149387815876261790986529844 55,
336.59441031934633373275739215318686474 90, none,
324.65521223407950944734782311523697588 06,
331.93806791711291890305332410106933120 75, none, none, none]

1 --> 2  target = [34.49522661167916674226620813322182908081,
3.8971313159738738535350371714919614528 22,
373.78081884716995539366050617568672943 85]
```

"Imaginary part neglected: ", $1.8899423791569025751853528309229169021 66 \times 10^{-17}$

```
one interval r = 21.06068473209856974323383911477523638288 ..
26.26979834291630944846109547118656653557
Time Approximations 0.035.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [.3e-37, .7e-37, .504e-34]Solution in 0.783s

Time Plot 0 s.
Exiting SolveHard() after 2.92r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493242269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67,
```

```
302.31384314246924913563135725270145775 72,
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91,
375.73285289266415992242151327513933828 54,
328.11709294014610388799668056780191540 71,
292.99969137869100569547619542797838798 51,
358.64341560717125878395635676208910138 28,
299.89866204838961089466038370686464522 04,
360.06173466149387815876261790986529844 55,
336.59441031934633373275739215318686474 90, none,
324.65521223407950944734782311523697588 06,
331.93806791711291890305332410106933120 75, none, none, none]

0 --> 2  target = [33.81362495406037942939407999718515878765,
3.72564899359006242957002536898002257400 3,
325.89209972637165577574930072355430029 26]
two intervals r = 18.55227049009039982257794589908492415144 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000 or r = 12.49196935778835679657421237655553541805 ..
19000000000002605748879876416195297633/10000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [.69e-37, -.3e-37, .131e-34]Solution in 2.519s

Time Plot 0 s.
Exiting SolveHard() after 5.352r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
```

```
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67,
302.31384314246924913563135725270145775 72,
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91,
375.73285289266415992242151327513933828 54,
328.11709294014610388799668056780191540 71,
292.99969137869100569547619542797838798 51,
358.64341560717125878395635676208910138 28,
299.89866204838961089466038370686464522 04,
360.06173466149387815876261790986529844 55,
336.59441031934633373275739215318686474 90, none,
324.65521223407950944734782311523697588 06,
331.93806791711291890305332410106933120 75, none, none,
289.54595772381252967825564131839042282 71]

1 --> 2  target = [33.81362495406037942939407999718515878765,
3.72564899359006242957002536898002257 4003,
325.89209972637165577574930072355430029 26]
```

"Imaginary part neglected: ", $1.8899423791569025751853528309229169021 66 \times 10^{-17}$

```
one interval r = 20.37468935106624348787339522561446342255 ..
25.37892165298972649640154423963752205083
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.2e-37, -.2e-37, .6e-36]Solution in 2.242s

Time Plot 0 s.
Exiting SolveHard() after 2.808r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
```

```
358.9736282391860705102980314771980737162,
398.3314710358270837210801044149324326986,
371.4838739465635194157150049177800661802,
336.6121584088636889932325942104974133414,
361.5088834710405397545818244354775105915,
324.6714499242463805021980452310092218867,
302.3138431424692491356313572527014577572,
328.4693851321922657562760957981164590178,
343.8134062494573863792889981727395227391,
375.7328528926641599224215132751393382854,
328.1170929401461038879966805678019154071,
292.9996913786910056954761954279783879851,
358.6434156071712587839563567620891013828,
299.8986620483896108946603837068646452204,
360.0617346614938781587626179098652984455,
336.5944103193463337327573921531868647490,
256.1075318569426023085483609366931065562,
324.6552122340795094473478231152369758806,
331.9380679171129189030533241010693312075, none, none,
289.5459577238125296782556413183904228271]

1 --> 0  target = [17.9304136971988102126046446591962987882 7,
4.6865087019260209336444031257658675176 27,
353.3054109465487850822976743281519946131]
```

"Imaginary part neglected: ", $1.8899423791569025751853528309229169021 66 \times 10^{-17}$

```
one interval r = 20.7315047908758141450347094569364406333 4 ..
25.9067535351893804334767095576407348164 2
Time Approximations 0.034.

hint used Hint := [25.4020715378188039710536173439704738275 4, 3, -1, 1,
17.0062444547662134530206245122467435705 3, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=6.8e-38
Equations at solution: [-.3e-37, -.68e-37, -.97e-35]Solution in 0.703s

Time Plot 0 s.
Exiting SolveHard() after 1.452r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349380735964760925047751910070244,
441.6429597323971115841604929390524539838,
436.9174816547554251130214586305731933835,
422.9849339724047291261889577245805234528,
```

```
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67,
302.31384314246924913563135725270145775 72,
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91,
375.73285289266415992242151327513933828 54,
328.11709294014610388799668056780191540 71,
292.99969137869100569547619542797838798 51,
358.64341560717125878395635676208910138 28,
299.89866204838961089466038370686464522 04,
360.06173466149387815876261790986529844 55,
336.59441031934633373275739215318686474 90,
256.10753185694260230854836093669310655 62,
324.65521223407950944734782311523697588 06,
331.93806791711291890305332410106933120 75,
304.79958324889255648683058913871941756 16, none,
289.54595772381252967825564131839042282 71]

2 --> 0  target = [17.93041369719881021260464465919629878827,
4.68650870192602093364440312576586751762 7,
353.30541094654878508229767432815199461 31]
one interval r = 31.37435486992462461038246147355156869949 ..
34.20127520021160098195086094012318060278
Time Approximations 0.014.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=1.2e-37
Equations at solution: [-.7e-37, .12e-36, -.77e-35]Solution in 0.339s

Time Plot 0 s.
Exiting SolveHard() after 2.06r=33.7963 in [32.25770943 .. 34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493807359647609250477519100702 44,
441.64295973239711158416049293905245398 38,
```

```
436.91748165475542511302145863057319338 35,
422.98493397240472912618895772458052345 28,
361.52580256061420202328081391677301905 71,
401.88173904148632814746174817402560614 01,
389.59001516062999574560172842407142191 29,
328.46939893146465627126973710815250547 22,
401.50757157844856887668007595961383212 73,
358.97362823918607051029803147719807371 62,
398.33147103582708372108010441493243269 86,
371.48387394656351941571500491778006618 02,
336.61215840886368899323259421049741334 14,
361.50888347104053975458182443547751059 15,
324.67144992424638050219804523100922188 67,
302.31384314246924913563135725270145775 72,
328.46938513219226575627609579811645901 78,
343.81340624945738637928899817273952273 91,
375.73285289266415992242151327513933828 54,
328.11709294014610388799668056780191540 71,
292.99969137869100569547619542797838798 51,
358.64341560717125878395635676208910138 28,
299.89866204838961089466038370686464522 04,
360.06173466149387815876261790986529844 55,
336.59441031934633373275739215318686474 90,
256.10753185694260230854836093669310655 62,
324.65521223407950944734782311523697588 06,
331.93806791711291890305332410106933120 75,
304.79958324889255648683058913871941756 16,
323.46169176394637650906253453390762824 97,
289.54595772381252967825564131839042282 71]

Cascade time 138.015
counts: 28, 28

 Iteration 27

Start Generation 1
1 --> 0  target = [12.00000000002481026476501846774756532900,
6.21701250297848107390462926997717640547 4,
485.54908089707002200433274765010870050 96]
one interval r = 23.40850301655205747723207749386089172550 ..
27.67578046421474036810508642916073880256
Time Approximations 0.039.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.2e-38
Equations at solution: [-.2e-37, .52e-37, -.198e-36]Solution in 2.489s

Time Plot 0 s.
```

```
Exiting SolveHard() after 3.617r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000002481026476501846774756532900,
6.217012502978481073904629269977176405474,
485.54908089707002200433274765010870050 96]
one interval r = 32.628147792157152855184612563173432322 79 ..
36.102483889460863357286721636063904842 24
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .15e-35]Solution in 0.625s

Time Plot 0 s.
Exiting SolveHard() after 1.055r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684473359541657909798559948843994,
6.583434721610159780480473027704083118465,
467.78730595808463352322001649105097007 27]
one interval r = 32.419789556655970338987539438902864799 24 ..
35.851524173761671931298983002804772325 87
Time Approximations 0.02.
```

```
hint used Hint := [34.9450788879772991463756999080445721474 1, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.53e-35]Solution in 0.619s

Time Plot 0 s.
Exiting SolveHard() after 2.334r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94, none, none,
401.88173904178135165635437327476692799 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684473359541657909798559948843994,
6.58343472161015978048047302770408311846 5,
467.78730595808463352322001649105097007 27]
two intervals r = 12.92327160831608882703627346004762236390 ..
9499999999992327378659001786173353 99/50000000000000000000000000000000000
0 or r = 18.39424858028151763118901978858971706634 ..
9499999999992327378659001786173353 99/50000000000000000000000000000000000
0
Time Approximations 0.038.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.1e-38
Equations at solution: [-.2e-37, -.21e-37, .952e-35]Solution in 36.502s

Time Plot 0 s.
Exiting SolveHard() after 39.467r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71, none,
401.88173904178135165635437327476692799 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962827995809797793839467691752105,
4.12565179679737011951735175923008816107 1,
440.67123064840825006852905313741238149 97]
two intervals r = 14.35659705126038418764342662993731725529 ..
94999999999923273786590017861733539 9/50000000000000000000000000000000000
0 or r = 17.70352613800909119804711381374013599032 ..
94999999999923273786590017861733539 9/50000000000000000000000000000000000
0
Time Approximations 0.044.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [-.57e-37, -.3e-37, .3223e-34]Solution in 1.335s

Time Plot 0 s.
Exiting SolveHard() after 3.868r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71, none,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962827995809797793839467691752105,
4.12565179679737011951735175923008816107 1,
440.67123064840825006852905313741238149 97]
one interval r = 22.39761154367688811631032577188814995750 ..
```

```
27.23722351590103083810992938941877752951
Time Approximations 0.035.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.239 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064375239866609508566613622577019, rm =
14.37818770286083396279231530720792932111}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., -.11813e-34]Solution in 7.868s

Time Plot 0 s.
Exiting SolveHard() after 10.237r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888802139552274924512445775365153,
4.00486908177872829368207241523226949134 6,
404.86224501238641421071581175562312544 83]
two intervals r = 16.08011007760564890984936001816564548606 ..
949999999999232737865900178617335399/500000000000000000000000000000000000
0 or r = 16.41579812688747798648713763689319750092 ..
949999999999232737865900178617335399/500000000000000000000000000000000000
0
Time Approximations 0.05.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
```

```
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.35e-37, .1e-37, -.246e-35]Solution in 3.405s

Time Plot 0 s.
Exiting SolveHard() after 6.287r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22, none, none,
358.97362823761046210071218472006676356 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888021395522749245124457753651 53,
4.0048690817787282936820724152322694913 46,
404.86224501238641421071581175562312544 83]
one interval r = 21.641943994183029728094324188344061414 19 ..
26.763306600355206374553280242899972288 97
Time Approximations 0.049.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=1.25e-37
Equations at solution: [.3e-37, .125e-36, -.26453e-34]Solution in
1.042s

Time Plot 0 s.
Exiting SolveHard() after 3.691r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80, none,
358.97362823761046210071218472006676356 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941778803877173353454121162135052,
5.5896371830060834434930960661349452598 20,
443.83065884621754116790364210092314671 23]
one interval r = 22.46725374484081784201187733340673152493 ..
27.27388428349185704783624057217158739651
Time Approximations 0.035.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.0e-38
Equations at solution: [-.1e-37, .80e-37, -.3630e-35]Solution in 0.948s

Time Plot 0 s.
Exiting SolveHard() after 3.478r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80, none,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

2 --> 0  target = [14.19258941778803877173353454121162135052,
5.5896371830060834434930960661349452598 20,
443.83065884621754116790364210092314671 23]
```

```
one interval r = 32.15575279505293737261754018779038429331 ..
35.50872228743598725819173090464383779240
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., -.32e-35]Solution in 0.441s

Time Plot 0 s.
Exiting SolveHard() after 2.648r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136521448470579132515930193496806,
5.18778357851932752657414801952145568294 1,
408.65773862405117031601119533802019618 32]
one interval r = 21.71840114652865565687496197438689197861 ..
26.81849303502424250825475314717029304416
Time Approximations 0.058.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
```

```
Accepted {r=26.4632, rm=15.9013} with Delta=5.3e-38
Equations at solution: [0., .53e-37, -.11204e-34]Solution in 2.582s

Time Plot 0 s.
Exiting SolveHard() after 3.686r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56, none, none,
361.50888346922661532219813988034252857 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136521448470579132515930193496806,
5.18778357851932752657414801952145568294 1,
408.65773862405117031601119533802019618 32]
one interval r = 31.80828598751960623024260949689626485269 ..
35.00011460046146340101264386885255323215
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [.3e-37, -.3e-37, -.359e-34]Solution in 0.437s

Time Plot 0 s.
Exiting SolveHard() after 0.75r=34.4952 in [32.91337941 .. 35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
```

```
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23, none,
361.50888346922661532219813988034252857 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110532312396102345118131254559839,
6.1962625653319829895464799184785027963 16,
385.44474379127188677770931774184297747 45]
one interval r = 31.608360975381311861605245123237324774 29 ..
34.663727956117307921747960499481880557 12
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .71e-35]Solution in 0.562s

Time Plot 0 s.
Exiting SolveHard() after 2.499r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23, none,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110532312396102345118131254559839,
6.196262565331982989546479918478502796316,
385.44474379127188677770931774184297747 45]
two intervals r = 16.875634087587014789796671647819408460 95 ..
94999999999923273786590017861733539 9/50000000000000000000000000000000000
0 or r = 15.556404937920278699609646403366963345 21 ..
94999999999923273786590017861733539 9/50000000000000000000000000000000000
0
Time Approximations 1.675.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 6.935 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175122202365588896232829906881460, rm
= 2.33653277406907633681954944478153099129 4}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.358e-37, 0., .222e-35]Solution in 27.321s

Time Plot 0 s.
Exiting SolveHard() after 30.045r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874739405220084678055995263294975,
```

```
4.883810779861836664142996036566700650306,
376.619678556232169115667815620715601897 0]
one interval r = 21.11001304885603514501408397448358569529 ..
26.31784243467530392755049665881357370465
Time Approximations 0.035.

hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=1.01e-37
Equations at solution: [.3e-37, .101e-36, .30671e-34]Solution in 0.834s

Time Plot 0 s.
Exiting SolveHard() after 1.523r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70, none,
328.46938513444963229136109000740850887 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874739405220084678055995263294975,
4.883810779861836664142996036566700650306,
376.619678556232169115667815620715601897 0]
one interval r = 31.53899497712474906186776572921444008738 ..
34.53618386095179749767019250220892549340
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
```

```
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=6.76e-36
Equations at solution: [-.519e-35, .676e-35, -.218e-34]Solution in
0.488s

Time Plot 0 s.
Exiting SolveHard() after 2.394r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70, none,
328.46938513444963229136109000740850887 99,
343.81340624563869745403884941349425938 51, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017536131428311122052943732137209,
6.0258135492930540902711884426067622064 67,
351.42702948326634718359443958076225498 37]
one interval r = 31.362302061158822672078797673447495910 22 ..
34.174466406178425745171034376160030487 34
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [-.3e-37, .6e-37, -.267e-34]Solution in 0.509s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.749r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70, none,
328.46938513444963229136109000740850887 99,
343.81340624563869745403884941349425938 51, none, none,
292.99969138246574732934141062038169967 88, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175361314283111220529437321372 09,
6.0258135492930540902711884426067622064 67,
351.42702948326634718359443958076225498 37]
two intervals r = 17.981355144420835769056438951927585593 06 ..
9499999999992327378659001786173353990/50000000000000000000000000000000000
0 or r = 13.846080154151061382536856230344329072 19 ..
9499999999992327378659001786173353990/50000000000000000000000000000000000
0
Time Approximations 0.044.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.249 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713674937720264233726087606351 53, rm
= 2.7345009931279629786339265905155007601 16}});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [.16e-37, 0., -.2141e-34]Solution in 17.13s
```

```
Time Plot 0 s.
Exiting SolveHard() after 19.795r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70,
302.31384314928882462644818578185465337 07,
328.46938513444963229136109000740850887 99,
343.81340624563869745403884941349425938 51, none, none,
292.99969138246574732934141062038169967 88, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418698021723048557099624206487 76,
6.3779438738944398462801816889489197770 85,
423.28832784138149579273920902196513049 87]
one interval r = 31.946618176018016725927125522208852973 00 ..
35.212123086588729120012747462620700737 36
Time Approximations 0.02.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .117e-34]Solution in 2.07s

Time Plot 0 s.
Exiting SolveHard() after 2.41r=34.3272 in [33.10127385 .. 35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70,
302.31384314928882462644818578185465337 07,
328.46938513444963229136109000740850887 99,
343.81340624563869745403884941349425938 51, none, none,
292.99969138246574732934141062038169967 88, none, none,
360.06173466954854099351147403117402669 99, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418698021723048557099624206487 76,
6.3779438738944398462801816889489197770 85,
423.28832784138149579273920902196513049 87]
two intervals r = 15.22886702425339108033405848317797098278 ..
949999999999232737865900178617335399/500000000000000000000000000000000000
0 or r = 17.12965777078079264659358377697681896769 ..
949999999999232737865900178617335399/500000000000000000000000000000000000
0
Time Approximations 0.063.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 8.434 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541202028926939149009586658628 51, rm
= 2.0640682986843933219916044776105208225 22}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.31e-37, -.1e-37, -.2549e-34]Solution in
28.173s

Time Plot 0 s.
Exiting SolveHard() after 29.509r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70,
302.31384314928882462644818578185465337 07,
328.46938513444963229136109000740850887 99,
343.81340624563869745403884941349425938 51,
375.73285290479321371705009136477391739 96, none,
292.99969138246574732934141062038169967 88, none, none,
360.06173466954854099351147403117402669 99, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234346046064262958088920472173357,
4.0035598154788277061721464975371641638 91,
404.47973593945693838688387660429921921 35]
two intervals r = 16.09683966371388950698068176071093014370 ..
9499999999992327378659001786173353 99/50000000000000000000000000000000000
0 or r = 16.39988649108702115115641496450796969017 ..
9499999999992327378659001786173353 99/50000000000000000000000000000000000
0
Time Approximations 0.054.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.17e-37, 0., .3224e-34]Solution in 3.263s

Time Plot 0 s.
Exiting SolveHard() after 4.301r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
```

```
441.642959731232388510755577606046110071 5,
436.917481650835716493723204889850556699 4,
422.984933975524295937466829977566508527 1,
361.525802558852344110289860347116616338 7,
401.881739041781351656354373274766927996 0,
389.590015155490428892864955553716813632 2,
328.469398933723011285716458869812919358 0,
401.507571579335921830491383567310004383 6,
358.973628237610462100712184720066763568 0,
398.331471042632838080354283174430745775 6,
371.483873939724082932556297648430762942 3,
336.612158411704666860909148044492460235 9,
361.508883469226615322198139880342528571 2,
324.671449924115411658086695522643869497 0,
302.313843149288824626448185781854653370 7,
328.469385134449632291361090007408508879 9,
343.813406245638697454038849413494259385 1,
375.732852904793213717050091364773917399 6, none,
292.999691382465747329341410620381699678 8,
358.643415606117859674350644502497653738 5, none,
360.061734669548540993511474031174026699 9, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343460460642629580889204721733 57,
4.003559815478827706172146497537164163891,
404.479735939456938386883876604299219213 5]
one interval r = 21.634296299956270588208636611598572782 79 ..
26.757681698853597264477984190516855808 41
Time Approximations 0.049.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.6633e-35]Solution in 2.56s

Time Plot 0 s.
Exiting SolveHard() after 5.275r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434935340923947032110875420272761 9,
441.642959731232388510755577606046110071 5,
436.917481650835716493723204889850556699 4,
422.984933975524295937466829977566508527 1,
361.525802558852344110289860347116616338 7,
```

```
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70,
302.31384314928882462644818578185465337 07,
328.46938513444963229136109000740850887 99,
343.81340624563869745403884941349425938 51,
375.73285290479321371705009136477391739 96,
328.11709294296250375281986786004030662 01,
292.99969138246574732934141062038169967 88,
358.64341560611785967435064450249765373 85, none,
360.06173466954854099351147403117402669 99, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954450497624213705571317991348613,
6.19617723018547865454876348146558200675 5,
385.42734025501819670319176322917948027 36]
one interval r = 31.60822049093492042068897395561754418865 ..
34.66347615049384648825060641299103975021
Time Approximations 0.016.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., .112e-34]Solution in 0.575s

Time Plot 0 s.
Exiting SolveHard() after 0.862r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
```

```
401.5075715793359218304913835673100043836,
358.9736282376104621007121847200667635680,
398.3314710426328380803542831744307457756,
371.4838739397240829325562976484307629423,
336.6121584117046668609091480444924602359,
361.5088834692266153221981398803425285712,
324.6714499241154116580866955226438694970,
302.3138431492888246264481857818546533707,
328.4693851344496322913610900074085088799,
343.8134062456386974540388494134942593851,
375.7328529047932137170500913647739173996,
328.1170929429625037528198678600403066201,
292.9996913824657473293414106203816996788,
358.6434156061178596743506445024976537385, none,
360.0617346695485409935114740311740266999, none, none,
324.6552122338986020386742074639481622949, none, none, none, none]

0 --> 1  target = [26.46318954450497624213705571317991348613,
6.196177230185478654548763481465582006755,
385.42734025501819670319176322917948027 36]
two intervals r = 16.87629600297357827298945238540638456867 ..
9499999999992327378659001786173353 99/50000000000000000000000000000000000
0 or r = 15.555590006405596179570516416700305906 58 ..
9499999999992327378659001786173353 99/50000000000000000000000000000000000
0
Time Approximations 0.058.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.019 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852533940666108006892785344081610, rm
= 2.33669042813447263354007597975138506986 0}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.537e-37, 0., .2235e-34]Solution in 26.941s

Time Plot 0 s.
Exiting SolveHard() after 29.826r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
```

```
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70,
302.31384314928882462644818578185465337 07,
328.46938513444963229136109000740850887 99,
343.81340624563869745403884941349425938 51,
375.73285290479321371705009136477391739 96,
328.11709294296250375281986786004030662 01,
292.99969138246574732934141062038169967 88,
358.64341560611785967435064450249765373 85, none,
360.06173466954854099351147403117402669 99,
336.59441032213263310685925412693391721 54, none,
324.65521223389860203867420746394816229 49, none, none, none, none]

0 --> 2  target = [34.49522661163563739637144459601620126354,
3.8971313158847854981248389749248005805 51,
373.78081884028675289411287896905089924 88]
two intervals r = 17.29769086236429321972940761603227259481 ..
94999999999923273786590017861733539 9/50000000000000000000000000000000000
0 or r = 14.99436407409173866567069920332856498760 ..
94999999999923273786590017861733539 9/50000000000000000000000000000000000
0
Time Approximations 0.081.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.108e-36, 0., .5103e-34]Solution in 1.167s

Time Plot 0 s.
Exiting SolveHard() after 5.042r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
```

```
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70,
302.31384314928882462644818578185465337 07,
328.46938513444963229136109000740850887 99,
343.81340624563869745403884941349425938 51,
375.73285290479321371705009136477391739 96,
328.11709294296250375281986786004030662 01,
292.99969138246574732934141062038169967 88,
358.64341560611785967435064450249765373 85, none,
360.06173466954854099351147403117402669 99,
336.59441032213263310685925412693391721 54, none,
324.65521223389860203867420746394816229 49,
331.93806790911696655472847364606657258 88, none, none, none]

1 --> 2  target = [34.49522661163563739637144459601620126354,
3.8971313158847854981248389749248005805 51,
373.78081884028675289411287896905089924 88]
one interval r = 21.06068473207416673028525032910907730321 ..
26.26979834271689176900218969519809800188
Time Approximations 0.033.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=4e-38
Equations at solution: [.2e-37, .4e-37, -.18431e-34]Solution in 0.771s

Time Plot 0 s.
Exiting SolveHard() after 3.03r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
```

```
389.5900151554904288928649555537168136322,
328.4693989337230112857164588698129193580,
401.5075715793359218304913835673100043836,
358.9736282376104621007121847200667635680,
398.3314710426328380803542831744307457756,
371.4838739397240829325562976484307629423,
336.6121584117046668609091480444924602359,
361.5088834692266153221981398803425285712,
324.6714499241154116580866955226438694970,
302.3138431492888246264481857818546533707,
328.4693851344496322913610900074085088799,
343.8134062456386974540388494134942593851,
375.7328529047932137170500913647739173996,
328.1170929429625037528198678600403066201,
292.9996913824657473293414106203816996788,
358.6434156061178596743506445024976537385,
299.8986620439788355452384979048308461834,
360.0617346695485409935114740311740266999,
336.5944103221326331068592541269339172154, none,
324.6552122338986020386742074639481622949,
331.9380679091169665547284736460665725888, none, none, none]

0 --> 2  target = [33.8136249541053623666106608857985973689 8,
3.7256489935254094824842023725062666731 40,
325.8920997263504200399352918840449715772]
two intervals r = 18.552270490062888198143616233291723443 55 ..
9499999999992327378659001786173353 99/500000000000000000000000000000000000
0 or r = 12.491969357772902002093772683544103378 58 ..
9499999999992327378659001786173353 99/500000000000000000000000000000000000
0
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [-.86e-37, .3e-37, .687e-35]Solution in 2.595s

Time Plot 0 s.
Exiting SolveHard() after 5.589r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349353409239470321108754202727619,
441.6429597312323885107555776060461100715,
436.9174816508357164937232048898505566994,
422.9849339755242959374668299775665085271,
```

```
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70,
302.31384314928882462644818578185465337 07,
328.46938513444963229136109000740850887 99,
343.81340624563869745403884941349425938 51,
375.73285290479321371705009136477391739 96,
328.11709294296250375281986786004030662 01,
292.99969138246574732934141062038169967 88,
358.64341560611785967435064450249765373 85,
299.89866204397883554523849790483084618 34,
360.06173466954854099351147403117402669 99,
336.59441032213263310685925412693391721 54, none,
324.65521223389860203867420746394816229 49,
331.93806790911696655472847364606657258 88, none, none,
289.54595772187252473104129842331273850 79]

1 --> 2  target = [33.81362495410536236661066088579859736898,
3.72564899352540948248420237250626667314 0,
325.89209972635042003993529188404497157 72]
one interval r = 20.37468935119055769616658324162802818904 ..
25.37892165293163744459494301466187143910
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.2e-37, .2e-37, -.10560e-34]Solution in 0.572s

Time Plot 0 s.
Exiting SolveHard() after 1.124r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
```

```
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70,
302.31384314928882462644818578185465337 07,
328.46938513444963229136109000740850887 99,
343.81340624563869745403884941349425938 51,
375.73285290479321371705009136477391739 96,
328.11709294296250375281986786004030662 01,
292.99969138246574732934141062038169967 88,
358.64341560611785967435064450249765373 85,
299.89866204397883554523849790483084618 34,
360.06173466954854099351147403117402669 99,
336.59441032213263310685925412693391721 54,
256.10753185885336045157291401821073341 11,
324.65521223389860203867420746394816229 49,
331.93806790911696655472847364606657258 88, none, none,
289.54595772187252473104129842331273850 79]

1 --> 0  target = [17.93041369708102997365481696382710418602,
4.68650870201134736508725070996087049182 5,
353.30541094958089981646973440581620784 74]
one interval r = 20.73150479102932806705541573419103460627 ..
25.90675353517273954500440448261118577946
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=6.9e-38
Equations at solution: [-.3e-37, -.69e-37, .35214e-34]Solution in
0.644s

Time Plot 0 s.
Exiting SolveHard() after 3.109r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70,
302.31384314928882462644818578185465337 07,
328.46938513444963229136109000740850887 99,
343.81340624563869745403884941349425938 51,
375.73285290479321371705009136477391739 96,
328.11709294296250375281986786004030662 01,
292.99969138246574732934141062038169967 88,
358.64341560611785967435064450249765373 85,
299.89866204397883554523849790483084618 34,
360.06173466954854099351147403117402669 99,
336.59441032213263310685925412693391721 54,
256.10753185885336045157291401821073341 11,
324.65521223389860203867420746394816229 49,
331.93806790911696655472847364606657258 88,
304.79958325583938264620267576967462901 46, none,
289.54595772187252473104129842331273850 79]

2 --> 0  target = [17.93041369708102997365481696382710418602,
4.6865087020113473650872507099608704918 25,
353.30541094958089981646973440581620784 74]
one interval r = 31.37435486995159764149361149614672415189 ..
34.20127520030612289974470994698063738731
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, .27e-35]Solution in 0.339s

Time Plot 0 s.
Exiting SolveHard() after 0.615r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493534092394703211087542027276 19,
441.64295973123238851075557760604611007 15,
436.91748165083571649372320488985055669 94,
422.98493397552429593746682997756650852 71,
361.52580255885234411028986034711661633 87,
401.88173904178135165635437327476692799 60,
389.59001515549042889286495555371681363 22,
328.46939893372301128571645886981291935 80,
401.50757157933592183049138356731000438 36,
358.97362823761046210071218472006676356 80,
398.33147104263283808035428317443074577 56,
371.48387393972408293255629764843076294 23,
336.61215841170466686090914804449246023 59,
361.50888346922661532219813988034252857 12,
324.67144992411541165808669552264386949 70,
302.31384314928882462644818578185465337 07,
328.46938513444963229136109000740850887 99,
343.81340624563869745403884941349425938 51,
375.73285290479321371705009136477391739 96,
328.11709294296250375281986786004030662 01,
292.99969138246574732934141062038169967 88,
358.64341560611785967435064450249765373 85,
299.89866204397883554523849790483084618 34,
360.06173466954854099351147403117402669 99,
336.59441032213263310685925412693391721 54,
256.10753185885336045157291401821073341 11,
324.65521223389860203867420746394816229 49,
331.93806790911696655472847364606657258 88,
304.79958325583938264620267576967462901 46,
323.46169176407032881344768375833397558 04,
289.54595772187252473104129842331273850 79]

Cascade time 228.887
counts: 28, 28

 Iteration 28

Start Generation 1
1 --> 0  target = [12.00000000001820672737459049395469675700,
6.21701250309960984051835084289993863851 5,
485.54908089754102345349106953005684533 91]
one interval r = 23.40850301649381182258401209303407363762 ..
27.67578046435065395291737130857509583329
Time Approximations 0.038.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
```

```
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=3.41e-37
Equations at solution: [-.10e-36, .341e-36, .28e-35]Solution in 0.952s

Time Plot 0 s.
Exiting SolveHard() after 3.728r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000001820672737459049395469675700,
6.217012503099609840518350842899938638515,
485.549080897541023453491069530056845339 1]
one interval r = 32.628147792082977954628950552789217157 68 ..
36.102483889401609328530052466859157294 62
Time Approximations 0.02.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .6e-36]Solution in 0.574s

Time Plot 0 s.
Exiting SolveHard() after 2.668r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844874290952302590352581271764 73,
6.583434721610364716466565806403994317715,
```

```
467.787305959261918982802608609346801492 1]
one interval r = 32.41978955658830461214564493707647068670 ..
35.85152417371208059428801394957041884211
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., -.256e-34]Solution in 2.623s

Time Plot 0 s.
Exiting SolveHard() after 2.985r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53, none, none,
401.88173904347408356864555936686458710 40, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684487429095230259035258127176473,
6.58343472161036471646656580640399431771 5,
467.787305959261918982802608609346801492 1]
two intervals r = 12.92327160828338974368559418532557492258 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000
0 or r = 18.39424858035779583291474569784980936128 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000
0
Time Approximations 0.041.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.2e-38
Equations at solution: [-.1e-37, -.12e-37, -.215e-35]Solution in
38.026s
```

```
Time Plot 0 s.
Exiting SolveHard() after 39.376r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91, none,
401.88173904347408356864555936686458710 40, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962821106819615360344405902141277,
4.12565179685372731038411119751408652334 5,
440.67123064829321345573395726115026579 27]
two intervals r = 14.35659705131058736996007336348838977462 ..
9500000000022537581594894541827655 91/50000000000000000000000000000000000
0 or r = 17.70352613805128118716536783204097730687 ..
9500000000022537581594894541827655 91/50000000000000000000000000000000000
0
Time Approximations 0.044.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=5e-38
Equations at solution: [-.99e-37, -.5e-37, .129e-35]Solution in 2.882s

Time Plot 0 s.
Exiting SolveHard() after 5.846r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91, none,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
1 --> 2  target = [35.46322962821106819615360344405902141277,
4.12565179685372731038411119751408652334 5,
440.67123064829321345573395726115026579 27]
one interval r = 22.39761154353633105463555148667590363550 ..
27.23722351602204706341566729816164192293
Time Approximations 0.039.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.303 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064385726662483971488941183429156, rm =
14.37818770356310665487598716529740310310}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.3e-38
Equations at solution: [.1e-37, .53e-37, -.92e-35]Solution in 10.431s

Time Plot 0 s.
Exiting SolveHard() after 11.32r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888797873979725815458573040583551,
4.00486908184132827119917443955775600028 4,
404.86224501418793244579591866066770348 69]
two intervals r = 16.08011007758254954832826142414042136141 ..
9500000000022537581594894541827655 91/50000000000000000000000000000000000
0 or r = 16.41579812699709711029127680031595864799 ..
9500000000022537581594894541827655 91/50000000000000000000000000000000000
0
Time Approximations 0.048.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
```

```
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.34e-37, 0., -.450e-35]Solution in 3.197s

Time Plot 0 s.
Exiting SolveHard() after 6.124r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12, none, none,
358.97362823847273561976613376052744927 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888797873979725815458573040583551,
4.0048690818413282711991744395577560002 84,
404.86224501418793244579591866066770348 69]
one interval r = 21.641943994024408928832686850580449547 41 ..
26.763306600488036316436338751042622602 48
Time Approximations 0.051.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, .494e-34]Solution in 2.686s

Time Plot 0 s.
Exiting SolveHard() after 3.686r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33, none,
358.97362823847273561976613376052744927 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941765516940762612076653575834018,
5.5896371831652104777841011095376663259 27,
443.83065884958916684896859848027014568 84]
one interval r = 22.46725374478229808016551074654241651126 ..
27.27388428365395113140038812811556199786
Time Approximations 0.041.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.08e-37
Equations at solution: [-.1e-37, .108e-36, .87e-35]Solution in 2.68s

Time Plot 0 s.
Exiting SolveHard() after 3.662r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33, none,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]
```

```
2 --> 0  target = [14.19258941765516940762612076653575834018,
5.589637183165210477784101109537666325927,
443.83065884958916684896859848027014568 84]
one interval r = 32.15575279500575811476967848605380788457 ..
35.50872228741739116947599383463150511696
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.293e-34]Solution in 0.477s

Time Plot 0 s.
Exiting SolveHard() after 0.828r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136531023382789547003563757272448,
5.187783578628759541939766464923647812644,
408.65773862313965342821694548469598899 12]
one interval r = 21.71840114632144107944621286276168491159 ..
26.81849303511953644111205007630424930561
Time Approximations 0.064.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=0
Equations at solution: [0., 0., .75e-35]Solution in 0.924s

Time Plot 0 s.
Exiting SolveHard() after 3.651r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49, none, none,
361.50888346987659392404109290627558171 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136531023382789547003563757272448,
5.18778357862875954193976646492364781 2644,
408.65773862313965342821694548469598899 12]
one interval r = 31.80828598742601155325440611571384432150 ..
35.00011460037964976736811564799755473484
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., -.30e-35]Solution in 0.404s

Time Plot 0 s.
Exiting SolveHard() after 2.365r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49,
371.48387393788733068779835190447117843 64, none,
361.50888346987659392404109290627558171 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110543012723189782159630149817744,
6.19626256533523225195625169866832571 2598,
385.44474379203895777244198507317796606 09]
one interval r = 31.60836097530078693591681313788575654240 ..
34.66372795605863674047259369107178000488
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .134e-34]Solution in 0.553s

Time Plot 0 s.
Exiting SolveHard() after 0.819r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
```

```
398.331471047538767907860011073277267664 9,
371.483873937887330687798351904471178436 4, none,
361.508883469876593924041092906275581716 4,
324.671449925446001949523628798034861904 4, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.463471105430127231897821596301498177 44,
6.196262565335232251956251698668325712598,
385.444743792038957772441985073177966060 9]
two intervals r = 16.875634087617261084148308962494989462 12 ..
95000000000225375815948945418276559 1/500000000000000000000000000000000000
0 or r = 15.556404937983489854896497489835262440 38 ..
95000000000225375815948945418276559 1/500000000000000000000000000000000000
0
Time Approximations 0.057.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.538e-37, 0., -.2527e-34]Solution in 2.822s

Time Plot 0 s.
Exiting SolveHard() after 5.61r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434935966488043162388174444332589 4,
441.642959732322965242596675064485285747 0,
436.917481650652302175507969704339001955 3,
422.984933978890959145625171873210833249 1,
361.525802559525706664337545320942500615 6,
401.881739043474083568645559366864587104 0,
389.590015154793647520119238704988390611 2,
328.469398936153371061341094692092285763 3,
401.507571581339272614562096269646987910 1,
358.973628238472735619766133760527449277 8,
398.331471047538767907860011073277267664 9,
371.483873937887330687798351904471178436 4,
336.612158414768414511642973620003331604 6,
361.508883469876593924041092906275581716 4,
324.671449925446001949523628798034861904 4, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747429031682080459370184602622 56,
4.883810779989293565814439591709348462616,
```

```
376.6196785569596596070067863291739503646]
one interval r = 21.11001304863150966025729148041414285969 ..
26.31784243477605631117447027251098373195
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=5.0e-38
Equations at solution: [.2e-37, .50e-37, -.109e-34]Solution in 0.833s

Time Plot 0 s.
Exiting SolveHard() after 1.532r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.6429597323229652425966750644852857470,
436.9174816506523021755079697043390019553,
422.9849339788909591456251718732108332491,
361.5258025595257066643375453209425006156,
401.8817390434740835686455593668645871040,
389.5900151547936475201192387049883906112,
328.4693989361533710613410946920922857633,
401.5075715813392726145620962696469879101,
358.9736282384727356197661337605274492778,
398.3314710475387679078600110732772676649,
371.4838739378873306877983519044711784364,
336.6121584147684145116429736200033316046,
361.5088834698765939240410929062755817164,
324.6714499254460019495236287980348619044, none,
328.4693851368796869472785122702236872849, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874742903168208045937018460262256,
4.883810779989293565814439591709348462616,
376.6196785569596596070067863291739503646]
one interval r = 31.53899497704307714597143514583820111836 ..
34.53618386089204549321643560259500688382
Time Approximations 0.015.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
```

```
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=6.92e-36
Equations at solution: [.532e-35, -.692e-35, .99e-35]Solution in 0.507s

Time Plot 0 s.
Exiting SolveHard() after 2.487r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49,
371.48387393788733068779835190447117843 64,
336.61215841476841451164297362000333160 46,
361.50888346987659392404109290627558171 64,
324.67144992544600194952362879803486190 44, none,
328.46938513687968694727851227022368728 49,
343.81340624507978610934041127273635350 64, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017547781592336256186797707298626,
6.0258135493074403888856349244498863238 66,
351.42702948584879925080827129224381854 68]
one interval r = 31.362302061086681925830236902597516728 35 ..
34.174466406143528016932973405042045084 83
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .159e-34]Solution in 0.512s

Time Plot 0 s.
Exiting SolveHard() after 0.754r=33.3686 in [32.23723258 ..
```

```
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49,
371.48387393788733068779835190447117843 64,
336.61215841476841451164297362000333160 46,
361.50888346987659392404109290627558171 64,
324.67144992544600194952362879803486190 44, none,
328.46938513687968694727851227022368728 49,
343.81340624507978610934041127273635350 64, none, none,
292.99969138550455489844040122647047189 52, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017547781592336256186797707298626,
6.0258135493074403888856349244498863238 66,
351.42702948584879925080827129224381854 68]
two intervals r = 17.981355144414314838982315255019674185 46 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000000
0 or r = 13.846080154298365430060707260679166301 92 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000000
0
Time Approximations 0.046.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=4e-38
Equations at solution: [.107e-36, -.4e-37, .79e-36]Solution in 2.604s

Time Plot 0 s.
Exiting SolveHard() after 5.435r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49,
371.48387393788733068779835190447117843 64,
336.61215841476841451164297362000333160 46,
361.50888346987659392404109290627558171 64,
324.67144992544600194952362879803486190 44,
302.31384315407943030726716171530322789 37,
328.46938513687968694727851227022368728 49,
343.81340624507978610934041127273635350 64, none, none,
292.99969138550455489844040122647047189 52, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941888098524572849364188383032810,
6.37794387391591473667171037094690892149 5,
423.28832784649068748007868072210945040 86]
one interval r = 31.94661817598361985343073695785584741880 ..
35.21212308659475653660422924021603360536
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .33e-35]Solution in 0.605s

Time Plot 0 s.
Exiting SolveHard() after 0.936r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
```

```
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49,
371.48387393788733068779835190447117843 64,
336.61215841476841451164297362000333160 46,
361.50888346987659392404109290627558171 64,
324.67144992544600194952362879803486190 44,
302.31384315407943030726716171530322789 37,
328.46938513687968694727851227022368728 49,
343.81340624507978610934041127273635350 64, none, none,
292.99969138550455489844040122647047189 52, none, none,
360.06173467493378467310767852262651406 79, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941888098524572849364188383032810,
6.3779438739159147366717103709469089214 95,
423.28832784649068748007868072210945040 86]
two intervals r = 15.22886702405665585927785254195636782858 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000
0 or r = 17.12965777100480074127248153803267697190 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000
0
Time Approximations 0.059.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [-.14e-37, 0., .594e-35]Solution in 2.78s

Time Plot 0 s.
Exiting SolveHard() after 5.843r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49,
```

```
371.48387393788733068779835190447117843 64,
336.61215841476841451164297362000333160 46,
361.50888346987659392404109290627558171 64,
324.67144992544600194952362879803486190 44,
302.31384315407943030726716171530322789 37,
328.46938513687968694727851227022368728 49,
343.81340624507978610934041127273635350 64,
375.73285291246253893659753325543124967 09, none,
292.99969138550455489844040122647047189 52, none, none,
360.06173467493378467310767852262651406 79, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343422390714877100273386987769 24,
4.0035598155425132602684456876885705735 53,
404.47973594157599437793746180733602732 98]
two intervals r = 16.096839663677197240380630931593734188 81 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000
0 or r = 16.399886491209933462974120929218716613 91 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000
0
Time Approximations 0.052.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.17e-37, 0., .4910e-34]Solution in 3.029s

Time Plot 0 s.
Exiting SolveHard() after 4.094r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49,
371.48387393788733068779835190447117843 64,
336.61215841476841451164297362000333160 46,
361.50888346987659392404109290627558171 64,
```

```
324.67144992544600194952362879803486190 44,
302.31384315407943030726716171530322789 37,
328.46938513687968694727851227022368728 49,
343.81340624507978610934041127273635350 64,
375.73285291246253893659753325543124967 09, none,
292.99969138550455489844040122647047189 52,
358.64341560725374669260602468496646148 52, none,
360.06173467493378467310767852262651406 79, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234342239071487710027338698776924,
4.0035598155425132602684456876885705735 53,
404.47973594157599437793746180733602732 98]
one interval r = 21.634296299803347194984178602206823006 54 ..
26.757681698990945103029376771008617409 41
Time Approximations 0.046.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=1.24e-37
Equations at solution: [-.3e-37, -.124e-36, -.88e-35]Solution in 1.019s

Time Plot 0 s.
Exiting SolveHard() after 3.652r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49,
371.48387393788733068779835190447117843 64,
336.61215841476841451164297362000333160 46,
361.50888346987659392404109290627558171 64,
324.67144992544600194952362879803486190 44,
302.31384315407943030726716171530322789 37,
328.46938513687968694727851227022368728 49,
343.81340624507978610934041127273635350 64,
```

```
375.7328529124625389365975332554312496709,
328.1170929456851837853411793418952700916,
292.9996913855045548984404012264704718952,
358.6434156072537466926060246849664614852, none,
360.0617346749337846731076785226265140679, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954461157985993952901728328677876,
6.196177230188610975234488518293831052187,
385.4273402557612177267699530837522628608]
one interval r = 31.60822049085419969238590646425380875625 ..
34.66347615043482636134978658477343033141
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, -.73e-35]Solution in 2.314s

Time Plot 0 s.
Exiting SolveHard() after 2.619r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349359664880431623881744443325894,
441.6429597323229652425966750644852857470,
436.9174816506523021755079697043390019553,
422.9849339788909591456251718732108332491,
361.5258025595257066643375453209425006156,
401.8817390434740835686455593668645871040,
389.5900151547936475201192387049883906112,
328.4693989361533710613410946920922857633,
401.5075715813392726145620962696469879101,
358.9736282384727356197661337605274492778,
398.3314710475387679078600110732772676649,
371.4838739378873306877983519044711784364,
336.6121584147684145116429736200033316046,
361.5088834698765939240410929062755817164,
324.6714499254460019495236287980348619044,
302.3138431540794303072671617153032278937,
328.4693851368796869472785122702236872849,
343.8134062450797861093404112727363535064,
375.7328529124625389365975332554312496709,
328.1170929456851837853411793418952700916,
292.9996913855045548984404012264704718952,
```

```
358.6434156072537466926060246849664614852, none,
360.0617346749337846731076785226265140679, none, none,
324.6552122352067624599729400022156938739, none, none, none, none]

0 --> 1  target = [26.46318954461157985993952901728328677876,
6.196177230188610975234488518293831052187,
385.42734025576121772676995308375226286 08]
two intervals r = 16.87629600300474597721309559059048653642 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000000
0 or r = 15.55559000646767713536085767802445777916 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000000
0
Time Approximations 0.058.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [.358e-37, -.1e-37, .3696e-34]Solution in 2.798s

Time Plot 0 s.
Exiting SolveHard() after 3.92r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.6429597323229652425966750644852857470,
436.9174816506523021755079697043390019553,
422.9849339788909591456251718732108332491,
361.5258025595257066643375453209425006156,
401.8817390434740835686455593668645871040,
389.5900151547936475201192387049883906112,
328.4693989361533710613410946920922857633,
401.5075715813392726145620962696469879101,
358.9736282384727356197661337605274492778,
398.3314710475387679078600110732772676649,
371.4838739378873306877983519044711784364,
336.6121584147684145116429736200033316046,
361.5088834698765939240410929062755817164,
324.6714499254460019495236287980348619044,
302.3138431540794303072671617153032278937,
328.4693851368796869472785122702236872849,
343.8134062450797861093404112727363535064,
375.7328529124625389365975332554312496709,
328.1170929456851837853411793418952700916,
292.9996913855045548984404012264704718952,
358.6434156072537466926060246849664614852, none,
```

```
360.06173467493378467310767852262651406 79,
336.59441032517181577729379544072408892 06, none,
324.65521223520676245997294000221569387 39, none, none, none, none]

0 --> 2  target = [34.49522661153914436283599531493186425351,
3.8971313159345830048704544463317289665 79,
373.78081883847813924595503548640700156 74]
two intervals r = 17.297690862487274633444397073906528834 35 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000000
0 or r = 14.994364074024435995094465731786865965 85 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000000
0
Time Approximations 0.079.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., -.1725e-34]Solution in 2.972s

Time Plot 0 s.
Exiting SolveHard() after 6.445r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49,
371.48387393788733068779835190447117843 64,
336.61215841476841451164297362000333160 46,
361.50888346987659392404109290627558171 64,
324.67144992544600194952362879803486190 44,
302.31384315407943030726716171530322789 37,
328.46938513687968694727851227022368728 49,
343.81340624507978610934041127273635350 64,
375.73285291246253893659753325543124967 09,
328.11709294568518378534117934189527009 16,
292.99969138550455489844040122647047189 52,
358.64341560725374669260602468496646148 52, none,
```

```
360.06173467493378467310767852262651406 79,
336.59441032517181577729379544072408892 06, none,
324.65521223520676245997294000221569387 39,
331.93806790678313761958493342111268050 85, none, none, none]

1 --> 2  target = [34.49522661153914436283599531493186425351,
3.8971313159345830048704544463317289665 79,
373.78081883478139245955035486407001567 4]
one interval r = 21.06068473180166390853069494333800816710 ..
26.26979834277256171644241419736108494735
Time Approximations 0.033.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [.1e-37, .3e-37, -.57e-35]Solution in 0.783s

Time Plot 0 s.
Exiting SolveHard() after 1.481r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49,
371.48387393788733068779835190447117843 64,
336.61215841476841451164297362000333160 46,
361.50888346987659392404109290627558171 64,
324.67144992544600194952362879803486190 44,
302.31384315407943030726716171530322789 37,
328.46938513687968694727851227022368728 49,
343.81340624507978610934041127273635350 64,
375.73285291246253893659753325543124967 09,
328.11709294568518378534117934189527009 16,
292.99969138550455489844040122647047189 52,
358.64341560725374669260602468496646148 52,
299.89866204308914108372125663689676116 35,
```

```
360.06173467493378467310767852262651406 79,
336.59441032517181577729379544072408892 06, none,
324.65521223520676245997294000221569387 39,
331.93806790678313761958493342111268050 85, none, none, none]

0 --> 2  target = [33.81362495405197155766982735834754317737,
3.72564899358628449177763155321863900453 4,
325.89209972777969908613255290245066917 63]
two intervals r = 18.552270490100499178668072941030717890 83 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000000
0 or r = 12.49196935784334504818539644711672175707 ..
95000000000225375815948945418276559 1/50000000000000000000000000000000000000
0
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=8e-38
Equations at solution: [.191e-36, -.8e-37, .979e-35]Solution in 2.767s

Time Plot 0 s.
Exiting SolveHard() after 6.169r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493596648804316238817444433258 94,
441.64295973232296524259667506448528574 70,
436.91748165065230217550796970433900195 53,
422.98493397889095914562517187321083324 91,
361.52580255952570666433754532094250061 56,
401.88173904347408356864555936686458710 40,
389.59001515479364752011923870498839061 12,
328.46939893615337106134109469209228576 33,
401.50757158133927261456209626964698791 01,
358.97362823847273561976613376052744927 78,
398.33147104753876790786001107327726766 49,
371.48387393788733068779835190447117843 64,
336.61215841476841451164297362000333160 46,
361.50888346987659392404109290627558171 64,
324.67144992544600194952362879803486190 44,
302.31384315407943030726716171530322789 37,
328.46938513687968694727851227022368728 49,
343.81340624507978610934041127273635350 64,
375.73285291246253893659753325543124967 09,
328.11709294568518378534117934189527009 16,
292.99969138550455489844040122647047189 52,
```

```
358.6434156072537466926060246849664614852,
299.8986620430891410837212566368967611635,
360.0617346749337846731076785226265140679,
336.5944103251718157772937954407240889206, none,
324.6552122352067624599729400022156938739,
331.9380679067831376195849334211126805085, none, none,
289.5459577223722736134823289982440973795]

1 --> 2  target = [33.8136249540519715576698273583475431773 7,
3.7256489935862844917776315532186390045 34,
325.8920997277796990861325529024506691763]
one interval r = 20.374689350906193851214047189069562835 83 ..
25.37892165300495968601585801772446429642
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=6e-38
Equations at solution: [-.4e-37, -.6e-37, .347e-34]Solution in 0.565s

Time Plot 0 s.
Exiting SolveHard() after 1.115r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349359664880431623881744443325894,
441.6429597323229652425966750644852857470,
436.9174816506523021755079697043390019553,
422.9849339788909591456251718732108332491,
361.5258025595257066643375453209425006156,
401.8817390434740835686455593668645871040,
389.5900151547936475201192387049883906112,
328.4693989361533710613410946920922857633,
401.5075715813392726145620962696469879101,
358.9736282384727356197661337605274492778,
398.3314710475387679078600110732772676649,
371.4838739378873306877983519044711784364,
336.6121584147684145116429736200033316046,
361.5088834698765939240410929062755817164,
324.6714499254460019495236287980348619044,
302.3138431540794303072671617153032278937,
328.4693851368796869472785122702236872849,
343.8134062450797861093404112727363535064,
375.7328529124625389365975332554312496709,
328.1170929456851837853411793418952700916,
```

```
292.99969138550455489844040122647047189 52,
358.6434156072537466926060246849664614852,
299.8986620430891410837212566368967611635,
360.0617346749337846731076785226265140679,
336.5944103251718157772937954407240889206,
256.1075318609375609359939161829381171342,
324.6552122352067624599729400022156938739,
331.9380679067831376195849334211126805085, none, none,
289.5459577223722736134823289982440973795]

1 --> 0  target = [17.9304136970606634828149393945696983412 0,
4.6865087021584612228180661071106565769 12,
353.3054109526037743876019729291756948256]
one interval r = 20.7315047908051856183053723637484453779 8 ..
25.9067535352986512390585244152490175019 3
Time Approximations 0.041.

hint used Hint := [25.4020715378188039710536173439704738275 4, 3, -1, 1,
17.0062444547662134530206245122467435705 3, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, -.137e-34]Solution in 0.648s

Time Plot 0 s.
Exiting SolveHard() after 2.964r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349359664880431623881744443325894,
441.6429597323229652425966750644852857470,
436.9174816506523021755079697043390019553,
422.9849339788909591456251718732108332491,
361.5258025595257066643375453209425006156,
401.8817390434740835686455593668645871040,
389.5900151547936475201192387049883906112,
328.4693989361533710613410946920922857633,
401.5075715813392726145620962696469879101,
358.9736282384727356197661337605274492778,
398.3314710475387679078600110732772676649,
371.4838739378873306877983519044711784364,
336.6121584147684145116429736200033316046,
361.5088834698765939240410929062755817164,
324.6714499254460019495236287980348619044,
302.3138431540794303072671617153032278937,
328.4693851368796869472785122702236872849,
343.8134062450797861093404112727363535064,
```

```
375.7328529124625389365975332554312496709,
328.1170929456851837853411793418952700916,
292.9996913855045548984404012264704718952,
358.6434156072537466926060246849664614852,
299.8986620430891410837212566368967611635,
360.0617346749337846731076785226265140679,
336.5944103251718157772937954407240889206,
256.1075318609375609359939161829381171342,
324.6552122352067624599729400022156938739,
331.9380679067831376195849334211126805085,
304.7995832606049575887602078552205794133, none,
289.5459577223722736134823289982440973795]

2 --> 0  target = [17.93041369706066348281493939456969834120,
4.6865087021584612228180661071106565769 12,
353.3054109526037743876019729291756948256]
one interval r = 31.37435486988265180381841189887081463852 ..
34.20127520027767234386022804513416150712
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=1.4e-37
Equations at solution: [.9e-37, -.14e-36, .74e-35]Solution in 2.001s

Time Plot 0 s.
Exiting SolveHard() after 2.281r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349359664880431623881744443325894,
441.6429597323229652425966750644852857470,
436.9174816506523021755079697043390019553,
422.9849339788909591456251718732108332491,
361.5258025595257066643375453209425006156,
401.8817390434740835686455593668645871040,
389.5900151547936475201192387049883906112,
328.4693989361533710613410946920922857633,
401.5075715813392726145620962696469879101,
358.9736282384727356197661337605274492778,
398.3314710475387679078600110732772676649,
371.4838739378873306877983519044711784364,
336.6121584147684145116429736200033316046,
361.5088834698765939240410929062755817164,
324.6714499254460019495236287980348619044,
```

```
302.31384315407943030726716171530322789 37,
328.46938513687968694727851227022368728 49,
343.81340624507978610934041127273635350 64,
375.73285291246253893659753325543124967 09,
328.11709294568518378534117934189527009 16,
292.99969138550455489844040122647047189 52,
358.64341560725374669260602468496646148 52,
299.89866204308914108372125663689676116 35,
360.06173467493378467310767852262651406 79,
336.59441032517181577729379544072408892 06,
256.10753186093756093599391618293811713 42,
324.65521223520676245997294000221569387 39,
331.93806790678313761958493342111268050 85,
304.79958326060495758876020785522057941 33,
323.46169176545535576475077342434661564 86,
289.54595772237227361348232899824409737 95]

Cascade time 144.457
counts: 28, 28

 Iteration 29

Start Generation 1
1 --> 0  target = [11.99999999987789805291952043232394516200,
6.217012502809421184379933369744066082161,
485.54908090084971994711437961599029651 55]
one interval r = 23.40850301661387713518685085330856872052 ..
27.67578046441842655022278467836790189391
Time Approximations 0.038.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=3.14e-37
Equations at solution: [-.10e-36, .314e-36, .31e-35]Solution in 0.994s

Time Plot 0 s.
Exiting SolveHard() after 3.832r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0  target = [11.999999999877898052919520432323945162 00,
6.2170125028094211843799333697440660821 61,
485.5490809008497199471143796159902965155]
one interval r = 32.628147792178601538038203560712881862 74 ..
36.102483889447560763411079672276965225 31
Time Approximations 0.021.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [.4e-37, -.2e-37, -.123e-34]Solution in 0.557s

Time Plot 0 s.
Exiting SolveHard() after 2.756r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844923495353076568940070554289 85,
6.5834347216246593419052703117016353111 32,
467.78730596024553690792533370059447823 02]
one interval r = 32.419789556659299309358890869195963707 97 ..
35.851524173729170514945395614112121811 74
Time Approximations 0.022.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.25e-35]Solution in 2.196s

Time Plot 0 s.
```

```
Exiting SolveHard() after 2.556r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10, none, none,
401.88173904613707581526945515679777950 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844923495353076568940070554289 85,
6.5834347216246593419052703117016353111 32,
467.78730596024553690792533370059447823 02]
two intervals r = 12.923271608235468029492042654136847765 88 ..
189999999999515823437444959459592286 89/100000000000000000000000000000000
00000 or r = 18.394248580289263277065405099303684258 60 ..
189999999999515823437444959459592286 89/100000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=8.6e-38
Equations at solution: [.7e-37, .86e-37, .321e-35]Solution in 39.067s

Time Plot 0 s.
Exiting SolveHard() after 40.389r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95, none,
401.88173904613707581526945515679777950 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628337266359612670984014853164 94,
4.1256517970430769709299368147242408133 48,
```

```
440.671230656440338301387169777547537913 0]
two intervals r = 14.35659705086501897037555196436415491969 ..
18999999999951582343744495945959228689/10000000000000000000000000000000
00000 or r = 17.70352613821485188788054187181124427625 ..
18999999999951582343744495945959228689/10000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.15e-37, .1e-37, -.320e-35]Solution in 2.875s

Time Plot 0 s.
Exiting SolveHard() after 5.427r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95, none,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962833726635961267098401485316494,
4.12565179704307697092993681472424081334 8,
440.671230656440338301387169777547537913 0]
one interval r = 22.39761154376748315009441927179225320686 ..
27.23722351615511650181454627347648842163
Time Approximations 0.04.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
```

```
in partial time = 2.822 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070644083184163242647652897039098 97, rm =
14.378187706585119187577358604597829577 70}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.59e-37
Equations at solution: [-.1e-37, -.159e-36, .170e-34]Solution in
10.192s

Time Plot 0 s.
Exiting SolveHard() after 11.103r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888034401743258555988420738696 18,
4.0048690820128076520218335022473476858 49,
404.86224501687422588987507255696224699 56]
two intervals r = 16.080110077411806068712693773986009583 80 ..
189999999999515823437444959459592286 89/100000000000000000000000000000000
00000 or r = 16.415798127042440907277034429422908241 60 ..
189999999999515823437444959459592286 89/100000000000000000000000000000000
00000
Time Approximations 0.047.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., .2642e-34]Solution in 3.422s

Time Plot 0 s.
Exiting SolveHard() after 6.327r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74, none, none,
358.97362824773316158578137356602197489 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888034401743258559884207386961 8,
4.0048690820128076520218335022473476858 49,
404.86224501687422588987507255696224699 56]
one interval r = 21.641943994136727446838564255141955407 59 ..
26.763306600562761190675225522572419500 22
Time Approximations 0.052.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.00e-37
Equations at solution: [-.5e-37, -.200e-36, -.93e-35]Solution in 2.7s

Time Plot 0 s.
Exiting SolveHard() after 3.723r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05, none,
358.97362824773316158578137356602197489 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589417914069696640788012358642876 84,
5.5896371827471281483231282765571629168 31,
```

```
443.83065884421122874617189169805771554 66]
one interval r = 22.46725374471493300369100139445921797765 ..
27.27388428362996569640623314706466951654
Time Approximations 0.039.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.89e-37
Equations at solution: [-.2e-37, .189e-36, .21e-35]Solution in 2.855s

Time Plot 0 s.
Exiting SolveHard() after 3.846r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05, none,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941791406969664078801235864287684,
5.589637182747128148323128276557162916831,
443.83065884421122874617189169805771554 66]
one interval r = 32.15575279501354847103387273619349886879 ..
35.50872228734814145707827043987042462764
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
```

```
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., .51e-35]Solution in 0.491s

Time Plot 0 s.
Exiting SolveHard() after 0.86r=34.9395 in [33.37332721 .. 35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136461405270650083319614462062433,
5.18778357842475666049774019576075128054 2,
408.65773863754320903727417277307894260 38]
one interval r = 21.71840114667139315738889007186246961797 ..
26.81849303536221819006704523249387483554
Time Approximations 0.056.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=5.3e-38
Equations at solution: [-.1e-37, -.53e-37, .90e-35]Solution in 2.68s

Time Plot 0 s.
Exiting SolveHard() after 5.403r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
```

```
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22, none, none,
361.50888347948521239552247380133262556 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136461405270650083319614462062433,
5.1877835784247566604977401957607512805 42,
408.65773863754320903727417277307894260 38]
one interval r = 31.80828598762903474397819090534571861976 ..
35.00011460060435415352804011240897608465
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.31e-35]Solution in 0.424s

Time Plot 0 s.
Exiting SolveHard() after 0.728r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76, none,
361.50888347948521239552247380133262556 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 1  target = [26.46347110562794489027557261143219445915,
6.19626256539500070665748244649834504096 2,
385.44474380212270847544181688504768122 63]
one interval r = 31.60836097545751463759898864157857398042 ..
34.66372795622565933015459036129614279373
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=8e-38
Equations at solution: [-.5e-37, .8e-37, .241e-34]Solution in 0.565s

Time Plot 0 s.
Exiting SolveHard() after 0.84r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76, none,
361.50888347948521239552247380133262556 88,
324.67144993671375856399180132710106006 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110562794489027557261143219445915,
6.19626256539500070665748244649834504096 2,
385.44474380212270847544181688504768122 63]
two intervals r = 16.87563408716901024827303131528421783373 ..
189999999999515823437444959459592286 89/10000000000000000000000000000000
00000 or r = 15.55640493840150945026003597229900213354 ..
189999999999515823437444959459592286 89/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.263 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175098426641090703524210080824283, rm
= 2.33653277413149585472135127719522563190 9}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.180e-37, 0., .1891e-34]Solution in 27.625s

Time Plot 0 s.
Exiting SolveHard() after 31.145r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76,
336.61215841798096853945367076729380504 63,
361.50888347948521239552247380133262556 88,
324.67144993671375856399180132710106006 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874700457998749240760137354434778,
4.88381077972389295773873919292881314651 7,
376.61967856704435721693699756849022609 9]
one interval r = 21.11001304887191860677448073416144577697 ..
26.31784243498026809488780621928561515661
Time Approximations 0.033.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
```

```
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.211e-34]Solution in 0.849s

Time Plot 0 s.
Exiting SolveHard() after 3.062r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76,
336.61215841798096853945367076729380504 63,
361.50888347948521239552247380133262556 88,
324.67144993671375856399180132710106006 78, none,
328.46938514156068840574499318027417283 21, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874700457998749240760137354434778,
4.8838107797238929577387391929288131465 17,
376.61967856704435721693699756844902260 99]
one interval r = 31.53899497719688620260522952904331775156 ..
34.53618386106076531774252257744097525384
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.046e-35
Equations at solution: [.804e-35, -.1046e-34, .226e-34]Solution in
2.074s

Time Plot 0 s.
Exiting SolveHard() after 2.372r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76,
336.61215841798096853945367076729380504 63,
361.50888347948521239552247380133262556 88,
324.67144993671375856399180132710106006 78, none,
328.46938514156068840574499318027417283 21,
343.81340626328604481906270907876826669 34, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017560314771675094175894083424031,
6.0258135493430287280377781745999810845 24,
351.42702949073758287648393704737181501 61]
one interval r = 31.36230206119806468339595745865938937678 ..
34.17446640624228280998874562263563154197
Time Approximations 0.015.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=1.5e-37
Equations at solution: [-.8e-37, .15e-36, .115e-34]Solution in 0.528s

Time Plot 0 s.
Exiting SolveHard() after 0.78r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
```

```
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76,
336.61215841798096853945367076729380504 63,
361.50888347948521239552247380133262556 88,
324.67144993671375856399180132710106006 78, none,
328.46938514156068840574499318027417283 21,
343.81340626328604481906270907876826669 34, none, none,
292.99969139195864914468346339545948348 32, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017560314771675094175894083424031,
6.0258135493430287280377781745999810845 24,
351.42702949073758287648393704737181501 61]
two intervals r = 17.981355144203741082132085546842484557 77 ..
189999999999515823437444959459592286 89/100000000000000000000000000000000
00000 or r = 13.846080154531460696280343624118500367 93 ..
189999999999515823437444959459592286 89/100000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 4.785 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713584181294519071188324552084 65, rm
= 2.7345009932884493717148993005137163033 52}});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [.53e-37, -.1e-37, -.93e-35]Solution in 17.501s

Time Plot 0 s.
Exiting SolveHard() after 20.166r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
```

```
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76,
336.61215841798096853945367076729380504 63,
361.50888347948521239552247380133262556 88,
324.67144993671375856399180132710106006 78,
302.31384315210777704406936822605164061 07,
328.46938514156068840574499318027417283 21,
343.81340626328604481906270907876826669 34, none, none,
292.99969139195864914468346339545948348 32, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941877796504898413028422176127917,
6.37794387387638029125008065369333243777 3,
423.28832783594475751483288955661430771 47]
one interval r = 31.94661817594934603338121551644625229750 ..
35.21212308645469338742576929587559719449
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=7e-38
Equations at solution: [.7e-37, -.7e-37, -.153e-34]Solution in 0.595s

Time Plot 0 s.
Exiting SolveHard() after 0.935r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76,
336.61215841798096853945367076729380504 63,
361.50888347948521239552247380133262556 88,
324.67144993671375856399180132710106006 78,
```

```
302.31384315210777704406936822605164061 07,
328.46938514156068840574499318027417283 21,
343.81340626328604481906270907876826669 34, none, none,
292.99969139195864914468346339545948348 32, none, none,
360.06173466685199233014871347407402994 95, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418777965048984130284221761279 17,
6.3779438738763802912500806536933324377 73,
423.28832783594475751483288955661430771 47]
two intervals r = 15.228867024527883529779833530549534531 96 ..
189999999999515823437444959459592286 89/100000000000000000000000000000000
00000 or r = 17.129657770551395005840643163793583965 43 ..
189999999999515823437444959459592286 89/100000000000000000000000000000000
00000
Time Approximations 0.062.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.066 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541345375075331709420371760472 66, rm
= 2.0640682988107246594926122980338058926 42}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.16e-37, .1e-37, .578e-35]Solution in 29.291s

Time Plot 0 s.
Exiting SolveHard() after 32.342r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76,
336.61215841798096853945367076729380504 63,
361.50888347948521239552247380133262556 88,
```

```
324.67144993671375856399180132710106006 78,
302.31384315210777704406936822605164061 07,
328.46938514156068840574499318027417283 21,
343.81340626328604481906270907876826669 34,
375.73285289423474669374944681640119616 58, none,
292.99969139195864914468346339545948348 32, none, none,
360.06173466685199233014871347407402994 95, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234347863671194007724070845653550,
4.0035598157141136311407105280739397256 15,
404.47973594429663494036763673248437824 26]
two intervals r = 16.09683966350498560766856515221639979200 ..
189999999999515823437444959459592286 89/10000000000000000000000000000000
00000 or r = 16.39988649125722061759172516599622901552 ..
189999999999515823437444959459592286 89/10000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [-.17e-37, 0., -.1003e-34]Solution in 3.232s

Time Plot 0 s.
Exiting SolveHard() after 4.288r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76,
336.61215841798096853945367076729380504 63,
361.50888347948521239552247380133262556 88,
324.67144993671375856399180132710106006 78,
302.31384315210777704406936822605164061 07,
328.46938514156068840574499318027417283 21,
```

```
343.8134062632860448190627090787682666934,
375.7328528942347466937494468164011961658, none,
292.9996913919586491446834633954594834832,
358.6434156165485446182691679487931869172, none,
360.0617346668519923301487134740740299495, none, none, none, none,
none, none, none]

1 --> 2  target = [34.9395323434786367119400772407084565355 0,
4.0035598157141136311407105280739397256 15,
404.4797359442966349403676367324843782426]
one interval r = 21.6342962999163470139586865128550377693 8 ..
26.7576816990662410982531635517529398517 4
Time Approximations 0.049.

hint used Hint := [25.8653228005422684844258072120286493757 6, 3, -1, 1,
16.7792152583743340383255391208949113668 0, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.301e-34]Solution in 1.042s

Time Plot 0 s.
Exiting SolveHard() after 3.76r=25.8653 in [23.83864811 .. 26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349386400081158106904644438778663,
441.6429597332208976694996742733765990249,
436.9174816586606927714192764463679096710,
422.9849339733951771631141275613973878295,
361.5258025692670632271057763084528454746,
401.8817390461370758152694551567977795032,
389.5900151681150608529699505663628372374,
328.4693989408392825981909927731998928205,
401.5075715840358469987472134486884218551,
358.9736282477331615857813735660219748999,
398.3314710372172558046683152098931415922,
371.4838739588386436880403686286723438976,
336.6121584179809685394536707672938050463,
361.5088834794852123955224738013326255688,
324.6714499367137585639918013271010600678,
302.3138431521077770440693682260516406107,
328.4693851415606884057449931802741728321,
343.8134062632860448190627090787682666934,
375.7328528942347466937494468164011961658,
328.1170929504026684750216523705164740615,
292.9996913919586491446834633954594834832,
358.6434156165485446182691679487931869172, none,
```

```
360.06173466685199233014871347407402994 95, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954480720122703797513037801968706,
6.196177230247711084722605289672898953123,
385.42734026570843422801292275371746537 39]
one interval r = 31.608220491009819731869606459327667506 75 ..
34.663476150599876921764217435766547100 22
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, .18e-35]Solution in 2.229s

Time Plot 0 s.
Exiting SolveHard() after 2.518r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76,
336.61215841798096853945367076729380504 63,
361.50888347948521239552247380133262556 88,
324.67144993671375856399180132710106006 78,
302.31384315210777704406936822605164061 07,
328.46938514156068840574499318027417283 21,
343.81340626328604481906270907876826669 34,
375.73285289423474669374944681640119616 58,
328.11709295040266847502165237051647406 15,
292.99969139195864914468346339545948348 32,
358.64341561654854461826916794879318691 72, none,
360.06173466685199233014871347407402994 95, none, none,
324.65521224634713737851978332237227196 21, none, none, none, none]
```

```
0 --> 1  target = [26.46318954480720122703797513037801968706,
6.196177230247711084722605289672898953123,
385.42734026570843422801292275371746537 39]
two intervals r = 16.87629600256173316422234907683256062657 ..
18999999999951582343744495945959228689/10000000000000000000000000000000
00000 or r = 15.55559000687935858745618524660993141233 ..
18999999999951582343744495945959228689/10000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 8.859 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852510472633492104831662433492403, rm
= 2.33669042819836477012713948133728550774 7}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.180e-37, 0., .1366e-34]Solution in 29.386s

Time Plot 0 s.
Exiting SolveHard() after 30.556r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76,
336.61215841798096853945367076729380504 63,
361.50888347948521239552247380133262556 88,
324.67144993671375856399180132710106006 78,
302.31384315210777704406936822605164061 07,
328.46938514156068840574499318027417283 21,
343.81340626328604481906270907876826669 34,
375.73285289423474669374944681640119616 58,
328.11709295040266847502165237051647406 15,
292.99969139195864914468346339545948348 32,
```

```
358.6434156165485446182691679487931869172, none,
360.0617346668519923301487134740740299495,
336.5944103282451037833364064172406800933, none,
324.6552122463471373785197833223722719621, none, none, none, none]

0 --> 2  target = [34.4952266118714203778261943159653634930 4,
3.8971313161720339824748883922793666775 80,
373.7808188598690402010238890399462076846]
two intervals r = 17.29769086168290871595660066429491507587 ..
18999999999951582343744495945959228689/1000000000000000000000000000000000
00000 or r = 14.99436407503722645019511027333100606820 ..
18999999999951582343744495945959228689/1000000000000000000000000000000000
00000
Time Approximations 0.085.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.18e-37, 0., -.2560e-34]Solution in 1.172s

Time Plot 0 s.
Exiting SolveHard() after 4.7r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349386400081158106904644438778663,
441.6429597332208976694996742733765990249,
436.9174816586606927714192764463679096710,
422.9849339733951771631141275613973878295,
361.5258025692670632271057763084528454746,
401.8817390461370758152694551567977795032,
389.5900151681150608529699505663628372374,
328.4693989408392825981909927731998928205,
401.5075715840358469987472134486884218551,
358.9736282477331615857813735660219748999,
398.3314710372172558046683152098931415922,
371.4838739588386436880403686286723438976,
336.6121584179809685394536707672938050463,
361.5088834794852123955224738013326255688,
324.6714499367137585639918013271010600678,
302.3138431521077770440693682260516406107,
328.4693851415606884057449931802741728321,
343.8134062632860448190627090787682666934,
375.7328528942347466937494468164011961658,
328.1170929504026684750216523705164740615,
292.9996913919586491446834633954594834832,
```

```
358.6434156165485446182691679487931869172, none,
360.0617346668519923301487134740740299495,
336.5944103282451037833364064172406800933, none,
324.6552122463471373785197833223722719621,
331.9380679327214681956537275173744698874, none, none, none]

1 --> 2  target = [34.4952266118714203778261943159653634930 4,
3.8971313161720339824748883922793666775 80,
373.7808188598690402010238890399462076846]
one interval r = 21.06068473223436991049520164646722995165 ..
26.26979834317128518594199212553908836889
Time Approximations 0.034.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=1.7e-37
Equations at solution: [.7e-37, .17e-36, -.438e-34]Solution in 0.778s

Time Plot 0 s.
Exiting SolveHard() after 3.363r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349386400081158106904644438778663,
441.6429597332208976694996742733765990249,
436.9174816586606927714192764463679096710,
422.9849339733951771631141275613973878295,
361.5258025692670632271057763084528454746,
401.8817390461370758152694551567977795032,
389.5900151681150608529699505663628372374,
328.4693989408392825981909927731998928205,
401.5075715840358469987472134486884218551,
358.9736282477331615857813735660219748999,
398.3314710372172558046683152098931415922,
371.4838739588386436880403686286723438976,
336.6121584179809685394536707672938050463,
361.5088834794852123955224738013326255688,
324.6714499367137585639918013271010600678,
302.3138431521077770440693682260516406107,
328.4693851415606884057449931802741728321,
343.8134062632860448190627090787682666934,
375.7328528942347466937494468164011961658,
328.1170929504026684750216523705164740615,
292.9996913919586491446834633954594834832,
358.6434156165485446182691679487931869172,
```

```
299.89866206492751101005223809313735687 37,
360.06173466685199233014871347407402994 95,
336.59441032824510378333640641724068009 33, none,
324.65521224634713737851978332237227196 21,
331.93806793272146819565372751737446988 74, none, none, none]

0 --> 2  target = [33.81362495424769962749177405230445859359,
3.7256489937908123059395945566800317183 52,
325.89209973927532450700286149541569028 94]
two intervals r = 18.552270489808079087943966466115847762 22 ..
189999999999515823437444959459592286 89/100000000000000000000000000000000
00000 or r = 12.49196935844774961656276686562469696142 ..
189999999999515823437444959459592286 89/100000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [-.139e-36, .6e-37, -.296e-34]Solution in 2.717s

Time Plot 0 s.
Exiting SolveHard() after 5.942r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493864000811581069046444387786 63,
441.64295973322089766949967427337659902 49,
436.91748165866069277141927644636790967 10,
422.98493397339517716311412756139738782 95,
361.52580256926706322710577630845284547 46,
401.88173904613707581526945515679777950 32,
389.59001516811506085296995056636283723 74,
328.46939894083928259819099277319989282 05,
401.50757158403584699874721344868842185 51,
358.97362824773316158578137356602197489 99,
398.33147103721725580466831520989314159 22,
371.48387395883864368804036862867234389 76,
336.61215841798096853945367076729380504 63,
361.50888347948521239552247380133262556 88,
324.67144993671375856399180132710106006 78,
302.31384315210777704406936822605164061 07,
328.46938514156068840574499318027417283 21,
343.81340626328604481906270907876826669 34,
375.73285289423474669374944681640119616 58,
328.11709295040266847502165237051647406 15,
```

```
292.99969139195864914468346339545948348 32,
358.6434156165485446182691679487931869172,
299.8986620649275110100522380931373568737,
360.0617346668519923301487134740740299495,
336.5944103282451037833364064172406800933, none,
324.6552122463471373785197833223722719621,
331.9380679327214681956537275173744698874, none, none,
289.5459577398079285102364296419519783621]

1 --> 2  target = [33.8136249542476996274917740523044585935 9,
3.7256489937908123059395945566800317183 52,
325.8920997392753245070028614954156902894]
one interval r = 20.374689351104852561719418531745972061 27 ..
25.37892165327103300549357252394080991507
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=8e-38
Equations at solution: [-.5e-37, -.8e-37, .265e-34]Solution in 0.587s

Time Plot 0 s.
Exiting SolveHard() after 1.121r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349386400081158106904644438778663,
441.6429597332208976694996742733765990249,
436.9174816586606927714192764463679096710,
422.9849339733951771631141275613973878295,
361.5258025692670632271057763084528454746,
401.8817390461370758152694551567977795032,
389.5900151681150608529699505663628372374,
328.4693989408392825981909927731998928205,
401.5075715840358469987472134486884218551,
358.9736282477331615857813735660219748999,
398.3314710372172558046683152098931415922,
371.4838739588386436880403686286723438976,
336.6121584179809685394536707672938050463,
361.5088834794852123955224738013326255688,
324.6714499367137585639918013271010600678,
302.3138431521077770440693682260516406107,
328.4693851415606884057449931802741728321,
343.8134062632860448190627090787682666934,
375.7328528942347466937494468164011961658,
```

```
328.1170929504026684750216523705164740615,
292.9996913919586491446834633954594834832,
358.6434156165485446182691679487931869172,
299.8986620649275110100522380931373568737,
360.0617346668519923301487134740740299495,
336.5944103282451037833364064172406800933,
256.1075318736321748984733917937615011647,
324.6552122463471373785197833223722719621,
331.9380679327214681956537275173744698874, none, none,
289.5459577398079285102364296419519783621]

1 --> 0  target = [17.9304136968786999913678837058478113443 1,
4.6865087018326929063962626306013683149 99,
353.3054109563592469530761412829641696718]
one interval r = 20.7315047909285715260205094377033060473 4 ..
25.9067535354024867873331604879931406153 2
Time Approximations 0.031.

hint used Hint := [25.4020715378188039710536173439704738275 4, 3, -1, 1,
17.0062444547662134530206245122467435705 3, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.6e-38
Equations at solution: [-.2e-37, -.46e-37, -.18e-35]Solution in 0.661s

Time Plot 0 s.
Exiting SolveHard() after 3.04r=25.4021 in [22.67806074 .. 25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349386400081158106904644438778663,
441.6429597332208976694996742733765990249,
436.9174816586606927714192764463679096710,
422.9849339733951771631141275613973878295,
361.5258025692670632271057763084528454746,
401.8817390461370758152694551567977795032,
389.5900151681150608529699505663628372374,
328.4693989408392825981909927731998928205,
401.5075715840358469987472134486884218551,
358.9736282477331615857813735660219748999,
398.3314710372172558046683152098931415922,
371.4838739588386436880403686286723438976,
336.6121584179809685394536707672938050463,
361.5088834794852123955224738013326255688,
324.6714499367137585639918013271010600678,
302.3138431521077770440693682260516406107,
328.4693851415606884057449931802741728321,
343.8134062632860448190627090787682666934,
```

```
375.7328528942347466937494468164011961658,
328.1170929504026684750216523705164740615,
292.9996913919586491446834633954594834832,
358.6434156165485446182691679487931869172,
299.8986620649275110100522380931373568737,
360.0617346668519923301487134740740299495,
336.5944103282451037833364064172406800933,
256.1075318736321748984733917937615011647,
324.6552122463471373785197833223722719621,
331.9380679327214681956537275173744698874,
304.7995832586945501966965237829175889625, none,
289.5459577398079285102364296419519783621]

2 --> 0  target = [17.93041369687869999136788370584781134431,
4.6865087018326929063962626306013683149 99,
353.3054109563592469530761412829641696718]
one interval r = 31.37435486998694296432678481343300963270 ..
34.20127520035988321738117173536762454482
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.329e-34]Solution in 0.34s

Time Plot 0 s.
Exiting SolveHard() after 0.624r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349386400081158106904644438778663,
441.6429597332208976694996742733765990249,
436.9174816586606927714192764463679096710,
422.9849339733951771631141275613973878295,
361.5258025692670632271057763084528454746,
401.8817390461370758152694551567977795032,
389.5900151681150608529699505663628372374,
328.4693989408392825981909927731998928205,
401.5075715840358469987472134486884218551,
358.9736282477331615857813735660219748999,
398.3314710372172558046683152098931415922,
371.4838739588386436880403686286723438976,
336.6121584179809685394536707672938050463,
361.5088834794852123955224738013326255688,
324.6714499367137585639918013271010600678,
```

```
302.31384315210777704406936822605164061 07,
328.46938514156068840574499318027417283 21,
343.81340626328604481906270907876826669 34,
375.73285289423474669374944681640119616 58,
328.11709295040266847502165237051647406 15,
292.99969139195864914468346339545948348 32,
358.64341561654854461826916794879318691 72,
299.89866206492751101005223809313735687 37,
360.06173466685199233014871347407402994 95,
336.59441032824510378333640641724068009 33,
256.10753187363217489847339179376150116 47,
324.65521224634713737851978332237227196 21,
331.93806793272146819565372751737446988 74,
304.79958325869455019669652378291758896 25,
323.46169177860897267273649379102075977 55,
289.54595773980792851023642964195197836 21]

Cascade time 238.568
counts: 28, 28

 Iteration 30

Start Generation 1
1 --> 0  target = [12.00000000011789483941707393457301460700,
6.21701250280597579459566906568270137077 6,
485.54908090210859111103216457277390080 28]
one interval r = 23.40850301659447364782632190464406123157 ..
27.67578046418520096221055328925234379973
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.12e-37
Equations at solution: [.7e-37, -.212e-36, -.13e-35]Solution in 1.022s

Time Plot 0 s.
Exiting SolveHard() after 4.037r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0  target = [12.000000000117894839417073934573014607 00,
6.2170125028059757945956690656827013707 76,
485.5490809021085911110321645727739008028]
one interval r = 32.628147792091761562173046228736697018 22 ..
36.102483889471839730317440871305892737 63
Time Approximations 0.022.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=6e-38
Equations at solution: [-.9e-37, .6e-37, -.59e-35]Solution in 2.384s

Time Plot 0 s.
Exiting SolveHard() after 2.811r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922 57324,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844705553075417450887476352384 47,
6.5834347216042597730651832039544640328 61,
467.78730596205930302052822255492140007 88]
one interval r = 32.419789556570654570555100370984410358 62 ..
35.851524173754634561319811588507904124 74
Time Approximations 0.021.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .150e-34]Solution in 0.65s

Time Plot 0 s.
```

```
Exiting SolveHard() after 1.043r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01, none, none,
401.88173904501329863196559445215306423 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684470555307541745088747635238447,
6.58343472160425977306518320395446403286 1,
467.78730596205930302052822255492140007 88]
two intervals r = 12.92327160845474163819904726212142204497 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000 or r = 18.39424858042170132425204333100148478576 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000
Time Approximations 0.042.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=0
Equations at solution: [0., 0., -.579e-35]Solution in 37.504s

Time Plot 0 s.
Exiting SolveHard() after 40.642r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83, none,
401.88173904501329863196559445215306423 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962829566978661554359201674303762,
4.12565179671995756151128071452959400027 4,
```

```
440.671230654338278879652247679113177382 4]
two intervals r = 14.35659705128266457595884542124981475380 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000 or r = 17.70352613823089458425379022593441955553 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [.43e-37, .2e-37, .2853e-34]Solution in 1.316s

Time Plot 0 s.
Exiting SolveHard() after 4.267r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83, none,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962829566978661554359201674303762,
4.12565179671995756151128071452959400027 4,
440.671230654338278879652247679113177382 4]
one interval r = 22.39761154372325107444162720433391337235 ..
27.23722351590953457116580933342253967850
Time Approximations 0.036.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
```

```
in partial time = 1.237 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643773282960261939516322992396 31, rm =
14.378187701967053664162136712141855110 85}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., -.398e-34]Solution in 8.185s

Time Plot 0 s.
Exiting SolveHard() after 10.697r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887987864219891394505152411479 26,
4.004869081691934920226474748375623571214,
404.86224501549215497285579667482930653 46]
two intervals r = 16.080110077749176087838172983548566643 73 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000 or r = 16.415798127049475391255326362425973547 55 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000
Time Approximations 0.051.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., .474e-35]Solution in 3.209s

Time Plot 0 s.
Exiting SolveHard() after 6.003r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56, none, none,
358.97362824209377039821642136119680282 79, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887987864219891394505152411479 26,
4.0048690816919349202264747483756235712 14,
404.86224501549215497285579667482930653 46]
one interval r = 21.641943994150765291200216585368775924 79 ..
26.763306600343268603100330829443285332 29
Time Approximations 0.051.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, -.213e-34]Solution in 2.792s

Time Plot 0 s.
Exiting SolveHard() after 3.8r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93, none,
358.97362824209377039821642136119680282 79, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589418074570364474563583982411886 93,
5.5896371827856029813531976116127075913 00,
443.83065884709596608504297975936392244 14]
one interval r = 22.467253744777109726518926907427279691 00 ..
```

```
27.27388428344040103631866321164972818742
Time Approximations 0.039.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.15e-37
Equations at solution: [-.2e-37, .215e-36, -.8e-36]Solution in 2.678s

Time Plot 0 s.
Exiting SolveHard() after 3.67r=27.0204 in [24.71083344 .. 27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93, none,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941807457036447456358398241188693,
5.58963718278560298135319761161270759 1300,
443.83065884709596608504297975936392244 14]
one interval r = 32.15575279492508169550010004037068254330 ..
35.50872228737964752958509813708007174668
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [-.8e-37, .7e-37, -.131e-34]Solution in 0.481s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.846r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136518501274068175920153194530790,
5.18778357837422311702687423586588075547 3,
408.65773863107206504406720788527973861 68]
one interval r = 21.71840114657774066066833018858527404861 ..
26.81849303506743799541308106708705402181
Time Approximations 0.056.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.58e-37
Equations at solution: [-.1e-37, -.158e-36, -.54e-35]Solution in 1.002s

Time Plot 0 s.
Exiting SolveHard() after 3.854r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
```

```
401.881739045013298631965594452153064232 5,
389.590015162317186001860340353973726875 6,
328.469398935457221165627828695876759409 3,
401.507571582137919507112181016192750838 3,
358.973628242093770398216421361196802827 9,
398.331471040927884110011526896135783183 0, none, none,
361.508883473624468415974042830126573699 1, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.911931365185012740681759201531945307 90,
5.187783578374223117026874235865880755473,
408.657738631072065044067207885279738616 8]
one interval r = 31.808285987437953401874961736109732877 41 ..
35.000114600485668588558237779434491943578
Time Approximations 0.019.

hint used Hint := [34.495226611650080820729149248019775312 21, 3, 1, 1,
15.763855890238725343023875204542137417 16, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, -.212e-34]Solution in 2.394s

Time Plot 0 s.
Exiting SolveHard() after 2.713r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434940300112435696343749389225732 4,
441.642959734971531112278792586609946755 3,
436.917481656831458784251913974194966600 1,
422.984933976535131786245178281454623608 3,
361.525802563213374968382139022954812663 1,
401.881739045013298631965594452153064232 5,
389.590015162317186001860340353973726875 6,
328.469398935457221165627828695876759409 3,
401.507571582137919507112181016192750838 3,
358.973628242093770398216421361196802827 9,
398.331471040927884110011526896135783183 0,
371.483873947975254060662069282381970172 9, none,
361.508883473624468415974042830126573699 1, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.463471105341220692869958696211034889 48,
6.196262565329857803023001667942349123258,
```

```
385.4447437958889468505511784381566695427]
one interval r = 31.60836097526762076035474709109278788943 ..
34.66372795610047262775011167200311018322
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.340e-34]Solution in 0.567s

Time Plot 0 s.
Exiting SolveHard() after 0.861r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29, none,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110534122069286995869621103488948,
6.196262565329857803023001667942349123258,
385.4447437958889468505511784381566695427]
two intervals r = 16.87563408766124438570886266569292635610 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000 or r = 15.55640493816876569837456486520289864103 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.1075e-36, 0., .2250e-34]Solution in 1.146s

Time Plot 0 s.
Exiting SolveHard() after 4.229r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874746950934914107112210199918722,
4.8838107796931962182771845246134430833 75,
376.61967856080005513971003685715218007 21]
one interval r = 21.11001304883352649365372842111293921188 ..
26.31784243469575985895078854712316357953
Time Approximations 0.033.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.15e-35]Solution in 0.816s

Time Plot 0 s.
Exiting SolveHard() after 3.44r=25.872 in [23.20517308 .. 26.31784245]
```

```
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83, none,
328.46938513618451211887343929965261622 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874746950934914107112210199918722,
4.8838107796931962182771845246134430833 75,
376.61967856080005513971003685715218007 21]
one interval r = 31.53899497700689283621290956540235524119 ..
34.53618386093173180957811217277473603725
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.000e-35
Equations at solution: [.768e-35, -.1000e-34, -.230e-34]Solution in
0.478s

Time Plot 0 s.
Exiting SolveHard() after 0.779r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
```

```
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83, none,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017534023977100631564412669379665,
6.0258135492777664199822253674806288885 00,
351.42702948518170571047377249271265178 72]
one interval r = 31.362302061013520597857916124020950679 08 ..
34.174466406112842161242886264450340314 05
Time Approximations 0.014.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, -.204e-34]Solution in 0.52s

Time Plot 0 s.
Exiting SolveHard() after 2.631r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
```

```
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83, none,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23, none, none,
292.99969138365722059338519240676361345 25, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175340239771006315644126693796 65,
6.0258135492777664199822253674806288885 00,
351.42702948518170571047377249271265178 72]
two intervals r = 17.981355144568587835066670773010335467 91 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000 or r = 13.846080154292864327797135795120132290 01 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [.88e-37, -.3e-37, .1138e-34]Solution in 2.878s

Time Plot 0 s.
Exiting SolveHard() after 3.913r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23, none, none,
```

```
292.99969138365722059338519240676361345 25, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941861753274558931859892093418661,
6.37794387386241453729206324410297754431 3,
423.28832783976166505134366948441555069 78]
one interval r = 31.94661817585987453135940175165198120941 ..
35.21212308649135214367423760328940951965
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.23e-35]Solution in 0.613s

Time Plot 0 s.
Exiting SolveHard() after 0.962r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23, none, none,
292.99969138365722059338519240676361345 25, none, none,
360.06173466748733716073167076512154694 13, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941861753274558931859892093418661,
6.37794387386241453729206324410297754431 3,
423.28832783976166505134366948441555069 78]
```

```
two intervals r = 15.22886702463835849040813830454540284662 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000 or r = 17.12965777075957850662516728843003863121 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.46e-37, -.1e-37, .646e-35]Solution in 2.995s

Time Plot 0 s.
Exiting SolveHard() after 6.023r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922 57324,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23,
375.73285289966301591330578778070333295 81, none,
292.99969138365722059338519240676361345 25, none, none,
360.06173466748733716073167076512154694 13, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343420457537719377783471218245 42,
4.0035598153905293802200322240013988122 10,
404.47973594212314099601844355084832745 52]
two intervals r = 16.09683966387636847585973860440450276116 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000 or r = 16.39988649123101001503591775922062818451 ..
```

```
475000000002261970642620216914510978 7/250000000000000000000000000000000
000
Time Approximations 0.055.

hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.7615110884558244986626642365610012490 6, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.17e-37, .1e-37, .2745e-34]Solution in 3.279s

Time Plot 0 s.
Exiting SolveHard() after 4.359r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23,
375.73285289966301591330578778070333295 81, none,
292.99969138365722059338519240676361345 25,
358.64341561022201875954079545364451250 68, none,
360.06173466748733716073167076512154694 13, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234342045753771937778347121824542,
4.00355981539052938022003222400139881221 0,
404.47973594212314099601844355084832745 52]
one interval r = 21.63429629991502475530768400627922676338 ..
26.75768169883530859004802783850998876428
Time Approximations 0.049.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
```

```
16.7792152583743340383255391208949113668O, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=1.01e-37
Equations at solution: [.2e-37, .101e-36, .116e-34]Solution in 1.025s

Time Plot 0 s.
Exiting SolveHard() after 3.909r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23,
375.73285289966301591330578778070333295 81,
328.11709294429175694191124504563606891 00,
292.99969138365722059338519240676361345 25,
358.64341561022201875954079545364451250 68, none,
360.06173466748733716073167076512154694 13, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954452369231925703340381743885066,
6.19617723018353950153994775803395758229 4,
385.42734025967313460365249386675100163 03]
one interval r = 31.60822049082152756079626074631221060672 ..
34.66347615047755435385146733565423228268
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .196e-34]Solution in 2.395s

Time Plot 0 s.
Exiting SolveHard() after 2.686r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23,
375.73285289966301591330578778070333295 81,
328.11709294429175694191124504563606891 00,
292.99969138365722059338519240676361345 25,
358.64341561022201875954079545364451250 68, none,
360.06173466748733716073167076512154694 13, none, none,
324.65521223767502181307697668147445582 00, none, none, none, none]

0 --> 1  target = [26.46318954452369231925703340381743885066,
6.19617723018353950153994775803395758229 4,
385.42734025967313460365249386675100163 03]
two intervals r = 16.87629600304636579617291193388812560833 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000 or r = 15.55559000665587779131280529346556257167 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.359e-37, 0., -.1883e-34]Solution in 2.993s

Time Plot 0 s.
Exiting SolveHard() after 4.161r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23,
375.73285289966301591330578778070333295 81,
328.11709294429175694191124504563606891 00,
292.99969138365722059338519240676361345 25,
358.64341561022201875954079545364451250 68, none,
360.06173466748733716073167076512154694 13,
336.59441032362539014299050563252963394 08, none,
324.65521223767502181307697668147445582 00, none, none, none, none]

0 --> 2  target = [34.49522661166767344545650148120490278810,
3.89713131581596508799542004633934127 7055,
373.78081884852524100341488868969150926 20]
two intervals r = 17.29769086231373785644968398203290930922 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000 or r = 14.99436407453301410053836408035631901465 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000
Time Approximations 0.081.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
```

```
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., .365e-35]Solution in 1.226s

Time Plot 0 s.
Exiting SolveHard() after 4.763r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23,
375.73285289966301591330578778070333295 81,
328.11709294429175694191124504563606891 00,
292.99969138365722059338519240676361345 25,
358.64341561022201875954079545364451250 68, none,
360.06173466748733716073167076512154694 13,
336.59441032362539014299050563252963394 08, none,
324.65521223767502181307697668147445582 00,
331.93806791811162973214644751333273639 70, none, none, none]

1 --> 2  target = [34.49522661166767344545650148120490278810,
3.89713131581596508799542004633934127 7055,
373.78081884852524100341488868969150926 20]
one interval r = 21.06068473211275489094850766259438040968 ..
26.26979834280080270602193869800227312071
Time Approximations 0.033.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
```

```
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=9e-38
Equations at solution: [.4e-37, .9e-37, -.157e-34]Solution in 0.777s

Time Plot 0 s.
Exiting SolveHard() after 3.49r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23,
375.73285289966301591330578778070333295 81,
328.11709294429175694191124504563606891 00,
292.99969138365722059338519240676361345 25,
358.64341561022201875954079545364451250 68,
299.89866205041153898465223474325330038 65,
360.06173466748733716073167076512154694 13,
336.59441032362539014299050563252963394 08, none,
324.65521223767502181307697668147445582 00,
331.93806791811162973214644751333273639 70, none, none, none]

0 --> 2  target = [33.81362495405594426633852285446905532004,
3.72564899344027144045882059264602127551 9,
325.89209972997578670796858797027643474 52]
two intervals r = 18.552270490162101224455629928731864905 75 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000 or r = 12.491969358017649230066362385380166482 28 ..
475000000002261970642620216914510978 7/2500000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
```

```
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1.2e-37
Equations at solution: [.277e-36, -.12e-36, .2831e-34]Solution in
2.974s

Time Plot 0 s.
Exiting SolveHard() after 6.241r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23,
375.73285289966301591330578778070333295 81,
328.11709294429175694191124504563606891 00,
292.99969138365722059338519240676361345 25,
358.64341561022201875954079545364451250 68,
299.89866205041153898465223474325330038 65,
360.06173466748733716073167070651215469 413,
336.59441032362539014299050563252963394 08, none,
324.65521223767502181307697668147445582 00,
331.93806791811162973214644751333273639 70, none, none,
289.54595772679427250324772175829981018 16]

1 --> 2  target = [33.81362495405594426633852285446905532004,
3.72564899344027144045882059264602127 5519,
325.89209972997578670796858797027643474 52]
one interval r = 20.37468935111579313214888682621663427045 ..
25.37892165294633792083462711445611572702
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=1e-38
Equations at solution: [-.1e-37, -.1e-37, -.22e-35]Solution in 0.575s

Time Plot 0 s.
Exiting SolveHard() after 1.111r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23,
375.73285289966301591330578778070333295 81,
328.11709294429175694191124504563606891 00,
292.99969138365722059338519240676361345 25,
358.64341561022201875954079545364451250 68,
299.89866205041153898465223474325330038 65,
360.06173466748733716073167076512154694 13,
336.59441032362539014299050563252963394 08,
256.10753186103837074394571126122638497 67,
324.65521223767502181307697668147445582 00,
331.93806791811162973214644751333273639 70, none, none,
289.54595772679427250324772175829981018 16]

1 --> 0  target = [17.93041369724393454441390371131588386114,
4.68650870181797144500401027485433823934 4,
353.30541095099751124448560891942098188 87]
one interval r = 20.73150479094161846105424854405284023329 ..
25.90675353514230454535164358087734783746
Time Approximations 0.03.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
```

```
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=7.1e-38
Equations at solution: [-.3e-37, -.71e-37, -.27e-35]Solution in 0.665s

Time Plot 0 s.
Exiting SolveHard() after 3.171r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23,
375.73285289966301591330578778070333295 81,
328.11709294429175694191124504563606891 00,
292.99969138365722059338519240676361345 25,
358.64341561022201875954079545364451250 68,
299.89866205041153898465223474325330038 65,
360.06173466748733716073167076512154694 13,
336.59441032362539014299050563252963394 08,
256.10753186103837074394571126122638497 67,
324.65521223767502181307697668147445582 00,
331.93806791811162973214644751333273639 70,
304.79958325436778668103323173612462357 71, none,
289.54595772679427250324772175829981018 16]

2 --> 0  target = [17.93041369724393454441390371131588386114,
4.68650870181797144500401027485433823934 4,
353.30541095099751124448560891942098188 87]
one interval r = 31.37435486980358498957482694600557486713 ..
34.20127520023397074373351518574683010275
```

```
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, .32e-35]Solution in 0.335s

Time Plot 0 s.
Exiting SolveHard() after 0.609r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494030011243569634374938922573 24,
441.64295973497153111227879258660994675 53,
436.91748165683145878425191397419496660 01,
422.98493397653513178624517828145462360 83,
361.52580256321337496838213902295481266 31,
401.88173904501329863196559445215306423 25,
389.59001516231718600186034035397372687 56,
328.46939893545722116562782869587675940 93,
401.50757158213791950711218101619275083 83,
358.97362824209377039821642136119680282 79,
398.33147104092788411001152689613578318 30,
371.48387394797525406066206928238197017 29,
336.61215841315875375075988433002832528 96,
361.50888347362446841597404283012657369 91,
324.67144992785651111603675363374310671 83,
302.31384314814782364534395537841121701 47,
328.46938513618451211887343929965261622 05,
343.81340625191518220394296821316832095 23,
375.73285289966301591330578778070333295 81,
328.11709294429175694191124504563606891 00,
292.99969138365722059338519240676361345 25,
358.64341561022201875954079545364451250 68,
299.89866205041153898465223474325330038 65,
360.06173466748733716073167076512154694 13,
336.59441032362539014299050563252963394 08,
256.10753186103837074394571126122638497 67,
324.65521223767502181307697668147445582 00,
331.93806791811162973214644751333273639 70,
304.79958325436778668103323173612462357 71,
323.46169176764514449186910170953808915 68,
289.54595772679427250324772175829981018 16]

Cascade time 141.784
```

```
counts: 28, 28

 Iteration 31

Start Generation 1
1 --> 0  target = [11.999999999992129147494599164758865834 00,
6.217012503061928102084581764663421933041,
485.549080898260598511014305631328735 2674]
one interval r = 23.408503016544663144572991584833118404 70 ..
27.675780464289821607735983300240408956 98
Time Approximations 0.038.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.04e-37
Equations at solution: [-.3e-37, .104e-36, .13e-35]Solution in 0.943s

Time Plot 0 s.
Exiting SolveHard() after 3.839r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999992129147494599164758865834 00,
6.217012503061928102084581764663421933041,
485.549080898260598511014305631328735 2674]
one interval r = 32.628147792060005953570776217212740673 14 ..
36.102483889436553236073328286283141533 16
Time Approximations 0.024.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
```

```
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., .67e-35]Solution in 2.692s

Time Plot 0 s.
Exiting SolveHard() after 3.113r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684481075489181614938394212989117,
6.58343472165188792423220576509884297317 0,
467.78730595946272988551768429461702208 91]
one interval r = 32.41978955655530115470242105429010146630 ..
35.85152417373598298885306718501535549527
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=9e-38
Equations at solution: [-.10e-36, .9e-37, .222e-34]Solution in 0.647s

Time Plot 0 s.
Exiting SolveHard() after 1.051r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20, none, none,
401.88173904396561726517977538190216363 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684481075489181614938394212989117,
6.58343472165188792423220576509884297317 0,
```

```
467.787305959462729885517684294617022089 1]
two intervals r = 12.92327160827651989414634870409896494869 ..
190000000000166053518483056118185116 47/100000000000000000000000000000000000
00000 or r = 18.39424858033350684240542179860771690703 ..
190000000000166053518483056118185116 47/100000000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.2e-38
Equations at solution: [.1e-37, .12e-37, -.1310e-34]Solution in 37.598s

Time Plot 0 s.
Exiting SolveHard() after 40.809r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72, none,
401.88173904396561726517977538190216363 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962825342163890700887166621114547,
4.12565179693385767438303303067394131751 7,
440.67123065018492302852503781353183351 93]
two intervals r = 14.35659705120796209190989839565935446407 ..
190000000000166053518483056118185116 47/100000000000000000000000000000000000
00000 or r = 17.70352613808129960537429517231659690725 ..
190000000000166053518483056118185116 47/100000000000000000000000000000000000
00000
Time Approximations 0.043.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
```

```
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.15e-37, .1e-37, .2733e-34]Solution in 1.31s

Time Plot 0 s.
Exiting SolveHard() after 3.965r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72, none,
401.88173904396561726517977538190216363 3,
389.59001515768336705548030786356630684 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962825342163890700887166621114547,
4.12565179693385767438303303067394131751 7,
440.67123065018492302852503781353183351 93]
one interval r = 22.39761154364811638815439687273839655268 ..
27.23722351598211996135910198497932638261
Time Approximations 0.036.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.239 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064387329404565236598732326478878, rm =
14.37818770514373606673498225801253088738}});
Accepted {r=26.4635, rm=16.5329} with Delta=7.9e-38
Equations at solution: [0., -.79e-37, -.375e-34]Solution in 8.336s

Time Plot 0 s.
Exiting SolveHard() after 11.198r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888799505726348796816439169609651,
4.0048690819174536292174280087646052927 71,
404.86224501483002009406352144094702939 75]
two intervals r = 16.08011007754061160825179263001134245971 ..
19000000000016605351848305611818511647/10000000000000000000000000000000
00000 or r = 16.41579812700066959773683851206164598201 ..
19000000000016605351848305611818511647/10000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., -.490e-35]Solution in 3.464s

Time Plot 0 s.
Exiting SolveHard() after 6.204r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19, none, none,
358.97362824045032695468839470093897160 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888799505726348796816439169609651,
4.0048690819174536292174280087646052927 71,
404.86224501483002009406352144094702939 75]
one interval r = 21.64194399413696332041207108562992260856 ..
```

```
26.763306600443002541146240243476717492 18
Time Approximations 0.054.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=1.24e-37
Equations at solution: [-.3e-37, -.124e-36, .312e-34]Solution in 2.816s

Time Plot 0 s.
Exiting SolveHard() after 3.823r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70, none,
358.97362824045032695468839470093897160 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941772084013413181168282113800302,
5.58963718309777714610287968644897354494 2,
443.83065884825409730869269489532829858 85]
one interval r = 22.46725374482039525520217125815781059811 ..
27.27388428357601364597472060565763786602
Time Approximations 0.038.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.34e-37
Equations at solution: [.1e-37, -.134e-36, .9e-36]Solution in 2.578s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.566r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 3,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70, none,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941772084013413181168282113800302,
5.58963718309777714610287968644897354 4942,
443.83065884825409730869269489532829858 85]
one interval r = 32.15575279495160157566702990865987998237 ..
35.50872228741397970959608082747656346490
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .208e-34]Solution in 0.479s

Time Plot 0 s.
Exiting SolveHard() after 0.839r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 3,
```

```
389.590015157683367055480307863566306841 9,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136514436845268518718406464590989,
5.18778357861277398514341546131405934955 5,
408.65773862654998135803920556124872935 57]
one interval r = 21.71840114648720335708617423634307857384 ..
26.81849303511314663174491294358636246174
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.85e-37
Equations at solution: [.1e-37, .185e-36, -.257e-34]Solution in 1.012s

Time Plot 0 s.
Exiting SolveHard() after 4.053r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.590015157683367055480307863566306841 9,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15, none, none,
361.50888347200804739902516797006166220 02, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136514436845268518718406464590989,
5.18778357861277398514341546131405934955 5,
408.65773862654998135803920556124872935 57]
one interval r = 31.80828598741084219338435132613871970587 ..
35.00011460043697363193138600359728406776
```

```
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., -.281e-34]Solution in 2.145s

Time Plot 0 s.
Exiting SolveHard() after 2.462r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77, none,
361.50888347200804739902516797006166220 02, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110541990667028614884820971549478,
6.196262565385115469567306149695820960030,
385.44474379441644476326853924105729093 99]
one interval r = 31.60836097526997171518225617462650411997 ..
34.66372795609557164128395392277613569520
Time Approximations 0.016.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
```

```
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.2e-36]Solution in 0.541s

Time Plot 0 s.
Exiting SolveHard() after 0.83r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766656580 722,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77, none,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110541990667028614884820971549478,
6.19626256538511546956730614969582096003 0,
385.44474379441644476326853924105729093 99]
two intervals r = 16.87563408750922899525832887348563984016 ..
19000000000016605351848305611818511647/10000000000000000000000000000000
00000 or r = 15.55640493807495058648585770178692384291 ..
19000000000016605351848305611818511647/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.537e-37, 0., .38e-36]Solution in 1.15s

Time Plot 0 s.
Exiting SolveHard() after 4.014r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874732400796264852521033523036224,
4.8838107799591318183074804496289109059 72,
376.61967855939877021436291092492555209 66]
one interval r = 21.11001304879630289715309907877046347144 ..
26.31784243477141682513404200214710051279
Time Approximations 0.033.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., .4e-36]Solution in 0.824s

Time Plot 0 s.
Exiting SolveHard() after 3.254r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
```

```
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25, none,
328.46938513789969514886139406893054381 82, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874732400796264852521033523036224,
4.8838107799591318183074804449628910905972,
376.61967855939877021436291092492555209 66]
one interval r = 31.538994977010437190458771873851934585 08 ..
34.536183860927742324754319954537559924 21
Time Approximations 0.017.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=9.57e-36
Equations at solution: [-.735e-35, .957e-35, -.64e-35]Solution in
0.479s

Time Plot 0 s.
Exiting SolveHard() after 0.786r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25, none,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017546435646033396085725836029879,
```

```
6.025813549350959700977958085444555739603,
351.42702948705088816529691878457933559 24]
one interval r = 31.362302061039771752278690364728218778 99 ..
34.174466406155298762522833191425859930 86
Time Approximations 0.015.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .365e-34]Solution in 0.553s

Time Plot 0 s.
Exiting SolveHard() after 2.689r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25, none,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06, none, none,
292.99969138700454737025029745529024573 03, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175464356460333960857258360298 79,
6.025813549350959700977958085444555739603,
351.42702948705088816529691878457933559 24]
two intervals r = 17.981355144359417871828161264172294951 47 ..
19000000000016605351848305611818511647/10000000000000000000000000000000
00000 or r = 13.846080154350210493145029095872526057 11 ..
19000000000016605351848305611818511647/10000000000000000000000000000000
00000
Time Approximations 0.045.
```

```
hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=4e-38
Equations at solution: [-.125e-36, .4e-37, .4541e-34]Solution in 2.973s

Time Plot 0 s.
Exiting SolveHard() after 4.035r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25,
302.31384315338661673730620275932917871 44,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06, none, none,
292.99969138700454737025029745529024573 03, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941878820367621740397949117152743,
6.37794387394312497762772606273213785818 1,
423.28832784392359395800824563258538816 16]
one interval r = 31.94661817591473939985115516189431695649 ..
35.21212308656879397991997388011831208759
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., .12e-35]Solution in 0.615s

Time Plot 0 s.
Exiting SolveHard() after 0.966r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25,
302.31384315338661673730620275932917871 44,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06, none, none,
292.99969138700454737025029745529024573 03, none, none,
360.06173467286554148049431431035721220 88, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418788203676217403979491171527 43,
6.3779438739431249776277260627321378581 81,
423.28832784392359395800824563258538816 16]
two intervals r = 15.228867024171463731535950627148908692 05 ..
190000000000166053518483056118185116 47/100000000000000000000000000000000
00000 or r = 17.129657770887358900096500348994134468 02 ..
190000000000166053518483056118185116 47/100000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
```

```
Equations at solution: [.16e-37, 0., .196e-35]Solution in 3.144s

Time Plot 0 s.
Exiting SolveHard() after 6.15r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25,
302.31384315338661673730620275932917871 44,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06,
375.73285290778858142791378449605916644 89, none,
292.99969138700454737025029745529024573 03, none, none,
360.06173467286554148049431431035721220 88, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343438488013269554729440261810 73,
4.0035598156186086592632775101487202249 13,
404.47973594220894231388565146371188268 02]
two intervals r = 16.096839663635645370373469686491461631 18 ..
19000000000016605351848305611818511647/10000000000000000000000000000000
00000 or r = 16.399886491213252440480767963425075608 92 ..
19000000000016605351848305611818511647/10000000000000000000000000000000
00000
Time Approximations 1.714.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.802e-35]Solution in 1.498s

Time Plot 0 s.
```

```
Exiting SolveHard() after 4.25r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25,
302.31384315338661673730620275932917871 44,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06,
375.73285290778858142791378449605916644 89, none,
292.99969138700454737025029745529024573 03,
358.64341560922469133350360192274437664 50, none,
360.06173467286554148049431431035721220 88, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234343848801326955472944026181073,
4.00355981561860865926327751014872022491 3,
404.47973594220894231388565146371188268 02]
one interval r = 21.63429629991601262681635470225050105560 ..
26.75768169894589893192482868527312363290
Time Approximations 0.05.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.137e-34]Solution in 1.033s

Time Plot 0 s.
Exiting SolveHard() after 4.061r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 3,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25,
302.31384315338661673730620275932917871 44,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06,
375.73285290778858142791378449605916644 89,
328.11709294669807535748472507925924745 01,
292.99969138700454737025029745529024573 03,
358.64341560922469133350360192274437664 50, none,
360.06173467286554148049431431035721220 88, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954460086009698072142787826163890,
6.19617723023834001313918779706577543740 0,
385.42734025810730755726592773399549507 30]
one interval r = 31.60822049082312647561700711560914645649 ..
34.66347615047130282977125783016867776163
Time Approximations 0.016.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .26e-35]Solution in 0.569s

Time Plot 0 s.
Exiting SolveHard() after 2.709r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25,
302.31384315338661673730620275932917871 44,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06,
375.73285290778858142791378449605916644 89,
328.11709294669807535748472507925924745 01,
292.99969138700454737025029745529024573 03,
358.64341560922469133350360192274437664 50, none,
360.06173467286554148049431431035721220 88, none, none,
324.65521223775949035696640350482956412 81, none, none, none, none]

0 --> 1  target = [26.46318954460086009698072142787826163890,
6.19617723023834001313918779706577543740 0,
385.42734025810730755726592773399549507 30]
two intervals r = 16.87629600289791779033844256524183674415 ..
19000000000016605351848305611818511647/10000000000000000000000000000000
00000 or r = 15.55559000655768113676310900995636368870 ..
19000000000016605351848305611818511647/10000000000000000000000000000000
00000
Time Approximations 0.062.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.359e-37, 0., -.1085e-34]Solution in 3.005s

Time Plot 0 s.
Exiting SolveHard() after 4.184r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
```

```
441.642959732360534129359011912754404992 3,
436.917481652367253547374353149278749122 0,
422.984933977267012521695805586537730027 2,
361.525802561687682651884910604952628146 4,
401.881739043965617265179775381902163633 3,
389.590015157683367055480307863566306841 9,
328.469398937174550243751732473120989727 0,
401.507571581821861590401464867666558072 2,
358.973628240450326954688394700938971600 8,
398.331471044887996055233037445426756111 5,
371.483873942762065848799803805459290897 7,
336.612158415243602166319200893366368296 4,
361.508883472008047399025167970061662200 2,
324.671449928028017586248796503402432752 5,
302.313843153386616737306202759329178714 4,
328.469385137899695148861394068930543818 2,
343.813406249362803324592376005849169790 6,
375.732852907788581427913784496059166448 9,
328.117092946698075357484725079259247450 1,
292.999691387004547370250297455290245730 3,
358.643415609224691333503601922744376645 0, none,
360.061734672865541480494314310357212208 8,
336.594410325614981644218625119848575651 5, none,
324.655212237759490356966403504829564128 1, none, none, none, none]

0 --> 2  target = [34.495226611612879388500892763501117814 62,
3.897131316026535093150821170386768409977,
373.780818843603129976612399127393036911 5]
two intervals r = 17.297690862291904838240765207029504353 66 ..
19000000000016605351848305611818511647/100000000000000000000000000000000
00000 or r = 14.994364074260519983652730215210936675 96 ..
19000000000016605351848305611818511647/100000000000000000000000000000000
00000
Time Approximations 0.082.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.36e-37, 0., -.1965e-34]Solution in 2.864s

Time Plot 0 s.
Exiting SolveHard() after 6.32r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
```

```
441.642959732360534129359011912754404992 3,
436.917481652367253547374353149278749122 0,
422.984933977267012521695805586537730027 2,
361.525802561687682651884910604952628146 4,
401.881739043965617265179775381902163633 3,
389.590015157683367055480307863566306841 9,
328.469398937174550243751732473120989727 0,
401.507571581821861590401464867666558072 2,
358.973628240450326954688394700938971600 8,
398.331471044887996055233037445426756111 5,
371.483873942762065848799803805459290897 7,
336.612158415243602166319200893366368296 4,
361.508883472008047399025167970061662200 2,
324.671449928028017586248796503402432752 5,
302.313843153386616737306202759329178714 4,
328.469385137899695148861394068930543818 2,
343.813406249362803324592376005849169790 6,
375.732852907788581427913784496059166448 9,
328.117092946698075357484725079259247450 1,
292.999691387004547370250297455290245730 3,
358.643415609224691333503601922744376645 0, none,
360.061734672865541480494314310357212208 8,
336.594410325614981644218625119848575651 5, none,
324.655212237759490356966403504829564128 1,
331.938067912766649398481378920717560919 2, none, none, none]

1 --> 2  target = [34.495226611612879388500892763501117814 62,
3.897131316026535093150821170386768409977,
373.780818843603129976612399127393036911 5]
one interval r = 21.060684732014151177076564862629878631 71 ..
26.269798342815163805278488367313382624 10
Time Approximations 0.034.

hint used Hint := [25.300466340350721366923008834208363590 84, 3, -1, 1,
16.974729527641787545085746138369074748 18, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, -.557e-34]Solution in 0.765s

Time Plot 0 s.
Exiting SolveHard() after 1.479r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936276336893915216898509022978 6,
441.642959732360534129359011912754404992 3,
```

```
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25,
302.31384315338661673730620275932917871 44,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06,
375.73285290778858142791378449605916644 89,
328.11709294669807535748472507925924745 01,
292.99969138700454737025029745529024573 03,
358.64341560922469133350360192274437664 50,
299.89866204823087696721882938338731012 79,
360.06173467286554148049431431035721220 88,
336.59441032561498164421862511984857565 15, none,
324.65521223775949035696640350482956412 81,
331.93806791276664939848137892071756091 92, none, none, none]

0 --> 2  target = [33.81362495407952958196250860871837021567,
3.72564899367036031667506143487607059826 9,
325.89209973056128993860402920989405408 57]
two intervals r = 18.552270490025366329392509628664593057 16 ..
19000000000016605351848305611818511647/10000000000000000000000000000000000
00000 or r = 12.49196935798567963249066689777236324077 ..
19000000000016605351848305611818511647/10000000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=4e-38
Equations at solution: [.104e-36, -.4e-37, .220e-35]Solution in 3.085s

Time Plot 0 s.
Exiting SolveHard() after 6.151r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25,
302.31384315338661673730620275932917871 44,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06,
375.73285290778858142791378449605916644 89,
328.11709294669807535748472507925924745 01,
292.99969138700454737025029745529024573 03,
358.64341560922469133350360192274437664 50,
299.89866204823087696721882938338731012 79,
360.06173467286554148049431431035721220 88,
336.59441032561498164421862511984857565 15, none,
324.65521223775949035696640350482956412 81,
331.93806791276664939848137892071756091 92, none, none,
289.54595772635430898247078430161814433 97]

1 --> 2  target = [33.81362495407952958196250860871837021567,
3.72564899367036031667506143487607059826 9,
325.89209973056128993860402920989405408 57]
one interval r = 20.37468935109239690573815731563934396153 ..
25.37892165303706225173686079445902379131
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, .147e-34]Solution in 0.567s

Time Plot 0 s.
Exiting SolveHard() after 1.095r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25,
302.31384315338661673730620275932917871 44,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06,
375.73285290778858142791378449605916644 89,
328.11709294669807535748472507925924745 01,
292.99969138700454737025029745529024573 03,
358.64341560922469133350360192274437664 50,
299.89866204823087696721882938338731012 79,
360.06173467286554148049431431035721220 88,
336.59441032561498164421862511984857565 15,
256.10753186391874086321427528507657487 43,
324.65521223775949035696640350482956412 81,
331.93806791276664939848137892071756091 92, none, none,
289.54595772635430898247078430161814433 97]

1 --> 0  target = [17.93041369701442749931890109501379035790,
4.68650870211328323606954823864306779 5462,
353.30541095347058153917175410773802281 26]
one interval r = 20.73150479095689283821191169126130769469 ..
25.90675353527806399715874194177060197744
Time Approximations 0.03.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.5e-38
Equations at solution: [.2e-37, .45e-37, -.507e-34]Solution in 0.635s

Time Plot 0 s.
Exiting SolveHard() after 3.306r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25,
302.31384315338661673730620275932917871 44,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06,
375.73285290778858142791378449605916644 89,
328.11709294669807535748472507925924745 01,
292.99969138700454737025029745529024573 03,
358.64341560922469133350360192274437664 50,
299.89866204823087696721882938338731012 79,
360.06173467286554148049431431035721220 88,
336.59441032561498164421862511984857565 15,
256.10753186391874086321427528507657487 43,
324.65521223775949035696640350482956412 81,
331.93806791276664939848137892071756091 92,
304.79958325999029853885036727986955566 67, none,
289.54595772635430898247078430161814433 97]

2 --> 0  target = [17.93041369701442749931890109501379035790,
4.68650870211328323606954823864306779546 2,
353.30541095347058153917175410773802281 26]
one interval r = 31.37435486983391573538003266231215885649 ..
34.20127520028511299048498178883712127736
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [-.5e-37, .8e-37, .115e-34]Solution in 0.319s

Time Plot 0 s.
Exiting SolveHard() after 0.582r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
```

```
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493627633689391521689850902297 86,
441.64295973236053412935901191275440499 23,
436.91748165236725354737435314927874912 20,
422.98493397726701252169580558653773002 72,
361.52580256168768265188491060495262814 64,
401.88173904396561726517977538190216363 33,
389.59001515768336705548030786356630684 19,
328.46939893717455024375173247312098972 70,
401.50757158182186159040146486766655807 22,
358.97362824045032695468839470093897160 08,
398.33147104488799605523303744542675611 15,
371.48387394276206584879980380545929089 77,
336.61215841524360216631920089336636829 64,
361.50888347200804739902516797006166220 02,
324.67144992802801758624879650340243275 25,
302.31384315338661673730620275932917871 44,
328.46938513789969514886139406893054381 82,
343.81340624936280332459237600584916979 06,
375.73285290778858142791378449605916644 89,
328.11709294669807535748472507925924745 01,
292.99969138700454737025029745529024573 03,
358.64341560922469133350360192274437664 50,
299.89866204823087696721882938338731012 79,
360.06173467286554148049431431035721220 88,
336.59441032561498164421862511984857565 15,
256.10753186391874086321427528507657487 43,
324.65521223775949035696640350482956412 81,
331.93806791276664939848137892071756091 92,
304.79958325999029853885036727986955566 67,
323.46169176848607530622057903310729770 54,
289.54595772635430898247078430161814433 97]

Cascade time 141.844
counts: 28, 28

 Iteration 32

Start Generation 1
1 --> 0  target = [12.00000000013237486813215101840604353100,
6.21701250279176267667863663086633999313 7,
485.54908089776723919999667618494360757 53]
one interval r = 23.40850301662332752911613345891890098846 ..
27.67578046444480727018889467764431495007
Time Approximations 0.042.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
```

```
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=8.0e-38
Equations at solution: [-.3e-37, .80e-37, 0.]Solution in 1.007s

Time Plot 0 s.
Exiting SolveHard() after 4.088r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000013237486813215101840604353100,
6.21701250279176267667863663086633999313 7,
485.54908089776723919999667618494360757 53]
one interval r = 32.62814779213858316946298346614454465136 ..
36.10248388947580745359576999063532410854
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., .101e-34]Solution in 2.93s

Time Plot 0 s.
Exiting SolveHard() after 3.363r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
```

```
2 --> 1  target = [27.52359684494119048116151363383641789947,
6.58343472163916914183184163078684429134 7,
467.78730595714762977240440297092070649 06]
one interval r = 32.41978955661604802040928707923285995773 ..
35.85152417375150057679770922398369017795
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .290e-34]Solution in 0.647s

Time Plot 0 s.
Exiting SolveHard() after 1.035r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53, none, none,
401.88173904043869923067323644956153138 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684494119048116151363383641789947,
6.58343472163916914183184163078684429134 7,
467.78730595714762977240440297092070649 06]
two intervals r = 12.92327160850127945376003744793403683990 ..
18999999999991069628390395885196635499/10000000000000000000000000000000
00000 or r = 18.39424858025313183421370974683492121917 ..
18999999999991069628390395885196635499/10000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.2e-38
```

```
Equations at solution: [.1e-37, .12e-37, .378e-35]Solution in 38.067s

Time Plot 0 s.
Exiting SolveHard() after 41.187r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31, none,
401.88173904043869923067323644956153138 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962831623553430736320589338558806,
4.12565179680043654820703543196698130854 6,
440.67123065092971096581404898934285472 31]
two intervals r = 14.35659705124634633574958487070110271060 ..
18999999999991069628390395885196635499/10000000000000000000000000000000
00000 or r = 17.70352613807399592675992088348927023265 ..
18999999999991069628390395885196635499/10000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.15e-37, .1e-37, .386e-35]Solution in 1.32s

Time Plot 0 s.
Exiting SolveHard() after 4.296r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31, none,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.463229628316235534307363205893385588 06,
4.1256517968004365482070354319669813085 46,
440.67123065092971096581404898934285472 31]
one interval r = 22.397611543748559505270366934375209210 40 ..
27.237223516133210960676863856260692176 53
Time Approximations 0.036.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 2.924 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643972357379863825344899595076 87, rm =
14.378187703584198595596947866510475401 57}});
Accepted {r=26.4635, rm=16.5329} with Delta=8.0e-38
Equations at solution: [.1e-37, .80e-37, -.73e-35]Solution in 10.702s

Time Plot 0 s.
Exiting SolveHard() after 13.273r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887994253440847926447678172330 32,
4.0048690817680825635860038463450834344 08,
404.86224501082430180517388238586022364 06]
two intervals r = 16.080110077760594946617325226346409840 46 ..
18999999999991069628390395885196635499/1000000000000000000000000000000000
00000 or r = 16.415798126814571679126982750877397335 74 ..
18999999999991069628390395885196635499/1000000000000000000000000000000000
00000
```

```
Time Approximations 0.053.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.33e-37, -.1e-37, -.2156e-34]Solution in
3.551s

Time Plot 0 s.
Exiting SolveHard() after 4.637r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80, none, none,
358.97362823881923935292889442802048767 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888799425344084792644767817233032,
4.00486908176808256358600384634508343440 8,
404.86224501082430180517388238586022364 06]
one interval r = 21.64194399413764882808805542488739773179 ..
26.76330660051450692899163632406659326943
Time Approximations 0.05.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., -.374e-34]Solution in 1.055s

Time Plot 0 s.
Exiting SolveHard() after 3.951r=25.8721 in [23.84730094 ..
```

```
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13, none,
358.97362823881923935292889442802048767 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941818061369627637261692746855509,
5.589637182750926448223517964174556220506,
443.83065884096498236827830677837430777 82]
one interval r = 22.46725374474372665457827741344000996202 ..
27.27388428363587980274794847634165546978
Time Approximations 0.037.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .10e-35]Solution in 0.973s

Time Plot 0 s.
Exiting SolveHard() after 3.707r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13, none,
358.97362823881923935292889442802048767 45,
```

```
398.331471034040423842213718424079767056 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941818061369627637261692746855509,
5.589637182750926448223517964174556220506,
443.83065884096498236827830677837430777 82]
one interval r = 32.15575279496521327443745638253103841077 ..
35.50872228736073128197119081973729068859
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .82e-35]Solution in 2.234s

Time Plot 0 s.
Exiting SolveHard() after 2.615r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.331471034040423842213718424079767056 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136509200093432074150277651637694,
5.187783578384518698008653988731531866464,
408.65773862878458424142732075937504298 66]
one interval r = 21.71840114661363508759502413433385716381 ..
26.81849303527687050042000915579176963979
Time Approximations 0.06.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, -.192e-34]Solution in 3.116s

Time Plot 0 s.
Exiting SolveHard() after 4.208r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69, none, none,
361.50888347067686713359977752718185902 35, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136509200093432074150277651637694,
5.18778357838451869800865398873153186 6464,
408.65773862878458424142732075937504298 66]
one interval r = 31.80828598752673257905262086989813045197 ..
35.00011460052608693256759278444732922227
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, .79e-35]Solution in 0.429s

Time Plot 0 s.
Exiting SolveHard() after 0.737r=34.4952 in [32.91337941 ..
35.00011460]
```

```
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43, none,
361.50888347067686713359977752718185902 35, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110551912131311187377722501127842,
6.19626256537826329106610253879745501408 7,
385.44474379286557364661864158853492613 80]
one interval r = 31.60836097535566089150964310183650322007 ..
34.66372795613338114719345392299195602168
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., .200e-34]Solution in 2.235s

Time Plot 0 s.
Exiting SolveHard() after 2.513r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
```

```
389.590015160042048752086858458744791948 0,
328.469398931376800912501887950239469851 3,
401.507571577607243929888137423928271219 5,
358.973628238819239352928894428020487674 5,
398.331471034040423842213718424079767056 9,
371.483873946934692346872462371923261784 3, none,
361.508883470676867133599777527181859023 5,
324.671449925098910292529402077816254467 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.463471105519121313111873777225011278 42,
6.196262565378263291066102538797455014087,
385.444743792865573646618641588534926138 0]
two intervals r = 16.875634087597283834104771233869442869 35 ..
189999999999910696283903958819663549 9/100000000000000000000000000000000
00000 or r = 15.556404937991767964023771110317555365 51 ..
189999999999910696283903958819663549 9/100000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.574 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175122058402150967231096955697562, rm
= 2.33653277415737096099001973640519113290 6}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.1074e-36, 0., .1728e-34]Solution in 28.59s

Time Plot 0 s.
Exiting SolveHard() after 29.711r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936077740479109799266179059685 9,
441.642959730232868516855573149872806796 2,
436.917481653488657445699607516425107175 3,
422.984933970614242849428941516182616683 1,
361.525802560295287104443058099169621895 6,
401.881739040438699230673236449561531385 3,
389.590015160042048752086858458744791948 0,
328.469398931376800912501887950239469851 3,
401.507571577607243929888137423928271219 5,
358.973628238819239352928894428020487674 5,
398.331471034040423842213718424079767056 9,
```

```
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874741489496755660305290443953489,
4.8838107796914107896659248588416878606 05,
376.61967855746013306968148060663170693 49]
one interval r = 21.11001304884140408376139555443967661498 ..
26.31784243486047373732567660060061196 2022
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.6e-38
Equations at solution: [.3e-37, .76e-37, -.139e-34]Solution in 2.571s

Time Plot 0 s.
Exiting SolveHard() after 3.283r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407975705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78, none,
328.46938513210308806649052557907598669 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874741489496755660305290443953489,
4.8838107796914107896659248588416878606 05,
376.61967855746013306968148060663170693 49]
one interval r = 31.53899497709477544470525613142314079722 ..
```

```
34.53618386096138623488000390570543494332
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.38e-36
Equations at solution: [.260e-35, -.338e-35, .202e-34]Solution in
0.494s

Time Plot 0 s.
Exiting SolveHard() after 0.779r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78, none,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017546613946422449659536167090008,
6.0258135493216568798489641519175675958 17,
351.42702948096728203001354688458255590 95]
one interval r = 31.362302061102611241466811036061587276 94 ..
34.174466406134164570373457193273910744 12
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
```

```
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.1e-37, -.3e-37, .167e-34]Solution in 0.516s

Time Plot 0 s.
Exiting SolveHard() after 0.756r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78, none,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31, none, none,
292.99969137981047066856800939420568307 73, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017546613946422449659536167090008,
6.0258135493216568798489641519175675958 17,
351.42702948096728203001354688458255590 95]
two intervals r = 17.98135514452858974982015637975472672034 ..
18999999999991069628390395885196635499/10000000000000000000000000000000
00000 or r = 13.84608015404147034317242465437036690798 ..
18999999999991069628390395885196635499/10000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.52 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.9135707137053976821782643980957906206 2, rm
= 2.7345009933347906655357363368047191498 53}});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.71e-37, -.2e-37, .740e-35]Solution in 17.646s

Time Plot 0 s.
Exiting SolveHard() after 20.86r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31, none, none,
292.99969137981047066856800939420568307 73, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941877577345296115011684264645 80,
6.37794387388950528578221168233081999331 5,
423.28832783265094324965962381457620186 90]
one interval r = 31.946618175898240465976923459083856976 16 ..
35.21212308646016441804887155494870572706
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.81e-35]Solution in 0.609s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.947r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31, none, none,
292.99969137981047066856800939420568307 73, none, none,
360.06173466088080800215474726460391238 65, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418775777345296115011684264645 80,
6.3779438738895052857822116823308199933 15,
423.28832783265094324965962381457620186 90]
two intervals r = 15.22886702477826177256728809214819055077 ..
189999999999910696283903958519663549 9/10000000000000000000000000000000
00000 or r = 17.12965777045726702412767180849931687335 ..
189999999999910696283903958519663549 9/10000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 8.106 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541529658614957557001984558 28397, rm
= 2.0640682987963503480294954254188678785 33}});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
```

```
Equations at solution: [-.46e-37, -.2e-37, -.1707e-34]Solution in
28.755s

Time Plot 0 s.
Exiting SolveHard() after 31.708r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31,
375.73285289139794853948044873877963272 07, none,
292.99969137981047066856800939420568307 73, none, none,
360.06173466088080800215474726460391238 65, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343427547594384864849930235396 32,
4.0035598154668293126165263780049973725 26,
404.47973593750018559310072053056786180 85]
two intervals r = 16.096839663885592658153049531655437001 39 ..
189999999999910696283903958851966354 99/10000000000000000000000000000000
00000 or r = 16.399886490997573799859397755151611171 51 ..
189999999999910696283903958851966354 99/10000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.35e-37, 0., .853e-35]Solution in 3.488s
```

```
Time Plot 0 s.
Exiting SolveHard() after 6.274r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31,
375.73285289139794853948044873877963272 07, none,
292.99969137981047066856800939420568307 73,
358.64341560698677296916728373986038482 96, none,
360.06173466088080800215474726460391238 65, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234342754759438486484993023539632,
4.0035598154668293126165263780049973725 26,
404.47973593750018559310072053056786180 85]
one interval r = 21.634296299902736185105433988342210090 10 ..
26.757681699006811261577702440605456906 65
Time Approximations 0.053.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.5e-38
Equations at solution: [.1e-37, .25e-37, -.12e-35]Solution in 2.949s

Time Plot 0 s.
Exiting SolveHard() after 3.924r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31,
375.73285289139794853948044873877963272 07,
328.11709294025273037343006969714687048 63,
292.99969137981047066856800939420568307 73,
358.64341560698677296916728373986038482 96, none,
360.06173466088080800215474726460391238 65, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954470108625718942824579497944744,
6.19617723023179702691139284315694585960 5,
385.42734025661940609153818295492429310 40]
one interval r = 31.60822049090932714383651342938326736877 ..
34.66347615051002620630211119979179464673
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.1e-37, -.3e-37, -.71e-35]Solution in 0.577s

Time Plot 0 s.
Exiting SolveHard() after 0.879r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31,
375.73285289139794853948044873877963272 07,
328.11709294025273037343006969714687048 63,
292.99969137981047066856800939420568307 73,
358.64341560698677296916728373986038482 96, none,
360.06173466088080800215474726460391238 65, none, none,
324.65521223488910768252450670099131039 97, none, none, none, none]

0 --> 1  target = [26.46318954470108625718942824579497944744,
6.19617723023179702691139284315694585960 5,
385.42734025661940609153818295492429310 40]
two intervals r = 16.87629600298355629001962102663898269044 ..
189999999999910696283903958851966354 99/100000000000000000000000000000000
00000 or r = 15.55559000647745005145604517644546859025 ..
189999999999910696283903958851966354 99/100000000000000000000000000000000
00000
Time Approximations 0.058.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 8.153 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852533782591590377055427738944514, rm
= 2.33669042822272110682061742039979475259 2}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.717e-37, 0., -.75e-36]Solution in 27.42s

Time Plot 0 s.
Exiting SolveHard() after 30.325r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31,
375.73285289139794853948044873877963272 07,
328.11709294025273037343006969714687048 63,
292.99969137981047066856800939420568307 73,
358.64341560698677296916728373986038482 96, none,
360.06173466088080800215474726460391238 65,
336.59441031961501539137620043115518381 94, none,
324.65521223488910768252450670099131039 97, none, none, none, none]

0 --> 2  target = [34.49522661173073171928366879272780708399,
3.8971313159046974726507113373882328578 33,
373.78081884747121440456085234816209686 48]
two intervals r = 17.297690862179792230756567382133773172 21 ..
189999999999910696283903958851966354 99/10000000000000000000000000000000
00000 or r = 14.994364074448183092089808007834327535 37 ..
189999999999910696283903958851966354 99/10000000000000000000000000000000
00000
Time Approximations 0.088.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.36e-37, 0., .729e-35]Solution in 3.088s

Time Plot 0 s.
Exiting SolveHard() after 6.702r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31,
375.73285289139794853948044873877963272 07,
328.11709294025273037343006969714687048 63,
292.99969137981047066856800939420568307 73,
358.64341560698677296916728373986038482 96, none,
360.06173466088080800215474726460391238 65,
336.59441031961501539137620043115518381 94, none,
324.65521223488910768252450670099131039 97,
331.93806791803209228378515027183594747 90, none, none, none]

1 --> 2  target = [34.49522661173073171928366879272780708399,
3.8971313159046974726507113373882328578 33,
373.78081884747121440456085234816209686 48]
one interval r = 21.06068473215971996794328997758962154538 ..
26.26979834300185406388851278951585712379
Time Approximations 0.035.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., .208e-34]Solution in 2.46s

Time Plot 0 s.
Exiting SolveHard() after 3.159r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31,
375.73285289139794853948044873877963272 07,
328.11709294025273037343006969714687048 63,
292.99969137981047066856800939420568307 73,
358.64341560698677296916728373986038482 96,
299.89866204965074170290874542481141495 64,
360.06173466088080800215474726460391238 65,
336.59441031961501539137620043115518381 94, none,
324.65521223488910768252450670099131039 97,
331.93806791803209228378515027183594747 90, none, none, none]

0 --> 2  target = [33.81362495410187814725629839870678109394,
3.7256489935230076966316223126929549496 26,
325.89209972716660207065671916157570423 69]
two intervals r = 18.552270490078582179401311979444805615 73 ..
189999999999910696283903958851966354 99/10000000000000000000000000000000
00000 or r = 12.491969357844237282475469781295922023 56 ..
189999999999910696283903958851966354 99/10000000000000000000000000000000
00000
Time Approximations 0.043.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [-.123e-36, .6e-37, .313e-35]Solution in 1.102s

Time Plot 0 s.
Exiting SolveHard() after 4.235r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31,
375.73285289139794853948044873877963272 07,
328.11709294025273037343006969714687048 63,
292.99969137981047066856800939420568307 73,
358.64341560698677296916728373986038482 96,
299.89866204965074170290874542481141495 64,
360.06173466088080800215474726460391238 65,
336.59441031961501539137620043115518381 94, none,
324.65521223488910768252450670099131039 97,
331.93806791803209228378515027183594747 90, none, none,
289.54595772518771885114265367112904133 81]

1 --> 2  target = [33.81362495410187814725629839870678109394,
3.7256489935230076966316223126929549496 26,
325.89209972716660207065671916157570423 69]
one interval r = 20.37468935113219210471630594318380994571 ..
25.37892165307695562476584826506435126777
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=3e-38
Equations at solution: [.3e-37, .3e-37, -.842e-34]Solution in 2.175s

Time Plot 0 s.
Exiting SolveHard() after 2.725r=24.3395 in [22.07732228 ..
25.37892164]
```

```
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31,
375.73285289139794853948044873877963272 07,
328.11709294025273037343006969714687048 63,
292.99969137981047066856800939420568307 73,
358.64341560698677296916728373986038482 96,
299.89866204965074170290874542481141495 64,
360.06173466088080800215474726460391238 65,
336.59441031961501539137620043115518381 94,
256.10753185866782588395591366162614867 87,
324.65521223488910768252450670099131039 97,
331.93806791803209228378515027183594747 90, none, none,
289.54595772518771885114265367112904133 81]

1 --> 0  target = [17.93041369720144156441827129981757671295,
4.6865087018092293586996829927991106058 86,
353.30541094692057171252802074674678655 48]
one interval r = 20.73150479093766370271523707086614583534 ..
25.90675353527250293431232970131219591092
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, .208e-34]Solution in 0.66s

Time Plot 0 s.
```

```
Exiting SolveHard() after 1.391r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31,
375.73285289139794853948044873877963272 07,
328.11709294025273037343006969714687048 63,
292.99969137981047066856800939420568307 73,
358.64341560698677296916728373986038482 96,
299.89866204965074170290874542481141495 64,
360.06173466088080800215474726460391238 65,
336.59441031961501539137620043115518381 94,
256.10753185866782588395591366162614867 87,
324.65521223488910768252450670099131039 97,
331.93806791803209228378515027183594747 90,
304.79958324952885946811540413540899872 66, none,
289.54595772518771885114265367112904133 81]

2 --> 0  target = [17.93041369720144156441827129981757671295,
4.68650870180922935869968299279911060588 6,
353.30541094692057171252802074674678655 48]
one interval r = 31.37435486989297841338270448694891445576 ..
34.20127520025691539304416773992439145788
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=5e-38
```

```
Equations at solution: [.4e-37, -.5e-37, -.37e-35]Solution in 0.355s

Time Plot 0 s.
Exiting SolveHard() after 2.869r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493607774047910979926617905968 59,
441.64295973023286851685557314987280679 62,
436.91748165348865744569960751642510717 53,
422.98493397061424284942894151618261668 31,
361.52580256029528710444305809916962189 56,
401.88173904043869923067323644956153138 53,
389.59001516004204875208685845874479194 80,
328.46939893137680091250188795023946985 13,
401.50757157760724392988813742392827121 95,
358.97362823881923935292889442802048767 45,
398.33147103404042384221371842407976705 69,
371.48387394693469234687246237192326178 43,
336.61215840917932684345198867003512087 77,
361.50888347067686713359977752718185902 35,
324.67144992509891029252940207781625446 78,
302.31384314300463028293436723876102941 51,
328.46938513210308806649052557907598669 51,
343.81340625011419726036174022084692764 31,
375.73285289139794853948044873877963272 07,
328.11709294025273037343006969714687048 63,
292.99969137981047066856800939420568307 73,
358.64341560698677296916728373986038482 96,
299.89866204965074170290874542481141495 64,
360.06173466088080800215474726460391238 65,
336.59441031961501539137620043115518381 94,
256.10753185866782588395591366162614867 87,
324.65521223488910768252450670099131039 97,
331.93806791803209228378515027183594747 90,
304.79958324952885946811540413540899872 66,
323.46169176527039814221882748547861278 79,
289.54595772518771885114265367112904133 81]

Cascade time 240.201
counts: 28, 28

 Iteration 33

Start Generation 1
1 --> 0  target = [12.00000000002706580168617141637782012200,
6.21701250285420689979051744517929858968 0,
485.54908089807946791815234637220717456 49]
one interval r = 23.40850301652179942185959113205039517119 ..
27.67578046422807423854279536514595011041
Time Approximations 0.039.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
```

```
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=8.0e-38
Equations at solution: [-.3e-37, .80e-37, -.1e-36]Solution in 0.977s

Time Plot 0 s.
Exiting SolveHard() after 4.189r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000002706580168617141637782012200,
6.217012502854206899790517445179298589680,
485.549080898079467918152346372207174 5649]
one interval r = 32.628147792087980948232892279631310 12632 ..
36.10248388939267457312122943089675982079
Time Approximations 0.021.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .5891e-35]Solution in 0.571s

Time Plot 0 s.
Exiting SolveHard() after 2.954r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
```

```
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684472570906132545853924828208121,
6.58343472184918621896912492473331850307 0,
467.78730595653969402716038158888056073 88]
one interval r = 32.41978955655662451666778779437933607493 ..
35.85152417365796444984728299861686157616
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, .21128e-34]Solution in 0.603s

Time Plot 0 s.
Exiting SolveHard() after 0.975r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383 68772,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80, none, none,
401.88173903882597779184287706118143181 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684472570906132545853924828208121,
6.58343472184918621896912492473331850307 0,
467.78730595653969402716038158888056073 88]
two intervals r = 12.92327160844422171210898674091572397701 ..
18999999999967222275228487138783179933/10000000000000000000000000000000
00000 or r = 18.39424858022774134756874422500496236507 ..
18999999999967222275228487138783179933/10000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
```

```
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.4e-38
Equations at solution: [.5e-37, .54e-37, -.1041e-34]Solution in 40.143s

Time Plot 0 s.
Exiting SolveHard() after 43.315r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47, none,
401.88173903882597779184287706118143181 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628251251071100994560328183226 92,
4.1256517969282216941493548526278986502 85,
440.67123065203736491860178544079905117 32]
two intervals r = 14.356597051106531934831930223581458649 48 ..
18999999999967222275228487138783179933/10000000000000000000000000000000
00000 or r = 17.703526138097431935544758803878318943 87 ..
18999999999967222275228487138783179933/10000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [.14e-37, 0., -.744e-35]Solution in 3.121s

Time Plot 0 s.
Exiting SolveHard() after 4.148r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47, none,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962825125107110099456032818322692,
4.1256517969282216941493548526278986502 85,
440.67123065203736491860178544079905117 32]
one interval r = 22.397611543677507136156445547595011395 72 ..
27.237223515950396689929434520552205731 44
Time Approximations 0.037.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.264 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643915142619107388573425392397 98, rm =
14.378187707776352640688737570382147545 73}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, .97e-35]Solution in 10.846s

Time Plot 0 s.
Exiting SolveHard() after 11.728r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888789878026474845523826460257363,
4.0048690818871611559610257181978382739 69,
```

```
404.86224500946999268241520502620204508 33]
two intervals r = 16.08011007775119393518377886397024990772 ..
189999999999672222752284871387831799 33/100000000000000000000000000000000
00000 or r = 16.41579812674953726318299129756521260874 ..
189999999999672222752284871387831799 33/100000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., -.1391e-34]Solution in 4.051s

Time Plot 0 s.
Exiting SolveHard() after 5.168r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383 68772,
441.64295972968119456650629415540399 29539,
436.91748165428308707778917795259627 24080,
422.98493396735947010437986342286652 46847,
361.52580256028458711902345515065161 28217,
401.88173903882597779184287706118143 18153,
389.59001516204909135562339660472216 29217, none, none,
358.97362823881303560114237476917176 93820, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888789878026474845523826460257363,
4.00486908188716115596102571819783827 3969,
404.86224500946999268241520502620204508 33]
one interval r = 21.64194399402370581666246430639600770655 ..
26.76330660031653526768903711319321270918
Time Approximations 0.049.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
```

```
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [.1e-37, .49e-37, .257e-34]Solution in 1.007s

Time Plot 0 s.
Exiting SolveHard() after 3.958r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05, none,
358.97362823881303560114237476917176938 20, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941827436314604142255460654495493,
5.5896371827568216855238478440572812930 57,
443.83065883756230890590545542875878032 69]
one interval r = 22.46725374457228295327622825561085191300 ..
27.27388428339943583592673802125962903558
Time Approximations 0.04.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, .96e-35]Solution in 0.997s

Time Plot 0 s.
Exiting SolveHard() after 4.203r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
```

```
361.525802560284587119023455150651612821 7,
401.881739038825977791842877061181431815 3,
389.590015162049091355623396604722162921 7,
328.469398929088941725699499753291157250 5, none,
358.973628238813035601142374769171769382 0,
398.331471028592187199069462548439682498 1, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589418274363146041422554606544954 93,
5.589637182756821685523847844057281293057,
443.830658837562308905905455428758780326 9]
one interval r = 32.155752794879125168561808128522561744 29 ..
35.508722287230365831366126953765226540 05
Time Approximations 0.021.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, -.210e-36]Solution in 2.226s

Time Plot 0 s.
Exiting SolveHard() after 2.605r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936309917501295400695853836877 2,
441.642959729681194566506294155403992953 9,
436.917481654283087077789177952596272408 0,
422.984933967359470104379863422866524684 7,
361.525802560284587119023455150651612821 7,
401.881739038825977791842877061181431815 3,
389.590015162049091355623396604722162921 7,
328.469398929088941725699499753291157250 5,
401.507571575580914323032805279216423753 1,
358.973628238813035601142374769171769382 0,
398.331471028592187199069462548439682498 1, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931364926071915865591578021203353 14,
5.187783578452261206160576973627368594823,
408.657738630918693608578601921657584605 2]
one interval r = 21.718401146569248968897021243299419903 23 ..
26.818493035127663254954548754078598431 95
```

```
Time Approximations 0.059.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.58e-37
Equations at solution: [-.1e-37, -.158e-36, .359e-34]Solution in 3.051s

Time Plot 0 s.
Exiting SolveHard() after 4.166r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917779525962724080,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81, none, none,
361.50888347070320688600462019285127971 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136492607191586559157802120335314,
5.1877835784522612061605769736273685948 23,
408.65773863091869360857860192165758460 52]
one interval r = 31.80828598749932646135623032940143313647 ..
35.00011460048066110644757408774405324294
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
```

```
Equations at solution: [-.2e-37, .3e-37, -.17820e-34]Solution in 0.41s

Time Plot 0 s.
Exiting SolveHard() after 0.697r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37, none,
361.50888347070320688600462019285127971 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110534917855461457690284713716986,
6.19626255586482325031435802223804558683,
385.44474379279699430893648774543853464 54]
one interval r = 31.60836097531004150667148686779612338625 ..
34.66372795605926229574579912120806993158
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, .23325e-34]Solution in 0.551s

Time Plot 0 s.
Exiting SolveHard() after 0.827r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
```

```
441.642959729681194566506294155403992 9539,
436.91748165428308707778917795259627 24080,
422.98493396735947010437986342286652 46847,
361.52580256028458711902345515065161 28217,
401.88173903882597779184287706118143 18153,
389.59001516204909135562339660472216 29217,
328.46939892908894172569949975329115 72505,
401.50757157558091432303280527921642 37531,
358.97362823881303560114237476917176 93820,
398.33147102859218719906946254843968 24981,
371.48387394984904600306070304134231 93837, none,
361.50888347070320688600462019285127 97170,
324.67144992390653060002597877065969 02146, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110534917855461457690284713716986,
6.19626255586482325031435802223804558683,
385.44474379279699430893648774543853464 54]
two intervals r = 16.87563408753919520648902752086945613755 ..
189999999999672222752284871387831799 33/10000000000000000000000000000000
00000 or r = 15.55640493797901743253909687186761490529 ..
189999999999672222752284871387831799 33/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.848 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175119288502534571034024607470825, rm
= 2.33653277397743964080675890747925182917 2}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.3734e-34]Solution in 27.135s

Time Plot 0 s.
Exiting SolveHard() after 30.004r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.642959729681194566506294155403992 9539,
436.91748165428308707778917795259627 24080,
422.98493396735947010437986342286652 46847,
361.52580256028458711902345515065161 28217,
401.88173903882597779184287706118143 18153,
```

```
389.590015162049091355623396604722162921 7,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874735726182723095087491675370569,
4.8838107797354355010571824005150003238 02,
376.61967855747486347689085741657042069 92]
one interval r = 21.11001304876016276699284346989947257675 ..
26.31784243469599494919884893474159248932
Time Approximations 0.034.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=4.9e-38
Equations at solution: [.1e-37, .49e-37, .6e-36]Solution in 0.812s

Time Plot 0 s.
Exiting SolveHard() after 3.393r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.590015162049091355623396604722162921 7,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46, none,
328.46938512981590355932265737599904458 00, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none]
```

```
2 --> 0  target = [17.19898874735726182723095087491675370569,
4.883810779735435501057182400515000323802,
376.61967855747486347689085741657042069 92]
one interval r = 31.53899497705046763514727786919188479227 ..
34.53618386088968720142406340116460893427
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.26e-36
Equations at solution: [-.98e-36, .126e-35, .15843e-34]Solution in
0.477s

Time Plot 0 s.
Exiting SolveHard() after 0.777r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383 68772,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46, none,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017526795625508489027115695030564,
6.025813549516233046189385059103833039306,
351.42702947854829871032219192152571348 03]
one interval r = 31.36230206104445324446603710007493819774 ..
34.17446640603115593865892048780611200655
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
```

```
12.1428029945613367595242565268234247214 4, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=1.7e-37
Equations at solution: [-.9e-37, .17e-36, .10797e-34]Solution in 2.468s

Time Plot 0 s.
Exiting SolveHard() after 2.694r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46, none,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84, none, none,
292.99969137639811018988891168019471391 94, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175267956255084890271156950305 64,
6.0258135495162330461893850591038330393 06,
351.42702947854829871032219192152571348 03]
two intervals r = 17.981355144546080929508918577905859119 29 ..
189999999999672222752284871387831799 33/10000000000000000000000000000000
00000 or r = 13.846080153900164054783303786281425696 91 ..
189999999999672222752284871387831799 33/10000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
```

```
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 7.422 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713698938603014934230604196658 73, rm
= 2.7345009930658941755055635178172319369 56}});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [-.18e-37, .1e-37, -.1251e-34]Solution in
18.983s

Time Plot 0 s.
Exiting SolveHard() after 20.022r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84, none, none,
292.99969137639811018988891168019471391 94, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418514088619164157118642282907 96,
6.3779438740733283356971419901947941964 69,
423.28832782700076996076858268061386508 61]
one interval r = 31.946618175794890283867862632206965573 30 ..
35.212123086299923781702869565818271118 96
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=8e-38
Equations at solution: [.6e-37, -.8e-37, -.9284e-35]Solution in 0.611s

Time Plot 0 s.
Exiting SolveHard() after 0.959r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84, none, none,
292.99969137639811018988891168019471391 94, none, none,
360.06173465449665065246252113258553407 80, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941851408861916415711864228290796,
6.37794387407332833569714199019479419 6469,
423.28832782700076996076858268061386508 61]
two intervals r = 15.22886702497580203625076033956222167935 ..
189999999999672222752284871387831799 33/10000000000000000000000000000000
00000 or r = 17.12965777024569045317528593317722628559 ..
189999999999672222752284871387831799 33/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
```

```
in partial time = 7.211 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541670214804788778276987081739 45, rm
= 2.0640682987209995194133466230704848956 74}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., -.472e-35]Solution in 28.454s

Time Plot 0 s.
Exiting SolveHard() after 31.657r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383 68772,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84,
375.73285288257479041478892728239561034 86, none,
292.99969137639811018988891168019471391 94, none, none,
360.06173465449665065246252113258553407 80, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343325996745993532937596191107 16,
4.0035598155844547961849206315890264246 41,
404.47973593572302312444366769712724862 99]
two intervals r = 16.096839663894677208962569903349923053 54 ..
189999999999672222752284871387831799 33/100000000000000000000000000000000
00000 or r = 16.399886490914768755777222880257557304 84 ..
189999999999672222752284871387831799 33/100000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.34e-37, 0., -.526e-35]Solution in 3.604s

Time Plot 0 s.
Exiting SolveHard() after 4.666r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84,
375.73285288257479041478892728239561034 86, none,
292.99969137639811018988891168019471391 94,
358.64341560661602596181602619619090029 55, none,
360.06173465449665065246252113258553407 80, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234332599674599353293759619110716,
4.0035598155844547961849206315890264246 41,
404.47973593572302312444366769712724862 99]
one interval r = 21.634296299780463854264742633911040591 38 ..
26.757681698802758967697189269877097045 49
Time Approximations 0.054.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, .81e-35]Solution in 2.9s

Time Plot 0 s.
```

```
Exiting SolveHard() after 3.934r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84,
375.73285288257479041478892728239561034 86,
328.11709293757543484496213482087025926 33,
292.99969137639811018988891168019471391 94,
358.64341560661602596181602619619090029 55, none,
360.06173465449665065246252113258553407 80, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954453176794261989376913145461477,
6.1961772304402020281859883346933672856 15,
385.42734025658892392100834706829652475 78]
one interval r = 31.608220490864016630585642441580065498 66 ..
34.663476150436460978228666281655934916 4
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=8e-38
Equations at solution: [-.6e-37, .8e-37, -.20880e-34]Solution in 0.582s

Time Plot 0 s.
Exiting SolveHard() after 0.897r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
```

```
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84,
375.73285288257479041478892728239561034 86,
328.11709293757543484496213482087025926 33,
292.99969137639811018988891168019471391 94,
358.64341560661602596181602619619090029 55, none,
360.06173465449665065246252113258553407 80, none, none,
324.65521223373225462830195942039837541 74, none, none, none, none]

0 --> 1  target = [26.46318954453176794261989376913145461477,
6.19617723044020202818598833469336728561 5,
385.42734025658892392100834706829652475 78]
two intervals r = 16.87629600292402555009934223080239683845 ..
189999999999672222752284871387831799 33/10000000000000000000000000000000
00000 or r = 15.55559000646648206891602625793841557507 ..
189999999999672222752284871387831799 33/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.376 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852530941448733479648336765234363, rm
= 2.33669042804239925404446830270706105642 5}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., -.1049e-34]Solution in 26.482s
```

```
Time Plot 0 s.
Exiting SolveHard() after 29.483r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84,
375.73285288257479041478892728239561034 86,
328.11709293757543484496213482087025926 33,
292.99969137639811018988891168019471391 94,
358.64341560661602596181602619619090029 55, none,
360.06173465449665065246252113258553407 80,
336.59441031673064207494086276099130275 12, none,
324.65521223373225462830195942039837541 74, none, none, none, none]

0 --> 2  target = [34.49522661170641491700842138366453509863,
3.8971313160395788046911136670265785925 23,
373.78081885074480587499812606438408877 40]
two intervals r = 17.29769086201151169538771623276194321713 ..
189999999999672222752284871387831799 33/10000000000000000000000000000000
00000 or r = 14.99436407459960543957691105858024079422 ..
189999999999672222752284871387831799 33/10000000000000000000000000000000
00000
Time Approximations 0.079.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [0., 0., .2424e-34]Solution in 2.969s
```

```
Time Plot 0 s.
Exiting SolveHard() after 6.655r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84,
375.73285288257479041478892728239561034 86,
328.11709293757543484496213482087025926 33,
292.99969137639811018988891168019471391 94,
358.64341560661602596181602619619090029 55, none,
360.06173465449665065246252113258553407 80,
336.59441031673064207494086276099130275 12, none,
324.65521223373225462830195942039837541 74,
331.93806792210841587329215738302947232 58, none, none, none]

1 --> 2  target = [34.49522661170641491700842138366453509863,
3.8971313160395788046911136670265785925 23,
373.78018885074480587499812606438408877 40]
one interval r = 21.06068473213508176577116596971498631034 ..
26.26979834289416814414831327611501316684
Time Approximations 0.033.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., -.188e-34]Solution in 0.787s

Time Plot 0 s.
Exiting SolveHard() after 1.503r=25.3005 in [23.14060343 ..
```

```
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84,
375.73285288257479041478892728239561034 86,
328.11709293757543484496213482087025926 33,
292.99969137639811018988891168019471391 94,
358.64341560661602596181602619619090029 55,
299.89866205161059914254369278218134335 63,
360.06173465449665065246252113258553407 80,
336.59441031673064207494086276099130275 12, none,
324.65521223373225462830195942039837541 74,
331.93806792210841587329215738302947232 58, none, none, none]

0 --> 2  target = [33.81362495402374125997886310469539794675,
3.72564899364252863028030488621132817331 6,
325.89209972623779234630304306142210178 80]
two intervals r = 18.552270490056865385094438549292095681 73 ..
189999999999672222752284871387831799 33/10000000000000000000000000000000
00000 or r = 12.491969357776973116621264970076913925 44 ..
189999999999672222752284871387831799 33/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [.70e-37, -.3e-37, .380e-35]Solution in 2.898s
```

```
Time Plot 0 s.
Exiting SolveHard() after 6.551r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84,
375.73285288257479041478892728239561034 86,
328.11709293757543484496213482087025926 33,
292.99969137639811018988891168019471391 94,
358.64341560661602596181602619619090029 55,
299.89866205161059914254369278218134335 63,
360.06173465449665065246252113258553407 80,
336.59441031673064207494086276099130275 12, none,
324.65521223373225462830195942039837541 74,
331.93806792210841587329215738302947232 58, none, none,
289.54595772556148429147823334940488897 20]

1 --> 2  target = [33.81362495402374125997886310469539794675,
3.72564899364252863028030488621132817 3316,
325.89209972623779234630304306142210178 80]
one interval r = 20.37468935104659863233173236596510452344 ..
25.37892165291597314175847018460766934963
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .164e-34]Solution in 0.546s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.09r=24.3395 in [22.07732228 .. 25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84,
375.73285288257479041478892728239561034 86,
328.11709293757543484496213482087025926 33,
292.99969137639811018988891168019471391 94,
358.64341560661602596181602619619090029 55,
299.89866205161059914254369278218134335 63,
360.06173465449665065246252113258553407 80,
336.59441031673064207494086276099130275 12,
256.10753185677564831127016863066170514 92,
324.65521223373225462830195942039837541 74,
331.93806792210841587329215738302947232 58, none, none,
289.54595772556148429147823334940488897 20]

1 --> 0  target = [17.93041369723646255898663241887366048588,
4.68650870182754631856748214685151815545 1,
353.30541094389404274065832700628560297 22]
one interval r = 20.731504790814608045771715362760363525 46 ..
25.906753535062593493239590785073379521 73
Time Approximations 0.031.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.6e-38
Equations at solution: [.2e-37, .46e-37, .116e-34]Solution in 0.669s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.396r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84,
375.73285288257479041478892728239561034 86,
328.11709293757543484496213482087025926 33,
292.99969137639811018988891168019471391 94,
358.64341560661602596181602619619090029 55,
299.89866205161059914254369278218134335 63,
360.06173465449665065246252113258553407 80,
336.59441031673064207494086276099130275 12,
256.10753185677564831127016863066170514 92,
324.65521223373225462830195942039837541 74,
331.93806792210841587329215738302947232 58,
304.79958324415108239012098223465165914 18, none,
289.54595772556148429147823334940488897 20]

2 --> 0  target = [17.93041369723646255898663241887366048588,
4.68650870182754631856748214685151815545 1,
353.30541094389404274065832700628560297 22]
one interval r = 31.37435486983053990662495742254846844207 ..
34.20127520014494024646285431467567896769
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
```

```
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.3046e-35]Solution in 0.342s

Time Plot 0 s.
Exiting SolveHard() after 0.621r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493630991750129540069585383687 72,
441.64295972968119456650629415540399295 39,
436.91748165428308707778917795259627240 80,
422.98493396735947010437986342286652468 47,
361.52580256028458711902345515065161282 17,
401.88173903882597779184287706118143181 53,
389.59001516204909135562339660472216292 17,
328.46939892908894172569949975329115725 05,
401.50757157558091432303280527921642375 31,
358.97362823881303560114237476917176938 20,
398.33147102859218719906946254843968249 81,
371.48387394984904600306070304134231938 37,
336.61215840625605567221372233669391859 96,
361.50888347070320688600462019285127971 70,
324.67144992390653060002597877065969021 46,
302.31384313784836545359133911648691200 08,
328.46938512981590355932265737599904458 00,
343.81340625134929673597776730682825486 84,
375.73285288257479041478892728239561034 86,
328.11709293757543484496213482087025926 33,
292.99969137639811018988891168019471391 94,
358.64341560661602596181602619619090029 55,
299.89866205161059914254369278218134335 63,
360.06173465449665065246252113258553407 80,
336.59441031673064207494086276099130275 12,
256.10753185677564831127016863066170514 92,
324.65521223373225462830195942039837541 74,
331.93806792210841587329215738302947232 58,
304.79958324415108239012098223465165914 18,
323.46169176390607177747096193129219264 18,
289.54595772556148429147823334940488897 20]

Cascade time 237.295
counts: 28, 28

 Iteration 34

Start Generation 1
1 --> 0  target = [12.00000000001605289961586139743182363300,
6.21701250281866960244253464985044019 0027,
485.54908089700115188804972914659975429 49]
one interval r = 23.40850301654743305303077966228228741379 ..
27.67578046440357377110258799432838444018
Time Approximations 0.041.
```

```
hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=0
Equations at solution: [0., 0., -.9e-36]Solution in 2.837s

Time Plot 0 s.
Exiting SolveHard() after 3.959r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000001605289961586139743182363300,
6.217012502818669602442534649850440190027,
485.54908089700115188804972914659975429 49]
one interval r = 32.62814779225971316531288789420102291899 ..
36.10248388945023032517108450524904258963
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., .62e-35]Solution in 0.595s

Time Plot 0 s.
Exiting SolveHard() after 1.027r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684491279199610844730526128515837,
6.58343472148628385600526522248393835254 1,
467.78730595757425605506380688998580914 47]
one interval r = 32.41978955676111691250870010156461057446 ..
35.85152417375208805571243981399549830731
Time Approximations 0.019.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .66e-35]Solution in 0.611s

Time Plot 0 s.
Exiting SolveHard() after 3.156r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28, none, none,
401.88173904313276798015255061685343308 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684491279199610844730526128515837,
6.58343472148628385600526522248393835254 1,
467.78730595757425605506380688998580914 47]
two intervals r = 12.92327160833540439002879438456758126628 ..
4749999999999412897786726541458563443/2500000000000000000000000000000
000 or r = 18.39424858028177037703340569454486946282 ..
4749999999999412897786726541458563443/2500000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
```

```
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=0
Equations at solution: [0., 0., .10568e-34]Solution in 37.573s

Time Plot 0 s.
Exiting SolveHard() after 40.918r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32, none,
401.88173904313276798015255061685343308 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962831965400652454970919574210550,
4.12565179685542440760689057779563993064 4,
440.67123065051989678104315547077815410 37]
two intervals r = 14.35659705114962474783824585619293170493 ..
47499999999994128977867265414585634 43/2500000000000000000000000000000000
000 or r = 17.70352613808128860180969086126159490522 ..
47499999999994128977867265414585634 43/2500000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [-.13e-37, 0., .10632e-34]Solution in 3.354s

Time Plot 0 s.
Exiting SolveHard() after 6.53r=15.9119 in [14.35659706 .. 18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32, none,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962831965400652454970919574210550,
4.1256517968554244076068905777956399306 44,
440.67123065051989678104315547077815410 37]
one interval r = 22.397611543662954200504880282378238261 72 ..
27.237223516099172830534287982025627922 53
Time Approximations 0.039.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.189 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643911885115295379873466740891 46, rm =
14.378187702314982226757959745398129784 06}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, -.602e-34]Solution in 11.025s

Time Plot 0 s.
Exiting SolveHard() after 11.9r=26.4635 in [24.64256576 .. 27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888806599205459299071031963459268,
4.0048690818343431524411720128574229103 29,
404.86224501372234104083296000939379467 95]
```

```
two intervals r = 16.08011007755191015369580378317471203165 ..
474999999999941289778672654145856344 3/250000000000000000000000000000000
000 or r = 16.41579812694715426576740118096404540132 ..
474999999999941289778672654145856344 3/250000000000000000000000000000000
000
Time Approximations 0.052.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., .37140e-34]Solution in 3.584s

Time Plot 0 s.
Exiting SolveHard() after 4.669r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76, none, none,
358.97362824166198879929469437029612733 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888806599205459299071031963459268,
4.00486908183434315244117201285742291032 9,
404.86224501372234104083296000939379467 95]
one interval r = 21.64194399410305295274955698128701848379 ..
26.76330660052963460686309015629427060949
Time Approximations 0.047.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
```

```
Equations at solution: [0., 0., -.968e-34]Solution in 0.986s

Time Plot 0 s.
Exiting SolveHard() after 3.996r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61, none,
358.97362824166198879929469437029612733 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941786129761226426740829164819559,
5.5896371828285514581758953940750922010 35,
443.83065884476841157636847121787294536 62]
one interval r = 22.46725374475254946737799096267312638522 ..
27.27388428364998004397575293757708459787
Time Approximations 0.035.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.07e-37
Equations at solution: [-.1e-37, .107e-36, .68e-35]Solution in 0.944s

Time Plot 0 s.
Exiting SolveHard() after 4.145r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
```

```
401.881739043132767980152550616853433085O,
389.590015159993855386210878744674869667 6,
328.469398937286627424447500999757078596 1, none,
358.973628241661988799294694370296127333 2,
398.331471040392956132566502980219035099 2, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589417861297612264267408291648195 59,
5.589637182828551458175895394075092201035,
443.830658844768411576368471217872945366 2]
one interval r = 32.155752795158355356036886091891800005 46 ..
35.508722287423031411542019939720597347 51
Time Approximations 0.019.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [-.8e-37, .7e-37, -.102e-34]Solution in 2.511s

Time Plot 0 s.
Exiting SolveHard() after 2.863r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434935205730498348528657785347854 2,
441.642959730676047120567121201630877178 7,
436.917481652947108625254266907701464652 8,
422.984933974085063838240589092107820493 2,
361.525802563148450680045193658794788139 9,
401.881739043132767980152550616853433085O,
389.590015159993855386210878744674869667 6,
328.469398937286627424447500999757078596 1,
401.507571581097991091093505080589468719 4,
358.973628241661988799294694370296127333 2,
398.331471040392956132566502980219035099 2, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365003592380525281007261652608 21,
5.187783578409802841235102722474413882065,
408.657738628849369325370543086969788952 6]
one interval r = 21.718401146524086740885060620282773582 19 ..
26.818493035250301656642359550159478250 28
Time Approximations 0.057.
```

```
hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.63e-37
Equations at solution: [-.2e-37, -.263e-36, -.134e-34]Solution in
3.425s

Time Plot 0 s.
Exiting SolveHard() after 4.499r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92, none, none,
361.50888347341404542397387858593882617 79, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136500359238052528100726165260821,
5.1877835784098028412351027224744138820 65,
408.65773862884936932537054308696978895 26]
one interval r = 31.80828598769730105909984182205679407930 ..
35.00011460055454083365201233485333478530
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, -.24e-35]Solution in 0.429s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.734r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29, none,
361.50888347341404542397387858593882617 79, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110553847520352731054349238527719,
6.19626256523774116842062169032713803 1001,
385.44474379577185596185125921465710227 78]
one interval r = 31.60836097555913573021124025536925506717 ..
34.66372795621653163516113878533489970111
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=5e-38
Equations at solution: [.3e-37, -.5e-37, -.143e-34]Solution in 0.56s

Time Plot 0 s.
Exiting SolveHard() after 0.839r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
```

```
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29, none,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110553847520352731054349238527719,
6.19626256523774116842062169032713803100 1,
385.44474379577185596185125921465710227 78]
two intervals r = 16.87563408742320282332886034008764550514 ..
47499999999994128977867265414585634 43/2500000000000000000000000000000000
000 or r = 15.55640493813200728123844649817624265555 ..
47499999999994128977867265414585634 43/2500000000000000000000000000000000
000
Time Approximations 0.058.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.707 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175113607442941459740085536303655, rm
= 2.33653277422481219185131265349133105664 2}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.16335e-34]Solution in 27.697s

Time Plot 0 s.
Exiting SolveHard() after 31.237r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
```

```
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874725079132763612865828686812230,
4.8838107797414313214788058675262874814 45,
376.61967856053555445699637783426007489 63]
one interval r = 21.11001304879010238724776960874142012637 ..
26.31784243488416462656834301910135002229
Time Approximations 0.032.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.238e-34]Solution in 0.847s

Time Plot 0 s.
Exiting SolveHard() after 3.729r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30, none,
328.46938513800991752537113619439009122 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874725079132763612865828686812230,
```

```
4.88381077974143132147880586752628748144 5,
376.61967856053555445699637783426007489 63]
one interval r = 31.53899497730182297069298688402732434337 ..
34.53618386105211189089305295805501798479
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=9.67e-36
Equations at solution: [-.743e-35, .967e-35, -.101e-34]Solution in
0.476s

Time Plot 0 s.
Exiting SolveHard() after 0.783r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30, none,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017554826755490091596529047639434,
6.02581354919793441764366619429209245249 1,
351.42702948702310369470840449747088370 81]
one interval r = 31.36230206133376741942418629619746767608 ..
34.17446640628197455234239964486582583366
Time Approximations 0.018.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
```

```
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [-.3e-37, .6e-37, -.136e-34]Solution in 2.554s

Time Plot 0 s.
Exiting SolveHard() after 2.81r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30, none,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92, none, none,
292.99969138794581159402048489408308496 97, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175548267554900915965290476394 34,
6.0258135491979344176436661942920924524 91,
351.42702948702310369470840449747088370 81]
two intervals r = 17.981355144330763222939916574274775842 56 ..
474999999999941289778672654145856344 3/2500000000000000000000000000000000
000 or r = 13.846080154343626654241994629470876298 91 ..
474999999999941289778672654145856344 3/2500000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.753 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713653633342211450194517952813 70, rm
= 2.7345009934038722606986611689973644044 15}});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [-.90e-37, .3e-37, .3042e-35]Solution in 18.483s

Time Plot 0 s.
Exiting SolveHard() after 19.556r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92, none, none,
292.99969138794581159402048489408308496 97, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418833580283416654276271499936 06,
6.3779438737652959601952763104032512257 31,
423.28832783915148779892727414312165897 87]
one interval r = 31.946618176124842162606785636664574909 24 ..
35.212123086573382320051363116075658571 28
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
```

```
Equations at solution: [-.4e-37, .5e-37, .282e-34]Solution in 0.593s

Time Plot 0 s.
Exiting SolveHard() after 0.941r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92, none, none,
292.99969138794581159402048489408308496 97, none, none,
360.06173466934204404059303545460086719 29, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418833580283416654276271499936 06,
6.3779438737652959601952763104032512257 31,
423.28832783915148779892727414312165897 87]
two intervals r = 15.22886702436316753073364130772290282672 ..
474999999999941289778672654145856344 3/2500000000000000000000000000000000
000 or r = 17.12965777070749539596154097497575412709 ..
474999999999941289778672654145856344 3/2500000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.308 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541261710676715320063984529 29415, rm
```

```
= 2.0640682988139250752575848938430928140000}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [-.15e-37, 0., -.7457e-35]Solution in 26.765s

Time Plot 0 s.
Exiting SolveHard() after 29.757r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92,
375.73285290148097423901705484001644165 83, none,
292.99969138794581159402048489408308496 97, none, none,
360.06173466934204404059303545460086719 29, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343511318377688733650368117548 07,
4.0035598155358818973844459239217471715 91,
404.47973594121268615664923401837547463 94]
two intervals r = 16.096839663642010117342969418957651873 24 ..
4749999999999412897786726541458563443/2500000000000000000000000000000000
000 or r = 16.399886491164277850343839085539967595 78 ..
4749999999999412897786726541458563443/2500000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.50e-37, .1e-37, -.4518e-35]Solution in 3.694s
```

```
Time Plot 0 s.
Exiting SolveHard() after 6.618r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92,
375.73285290148097423901705484001644165 83, none,
292.99969138794581159402048489408308496 97,
358.64341561053349474612186403403001384 48, none,
360.06173466934204404059303545460086719 29, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234351131837768873365036811754807,
4.0035598155358818973844459239217471715 91,
404.47973594121268615664923401837547463 94]
one interval r = 21.634296299884158324025626330889093305 69 ..
26.757681699034019347999185206553514193 22
Time Approximations 0.053.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .154e-34]Solution in 2.976s

Time Plot 0 s.
Exiting SolveHard() after 3.993r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92,
375.73285290148097423901705484001644165 83,
328.11709294691264283290615154417818731 25,
292.99969138794581159402048489408308496 97,
358.64341561053349474612186403403001384 48, none,
360.06173466934204404059303545460086719 29, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954471851308456230143962603345478,
6.19617723009069021816622967435294714021 8,
385.42734025940638328976031943218475401 63]
one interval r = 31.60822049111184367521453150122219321233 ..
34.66347615059146064089319399528529032216
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=1.2e-37
Equations at solution: [-.8e-37, .12e-36, .165e-34]Solution in 0.579s

Time Plot 0 s.
Exiting SolveHard() after 0.885r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92,
375.73285290148097423901705484001644165 83,
328.11709294691264283290615154417818731 25,
292.99969138794581159402048489408308496 97,
358.64341561053349474612186403403001384 48, none,
360.06173466934204404059303545460086719 29, none, none,
324.65521223992741448865315399945489308 36, none, none, none, none]

0 --> 1  target = [26.46318954471851308456230143962603345478,
6.19617723009069021816622967435294714021 8,
385.42734025940638328976031943218475401 63]
two intervals r = 16.87629600281404400741373833528214664204 ..
47499999999994128977867265414585634 43/25000000000000000000000000000000000
000 or r = 15.55559000661210645116898105585259674743 ..
47499999999994128977867265414585634 43/25000000000000000000000000000000000
000
Time Approximations 0.06.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.651 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852525558653856505428935327552909, rm
= 2.33669042829126130870136141552552140955 5}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.179e-37, 0., .26168e-34]Solution in 27.282s

Time Plot 0 s.
Exiting SolveHard() after 30.44r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92,
375.73285290148097423901705484001644165 83,
328.11709294691264283290615154417818731 25,
292.99969138794581159402048489408308496 97,
358.64341561053349474612186403403001384 48, none,
360.06173466934204404059303545460086719 29,
336.59441032563503838561944634541787696 75, none,
324.65521223992741448865315399945489308 36, none, none, none, none]

0 --> 2  target = [34.49522661178692340654979518565433851036,
3.89713131596314158454798835162307268 3527,
373.78081884803754249823098764639115822 96]
two intervals r = 17.29769086210703035122106932716909715588 ..
4749999999999412897786726541458563443/2500000000000000000000000000000000
000 or r = 14.99436407447401217287558601177612164224 ..
4749999999999412897786726541458563443/2500000000000000000000000000000000
000
Time Approximations 0.078.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.18e-37, 0., -.36904e-34]Solution in 3.429s

Time Plot 0 s.
Exiting SolveHard() after 6.915r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92,
375.73285290148097423901705484001644165 83,
328.11709294691264283290615154417818731 25,
292.99969138794581159402048489408308496 97,
358.64341561053349474612186403403001384 48, none,
360.06173466934204404059303545460086719 29,
336.59441032563503838561944634541787696 75, none,
324.65521223992741448865315399945489308 36,
331.93806791888439303670812003085319710 76, none, none, none]

1 --> 2  target = [34.49522661178692340654979518565433851036,
3.8971313159631415845479883516230726835 27,
373.78081884803754249823098764639115822 96]
one interval r = 21.06068473206305058632325626625998794027 ..
26.26979834298329083984506624668710145317
Time Approximations 0.032.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, .242e-34]Solution in 0.777s

Time Plot 0 s.
Exiting SolveHard() after 1.464r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92,
375.73285290148097423901705484001644165 83,
328.11709294691264283290615154417818731 25,
292.99969138794581159402048489408308496 97,
358.64341561053349474612186403403001384 48,
299.89866205341796739465474970168208691 22,
360.06173466934204404059303545460086719 29,
336.59441032563503838561944634541787696 75, none,
324.65521223992741448865315399945489308 36,
331.93806791888439303670812003085319710 76, none, none, none]

0 --> 2  target = [33.81362495425446330591857489518629530632,
3.72564899359966328033225566083513163547 9,
325.89209973258326617455432131468782135 03]
two intervals r = 18.552270489963565539403557562681711409 87 ..
474999999999941289778672654145856344 3/2500000000000000000000000000000000
000 or r = 12.491969358086581183067085996979601883 01 ..
474999999999941289778672654145856344 3/2500000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [.70e-37, -.3e-37, .36025e-34]Solution in 2.931s

Time Plot 0 s.
Exiting SolveHard() after 6.806r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92,
375.73285290148097423901705484001644165 83,
328.11709294691264283290615154417818731 25,
292.99969138794581159402048489408308496 97,
358.64341561053349474612186403403001384 48,
299.89866205341796739465474970168208691 22,
360.06173466934204404059303545460086719 29,
336.59441032563503838561944634541787696 75, none,
324.65521223992741448865315399945489308 36,
331.93806791888439303670812003085319710 76, none, none,
289.54595773036821916739486518573704886 19]

1 --> 2  target = [33.81362495425446330591857489518629530632,
3.72564899359966328033225566083513163547 9,
325.89209973258326617455432131468782135 03]
one interval r = 20.37468935106715907776707813357435329104 ..
25.37892165315181235521733821500290768454
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., -.943e-34]Solution in 0.559s

Time Plot 0 s.
Exiting SolveHard() after 1.106r=24.3395 in [22.07732228 ..
25.37892164]
```

```
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92,
375.73285290148097423901705484001644165 83,
328.11709294691264283290615154417818731 25,
292.99969138794581159402048489408308496 97,
358.64341561053349474612186403403001384 48,
299.89866205341796739465474970168208691 22,
360.06173466934204404059303545460086719 29,
336.59441032563503838561944634541787696 75,
256.10753186686889687038240908244242531 92,
324.65521223992741448865315399945489308 36,
331.93806791888439303670812003085319710 76, none, none,
289.54595773036821916739486518573704886 19]

1 --> 0  target = [17.93041369698610154691443756608993451888,
4.6865087018845233505656799636712146956 66,
353.30541095342125354920223240759740787 25]
one interval r = 20.731504790919272476462619879941609309 08 ..
25.906753535362297628201785455675388128 73
Time Approximations 0.029.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.8e-38
Equations at solution: [.2e-37, .48e-37, .110e-34]Solution in 0.641s

Time Plot 0 s.
```

```
Exiting SolveHard() after 3.259r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92,
375.73285290148097423901705484001644165 83,
328.11709294691264283290615154417818731 25,
292.99969138794581159402048489408308496 97,
358.64341561053349474612186403403001384 48,
299.89866205341796739465474970168208691 22,
360.06173466934204404059303545460086719 29,
336.59441032563503838561944634541787696 75,
256.10753186686889687038240908244242531 92,
324.65521223992741448865315399945489308 36,
331.93806791888439303670812003085319710 76,
304.79958325886533755868019611777884096 40, none,
289.54595773036821916739486518573704886 19]

2 --> 0  target = [17.93041369698610154691443756608993451888,
4.68650870188452335056567996367121469566 6,
353.30541095342125354920223240759740787 25]
one interval r = 31.37435487012698257156330683048511457781 ..
34.20127520041004600127395053010195841097
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=1.6e-37
```

```
Equations at solution: [.11e-36, -.16e-36, -.324e-34]Solution in 0.345s

Time Plot 0 s.
Exiting SolveHard() after 0.62r=33.7963 in [32.25770943 .. 34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493520573049834852865778534785 42,
441.64295973067604712056712120163087717 87,
436.91748165294710862525426690770146465 28,
422.98493397408506383824058909210782049 32,
361.52580256314845068004519365879478813 99,
401.88173904313276798015255061685343308 50,
389.59001515999385538621087874467486966 76,
328.46939893728662742444750099975707859 61,
401.50757158109799109109350508058946871 94,
358.97362824166198879929469437029612733 32,
398.33147104039295613256650298021903509 92,
371.48387394734692119284548685875266042 29,
336.61215841532111432892740447118298811 56,
361.50888347341404542397387858593882617 79,
324.67144993024850177288554791064308570 30,
302.31384315208806249964218697168044772 10,
328.46938513800991752537113619439009122 32,
343.81340625320176671044738760136626252 92,
375.73285290148097423901705484001644165 83,
328.11709294691264283290615154417818731 25,
292.99969138794581159402048489408308496 97,
358.64341561053349474612186403403001384 48,
299.89866205341796739465474970168208691 22,
360.06173466934204404059303545460086719 29,
336.59441032563503838561944634541787696 75,
256.10753186686889687038240908244242531 92,
324.65521223992741448865315399945489308 36,
331.93806791888439303670812003085319710 76,
304.79958325886533755868019611777884096 40,
323.46169177140949836865366722372152322 41,
289.54595773036821916739486518573704886 19]

Cascade time 240.215
counts: 28, 28

 Iteration 35

Start Generation 1
1 --> 0  target = [12.00000000010795767555945358330305593900,
6.21701250280055306529360327848533891238 3,
485.54908089379945118936970326038824744 68]
one interval r = 23.40850301641012656265184559404796382080 ..
27.67578046411385384462695583006806837053
Time Approximations 0.037.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
```

```
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=0
Equations at solution: [0., 0., .1e-36]Solution in 0.973s

Time Plot 0 s.
Exiting SolveHard() after 3.974r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000010795767555945358330305593900,
6.217012502800553065293603278485338912383,
485.549080893799451189369703260388247446 8]
one interval r = 32.628147792062287496105157685386163094 19 ..
36.102483889385061113489657052630500715 60
Time Approximations 0.021.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.26e-35]Solution in 0.57s

Time Plot 0 s.
Exiting SolveHard() after 0.989r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
```

```
436.91748165059238855969925230717973120 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684461527877149373974510464332173,
6.58343472170100049122632185101186123602 7,
467.78730595311075031010913781016368589 80]
one interval r = 32.41978955654150606095296351356953129497 ..
35.85152417365966921683834582869247030281
Time Approximations 0.019.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .91e-35]Solution in 0.609s

Time Plot 0 s.
Exiting SolveHard() after 3.148r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55, none, none,
401.88173903731118993826322689471932612 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684461527877149373974510464332173,
6.58343472170100049122632185101186123602 7,
467.78730595311075031010913781016368589 80]
two intervals r = 12.92327160850474815382393911328959567173 ..
18999999999982140821723374444546246973/10000000000000000000000000000000
00000 or r = 18.39424858018066098385574071822303335332 ..
18999999999982140821723374444546246973/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
```

```
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=6.6e-38
Equations at solution: [-.6e-37, -.66e-37, .5862e-35]Solution in
40.319s

Time Plot 0 s.
Exiting SolveHard() after 43.626r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99, none,
401.88173903731118993826322689471932612 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628244505851049944814500399194 75,
4.1256517969305057937774379785198429982 16,
440.67123064828084063052472221057190237 09]
two intervals r = 14.356597051204149098307043519733486270 24 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000 or r = 17.703526138006657034274331485428281658 99 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=4e-38
Equations at solution: [-.71e-37, -.4e-37, -.15040e-34]Solution in
3.534s

Time Plot 0 s.
Exiting SolveHard() after 4.578r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99, none,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962824450585104994481450039919475,
4.1256517969305057937774379785198429982 16,
440.67123064828084063052472221057190237 09]
one interval r = 22.397611543598141675907868988692606586 87 ..
27.237223515832680613451422352274587369 87
Time Approximations 0.038.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.112 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643755294913867023186194710427 78, rm =
14.378187705877083833219359203707089415 32}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., .86e-35]Solution in 11.298s

Time Plot 0 s.
Exiting SolveHard() after 12.168r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888791990135104472474589931565797,
4.0048690818969181225426401830999416674 36,
```

```
404.8622450079372137014297562247875586624]
two intervals r = 16.08011007775562996636234656613322895854 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000 or r = 16.41579812669905734280772191748243936419 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., .7458e-35]Solution in 3.598s

Time Plot 0 s.
Exiting SolveHard() after 4.647r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178 64370,
441.64295972631242461935703111977877 04593,
436.91748165059238855969925230717973 12055,
422.98493396589246094022321713354155 56399,
361.52580255905122648003457702876280 21210,
401.88173903731118993826322689471932 61269,
389.59001515859169699584749215725500 28396, none, none,
358.97362823724325784122963787618360 88571, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888791990135104472474589931565797,
4.0048690818969181225426401830999416674 36,
404.8622450079372137014297562247875586624]
one interval r = 21.64194399401008878046201490560305856307 ..
26.76330660022774442340588021488740028915
Time Approximations 0.046.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
```

```
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, .67e-35]Solution in 1.04s

Time Plot 0 s.
Exiting SolveHard() after 4.529r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98, none,
358.97362823724325784122963787618360885 71, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941823766188256873003447316846420,
5.5896371827536933874479764802580272792 89,
443.83065883635525208311188937376866630 40]
one interval r = 22.46725374454759185891085762089881811744 ..
27.27388428331139008241593056168326112426
Time Approximations 0.038.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., -.103e-34]Solution in 0.966s

Time Plot 0 s.
Exiting SolveHard() after 4.036r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
```

```
361.525802559051226480034577028762802121 0,
401.881739037311189938263226894719326126 9,
389.590015158591696995847492157255002839 6,
328.469398929949446161503688743265274199 8, none,
358.973628237243257841229637876183608857 1,
398.331471029185571285221214606580193688 5, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589418237661882568730034473168464 20,
5.589637182753693387447976480258027279289,
443.830658836355252083111889373768666304 0]
one interval r = 32.155752794888435606836585518962748819 77 ..
35.508722287260730028473331705570112592 69
Time Approximations 0.02.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [-.6e-37, .5e-37, -.42e-35]Solution in 2.273s

Time Plot 0 s.
Exiting SolveHard() after 2.644r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434931965938559101898761411786437 0,
441.642959726312424619357031119778770459 3,
436.917481650592388559699252307179731205 5,
422.984933965892460940223217133541555639 9,
361.525802559051226480034577028762802121 0,
401.881739037311189938263226894719326126 9,
389.590015158591696995847492157255002839 6,
328.469398929949446161503688743265274199 8,
401.507571574570728345016683478668084996 9,
358.973628237243257841229637876183608857 1,
398.331471029185571285221214606580193688 5, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365007784349292025523379557008 16,
5.187783578427388187108379425240750200920,
408.657738627620772477666005258519948690 1]
one interval r = 21.718401146517988042021328604416359683 62 ..
26.818493035013093948664797962212934398 51
```

```
Time Approximations 0.061.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .42e-35]Solution in 3.008s

Time Plot 0 s.
Exiting SolveHard() after 4.109r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713353415556 399,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85, none, none,
361.50888346939400515641192702282340269 93, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136500778434929202552337955700816,
5.18778357842738818710837942524075020092 0,
408.65773862762077247766600525851994869 01]
one interval r = 31.80828598748857664368030186883053635909 ..
35.00011460047661421580347532001014643253
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=7e-38
```

```
Equations at solution: [.8e-37, -.7e-37, -.4e-36]Solution in 0.438s

Time Plot 0 s.
Exiting SolveHard() after 0.753r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713353415556399,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62, none,
361.50888346939400515641192702282340269 93, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110526729392468158352064758570576,
6.19626256544575439092218063703592416649 7,
385.44474379155184371655499401654719460 47]
one interval r = 31.60836097531780718414947531767529167061 ..
34.66372795608242993296137387277387993900
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .238e-34]Solution in 0.543s

Time Plot 0 s.
Exiting SolveHard() after 0.821r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
```

```
441.642959726312424619357031119778770459 3,
436.917481650592388559699252307179731205 5,
422.984933965892460940223217133541555639 9,
361.525802559051226480034577028762802121 0,
401.881739037311189938263226894719326126 9,
389.590015158591696995847492157255002839 6,
328.469398929949446161503688743265274199 8,
401.507571574570728345016683478668084996 9,
358.973628237243257841229637876183608857 1,
398.331471029185571285221214606580193688 5,
371.483873947077892241178633323201505316 2, none,
361.508883469394005156411927022823402699 3,
324.671449924616714968308341630996782126 6, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.463471105267293924681583520647585705 76,
6.196262565445754390922180637035924166497,
385.444743791551843716554994016547194604 7]
two intervals r = 16.875634087540529197426201899339580603 30 ..
189999999999821408217233744445462469 73/100000000000000000000000000000000
00000 or r = 15.556404937925135025287893738007324775 94 ..
189999999999821408217233744445462469 73/100000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.545 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.468341751198989704555947315027482879 33, rm
= 2.336532774078312973505823386735545457787}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.179e-37, 0., .23445e-34]Solution in 27.671s

Time Plot 0 s.
Exiting SolveHard() after 30.719r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434931965938559101898761411786437 0,
441.642959726312424619357031119778770459 3,
436.917481650592388559699252307179731205 5,
422.984933965892460940223217133541555639 9,
361.525802559051226480034577028762802121 0,
401.881739037311189938263226894719326126 9,
```

```
389.590015158591696995847492157255002839 6,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874736461413083038697028050442767,
4.8838107797316431152345004955598977432 91,
376.61967855615894523612460453242244740 73]
one interval r = 21.11001304876437479391374585722537060592 ..
26.31784243461430653500027766472836515910
Time Approximations 0.031.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.4e-38
Equations at solution: [0., .24e-37, .15e-35]Solution in 0.806s

Time Plot 0 s.
Exiting SolveHard() after 3.585r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713353415556 399,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.590015158591696995847492157255002839 6,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66, none,
328.46938513067442237734613867607809925 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
2 --> 0  target = [17.19898874736461413083038697028050442767,
4.8838107797316431152345004955598977432 91,
376.61967855615894523612460453242244740 73]
one interval r = 31.53899497705767403761155649060719425191 ..
34.53618386091089977332083929541271434076
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.25e-36
Equations at solution: [-.249e-35, .325e-35, .265e-34]Solution in
0.466s

Time Plot 0.001 s.
Exiting SolveHard() after 0.769r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178 64370,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66, none,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017523190503768039529004511313771,
6.0258135493855154041446288475157119906 08,
351.42702947945510115747901855345769743 02]
one interval r = 31.36230206106607961943105204161587997870 ..
34.17446640608153903822146184803574150968
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
```

```
12.1428029945613367595242565268234247214 4, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .40e-35]Solution in 2.95s

Time Plot 0 s.
Exiting SolveHard() after 3.167r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178 64370,
441.64295972631242461935703111977877 04593,
436.91748165059238855969925230717973 12055,
422.98493396589246094022321713354155 56399,
361.52580255905122648003457702876280 21210,
401.88173903731118993826322689471932 61269,
389.59001515859169699584749215725500 28396,
328.46939892994944616150368874326527 41998,
401.50757157457072834501668347866808 49969,
358.97362823724325784122963787618360 88571,
398.33147102918557128522121460658019 36885,
371.48387394707789224117863332320150 53162,
336.61215840734383294633305218283889 87702,
361.50888346939400515641192702282340 26993,
324.67144992461671496830834163099678 21266, none,
328.46938513067442237734613867607809 92589,
343.81340625029278906949577813310822 56012, none, none,
292.99969137917773321697102373032860 40100, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017523190503768039529004511313771,
6.02581354938551540414462884751571199 0608,
351.42702947945510115747901855345769 74302]
two intervals r = 17.98135514450079304107950394961922488888 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000 or r = 13.84608015392979084198595145703262160588 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
```

```
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.773 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713695679327857125430939079250 16, rm
= 2.7345009932059386886657900176807775980 56}});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.54e-37, -.2e-37, .24950e-34]Solution in
18.903s

Time Plot 0 s.
Exiting SolveHard() after 19.956r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863333232015053 162,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12, none, none,
292.99969137917773321697102373032860401 00, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418452277359099138506524088119 93,
6.3779438739425938835103798239199312641 60,
423.28832782763747238595942667595285365 99]
one interval r = 31.946618175821574358957377828864678405 67 ..
35.212123086354556550794257165212255614 51
Time Approximations 0.02.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, -.289e-34]Solution in 0.62s

Time Plot 0 s.
Exiting SolveHard() after 0.969r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863333232015053 162,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12, none, none,
292.99969137917773321697102373032860401 00, none, none,
360.06173465689426792203419058203650925 28, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941845227735909913850652408811993,
6.37794387394259388351037982391993126416 0,
423.28832782763747238595942667595285365 99]
two intervals r = 15.22886702486517440845639650873951152131 ..
18999999999982140821723374444546246973/10000000000000000000000000000000
00000 or r = 17.12965777028750074357413744272446224589 ..
18999999999982140821723374444546246973/10000000000000000000000000000000
00000
Time Approximations 0.062.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
```

```
in partial time = 7.24 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541602989389934411242127961982 57, rm
= 2.0640682987694026643504239915204217356 59}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.15e-37, -.1e-37, -.43437e-34]Solution in
28.553s

Time Plot 0 s.
Exiting SolveHard() after 32.1r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12,
375.73285288566296900744942625780766987 50, none,
292.99969137917773321697102373032860401 00, none, none,
360.06173465689426792203419058203650925 28, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343354555216529971353912853689 02,
4.0035598155959784554520670237247752499 82,
404.47973593470609923037680241531925056 30]
two intervals r = 16.096839663876759167575261202975774048 15 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000 or r = 16.399886490885458928183009369796333343 81 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.32e-37, -.1e-37, .7228e-35]Solution in 3.823s

Time Plot 0 s.
Exiting SolveHard() after 4.818r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713353415556399,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12,
375.73285288566296900744942625780766987 50, none,
292.99969137917773321697102373032860401 00,
358.64341560549160584868895756619490510 24, none,
360.06173465689426792203419058203650925 28, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343354555216529971353912853689 02,
4.0035598155959784554520670237247752499 82,
404.47973593470609923037680241531925056 30]
one interval r = 21.634296299777340359942779836733206420 12 ..
26.757681698721601013241039730437834332 76
Time Approximations 0.052.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.2e-38
Equations at solution: [.2e-37, .52e-37, .246e-34]Solution in 2.868s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.887r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12,
375.73285288566296900744942625780766987 50,
328.11709293891140947366581314499145711 93,
292.99969137917773321697102373032860401 00,
358.64341560549160584868895756619490510 24, none,
360.06173465689426792203419058203650925 28, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954444862374232661645768783024472,
6.19617723029909103613184124156938420998 5,
385.42734025526575868236583540398899369 53]
one interval r = 31.608220490871152515796026768504242203 24 ..
34.663476150458497989964179053758765695 88
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, .47e-35]Solution in 0.544s

Time Plot 0 s.
Exiting SolveHard() after 0.848r=33.8134 in [32.62668594 ..
34.66347615]
```

```
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12,
375.73285288566296900744942625780766987 50,
328.11709293891140947366581314499145711 93,
292.99969137917773321697102373032860401 00,
358.64341560549160584868895756619490510 24, none,
360.06173465689426792203419058203650925 28, none, none,
324.65521223436968593318991268677203614 36, none, none, none, none]

0 --> 1  target = [26.46318954444862374232661645768783024472,
6.19617723029909103613184124156938420998 5,
385.42734025526575868236583540399899369 53]
two intervals r = 16.87629600292833391672908951832968200459 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000 or r = 15.55559000640893189083266180898495169203 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.682 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852531698848767189072678200225738, rm
= 2.33669042814400545183379319510729829431 1}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
```

```
Equations at solution: [-.359e-37, 0., -.44217e-34]Solution in 26.978s

Time Plot 0 s.
Exiting SolveHard() after 30.126r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12,
375.73285288566296900744942625780766987 50,
328.11709293891140947366581314499145711 93,
292.99969137917773321697102373032860401 00,
358.64341560549160584868895756619490510 24, none,
360.06173465689426792203419058203650925 28,
336.59441031773883040042866061874728933 97, none,
324.65521223436968593318991268677203614 36, none, none, none, none]

0 --> 2  target = [34.49522661170567959798459041489857171066,
3.8971313160448759556346048740445885222 64,
373.78018884792732854640193827117914286 33]
two intervals r = 17.29769086207134458041862408523944975199 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000 or r = 14.99436407445794816000369221567572511484 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000
Time Approximations 0.077.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
```

```
Equations at solution: [-.126e-36, .1e-37, -.24824e-34]Solution in
3.08s

Time Plot 0 s.
Exiting SolveHard() after 6.666r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12,
375.73285288566296900744942625780766987 50,
328.11709293891140947366581314499145711 93,
292.99969137917773321697102373032860401 00,
358.64341560549160584868895756619490510 24, none,
360.06173465689426792203419058203650925 28,
336.59441031773883040042866061874728933 97, none,
324.65521223436968593318991268677203614 36,
331.93806791940770652341872393056726500 76, none, none, none]

1 --> 2  target = [34.49522661170567959798459041489857171066,
3.89713131604487595563460487404458852 2264,
373.78081884792732854640193827117914286 33]
one interval r = 21.06068473211478428097040983872283976768 ..
26.26979834278737041784906141165121645258
Time Approximations 0.035.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, .701e-34]Solution in 0.779s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.485r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12,
375.73285288566296900744942625780766987 50,
328.11709293891140947366581314499145711 93,
292.99969137917773321697102373032860401 00,
358.64341560549160584868895756619490510 24,
299.89866205131980767611140944957417319 95,
360.06173465689426792203419058203650925 28,
336.59441031773883040042866061874728933 97, none,
324.65521223436968593318991268677203614 36,
331.93806791940770652341872393056726500 76, none, none, none]

0 --> 2  target = [33.81362495406885458048332679692962480774,
3.72564899366069521667583637897705447 7313,
325.89209972698211203683143904045163845 84]
two intervals r = 18.552270490037729275891029501646741005 35 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000 or r = 12.491969357774609615278271006335076670 98 ..
189999999999821408217233744445462469 73/10000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [-.70e-37, .3e-37, .9547e-35]Solution in 2.977s

Time Plot 0 s.
Exiting SolveHard() after 6.277r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12,
375.73285288566296900744942625780766987 50,
328.11709293891140947366581314499145711 93,
292.99969137917773321697102373032860401 00,
358.64341560549160584868895756619490510 24,
299.89866205131980767611140944957417319 95,
360.06173465689426792203419058203650925 28,
336.59441031773883040042866061874728933 97, none,
324.65521223436968593318991268677203614 36,
331.93806791940770652341872393056726500 76, none, none,
289.54595772599879331503271067439421531 44]

1 --> 2  target = [33.81362495406885458048332679692962480774,
3.72564899366069521667583637897705447731 3,
325.89209972698211203683143904045163845 84]
one interval r = 20.37468935109701037807240828313801055264 ..
25.37892165288548384474457383854863234387
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
```

```
Accepted {r=24.3395, rm=17.2722} with Delta=1e-38
Equations at solution: [-.1e-37, -.1e-37, .200e-34]Solution in 0.575s

Time Plot 0 s.
Exiting SolveHard() after 1.114r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713353415556 399,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12,
375.73285288566296900744942625780766987 50,
328.11709293891140947366581314499145711 93,
292.99969137917773321697102373032860401 00,
358.64341560549160584868895756619490510 24,
299.89866205131980767611140944957417319 95,
360.06173465689426792203419058203650925 28,
336.59441031773883040042866061874728933 97,
256.10753185977436461680105438500672756 27,
324.65521223436968593318991268677203614 36,
331.93806791940770652341872393056726500 76, none, none,
289.54595772599879331503271067439421531 44]

1 --> 0  target = [17.93041369717435036866120886464269648554,
4.68650870184701003474771725402276358 5129,
353.30541094534750326225633768756729117 83]
one interval r = 20.73150479087102117475478780229927836591 ..
25.90675353503597776602809473179040036562
Time Approximations 0.03.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.6e-38
Equations at solution: [.2e-37, .46e-37, -.96e-35]Solution in 0.618s

Time Plot 0 s.
Exiting SolveHard() after 3.618r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863333220150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12,
375.73285288566296900744942625780766987 50,
328.11709293891140947366581314499145711 93,
292.99969137917773321697102373032860401 00,
358.64341560549160584868895756619490510 24,
299.89866205131980767611140944957417319 95,
360.06173465689426792203419058203650925 28,
336.59441031773883040042866061874728933 97,
256.10753185977436461680105438500672756 27,
324.65521223436968593318991268677203614 36,
331.93806791940770652341872393056726500 76,
304.79958324783777291044828871012952061 02, none,
289.54595772599879331503271067439421531 44]

2 --> 0  target = [17.93041369717435036866120886464269648554,
4.68650870184701003474771725402276358512 9,
353.30541094534750326225633768756729117 83]
one interval r = 31.37435486985588717749896724793330327018 ..
34.20127520020334871666518215014436397123
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
```

```
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .233e-34]Solution in 0.345s

Time Plot 0 s.
Exiting SolveHard() after 0.621r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493196593855910189876141178643 70,
441.64295972631242461935703111977877045 93,
436.91748165059238855969925230717973120 55,
422.98493396589246094022321713354155563 99,
361.52580255905122648003457702876280212 10,
401.88173903731118993826322689471932612 69,
389.59001515859169699584749215725500283 96,
328.46939892994944616150368874326527419 98,
401.50757157457072834501668347866808499 69,
358.97362823724325784122963787618360885 71,
398.33147102918557128522121460658019368 85,
371.48387394707789224117863332320150531 62,
336.61215840734383294633305218283889877 02,
361.50888346939400515641192702282340269 93,
324.67144992461671496830834163099678212 66,
302.31384314100876568534194899032907988 81,
328.46938513067442237734613867607809925 89,
343.81340625029278906949577813310822560 12,
375.73285288566296900744942625780766987 50,
328.11709293891140947366581314499145711 93,
292.99969137917773321697102373032860401 00,
358.64341560549160584868895756619490510 24,
299.89866205131980767611140944957417319 95,
360.06173465689426792203419058203650925 28,
336.59441031773883040042866061874728933 97,
256.10753185977436461680105438500672756 27,
324.65521223436968593318991268677203614 36,
331.93806791940770652341872393056726500 76,
304.79958324783777291044828871012952061 02,
323.46169176527884401650727472055724210 05,
289.54595772599879331503271067439421531 44]

Cascade time 240.807
counts: 28, 28

 Iteration 36

Start Generation 1
1 --> 0  target = [12.00000000010999875913815060085694357200,
6.21701250309577935047559619637805566 5123,
```

```
485.549080893534804462908236796714796930 0]
one interval r = 23.40850301641820294971018383274053018470 ..
27.67578046402923955436542458545720547374
Time Approximations 0.037.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.3e-38
Equations at solution: [-.1e-37, .53e-37, .10e-35]Solution in 0.997s

Time Plot 0 s.
Exiting SolveHard() after 4.378r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000010999875913815060085694357200,
6.217012503095779350475596196378055665123,
485.549080893534804462908236796714796930 0]
one interval r = 32.62814779204906083712697251618289007469 ..
36.10248389934870183845746449828945501259
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=6e-38
Equations at solution: [.9e-37, -.6e-37, .23e-35]Solution in 0.556s

Time Plot 0 s.
Exiting SolveHard() after 0.994r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
```

```
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684454696472792819246039568316781,
6.58343472178990502963811258933389131 7916,
467.78730595437286153341729825492584132 16]
one interval r = 32.41978955654737812941975828022042798382 ..
35.85152417364653449382327826912498141615
Time Approximations 0.02.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, .68e-35]Solution in 0.631s

Time Plot 0 s.
Exiting SolveHard() after 2.914r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74, none, none,
401.88173903802150935842343943775704305 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684454696472792819246039568316781,
6.58343472178990502963811258933389131 7916,
467.78730595437286153341729825492584132 16]
```

"Imaginary part neglected: ", $1.1031121148994399547873100307871269247 06 \times 10^{-17}$

```
two intervals r = 12.92327160844148147700388523750722667715 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000 or r = 18.39424858025745469622680615712284138627 ..
```

```
190000000000407390501016628071252075 09/100000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.4e-38
Equations at solution: [.5e-37, .54e-37, -.383e-35]Solution in 40.653s

Time Plot 0 s.
Exiting SolveHard() after 43.909r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53, none,
401.88173903802150935842343943775704305 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962817434744660369043523706124 54,
4.12565179696155870838596930022083878364 6,
440.67123064539942340968486781283446285 37]

"Imaginary part neglected: ", 1.10311211489943995478731003078712692 4706 × 10^-17
two intervals r = 14.35659705137590124762485200605281083736 ..
190000000000407390501016628071252075 09/100000000000000000000000000000000
00000 or r = 17.70352613796331066263914137720542144186 ..
190000000000407390501016628071252075 09/100000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
```

```
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.42e-37, -.2e-37, .543e-35]Solution in 3.211s

Time Plot 0 s.
Exiting SolveHard() after 4.289r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53, none,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962817434744660369043523706124 54,
4.12565179696155870838596930022083878364 6,
440.67123064539942340968486781283446285 37]
one interval r = 22.39761154357830351924001069062879796337 ..
27.23722351572371120236504398139790736284
Time Approximations 0.038.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.125 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064367710170567641970791436136573, rm =
14.37818770705454821172701167000798100869}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.05e-37
Equations at solution: [.1e-37, .105e-36, .469e-34]Solution in 11.544s

Time Plot 0 s.
Exiting SolveHard() after 12.456r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
```

```
441.642959727657379985129954243133678215 6,
436.917481647683630738193096396576952947 4,
422.984933971155786705205032796078580115 3,
361.525802555969310581291225367418927021 6,
401.881739038021509358423439437757043059 7,
389.590015152811455228594481158345525101 2, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887906361569564142354428815855 20,
4.004869081940198160831547202193867887045,
404.862245008753716301973329825684014035 2]
```

"Imaginary part neglected: ", $1.10311211489943995478731003078712692 4706 \times 10^{-17}$

```
two intervals r = 16.08011007776049841342702810162579224147 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000 or r = 16.41579812676295571048190934488644833526 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000
Time Approximations 0.058.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.18e-37, .1e-37, .5495e-34]Solution in 3.453s

Time Plot 0 s.
Exiting SolveHard() after 4.555r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434931852200045198958147499115073 3,
441.642959727657379985129954243133678215 6,
436.917481647683630738193096396576952947 4,
422.984933971155786705205032796078580115 3,
361.525802555969310581291225367418927021 6,
401.881739038021509358423439437757043059 7,
389.590015152811455228594481158345525101 2, none, none,
358.973628234327179891005630103372627298 4, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887906361569564142354428815855 20,
4.004869081940198160831547202193867887045,
```

```
404.8622450087537163019733298256840140352]
one interval r = 21.641943994093652089311739474717437022 72 ..
26.763306600173621821127178646169215300 18
Time Approximations 0.048.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.62e-35]Solution in 1.027s

Time Plot 0 s.
Exiting SolveHard() after 4.025r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92, none,
358.97362823432717989100563010337262729 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941797558335801170220065954005806,
5.5896371831241626304968216681794059941 90,
443.83065884178335558978563496060056469 15]
one interval r = 22.467253744708951204939280611995645116 60 ..
27.273884283297834235858615170153649349 45
Time Approximations 0.036.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
```

```
Equations at solution: [0., 0., .73e-35]Solution in 0.952s

Time Plot 0 s.
Exiting SolveHard() after 3.748r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92, none,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589417975583358011702200659540058 06,
5.5896371831241626304968216681794059941 90,
443.83065884178353558978563496060056469 15]
one interval r = 32.155752794939231327863836558067819185 33 ..
35.508722287309914920197777183297405807 93
Time Approximations 0.021.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .142e-34]Solution in 2.375s

Time Plot 0 s.
Exiting SolveHard() after 2.749r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
```

```
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136532470422776021963567605402117,
5.18778357866684140548034578723946190301 4,
408.65773862142268777956597401271454603 03]
one interval r = 21.71840114645705810754567897750176068684 ..
26.81849303485670099078957745221695124822
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.38e-37
Equations at solution: [-.2e-37, -.238e-36, -.15e-35]Solution in 3.004s

Time Plot 0 s.
Exiting SolveHard() after 4.095r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31, none, none,
361.50888346636023720488456652000682590 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136532470422776021963567605402117,
5.18778357866684140548034578723946190301 4,
408.65773862142268777956597401271454603 03]
```

```
one interval r = 31.80828598742788480403693416879838443972 ..
35.00011460036101585055484171093203593050
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .78e-35]Solution in 0.433s

Time Plot 0 s.
Exiting SolveHard() after 0.738r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08, none,
361.50888346636023720488456652000682590 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110515388750498440415922941943504,
6.196262565510346066868834739551891170316,
385.44474378825323948090647760390094716 37]
one interval r = 31.60836097528868683487995757327948145648 ..
34.66372795601111214988399040016460133956
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., -.143e-34]Solution in 0.559s

Time Plot 0 s.
Exiting SolveHard() after 0.839r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08, none,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110515388750498440415922941943504,
6.19626256551034606686883473955189117031 6,
385.44474378825323948090647760390094716 37]

"Imaginary part neglected: ", 1.10311211489943995478731003078712692 4706 × 10^-17
two intervals r = 16.87563408771330586489902875071788307729 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000 or r = 15.55640493779109498703676108653881528778 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.359e-37, 0., -.2303e-34]Solution in 1.159s

Time Plot 0 s.
```

```
Exiting SolveHard() after 4.47r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695 29474,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874752610904136634606346785294333,
4.8838107800086371277226226391767422835 65,
376.61967855298751398521595741813016139 42]
one interval r = 21.11001304879365938270027457413899950136 ..
26.31784243450395148082577153134754590494
Time Approximations 0.033.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=9.8e-38
Equations at solution: [.3e-37, .98e-37, .33e-35]Solution in 0.839s

Time Plot 0 s.
Exiting SolveHard() after 3.477r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695 29474,
422.98493397115578670520503279607858011 53,
```

```
361.525802555969310581291225367418927021 6,
401.881739038021509358423439437757043059 7,
389.590015152811455228594481158345525101 2,
328.469398930281888044283302743111419029 2,
401.507571575424422538924445843094332324 3,
358.973628234327179891005630103372627298 4,
398.331471037863591820118724697804492293 1,
371.483873937307844565767036232061862860 8,
336.612158408238284981501905652133243487 1,
361.508883466360237204884566520006825906 3,
324.671449921021381988993904723838823002 3, none,
328.469385131008899745619711095678502332 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747526109041366346063467852943 33,
4.883810780008637127722622639176742283565,
376.619678552987513985215957418130161394 2]
one interval r = 31.538994977031488636386404968488817696 44 ..
34.536183860842346555959438358059175257 53
Time Approximations 0.02.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.87e-36
Equations at solution: [-.144e-35, .187e-35, -.330e-34]Solution in
2.426s

Time Plot 0 s.
Exiting SolveHard() after 2.73r=34.0898 in [32.52213872 .. 34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434931852200045198958147499115073 3,
441.642959727657379985129954243133678215 6,
436.917481647683630738193096396576952947 4,
422.984933971155786705205032796078580115 3,
361.525802555969310581291225367418927021 6,
401.881739038021509358423439437757043059 7,
389.590015152811455228594481158345525101 2,
328.469398930281888044283302743111419029 2,
401.507571575424422538924445843094332324 3,
358.973628234327179891005630103372627298 4,
398.331471037863591820118724697804492293 1,
371.483873937307844565767036232061862860 8,
336.612158408238284981501905652133243487 1,
361.508883466360237204884566520006825906 3,
```

```
324.671449921021381988993904723838823 0023, none,
328.46938513100889974561971109567850 23329,
343.81340624263484391176468156061785 48641, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017518843220936566596267586706211,
6.0258135494664922279582412442467620 65786,
351.42702947966575218089781330713217 32390]
one interval r = 31.36230206106694072335647076797412682566 ..
34.17446640606409710854718491431442558198
Time Approximations 0.015.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.126e-34]Solution in 0.552s

Time Plot 0 s.
Exiting SolveHard() after 0.818r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.671449921021381988993904723838823 0023, none,
328.46938513100889974561971109567850 23329,
343.81340624263484391176468156061785 48641, none, none,
292.99969137883480013606279242764201454 80, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017518843220936566596267586706211,
6.0258135494664922279582412442467620 65786,
351.42702947966575218089781330713217 32390]
```

"Imaginary part neglected: ", $1.10311211489943995478731003078712692470 6 \times 10^{-17}$

```
two intervals r = 17.98135514455020328410458185078807188491 ..
190000000000407390501016628071252075 09/100000000000000000000000000000000
00000 or r = 13.84608015394018227332333296835454266162 ..
190000000000407390501016628071252075 09/100000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.73e-37, -.2e-37, -.130e-35]Solution in 1.121s

Time Plot 0 s.
Exiting SolveHard() after 4.378r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41, none, none,
292.99969137883480013606279242764201454 80, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941849727054255670479343374112894,
6.37794387406582638626791966560519183109 0,
423.28832783643232910421797879129569361 58]
one interval r = 31.94661817590235946774052990324565016491 ..
35.21212308645480855536627004300438766766
Time Approximations 0.019.
```

```
hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .71e-35]Solution in 2.514s

Time Plot 0 s.
Exiting SolveHard() after 2.854r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41, none, none,
292.99969137883480013606279242764201454 80, none, none,
360.06173466463960340655202602328504028 76, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941849727054255670479343374112894,
6.37794387406582638626791966560519183109 0,
423.28832783643232910421797879129569361 58]

"Imaginary part neglected: ", 1.10311211489943995478731003078712692 4706 × 10^-17
two intervals r = 15.22886702447156985323614611053897598024 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000 or r = 17.12965777064039780483914392922200022886 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000
Time Approximations 0.064.
```

```
hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [.15e-37, 0., .124e-35]Solution in 3.237s

Time Plot 0 s.
Exiting SolveHard() after 4.575r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41,
375.73285289919148093851846556337558332 05, none,
292.99969137883480013606279242764201454 80, none, none,
360.06173466463960340655202602328504028 76, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343343175837922726932418467080 24,
4.0035598156397596499710328120098133631 42,
404.47973593566917121646965909301792460 56]

"Imaginary part neglected: ", 1.1031121148994399547873100307871269247 06 × 10^-17
two intervals r = 16.096839663875461767440202797319159993 55 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000 or r = 16.399886490955382105521041465305424524 96 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000
Time Approximations 0.059.
```

```
hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.4351e-34]Solution in 3.433s

Time Plot 0 s.
Exiting SolveHard() after 4.529r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41,
375.73285289919148093851846556337558332 05, none,
292.99969137883480013606279242764201454 80,
358.64341560269986480192753709360550310 90, none,
360.06173466463960340655202602328504028 76, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234334317583792272693241846708024,
4.0035598156397596499710328120098133631 42,
404.47973593566917121646965909301792460 56]
one interval r = 21.63429629986405702943312398165104103426 ..
26.75768169866977218844263563189339466825
Time Approximations 0.049.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
```

```
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, .200e-34]Solution in 1.059s

Time Plot 0 s.
Exiting SolveHard() after 4.132r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41,
375.73285289919148093851846556337558332 05,
328.11709293937875833932977363894815923 02,
292.99969137883480013606279242764201454 80,
358.64341560269986480192753709360550310 90, none,
360.06173466463960340655202602328504028 76, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954433602012443280077669402577051,
6.19617723036392473676451913007522589449 2,
385.42734025201667796186760883847840522 09]
one interval r = 31.60822049084243576298470503309078554660 ..
34.66347615038789853323637322144296618856
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., .138e-34]Solution in 2.634s

Time Plot 0 s.
Exiting SolveHard() after 2.924r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41,
375.73285289919148093851846556337558332 05,
328.11709293937875833932977363894815923 02,
292.99969137883480013606279242764201454 80,
358.64341560269986480192753709360550310 90, none,
360.06173466463960340655202602328504028 76, none, none,
324.65521223082055570494356824992328247 19, none, none, none, none]

0 --> 1  target = [26.46318954433602012443280077669402577051,
6.19617723036392473676451913007522589449 2,
385.42734025201667796186760883847840522 09]
```

"Imaginary part neglected: ", $1.10311211489943995478731003078712692 4706 \times 10^{-17}$

```
two intervals r = 16.87629600309921442387177904212350810306 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000 or r = 15.55559000627718741400938299453483552175 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000
Time Approximations 0.063.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
```

```
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., .1726e-34]Solution in 3.174s

Time Plot 0 s.
Exiting SolveHard() after 4.34r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41,
375.73285289919148093851846556337558332 05,
328.11709293937875833932977363894815923 02,
292.99969137883480013606279242764201454 80,
358.64341560269986480192753709360550310 90, none,
360.06173466463960340655202602328504028 76,
336.59441031868377216013322611087255793 65, none,
324.65521223082055570494356824992328247 19, none, none, none, none]

0 --> 2  target = [34.49522661154037930470995938312339608235,
3.89713131605048465652481677088628792650 1,
373.78081883802558630608934951050101089 24]
```

"Imaginary part neglected: ", $1.10311211489943995478731003078712692 4706 \times 10^{-17}$

```
two intervals r = 17.29769086246152625628479945317254067591 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000 or r = 14.99436407398192905408964693025911589073 ..
19000000000040739050101662807125207509/10000000000000000000000000000000
00000
Time Approximations 0.082.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., .1358e-34]Solution in 1.194s

Time Plot 0 s.
Exiting SolveHard() after 4.724r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41,
375.73285289919148093851846556337558332 05,
328.11709293937875833932977363894815923 02,
292.99969137883480013606279242764201454 80,
358.64341560269986480192753709360550310 90, none,
360.06173466463960340655202602328504028 76,
336.59441031868377216013322611087255793 65, none,
324.65521223082055570494356824992328247 19,
331.93806790696815106777667011272565439 31, none, none, none]

1 --> 2  target = [34.49522661154037930470995938312339608235,
3.8971313160504846565248167708862879265 01,
373.78081883802558630608934951050101089 24]
one interval r = 21.06068473203073258380493337870011605746 ..
26.26979834256280369408290599375337616774
Time Approximations 0.044.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, -.396e-34]Solution in 0.784s

Time Plot 0 s.
Exiting SolveHard() after 3.697r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41,
375.73285289919148093851846556337558332 05,
328.11709293937875833932977363894815923 02,
292.99969137883480013606279242764201454 80,
358.64341560269986480192753709360550310 90,
299.89866204181164075423596589243205914 27,
360.06173466463960340655202602328504028 76,
336.59441031868377216013322611087255793 65, none,
324.65521223082055570494356824992328247 19,
331.93806790696815106777667011272565439 31, none, none, none]

0 --> 2  target = [33.81362495400058154895610580862944463653,
3.72564899368780980668212698336504483782 4,
325.89209972339241867002903674719183755 88]
```

"Imaginary part neglected: ", $1.10311211489943995478731003078712692 4706 \times 10^{-17}$

```
two intervals r = 18.552270490160034338127557314608018660 33 ..
190000000000407390501016628071252075 09/100000000000000000000000000000000
00000 or r = 12.491969357567046867962582963176583165 10 ..
190000000000407390501016628071252075 09/100000000000000000000000000000000
00000
```

```
Time Approximations 0.044.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=7e-38
Equations at solution: [-.174e-36, .7e-37, .3315e-34]Solution in 1.209s

Time Plot 0 s.
Exiting SolveHard() after 4.758r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41,
375.73285289919148093851846556337558332 05,
328.11709293937875833932977363894815923 02,
292.99969137883480013606279242764201454 80,
358.64341560269986480192753709360550310 90,
299.89866204181164075423596589243205914 27,
360.06173466463960340655202602328504028 76,
336.59441031868377216013322611087255793 65, none,
324.65521223082055570494356824992328247 19,
331.93806790696815106777667011272565439 31, none, none,
289.54595771902698400264624433488358192 84]

1 --> 2  target = [33.81362495400058154895610580862944463653,
3.72564899368780980668212698336504483782 4,
325.89209972339241867002903674719183755 88]
one interval r = 20.37468935116776451072785322753965368903 ..
25.37892165277733987367255355799212494134
```

```
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .319e-34]Solution in 0.581s

Time Plot 0 s.
Exiting SolveHard() after 3.17r=24.3395 in [22.07732228 .. 25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41,
375.73285289919148093851846556337558332 05,
328.11709293937875833932977363894815923 02,
292.99969137883480013606279242764201454 80,
358.64341560269986480192753709360550310 90,
299.89866204181164075423596589243205914 27,
360.06173466463960340655202602328504028 76,
336.59441031868377216013322611087255793 65,
256.10753185606469023474830663732592502 99,
324.65521223082055570494356824992328247 19,
331.93806790696815106777667011272565439 31, none, none,
289.54595771902698400264624433488358192 84]

1 --> 0  target = [17.93041369720689536152340062760117507074,
4.68650870215995039103429407885429386 2533,
353.30541094615640102574356734155292393 51]
one interval r = 20.73150479098091502039840053244887295028 ..
```

```
25.906753535002729441663728166058766062 48
Time Approximations 0.034.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.8e-38
Equations at solution: [.2e-37, .48e-37, .142e-34]Solution in 2.591s

Time Plot 0 s.
Exiting SolveHard() after 3.308r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41,
375.73285289919148093851846556337558332 05,
328.11709293937875833932977363894815923 02,
292.99969137883480013606279242764201454 80,
358.64341560269986480192753709360550310 90,
299.89866204181164075423596589243205914 27,
360.06173466463960340655202602328504028 76,
336.59441031868377216013322611087255793 65,
256.10753185606469023474830663732592502 99,
324.65521223082055570494356824992328247 19,
331.93806790696815106777667011272565439 31,
304.79958325227618912752095369097103348 86, none,
289.54595771902698400264624433488358192 84]

2 --> 0  target = [17.930413697206895361523400627601175070 74,
```

```
4.686508702159950391034294078854293862533,
353.30541094615640102574356734155292393 51]
one interval r = 31.374354869860535081843337163881257150 11 ..
34.201275200194279817636974470572652259 18
Time Approximations 0.016.

hint used Hint := [33.796329522691901201472146203888695425 08, 2, 1, -1,
17.863534069716629640044658426895081929 62, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.269e-34]Solution in 0.376s

Time Plot 0 s.
Exiting SolveHard() after 0.647r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493185220004519895814749911507 33,
441.64295972765737998512995424313367821 56,
436.91748164768363073819309639657695294 74,
422.98493397115578670520503279607858011 53,
361.52580255596931058129122536741892702 16,
401.88173903802150935842343943775704305 97,
389.59001515281145522859448115834552510 12,
328.46939893028188804428330274311141902 92,
401.50757157542442253892444584309433232 43,
358.97362823432717989100563010337262729 84,
398.33147103786359182011872469780449229 31,
371.48387393730784456576703623206186286 08,
336.61215840823828498150190565213324348 71,
361.50888346636023720488456652000682590 63,
324.67144992102138198899390472383882300 23,
302.31384314534390287193809860073354836 90,
328.46938513100889974561971109567850233 29,
343.81340624263484391176468156061785486 41,
375.73285289919148093851846556337558332 05,
328.11709293937875833932977363894815923 02,
292.99969137883480013606279242764201454 80,
358.64341560269986480192753709360550310 90,
299.89866204181164075423596589243205914 27,
360.06173466463960340655202602328504028 76,
336.59441031868377216013322611087255793 65,
256.10753185606469023474830663732592502 99,
324.65521223082055570494356824992328247 19,
331.93806790696815106777667011272565439 31,
304.79958325227618912752095369097103348 86,
```

```
323.46169176086183942064087015794203819 80,
289.54595771902698400264624433488358192 84]

Cascade time 149.28
counts: 28, 28

 Iteration 37

Start Generation 1
1 --> 0  target = [12.00000000006039800142464046637694742000,
6.21701250307767733501719597165515396318 1,
485.54908090046196552720032439815321663 85]
one interval r = 23.40850301650241603454035177202320565237 ..
27.67578046421120582725045579230933931783
Time Approximations 0.041.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=7.8e-38
Equations at solution: [-.2e-37, .78e-37, .480e-36]Solution in 3.686s

Time Plot 0 s.
Exiting SolveHard() after 4.82r=27.5236 in [25.56992694 .. 27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000006039800142464046637694742000,
6.21701250307767733501719597165515396318 1,
485.54908090046196552720032439815321663 85]
one interval r = 32.62814779215389845180998612483809910619 ..
36.10248388945091151663588358059881284502
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=8e-38
Equations at solution: [-.12e-36, .8e-37, -.146e-34]Solution in 0.6s

Time Plot 0 s.
Exiting SolveHard() after 1.014r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684473884614088976325254377577216,
6.58343472171609631523962067562452759064 7,
467.78730596145580919324550065632738756 21]
one interval r = 32.41978955665115225239041917499835083919 ..
35.85152417375328987199835299757823826911
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, .310e-34]Solution in 0.641s

Time Plot 0 s.
Exiting SolveHard() after 1.014r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01, none, none,
401.88173904450324490667511203934784662 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
0 --> 1  target = [27.52359684473884614088976325254377577216,
6.583434721716096315239620675624527590647,
467.78730596145580919324550065632738756 21]
two intervals r = 12.92327160833082843871987241490080929432 ..
189999999999814381290521999498534380 97/10000000000000000000000000000000
00000 or r = 18.39424858032507698727488132340991030290 ..
189999999999814381290521999498534380 97/10000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.5e-38
Equations at solution: [.4e-37, .55e-37, .29e-35]Solution in 38.158s

Time Plot 0 s.
Exiting SolveHard() after 41.445r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39, none,
401.88173904450324490667511203934784662 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962826793227315727544715254000695,
4.125651796853705739604287356710974821761,
440.67123065135376157314564749424862570 71]
two intervals r = 14.35659705127165779595418747345042834831 ..
189999999999814381290521999498534380 97/10000000000000000000000000000000
00000 or r = 17.70352613806818718090340189321533529299 ..
189999999999814381290521999498534380 97/10000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.15e-37, -.1e-37, -.207e-34]Solution in 1.325s

Time Plot 0 s.
Exiting SolveHard() after 4.203r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39, none,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962826793227315727544715254000695,
4.12565179685370573960428735671097482176 1,
440.67123065135376157314564749424862570 71]
one interval r = 22.39761154356675119858230010423659249012 ..
27.23722351591176090915532466606045564152
Time Approximations 0.037.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.222 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064381188282124884727279388306508, rm =
14.37818770487851002956552493706279549906}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.2e-38
Equations at solution: [.1e-37, .52e-37, -.7866e-35]Solution in 8.883s

Time Plot 0 s.
Exiting SolveHard() after 12.065r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888009855520004337786585839958 76,
4.004869081834610624437703435073157272793,
404.86224501518955475813463668345408009 06]
two intervals r = 16.080110077600777215324872906461474817 02 ..
189999999999814381290521999498534380 97/10000000000000000000000000000000
00000 or r = 16.415798126979005007861183580459221698 01 ..
189999999999814381290521999498534380 97/10000000000000000000000000000000
00000
Time Approximations 0.054.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.34e-37, 0., -.85e-35]Solution in 3.931s

Time Plot 0 s.
Exiting SolveHard() after 5.034r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73, none, none,
358.97362823984698758851003125666323645 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888009855520004337786585839958 76,
4.004869081834610624437703435073157272793,
```

```
404.86224501518955475813463668345408009 06]
one interval r = 21.641943994023581889586637643220746470 27 ..
26.763306600370199983568286434571051775 18
Time Approximations 0.051.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=5.2e-38
Equations at solution: [.1e-37, .52e-37, -.5961e-35]Solution in 1.043s

Time Plot 0 s.
Exiting SolveHard() after 4.114r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84, none,
358.97362823984698758851003125666323645 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941778733165921491291266100103984,
5.5896371831024370753522180312240038549 24,
443.83065884944345406205052465822810668 14]
one interval r = 22.467253744740752453573799732476556901 75 ..
27.273884283504901105705348102177204786 32
Time Approximations 0.034.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
```

```
Equations at solution: [0., 0., .6296e-35]Solution in 0.977s

Time Plot 0 s.
Exiting SolveHard() after 4.027r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84, none,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941778733165921491291266100103984,
5.58963718310243707535221803122400385492 4,
443.83065884944345406205052465822810668 14]
one interval r = 32.15575279504441652427569105972010683916 ..
35.50872228742773650729901063352594594772
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., .63e-35]Solution in 0.449s

Time Plot 0 s.
Exiting SolveHard() after 0.827r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
```

```
361.525802560676673921180844407571325641 1,
401.881739044503244906675112039347846622 6,
389.590015157863230375757014304227128697 3,
328.469398935394141363107956106971879208 4,
401.507571581924556896826479132830721934 1,
358.973628239846987588510031256663236459 6,
398.331471045347091099964429235341032826 4, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365226830983423360847632516520 83,
5.187783578605988770121732311018345048611,
408.657738626512466319319596610177717780 9]
one interval r = 21.718401146367335693867452939052175998 ..
26.818493035034030447790133867456301640 19
Time Approximations 0.059.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=5.2e-38
Equations at solution: [.1e-37, .52e-37, .6521e-35]Solution in 1.013s

Time Plot 0 s.
Exiting SolveHard() after 3.915r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434938596130719431039328547630273 4,
441.642959734235441168297404967947025406 1,
436.917481653697058873762483139909220680 1,
422.984933978621859826963390794779568373 9,
361.525802560676673921180844407571325641 1,
401.881739044503244906675112039347846622 6,
389.590015157863230375757014304227128697 3,
328.469398935394141363107956106971879208 4,
401.507571581924556896826479132830721934 1,
358.973628239846987588510031256663236459 6,
398.331471045347091099964429235341032826 4, none, none,
361.508883471078914405423481434126032457 5, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.911931365226830983423360847632516520 83,
5.187783578605988770121732311018345048611,
408.657738626512466319319596610177717780 9]
```

```
one interval r = 31.80828598750033609951070892732422757906 ..
35.00011460044473196042990117993269879474
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., -.81e-35]Solution in 0.414s

Time Plot 0 s.
Exiting SolveHard() after 2.933r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27, none,
361.50888347107891440542348143412603245 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110532798967018731063562704884908,
6.19626256543433958583523639333147612363 9,
385.44474379343940459322403549303154906 75]
one interval r = 31.60836097535752926247667725988308549622 ..
34.66372795609764091957080520109742089943
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, .52e-35]Solution in 0.55s

Time Plot 0 s.
Exiting SolveHard() after 0.822r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27, none,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110532798967018731063562704884908,
6.19626256543433958583523639333147612363 9,
385.44474379343940459322403549303154906 75]
two intervals r = 16.875634087598673083959714302413706513 79 ..
189999999999814381290521999498534380 97/100000000000000000000000000000000
00000 or r = 15.556404938004441372613449063838541864 50 ..
189999999999814381290521999498534380 97/100000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 8.083 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175122759660157946900593467333321, rm
= 2.33653277393777219439186904098295812478 5}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.1076e-36, 0., -.261e-34]Solution in 28.387s
```

```
Time Plot 0 s.
Exiting SolveHard() after 32.255r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874739623633705085184562649002421,
4.8838107799464515326306334530272785749 38,
376.61967855854974450643541890511604015 63]
one interval r = 21.11001304864998136642480261094224153790 ..
26.31784243468161584684956950898468237776
Time Approximations 0.032.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.16340e-34]Solution in 0.794s

Time Plot 0 s.
Exiting SolveHard() after 1.455r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
```

```
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77, none,
328.46938513612156645524663587388097276 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874739623633705085184562649002421,
4.8838107799464515326306334530272785749 38,
376.61967855854974450643541890511604015 63]
one interval r = 31.53899497710134887284013020641281165008 ..
34.53618386093473027149168858522285912772
Time Approximations 0.024.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=2.81e-36
Equations at solution: [.216e-35, -.281e-35, .201e-34]Solution in
2.429s

Time Plot 0 s.
Exiting SolveHard() after 2.701r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
```

```
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77, none,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017535662029304410735821564834711,
6.0258135493954407177316048421927094756 53,
351.42702948527900875013378337389685690 82]
one interval r = 31.362302061130906182173750408127569687 91 ..
34.174466406158003975448723805561929943 42
Time Approximations 0.015.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, -.76e-35]Solution in 0.543s

Time Plot 0 s.
Exiting SolveHard() after 0.807r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77, none,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89, none, none,
292.99969138380500624080641325100694576 84, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017535662029304410735821564834711,
```

```
6.025813549395440717731604842192709475653,
351.42702948527900875013378337389685690 82]
two intervals r = 17.98135514442359892763270952917526876986 ..
189999999999814381290521999498534380 97/100000000000000000000000000000000
00000 or r = 13.84608015426361574557569963696216437527 ..
189999999999814381290521999498534380 97/100000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.895 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.91357071367466302821224562013346597706, rm
= 2.7345009929060316183377976845210642257 34}});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [-.106e-36, .3e-37, .84e-35]Solution in 18.659s

Time Plot 0 s.
Exiting SolveHard() after 19.721r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89, none, none,
292.99969138380500624080641325100694576 84, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418714495014368687295502487892 24,
```

```
6.37794387400016494725510213511363694460 1,
423.28832784446031181519446130151647580 94]
one interval r = 31.94661817600597037710686460902201826312 ..
35.21212308657996344850898470032110652790
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.257e-34]Solution in 0.607s

Time Plot 0 s.
Exiting SolveHard() after 0.952r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89, none, none,
292.99969138380500624080641325100694576 84, none, none,
360.06173467194344381712329267479788869 77, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941871449501436868729550248789224,
6.37794387400016494725510213511363694460 1,
423.28832784446031181519446130151647580 94]
two intervals r = 15.22886702424566230034623327211557952308 ..
189999999999814381290521999498534380 97/100000000000000000000000000000000
00000 or r = 17.12965777086246276300854331097560212678 ..
189999999999814381290521999498534380 97/100000000000000000000000000000000
```

```
00000
Time Approximations 0.059.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.787 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541194392450922303620988616966 61, rm
= 2.0640682986020879523744042858743522199 17}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [.15e-37, 0., -.152e-34]Solution in 27.981s

Time Plot 0 s.
Exiting SolveHard() after 31.325r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89,
375.73285290749743890802217652009619248 78, none,
292.99969138380500624080641325100694576 84, none, none,
360.06173467194344381712329267479788869 77, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343446968485471548648806052263 02,
4.0035598155342439784613890023356931872 60,
404.47973594212385127127443525785217306 54]
two intervals r = 16.096839663714785212736590838347614244 34 ..
189999999999814381290521999498534380 97/10000000000000000000000000000000
```

```
00000 or r = 16.399886491173300814414761476360552684 36 ..
189999999999814381290521999498534380 97/100000000000000000000000000000000
00000
Time Approximations 0.063.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.221e-34]Solution in 1.623s

Time Plot 0 s.
Exiting SolveHard() after 5.479r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89,
375.73285290749743890802217652009619248 78, none,
292.99969138380500624080641325100694576 84,
358.64341560823671704539634935360662107 91, none,
360.06173467194344381712329267479788869 77, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234344696848547154864880605226302,
4.00355981553424397846138900233569318726 0,
404.47973594212385127127443525785217306 54]
one interval r = 21.63429629979358961706992349631674966032 ..
26.75768169886660985542086304416728389387
Time Approximations 0.05.
```

```
hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., .42719e-34]Solution in 1.044s

Time Plot 0 s.
Exiting SolveHard() after 4.287r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89,
375.73285290749743890802217652009619248 78,
328.11709294450797738857837837383468831 44,
292.99969138380500624080641325100694576 84,
358.64341560823671704539634935360662107 91, none,
360.06173467194344381712329267479788869 77, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954451030576874007259257612833895,
6.19617723028797675907584457723033381458 4,
385.42734025721448618456352302451917005 31]
one interval r = 31.608220490911368588323555076250914339 11 ..
34.663476150474596666137226752849654306 97
Time Approximations 0.018.

hint used Hint := [33.813392675100157147469018161182119369 13, 3, 1, 1,
11.783209121413521870719438174471899656 27, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .36e-35]Solution in 2.471s

Time Plot 0 s.
Exiting SolveHard() after 2.773r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89,
375.73285290749743890802217652009619248 78,
328.11709294450797738857837837383468831 44,
292.99969138380500624080641325100694576 84,
358.64341560823671704539634935360662107 91, none,
360.06173467194344381712329267479788869 77, none, none,
324.65521223543285727122926076218339474 26, none, none, none, none]

0 --> 1  target = [26.46318954451030576874007259257612833895,
6.19617723028797675907584457723033381458 4,
385.42734025721448618456352302451917005 31]
two intervals r = 16.87629600298413352881426125397385788404 ..
189999999999814381290521999498534380 97/100000000000000000000000000000000
00000 or r = 15.55559000649112405837819996285502755241 ..
189999999999814381290521999498534380 97/100000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.821 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852534443431556538174621283192454, rm
= 2.33669042800287394296588160817373120062 8}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.896e-37, 0., .234e-34]Solution in 28.306s

Time Plot 0 s.
Exiting SolveHard() after 29.456r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89,
375.73285290749743890802217652009619248 78,
328.11709294450797738857837837383468831 44,
292.99969138380500624080641325100694576 84,
358.64341560823671704539634935360662107 91, none,
360.06173467194344381712329267479788869 77,
336.59441032389420472099520306178374520 91, none,
324.65521223543285727122926076218339474 26, none, none, none, none]

0 --> 2  target = [34.49522661161309914882888173638293679189,
3.897131315937629046911836451347314440689,
373.78018884221597557965424928245411692 12]
two intervals r = 17.297690862378714138534285332009680367 78 ..
18999999999981438129052199949853438097/10000000000000000000000000000000
00000 or r = 14.994364074176372662055225993506128517 57 ..
18999999999981438129052199949853438097/10000000000000000000000000000000
00000
```

```
Time Approximations 0.084.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [0., 0., .210e-34]Solution in 3.314s

Time Plot 0 s.
Exiting SolveHard() after 7.265r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89,
375.73285290749743890802217652009619248 78,
328.11709294450797738857837837383468831 44,
292.99969138380500624080641325100694576 84,
358.64341560823671704539634935360662107 91, none,
360.06173467194344381712329267479788869 77,
336.59441032389420472099520306178374520 91, none,
324.65521223543285727122926076218339474 26,
331.93806791059065574315689907209320935 57, none, none, none]

1 --> 2  target = [34.49522661161309914882888173638293679189,
3.8971313159376290469118364513473144406 89,
373.78018884221597557965424928245411692 12]
one interval r = 21.06068473185774138006364933055081832836 ..
26.26979834271617870566455148591099283330
Time Approximations 0.033.
```

```
hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., .40913e-34]Solution in 0.783s

Time Plot 0 s.
Exiting SolveHard() after 1.466r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89,
375.73285290749743890802217652009619248 78,
328.11709294450797738857837837383468831 44,
292.99969138380500624080641325100694576 84,
358.64341560823671704539634935360662107 91,
299.89866204483603578830550532083983623 29,
360.06173467194344381712329267479788869 77,
336.59441032389420472099520306178374520 91, none,
324.65521223543285727122926076218339474 26,
331.93806791059065574315689907209320935 57, none, none, none]

0 --> 2  target = [33.81362495407936227536220317348771312183,
3.72564899357684490543280256031500648 2738,
325.89209972791123622282929361345130969 00]
two intervals r = 18.552270490067763594679526810645929443 77 ..
189999999999814381290521999498534380 97/10000000000000000000000000000000
00000 or r = 12.491969357887671374079694724164169942 87 ..
189999999999814381290521999498534380 97/10000000000000000000000000000000
```

```
00000
Time Approximations 0.041.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [.104e-36, -.5e-37, .53e-35]Solution in 1.099s

Time Plot 0 s.
Exiting SolveHard() after 4.787r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89,
375.73285290749743890802217652009619248 78,
328.11709294450797738857837837383468831 44,
292.99969138380500624080641325100694576 84,
358.64341560823671704539634935360662107 91,
299.89866204483603578830550532083983623 29,
360.06173467194344381712329267479788869 77,
336.59441032389420472099520306178374520 91, none,
324.65521223543285727122926076218339474 26,
331.93806791059065574315689907209320935 57, none, none,
289.54595772303459597585890827622523693 73]

1 --> 2  target = [33.81362495407936227536220317348771312183,
3.72564899357684490543280256031500648 2738,
325.89209972791123622282929361345130969 00]
one interval r = 20.37468935091447020570020796876287510171 ..
```

```
25.37892165290804515396215517216710890357
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=3e-38
Equations at solution: [.3e-37, .3e-37, .30694e-34]Solution in 0.558s

Time Plot 0 s.
Exiting SolveHard() after 3.134r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89,
375.73285290749743890802217652009619248 78,
328.11709294450797738857837837383468831 44,
292.99969138380500624080641325100694576 84,
358.64341560823671704539634935360662107 91,
299.89866204483603578830550532083983623 29,
360.06173467194344381712329267479788869 77,
336.59441032389420472099520306178374520 91,
256.10753185937463337532277230565190171 41,
324.65521223543285727122926076218339474 26,
331.93806791059065574315689907209320935 57, none, none,
289.54595772303459597585890827622523693 73]

1 --> 0  target = [17.93041369709048744222363017722593904202,
4.68650870209103327118140720088397275 0141,
```

```
353.30541095137174931663860057243032642 32]
one interval r = 20.731504790785785155304282917969650969 42 ..
25.906753535164529248133827061682611068 67
Time Approximations 0.034.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., -.1116e-35]Solution in 2.808s

Time Plot 0 s.
Exiting SolveHard() after 3.518r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89,
375.73285290749743890802217652009619248 78,
328.11709294450797738857837837383468831 44,
292.99969138380500624080641325100694576 84,
358.64341560823671704539634935360662107 91,
299.89866204483603578830550532083983623 29,
360.06173467194344381712329267479788869 77,
336.59441032389420472099520306178374520 91,
256.10753185937463337532277230565190171 41,
324.65521223543285727122926076218339474 26,
331.93806791059065574315689907209320935 57,
304.79958325696516439742564179465169879 80, none,
289.54595772303459597585890827622523693 73]
```

```
2 --> 0  target = [17.93041369709048744222363017722593904202,
4.68650870209103327118140720088397275014 1,
353.30541095137174931663860057243032642 32]
one interval r = 31.37435486992241924879029326261147546838 ..
34.20127520028246628157988250523728135170
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., .186e-34]Solution in 0.38s

Time Plot 0 s.
Exiting SolveHard() after 0.646r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493859613071943103932854763027 34,
441.64295973423544116829740496794702540 61,
436.91748165369705887376248313990922068 01,
422.98493397862185982696339079477956837 39,
361.52580256067667392118084440757132564 11,
401.88173904450324490667511203934784662 26,
389.59001515786323037575701430422712869 73,
328.46939893539414136310795610697187920 84,
401.50757158192455689682647913283072193 41,
358.97362823984698758851003125666323645 96,
398.33147104534709109996442923534103282 64,
371.48387394159129215236066817663259149 27,
336.61215841343690221894688856165131194 28,
361.50888347107891440542348143412603245 75,
324.67144992562283647225916108923196766 77,
302.31384315086127760146703928917028139 14,
328.46938513612156645524663587388097276 58,
343.81340624734490042276201691724772923 89,
375.73285290749743890802217652009619248 78,
328.11709294450797738857837837383468831 44,
292.99969138380500624080641325100694576 84,
358.64341560823671704539634935360662107 91,
299.89866204483603578830550532083983623 29,
360.06173467194344381712329267479788869 77,
336.59441032389420472099520306178374520 91,
256.10753185937463337532277230565190171 41,
324.65521223543285727122926076218339474 26,
331.93806791059065574315689907209320935 57,
```

```
304.79958325696516439742564179465169879 80,
323.46169176530270287358699207629096287 84,
289.54595772303459597585890827622523693 73]

Cascade time 238.324
counts: 28, 28

 Iteration 38

Start Generation 1
1 --> 0  target = [11.99999999995543309136207669663150889600,
6.21701250293349019144852014473918019250 2,
485.54908089899336179996446331136468593 76]
one interval r = 23.40850301652132949856747321512889437871 ..
27.67578046435953109766827992092955754760
Time Approximations 0.038.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.10e-37
Equations at solution: [.7e-37, -.210e-36, -.5e-36]Solution in 2.868s

Time Plot 0 s.
Exiting SolveHard() after 3.982r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999995543309136207669663150889600,
6.21701250293349019144852014473918019250 2,
485.54908089899336179996446331136468593 76]
one interval r = 32.62814779214895478579946439372652657872 ..
36.10248388940497663474191916018307429192
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
```

```
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.11e-35]Solution in 0.601s

Time Plot 0 s.
Exiting SolveHard() after 1.01r=35.4632 in [33.94922194 .. 36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684487311950136579697266187001384,
6.58343472162475020284025420467243729851 2,
467.78730595938475905845699381703593456 27]
one interval r = 32.41978955664255122990999722938628052152 ..
35.85152417370116841720834833505656050620
Time Approximations 0.023.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, .43e-35]Solution in 0.64s

Time Plot 0 s.
Exiting SolveHard() after 1.015r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32, none, none,
401.88173904457371797795238687090389338 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844873119501365796972661870013 84,
6.5834347216247502028402542046724372985 12,
467.78730595938475905845699381703593456 27]
two intervals r = 12.923271608283303110312954540748950355 47 ..
190000000000281449938673493771197160 37/100000000000000000000000000000000
00000 or r = 18.394248580342380498986630719775908362 39 ..
190000000000281449938673493771197160 37/100000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.1e-38
Equations at solution: [.2e-37, .21e-37, -.100e-35]Solution in 39.322s

Time Plot 0 s.
Exiting SolveHard() after 43.665r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95, none,
401.88173904457371797795238687090389338 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628262159742510688261100318489 71,
4.1256517969342992868015349260763452266 20,
440.67123065224225734956289103505104201 14]
two intervals r = 14.356597051107375626167329603828791081 65 ..
190000000000281449938673493771197160 37/100000000000000000000000000000000
00000 or r = 17.703526138152768972362526714508232048 88 ..
190000000000281449938673493771197160 37/100000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
```

```
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [.56e-37, .3e-37, -.456e-35]Solution in 1.318s

Time Plot 0 s.
Exiting SolveHard() after 4.165r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95, none,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962826215974251068826110031848971,
4.1256517969342992868015349260763452266 20,
440.67123065224225734956289103505104201 14]
one interval r = 22.397611543616219896199309257079954203 76 ..
27.237223516066112587828975540422669665 05
Time Approximations 0.037.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.236 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064394721950578509096321320737455, rm =
14.37818770499270686868175373017157680610}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.3e-38
Equations at solution: [0., -.53e-37, .628e-34]Solution in 8.696s

Time Plot 0 s.
Exiting SolveHard() after 11.482r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887993131959741056100473923894 89,
4.0048690819112860695599333033216038278 25,
404.86224501493547987372917798649095777 53]
two intervals r = 16.080110077544591247139332647652088725 64 ..
19000000000028144993867349377119716037/1000000000000000000000000000000
00000 or r = 16.415798127012601050465131392065124218 14 ..
19000000000028144993867349377119716037/1000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., .76e-36]Solution in 1.459s

Time Plot 0 s.
Exiting SolveHard() after 4.755r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85, none, none,
358.97362824253624081949069924283828754 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]
```

```
1 --> 2  target = [34.94507888799313195974105610047392389489,
4.004869081911286069559933303321603827825,
404.86224501493547987372917798649095777753]
one interval r = 21.641943994031327937584471262540284708 90 ..
26.763306600496597629778469587282794161 09
Time Approximations 0.05.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [-.2e-37, -.49e-37, .285e-34]Solution in 1.052s

Time Plot 0 s.
Exiting SolveHard() after 4.774r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34, none,
358.97362824253624081949069924283828754 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941784127930220733739564856157392,
5.589637182928535353828299459870707597108,
443.83065884604773692192445379090677249 87]
one interval r = 22.467253744697204892073702414459958987 31 ..
27.273884283611047939783392920262973287 23
Time Approximations 0.038.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
```

```
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, -.146e-34]Solution in 0.978s

Time Plot 0 s.
Exiting SolveHard() after 4.138r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34, none,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941784127930220733739564856157392,
5.5896371829285353538282994598707075971 08,
443.83065884604773692192445379090677249 87]
one interval r = 32.15575279502663582845654442707238394670 ..
35.50872228735981471638584054921126677535
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [-.6e-37, .5e-37, .46e-35]Solution in 0.461s

Time Plot 0 s.
Exiting SolveHard() after 2.678r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
```

```
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136498604996899765478305242643485,
5.18778357851032712679383955346421267019 0,
408.65773863026832766046802343770109442 51]
one interval r = 21.71840114645807601195808044598914646304 ..
26.81849303521971515586656809888187826932
Time Approximations 0.059.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.37e-37
Equations at solution: [-.2e-37, -.237e-36, .156e-34]Solution in 2.984s

Time Plot 0 s.
Exiting SolveHard() after 4.094r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18, none, none,
361.50888347426302917073561422599850823 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136498604996899765478305242643485,
```

```
5.187783578510327126793839553464212670190,
408.657738630268327660468023437701094425 1]
one interval r = 31.80828598755687196939137476101228599889 ..
35.00011460048556057176639843488872344299
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, .259e-34]Solution in 0.441s

Time Plot 0 s.
Exiting SolveHard() after 0.749r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57, none,
361.50888347426302917073561422599850823 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110550195558371137102009014628337,
6.196262565371604002498123801289694009323,
385.44474379663779218954287150786743096 06]
one interval r = 31.60836097540790005373231681080284796791 ..
34.66372795613391229478634002345315459687
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
```

```
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=8e-38
Equations at solution: [-.6e-37, .8e-37, -.5e-36]Solution in 0.552s

Time Plot 0 s.
Exiting SolveHard() after 0.83r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57, none,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110550195558371137102009014628337,
6.19626256537160400249812380128969400932 3,
385.44474379663779218954287150786743096 06]
two intervals r = 16.87563408743438751793573921537805516418 ..
19000000000028144993867349377119716037/10000000000000000000000000000000
00000 or r = 15.55640493818508240779644642545676564291 ..
19000000000028144993867349377119716037/10000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.538e-37, 0., .1640e-34]Solution in 2.985s

Time Plot 0 s.
Exiting SolveHard() after 6.384r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
```

```
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874726042444171596767770609928767,
4.8838107798373532112037951914298919624 99,
376.61967856148853656846427924197185525 43]
one interval r = 21.11001304870202522668938151583087384650 ..
26.31784243484958449707565152084336494967
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.3e-38
Equations at solution: [0., -.23e-37, -.145e-34]Solution in 0.846s

Time Plot 0 s.
Exiting SolveHard() after 1.546r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
```

```
389.590015161312497327222451669597574328 5,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65, none,
328.46938513835032130598719281106004969 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874726042444171596767770609928767,
4.88381077983735321120379519142989196249 9,
376.61967856148853656846427924197185525 43]
one interval r = 31.53899497714918021503340660470281523580 ..
34.53618386096852715934010585611984699194
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=7.84e-36
Equations at solution: [-.602e-35, .784e-35, -.344e-34]Solution in
2.403s

Time Plot 0 s.
Exiting SolveHard() after 2.682r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.590015161312497327222451669597574328 5,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65, none,
```

```
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017550354544581414393180955491324,
6.02581354932919066642726996042926093709 7,
351.42702948739720682758245248843052391 32]
one interval r = 31.36230206117181249657406549325197028412 ..
34.17446640618354455818648530514509219741
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .136e-34]Solution in 0.539s

Time Plot 0 s.
Exiting SolveHard() after 0.806r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65, none,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13, none, none,
292.99969138797133687699722889963149928 80, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017550354544581414393180955491324,
6.02581354932919066642726996042926093709 7,
351.42702948739720682758245248843052391 32]
two intervals r = 17.98135514436100873172098913691162305627 ..
```

```
190000000000281449938673493771197160 37/100000000000000000000000000000000
00000 or r = 13.846080154371326489486767056833337195 72 ..
190000000000281449938673493771197160 37/100000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=4e-38
Equations at solution: [-.125e-36, .4e-37, -.1978e-34]Solution in 1.15s

Time Plot 0 s.
Exiting SolveHard() after 4.017r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13, none, none,
292.99969138797133687699722889963149928 80, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418792754509754507314203340260 66,
6.377943873899112910229736270585173385161,
423.28832783998383409723407437294293849 01]
one interval r = 31.946618175982851414700860178210728643 50 ..
35.212123086499260337989252061307414664 73
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
```

```
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .103e-34]Solution in 0.586s

Time Plot 0 s.
Exiting SolveHard() after 0.935r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13, none, none,
292.99969138797133687699722889963149928 80, none, none,
360.06173466983403626912345656995685020 50, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418792754509754507314203340260 66,
6.3779438738991129102297362705851733851 61,
423.28832783998383409723407437294293849 01]
two intervals r = 15.22886702436961104533303551958529690347 ..
19000000000028144993867349377119716037/10000000000000000000000000000000
00000 or r = 17.12965777075474209440142072346738086131 ..
19000000000028144993867349377119716037/10000000000000000000000000000000
00000
Time Approximations 0.063.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
```

```
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [-.63e-37, -.2e-37, -.2278e-34]Solution in 3.41s

Time Plot 0 s.
Exiting SolveHard() after 6.762r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13,
375.73285290154605346640238320838257047 14, none,
292.99969138797133687699722889963149928 80, none, none,
360.06173466983403626912345656995685020 50, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234343566520856477283069351157648,
4.0035598156121839098353459450739890082 80,
404.47973594223946705230364104175733758 20]
two intervals r = 16.09683966364293328507676533870319592562 ..
19000000000028144993867349371197160 37/10000000000000000000000000000000
00000 or r = 16.39988649122204899777392707837659708731 ..
19000000000028144993867349371197160 37/10000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [-.17e-37, 0., .2591e-34]Solution in 3.359s

Time Plot 0 s.
Exiting SolveHard() after 4.446r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13,
375.73285290154605346640238320838257047 14, none,
292.99969138797133687699722889963149928 80,
358.64341561124674301131178830298682938 77, none,
360.06173466983403626912345656995685020 50, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343435665208564772830693511576 48,
4.0035598156121839098353459450739890082 80,
404.47973594223946705230364104175733758 20]
one interval r = 21.634296299808553926540927990905168543 84 ..
26.757681698998288943702690594885136959 89
Time Approximations 0.046.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.859e-34]Solution in 1.033s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.005r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13,
375.73285290154605346640238320838257047 14,
328.11709294708044818854438010476523036 97,
292.99969138797133687699722889963149928 80,
358.64341561124674301131178830298682938 77, none,
360.06173466983403626912345656995685020 50, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954468252964599447217002291808140,
6.19617723022471517503276250813093801111 6,
385.42734026030539215021234771086036493 70]
one interval r = 31.60822049096087079977698906332467482146 ..
34.66347615050931547854696880793261702991
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, -.239e-34]Solution in 0.55s

Time Plot 0 s.
Exiting SolveHard() after 0.857r=33.8134 in [32.62668594 ..
34.66347615]
```

```
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13,
375.73285290154605346640238320838257047 14,
328.11709294708044818854438010476523036 97,
292.99969138797133687699722889963149928 80,
358.64341561124674301131178830298682938 77, none,
360.06173466983403626912345656995685020 50, none, none,
324.65521224045995379991144151624171431 00, none, none, none, none]

0 --> 1  target = [26.46318954468252964599447217002291808140,
6.19617723022471517503276250813093801111 6,
385.42734026030539215021234771086036493 70]
two intervals r = 16.87629600282397390705241899510444178692 ..
19000000000028144993867349377119716037/10000000000000000000000000000000
00000 or r = 15.55559000666673172241044021918819616351 ..
19000000000028144993867349377119716037/10000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [.717e-37, -.1e-37, .438e-35]Solution in 1.153s

Time Plot 0 s.
Exiting SolveHard() after 4.233r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
```

```
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13,
375.73285290154605346640238320838257047 14,
328.11709294708044818854438010476523036 97,
292.99969138797133687699722889963149928 80,
358.64341561124674301131178830298682938 77, none,
360.06173466983403626912345656995685020 50,
336.59441032578253537592768648155535862 55, none,
324.65521224045995379991144151624171431 00, none, none, none, none]

0 --> 2  target = [34.49522661170475527551440157647776171971,
3.8971313160394254402666335486293688538 21,
373.78081884913905603632102121957367597 27]
two intervals r = 17.297690862112137216897306759642016838 62 ..
19000000000028144993867349377119716037/10000000000000000000000000000000
00000 or r = 14.994364074539485050813921304936891439 97 ..
19000000000028144993867349377119716037/10000000000000000000000000000000
00000
Time Approximations 0.087.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.109e-36, 0., .856e-35]Solution in 2.989s

Time Plot 0 s.
Exiting SolveHard() after 6.618r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13,
375.73285290154605346640238320838257047 14,
328.11709294708044818854438010476523036 97,
292.99969138797133687699722889963149928 80,
358.64341561124674301131178830298682938 77, none,
360.06173466983403626912345656995685020 50,
336.59441032578253537592768648155535862 55, none,
324.65521224045995379991144151624171431 00,
331.93806791965469140997582052705290097 17, none, none, none]

1 --> 2  target = [34.49522661170475527551440157647776171971,
3.8971313160394254402666335486293688538 21,
373.78081884913905603632102121957367597 27]
one interval r = 21.06068473197640736927786147582344245306 ..
26.26979834295137484847364927998862528964
Time Approximations 0.037.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [-.2e-37, -.5e-37, -.564e-34]Solution in 2.881s

Time Plot 0 s.
Exiting SolveHard() after 3.604r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13,
375.73285290154605346640238320838257047 14,
328.11709294708044818854438010476523036 97,
292.99969138797133687699722889963149928 80,
358.64341561124674301131178830298682938 77,
299.89866205365172911768714242371649920 33,
360.06173466983403626912345656995685020 50,
336.59441032578253537592768648155535862 55, none,
324.65521224045995379991144151624171431 00,
331.93806791965469140997582052705290097 17, none, none, none]

0 --> 2  target = [33.81362495414745064080064434029941069375,
3.72564899367227587023683831841821217868 3,
325.89209973283146214227451526920003295 60]
two intervals r = 18.552270489996158154818066844151432509 82 ..
19000000000028144993867349377119716037/10000000000000000000000000000000000
00000 or r = 12.491969358103909624862589257518914173 99 ..
19000000000028144993867349377119716037/10000000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [.138e-36, -.6e-37, .244e-35]Solution in 3.233s

Time Plot 0 s.
Exiting SolveHard() after 4.627r=18.8546 in [18.55227050 .. 19]
```

```
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13,
375.73285290154605346640238320838257047 14,
328.11709294708044818854438010476523036 97,
292.99969138797133687699722889963149928 80,
358.64341561124674301131178830298682938 77,
299.89866205365172911768714242371649920 33,
360.06173466983403626912345656995685020 50,
336.59441032578253537592768648155535862 55, none,
324.65521224045995379991144151624171431 00,
331.93806791965469140997582052705290097 17, none, none,
289.54595773037887981782955709793411196 97]

1 --> 2  target = [33.81362495414745064080064434029941069375,
3.72564899367227587023683831841821217868 3,
325.89209973283146214227451526920003295 60]
one interval r = 20.37468935095216707351706657092003445439 ..
25.37892165310284594079145980147676484684
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, -.405e-34]Solution in 0.565s

Time Plot 0 s.
Exiting SolveHard() after 1.088r=24.3395 in [22.07732228 ..
```

```
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13,
375.73285290154605346640238320838257047 14,
328.11709294708044818854438010476523036 97,
292.99969138797133687699722889963149928 80,
358.64341561124674301131178830298682938 77,
299.89866205365172911768714242371649920 33,
360.06173466983403626912345656995685020 50,
336.59441032578253537592768648155535862 55,
256.10753186632367145412546489661521460 33,
324.65521224045995379991144151624171431 00,
331.93806791965469140997582052705290097 17, none, none,
289.54595773037887981782955709793411196 97]

1 --> 0  target = [17.93041369702216760636323788607226857142,
4.68650870197423202386974170116450763023 0,
353.30541095358286935850400548100863964 20]
one interval r = 20.73150479081031417979055158425060051127 ..
25.90675353531373685016941727603305650772
Time Approximations 0.03.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., .155e-34]Solution in 0.641s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.52r=25.4021 in [22.67806074 .. 25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13,
375.73285290154605346640238320838257047 14,
328.11709294708044818854438010476523036 97,
292.99969138797133687699722889963149928 80,
358.64341561124674301131178830298682938 77,
299.89866205365172911768714242371649920 33,
360.06173466983403626912345656995685020 50,
336.59441032578253537592768648155535862 55,
256.10753186632367145412546489661521460 33,
324.65521224045995379991144151624171431 00,
331.93806791965469140997582052705290097 17,
304.79958325840493799547985632686362373 67, none,
289.54595773037887981782955709793411196 97]

2 --> 0  target = [17.93041369702216760636323788607226857142,
4.68650870197423202386974170116450763023 0,
353.30541095358286935850400548100863964 20]
one interval r = 31.37435486996399465192465051382486374661 ..
34.20127520030907456621363484382769346467
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
```

```
Equations at solution: [.1e-37, -.3e-37, .40e-35]Solution in 0.342s

Time Plot 0 s.
Exiting SolveHard() after 0.617r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493704639553297012998031739486 93,
441.64295973241128004114499431493676734 01,
436.91748165488523988516645940967421938 32,
422.98493397526757156978916637281136918 95,
361.52580256396528222422075981766738662 12,
401.88173904457371797795238687090389338 29,
389.59001516131249732722245166959757432 85,
328.46939893762612380210327676530802789 34,
401.50757158235666151879207435852848827 65,
358.97362824253624081949069924283828754 69,
398.33147104117664732337732062981732420 18,
371.48387394867997492856383594714175449 57,
336.61215841543487203736546128478769121 99,
361.50888347426302917073561422599850823 10,
324.67144993075019383946606041337774408 65,
302.31384315174049306798183488049838830 65,
328.46938513835032130598719281106004969 44,
343.81340625410081778746357467072779306 13,
375.73285290154605346640238320838257047 14,
328.11709294708044818854438010476523036 97,
292.99969138797133687699722889963149928 80,
358.64341561124674301131178830298682938 77,
299.89866205365172911768714242371649920 33,
360.06173466983403626912345656995685020 50,
336.59441032578253537592768648155535862 55,
256.10753186632367145412546489661521460 33,
324.65521224045995379991144151624171431 00,
331.93806791965469140997582052705290097 17,
304.79958325840493799547985632686362373 67,
323.46169177159625081353115138115065431 49,
289.54595773037887981782955709793411196 97]

Cascade time 144.15
counts: 28, 28

 Iteration 39

Start Generation 1
1 --> 0  target = [12.00000000016583361598964045854155191200,
6.21701250286170985868823867482301538848 9,
485.54908089712438881179961022641286061 57]
one interval r = 23.40850301645757837569816266707946442560 ..
27.67578046411576036581710234125343759437
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
```

```
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.05e-37
Equations at solution: [.3e-37, -.105e-36, .3e-36]Solution in 3.053s

Time Plot 0 s.
Exiting SolveHard() after 4.207r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.000000000165833615989640458541551912 00,
6.217012502861709858688238674823015388489,
485.549080897124388811799610226412860615 7]
one interval r = 32.628147792143131889110693707537502404 45 ..
36.102483889443431848940246763453750018 93
Time Approximations 0.023.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .77e-35]Solution in 0.622s

Time Plot 0 s.
Exiting SolveHard() after 1.064r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
```

```
441.642959730469099701747700333426204862 6,
436.91748165375614150938133088464593474 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684463308722362025294479380044880,
6.58343472162821228731424362463637071355 9,
467.78730595741542051182535377780339668 24]
one interval r = 32.41978955663376595905924642296977872762 ..
35.85152417373432142343451709692233365595
Time Approximations 0.019.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=4e-38
Equations at solution: [.6e-37, -.4e-37, .225e-34]Solution in 0.622s

Time Plot 0 s.
Exiting SolveHard() after 3.175r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.642959730469099701747700333426204862 6,
436.91748165375614150938133088464593474 08, none, none,
401.88173904303024547594122533894021544 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684463308722362025294479380044880,
6.58343472162821228731424362463637071355 9,
467.78730595741542051182535377780339668 24]
two intervals r = 12.92327160847676159294567556123804862407 ..
9499999999969541755232771274693752419/50000000000000000000000000000000
000 or r = 18.39424858021121897594221235477789652793 ..
9499999999969541755232771274693752419/50000000000000000000000000000000
000
Time Approximations 0.039.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
```

```
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.3e-38
Equations at solution: [.5e-37, .53e-37, .478e-35]Solution in 39.325s

Time Plot 0 s.
Exiting SolveHard() after 42.624r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50, none,
401.88173904303024547594122533894021544 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628308440791970288021838089223 78,
4.1256517969997146671930964234053931117 95,
440.67123065158754665739854190103664542 11]
two intervals r = 14.356597051187767223585284997329774560 75 ..
9499999999969541755232771274693752419/50000000000000000000000000000000
000 or r = 17.703526138050787833282306106271702281 34 ..
9499999999969541755232771274693752419/50000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.28e-37, .1e-37, -.5590e-34]Solution in 3.236s

Time Plot 0 s.
Exiting SolveHard() after 4.312r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50, none,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962830844079197028802183808922378,
4.1256517969997146671930964234053931117 95,
440.67123065158754665739854190103664542 11]
one interval r = 22.397611543636541361004759545675545669 94 ..
27.237223515850651322549153450357244693 86
Time Approximations 0.037.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.407 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643786817113317620904806342223 17, rm =
14.378187705803477507154040676473544323 92}});
Accepted {r=26.4635, rm=16.5329} with Delta=7.9e-38
Equations at solution: [.1e-37, .79e-37, .22e-35]Solution in 11.171s

Time Plot 0 s.
Exiting SolveHard() after 12.075r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888802603197919068669024250288668,
4.0048690819754594739861858254551160872 80,
```

```
404.86224501385794580422191101199123794 39]
two intervals r = 16.08011007759002278745903768646266160059 ..
94999999999695417552327712746937524 19/50000000000000000000000000000000000
000 or r = 16.41579812690636756940217396578334163129 ..
94999999999695417552327712746937524 19/50000000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.69e-37, -.1e-37, -.1278e-34]Solution in
3.711s

Time Plot 0 s.
Exiting SolveHard() after 4.744r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86, none, none,
358.97362824312742963799516946103119044 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888802603197919068669024250288668,
4.00486908197545947398618582545511608728 0,
404.86224501385794580422191101199123794 39]
one interval r = 21.64194399408889328150910062105621573392 ..
26.76330660029516501245072298055662456456
Time Approximations 0.048.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
```

```
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.87e-35]Solution in 1.002s

Time Plot 0 s.
Exiting SolveHard() after 4.984r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33, none,
358.97362824312742963799516946103119044 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941803822570380612334886509121618,
5.5896371828600430198921403237282646797 72,
443.83065884325335245208127750838957329 00]
one interval r = 22.46725374466619193338724924007139250344 ..
27.27388428336948608966646046422111108952
Time Approximations 2.032.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [-.1e-37, .81e-37, -.55e-35]Solution in 0.96s

Time Plot 0 s.
Exiting SolveHard() after 3.899r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
```

```
422.984933972737311906536943970453480435 0,
361.525802564356974187498113578266716794 4,
401.881739043030245475941225338940215447 6,
389.590015162158121162517621711958946878 6,
328.469398937665456138608770881918233863 3, none,
358.973628243127429637995169461031190449 9,
398.331471037867701391119511442653338705 2, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589418038225703806123348865091216 18,
5.589637182860043019892140323728264679772,
443.830658843253352452081277508389573290 0]
one interval r = 32.155752795007164508067676659605791277 11 ..
35.508722287375950658975131944649446738 77
Time Approximations 0.02.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., .129e-34]Solution in 0.448s

Time Plot 0 s.
Exiting SolveHard() after 0.827r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434935435737488730437194351713601 3,
441.642959730469099701747700333426204862 6,
436.917481653756141509381330884645934740 8,
422.984933972737311906536943970453480435 0,
361.525802564356974187498113578266716794 4,
401.881739043030245475941225338940215447 6,
389.590015162158121162517621711958946878 6,
328.469398937665456138608770881918233863 3,
401.507571580852815215718056982422448818 9,
358.973628243127429637995169461031190449 9,
398.331471037867701391119511442653338705 2, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931364954198184130188514396009461 23,
5.187783578485691208120028800981445118948,
408.657738631042261844520033869438408332 5]
one interval r = 21.718401146548291606155153523961374061 13 ..
```

```
26.81849303504268391274607607437859254976
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.59e-37
Equations at solution: [-.2e-37, -.159e-36, .215e-34]Solution in 0.992s

Time Plot 0 s.
Exiting SolveHard() after 4.278r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52, none, none,
361.50888347461910035081526222431755843 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136495419818413018851439600946123,
5.18778357848569120812002880098144511894 8,
408.65773863104226184452003386943840833 25]
one interval r = 31.80828598756925799084424727887806908832 ..
35.00011460054606822994643570303551203708
Time Approximations 0.027.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
```

```
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.124e-34]Solution in 2.989s

Time Plot 0 s.
Exiting SolveHard() after 3.315r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77, none,
361.50888347461910035081526222431755843 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110533684341880248447047035549377,
6.19626256538014830816946286379680238951 6,
385.44474379704413766050030213341802943 87]
one interval r = 31.60836097541374977110989804073006850970 ..
34.66372795618443244806383303760406865117
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., -.219e-34]Solution in 0.577s

Time Plot 0 s.
Exiting SolveHard() after 0.879r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77, none,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110533684341880248447047035549377,
6.19626256538014830816946286379680238951 6,
385.44474379704413766050030213341802943 87]
two intervals r = 16.87563408739568308490868894315338632280 ..
94999999999695417552327712746937524 19/50000000000000000000000000000000000
000 or r = 15.55640493815969134068389136062995120843 ..
94999999999695417552327712746937524 19/50000000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 8.133 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175108880640005874783282299275616, rm
= 2.33653277407710580406641775027306813695 1}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.474e-35]Solution in 28.263s

Time Plot 0 s.
Exiting SolveHard() after 31.546r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
```

```
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874721081556631207175675250334733,
4.88381077980773742211255143688353008218 7,
376.61967856199025283168955788381093837 00]
one interval r = 21.11001304882267222268938901617266026747 ..
26.31784243469310701920147905380164188536
Time Approximations 0.038.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=4.9e-38
Equations at solution: [.1e-37, .49e-37, -.46e-35]Solution in 0.83s

Time Plot 0 s.
Exiting SolveHard() after 1.517r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54, none,
328.46938513838831386301633834145560649 79, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874721081556631207175675250334733,
4.883810779807737422112551436883530082187,
376.61967856199025283168955788381093837 00]
one interval r = 31.53899497715463789026415833061246522578 ..
34.53618386101865226194371795865539466365
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=7.44e-36
Equations at solution: [.571e-35, -.744e-35, .74e-35]Solution in 2.687s

Time Plot 0 s.
Exiting SolveHard() after 2.965r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54, none,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017536014515152290057202438691510,
6.025813549333785582081372958575707349502,
351.42702948743346316755613813788062088 59]
one interval r = 31.36230206117126878985930326576883842062 ..
34.17446640622196698955335674177260778879
Time Approximations 0.016.
```

```
hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .241e-34]Solution in 0.541s

Time Plot 0 s.
Exiting SolveHard() after 0.801r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54, none,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65, none, none,
292.99969138844188528463437990871092908 54, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017536014515152290057202438691510,
6.02581354933378558208137295857570734950 2,
351.42702948743346316755613813788062088 59]
two intervals r = 17.98135514430683865866246007162550247310 ..
94999999999695417552327712746937524 19/50000000000000000000000000000000
000 or r = 13.84608015437032355163703162547564612474 ..
94999999999695417552327712746937524 19/50000000000000000000000000000000
000
Time Approximations 2.042.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
```

```
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 5.277 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713604198350803508980443962554 17, rm
= 2.7345009931702200442969856678678603884 27}});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [0., 0., -.90e-35]Solution in 17.527s

Time Plot 0 s.
Exiting SolveHard() after 20.58r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65, none, none,
292.99969138844188528463437990871092908 54, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418550453788802978858915108788 74,
6.3779438738923140954496832629230283612 12,
423.28832783659594735539812374723607156 36]
one interval r = 31.946618175956985535371345374874840412 23 ..
35.212123086502438600776231711631583446 41
Time Approximations 0.018.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.39e-35]Solution in 0.608s

Time Plot 0 s.
Exiting SolveHard() after 0.947r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65, none, none,
292.99969138844188528463437990871092908 54, none, none,
360.06173466702333607993320496968841093 11, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418550453788802978858915108788 74,
6.3779438738923140954496832629230283612 12,
423.28832783659594735539812374723607156 36]
two intervals r = 15.228867024555466536546715626145996232 24 ..
9499999999969541755232771274693752419/50000000000000000000000000000000
000 or r = 17.129657770559506844522325180072731100 85 ..
9499999999969541755232771274693752419/50000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
```

```
in partial time = 7.584 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.5153705413537094117552067793787365265 6, rm
= 2.0640682987467319834621495123351031734 38}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.31e-37, -.1e-37, -.1470e-34]Solution in
28.055s

Time Plot 0 s.
Exiting SolveHard() after 31.396r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65,
375.73285289751309194462972595589005357 20, none,
292.99969138844188528463437990871092908 54, none, none,
360.06173466702333607993320496968841093 11, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343469076095896397020312654942 35,
4.0035598156764975380846983030221487834 26,
404.47973594120244816397032691935067782 72]
two intervals r = 16.0968396636860247913356471893257322422 1 ..
9499999999969541755232771274693752419/5000000000000000000000000000000000
000 or r = 16.3998864911176758213240881466213892730 0 ..
9499999999969541755232771274693752419/5000000000000000000000000000000000
000
Time Approximations 0.051.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.35e-37, .1e-37, .623e-35]Solution in 3.768s

Time Plot 0 s.
Exiting SolveHard() after 6.862r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65,
375.73285289751309194462972595589005357 20, none,
292.99969138844188528463437990871092908 54,
358.64341561187384085373111624544618641 36, none,
360.06173466702333607993320496968841093 11, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234346907609589639702031265494235,
4.0035598156764975380846983030221487834 26,
404.47973594120244816397032691935067782 72]
one interval r = 21.63429629986742081694047235582850518385 ..
26.75768169879768147629911279808727584345
Time Approximations 0.051.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.2e-38
Equations at solution: [-.1e-37, -.52e-37, .436e-34]Solution in 3.11s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.121r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65,
375.73285289751309194462972595589005357 20,
328.11709294715746593637647633953847516 87,
292.99969138844188528463437990871092908 54,
358.64341561187384085373111624544618641 36, none,
360.06173466702333607993320496968841093 11, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954451683820515681252283758470183,
6.19617723023307869150172613714070664307 0,
385.42734026067509288081670975917119479 76]
one interval r = 31.60822049096642246974693878423643877379 ..
34.66347615055930193883233728999161088967
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., .349e-34]Solution in 0.563s

Time Plot 0 s.
Exiting SolveHard() after 0.875r=33.8134 in [32.62668594 ..
34.66347615]
```

```
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65,
375.73285289751309194462972595589005357 20,
328.11709294715746593637647633953847516 87,
292.99969138844188528463437990871092908 54,
358.64341561187384085373111624544618641 36, none,
360.06173466702333607993320496968841093 11, none, none,
324.65521224120965589516714413570202555 69, none, none, none, none]

0 --> 1  target = [26.46318954451683820515681252283758470183,
6.19617723023307869150172613714070664307 0,
385.42734026067509288081670975917119479 76]
two intervals r = 16.87629600278664752358716260301990771945 ..
94999999999695417552327712746937524 19/50000000000000000000000000000000000
000 or r = 15.55559000663964373804780644731516323911 ..
94999999999695417552327712746937524 19/50000000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.996 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852520837943669770185221584936332, rm
= 2.33669042814355093225039870566098513525 5}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
```

```
Equations at solution: [-.179e-37, 0., -.1271e-34]Solution in 28.824s

Time Plot 0 s.
Exiting SolveHard() after 29.974r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65,
375.73285289751309194462972595589005357 20,
328.11709294715746593637647633953847516 87,
292.99969138844188528463437990871092908 54,
358.64341561187384085373111624544618641 36, none,
360.06173466702333607993320496968841093 11,
336.59441032569511671259082621056567872 85, none,
324.65521224120965589516714413570202555 69, none, none, none, none]

0 --> 2  target = [34.49522661178101348259495273755045055685,
3.8971313161157527681640127663438363319 19,
373.78018885149261142885505063412300695 74]
two intervals r = 17.29769086199680734478159581076381403166 ..
94999999999695417552327712746937524 19/50000000000000000000000000000000000
000 or r = 14.99436407462398066072710255829600348063 ..
94999999999695417552327712746937524 19/50000000000000000000000000000000000
000
Time Approximations 0.086.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
```

```
Equations at solution: [.36e-37, -.1e-37, .1762e-34]Solution in 3.236s

Time Plot 0 s.
Exiting SolveHard() after 7.014r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65,
375.73285289751309194462972595589005357 20,
328.11709294715746593637647633953847516 87,
292.99969138844188528463437990871092908 54,
358.64341561187384085373111624544618641 36, none,
360.06173466702333607993320496968841093 11,
336.59441032569511671259082621056567872 85, none,
324.65521224120965589516714413570202555 69,
331.93806792331323556540535560724295004 56, none, none, none]

1 --> 2  target = [34.49522661178101348259495273755045055685,
3.89713131611575276816401276634383633191 9,
373.78081885149261142885505063412300695 74]
one interval r = 21.06068473213190793213621711605944775173 ..
26.26979834282876266903352831246795540114
Time Approximations 0.033.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [-.3e-37, -.7e-37, -.206e-34]Solution in 2.686s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.368r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65,
375.73285289751309194462972595589005357 20,
328.11709294715746593637647633953847516 87,
292.99969138844188528463437990871092908 54,
358.64341561187384085373111624544618641 36,
299.89866205686548149537895230485986314 58,
360.06173466702333607993320496968841093 11,
336.59441032569511671259082621056567872 85, none,
324.65521224120965589516714413570202555 69,
331.93806792331323556540535560724295004 56, none, none, none]

0 --> 2  target = [33.81362495419863376163006812552205109062,
3.72564899374535810392425097528877676335 8,
325.89209973413617715829743019944032485 47]
two intervals r = 18.552270489903058122684771847774075674 66 ..
94999999999695417552327712746937524 19/50000000000000000000000000000000
000 or r = 12.491969358210267882424169849670148731 29 ..
94999999999695417552327712746937524 19/50000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [-.87e-37, .3e-37, .130e-34]Solution in 3.273s

Time Plot 0 s.
Exiting SolveHard() after 4.681r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65,
375.73285289751309194462972595589005357 20,
328.11709294715746593637647633953847516 87,
292.99969138844188528463437990871092908 54,
358.64341561187384085373111624544618641 36,
299.89866205686548149537895230485986314 58,
360.06173466702333607993320496968841093 11,
336.59441032569511671259082621056567872 85, none,
324.65521224120965589516714413570202555 69,
331.93806792331323556540535560724295004 56, none, none,
289.54595773317886703206823693241751063 69]

1 --> 2  target = [33.81362495419863376163006812552205109062,
3.72564899374535810392425097528877676335 8,
325.89209973413617715829743019944032485 47]
one interval r = 20.37468935112573165196311826414835903727 ..
25.37892165300833928674111650933002810458
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
```

```
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, .336e-34]Solution in 0.591s

Time Plot 0 s.
Exiting SolveHard() after 1.134r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65,
375.73285289751309194462972595589005357 20,
328.11709294715746593637647633953847516 87,
292.99969138844188528463437990871092908 54,
358.64341561187384085373111624544618641 36,
299.89866205686548149537895230485986314 58,
360.06173466702333607993320496968841093 11,
336.59441032569511671259082621056567872 85,
256.10753186858904796060466758474619935 27,
324.65521224120965589516714413570202555 69,
331.93806792331323556540535560724295004 56, none, none,
289.54595773317886703206823693241751063 69]

1 --> 0  target = [17.93041369697508248188477389490827972473,
4.68650870193726325573498632957762062699 4,
353.30541095341154425141043183257746085 46]
one interval r = 20.73150479094367713954316965115743157395 ..
25.90675353516487777835102780169269382174
Time Approximations 0.033.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, -.190e-34]Solution in 0.664s

Time Plot 0 s.
Exiting SolveHard() after 3.463r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65,
375.73285289751309194462972595589005357 20,
328.11709294715746593637647633953847516 87,
292.99969138844188528463437990871092908 54,
358.64341561187384085373111624544618641 36,
299.89866205686548149537895230485986314 58,
360.06173466702333607993320496968841093 11,
336.59441032569511671259082621056567872 85,
256.10753186858904796060466758474619935 27,
324.65521224120965589516714413570202555 69,
331.93806792331323556540535560724295004 56,
304.79958325774524425834035380177522519 72, none,
289.54595773317886703206823693241751063 69]

2 --> 0  target = [17.93041369697508248188477389490827972473,
4.68650870193726325573498632957762062699 4,
353.30541095341154425141043183257746085 46]
one interval r = 31.37435486996227467839459244930400923577 ..
34.20127520034490194400455807388437910110
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
```

```
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., .100e-34]Solution in 0.343s

Time Plot 0 s.
Exiting SolveHard() after 0.622r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493543573748873043719435171360 13,
441.64295973046909970174770033342620486 26,
436.91748165375614150938133088464593474 08,
422.98493397273731190653694397045348043 50,
361.52580256435697418749811357826671679 44,
401.88173904303024547594122533894021544 76,
389.59001516215812116251762171195894687 86,
328.46939893766545613860877088191823386 33,
401.50757158085281521571805698242244881 89,
358.97362824312742963799516946103119044 99,
398.33147103786770139111951144265333870 52,
371.48387395049112006000995916423470473 77,
336.61215841538481806877452388144380869 76,
361.50888347461910035081526222431755843 75,
324.67144993153406931589526988767385969 54,
302.31384315133008537068275068270599623 78,
328.46938513838831386301633834145560649 79,
343.81340625587953383121489880709102048 65,
375.73285289751309194462972595589005357 20,
328.11709294715746593637647633953847516 87,
292.99969138844188528463437990871092908 54,
358.64341561187384085373111624544618641 36,
299.89866205686548149537895230485986314 58,
360.06173466702333607993320496968841093 11,
336.59441032569511671259082621056567872 85,
256.10753186858904796060466758474619935 27,
324.65521224120965589516714413570202555 69,
331.93806792331323556540535560724295004 56,
304.79958325774524425834035380177522519 72,
323.46169177289608198261139589342637380 65,
289.54595773317886703206823693241751063 69]

Cascade time 242.312
counts: 28, 28

 Iteration 40

Start Generation 1
1 --> 0  target = [12.00000000007456875484734790054772495400,
6.21701250283110986228886307453362449592 8,
```

```
485.549080886168762949455832541479113016 2]
one interval r = 23.40850301627286760499127243681795552470 ..
27.67578046417110928897303810778279635919
Time Approximations 0.048.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.8e-38
Equations at solution: [.1e-37, -.28e-37, .1e-36]Solution in 1.003s

Time Plot 0 s.
Exiting SolveHard() after 4.305r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000007456875484734790054772495400,
6.217012502831109862288863074533624495928,
485.549080886168762949455832541479113016 2]
one interval r = 32.62814779193072258320069066777888762675 ..
36.10248388923144724249182711945310849744
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.14786e-34]Solution in 0.573s

Time Plot 0 s.
Exiting SolveHard() after 1.014r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
```

```
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684465741017261140638251322083472,
6.58343472169769598544741063365785182885 3,
467.78730594564665953830258668629507039 91]
one interval r = 32.41978955641649450016108682263218496514 ..
35.85152417350769479827673120827067998036
Time Approximations 0.019.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.7764e-35]Solution in 0.632s

Time Plot 0 s.
Exiting SolveHard() after 3.535r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31, none, none,
401.88173903040357881756748988308621793 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684465741017261140638251322083472,
6.58343472169769598544741063365785182885 3,
467.78730594564665953830258668629507039 91]
two intervals r = 12.92327160847787914874019713679720967032 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000 or r = 18.39424857995030159329038275032657492161 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000
Time Approximations 0.039.
```

```
hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=6.5e-38
Equations at solution: [.5e-37, .65e-37, .3969e-35]Solution in 40.423s

Time Plot 0 s.
Exiting SolveHard() after 43.924r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78, none,
401.88173903040357881756748988308621793 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962809161333331279322627461977633,
4.12565179695779630301288156760456713485 6,
440.67123064078919644525720973805468098 07]
two intervals r = 14.35659705119066770504364689323590353890 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000 or r = 17.70352613772446719972972152745145007850 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.14e-37, -.1e-37, -.25118e-34]Solution in
1.326s

Time Plot 0 s.
Exiting SolveHard() after 4.333r=15.9119 in [14.35659706 ..
18.96093397]
```

```
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78, none,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962809161333331279322627461977633,
4.1256517969577963030128815676045671348 56,
440.67123064078919644525720973805468098 07]
one interval r = 22.397611543453172769000966365003943367 85 ..
27.237223515849580029998227271525548251 17
Time Approximations 0.035.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.243 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643729458126288768263958311417 82, rm =
14.378187706148637813384410334819392763 10}});
Accepted {r=26.4635, rm=16.5329} with Delta=7.9e-38
Equations at solution: [-.1e-37, -.79e-37, -.480e-34]Solution in 9.477s

Time Plot 0 s.
Exiting SolveHard() after 12.593r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887774505012126695045174837670 75,
4.004869081924931628143885831601885572325,
404.86224500093418912113672475455758775 43]
two intervals r = 16.08011007771862520078380861681289963145 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000 or r = 16.41579812636633183052143942994288400749 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.34e-37, -.1e-37, .4034e-35]Solution in 1.67s

Time Plot 0 s.
Exiting SolveHard() after 4.917r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02, none, none,
358.97362823060457291594184267456973695 01, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887774505012126695045174837670 75,
4.004869081924931628143885831601885572325,
404.86224500093418912113672475455758775 43]
one interval r = 21.64194399387689892842209764507509661874 ..
26.76330660021949376203430344768635731077
Time Approximations 0.047.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [.1e-37, .49e-37, .191e-34]Solution in 0.994s

Time Plot 0.001 s.
Exiting SolveHard() after 4.166r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14, none,
358.97362823060457291594184267456973695 01, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941819900182819802849526554142 36,
5.58963718279522723710532929311526997969 4,
443.83065882950717839007140939519473837 70]
one interval r = 22.46725374441715775728751556445351372146 ..
27.27388428333856400208239588961516056996
Time Approximations 0.038.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.8e-38
Equations at solution: [0., -.28e-37, -.56e-35]Solution in 0.984s

Time Plot 0 s.
Exiting SolveHard() after 4.237r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14, none,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941819900182819802849526554142 36,
5.589637182795227237105329293115269979694,
443.83065882950717839007140939519473837 70]
one interval r = 32.15575279477675366733403491972155262244 ..
35.50872228711712434326215003853754430538
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., -.4358e-35]Solution in 0.453s

Time Plot 0 s.
Exiting SolveHard() after 2.767r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]
```

```
1 --> 0  target = [15.91193136499235125161321264039527441791,
5.1877835784619403433051178375524168639 76,
408.65773862014886755262323611471993510 11]
one interval r = 21.71840114637470156352297728995584366264 ..
26.81849303500132499655603598023211574429
Time Approximations 0.059.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.58e-37
Equations at solution: [.2e-37, .158e-36, .290e-34]Solution in 3.104s

Time Plot 0 s.
Exiting SolveHard() after 4.225r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02, none, none,
361.50888346278201046309606763837697658 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136499235125161321264039527441791,
5.1877835784619403433051178375524168639 76,
408.65773862014886755262323611471993510 11]
one interval r = 31.80828598738164483896970913811333935127 ..
35.00011460032431209771270780018650698383
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
```

```
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.24836e-34]Solution in 0.427s

Time Plot 0 s.
Exiting SolveHard() after 0.723r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12, none,
361.50888346278201046309606763837697658 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110524504146134165813202399608039,
6.19626256544527120433901405536006925 3440,
385.44474378469036780661726591339282391 98]
one interval r = 31.60836097522411508903509106424585472297 ..
34.66372795593985911800727669438929004835
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=9e-38
Equations at solution: [-.6e-37, .9e-37, .4055e-35]Solution in 0.568s

Time Plot 0 s.
Exiting SolveHard() after 0.842r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12, none,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110524504146134165813202399608039,
6.19626256544527120433901405536006925344 0,
385.44474378469036780661726591339282391 98]
two intervals r = 16.875634087492839532437264177521001470 05 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000 or r = 15.556404937564479248631307646360989044 93 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 8.772 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175111954884688558096319021166616, rm
= 2.33653277404934285925135421570595491238 3}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.538e-37, 0., -.3713e-35]Solution in 30.159s

Time Plot 0 s.
Exiting SolveHard() after 33.309r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
```

```
441.642959719127344775008863735274850310 4,
436.917481643202746334097897668173258273 1,
422.984933959459872772076771820128701677 8,
361.525802552453432622572642919748736021 8,
401.881739030403578817567489883086217934 8,
389.590015151566788286415540646708833170 2,
328.469398923821418171965404617988936341 4,
401.507571567784303844512223490720428361 2,
358.973628230604572915941842674569736950 1,
398.331471023110037035577114795693101650 2,
371.483873939708151192976085071914477011 2,
336.612158401699737475978995491293338776 6,
361.508883462782010463096067638376976583 0,
324.671449918345178505545213816761154722 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747320859901562160035221101399 73,
4.883810779770312040377116050489032193145,
376.619678549092187965837179604621955107 0]
one interval r = 21.110013048638287192920713903252174320 51 ..
26.317842434581432736154398904886131297 21
Time Approximations 0.036.

hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.761063912186698964426645072343113601 29, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=5.0e-38
Equations at solution: [-.2e-37, -.50e-37, -.123e-34]Solution in 2.97s

Time Plot 0 s.
Exiting SolveHard() after 3.7r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434924794600952298557698400813965 4,
441.642959719127344775008863735274850310 4,
436.917481643202746334097897668173258273 1,
422.984933959459872772076771820128701677 8,
361.525802552453432622572642919748736021 8,
401.881739030403578817567489883086217934 8,
389.590015151566788286415540646708833170 2,
328.469398923821418171965404617988936341 4,
401.507571567784303844512223490720428361 2,
358.973628230604572915941842674569736950 1,
398.331471023110037035577114795693101650 2,
371.483873939708151192976085071914477011 2,
```

```
336.61215840169973747597899549129333387766,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23, none,
328.46938512454608772992890170160353727 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874732085990156216003522110139973,
4.8838107797703120403771160504890321931 45,
376.61967854909218796583717960462195510 70]
one interval r = 31.538994976965469899546447617801137454 00 ..
34.536183860765844774321013776145826765 66
Time Approximations 0.017.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.6e-37
Equations at solution: [.27e-36, -.36e-36, -.11284e-34]Solution in
0.502s

Time Plot 0 s.
Exiting SolveHard() after 0.797r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333387766,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23, none,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017519154827950016648490523156562,
6.0258135493869296337171338946060320634 95,
```

```
351.42702947307460057071048263222315471 00]
one interval r = 31.362302060987635862358985063089269022 57 ..
34.174466405948132623716412464786508214 01
Time Approximations 0.016.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .18058e-34]Solution in 0.522s

Time Plot 0 s.
Exiting SolveHard() after 0.76r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23, none,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86, none, none,
292.99969137338353103348952200534336310 43, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017519154827950016648490523156562,
6.0258135493869296337171338946060320634 95,
351.42702947307460057071048263222315471 00]
two intervals r = 17.981355144423537852678293450930365783 73 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000 or r = 13.846080153546667443729964269029186247 55 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
```

```
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 5.191 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.91357071358146518493006618521494924139, rm
= 2.7345009931504682082065046381460816308 46}});
Accepted {r=18.6878, rm=15.3648} with Delta=6e-38
Equations at solution: [.196e-36, -.6e-37, .14246e-34]Solution in
17.703s

Time Plot 0 s.
Exiting SolveHard() after 21.046r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86, none, none,
292.99969137338353103348952200534336310 43, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941846878256586409528124077674331,
6.3779438739448470517179798146374610280 94,
423.28832782131404673658684319470757286 41]
one interval r = 31.946618175720947302277688625455015029 29 ..
35.212123086218610786096130226561281403 81
Time Approximations 0.033.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, -.15841e-34]Solution in 0.586s

Time Plot 0 s.
Exiting SolveHard() after 3.081r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86, none, none,
292.99969137338353103348952200534336310 43, none, none,
360.06173465109492827479323804435056415 49, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418468782565864095281240776743 31,
6.3779438739448470517179798146374610280 94,
423.28832782131404673658684319470757286 41]
two intervals r = 15.228867024796354002538288893348403064 29 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000 or r = 17.129657770012931508070371142141567963 57 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000
Time Approximations 0.065.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
```

```
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.874 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541532875942979980251845081354 33, rm
= 2.0640682987491824369042652210318156584 35}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [.15e-37, 0., -.832e-36]Solution in 30.861s

Time Plot 0 s.
Exiting SolveHard() after 32.219r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86,
375.73285288055490876503591851684883110 45, none,
292.99969137338353103348952200534336310 43, none, none,
360.06173465109492827479323804435056415 49, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343210966289138667792370512436 23,
4.0035598156244061069645905690905042452 14,
404.47973592782692052659475678384366841 86]
two intervals r = 16.096839663834279494552168295171562733 04 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000 or r = 16.399886490557194444904292743522176291 47 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
```

```
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.52e-37, -.1e-37, .31277e-34]Solution in
3.958s

Time Plot 0 s.
Exiting SolveHard() after 5.004r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86,
375.73285288055490876503591851684883110 45, none,
292.99969137338353103348952200534336310 43,
358.64341559895957332594929552110870272 47, none,
360.06173465109492827479323804435056415 49, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343210966289138667792370512436 23,
4.0035598156244061069645905690905042452 14,
404.47973592782692052659475678384366841 86]
one interval r = 21.634296299646690786549819612377340316 48 ..
26.757681698714848757934153261163066734 64
Time Approximations 0.052.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., .547e-34]Solution in 3.676s

Time Plot 0 s.
Exiting SolveHard() after 4.718r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86,
375.73285288055490876503591851684883110 45,
328.11709293289764684008948245878819460 91,
292.99969137338353103348952200534336310 43,
358.64341559895957332594929552110870272 47, none,
360.06173465109492827479323804435056415 49, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954442612069859087962923386844974,
6.19617723029853602433978737528132789018 3,
385.42734024838967432708257309751304767 08]
one interval r = 31.60822049077734845838404821660447659163 ..
34.66347615031571667562427575647559734456
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
```

```
Equations at solution: [0., 0., -.9000e-35]Solution in 0.553s

Time Plot 0 s.
Exiting SolveHard() after 0.843r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86,
375.73285288055490876503591851684883110 45,
328.11709293289764684008948245878819460 91,
292.99969137338353103348952200534336310 43,
358.64341559895957332594929552110870272 47, none,
360.06173465109492827479323804435056415 49, none, none,
324.65521222808453481135870746178594760 66, none, none, none, none]

0 --> 1  target = [26.46318954442612069859087962923386844974,
6.19617723029853602433978737528132789018 3,
385.42734024838967432708257309751304767 08]
two intervals r = 16.87629600288119406816120450997702866068 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000 or r = 15.55559000604756284657997438432104626843 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000
Time Approximations 0.058.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
```

```
in partial time = 8.176 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852523780759898539986325678362734, rm
= 2.33669042811516044918841506272562072782 2}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.358e-37, 0., .9288e-35]Solution in 28.846s

Time Plot 0 s.
Exiting SolveHard() after 31.981r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86,
375.73285288055490876503591851684883110 45,
328.11709293289764684008948245878819460 91,
292.99969137338353103348952200534336310 43,
358.64341559895957332594929552110870272 47, none,
360.06173465109492827479323804435056415 49,
336.59441031207984606545757784197387474 06, none,
324.65521222808453481135870746178594760 66, none, none, none, none]

0 --> 2  target = [34.49522661155485210956041477607842494287,
3.89713131607040770106257017631302252285 7,
373.78081884044818406601194686837356738 32]
two intervals r = 17.29769086204032948498501383039012589648 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000 or r = 14.99436407404793608576076396383357168807 ..
189999999998467396263664128578048574 99/100000000000000000000000000000000
00000
Time Approximations 0.075.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
```

```
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [.72e-37, -.1e-37, -.38147e-34]Solution in
1.166s

Time Plot 0 s.
Exiting SolveHard() after 4.876r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86,
375.73285288055490876503591851684883110 45,
328.11709293289764684008948245878819460 91,
292.99969137338353103348952200534336310 43,
358.64341559895957332594929552110870272 47, none,
360.06173465109492827479323804435056415 49,
336.59441031207984606545757784197387474 06, none,
324.65521222808453481135870746178594760 66,
331.93806791233207218066094459508320619 39, none, none, none]

1 --> 2  target = [34.49522661155485210956041477607842494287,
3.8971313160704077010625701763130225228 57,
373.78081884044818406601194686837356738 32]
one interval r = 21.06068473198282187775075682976901106926 ..
26.26979834274514976642488850732992687350
Time Approximations 0.032.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, -.259e-34]Solution in 0.752s

Time Plot 0 s.
Exiting SolveHard() after 3.782r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86,
375.73285288055490876503591851684883110 45,
328.11709293289764684008948245878819460 91,
292.99969137338353103348952200534336310 43,
358.64341559895957332594929552110870272 47,
299.89866204477223866796600569172054815 35,
360.06173465109492827479323804435056415 49,
336.59441031207984606545757784197387474 06, none,
324.65521222808453481135870746178594760 66,
331.93806791233207218066094459508320619 39, none, none, none]

0 --> 2  target = [33.81362495393815373271174177378276230139,
3.72564899368895690412037884097007181830 0,
325.89209972062098399148160245813568331 06]
two intervals r = 18.55227048994532050688302090692803376097 ..
189999999998467396263664128578048574 99/10000000000000000000000000000000
00000 or r = 12.49196935738120798132537047227333741620 ..
189999999998467396263664128578048574 99/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
```

```
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [-.87e-37, .3e-37, -.3762e-35]Solution in 1.125s

Time Plot 0 s.
Exiting SolveHard() after 4.725r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86,
375.73285288055490876503591851684883110 45,
328.11709293289764684008948245878819460 91,
292.99969137338353103348952200534336310 43,
358.64341559895957332594929552110870272 47,
299.89866204477223866796600569172054815 35,
360.06173465109492827479323804435056415 49,
336.59441031207984606545757784197387474 06, none,
324.65521222808453481135870746178594760 66,
331.93806791233207218066094459508320619 39, none, none,
289.54595771989602682639872240273822923 94]

1 --> 2  target = [33.81362495393815373271174177378276230139,
3.72564899368895690412037884097007181830 0,
325.89209972062098399148160245813568331 06]
one interval r = 20.37468935101150707648451767469982375936 ..
25.37892165282729096088205862366674987312
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., -.254e-34]Solution in 2.725s

Time Plot 0.001 s.
Exiting SolveHard() after 3.269r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86,
375.73285288055490876503591851684883110 45,
328.11709293289764684008948245878819460 91,
292.99969137338353103348952200534336310 43,
358.64341559895957332594929552110870272 47,
299.89866204477223866796600569172054815 35,
360.06173465109492827479323804435056415 49,
336.59441031207984606545757784197387474 06,
256.10753185427888162110405804747882437 48,
324.65521222808453481135870746178594760 66,
331.93806791233207218066094459508320619 39, none, none,
289.54595771989602682639872240273822923 94]

1 --> 0  target = [17.93041369708840453247095056983500809583,
4.68650870189385540935374104646810404063 9,
353.30541093931580722501213569913965568 49]
one interval r = 20.73150479077254833067005073127996394953 ..
25.90675353500343071751025576283263220193
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
```

```
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [.2e-37, .23e-37, .197e-34]Solution in 0.688s

Time Plot 0 s.
Exiting SolveHard() after 1.416r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86,
375.73285288055490876503591851684883110 45,
328.11709293289764684008948245878819460 91,
292.99969137338353103348952200534336310 43,
358.64341559895957332594929552110870272 47,
299.89866204477223866796600569172054815 35,
360.06173465109492827479323804435056415 49,
336.59441031207984606545757784197387474 06,
256.10753185427888162110405804747882437 48,
324.65521222808453481135870746178594760 66,
331.93806791233207218066094459508320619 39,
304.79958324278715034768838093217778092 84, none,
289.54595771989602682639872240273822923 94]

2 --> 0  target = [17.93041369708840453247095056983500809583,
4.68650870189385540935374104646810404063 9,
353.30541093931580722501213569913965568 49]
one interval r = 31.37435486977904607938536907623548735280 ..
34.20127520007477125919464984172101281784
Time Approximations 0.016.
```

```
hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [.6e-37, -.8e-37, -.6158e-35]Solution in 2.532s

Time Plot 0 s.
Exiting SolveHard() after 2.792r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492479460095229855769840081396 54,
441.64295971912734477500886373527485031 04,
436.91748164320274633409789766817325827 31,
422.98493395945987277207677182012870167 78,
361.52580255245343262257264291974873602 18,
401.88173903040357881756748988308621793 48,
389.59001515156678828641554064670883317 02,
328.46939892382141817196540461798893634 14,
401.50757156778430384451222349072042836 12,
358.97362823060457291594184267456973695 01,
398.33147102311003703557711479569310165 02,
371.48387393970815119297608507191447701 12,
336.61215840169973747597899549129333877 66,
361.50888346278201046309606763837697658 30,
324.67144991834517850554521381676115472 23,
302.31384313585893457694879328345199270 89,
328.46938512454608772992890170160353727 42,
343.81340624323224493881924760705721604 86,
375.73285288055490876503591851684883110 45,
328.11709293289764684008948245878819460 91,
292.99969137338353103348952200534336310 43,
358.64341559895957332594929552110870272 47,
299.89866204477223866796600569172054815 35,
360.06173465109492827479323804435056415 49,
336.59441031207984606545757784197387474 06,
256.10753185427888162110405804747882437 48,
324.65521222808453481135870746178594760 66,
331.93806791233207218066094459508320619 39,
304.79958324278715034768838093217778092 84,
323.46169175910365735368746287788395695 91,
289.54595771989602682639872240273822923 94]

Cascade time 249.961
counts: 28, 28
```

```
 Iteration 41

Start Generation 1
1 --> 0  target = [12.00000000012035533257607484323308599900,
6.21701250290142350333952827720499090831 0,
485.54908088956612140589420940979701484 02]
one interval r = 23.40850301640245694837539735789195159642 ..
27.67578046433021603918480434280914624374
Time Approximations 0.039.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.8e-38
Equations at solution: [-.1e-37, .28e-37, .10e-35]Solution in 3.716s

Time Plot 0 s.
Exiting SolveHard() after 4.865r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000012035533257607484323308599900,
6.21701250290142350333952827720499090831 0,
485.54908088956612140589420940979701484 02]
one interval r = 32.62814779216249976475408261453661915680 ..
36.10248388935227675705763323706259516919
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.63e-35]Solution in 0.623s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.025r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684483506406348633575547728725555,
6.58343472147314922406279468138637620 3265,
467.78730595098813589643686035556879241 06]
one interval r = 32.41978955667684982572505000404369392134 ..
35.85152417366395133368622608309110007743
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.10e-35]Solution in 3.679s

Time Plot 0 s.
Exiting SolveHard() after 4.051r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30, none, none,
401.88173903697858206431044783005700340 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684483506406348633575547728725555,
6.58343472147314922406279468138637620 3265,
467.78730595098813589643686035556879241 06]
```

"Imaginary part neglected: ", $1.10311211489793710385225880154477596873 4 \times 10^{-17}$

```
two intervals r = 12.92327160843655589891466919639938933685 ..
190000000000136669188156498092190268 47/100000000000000000000000000000000
00000 or r = 18.39424858018484512634659031046255481833 ..
190000000000136669188156498092190268 47/100000000000000000000000000000000
00000
Time Approximations 0.047.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=0
Equations at solution: [0., 0., -.479e-35]Solution in 42.083s

Time Plot 0 s.
Exiting SolveHard() after 43.49r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32, none,
401.88173903697858206431044783005700340 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962820315080638472104158219343535,
4.12565179684181245226015066898439217378 8,
440.67123064215393269646578636773995137 31]
```

"Imaginary part neglected: ", $1.10311211489793710385225880154477596873 4 \times 10^{-17}$

```
two intervals r = 14.35659705137830402702394025358232845789 ..
190000000000136669188156498092190268 47/100000000000000000000000000000000
00000 or r = 17.70352613786915026639135871433006237808 ..
190000000000136669188156498092190268 47/100000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
```

```
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.29e-37, -.2e-37, .3111e-34]Solution in 3.297s

Time Plot 0 s.
Exiting SolveHard() after 4.349r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32, none,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962820315080638472104158219343535,
4.1256517968418124522601506689843921737 88,
440.67123064215393269646578636773995137 31]
one interval r = 22.39761154352022330980247082268238042009 ..
27.23722351598520965831310782091933730285
Time Approximations 0.039.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.394 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064375745966393411593638275307163, rm =
14.37818770178735397998682771831629539247}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.3e-38
Equations at solution: [-.1e-37, -.53e-37, .186e-34]Solution in 11.63s

Time Plot 0 s.
Exiting SolveHard() after 12.507r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887975786476303129157956940743 55,
4.0048690818268021257669374659373600692 66,
404.86224500736186182060230146770044303 73]
```

"Imaginary part neglected: ", $1.10311211489793710385225880154477596873 4 \times 10^{-17}$

```
two intervals r = 16.0801100776903774818679161933404241657 1 ..
19000000000013666918815649809219026847/10000000000000000000000000000000
00000 or r = 16.4157981267101015682959924036075030627 2 ..
19000000000013666918815649809219026847/10000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.49e-37, -.1e-37, -.4251e-34]Solution in 3.49s

Time Plot 0 s.
Exiting SolveHard() after 4.561r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31, none, none,
358.97362823361007903889450154650182340 94, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888797578647630312915795694074355,
4.004869081826802125766937465937360069266,
404.86224500736186182060230146770044303 73]
one interval r = 21.641943994031188407623483174360055203 86 ..
26.763306600422558649039936669632878373 56
Time Approximations 0.054.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [0., .26e-37, .124e-34]Solution in 3.17s

Time Plot 0 s.
Exiting SolveHard() after 4.173r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02, none,
358.97362823361007903889450154650182340 94, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941787499206862754129795725822540,
5.589637182967888045058622883463301208878,
443.83065884048861294030374334542478393 79]
one interval r = 22.467253744697926687335484399704110511 95 ..
27.273884283585054178928152801187245045 22
Time Approximations 0.04.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
```

```
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [-.1e-37, .27e-37, .19e-35]Solution in 3.042s

Time Plot 0 s.
Exiting SolveHard() after 4.032r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02, none,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589417874992068627541297957258225 40,
5.5896371829678880450586228834633012088 78,
443.83065884048861294030374334542478393 79]
one interval r = 32.155752795102868533236470084851898814 10 ..
35.508722287365645848148690072146034761 68
Time Approximations 0.02.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, -.203e-34]Solution in 0.487s

Time Plot 0 s.
Exiting SolveHard() after 0.855r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365298646732603796192014051178 79,
5.1877835784953053247076341747676787394 50,
408.65773861899608319585216418683740173 12]
one interval r = 21.718401146378166762656418283241783281 38 ..
26.818493035094560772392671018221374116 15
Time Approximations 0.058.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.32e-37
Equations at solution: [-.2e-37, -.132e-36, -.34e-35]Solution in 0.993s

Time Plot 0 s.
Exiting SolveHard() after 4.255r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64, none, none,
361.50888346566391678458313765616911321 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0  target = [15.91193136529864673260379619201405117879,
5.1877835784953053247076341747676787394 50,
408.65773861899608319585216418683740173 12]
one interval r = 31.80828598759532041736943808135380207686 ..
35.00011460041391140750451332182393573322
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., .159e-34]Solution in 3.407s

Time Plot 0 s.
Exiting SolveHard() after 3.724r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969 68931,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32, none,
361.50888346566391678458313765616911321 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110539769421963053449715312757321,
6.19626256521449253421020493840901955507 7,
385.44474378778917030662404544828097290 33]
one interval r = 31.60836097548171848274159284556314974865 ..
34.66372795610193784745906874089119304978
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
```

```
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, -.233e-34]Solution in 0.57s

Time Plot 0 s.
Exiting SolveHard() after 0.867r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32, none,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110539769421963053449715312757321,
6.19626256521449253421020493840901955507 7,
385.44474378778917030662404544828097290 33]
```

"Imaginary part neglected: ", $1.10311211489793710385225880154477596873 4 \times 10^{-17}$

```
two intervals r = 16.875634087618931922862369669008665748 99 ..
19000000000013666918815649809219026847/100000000000000000000000000000000
00000 or r = 15.556404937771442994675672904277740907 31 ..
19000000000013666918815649809219026847/100000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
```

```
Equations at solution: [-.897e-37, .1e-37, -.430e-35]Solution in 1.128s

Time Plot 0 s.
Exiting SolveHard() after 4.509r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874744809861628661933489917670437,
4.88381077985205388227904175618762909351 4,
376.61967855226104723736534326197186335 08]
one interval r = 21.11001304871142723498016248056011345208 ..
26.31784243473444129822574382179803645645
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.191e-34]Solution in 3.776s

Time Plot 0 s.
Exiting SolveHard() after 4.497r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
```

```
441.642959724252555331266898265551707822 0,
436.917481644827727121827072535666956443 0,
422.984933969875965470420454270368742763 2,
361.525802555380073060008987434439883429 6,
401.881739036978582064310447830057003408 0,
389.590015150465115977762894013160429193 1,
328.469398931403901125321698523503441020 2,
401.507571575054266861290941801224571300 5,
358.973628233610079038894501546501823409 4,
398.331471038103661707921643569485899326 4,
371.483873936105217980523320542514473303 2,
336.612158410149589473199194246761893012 2,
361.508883465663916784583137656169113215 3,
324.671449922860985572181079504661849243 5, none,
328.469385132128085589572498842060083400 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747448098616286619334899176704 37,
4.883810779852053882279041756187629093514,
376.619678552261047237365343261971863350 8]
one interval r = 31.538994977225213247258378570620788324 75 ..
34.536183860933066773595775883561344263 39
Time Approximations 0.017.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=8.98e-36
Equations at solution: [.689e-35, -.898e-35, -.268e-34]Solution in
0.518s

Time Plot 0 s.
Exiting SolveHard() after 0.82r=34.0898 in [32.52213872 .. 34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434928053662198967772423729696893 1,
441.642959724252555331266898265551707822 0,
436.917481644827727121827072535666956443 0,
422.984933969875965470420454270368742763 2,
361.525802555380073060008987434439883429 6,
401.881739036978582064310447830057003408 0,
389.590015150465115977762894013160429193 1,
328.469398931403901125321698523503441020 2,
401.507571575054266861290941801224571300 5,
358.973628233610079038894501546501823409 4,
398.331471038103661707921643569485899326 4,
```

```
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35, none,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017542948460766796498038521613565,
6.0258135491825000135248320846917609542 81,
351.42702948097353659856805981102970612 12]
one interval r = 31.362302061280938491516345182117027357 88 ..
34.174466406194588748650333840722212535 02
Time Approximations 0.016.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, -.2e-36]Solution in 0.527s

Time Plot 0 s.
Exiting SolveHard() after 0.778r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35, none,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03, none, none,
292.99969138238636669170748693960608530 07, none, none, none, none,
none, none, none, none, none, none]
```

```
0 --> 1  target = [25.872050175429484607667964980385216135 65,
6.0258135491825000135248320846917609542 81,
351.42702948097353659856805981102970612 12]
```

"Imaginary part neglected: ", $1.10311211489793710385225880154477596873 4 \times 10^{-17}$

```
two intervals r = 17.981355144434331672868617290710542533 98 ..
19000000000013666918815649809219026847/10000000000000000000000000000000
00000 or r = 13.846080153998890856502153975825587283 36 ..
19000000000013666918815649809219026847/10000000000000000000000000000000
00000
Time Approximations 0.047.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [.107e-36, -.3e-37, -.2599e-34]Solution in
3.369s

Time Plot 0 s.
Exiting SolveHard() after 6.894r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03, none, none,
292.99969138238636669170748693960608530 07, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418787664697777792324276481275 37,
```

```
6.37794387377055320698036547720939666152 2,
423.28832783680808970737316841151748594 22]
one interval r = 31.94661817609072506139207067639211715642 ..
35.21212308654252483954725350714753292497
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., .188e-34]Solution in 0.59s

Time Plot 0 s.
Exiting SolveHard() after 0.937r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03, none, none,
292.99969138238636669170748693960608530 07, none, none,
360.06173466720309293517909440618957378 13, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941878766469777779232427648127537,
6.37794387377055320698036547720939666152 2,
423.28832783680808970737316841151748594 22]
```

"Imaginary part neglected: ", $1.10311211489793710385225880154477596873 4 \times 10^{-17}$

```
two intervals r = 15.22886702430416038844607411217180915898 ..
```

```
190000000000136669188156498092190268 47/1000000000000000000000000000000000
00000 or r = 17.12965777065913539950399141585360682750 ..
190000000000136669188156498092190268 47/1000000000000000000000000000000000
00000
Time Approximations 0.062.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [-.48e-37, -.2e-37, .2291e-34]Solution in 1.271s

Time Plot 0 s.
Exiting SolveHard() after 4.883r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332052451447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03,
375.73285290209490174378206184968993970 80, none,
292.99969138238636669170748693960608530 07, none, none,
360.06173466720309293517909440618957378 13, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343422728304223060379910616052 62,
4.0035598155287207539160057672070118476 80,
404.47973593496510232699087754415440851 63]

"Imaginary part neglected: ", 1.10311211489793710385225880154477596873 4 × 10^-17
two intervals r = 16.09683966377552709691567615598085095095 ..
```

```
190000000000136669188156498092190268 47/100000000000000000000000000000000
00000 or r = 16.39988649093099057291186979070196130419 ..
190000000000136669188156498092190268 47/100000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.51e-37, .1e-37, -.1965e-34]Solution in 3.682s

Time Plot 0 s.
Exiting SolveHard() after 6.838r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969 68931,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03,
375.73285290209490174378206184968993970 80, none,
292.99969138238636669170748693960608530 07,
358.64341560257705003477707898928672209 22, none,
360.06173466720309293517909440618957378 13, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234342272830422306037991061605262,
4.0035598155287207539160057672070118476 80,
404.47973593496510232699087754415440851 63]
one interval r = 21.63429629981487956568782176651233542307 ..
26.75768169892844699296144978127677408965
Time Approximations 0.051.
```

```
hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, .69e-35]Solution in 3.106s

Time Plot 0 s.
Exiting SolveHard() after 4.088r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03,
375.73285290209490174378206184968993970 80,
328.11709294113421172257853212579980853 85,
292.99969138238636669170748693960608530 07,
358.64341560257705003477707898928672209 22, none,
360.06173466720309293517909440618957378 13, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954457802749844166385826257406863,
6.19617723006753261696129773842677368987 4,
385.42734025144246519415858099721988274 93]
one interval r = 31.608220491034583802032127399671708587 61 ..
34.663476150477137825758827408891103518 35
Time Approximations 0.016.

hint used Hint := [33.813392675100157147469018161182119369 13, 3, 1, 1,
11.783209121413521870719438174471899656 27, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., -.31e-35]Solution in 0.563s

Time Plot 0 s.
Exiting SolveHard() after 0.872r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03,
375.73285290209490174378206184968993970 80,
328.11709294113421172257853212579980853 85,
292.99969138238636669170748693960608530 07,
358.64341560257705003477707898928672209 22, none,
360.06173466720309293517909440618957378 13, none, none,
324.65521223255742539039089382971662246 86, none, none, none, none]

0 --> 1  target = [26.46318954457802749844166385826257406863,
6.19617723006753261696129773842677368987 4,
385.42734025144246519415858099721988274 93]
```

"Imaginary part neglected: ", $1.10311211489793710385225880154477596873 4 \times 10^{-17}$

```
two intervals r = 16.87629600300904358449814878903420428116 ..
19000000000013666918815649809219026847/10000000000000000000000000000000
00000 or r = 15.55559000625237177749402250118943105170 ..
19000000000013666918815649809219026847/10000000000000000000000000000000
00000
Time Approximations 0.063.
```

```
hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.359e-37, 0., .1244e-34]Solution in 3.222s

Time Plot 0 s.
Exiting SolveHard() after 4.375r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03,
375.73285290209490174378206184968993970 80,
328.11709294113421172257853212579980853 85,
292.99969138238636669170748693960608530 07,
358.64341560257705003477707898928672209 22, none,
360.06173466720309293517909440618957378 13,
336.59441032048268768462501309035202874 66, none,
324.65521223255742539039089382971662246 86, none, none, none, none]

0 --> 2  target = [34.49522661162036458362155542424394273327,
3.89713131593677470901497946872839210970 8,
373.78081883647171866933168560293175124 73]
```

"Imaginary part neglected: ", $1.10311211489793710385225880154477596873 4 \times 10^{-17}$

```
two intervals r = 17.29769086241389686020657539469020727802 ..
19000000000013666918815649809219026847/10000000000000000000000000000000
00000 or r = 14.99436407390399202445947920705783729300 ..
19000000000013666918815649809219026847/10000000000000000000000000000000
```

```
00000
Time Approximations 0.083.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., -.604e-35]Solution in 1.206s

Time Plot 0 s.
Exiting SolveHard() after 4.98r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03,
375.73285290209490174378206184968993970 80,
328.11709294113421172257853212579980853 85,
292.99969138238636669170748693960608530 07,
358.64341560257705003477707898928672209 22, none,
360.06173466720309293517909440618957378 13,
336.59441032048268768462501309035202874 66, none,
324.65521223255742539039089382971662246 86,
331.93806790613620657062439316163188414 08, none, none, none]

1 --> 2  target = [34.49522661162036458362155542424394273327,
3.897131315936774709014979468728392109708,
373.78081883647171866933168560293175124 73]
one interval r = 21.06068473193117476377997624956588394639 ..
26.26979834277624028486539120598702348624
Time Approximations 0.034.
```

```
hint used Hint := [25.3004663403507213669230088342083635908 4, 3, -1, 1,
16.9747295276417875450857461383690747481 8, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [-.2e-37, -.5e-37, -.295e-34]Solution in 0.758s

Time Plot 0 s.
Exiting SolveHard() after 3.53r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03,
375.73285290209490174378206184968993970 80,
328.11709294113421172257853212579980853 85,
292.99969138238636669170748693960608530 07,
358.64341560257705003477707898928672209 22,
299.89866204278518438815331624028384128 89,
360.06173466720309293517909440618957378 13,
336.59441032048268768462501309035202874 66, none,
324.65521223255742539039089382971662246 86,
331.93806790613620657062439316163188414 08, none, none, none]

0 --> 2  target = [33.81362495414520165181978754467516229888,
3.72564899358604051086562811483181829163 0,
325.89209972495440787616834908211132938 82]

"Imaginary part neglected: ", 1.10311211489793710385225880154477596873 4 × 10^-17
two intervals r = 18.55227049007204752285513195850057679930 ..
19000000000013666918815649809219026847/10000000000000000000000000000000
```

```
00000 or r = 12.4919693576221618347327710561843390903 6 ..
190000000000136669188156498092190268 47/100000000000000000000000000000000
00000
Time Approximations 0.043.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [-.122e-36, .5e-37, .1827e-34]Solution in 1.229s

Time Plot 0 s.
Exiting SolveHard() after 4.863r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03,
375.73285290209490174378206184968993970 80,
328.11709294113421172257853212579980853 85,
292.99969138238636669170748693960608530 07,
358.64341560257705003477707898928672209 22,
299.89866204278518438815331624028384128 89,
360.06173466720309293517909440618957378 13,
336.59441032048268768462501309035202874 66, none,
324.65521223255742539039089382971662246 86,
331.93806790613620657062439316163188414 08, none, none,
289.54595772096912185295465073189713303 14]

1 --> 2  target = [33.81362495414520165181978754467516229888,
3.72564899358604051086562811483181829163 0,
```

```
325.8920997249544078761683490821113293882]
one interval r = 20.37468935106575233909031246602023265672 ..
25.37892165299722684739347621447254677109
Time Approximations 0.024.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=1e-38
Equations at solution: [-.1e-37, -.1e-37, .77e-35]Solution in 0.549s

Time Plot 0 s.
Exiting SolveHard() after 3.023r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332052451447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03,
375.73285290209490174378206184968993970 80,
328.11709294113421172257853212579980853 85,
292.99969138238636669170748693960608530 07,
358.64341560257705003477707898928672209 22,
299.89866204278518438815331624028384128 89,
360.06173466720309293517909440618957378 13,
336.59441032048268768462501309035202874 66,
256.10753185989075094457403213545515566 10,
324.65521223255742539039089382971662246 86,
331.93806790613620657062439316163188414 08, none, none,
289.54595772096912185295465073189713303 14]
```

```
1 --> 0  target = [17.93041369706938335881820961350535485187,
4.68650870202455425229291261371909765890 2,
353.30541094816286357209742781036924457 30]
one interval r = 20.73150479091579228904030221737579569840 ..
25.90675353525848885528263390648737545325
Time Approximations 0.034.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.5e-38
Equations at solution: [-.1e-37, -.25e-37, .144e-34]Solution in 2.706s

Time Plot 0 s.
Exiting SolveHard() after 3.425r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03,
375.73285290209490174378206184968993970 80,
328.11709294113421172257853212579980853 85,
292.99969138238636669170748693960608530 07,
358.64341560257705003477707898928672209 22,
299.89866204278518438815331624028384128 89,
360.06173466720309293517909440618957378 13,
336.59441032048268768462501309035202874 66,
256.10753185989075094457403213545515566 10,
324.65521223255742539039089382971662246 86,
331.93806790613620657062439316163188414 08,
```

```
304.79958325614524065276503989565387852 35, none,
289.54595772096912185295465073189713303 14]

2 --> 0  target = [17.93041369706938335881820961350535485187,
4.68650870202455425229291261371909765890 2,
353.30541094816286357209742781036924457 30]
one interval r = 31.37435487007874223446854825945173714837 ..
34.20127520033397061136667906675234432050
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, .281e-34]Solution in 0.369s

Time Plot 0 s.
Exiting SolveHard() after 0.627r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492805366219896777242372969689 31,
441.64295972425255533126689826555170782 20,
436.91748164482772712182707253566695644 30,
422.98493396987596547042045427036874276 32,
361.52580255538007306000898743443988342 96,
401.88173903697858206431044783005700340 80,
389.59001515046511597776289401316042919 31,
328.46939893140390112532169852350344102 02,
401.50757157505426686129094180122457130 05,
358.97362823361007903889450154650182340 94,
398.33147103810366170792164356948589932 64,
371.48387393610521798052332054251447330 32,
336.61215841014958947319919424676189301 22,
361.50888346566391678458313765616911321 53,
324.67144992286098557218107950466184924 35,
302.31384314888914343912551191398218831 46,
328.46938513212808558957249884206008340 03,
343.81340624298722046826205238599015143 03,
375.73285290209490174378206184968993970 80,
328.11709294113421172257853212579980853 85,
292.99969138238636669170748693960608530 07,
358.64341560257705003477707898928672209 22,
299.89866204278518438815331624028384128 89,
360.06173466720309293517909440618957378 13,
336.59441032048268768462501309035202874 66,
```

```
256.10753185989075094457403213545515566 10,
324.65521223255742539039089382971662246 86,
331.93806790613620657062439316163188414 08,
304.79958325614524065276503989565387852 35,
323.46169176365750729535047779883288045 71,
289.54595772096912185295465073189713303 14]

Cascade time 152.756
counts: 28, 28

 Iteration 42

Start Generation 1
1 --> 0  target = [11.99999999995348119825350539616722694200,
6.21701250307805932696903414447173660018 7,
485.54908090129463261138399163154625720 01]
one interval r = 23.40850301667935041514900976709001616287 ..
27.67578046422834494597230485335915506470
Time Approximations 0.041.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.8e-38
Equations at solution: [0., -.28e-37, -.7e-36]Solution in 3.323s

Time Plot 0 s.
Exiting SolveHard() after 4.459r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999995348119825350539616722694200,
6.21701250307805932696903414447173660018 7,
485.54908090129463261138399163154625720 01]
one interval r = 32.62814779222496950641630841262229401405 ..
36.10248388947404205476800424887671811376
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
```

```
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., -.95e-35]Solution in 0.613s

Time Plot 0 s.
Exiting SolveHard() after 1.039r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684475333681505089910487485773959,
6.58343472170777124322704676419452642 6473,
467.78730596223899980782741828997182996 35]
one interval r = 32.41978955672465970849154624330920336370 ..
35.85152417377894002667016216831266942428
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.31e-35]Solution in 2.613s

Time Plot 0 s.
Exiting SolveHard() after 2.971r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
```

```
441.642959735315948647256242300505585232 5,
436.917481654026360309034368075808038923 5, none, none,
401.881739044740314334425575403777191736 5, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844753336815050899104874857739 59,
6.583434721707771243227046764194526426473,
467.787305962238999807827418289971829963 5]
two intervals r = 12.923271608221644229849751738624556998 78 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000 or r = 18.394248580327685017474255586724494213 42 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=0
Equations at solution: [0., 0., -.639e-35]Solution in 39.727s

Time Plot 0 s.
Exiting SolveHard() after 41.098r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434939170206833479643045765159840 9,
441.642959735315948647256242300505585232 5,
436.917481654026360309034368075808038923 5,
422.984933979118712286816799598751214089 3, none,
401.881739044740314334425575403777191736 5, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628293865591627064734902886020 28,
4.125651796812358316960001633787514791021,
440.671230651805009152065939194960465417 1]
two intervals r = 14.356597051182515771761953871771667499 95 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000 or r = 17.703526138072387697179562748766976595 55 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
```

```
15.8447847361863649834121554328181430355 9, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [.29e-37, .2e-37, .911e-35]Solution in 3.666s

Time Plot 0 s.
Exiting SolveHard() after 4.729r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93, none,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962829386559162706473490288602028,
4.12565179681235831696000163378751479102 1,
440.67123065180500915206593919496046541 71]
one interval r = 22.39761154380172546179278234013273512090 ..
27.23722351592278573043223110817825817451
Time Approximations 0.039.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.649 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064382403241932315333849729059108, rm =
14.37818770422870256144367693240198391189}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.3e-38
Equations at solution: [0., -.53e-37, .195e-34]Solution in 9.994s

Time Plot 0 s.
```

```
Exiting SolveHard() after 10.888r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888804132626559026521225770661685,
4.0048690817930579376391888393184813566 69,
404.86224501554712851625061839497119671 24]
two intervals r = 16.08011007752688795687358409409030756803 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000 or r = 16.41579812698885081220993039329594110466 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.84e-37, -.1e-37, -.2478e-34]Solution in
3.783s

Time Plot 0 s.
Exiting SolveHard() after 7.16r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
```

```
389.590015157787925912473376045383179381 7, none, none,
358.97362823970204535064013391932768788 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888804132626559026521225770661685,
4.0048690817930579376391888393184813566 69,
404.86224501554712851625061839497119671 24]
one interval r = 21.641943994312213778028526741311476445 60 ..
26.763306600386825978287069923554721456 38
Time Approximations 0.052.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, -.317e-34]Solution in 3.974s

Time Plot 0 s.
Exiting SolveHard() after 5.005r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.590015157787925912473376045383179381 7,
328.46939893557582161143475195276425204 09, none,
358.97362823970204535064013391932768788 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941768339856531246531841784171625,
5.5896371830915073890649995856000518594 72,
443.83065885016610571767843947765853058 09]
one interval r = 22.467253744976848752237447882612978355 60 ..
27.273884283518832947813164780496636482 46
Time Approximations 0.037.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
```

```
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., -.70e-35]Solution in 3.52s

Time Plot 0 s.
Exiting SolveHard() after 4.467r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09, none,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589417683398565312465318417841 71625,
5.5896371830915073890649995858600051859472,
443.83065885016610571767843947765853058 09]
one interval r = 32.155752795121105110226749992141295179 58 ..
35.508722287456981950304541694510333142 19
Time Approximations 0.02.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, .83e-35]Solution in 0.474s

Time Plot 0 s.
Exiting SolveHard() after 0.833r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136516286715723575392595011191248,
5.1877835785838598580983668166074902244 78,
408.65773862661919043887019705436067889 07]
one interval r = 21.71840114664501927726199553096328646832 ..
26.81849303504592643285974744266666600395
Time Approximations 0.059.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [.1e-37, .79e-37, .275e-34]Solution in 3.334s

Time Plot 0 s.
Exiting SolveHard() after 4.443r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74, none, none,
```

```
361.5088834713682337036966627254281113637, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136516286715723575392595011191248,
5.1877835785838598580983668166074902244 78,
408.65773862661919043887019705436067889 07]
one interval r = 31.80828598757642832615891403587981158190 ..
35.00011460047182133192334805603830669387
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.373e-34]Solution in 0.438s

Time Plot 0 s.
Exiting SolveHard() after 0.764r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48, none,
361.5088834713682337036966627254281113637, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110534530788404978303712659459005,
6.19626256542015115880203267396274626286 4,
385.44474379338996625755738648039270708 32]
one interval r = 31.60836097543583454787227212610326087897 ..
34.66372795612707294246070906867128798489
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
```

```
11.783029068742676904624149068588691721 32, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, -.248e-34]Solution in 2.721s

Time Plot 0 s.
Exiting SolveHard() after 3r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48, none,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110534530788404978303712659459005,
6.196262565420151158802032673962746262864,
385.44474379338996625755738648039270708 32]
two intervals r = 16.875634087547424905711847495529516114 38 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000 or r = 15.556404937998790535191804621458784998 36 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
```

```
in partial time = 8.279 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175118740659848094157151173832960, rm
= 2.33653277396679578748041439296451853953 5}});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [.897e-37, -.1e-37, .374e-35]Solution in 29.611s

Time Plot 0 s.
Exiting SolveHard() after 30.757r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874734454142430385968358700411940,
4.8838107799216681900001877253243860725 44,
376.61967855857707783561134797181636060 77]
one interval r = 21.11001304897498761560283230998256391611 ..
26.31784243470413034017243333221954120490
Time Approximations 0.034.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .51e-35]Solution in 0.853s

Time Plot 0 s.
Exiting SolveHard() after 1.551r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69, none,
328.46938513630359810756534222447024778 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874734454142430385968358700411940,
4.8838107799216681900001877253243860725 44,
376.61967855857707783561134797181636060 77]
one interval r = 31.53899497718152416703612973949038358132 ..
34.53618386096705510171863554124766037132
Time Approximations 0.018.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=9.11e-36
Equations at solution: [-.700e-35, .911e-35, -.231e-34]Solution in
2.687s

Time Plot 0 s.
Exiting SolveHard() after 2.967r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
```

```
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69, none,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017539016422788511495851877936006,
6.0258135493800481229862255300484921869 40,
351.42702948512938048333122963104349379 64]
one interval r = 31.362302061213179766320822328181549575 41 ..
34.174466406192966788896620141162228328 41
Time Approximations 0.016.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, .292e-34]Solution in 0.549s

Time Plot 0 s.
Exiting SolveHard() after 0.82r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69, none,
```

```
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88, none, none,
292.99969138311589069970614313631314662 44, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175390164227885114958518779360 06,
6.0258135493800481229862255300484921869 40,
351.42702948512938048333122963104349379 64]
two intervals r = 17.981355144383743736792664245535147224 61 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000 or r = 13.846080154253773582291196607110317326 75 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 8.178 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713644570475225799053129542921 01, rm
= 2.7345009929592694646457014171734695345 57}});
Accepted {r=18.6878, rm=15.3648} with Delta=5e-38
Equations at solution: [-.144e-36, .5e-37, -.9e-37]Solution in 20.441s

Time Plot 0 s.
Exiting SolveHard() after 21.505r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
```

```
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88, none, none,
292.99969138311589069970614313631314662 44, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941873061270893078005495610541962,
6.37794387399026148023231654600574813469 1,
423.28832784512212536760127405269817089 68]
one interval r = 31.94661817608510799800908642081586141505 ..
35.21212308661225574759883892576965085264
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, .282e-34]Solution in 0.602s

Time Plot 0 s.
Exiting SolveHard() after 0.92r=34.3272 in [33.10127385 .. 35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88, none, none,
292.99969138311589069970614313631314662 44, none, none,
360.06173467205394564581722582546252233 87, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941873061270893078005495610541962,
6.37794387399026148023231654600574813469 1,
```

```
423.2883278451221253676012740526981708968]
two intervals r = 15.22886702415101975173953527484882325635 ..
189999999999563487820532062363305315 89/10000000000000000000000000000000000
00000 or r = 17.12965777087912115014113009866235935419 ..
189999999999563487820532062363305315 89/10000000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 5.761 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541120362327379174270283522313 26, rm
= 2.06406829861403444742011577915728046965 8}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., .1519e-34]Solution in 28.73s

Time Plot 0 s.
Exiting SolveHard() after 32.204r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88,
375.73285290782389704376214143152247977 50, none,
292.99969138311589069970614313631314662 44, none, none,
360.06173467205394564581722582546252233 87, none, none, none, none,
none, none, none]
```

```
0 --> 2  target = [34.939532343477651976792175935108656282 94,
4.0035598154924891091990631715841688480 86,
404.47973594242194997301251426253880657 00]
two intervals r = 16.09683966364362877000304320787369452277 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000 or r = 16.39988649118074752397209300520150516928 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000
Time Approximations 0.054.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.3644e-34]Solution in 3.783s

Time Plot 0 s.
Exiting SolveHard() after 4.835r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88,
375.73285290782389704376214143152247977 50, none,
292.99969138311589069970614313631314662 44,
358.64341560804054818314593602519846921 62, none,
360.06173467205394564581722582546252233 87, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343477651976792175935108656282 94,
4.0035598154924891091990631715841688480 86,
```

```
404.47973594242194997301251426253880657 00]
one interval r = 21.63429630008162229457132546561626321844 ..
26.75768169888247872816686824718399527022
Time Approximations 0.052.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=1.01e-37
Equations at solution: [-.2e-37, -.101e-36, .116e-34]Solution in 3.287s

Time Plot 0 s.
Exiting SolveHard() after 4.32r=25.8653 in [23.83864811 .. 26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88,
375.73285290782389704376214143152247977 50,
328.11709294463474586596213394999188163 69,
292.99969138311589069970614313631314662 44,
358.64341560804054818314593602519846921 62, none,
360.06173467205394564581722582546252233 87, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954452783684994541929091730239922,
6.19617723027385027432821677598547624895 5,
385.42734025717776221857302008757532253 24]
one interval r = 31.60822049098977908851325866113991622419 ..
34.66347615050421615126189814313826600123
Time Approximations 0.017.
```

```
hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=8e-38
Equations at solution: [-.6e-37, .8e-37, .89e-35]Solution in 0.572s

Time Plot 0 s.
Exiting SolveHard() after 0.873r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88,
375.73285290782389704376214143152247977 50,
328.11709294463474586596213394999188163 69,
292.99969138311589069970614313631314662 44,
358.64341560804054818314593602519846921 62, none,
360.06173467205394564581722582546252233 87, none, none,
324.65521223487217630214816762206033469 30, none, none, none, none]

0 --> 1  target = [26.46318954452783684994541929091730239922,
6.19617723027385027432821677598547624895 5,
385.42734025717776221857302008757532253 24]
two intervals r = 16.87629600293240624369401597649731513243 ..
18999999999956348782053206236330531589/10000000000000000000000000000000
00000 or r = 15.55559000648606963737559445878952476260 ..
18999999999956348782053206236330531589/10000000000000000000000000000000
00000
Time Approximations 0.057.
```

```
hint used Hint := [17.9309208304359097016944008190092163318 1, 3, 1, 1,
15.7008606270133975112531574051596448801 2, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 6.02 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852530401066849775267868737666635, rm
= 2.33669042803179055534113984429696835050 3}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.359e-37, 0., .4216e-34]Solution in 27.974s

Time Plot 0 s.
Exiting SolveHard() after 31.385r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88,
375.73285290782389704376214143152247977 50,
328.11709294463474586596213394999188163 69,
292.99969138311589069970614313631314662 44,
358.64341560804054818314593602519846921 62, none,
360.06173467205394564581722582546252233 87,
336.59441032347791439280042856175541377 56, none,
324.65521223487217630214816762206033469 30, none, none, none, none]

0 --> 2  target = [34.49522661164185947768358028175339121158,
3.89713131589404582508250359663488372641 2,
373.78081884195614722576836880188467984 47]
two intervals r = 17.29769086233767095580407660548615124014 ..
189999999999563487820532062363305315 89/10000000000000000000000000000000
```

```
00000 or r = 14.994364074160903579701135742768148121 17 ..
189999999999563487820532062363305315 89/100000000000000000000000000000000
00000
Time Approximations 0.084.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [.91e-37, -.1e-37, -.4225e-34]Solution in 1.2s

Time Plot 0 s.
Exiting SolveHard() after 5.201r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88,
375.73285290782389704376214143152247977 50,
328.11709294463474586596213394999188163 69,
292.99969138311589069970614313631314662 44,
358.64341560804054818314593602519846921 62, none,
360.06173467205394564581722582546252233 87,
336.59441032347791439280042856175541377 56, none,
324.65521223487217630214816762206033469 30,
331.93806790991598112654289787996758183 07, none, none, none]

1 --> 2  target = [34.495226611641859477683580281753391211 58,
3.897131315894045825082503596634883726 12,
373.78081884195614722576836880188467984 47]
one interval r = 21.060684732181928345125330120475389634 64 ..
```

```
26.26979834273519269579729806308713718568
Time Approximations 0.034.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., -.167e-34]Solution in 0.768s

Time Plot 0 s.
Exiting SolveHard() after 3.725r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88,
375.73285290782389704376214143152247977 50,
328.11709294463474586596213394999188163 69,
292.99969138311589069970614313631314662 44,
358.64341560804054818314593602519846921 62,
299.89866204443704494153211519770682466 02,
360.06173467205394564581722582546252233 87,
336.59441032347791439280042856175541377 56, none,
324.65521223487217630214816762206033469 30,
331.93806790991598112654289787996758183 07, none, none, none]

0 --> 2  target = [33.81362495411522450259948514191455666967,
3.72564899353271468504217952346635764809 5,
325.89209972744248947291108760513275807 02]
two intervals r = 18.55227049003903452494936827002813556822 ..
```

```
1899999999995634878205320623633053158 9/10000000000000000000000000000000
00000 or r = 12.49196935786066377536411343805523286433 ..
1899999999995634878205320623633053158 9/10000000000000000000000000000000
00000
Time Approximations 0.038.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [.120e-36, -.5e-37, -.182e-35]Solution in 1.092s

Time Plot 0 s.
Exiting SolveHard() after 4.608r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88,
375.73285290782389704376214143152247977 50,
328.11709294463474586596213394999188163 69,
292.99969138311589069970614313631314662 44,
358.64341560804054818314593602519846921 62,
299.89866204443704494153211519770682466 02,
360.06173467205394564581722582546252233 87,
336.59441032347791439280042856175541377 56, none,
324.65521223487217630214816762206033469 30,
331.93806790991598112654289787996758183 07, none, none,
289.54595772218613859356663288763403839 65]

1 --> 2  target = [33.81362495411522450259948514191455666967,
```

```
3.725648993532714685042179523466357648095,
325.892099727442489472911087605132758070 2]
one interval r = 20.37468935129809439773790434283924103849 ..
25.37892165295401035952307000031069790157
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, .513e-34]Solution in 0.554s

Time Plot 0 s.
Exiting SolveHard() after 3.085r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88,
375.73285290782389704376214143152247977 50,
328.11709294463474586596213394999188163 69,
292.99969138311589069970614313631314662 44,
358.64341560804054818314593602519846921 62,
299.89866204443704494153211519770682466 02,
360.06173467205394564581722582546252233 87,
336.59441032347791439280042856175541377 56,
256.10753185875106084182084492628913528 98,
324.65521223487217630214816762206033469 30,
331.93806790991598112654289787996758183 07, none, none,
289.54595772218613859356663288763403839 65]
```

```
1 --> 0  target = [17.93041369705358196479023174384930745378,
4.686508702062860489777766235585864813009,
353.30541095109963011461716808004817622 78]
one interval r = 20.731504791138646944151601987554282510 83 ..
25.906753535194617057115278526629903108 99
Time Approximations 0.034.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, .128e-34]Solution in 0.658s

Time Plot 0 s.
Exiting SolveHard() after 1.37r=25.4021 in [22.67806074 .. 25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88,
375.73285290782389704376214143152247977 50,
328.11709294463474586596213394999188163 69,
292.99969138311589069970614313631314662 44,
358.64341560804054818314593602519846921 62,
299.89866204443704494153211519770682466 02,
360.06173467205394564581722582546252233 87,
336.59441032347791439280042856175541377 56,
256.10753185875106084182084492628913528 98,
324.65521223487217630214816762206033469 30,
331.93806790991598112654289787996758183 07,
304.79958325682578719522934722044934676 01, none,
```

```
289.5459577221861385935666328876340383965]

2 --> 0  target = [17.93041369705358196479023174384930745378,
4.68650870206286048977776623558586481300 9,
353.30541095109963011461716808004817622 78]
one interval r = 31.37435487000366149483926902736085187903 ..
34.20127520031528886086867406427999600428
Time Approximations 0.014.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=1.1e-37
Equations at solution: [.7e-37, -.11e-36, .432e-34]Solution in 0.359s

Time Plot 0 s.
Exiting SolveHard() after 2.728r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493917020683347964304576515984 09,
441.64295973531594864725624230050558523 25,
436.91748165402636030903436807580803892 35,
422.98493397911871228681679959875121408 93,
361.52580256095363150243334470381468724 68,
401.88173904474031433442557540377719173 65,
389.59001515778792591247337604538317938 17,
328.46939893557582161143475195276425204 09,
401.50757158210344118489733768220736728 66,
358.97362823970204535064013391932768788 18,
398.33147104631295675001675534481329771 74,
371.48387394122397690817496713504542655 48,
336.61215841300765064975772455261691262 62,
361.50888347136823370369666272542811136 37,
324.67144992505030261519215284608024874 69,
302.31384315032054284971661822820372934 09,
328.46938513630359810756534222447024778 99,
343.81340624690016637996634549913704972 88,
375.73285290782389704376214143152247977 50,
328.11709294463474586596213394999188163 69,
292.99969138311589069970614313631314662 44,
358.64341560804054818314593602519846921 62,
299.89866204443704494153211519770682466 02,
360.06173467205394564581722582546252233 87,
336.59441032347791439280042856175541377 56,
256.10753185875106084182084492628913528 98,
```

```
324.65521223487217630214816762206033469 30,
331.93806790991598112654289787996758183 07,
304.79958325682578719522934722044934676 01,
323.46169176459425350438706791976636718 67,
289.54595772218613859356663288763403839 65]

Cascade time 243.783
counts: 28, 28

 Iteration 43

Start Generation 1
1 --> 0  target = [12.00000000012034228266109554867795033600,
6.21701250300369656312980054939222115179 5,
485.54908089787753833510149102277072051 44]
one interval r = 23.40850301658232032722839544506968134374 ..
27.67578046442240258870108281320275687637
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=0
Equations at solution: [0., 0., -.3e-36]Solution in 2.974s

Time Plot 0 s.
Exiting SolveHard() after 4.119r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000012034228266109554867795033600,
6.21701250300369656312980054939222115179 5,
485.54908089787753833510149102277072051 44]
one interval r = 32.62814779202880259961092835832503259598 ..
36.10248388942614878759195833925016435443
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., .4902e-35]Solution in 0.626s

Time Plot 0 s.
Exiting SolveHard() after 1.048r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684494090627399264318024848564219,
6.58343472154882692077244782937274190114 6,
467.78730595938096779979410143734047195 95]
one interval r = 32.41978955652614061610595442531161655375 ..
35.85152417372854501164493522609909006139
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=6e-38
Equations at solution: [-.8e-37, .6e-37, .24246e-34]Solution in 2.705s

Time Plot 0 s.
Exiting SolveHard() after 3.077r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
```

```
436.917481652445830107423173906202136768 4, none, none,
401.881739044852245676596689646378527718 6, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844940906273992643180248485642 19,
6.583434721548826920772447829372741901146,
467.787305959380967799794101437340471959 5]
```

"Imaginary part neglected: ", $1.10311211490015734205772292130214567158 1 \times 10^{-17}$

```
two intervals r = 12.923271608393797393043436566871952053 35 ..
190000000000625421938415328206763740 99/100000000000000000000000000000000
00000 or r = 18.394248580361172446180542892928094505 22 ..
190000000000625421938415328206763740 99/100000000000000000000000000000000
00000
Time Approximations 0.047.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=0
Equations at solution: [0., 0., .1127e-34]Solution in 40.827s

Time Plot 0 s.
Exiting SolveHard() after 42.199r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936028501079661169925473033662 0,
441.642959732455536406547409084893857658 6,
436.917481652445830107423173906202136768 4,
422.984933978174483828193732384102709243 3, none,
401.881739044852245676596689646378527718 6, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628241785498941156540573249741 51,
4.125651796897463175922535237716333614976,
440.671230649953411715061317722771297411 8]
```

"Imaginary part neglected: ", $1.10311211490015734205772292130214567158 1 \times 10^{-17}$

```
two intervals r = 14.356597051329829265665946030348591082 92 ..
190000000000625421938415328206763740 99/100000000000000000000000000000000
00000 or r = 17.703526138097878650101014172308243068 56 ..
```

```
190000000000625421938415328206763740 99/100000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., -.4085e-34]Solution in 3.549s

Time Plot 0 s.
Exiting SolveHard() after 4.627r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33, none,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962824178549894115654057324974151,
4.12565179689746317592253523771633361 4976,
440.67123064995341171506131772277129741 18]
one interval r = 22.39761154366919925120028307199930777629 ..
27.23722351610272717719668256990872960544
Time Approximations 0.039.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.624 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064393750215765681249723395595001, rm =
```

```
14.37818770346866994155384083794771206406}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, -.143e-34]Solution in 10.124s

Time Plot 0 s.
Exiting SolveHard() after 11.011r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888799249714472310877878010661584,
4.0048690818838391886045295554011217001 57,
404.86224501542432519883076728004719020 89]
```

"Imaginary part neglected: ", $1.10311211490015734205772292130214567158 1 \times 10^{-17}$

```
two intervals r = 16.08011007761217047991337979785805545570 ..
19000000000062542193841532820676374099/10000000000000000000000000000000
00000 or r = 16.41579812704333094458249653426867618245 ..
19000000000062542193841532820676374099/10000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.51e-37, -.1e-37, -.561e-35]Solution in 3.805s

Time Plot 0 s.
Exiting SolveHard() after 7.042r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26, none, none,
358.97362824101783190468746478322795347 07, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888799249714472310877878010661584,
4.0048690818838391886045295554011217001 57,
404.86224501542432519883076728004719020 89]
one interval r = 21.641943994159194891838765488353036118 75 ..
26.763306600564084351730831251516778879 29
Time Approximations 0.051.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [0., .26e-37, .177e-34]Solution in 3.225s

Time Plot 0 s.
Exiting SolveHard() after 4.243r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86, none,
358.97362824101783190468746478322795347 07, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941778602149851632923287379989789,
5.5896371830672024857366218247158915879 41,
443.83065884912637355124565900878867236 48]
one interval r = 22.467253744867267747516471306560544578 00 ..
27.273884283710258175307315510420587084 31
```

```
Time Approximations 0.038.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, -.54e-35]Solution in 0.998s

Time Plot 0 s.
Exiting SolveHard() after 1.954r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86, none,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941778602149851632923287379989789,
5.589637183067202485736621824715891587941,
443.83065884912637355124565900878867236 48]
one interval r = 32.15575279493048114925362778177033651946 ..
35.50872228741847446870240562529257575326
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., -.1727e-35]Solution in 0.448s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.949r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136525138582850938891941446602669,
5.18778357857429210893073720117273608 1656,
408.65773862637360508795059077063890 91552]
one interval r = 21.71840114649560524923300280041436835719 ..
26.81849303522417439181979626819263667793
Time Approximations 0.062.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.59e-37
Equations at solution: [-.2e-37, -.159e-36, .38e-35]Solution in 3.079s

Time Plot 0 s.
Exiting SolveHard() after 4.19r=26.4632 in [23.93303356 .. 26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
```

```
389.590015157582762826467886467133386612 6,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31, none, none,
361.50888347282560104796751315316221793 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136525138582850938891941446602669,
5.18778357857429210893073720117273608165 6,
408.65773862637360508795059077063890915 52]
one interval r = 31.80828598737593561700254471367881399890 ..
35.00011460042355347795864375039098606548
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.16525e-34]Solution in 0.434s

Time Plot 0 s.
Exiting SolveHard() after 0.744r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.590015157582762826467886467133386612 6,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13, none,
361.50888347282560104796751315316221793 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110553724328497341939481772827239,
6.19626256528895659752663589189269451838 4,
385.44474379512038678375283097726878016 35]
```

```
one interval r = 31.60836097524057346648544915837149110746 ..
34.66372795609293301090362816949098676391
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .420e-36]Solution in 0.558s

Time Plot 0 s.
Exiting SolveHard() after 0.83r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13, none,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110553724328497341939481772827239,
6.196262565288956597526635891892694518384,
385.44474379512038678375283097726878016 35]

"Imaginary part neglected: ", 1.10311211490015734205772292130214567158 1 × 10^-17
two intervals r = 16.87563408757186186321601571468549653864 ..
19000000000062542193841532820676374099/10000000000000000000000000000000
00000 or r = 15.55640493812396684463156280695113944328 ..
19000000000062542193841532820676374099/10000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.179e-37, 0., -.1193e-34]Solution in 3.363s

Time Plot 0 s.
Exiting SolveHard() after 6.887r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874739085073745446871879773926370,
4.8838107799301555770120465378635109466 73,
376.61967855993101206755107670686709779 52]
one interval r = 21.11001304880430698532334612158502023221 ..
26.31784243488330328157726412124576204239
Time Approximations 0.034.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., .16e-35]Solution in 0.85s

Time Plot 0 s.
```

```
Exiting SolveHard() after 1.547r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01, none,
328.46938513950578611173042053692621196 09, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874739085073745446871879773926370,
4.88381077993015557701204653786351094667 3,
376.61967855993101206755107670686709779 52]
one interval r = 31.53899497697878558622022579725794947111 ..
34.53618386092183537735853426347080093925
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.054e-35
Equations at solution: [.810e-35, -.1054e-34, -.17529e-34]Solution in
2.731s

Time Plot 0 s.
Exiting SolveHard() after 3.014r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
```

```
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01, none,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017558594767921135483633456766289,
6.0258135492601529616188435805896162218 14,
351.42702948854143215717719752254699158 93]
one interval r = 31.362302061011848563380515480783473383 09 ..
34.174466406160731917665904579257432165 69
Time Approximations 0.016.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, .8755e-35]Solution in 0.541s

Time Plot 0 s.
Exiting SolveHard() after 0.811r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
```

```
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01, none,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93, none, none,
292.99969138943199645538518558990490572 14, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175585947679211354836334567662 89,
6.0258135492601529616188435805896162218 14,
351.42702948854143215717719752254699158 93]
```

"Imaginary part neglected: ", $1.10311211490015734205772292130214567158 1 \times 10^{-17}$

```
two intervals r = 17.981355144394224778142163989538292452 30 ..
19000000000062542193841532820676374099/10000000000000000000000000000000
00000 or r = 13.846080154443776232280598018340988012 35 ..
19000000000062542193841532820676374099/10000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [-.17e-37, 0., .381e-35]Solution in 3.585s

Time Plot 0 s.
Exiting SolveHard() after 4.633r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
```

```
324.67144992970281241376080933920649423 01,
302.31384315598777213882717576172406602 98,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93, none, none,
292.99969138943199645538518558990490572 14, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941892813313577049940557013308923,
6.3779438738499127762840942884416185758 23,
423.28832784557466384669525699416541906 43]
one interval r = 31.946618175898761431250155236560002687 85 ..
35.212123086583025465207300978129992680 58
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., .14092e-34]Solution in 0.614s

Time Plot 0 s.
Exiting SolveHard() after 0.961r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01,
302.31384315598777213882717576172406602 98,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93, none, none,
292.99969138943199645538518558990490572 14, none, none,
360.06173467540565095595548974031499841 68, none, none, none, none,
none, none, none]
```

```
0 --> 1  target = [27.020379418928133135770499405570133089 23,
6.3779438738499127762840942884416185758 23,
423.28832784557466384669525699416541906 43]
```

"Imaginary part neglected: ", $1.10311211490015734205772292130214567158 1 \times 10^{-17}$

```
two intervals r = 15.22886702419610207312284622869546578089 ..
19000000000062542193841532820676374099/10000000000000000000000000000000
00000 or r = 17.12965777096709243280238627777312880467 ..
19000000000062542193841532820676374099/10000000000000000000000000000000
00000
Time Approximations 0.063.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.48e-37, -.1e-37, .3589e-34]Solution in 1.294s

Time Plot 0 s.
Exiting SolveHard() after 5.058r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01,
302.31384315598777213882717576172406602 98,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93,
375.73285291065790382222766434725801558 45, none,
292.99969138943199645538518558990490572 14, none, none,
360.06173467540565095595548974031499841 68, none, none, none, none,
none, none, none]
```

```
0 --> 2  target = [34.93953234343913697427354812678471032561,
4.0035598155857596461237582851947977809 75,
404.47973594302675768937462785928203655 60]
```

"Imaginary part neglected: ", $1.10311211490015734205772292130214567158 1 \times 10^{-17}$

```
two intervals r = 16.09683966369734799442163456172102023067 ..
19000000000062542193841532820676374099/10000000000000000000000000000000
00000 or r = 16.39988649126524488922539082641524543547 ..
19000000000062542193841532820676374099/10000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.34e-37, .1e-37, .2058e-34]Solution in 3.682s

Time Plot 0 s.
Exiting SolveHard() after 7.026r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01,
302.31384315598777213882717576172406602 98,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93,
375.73285291065790382222766434725801558 45, none,
292.99969138943199645538518558990490572 14,
358.64341560998487604560330363574251285 80, none,
360.06173467540565095595548974031499841 68, none, none, none, none,
none, none, none]
```

```
1 --> 2  target = [34.93953234343913697427354812678471032561,
4.0035598155857596461237582851947977809 75,
404.47973594302675768937462785928203655 60]
one interval r = 21.63429629994252880641142860337376832196 ..
26.75768169907016575764752597396215772731
Time Approximations 0.05.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=9.8e-38
Equations at solution: [.3e-37, .98e-37, -.275e-34]Solution in 3.212s

Time Plot 0 s.
Exiting SolveHard() after 4.227r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01,
302.31384315598777213882717576172406602 98,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93,
375.73285291065790382222766434725801558 45,
328.11709294851069421820488831780323447 25,
292.99969138943199645538518558990490572 14,
358.64341560998487604560330363574251285 80, none,
360.06173467540565095595548974031499841 68, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954471769251117097926875678634985,
6.19617723014203052583824347516991516300 2,
385.42734025878039307606657962588120594 13]
```

```
one interval r = 31.6082204907934772913379951731443746548 8 ..
34.66347615046821620972210432183315580413
Time Approximations 0.016.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=5e-38
Equations at solution: [.3e-37, -.5e-37, .4342e-35]Solution in 0.571s

Time Plot 0 s.
Exiting SolveHard() after 0.874r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01,
302.31384315598777213882717576172406602 98,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93,
375.73285291065790382222766434725801558 45,
328.11709294851069421820488831780323447 25,
292.99969138943199645538518558990490572 14,
358.64341560998487604560330363574251285 80, none,
360.06173467540565095595548974031499841 68, none, none,
324.65521223940550645361741490860068762 48, none, none, none, none]

0 --> 1  target = [26.46318954471769251117097926875678634985,
6.19617723014203052583824347516991516300 2,
385.42734025878039307606657962588120594 13]
```

"Imaginary part neglected: ", $1.10311211490015734205772292130214567158 1 \times 10^{-17}$

```
two intervals r = 16.876296002961720485512739621159345428 35 ..
190000000000625421938415328206763740 99/100000000000000000000000000000000000
00000 or r = 15.555590006605254133582272872101938126 87 ..
190000000000625421938415328206763740 99/100000000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.538e-37, 0., .890e-35]Solution in 1.133s

Time Plot 0 s.
Exiting SolveHard() after 4.522r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01,
302.31384315598777213882717576172406602 98,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93,
375.73285291065790382222766434725801558 45,
328.11709294851069421820488831780323447 25,
292.99969138943199645538518558990490572 14,
358.64341560998487604560330363574251285 80, none,
360.06173467540565095595548974031499841 68,
336.59441032744350481966806735333986742 76, none,
324.65521223940550645361741490860068762 48, none, none, none, none]

0 --> 2  target = [34.49522661159702742142134198235639689852,
3.89713131599035770040868083370159103743 0,
373.78081884346346063316584471556214893 27]
```

"Imaginary part neglected: ", $1.10311211490015734205772292130214567158 1 \times 10^{-17}$

```
two intervals r = 17.29769086238105889480848334998523659333 ..
190000000000625421938415328206763740 99/100000000000000000000000000000000
00000 or r = 14.99436407426892738901316141423940238817 ..
190000000000625421938415328206763740 99/100000000000000000000000000000000
00000
Time Approximations 0.085.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [0., 0., .423e-35]Solution in 1.208s

Time Plot 0 s.
Exiting SolveHard() after 5.319r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01,
302.31384315598777213882717576172406602 98,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93,
375.73285291065790382222766434725801558 45,
328.11709294851069421820488831780323447 25,
292.99969138943199645538518558990490572 14,
358.64341560998487604560330363574251285 80, none,
360.06173467540565095595548974031499841 68,
336.59441032744350481966806735333986742 76, none,
324.65521223940550645361741490860068762 48,
331.93806791267024947872414936977541814 97, none, none, none]
```

```
1 --> 2  target = [34.49522661159702742142134198235639689852,
3.89713131599035770040868083370159103743 0,
373.78081884346346063316584471556214893 27]
one interval r = 21.06068473200931398819497868475798379940 ..
26.26979834291473493223874655313796989072
Time Approximations 0.033.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=4e-38
Equations at solution: [-.2e-37, -.4e-37, -.467e-34]Solution in 0.769s

Time Plot 0 s.
Exiting SolveHard() after 4.111r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01,
302.31384315598777213882717576172406602 98,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93,
375.73285291065790382222766434725801558 45,
328.11709294851069421820488831780323447 25,
292.99969138943199645538518558990490572 14,
358.64341560998487604560330363574251285 80,
299.89866204916021975460157888470978455 78,
360.06173467540565095595548974031499841 68,
336.59441032744350481966806735333986742 76, none,
324.65521223940550645361741490860068762 48,
331.93806791267024947872414936977541814 97, none, none, none]
```

```
0 --> 2  target = [33.81362495408094860026634847877128189827,
3.72564899363968989951752463307963668353 7,
325.89209973196201425522008432562867018 80]
```

"Imaginary part neglected: ", $1.10311211490015734205772292130214567158 1 \times 10^{-17}$

```
two intervals r = 18.552270490064548368831584673666360913 88 ..
190000000000625421938415328206763740 99/100000000000000000000000000000000
00000 or r = 12.491969358074697470114751136373800859 13 ..
190000000000625421938415328206763740 99/100000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=0
Equations at solution: [.17e-37, 0., .2947e-34]Solution in 1.194s

Time Plot 0 s.
Exiting SolveHard() after 4.757r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01,
302.31384315598777213882717576172406602 98,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93,
375.73285291065790382222766434725801558 45,
328.11709294851069421820488831780323447 25,
292.99969138943199645538518558990490572 14,
358.64341560998487604560330363574251285 80,
299.89866204916021975460157888470978455 78,
```

```
360.06173467540565095595548974031499841 68,
336.59441032744350481966806735333986742 76, none,
324.65521223940550645361741490860068762 48,
331.93806791267024947872414936977541814 97, none, none,
289.54595772760229529245715333709271038 07]

1 --> 2  target = [33.81362495408094860026634847877128189827,
3.7256489936396898995175246330796366835 37,
325.89209973196201425522008432562867018 80]
one interval r = 20.37468935108938833500299665742351693888 ..
25.37892165314947421505856185544583013387
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.2e-37, .2e-37, -.121e-34]Solution in 0.575s

Time Plot 0 s.
Exiting SolveHard() after 3.292r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01,
302.31384315598777213882717576172406602 98,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93,
375.73285291065790382222766434725801558 45,
328.11709294851069421820488831780323447 25,
292.99969138943199645538518558990490572 14,
358.64341560998487604560330363574251285 80,
```

```
299.89866204916021975460157888470978455 78,
360.06173467540565095595548974031499841 68,
336.59441032744350481966806735333986742 76,
256.10753186625132629155570784963535634 00,
324.65521223940550645361741490860068762 48,
331.93806791267024947872414936977541814 97, none, none,
289.54595772760229529245715333709271038 07]

1 --> 0  target = [17.93041369703831746461759983558888072797,
4.68650870209649900769413670008131406223 4,
353.30541095535300278266337757445318176 18]
one interval r = 20.73150479097488747659165445377756406520 ..
25.90675353540737880910802274880270539597
Time Approximations 0.033.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., -.277e-34]Solution in 0.649s

Time Plot 0 s.
Exiting SolveHard() after 1.356r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.64295973245553640654740908489385765 86,
436.91748165244583010742317390620213676 84,
422.98493397817448382819373238410270924 33,
361.52580256253523637112715064424740675 22,
401.88173904485224567659668964637852771 86,
389.59001515758276282646788646713338661 26,
328.46939893878136455259853593728404943 86,
401.50757158292712487529345518348686139 51,
358.97362824101783190468746478322795347 07,
398.33147104666263147097198489013091245 31,
371.48387394293098182443604342671397549 13,
336.61215841710361035456223231490968689 63,
361.50888347282560104796751315316221793 81,
324.67144992970281241376080933920649423 01,
302.31384315598777213882717576172406602 98,
328.46938513950578611173042053692621196 09,
343.81340625010822932472724113318642214 93,
375.73285291065790382222766434725801558 45,
328.11709294851069421820488831780323447 25,
```

```
292.99969138943199645538518558990490572 14,
358.6434156099848760456033036357425128580,
299.8986620491602197546015788847097845578,
360.0617346754056509559554897403149984168,
336.5944103274435048196680673533398674276,
256.1075318662513262915557078496353563400,
324.6552122394055064536174149086006876248,
331.9380679126702494787241493697754181497,
304.7995832629724302496141278873029345706, none,
289.5459577276022952924571533370927103807]

2 --> 0  target = [17.93041369703831746461759983558888072797,
4.6865087020964990076941367000813140622 34,
353.3054109553530027826633775744531817618]
one interval r = 31.37435486980879156818273616510144639424 ..
34.20127520029633615646863534968287513315
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=5e-38
Equations at solution: [.3e-37, -.5e-37, .5426e-35]Solution in 0.346s

Time Plot 0 s.
Exiting SolveHard() after 2.901r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493602850107966116992547303366 20,
441.6429597324555364065474090848938576586,
436.9174816524458301074231739062021367684,
422.9849339781744838281937323841027092433,
361.5258025625352363711271506442474067522,
401.8817390448522456765966896463785277186,
389.5900151575827628264678864671333866126,
328.4693989387813645525985359372840494386,
401.5075715829271248752934551834868613951,
358.9736282410178319046874647832279534707,
398.3314710466626314709719848901309124531,
371.4838739429309818244360434267139754913,
336.6121584171036103545622323149096868963,
361.5088834728256010479675131531622179381,
324.6714499297028124137608093392064942301,
302.3138431559877721388271757617240660298,
328.4693851395057861117304205369262119609,
```

```
343.8134062501082293247272411331864221493,
375.7328529106579038222276643472580155845,
328.1170929485106942182048883178032344725,
292.9996913894319964553851855899049057214,
358.6434156099848760456033036357425128580,
299.8986620491602197546015788847097845578,
360.0617346754056509559554897403149984168,
336.5944103274435048196680673533398674276,
256.1075318662513262915557078496353563400,
324.6552122394055064536174149086006876248,
331.9380679126702494787241493697754181497,
304.7995832629724302496141278873029345706,
323.4616917703960956118251094235664737231,
289.5459577276022952924571533370927103807]

Cascade time 149.387
counts: 28, 28

 Iteration 44

Start Generation 1
1 --> 0  target = [11.999999999979293363784097098916910771 00,
6.217012502746203523099430986588023254565,
485.5490809055684723082749245845895073620]
one interval r = 23.40850301663490073245473516484426943433 ..
27.67578046422707779690458887236724065223
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=8.0e-38
Equations at solution: [-.3e-37, .80e-37, -.10e-35]Solution in 3.264s

Time Plot 0 s.
Exiting SolveHard() after 4.41r=27.5236 in [25.56992694 .. 27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349433548676707667953079802689847,
441.6429597373937558922784783002984194567, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999979293363784097098916910771 00,
6.217012502746203523099430986588023254565,
```

```
485.549080905568472308274924584589507362 0]
```

"Imaginary part neglected: ", $3.183223432212849731105990893780303693505 \times 10^{-17}$

```
one interval r = 32.628147792192906694385835432094707032 71 ..
36.102483889475678593244736251616204967 76
Time Approximations 0.024.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=6e-38
Equations at solution: [.9e-37, -.6e-37, -.103e-34]Solution in 0.618s

Time Plot 0 s.
Exiting SolveHard() after 1.043r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844744912264887176995102253906 43,
6.583434721650855476938178010786895138011,
467.787305964603918758964230738594308412 5]
```

"Imaginary part neglected: ", $3.183223432212849731105990893780303693505 \times 10^{-17}$

```
one interval r = 32.419789556666232470975075570537202943 35 ..
35.851524173753047033170922985571705415 44
Time Approximations 0.025.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
```

```
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., -.11e-35]Solution in 2.842s

Time Plot 0 s.
Exiting SolveHard() after 3.217r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32, none, none,
401.88173904826890813698462346159133935 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684474491226488717699510225390643,
6.58343472165085547693817801078689513801 1,
467.78730596460391875896423073859430841 25]
two intervals r = 12.92327160832829737090374421415513331954 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000 or r = 18.39424858037852532674152307108352381868 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.2e-38
Equations at solution: [0., .12e-37, -.1279e-34]Solution in 42.423s

Time Plot 0 s.
Exiting SolveHard() after 43.763r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82, none,
401.88173904826890813698462346159133935 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962834794605170823451872116336938,
4.12565179685635894532340523640957038559 7,
440.67123065982871029099662250131260078 58]
two intervals r = 14.35659705097240191780278978087763520599 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000 or r = 17.70352613831097699919412886682687485286 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [.72e-37, .3e-37, -.1384e-34]Solution in 3.563s

Time Plot 0 s.
Exiting SolveHard() after 4.613r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82, none,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962834794605170823451872116336938,
4.12565179685635894532340523640957038559 7,
440.67123065982871029099662250131260078 58]
one interval r = 22.39761154378346300444703793487280202393 ..
27.23722351598907946200395139947320833687
Time Approximations 0.038.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.419 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643921814616482578929889065562 34, rm =
14.37818770446726556114970720509368906665}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., .354e-34]Solution in 11.992s

Time Plot 0 s.
Exiting SolveHard() after 12.886r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888025587534152593325081111559 73,
4.0048690818216443771693291441362982098 99,
404.86224501891637689060320012996345774 29]
two intervals r = 16.080110077538155088655890342019986092 32 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000 or r = 16.415798127128184148479980983364479985 85 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000
Time Approximations 0.053.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.17e-37, .1e-37, -.2602e-34]Solution in 3.833s

Time Plot 0 s.
Exiting SolveHard() after 4.925r=17.199 in [16.08011004 .. 19]
```

```
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44, none, none,
358.97362824838604073424653871909436831 86, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888025587534152593325081111559 73,
4.0048690818216443771693291441362982098 99,
404.86224501891637689060320012996345774 29]
one interval r = 21.641943994140253144585310226107199344 50 ..
26.763306600405775674044118568978193265 81
Time Approximations 0.056.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, -.270e-34]Solution in 3.296s

Time Plot 0 s.
Exiting SolveHard() after 4.339r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30, none,
358.97362824838604073424653871909436831 86, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]
```

```
1 --> 0  target = [14.19258941804910620604521465862856515852,
5.589637182664775045718855746357280322492,
443.83065884714780485023596339444566775 54]
one interval r = 22.46725374471938123196907551294239268098 ..
27.27388428345621514187662248126857182366
Time Approximations 0.038.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, -.109e-34]Solution in 3.106s

Time Plot 0 s.
Exiting SolveHard() after 4.102r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30, none,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941804910620604521465862856515852,
5.589637182664775045718855746357280322492,
443.83065884714780485023596339444566775 54]
```

"Imaginary part neglected: ", $3.18322343221284973110599089378030369 3505 \times 10^{-17}$

```
one interval r = 32.15575279500091251085440016983797502736 ..
35.50872228735227554518547904691485036528
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
```

```
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=3e-38
Equations at solution: [.3e-37, -.3e-37, .44e-35]Solution in 0.478s

Time Plot 0 s.
Exiting SolveHard() after 0.832r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931364755843736949334977677965215 76,
5.1877835783347451817324999078803918963 78,
408.65773863935238190095368729993669933 79]
one interval r = 21.718401146668585380029820556462083991 01 ..
26.818493035199223790576519229004704269 48
Time Approximations 0.057.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.38e-37
Equations at solution: [-.2e-37, -.238e-36, .146e-34]Solution in 0.969s

Time Plot 0 s.
Exiting SolveHard() after 4.485r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74, none, none,
361.50888347982936399375776457874035107 94, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136475584373694933497767796521576,
5.18778357833474518173249990788039189637 8,
408.65773863935238190095368729993669933 79]
```

"Imaginary part neglected: ", $3.18322343221284973110599089378030369350 5 \times 10^{-17}$

```
one interval r = 31.80828598760071116653047584411106820627 ..
35.00011460059220181006342779014007568514
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, -.3e-36]Solution in 0.399s

Time Plot 0 s.
Exiting SolveHard() after 0.712r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
```

```
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04, none,
361.50888347982936399375776457874035107 94, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110545615504442587134914586659303,
6.19626256539787694029057332685520394469 5,
385.44474380240585495983773178174598364 29]
```

"Imaginary part neglected: ", $3.18322343221284973110599089378030369350 5 \times 10^{-17}$

```
one interval r = 31.60836097541421484408446428958026162409 ..
34.66372795619107886645706192769323876866
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .262e-34]Solution in 2.831s

Time Plot 0 s.
Exiting SolveHard() after 3.1r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04, none,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110545615504442587134914586659303,
```

```
6.196262565397876940290573326855203944695,
385.44474380240585495983773178174598364 29]
two intervals r = 16.875634087338551323903681501346412300 45 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000 or r = 15.556404938426534108056381257595150315 18 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 9.164 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.468341751065290054295811025511669488 42, rm
= 2.336532774198364563113385967675732315219}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.2797e-34]Solution in 30.88s

Time Plot 0 s.
Exiting SolveHard() after 32.039r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747152245141654298843213710507 23,
4.883810779625078497360296390382327509575,
376.61967856753363699109501997958876104 34]
one interval r = 21.110013048858356674385171309794473979 32 ..
```

```
26.31784243481704427678680371631208786367
Time Approximations 0.034.

hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=4.9e-38
Equations at solution: [.1e-37, .49e-37, -.289e-34]Solution in 4.143s

Time Plot 0 s.
Exiting SolveHard() after 4.838r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27, none,
328.46938514052562610143508827171103164 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874715224514165429884321371050723,
4.88381077962507849736029639038232750957 5,
376.61967856753363699109501997958876104 34]
```

"Imaginary part neglected: ", $3.18322343221284973110599089378030369350 5 \times 10^{-17}$

```
one interval r = 31.53899497715477338019574258765480463767 ..
34.53618386102901552909231231991253005975
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
```

```
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=6.65e-36
Equations at solution: [-.511e-35, .665e-35, -.295e-34]Solution in
0.498s

Time Plot 0 s.
Exiting SolveHard() after 0.778r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27, none,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017542709026623494446584693265296,
6.0258135493374311623001867929277619187 73,
351.42702948971560459505099664264751956 88]
```

"Imaginary part neglected: ", $3.18322343221284973110599089378030369350 5 \times 10^{-17}$

```
one interval r = 31.362302061144973997251861785828585291 01 ..
34.174466406188440103470007914908409566 23
Time Approximations 0.017.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
```

```
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, -.189e-34]Solution in 0.5s

Time Plot 0 s.
Exiting SolveHard() after 0.737r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27, none,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31, none, none,
292.99969138878545365564255363979725322 47, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175427090266234944465846932652 96,
6.0258135493374311623001867929277619187 73,
351.42702948971560459505099664264751956 88]
two intervals r = 17.981355144355166195604426762730091702 19 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000 or r = 13.846080154522088514741016105303828198 89 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000
Time Approximations 0.042.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 5.206 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713643044113077536194027999941 53, rm
= 2.7345009934232138222665083676286058587 25}});
```

```
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [-.89e-37, .2e-37, -.93e-35]Solution in 17.802s

Time Plot 0 s.
Exiting SolveHard() after 21.091r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31, none, none,
292.99969138878545365564255363979725322 47, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941860492551923379737482403847499,
6.37794387388852449207886333847873828 8779,
423.28832783774429854644861190739941478 75]
```

"Imaginary part neglected: ", $3.18322343221284973110599089378030369350 5 \times 10^{-17}$

```
one interval r = 31.94661817592253418624456974123606043196 ..
35.21212308644249524428341935724321573797
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, .104e-34]Solution in 0.603s

Time Plot 0 s.
Exiting SolveHard() after 0.952r=34.3272 in [33.10127385 ..
```

```
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31, none, none,
292.99969138878545365564255363979725322 47, none, none,
360.06173466647150786976289829201528170 01, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941860492551923379737482403847499,
6.37794387388852449207886333847873828877 9,
423.28832783774429854644861190739941478 75]
two intervals r = 15.22886702469120106082523488232759215352 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000
000 or r = 17.12965777061130905448697762031338674016 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000
000
Time Approximations 0.059.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.871 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.51537054142696935844541553644948482948, rm
= 2.06406829884168511331173207128865007856 7}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., .916e-35]Solution in 28.814s
```

```
Time Plot 0 s.
Exiting SolveHard() after 32.232r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31,
375.73285289439320053029826884919030349 85, none,
292.99969138878545365564255363979725322 47, none, none,
360.06173466647150786976289829201528170 01, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343459860029370188896429946910 58,
4.0035598155205996933662392384410254781 34,
404.47973594565204268098477346376317240 24]
two intervals r = 16.096839663660836268553375057937431070 73 ..
37999999999970247838051247970973165 63/2000000000000000000000000000000000
000 or r = 16.399886491314749612293089004844783233 60 ..
37999999999970247838051247970973165 63/2000000000000000000000000000000000
000
Time Approximations 0.055.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.18e-37, 0., -.2816e-34]Solution in 3.821s

Time Plot 0 s.
Exiting SolveHard() after 4.892r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31,
375.73285289439320053029826884919030349 85, none,
292.99969138878545365564255363979725322 47,
358.64341561660743199939225154124285923 29, none,
360.06173466647150786976289829201528170 01, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234345986002937018889642994691058,
4.0035598155205996933662392384410254781 34,
404.47973594565204268098477346376317240 24]
one interval r = 21.63429629990631110190516663930576895269 ..
26.75768169889941444677084428231243960870
Time Approximations 0.046.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., -.221e-34]Solution in 0.999s

Time Plot 0 s.
Exiting SolveHard() after 4.313r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31,
375.73285289439320053029826884919030349 85,
328.11709294873160019456053889636118470 37,
292.99969138878545365564255363979725322 47,
358.64341561660743199939225154124285923 29, none,
360.06173466647150786976289829201528170 01, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954463743925061024964611517208940,
6.19617723025119792718986810974734480 1856,
385.42734026611630185651049535980198241 06]
```

"Imaginary part neglected: ", $3.18322343221284973110599089378030369 3505 \times 10^{-17}$

```
one interval r = 31.60822049096752588833243133534009731338 ..
34.66347615056710073163759244061442337085
Time Approximations 0.02.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, .182e-34]Solution in 0.544s

Time Plot 0 s.
Exiting SolveHard() after 0.851r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31,
375.73285289439320053029826884919030349 85,
328.11709294873160019456053889636118470 37,
292.99969138878545365564255363979725322 47,
358.64341561660743199939225154124285923 29, none,
360.06173466647150786976289829201528170 01, none, none,
324.65521224458036687706005432958859511 07, none, none, none, none]

0 --> 1  target = [26.46318954463743925061024964611517208940,
6.19617723025119792718986810974734480185 6,
385.42734026611630185651049535980198241 06]
two intervals r = 16.87629600272650166889482615773195772 00 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000 or r = 15.55559000691023707502048937818244977641 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000
Time Approximations 0.058.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 8.294 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852518338941970752460768336704772, rm
= 2.33669042826412086024786819850522390199 0}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.538e-37, 0., -.1829e-34]Solution in 30.075s

Time Plot 0 s.
Exiting SolveHard() after 33.731r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31,
375.73285289439320053029826884919030349 85,
328.11709294873160019456053889636118470 37,
292.99969138878545365564255363979725322 47,
358.64341561660743199939225154124285923 29, none,
360.06173466647150786976289829201528170 01,
336.59441032706119108472094609326451501 10, none,
324.65521224458036687706005432958859511 07, none, none, none, none]

0 --> 2  target = [34.49522661183347169729256753528526300670,
3.89713131597394659008541288074721298205 0,
373.78081885993211376559001714153832216 22]
two intervals r = 17.29769086184080258506908953049120528564 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000 or r = 14.99436407506132778771727115577995982959 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000
Time Approximations 0.084.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.36e-37, 0., .964e-35]Solution in 3.644s

Time Plot 0 s.
Exiting SolveHard() after 5.259r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31,
375.73285289439320053029826884919030349 85,
328.11709294873160019456053889636118470 37,
292.99969138878545365564255363979725322 47,
358.64341561660743199939225154124285923 29, none,
360.06173466647150786976289829201528170 01,
336.59441032706119108472094609326451501 10, none,
324.65521224458036687706005432958859511 07,
331.93806793157771246491093254090105273 29, none, none, none]

1 --> 2  target = [34.49522661183347169729256753528526300670,
3.8971313159739465900854128807472129820 50,
373.78081885993211376559001714153832216 22]
one interval r = 21.06068473221474552103736824428064939129 ..
26.26979834300249780803352772200441900601
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, .564e-34]Solution in 3.115s

Time Plot 0 s.
Exiting SolveHard() after 3.838r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31,
375.73285289439320053029826884919030349 85,
328.11709294873160019456053889636118470 37,
292.99969138878545365564255363979725322 47,
358.64341561660743199939225154124285923 29,
299.89866206206139467079143810519866312 90,
360.06173466647150786976289829201528170 01,
336.59441032706119108472094609326451501 10, none,
324.65521224458036687706005432958859511 07,
331.93806793157771246491093254090105273 29, none, none, none]

0 --> 2  target = [33.81362495417899683673476563085578115570,
3.72564899358469645795313431198628357389 8,
325.89209973720917894651604057902231134 42]
two intervals r = 18.552270489932910392136481991402075646 18 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000 or r = 12.491969358420582594449299650252732466 60 ..
37999999999970247838051247970973165 63/20000000000000000000000000000000000
000
Time Approximations 0.039.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1.0e-37
Equations at solution: [.243e-36, -.10e-36, .298e-34]Solution in 3.269s

Time Plot 0 s.
Exiting SolveHard() after 4.655r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31,
375.73285289439320053029826884919030349 85,
328.11709294873160019456053889636118470 37,
292.99969138878545365564255363979725322 47,
358.64341561660743199939225154124285923 29,
299.89866206206139467079143810519866312 90,
360.06173466647150786976289829201528170 01,
336.59441032706119108472094609326451501 10, none,
324.65521224458036687706005432958859511 07,
331.93806793157771246491093254090105273 29, none, none,
289.54595773673674759291799179240475335 00]

1 --> 2  target = [33.81362495417899683673476563085578115570,
3.72564899358469645795313431198628357389 8,
325.89209973720917894651604057902231134 42]
one interval r = 20.37468935108142482085148004410520826110 ..
25.37892165308747552659533537697678296466
Time Approximations 0.025.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .259e-34]Solution in 0.541s

Time Plot 0 s.
Exiting SolveHard() after 1.046r=24.3395 in [22.07732228 ..
25.37892164]
```

```
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31,
375.73285289439320053029826884919030349 85,
328.11709294873160019456053889636118470 37,
292.99969138878545365564255363979725322 47,
358.64341561660743199939225154124285923 29,
299.89866206206139467079143810519866312 90,
360.06173466647150786976289829201528170 01,
336.59441032706119108472094609326451501 10,
256.10753186881754263829490665440437970 41,
324.65521224458036687706005432958859511 07,
331.93806793157771246491093254090105273 29, none, none,
289.54595773673674759291799179240475335 00]

1 --> 0  target = [17.93041369704809970784910288843336920284,
4.68650870171917787137679117045535295464 8,
353.30541095481101275174871058990216160 14]
one interval r = 20.73150479089444357991934524801910756423 ..
25.90675353521577730935660619395266916684
Time Approximations 0.031.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.5e-38
Equations at solution: [.2e-37, .45e-37, .109e-34]Solution in 0.65s

Time Plot 0 s.
```

```
Exiting SolveHard() after 3.63r=25.4021 in [22.67806074 .. 25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31,
375.73285289439320053029826884919030349 85,
328.11709294873160019456053889636118470 37,
292.99969138878545365564255363979725322 47,
358.64341561660743199939225154124285923 29,
299.89866206206139467079143810519866312 90,
360.06173466647150786976289829201528170 01,
336.59441032706119108472094609326451501 10,
256.10753186881754263829490665440437970 41,
324.65521224458036687706005432958859511 07,
331.93806793157771246491093254090105273 29,
304.79958325562665951904944226086082845 85, none,
289.54595773673674759291799179240475335 00]

2 --> 0  target = [17.93041369704809970784910288843336920284,
4.68650870171917787137679117045535295464 8,
353.30541095481101275174871058990216160 14]
```

"Imaginary part neglected: ", $3.18322343221284973110599089378030369350 5 \times 10^{-17}$

```
one interval r = 31.37435486993040166252461750773963307092 ..
34.20127520029854567961452251968644919635
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
```

```
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, .2e-36]Solution in 0.337s

Time Plot 0 s.
Exiting SolveHard() after 0.613r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494335486767076679530798026898 47,
441.64295973739375589227847830029841945 67,
436.91748166211008468676934591913989258 32,
422.98493397641184066845536082424193270 82,
361.52580256948996205089229161401390510 70,
401.88173904826890813698462346159133935 97,
389.59001517004313727117840817766416749 44,
328.46939893980087181012506786991809594 30,
401.50757158549591258553521715377777505 18,
358.97362824838604073424653871909436831 86,
398.33147103890775705087434034712421252 74,
371.48387395903520253841397065931884719 04,
336.61215841666980424602300376984438900 40,
361.50888347982936399375776457874035107 94,
324.67144993483066031160749214618060869 27,
302.31384314961507401781973890823739083 32,
328.46938514052562610143508827171103164 84,
343.81340626214226776632196774673367386 31,
375.73285289439320053029826884919030349 85,
328.11709294873160019456053889636118470 37,
292.99969138878545365564255363979725322 47,
358.64341561660743199939225154124285923 29,
299.89866206206139467079143810519866312 90,
360.06173466647150786976289829201528170 01,
336.59441032706119108472094609326451501 10,
256.10753186881754263829490665440437970 41,
324.65521224458036687706005432958859511 07,
331.93806793157771246491093254090105273 29,
304.79958325562665951904944226086082845 85,
323.46169177557817479906938609640088472 83,
289.54595773673674759291799179240475335 00]

Cascade time 247.986
counts: 28, 28

 Iteration 45

Start Generation 1
1 --> 0  target = [11.99999999996653457745400212109715883500,
6.21701250280766403847391374650087353706 1,
485.54908090055139506448000565841478861 30]
```

"Imaginary part neglected: ", $1.8899423791490658534518767433428795060 73 \times 10^{-17}$

```
one interval r = 23.40850301656281854749435150151203091662 ..
27.67578046429931203967108368657013378865
Time Approximations 0.043.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.5e-38
Equations at solution: [-.1e-37, .55e-37, .13e-35]Solution in 1.001s

Time Plot 0 s.
Exiting SolveHard() after 4.376r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999996653457745400212109715883500,
6.217012502807664038473913746500873537061,
485.549080900551395064480005658414788613 0]
one interval r = 32.62814779215699516557938142910448703113 ..
36.10248388951175078910053250614494748244
Time Approximations 0.024.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=3e-38
Equations at solution: [.3e-37, -.3e-37, .45e-35]Solution in 3.025s

Time Plot 0 s.
Exiting SolveHard() after 3.45r=35.4632 in [33.94922194 .. 36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844811937197888951174072671840 97,
6.583434721593612034564098956688410302363,
467.787305960345893857133219573861236610 6]
one interval r = 32.419789556636907015552323177711981690 05 ..
35.851524173792995703374850101911080066 05
Time Approximations 0.021.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., .172e-34]Solution in 0.64s

Time Plot 0 s.
Exiting SolveHard() after 1.023r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57, none, none,
401.88173904454101127363869747840896698 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844811937197888951174072671840 97,
6.583434721593612034564098956688410302363,
467.787305960345893857133219573861236610 6]
two intervals r = 12.923271608305358358191065254486461647 68 ..
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000 or r = 18.394248580313276520969898101320502081 16 ..
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
```

```
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.1e-38
Equations at solution: [-.2e-37, -.21e-37, -.233e-35]Solution in
39.106s

Time Plot 0 s.
Exiting SolveHard() after 43.254r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12, none,
401.88173904454101127363869747840896698 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962835975290725896982518449619900,
4.12565179686937843931714717595650372382 8,
440.67123065431583198100739325285641581 13]
two intervals r = 14.35659705104685000661834556494362730508 ..
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000 or r = 17.70352613817038632906813245472763205019 ..
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [.58e-37, .3e-37, -.2155e-34]Solution in 1.331s

Time Plot 0 s.
Exiting SolveHard() after 4.67r=15.9119 in [14.35659706 .. 18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12, none,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962835975290725896982518449619900,
4.12565179686937843931714717595650372382 8,
440.67123065431583198100739325285641581 13]
```

"Imaginary part neglected: ", $1.8899423791490658534518767433428795060 73 \times 10^{-17}$

```
one interval r = 22.39761154369119799125719778683763867831 ..
27.23722351602451991154366084231398871600
Time Approximations 0.042.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.227 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064390327449987451176696294448662, rm =
14.37818770385288372698134426844769258393}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, .123e-34]Solution in 9.038s

Time Plot 0 s.
Exiting SolveHard() after 12.035r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
```

```
389.590015164184266161614109042958137926 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888054340849742836104410424594 69,
4.004869081841578698780291420890760600156,
404.862245015452393204235543685879656302 6]
two intervals r = 16.080110077537085708404166742299330549 70 ..
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000 or r = 16.415798127001389402161681248274382682 80 ..
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000
Time Approximations 0.047.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., -.2349e-34]Solution in 1.648s

Time Plot 0 s.
Exiting SolveHard() after 5.166r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434938683963952048652413964364 5864,
441.642959733327553078766192802611600596 8,
436.917481656377361672580833797600918625 7,
422.984933974632530438067432115397344451 2,
361.525802565337187187908737346537104742 4,
401.881739044541011273638697478408966987 3,
389.590015164184266161614109042958137926 3, none, none,
358.973628244189003102724721797080492876 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888054340849742836104410424594 69,
4.004869081841578698780291420890760600156,
404.862245015452393204235543685879656302 6]
```

"Imaginary part neglected: ", $1.8899423791490658534518767433428795060 73 \times 10^{-17}$

```
one interval r = 21.641943994089921528611766148780629657 41 ..
26.763306600445749942771441652739414678 09
Time Approximations 0.051.
```

```
hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .376e-34]Solution in 1.049s

Time Plot 0 s.
Exiting SolveHard() after 4.424r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94, none,
358.97362824418900310272472179708049287 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589417935262929642588292538036731 03,
5.5896371827696503354605100583799332175 53,
443.83065884519790217510131720248776333 63]
```

"Imaginary part neglected: ", $1.8899423791490658534518767433428795060 73 \times 10^{-17}$

```
one interval r = 22.467253744706138381597588225816684520 04 ..
27.273884283534809837717405029948038700 98
Time Approximations 0.041.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.0e-38
Equations at solution: [-.1e-37, .80e-37, -.124e-34]Solution in 0.968s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.283r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94, none,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941793526292964258829253803673103,
5.589637182769650335460510058379933217553,
443.83065884519790217510131720248776333 63]
one interval r = 32.15575279499339172931871404284567916807 ..
35.50872228741652906465977471616172354176
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., -.256e-34]Solution in 2.57s

Time Plot 0 s.
Exiting SolveHard() after 2.943r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
```

```
389.590015164184266161614109042958137926 3,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931364876869626140033098488454253 76,
5.1877835783924001457528987941301754017 07,
408.65773863310966403937480247790864340 02]
```

"Imaginary part neglected: ", $1.8899423791490658534518767433428795060 73 \times 10^{-17}$

```
one interval r = 21.718401146561980251406417414413738211362 ..
26.818493035201261665183483615613962182 90
Time Approximations 0.06.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=5.3e-38
Equations at solution: [0., -.53e-37, .122e-34]Solution in 1.024s

Time Plot 0 s.
Exiting SolveHard() after 2.13r=26.4632 in [23.93303356 .. 26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.590015164184266161614109042958137926 3,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99, none, none,
361.50888347564554029255283152423430949 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.911931364876869626140033098488454253 76,
5.1877835783924001457528987941301754017 07,
408.65773863310966403937480247790864340 02]
```

```
one interval r = 31.80828598754851327042988199205277724400 ..
35.00011460058196476120082760277047647343
Time Approximations 0.016.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.249e-34]Solution in 0.414s

Time Plot 0 s.
Exiting SolveHard() after 3.173r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79, none,
361.50888347564554029255283152423430949 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110547151924512845115453628533707,
6.19626256534104150040963801396009743513 7,
385.44474379806141109058994387637091347 26]
one interval r = 31.60836097537925176758499339147822736696 ..
34.66372795620057487256980650723667050389
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.1e-37, -.3e-37, .67e-35]Solution in 0.545s

Time Plot 0 s.
Exiting SolveHard() after 0.815r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79, none,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110547151924512845115453628533707,
6.19626256534104150040963801396009743513 7,
385.44474379806141109058994387637091347 26]
two intervals r = 16.87563408738093068121413696459788944006 ..
189999999999820538271526785797497899 61/1000000000000000000000000000000
00000 or r = 15.55640493822935627702630495159195351137 ..
189999999999820538271526785797497899 61/1000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 8.597 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175110090764540161911527374798248, rm
= 2.33653277418032197729274467308147974034 4}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.2463e-34]Solution in 28.707s
```

```
Time Plot 0 s.
Exiting SolveHard() after 32.089r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874720044384990873756236669334105,
4.88381077970358955646335873685136399999 2,
376.61967856304306466470085678749906809 38]
```

"Imaginary part neglected: ", $1.8899423791490658534518767433428795060 73 \times 10^{-17}$

```
one interval r = 21.11001304879155379835270851082907899011 ..
26.31784243482474231203377377874963539646
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.3e-38
Equations at solution: [0., .23e-37, .176e-34]Solution in 0.824s

Time Plot 0 s.
Exiting SolveHard() after 3.635r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84, none,
328.46938513829873334622847167792813268 28, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874720044384990873756236669334105,
4.8838107779703589556463358736851363999992,
376.61967856304306466470085678749906809 38]
one interval r = 31.53899497711890704217180711114464552506 ..
34.53618386103355044990259617841667311451
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.060e-35
Equations at solution: [.814e-35, -.1060e-34, .47e-35]Solution in
0.486s

Time Plot 0 s.
Exiting SolveHard() after 0.785r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
```

```
358.9736282441890031027247217970804928769,
398.3314710389147765293917657140809970399,
371.4838739518021492195492464114156757379,
336.6121584150944118543189234130847248647,
361.5088834756455402925528315242343094975,
324.6714499311502718312927503532235509884, none,
328.4693851382987333462284716779281326828,
343.8134062562789035110123697789650964618, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017545955211722884322853731924763,
6.025813549290683192495004023623033465862,
351.4270294873539063884557943907056089832]
one interval r = 31.36230206112424906171647550519697148237 ..
34.17446640621559632996558046812597647637
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, -.88e-35]Solution in 0.527s

Time Plot 0 s.
Exiting SolveHard() after 2.864r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349386839639520486524139643645864,
441.6429597333275530787661928026116005968,
436.9174816563773616725808337976009186257,
422.9849339746325304380674321153973444512,
361.5258025653371871879087373465371047424,
401.8817390445410112736386974784089669873,
389.5900151641842661616141090429581379263,
328.4693989375745360631856469090286774094,
401.5075715821110294388543911327189924893,
358.9736282441890031027247217970804928769,
398.3314710389147765293917657140809970399,
371.4838739518021492195492464114156757379,
336.6121584150944118543189234130847248647,
361.5088834756455402925528315242343094975,
324.6714499311502718312927503532235509884, none,
328.4693851382987333462284716779281326828,
343.8134062562789035110123697789650964618, none, none,
292.9996913870017546901555890864884605316, none, none, none, none,
```

```
none, none, none, none, none, none]

0 --> 1  target = [25.872050175459552117228843228537319247 63,
6.025813549290683192495004023623033465862,
351.42702948735390638845579439070560898 32]
two intervals r = 17.981355144342272062721805702499415703 85 ..
189999999999820538271526785797497899 61/10000000000000000000000000000000
00000 or r = 13.846080154372081552495418975231272406 21 ..
189999999999820538271526785797497899 61/10000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 7.456 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713643433821833476701747163713 83, rm
= 2.7345009933600364300116614358754662621 49}});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [.107e-36, -.3e-37, -.1074e-34]Solution in
18.567s

Time Plot 0 s.
Exiting SolveHard() after 19.622r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84,
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18, none, none,
```

```
292.99969138700175469015558908648846053 16, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941869970534029673322532181187046,
6.37794387385160884002856776057311545251 9,
423.28832783767484157058387700655078573 17]
one interval r = 31.94661817592999653020386098948128317994 ..
35.21212308652677708702877663491580871714
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .136e-34]Solution in 0.593s

Time Plot 0 s.
Exiting SolveHard() after 3.461r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84,
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18, none, none,
292.99969138700175469015558908648846053 16, none, none,
360.06173466673869285856699463924511217 50, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941869970534029673322532181187046,
6.37794387385160884002856776057311545251 9,
423.28832783767484157058387700655078573 17]
```

```
two intervals r = 15.22886702451208794854715353822163749887 ..
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000 or r = 17.12965777063198432845682334640658607371 ..
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 5.92 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541339755216090188130489909276 36, rm
= 2.0640682988079960628810866112437794961 07}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.48e-37, -.1e-37, -.917e-35]Solution in
27.158s

Time Plot 0 s.
Exiting SolveHard() after 31.202r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84,
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18,
375.73285289728924067269399650197838598 03, none,
292.99969138700175469015558908648846053 16, none, none,
360.06173466673869285856699463924511217 50, none, none, none, none,
none, none, none]
```

```
0 --> 2  target = [34.93953234349356748226980443212306932810,
4.00355981554173514115904809947780700687 4,
404.47973594253872153146757754943039670 11]
two intervals r = 16.09683966364468722902115304624249192811 ..
18999999999982053827152678579749789961/10000000000000000000000000000000
00000 or r = 16.39988649120200477797045246880624721115 ..
18999999999982053827152678579749789961/10000000000000000000000000000000
00000
Time Approximations 0.047.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., .1020e-34]Solution in 3.835s

Time Plot 0 s.
Exiting SolveHard() after 8.144r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84,
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18,
375.73285289728924067269399650197838598 03, none,
292.99969138700175469015558908648846053 16,
358.64341561271249220953209229572787410 99, none,
360.06173466673869285856699463924511217 50, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234349356748226980443212306932810,
4.00355981554173514115904809947780700687 4,
```

```
404.47973594253872153146757754943039670 11]
```

"Imaginary part neglected: ", $1.8899423791490658534518767433428795060 73 \times 10^{-17}$

```
one interval r = 21.634296299862981537989121505350457989 83 ..
26.757681698944344587618851548513703178 21
Time Approximations 0.053.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [0., .26e-37, .206e-34]Solution in 3.159s

Time Plot 0 s.
Exiting SolveHard() after 4.2r=25.8653 in [23.83864811 .. 26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84,
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18,
375.73285289728924067269399650197838598 03,
328.11709294682855359438392181624646055 12,
292.99969138700175469015558908648846053 16,
358.64341561271249220953209229572787410 99, none,
360.06173466673869285856699463924511217 50, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954465227639080004287963417343110,
6.19617723019420590920693929194204539796 8,
385.42734026173992004765474039186345722 82]
```

```
one interval r = 31.6082204909323052982065656667546530603 3 ..
34.66347615057612895343850924122091743558
Time Approximations 0.019.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.9e-36]Solution in 0.571s

Time Plot 0 s.
Exiting SolveHard() after 0.869r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84,
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18,
375.73285289728924067269399650197838598 03,
328.11709294682855359438392181624646055 12,
292.99969138700175469015558908648846053 16,
358.64341561271249220953209229572787410 99, none,
360.06173466673869285856699463924511217 50, none, none,
324.65521224087020756063500127215304163 82, none, none, none, none]

0 --> 1  target = [26.46318954465227639080004287963417343110,
6.19617723019420590920693929194204539796 8,
385.42734026173992004765474039186345722 82]
two intervals r = 16.87629600277009788247988926998317959141 ..
189999999999820538271526785797497899 61/10000000000000000000000000000000
00000 or r = 15.55559000671153308006044338351525598208 ..
```

```
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000
Time Approximations 0.058.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 6.223 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852521959824066473395696432072945, rm
= 2.33669042824636206555081633319958061266 3}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.359e-37, 0., .1295e-34]Solution in 28.589s

Time Plot 0 s.
Exiting SolveHard() after 31.991r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84,
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18,
375.73285289728924067269399650197838598 03,
328.11709294682855359438392181624646055 12,
292.99969138700175469015558908648846053 16,
358.64341561271249220953209229572787410 99, none,
360.06173466673869285856699463924511217 50,
336.59441032545321521815846366466385831 48, none,
324.65521224087020756063500127215304163 82, none, none, none, none]

0 --> 2  target = [34.49522661180021591151053143895096362534,
```

```
3.897131315981320905047662451055787909952,
373.780818852883409420326817285337232150 1]
two intervals r = 17.29769086197665090528327867925763001767 ..
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000 or r = 14.99436407470994337951121642080068618354 ..
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000
Time Approximations 0.085.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.72e-37, 0., .1096e-34]Solution in 3.394s

Time Plot 0 s.
Exiting SolveHard() after 7.266r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84,
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18,
375.73285289728924067269399650197838598 03,
328.11709294682855359438392181624646055 12,
292.99969138700175469015558908648846053 16,
358.64341561271249220953209229572787410 99, none,
360.06173466673869285856699463924511217 50,
336.59441032545321521815846366466385831 48, none,
324.65521224087020756063500127215304163 82,
331.93806792419697033469290561875368842 80, none, none, none]
```

```
1 --> 2  target = [34.49522661180021591151053143895096362534,
3.89713131598132090504766245105578790995 2,
373.78081885288340942032681728533723215 01]
```

"Imaginary part neglected: ", $1.8899423791490658534518767433428795060 73 \times 10^{-17}$

```
one interval r = 21.06068473210453718105869753715412457195 ..
26.26979834296495386477365822113795472146
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [.1e-37, .3e-37, -.484e-34]Solution in 0.776s

Time Plot 0 s.
Exiting SolveHard() after 1.498r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84,
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18,
375.73285289728924067269399650197838598 03,
328.11709294682855359438392181624646055 12,
292.99969138700175469015558908648846053 16,
358.64341561271249220953209229572787410 99,
299.89866205657245625154243238802738320 97,
360.06173466673869285856699463924511217 50,
336.59441032545321521815846366466385831 48, none,
```

```
324.65521224087020756063500127215304163 82,
331.93806792419697033469290561875368842 80, none, none, none]

0 --> 2  target = [33.81362495418359603876620636515396131726,
3.7256489936047509118093801991666237922 53,
325.89209973379318000807329204792614813 47]
two intervals r = 18.552270489947336046581230831698985968 28 ..
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000 or r = 12.491969358182534033571033701891767749 71 ..
189999999999820538271526785797497899 61/100000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=4e-38
Equations at solution: [.87e-37, -.4e-37, -.38e-36]Solution in 1.107s

Time Plot 0 s.
Exiting SolveHard() after 5.729r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84,
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18,
375.73285289728924067269399650197838598 03,
328.11709294682855359438392181624646055 12,
292.99969138700175469015558908648846053 16,
358.64341561271249220953209229572787410 99,
299.89866205657245625154243238802738320 97,
```

```
360.06173466673869285856699463924511217 50,
336.59441032545321521815846366466385831 48, none,
324.65521224087020756063500127215304163 82,
331.93806792419697033469290561875368842 80, none, none,
289.54595773252594865542019936221159482 67]

1 --> 2  target = [33.81362495418359603876620636515396131726,
3.72564899360475091180938019916662379225 3,
325.89209973379318000807329204792614813 47]
```

"Imaginary part neglected: ", $1.8899423791490658534518767433428795060 73 \times 10^{-17}$

```
one interval r = 20.37468935104558692672608120091899638113 ..
25.37892165308795342383842025163600088390
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.2e-37, -.2e-37, -.48e-35]Solution in 0.542s

Time Plot 0 s.
Exiting SolveHard() after 3.3r=24.3395 in [22.07732228 .. 25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84,
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18,
375.73285289728924067269399650197838598 03,
328.11709294682855359438392181624646055 12,
```

```
292.99969138700175469015558908648846053 16,
358.6434156127124922095320922957278741099,
299.8986620565724562515424323880273832097,
360.0617346667386928585669946392451121750,
336.5944103254532152181584636646638583148,
256.1075318666998013752681115962988921063,
324.6552122408702075606350012721530416382,
331.9380679241969703346929056187536884280, none, none,
289.5459577325259486554201993622115948267]

1 --> 0  target = [17.9304136970177071366596653173264265 0699,
4.6865087018217840862541569748281117049 72,
353.3054109530562709414660578306622829537]
```

"Imaginary part neglected: ", $1.8899423791490658534518767433428795060 73 \times 10^{-17}$

```
one interval r = 20.7315047908751680559575033439910804325 8 ..
25.9067535352602295397371582496598919398 2
Time Approximations 0.034.

hint used Hint := [25.4020715378188039710536173439704738275 4, 3, -1, 1,
17.0062444547662134530206245122467435705 3, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, -.64e-35]Solution in 3.292s

Time Plot 0 s.
Exiting SolveHard() after 3.986r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349386839639520486524139643645864,
441.6429597333275530787661928026116005968,
436.9174816563773616725808337976009186257,
422.9849339746325304380674321153973444512,
361.5258025653371871879087373465371047424,
401.8817390445410112736386974784089669873,
389.5900151641842661616141090429581379263,
328.4693989375745360631856469090286774094,
401.5075715821110294388543911327189924893,
358.9736282441890031027247217970804928769,
398.3314710389147765293917657140809970399,
371.4838739518021492195492464114156757379,
336.6121584150944118543189234130847248647,
361.5088834756455402925528315242343094975,
324.6714499311502718312927503532235509884,
```

```
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18,
375.73285289728924067269399650197838598 03,
328.11709294682855359438392181624646055 12,
292.99969138700175469015558908648846053 16,
358.64341561271249220953209229572787410 99,
299.89866205657245625154243238802738320 97,
360.06173466673869285856699463924511217 50,
336.59441032545321521815846366466385831 48,
256.10753186669980137526811159629889210 63,
324.65521224087020756063500127215304163 82,
331.93806792419697033469290561875368842 80,
304.79958325621535706170747517866172140 95, none,
289.54595773252594865542019936221159482 67]

2 --> 0  target = [17.93041369701770713665966531732642650699,
4.6865087018217840862541569748281117049 72,
353.30541095305627094146057830662282953 7]
one interval r = 31.37435486991365101166671587404537723006 ..
34.20127520033496134195662797259298935205
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=1.1e-37
Equations at solution: [.8e-37, -.11e-36, .332e-34]Solution in 0.369s

Time Plot 0 s.
Exiting SolveHard() after 0.634r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493868396395204865241396436458 64,
441.64295973332755307876619280261160059 68,
436.91748165637736167258083379760091862 57,
422.98493397463253043806743211539734445 12,
361.52580256533718718790873734653710474 24,
401.88173904454101127363869747840896698 73,
389.59001516418426616161410904295813792 63,
328.46939893757453606318564690902867740 94,
401.50757158211102943885439113271899248 93,
358.97362824418900310272472179708049287 69,
398.33147103891477652939176571408099703 99,
371.48387395180214921954924641141567573 79,
```

```
336.61215841509441185431892341308472486 47,
361.50888347564554029255283152423430949 75,
324.67144993115027183129275035322355098 84,
302.31384314997659374259614858018801057 98,
328.46938513829873334622847167792813268 28,
343.81340625627890351101236977896509646 18,
375.73285289728924067269399650197838598 03,
328.11709294682855359438392181624646055 12,
292.99969138700175469015558908648846053 16,
358.64341561271249220953209229572787410 99,
299.89866205657245625154243238802738320 97,
360.06173466673869285856699463924511217 50,
336.59441032545321521815846366466385831 48,
256.10753186669980137526811159629889210 63,
324.65521224087020756063500127215304163 82,
331.93806792419697033469290561875368842 80,
304.79958325621535706170747517866172140 95,
323.46169177203948020488022175078401964 15,
289.54595773252594865542019936221159482 67]

Cascade time 253.078
counts: 28, 28

 Iteration 46

Start Generation 1
1 --> 0  target = [11.99999999992233579365252977325446269200,
6.21701250300388736044729257321083879751 0,
485.54908091040481205881757183652715045 60]
one interval r = 23.40850301680330950988609063550953734545 ..
27.67578046454072813415484735112245573631
Time Approximations 0.048.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.37e-37
Equations at solution: [.7e-37, -.237e-36, .1032e-35]Solution in 3.46s

Time Plot 0 s.
Exiting SolveHard() after 4.651r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999922335793652529773254462692 00,
6.2170125030038873604472925732108387975 10,
485.54908091040481205881757183652715045 60]
one interval r = 32.628147792351174795256630817494373945 74 ..
36.102483889665719121261768773504130535 65
Time Approximations 0.023.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [.6e-37, -.4e-37, -.182e-34]Solution in 0.625s

Time Plot 0 s.
Exiting SolveHard() after 1.052r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596845070757318253683172271052577 87,
6.5834347216414179509900081336587905873 63,
467.78730597094713570177104002090368232 09]
one interval r = 32.419789556838153156673149750962506320 58 ..
35.851524173963166055323274882047315842 42
Time Approximations 0.022.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
```

```
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., .250e-34]Solution in 2.835s

Time Plot 0 s.
Exiting SolveHard() after 3.208r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75, none, none,
401.88173905563068411591192822042168839 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684507075731825368317227105257787,
6.58343472164141795099000813365879058736 3,
467.78730597094713570177104002090368232 09]
two intervals r = 12.92327160819674859343685349439453382544 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000000
000 or r = 18.39424858060605457614269128225359007416 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000000
000
Time Approximations 0.043.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.3e-38
Equations at solution: [.2e-37, .33e-37, -.12e-35]Solution in 39.723s

Time Plot 0 s.
Exiting SolveHard() after 41.092r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56, none,
401.88173905563068411591192822042168839 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962851261254749905636829928830443,
4.12565179697768225200703773526496327 5647,
440.67123066315307925280122686511746681 98]
two intervals r = 14.35659705101587498477561583358592522516 ..
475000000002908357101651458711257 5143/250000000000000000000000000000000
000 or r = 17.70352613849116520751710872708993435739 ..
475000000002908357101651458711257 5143/250000000000000000000000000000000
000
Time Approximations 0.048.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.14e-37, -.1e-37, .36e-35]Solution in 1.334s

Time Plot 0 s.
Exiting SolveHard() after 4.683r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56, none,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962851261254749905636829928830443,
4.12565179697768225200703773526496327 5647,
440.67123066315307925280122686511746681 98]
one interval r = 22.39761154386114434656320712399013657014 ..
27.23722351627803690943470916870228980281
Time Approximations 0.034.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.178 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070644204658636737391896395750099 87, rm =
14.37818770586063728265824510792998689438}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.05e-37
Equations at solution: [0., -.105e-36, -.18636e-34]Solution in 9.588s

Time Plot 0 s.
Exiting SolveHard() after 12.894r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888247166094716511430737585531 30,
4.0048690819581036266774704620506671359 59,
404.86224502640920791237855990245377267 87]
two intervals r = 16.0801100774179102739504518719552470228 0 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000
000 or r = 16.4157981274991169591761476576636861749 7 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000
000
Time Approximations 0.053.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., -.57e-35]Solution in 3.837s

Time Plot 0 s.
Exiting SolveHard() after 4.91r=17.199 in [16.08011004 .. 19]
```

```
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61, none, none,
358.97362825336888260803939061097948688 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888824716609471651143073758553130,
4.0048690819581036266774704620506671359 59,
404.86224502640920791237855990245377267 87]
one interval r = 21.641943994252604991355288944576743397 74 ..
26.763306600740748241974447454094020144 83
Time Approximations 0.053.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=1.26e-37
Equations at solution: [-.3e-37, -.126e-36, -.18753e-34]Solution in
4.019s

Time Plot 0 s.
Exiting SolveHard() after 5.053r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52, none,
358.97362825336888260803939061097948688 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none]

1 --> 0  target = [14.19258941768946795516890156260383598781,
5.589637182996506393284772503117840459664,
443.83065885817389326621595594420037493 68]
one interval r = 22.46725374497143356316902381296858768765 ..
27.27388428383462490008230971421334749060
Time Approximations 0.04.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.0e-38
Equations at solution: [-.1e-37, .80e-37, .4361e-35]Solution in 3.368s

Time Plot 0 s.
Exiting SolveHard() after 4.343r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52, none,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941768946795516890156260383598781,
5.589637182996506393284772503117840459664,
443.83065885817389326621595594420037493 68]
one interval r = 32.15575279521594972551458238603968805681 ..
35.50872228762807340851722975951799171877
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
```

```
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .24e-35]Solution in 0.485s

Time Plot 0 s.
Exiting SolveHard() after 0.842r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136489316995312553161282856330307,
5.18778357855260355507332916983055165271 8,
408.65773864095882974963008130993590227 40]
one interval r = 21.71840114666795468471810067886995253393 ..
26.81849303545006965713866519418460632371
Time Approximations 0.066.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.63e-37
Equations at solution: [-.2e-37, -.263e-36, .21136e-34]Solution in
3.272s

Time Plot 0 s.
Exiting SolveHard() after 4.366r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66, none, none,
361.50888348473561811477055797030226119 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136489316995312553161282856330307,
5.18778357855260355507332916983055165271 8,
408.65773864095882974963008130993590227 40]
one interval r = 31.80828598771253735240732728619093142204 ..
35.00011460072881195594470330857806616270
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, .77e-35]Solution in 0.444s

Time Plot 0 s.
Exiting SolveHard() after 0.75r=34.4952 in [32.91337941 .. 35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
```

```
371.48387395881062351772134952422780260 30, none,
361.50888348473561811477055797030226119 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110574876288840633568978955497772,
6.19626256539135661138329912090335875702 0,
385.44474380752603574385553101360304019 85]
one interval r = 31.60836097555207140712345168807711895534 ..
34.66372795637617695906933339470870215433
Time Approximations 0.027.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., -.6e-36]Solution in 0.554s

Time Plot 0 s.
Exiting SolveHard() after 0.84r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30, none,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110574876288840633568978955497772,
6.19626256539135661138329912090335875702 0,
385.44474380752603574385553101360304019 85]
two intervals r = 16.87563408734257691921643402620683401374 ..
4750000000029083571016514587112575143/2500000000000000000000000000000000
000 or r = 15.55640493871501323060677135558659379615 ..
4750000000029083571016514587112575143/2500000000000000000000000000000000
000
```

```
Time Approximations 0.057.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [.359e-37, -.1e-37, -.45e-35]Solution in 1.175s

Time Plot 0 s.
Exiting SolveHard() after 4.533r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874716119407869760831235637005100,
4.88381077987976044006046545578947345103 1,
376.61967857278668443428924049117537682 51]
one interval r = 21.11001304888195471797680006544035234247 ..
26.31784243510795023902889543935067980579
Time Approximations 0.033.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
```

```
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.8028e-35]Solution in 0.828s

Time Plot 0 s.
Exiting SolveHard() after 3.935r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10, none,
328.46938514933015358819164942414745595 14, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874716119407869760831235637005100,
4.8838107798797604400604654557894734510 31,
376.61967857278668443428924049117537682 51]
one interval r = 31.53899497729125700397295151354006448720 ..
34.53618386121502461315595920613476360454
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.98e-36
Equations at solution: [.305e-35, -.398e-35, .58e-35]Solution in 0.492s

Time Plot 0 s.
Exiting SolveHard() after 0.789r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10, none,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017577603369790617723660050449463,
6.0258135493547886390033639402342990296 65,
351.42702949879574272393414914343561980 54]
one interval r = 31.362302061298628884114384214819343502 75 ..
34.174466406425910632892433182409674523 28
Time Approximations 0.014.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=9e-38
Equations at solution: [-.5e-37, .9e-37, .29e-35]Solution in 0.525s

Time Plot 0 s.
Exiting SolveHard() after 3.24r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
```

```
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10, none,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61, none, none,
292.99969139879739071272372492269269927 16, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175776033697906177236600504 49463,
6.0258135493547886390033639402342990296 65,
351.42702949879574272393414914343561980 54]
two intervals r = 17.981355144290935921202348652628802 27080 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000
000 or r = 13.846080155031417276681485519814507517 07 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=6e-38
Equations at solution: [-.213e-36, .6e-37, -.34e-35]Solution in 3.444s

Time Plot 0 s.
Exiting SolveHard() after 4.508r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
```

```
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61, none, none,
292.99969139879739071272372492269269927 16, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941903748300327312811641393693654,
6.3779438739232478860022109806002820649 81,
423.28832785246333353731040946707472806 37]
one interval r = 31.946618176163877589514070209810524394 04 ..
35.21212308677042919500696639644199314439
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, .284e-34]Solution in 0.61s

Time Plot 0 s.
Exiting SolveHard() after 0.955r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61, none, none,
292.99969139879739071272372492269269927 16, none, none,
360.06173468172036231782324791072384005 50, none, none, none, none,
none, none, none]
```

```
0 --> 1  target = [27.020379419037483003273128116413936936 54,
6.3779438739232478860022109806002820649 81,
423.28832785246333353731040946707472806 37]
two intervals r = 15.22886702419191118293802554986485148779 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000
000 or r = 17.12965777120919995011588896614563521202 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000
000
Time Approximations 0.06.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [-.17e-37, 0., .192e-34]Solution in 1.23s

Time Plot 0 s.
Exiting SolveHard() after 5.004r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61,
375.73285291393068834560325211392471012 46, none,
292.99969139879739071272372492269269927 16, none, none,
360.06173468172036231782324791072384005 50, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343692165338497817984426935974 01,
4.0035598156596145143221884909796862330 45,
404.47973595388733462302714686887590795 07]
```

```
two intervals r = 16.09683966350879943155795691875772811415 ..
475000000002908357101651458711257514 3/250000000000000000000000000000000
000 or r = 16.39988649171723368001162816268173145243 ..
475000000002908357101651458711257514 3/250000000000000000000000000000000
000
Time Approximations 0.055.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.50e-37, -.1e-37, -.173e-34]Solution in 1.493s

Time Plot 0 s.
Exiting SolveHard() after 5.021r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61,
375.73285291393068834560325211392471012 46, none,
292.99969139879739071272372492269269927 16,
358.64341562223099124939959987002300591 88, none,
360.06173468172036231782324791072384005 50, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343692165338497817984426935974 01,
4.0035598156596145143221884909796862330 45,
404.47973595388733462302714686887590795 07]
one interval r = 21.63429630003284950013945494726875133580 ..
26.75768169924525482886508864834758232236
```

```
Time Approximations 0.046.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, .14128e-34]Solution in
1.011s

Time Plot 0 s.
Exiting SolveHard() after 4.35r=25.8653 in [23.83864811 .. 26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61,
375.73285291393068834560325211392471012 46,
328.11709295822092048010125208247215184 44,
292.99969139879739071272372492269269927 16,
358.64341562223099124939959987002300591 88, none,
360.06173468172036231782324791072384005 50, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954492893669340311855899181695995,
6.19617723024434514794069126430022387012 6,
385.42734027116831007421680608327095064 24]
one interval r = 31.60822049110482750791896524802225549305 ..
34.66347615075121051567602990974779618581
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
```

```
11.7832091214135218707194381744718996562 7, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.1e-37, -.3e-37, .142e-34]Solution in 2.865s

Time Plot 0 s.
Exiting SolveHard() after 3.167r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61,
375.73285291393068834560325211392471012 46,
328.11709295822092048010125208247215184 44,
292.99969139879739071272372492269269927 16,
358.64341562223099124939959987002300591 88, none,
360.06173468172036231782324791072384005 50, none, none,
324.65521225080824306374501052613584298 39, none, none, none, none]

0 --> 1  target = [26.46318954492893669340311855899181695995,
6.19617723024434514794069126430022387012 6,
385.42734027116831007421680608327095064 24]
two intervals r = 16.876296002733138668171400276142242264 25 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000
000 or r = 15.555590007195536015038767381530407810 11 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000
000
Time Approximations 0.058.

hint used Hint := [17.930920830435909701694400819009216331 81, 3, 1, 1,
```

```
15.7008606270133975112531574051596448801 2, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.180e-37, 0., -.9e-36]Solution in 3.365s

Time Plot 0 s.
Exiting SolveHard() after 4.512r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61,
375.73285291393068834560325211392471012 46,
328.11709295822092048010125208247215184 44,
292.99969139879739071272372492269269927 16,
358.64341562223099124939959987002300591 88, none,
360.06173468172036231782324791072384005 50,
336.59441033630247726680535228137917197 93, none,
324.65521225080824306374501052613584298 39, none, none, none, none]

0 --> 2  target = [34.49522661193963602757798697214390065334,
3.897131316084322517168401030598383168686,
373.78081885967139752766154932972068260 00]
two intervals r = 17.297690862042157190729162306779335839 94 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000
000 or r = 14.994364075090618939316653834072011062 21 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000
000
Time Approximations 0.087.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
```

```
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.54e-37, 0., -.47e-35]Solution in 3.978s

Time Plot 0.001 s.
Exiting SolveHard() after 5.628r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61,
375.73285291393068834560325211392471012 46,
328.11709295822092048010125208247215184 44,
292.99969139879739071272372492269269927 16,
358.64341562223099124939959987002300591 88, none,
360.06173468172036231782324791072384005 50,
336.59441033630247726680535228137917197 93, none,
324.65521225080824306374501052613584298 39,
331.93806792975991203656863538457789579 46, none, none, none]

1 --> 2  target = [34.49522661193963602757798697214390065334,
3.89713131608432251716840103059838316868 6,
373.78081885967139752766154932972068260 00]
one interval r = 21.06068473213888328999751996505194471036 ..
26.26979834319822146177169073255113023473
Time Approximations 0.035.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [.3e-37, .7e-37, .37301e-34]Solution in 3.351s

Time Plot 0 s.
Exiting SolveHard() after 4.087r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61,
375.73285291393068834560325211392471012 46,
328.11709295822092048010125208247215184 44,
292.99969139879739071272372492269269927 16,
358.64341562223099124939959987002300591 88,
299.89866206373488430715334985806433024 14,
360.06173468172036231782324791072384005 50,
336.59441033630247726680535228137917197 93, none,
324.65521225080824306374501052613584298 39,
331.93806792975991203656863538457789579 46, none, none, none]

0 --> 2  target = [33.81362495437449513678111106794786742969,
3.72564899372013274520996226165674843012 4,
325.89209974361928747154246401076783582 38]
two intervals r = 18.552270489981316329535924733965079177 29 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000
000 or r = 12.491969358771302259404336250339211390 31 ..
475000000002908357101651458711257514 3/2500000000000000000000000000000000
000
Time Approximations 0.039.
```

```
hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [.138e-36, -.6e-37, -.195e-34]Solution in 3.445s

Time Plot 0 s.
Exiting SolveHard() after 4.82r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61,
375.73285291393068834560325211392471012 46,
328.11709295822092048010125208247215184 44,
292.99969139879739071272372492269269927 16,
358.64341562223099124939959987002300591 88,
299.89866206373488430715334985806433024 14,
360.06173468172036231782324791072384005 50,
336.59441033630247726680535228137917197 93, none,
324.65521225080824306374501052613584298 39,
331.93806792975991203656863538457789579 46, none, none,
289.54595774081762296001499478618404108 99]

1 --> 2  target = [33.81362495437449513678111106794786742969,
3.72564899372013274520996226165674843012 4,
325.89209974361928747154246401076783582 38]
one interval r = 20.37468935103941631652497449470867538407 ..
25.37892165337257098920462315167634224250
Time Approximations 0.027.
```

```
hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=1.3e-37
Equations at solution: [-.10e-36, -.13e-36, .19604e-34]Solution in
0.57s

Time Plot 0 s.
Exiting SolveHard() after 1.119r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61,
375.73285291393068834560325211392471012 46,
328.11709295822092048010125208247215184 44,
292.99969139879739071272372492269269927 16,
358.64341562223099124939959987002300591 88,
299.89866206373488430715334985806433024 14,
360.06173468172036231782324791072384005 50,
336.59441033630247726680535228137917197 93,
256.10753187659262124870619937004267889 78,
324.65521225080824306374501052613584298 39,
331.93806792975991203656863538457789579 46, none, none,
289.54595774081762296001499478618404108 99]

1 --> 0  target = [17.93041369696050918077262515028910369791,
4.68650870201087317161373062970490901314 6,
353.30541096457132443360541350296772146 14]
one interval r = 20.73150479094835497271271047835650551146 ..
```

```
25.906753535577845973800624474468288337 64
Time Approximations 0.036.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., -.7546e-35]Solution in 2.862s

Time Plot 0 s.
Exiting SolveHard() after 3.558r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61,
375.73285291393068834560325211392471012 46,
328.11709295822092048010125208247215184 44,
292.99969139879739071272372492269269927 16,
358.64341562223099124939959987002300591 88,
299.89866206373488430715334985806433024 14,
360.06173468172036231782324791072384005 50,
336.59441033630247726680535228137917197 93,
256.10753187659262124870619937004267889 78,
324.65521225080824306374501052613584298 39,
331.93806792975991203656863538457789579 46,
304.79958326897356014813070994795958330 21, none,
289.54595774081762296001499478618404108 99]

2 --> 0  target = [17.93041369696050918077262515028910369791,
```

```
4.686508702010873171613730629704909013146,
353.305410964571324433605413502967721461 4]
one interval r = 31.374354870089376873536066818267613206 59 ..
34.201275200546038215766902530435668305 97
Time Approximations 0.016.

hint used Hint := [33.796329522691901201472146203888695425 08, 2, 1, -1,
17.863534069716629640044658426895081929 62, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., -.211e-34]Solution in 0.374s

Time Plot 0 s.
Exiting SolveHard() after 0.639r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494791608318296505166646174978 39,
441.64295974358891924727373817275483447 05,
436.91748166539797544963933522347499079 75,
422.98493398683957923942007113500370131 56,
361.52580257446249779228736699591656235 14,
401.88173905563068411591192822042168839 58,
389.59001517154721524625121362874840887 61,
328.46939894860662048698205698767757559 52,
401.50757159358389788741793799630906296 98,
358.97362825336888260803939061097948688 78,
398.33147105323417007852901039769978774 66,
371.48387395881062351772134952422780260 30,
336.61215842598072042052322286660795640 90,
361.50888348473561811477055797030226119 05,
324.67144994112212257893942654847301654 10,
302.31384316264313004988382984808871444 08,
328.46938514933015358819164942414745595 14,
343.81340626487440076143109626449290194 61,
375.73285291393068834560325211392471012 46,
328.11709295822092048010125208247215184 44,
292.99969139879739071272372492269269927 16,
358.64341562223099124939959987002300591 88,
299.89866206373488430715334985806433024 14,
360.06173468172036231782324791072384005 50,
336.59441033630247726680535228137917197 93,
256.10753187659262124870619937004267889 78,
324.65521225080824306374501052613584298 39,
331.93806792975991203656863538457789579 46,
304.79958326897356014813070994795958330 21,
```

```
323.46169178217999921544025465809330862 00,
289.54595774081762296001499478618404108 99]

Cascade time 148.607
counts: 28, 28

 Iteration 47

Start Generation 1
1 --> 0  target = [12.00000000014260835968575655344766828000,
6.21701250302657352473017835203805758549 1,
485.54908089591667593243292260300838478 66]
one interval r = 23.40850301645061060402760232753357952873 ..
27.67578046413454477310508260130543957132
Time Approximations 0.042.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.08e-37
Equations at solution: [.3e-37, -.108e-36, -.3e-36]Solution in 3.272s

Time Plot 0 s.
Exiting SolveHard() after 4.43r=27.5236 in [25.56992694 .. 27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000014260835968575655344766828000,
6.21701250302657352473017835203805758549 1,
485.54908089591667593243292260300838478 66]
one interval r = 32.62814779204326101393575123978313802554 ..
36.10248388934167493237008458386427189652
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, -.104e-34]Solution in 0.615s

Time Plot 0 s.
Exiting SolveHard() after 1.038r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684465138705530929478266392852054,
6.58343472174545999049034788829961241329 1,
467.78730595650448747155795228295608683 60]
one interval r = 32.41978955653753299967236041076926165763 ..
35.85152417363704875256404674377009956338
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=9e-38
Equations at solution: [.11e-36, -.9e-37, .74e-35]Solution in 2.94s

Time Plot 0 s.
Exiting SolveHard() after 3.314r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07, none, none,
401.88173903873748253098703786510211487 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
0 --> 1  target = [27.523596844651387055309294782663928520 54,
6.5834347217454599904903478882996124132 91,
467.78730595650448747155795228295608683 60]
two intervals r = 12.923271608479711832235165193404552554 53 ..
237500000001076095319317420821248413 9/125000000000000000000000000000000000
000 or r = 18.394248580331617160454387032244685796 89 ..
237500000001076095319317420821248413 9/125000000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=8.8e-38
Equations at solution: [-.7e-37, -.88e-37, .193e-35]Solution in 39.666s

Time Plot 0 s.
Exiting SolveHard() after 41.034r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32, none,
401.88173903873748253098703786510211487 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628156653332587743742464375952 46,
4.1256517968026834389492731357832204341 43,
440.67123064686665461283030051824835364 19]
two intervals r = 14.356597051439807226565903294581869657 67 ..
237500000001076095319317420821248413 9/125000000000000000000000000000000000
000 or r = 17.703526138034336447426395978847984102 69 ..
237500000001076095319317420821248413 9/125000000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.27e-37, .1e-37, .735e-35]Solution in 1.323s

Time Plot 0 s.
Exiting SolveHard() after 4.541r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32, none,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962815665333258774374246437595246,
4.12565179680268343894927313578322043414 3,
440.67123064686665461283030051824835364 19]
one interval r = 22.39761154355448653796278615263431623936 ..
27.23722351582024572432919989821059731525
Time Approximations 0.035.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.227 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064370918029277606511682757370260, rm =
14.37818770455753867884086158611784031195}});
Accepted {r=26.4635, rm=16.5329} with Delta=7.9e-38
Equations at solution: [.1e-37, .79e-37, -.339e-34]Solution in 9.354s

Time Plot 0 s.
Exiting SolveHard() after 12.62r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888787904295188393740046077871308,
4.00486908177871811949468930387821088658 1,
404.86224500947026331070208986514574014 40]
two intervals r = 16.08011007784575861884658383800372826029 ..
23750000000107609531931742082124841 39/125000000000000000000000000000000
000 or r = 16.41579812681992968958357795430325881672 ..
23750000000107609531931742082124841 39/125000000000000000000000000000000
000
Time Approximations 0.054.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., -.14e-36]Solution in 3.94s

Time Plot 0 s.
Exiting SolveHard() after 5.018r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05, none, none,
358.97362823423642453287471498799122021 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888787904295188393740046077871308,
4.00486908177871811949468930387821088658 1,
```

```
404.86224500947026331070208986514574014 40]
one interval r = 21.641943994024164383301255416527138190 21 ..
26.763306600253731147589688138020217907 24
Time Approximations 0.053.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [.1e-37, .49e-37, -.313e-34]Solution in 3.433s

Time Plot 0 s.
Exiting SolveHard() after 4.477r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18, none,
358.97362823423642453287471498799122021 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941803684112293741435625712307283,
5.589637183045278442086631674891641195752,
443.83065884332329766465158621968686781 69]
one interval r = 22.467253744689366917760956264143148694 30 ..
27.273884283395499825567933928258383274 18
Time Approximations 0.04.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.08e-37
```

```
Equations at solution: [-.1e-37, .108e-36, .83e-35]Solution in 3.346s

Time Plot 0 s.
Exiting SolveHard() after 4.31r=27.0204 in [24.71083344 .. 27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.642959729529503753558343795535287399 6,
436.917481649133783929259764418563478160 7,
422.984933972949234832251336262072557263 2,
361.525802555402422970815855551178727580 6,
401.881739038737482530987037865102114872 6,
389.590015153463525981708622482243084940 5,
328.469398928994440624617902939320607471 8, none,
358.973628234236424532874714987991220211 2,
398.331471038667087237842171841157324239 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941803684112293741435625712307283,
5.589637183045278442086631674891641195752,
443.83065884332329766465158621968686781 69]
one interval r = 32.15575279492147698552762210755664594728 ..
35.50872228729313272449400716870975834664
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, .111e-34]Solution in 0.476s

Time Plot 0 s.
Exiting SolveHard() after 0.821r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.642959729529503753558343795535287399 6,
436.917481649133783929259764418563478160 7,
422.984933972949234832251336262072557263 2,
361.525802555402422970815855551178727580 6,
```

```
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136542826239080944773207246661309,
5.18778357857785247044359456511525454632 5,
408.65773862177207888951737028354857134 26]
one interval r = 21.71840114638321445145703187927643819557 ..
26.81849303493250300429196322434540986042
Time Approximations 0.058.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.11e-37
Equations at solution: [.2e-37, .211e-36, .23e-35]Solution in 3.508s

Time Plot 0 s.
Exiting SolveHard() after 4.589r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98, none, none,
361.50888346586056177383659174673221952 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136542826239080944773207246661309,
5.18778357857785247044359456511525454632 5,
408.65773862177207888951737028354857134 26]
one interval r = 31.80828598739721090640619502809312139954 ..
```

```
35.00011460032826118135403832918549499077
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, .77e-35]Solution in 0.434s

Time Plot 0 s.
Exiting SolveHard() after 0.736r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42, none,
361.50888346586056177383659174673221952 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110521088310145465659807831689094,
6.19626256545708783875215643953277898972 3,
385.44474378789118822837555193207928749 6]
one interval r = 31.60836097525195044426136925207602664778 ..
34.66372795596868032549038296120070499959
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
```

```
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, .33e-35]Solution in 0.557s

Time Plot 0 s.
Exiting SolveHard() after 0.845r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42, none,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110521088310145465659807831689094,
6.19626256545708783875215643953277898972 3,
385.44474378789118822837555193207928749 6]
two intervals r = 16.87563408783063854080155495283441022283 ..
23750000000107609531931742082124841 39/12500000000000000000000000000000000
000 or r = 15.55640493780374001472005849642002232304 ..
23750000000107609531931742082124841 39/12500000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.1076e-36, 0., .11e-36]Solution in 1.178s

Time Plot 0 s.
Exiting SolveHard() after 4.877r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899440624617902939320607471 8,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874763841556127248691502623537190,
4.88381077991567355070895205441548395 6478,
376.61967855257126558526269519766514190 71]
one interval r = 21.11001304868183735818172296633016251836 ..
26.31784243455645756125239830665118231193
Time Approximations 0.034.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.5e-38
Equations at solution: [0., -.25e-37, -.316e-34]Solution in 0.838s

Time Plot 0 s.
Exiting SolveHard() after 4.208r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899440624617902939320607471 8,
```

```
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10, none,
328.46938512972342675374086857432547606 85, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874763841556127248691502623537190,
4.8838107799156735507089520544154839564 78,
376.61967855257126558526269519766514190 71]
one interval r = 31.53899497699452591558216153286273080997 ..
34.53618386079940790711145573568932887696
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=9.16e-36
Equations at solution: [-.704e-35, .916e-35, -.204e-34]Solution in
0.497s

Time Plot 0 s.
Exiting SolveHard() after 0.804r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899440624617902939320607471 8,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10, none,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017521669931873276334237001920274,
6.02581354941032018802987145968490324193 8,
351.42702947856279158216070603627211282 61]
one interval r = 31.36230206102615828026206354926706696941 ..
34.17446640601204865698723987497290390480
Time Approximations 0.015.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, -.138e-34]Solution in 0.526s

Time Plot 0 s.
Exiting SolveHard() after 3.122r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899440624617902939320607471 8,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10, none,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74, none, none,
292.99969137635449773224173436893004550 23, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017521669931873276334237001920274,
6.02581354941032018802987145968490324193 8,
351.42702947856279158216070603627211282 61]
two intervals r = 17.98135514466161108549856031688339659344 ..
23750000000107609531931742082124841 39/1250000000000000000000000000000000
000 or r = 13.84608015391994286005868424925390106241 ..
```

```
2375000000010760953193174208212484139/125000000000000000000000000000000
000
Time Approximations 0.048.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.54e-37, -.2e-37, -.1409e-34]Solution in 3.332s

Time Plot 0 s.
Exiting SolveHard() after 4.402r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74, none, none,
292.99969137635449773224173436893004550 23, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418586014646256430120600822267 84,
6.3779438740180736848569652190590011040 28,
423.28832783747268011733968013951941140 08]
one interval r = 31.946618175878587724666617057383836708 92 ..
35.212123086431600924508895469955582748 15
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .276e-34]Solution in 0.603s

Time Plot 0 s.
Exiting SolveHard() after 0.928r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74, none, none,
292.99969137635449773224173436893004550 23, none, none,
360.06173466425700536920317984602776101 19, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941858601464625643012060082226784,
6.37794387401807368485696521905900110402 8,
423.28832783747268011733968013951941140 08]
two intervals r = 15.22886702455018556561524553699015360400 ..
23750000000107609531931742082124841 39/12500000000000000000000000000000000
000 or r = 17.12965777070405516477346397978334906440 ..
23750000000107609531931742082124841 39/12500000000000000000000000000000000
000
Time Approximations 0.06.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.32e-37, -.1e-37, -.119e-35]Solution in 1.291s

Time Plot 0 s.
Exiting SolveHard() after 5.109r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74,
375.73285289976932967218496500260112577 81, none,
292.99969137635449773224173436893004550 23, none, none,
360.06173466425700536920317984602776101 19, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234331097742565677280987907960200,
4.0035598154771237604012895916113425158 42,
404.47973593604842152101707697853362780 73]
two intervals r = 16.09683966397527179581653190641251493151 ..
23750000000107609531931742082124841 39/12500000000000000000000000000000000
000 or r = 16.39988649099841972254810198189359200650 ..
23750000000107609531931742082124841 39/12500000000000000000000000000000000
000
Time Approximations 0.053.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.52e-37, -.1e-37, -.1052e-34]Solution in
1.492s

Time Plot 0 s.
Exiting SolveHard() after 5.108r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899440624617902939320607471 8,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74,
375.73285289976932967218496500260112577 81, none,
292.99969137635449773224173436893004550 23,
358.64341560231699284724003731201087233 82, none,
360.06173466425700536920317984602776101 19, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234331097742565677280987907960200,
4.0035598154771237604012895916113425158 42,
404.47973593604842152101707697853362780 73]
one interval r = 21.63429629978751947116369452369761983529 ..
26.75768169874481579872114516686784318911
Time Approximations 0.047.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, -.140e-34]Solution in 1.025s

Time Plot 0 s.
```

```
Exiting SolveHard() after 4.125r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74,
375.73285289976932967218496500260112577 81,
328.11709293778041450278874643169064390 73,
292.99969137635449773224173436893004550 23,
358.64341560231699284724003731201087233 82, none,
360.06173466425700536920317984602776101 19, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954439412738625488257239911836997,
6.19617723031100600011360092213628794073 6,
385.42734025172169569623862041238953940 55]
one interval r = 31.608220490806257781704123810308896703 46 ..
34.663476150346467534795013909527025083 82
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=1.2e-37
Equations at solution: [-.8e-37, .12e-36, -.76e-35]Solution in 2.918s

Time Plot 0 s.
Exiting SolveHard() after 3.217r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
```

```
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74,
375.73285289976932967218496500260112577 81,
328.11709293778041450278874643169064390 73,
292.99969137635449773224173436893004550 23,
358.64341560231699284724003731201087233 82, none,
360.06173466425700536920317984602776101 19, none, none,
324.65521222913533921748679365998117220 18, none, none, none, none]

0 --> 1  target = [26.46318954439412738625488257239911836997,
6.19617723031100600011360092213628794073 6,
385.42734025172169569623862041238953940 55]
two intervals r = 16.87629600321390410487287079201890560287 ..
23750000000107609531931742082124841 39/12500000000000000000000000000000000
000 or r = 15.55559000629307145305625699510985015075 ..
23750000000107609531931742082124841 39/12500000000000000000000000000000000
000
Time Approximations 0.058.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., .4433e-34]Solution in 3.454s

Time Plot 0 s.
Exiting SolveHard() after 4.615r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74,
375.73285289976932967218496500260112577 81,
328.11709293778041450278874643169064390 73,
292.99969137635449773224173436893004550 23,
358.64341560231699284724003731201087233 82, none,
360.06173466425700536920317984602776101 19,
336.59441031791861986611284878000267708 99, none,
324.65521222913533921748679365998117220 18, none, none, none, none]

0 --> 2  target = [34.49522661149062283150753305201255455389,
3.89713131588325490211149058638724303661 9,
373.78081883713108414735350559921583333 76]
two intervals r = 17.29769086258800620375096237169655194163 ..
23750000000107609531931742082124841 39/12500000000000000000000000000000000
000 or r = 14.99436407396966966903648971292890652061 ..
23750000000107609531931742082124841 39/12500000000000000000000000000000000
000
Time Approximations 0.087.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.55e-37, 0., -.436e-35]Solution in 3.595s

Time Plot 0 s.
Exiting SolveHard() after 5.169r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74,
375.73285289976932967218496500260112577 81,
328.11709293778041450278874643169064390 73,
292.99969137635449773224173436893004550 23,
358.64341560231699284724003731201087233 82, none,
360.06173466425700536920317984602776101 19,
336.59441031791861986611284878000267708 99, none,
324.65521222913533921748679365998117220 18,
331.93806790540937809731979611878200648 50, none, none, none]

1 --> 2  target = [34.49522661149062283150753305201255455389,
3.89713131588325490211149058638724303661 9,
373.78081883713108414735350559921583333 76]
one interval r = 21.06068473190882422477742823564699504932 ..
26.26979834260595197710542909790626639275
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [-.1e-37, -.3e-37, .660e-34]Solution in 3.272s

Time Plot 0 s.
Exiting SolveHard() after 4.008r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899440624617902939320607471 8,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74,
375.73285289976932967218496500260112577 81,
328.11709293778041450278874643169064390 73,
292.99969137635449773224173436893004550 23,
358.64341560231699284724003731201087233 82,
299.89866203902928717875916833929678031 20,
360.06173466425700536920317984602776101 19,
336.59441031791861986611284878000267708 99, none,
324.65521222913533921748679365998117220 18,
331.93806790540937809731979611878200648 50, none, none, none]

0 --> 2  target = [33.81362495393954466001923689298769803840,
3.72564899351687377734251334375902829749 4,
325.89209972158483496940766461838835544 09]
two intervals r = 18.552270490259895776975130344429460798 62 ..
237500000001076095319317420821248413 9/125000000000000000000000000000000
000 or r = 12.491969357519603128436793385868506662 83 ..
237500000001076095319317420821248413 9/125000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [-.139e-36, .5e-37, -.570e-35]Solution in 3.593s

Time Plot 0 s.
Exiting SolveHard() after 5.018r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74,
375.73285289976932967218496500260112577 81,
328.11709293778041450278874643169064390 73,
292.99969137635449773224173436893004550 23,
358.64341560231699284724003731201087233 82,
299.89866203902928717875916833929678031 20,
360.06173466425700536920317984602776101 19,
336.59441031791861986611284878000267708 99, none,
324.65521222913533921748679365998117220 18,
331.93806790540937809731979611878200648 50, none, none,
289.54595771661288680200210406352343133 52]

1 --> 2  target = [33.81362495393954466001923689298769803840,
3.72564899351687377734251334375902829749 4,
325.89209972158483496940766461838835544 09]
one interval r = 20.37468935101300121915779107573029785472 ..
25.37892165277713115549453224362140143788
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.2e-37, .2e-37, .320e-34]Solution in 0.563s

Time Plot 0 s.
Exiting SolveHard() after 1.112r=24.3395 in [22.07732228 ..
25.37892164]
```

```
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74,
375.73285289976932967218496500260112577 81,
328.11709293778041450278874643169064390 73,
292.99969137635449773224173436893004550 23,
358.64341560231699284724003731201087233 82,
299.89866203902928717875916833929678031 20,
360.06173466425700536920317984602776101 19,
336.59441031791861986611284878000267708 99,
256.10753185244322569495677539392151476 28,
324.65521222913533921748679365998117220 18,
331.93806790540937809731979611878200648 50, none, none,
289.54595771661288680200210406352343133 52]

1 --> 0  target = [17.93041369732239193657510892758232424949,
4.6865087020626030269432269499567535326 06,
353.30541094497131547483562350983345702 00]
one interval r = 20.73150479084344609550568719916864564980 ..
25.906753535030481408966974928940544358 54
Time Approximations 2.32.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.9e-38
Equations at solution: [-.2e-37, -.49e-37, -.157e-34]Solution in 0.649s

Time Plot 0 s.
```

```
Exiting SolveHard() after 3.639r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74,
375.73285289976932967218496500260112577 81,
328.11709293778041450278874643169064390 73,
292.99969137635449773224173436893004550 23,
358.64341560231699284724003731201087233 82,
299.89866203902928717875916833929678031 20,
360.06173466425700536920317984602776101 19,
336.59441031791861986611284878000267708 99,
256.10753185244322569495677539392151476 28,
324.65521222913533921748679365998117220 18,
331.93806790540937809731979611878200648 50,
304.79958325023452407947201359325745475 27, none,
289.54595771661288680200210406352343133 52]

2 --> 0  target = [17.93041369732239193657510892758232424949,
4.68650870206260302694322694995675353260 6,
353.30541094497131547483562350983345702 00]
one interval r = 31.37435486981911313274603374549610536700 ..
34.20127520014099720967220958903880711633
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
```

```
Equations at solution: [-.4e-37, .6e-37, .525e-34]Solution in 0.347s

Time Plot 0 s.
Exiting SolveHard() after 0.631r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493444559202834941256862746767 07,
441.64295972952950375355834379553528739 96,
436.91748164913378392925976441856347816 07,
422.98493397294923483225133626207255726 32,
361.52580255540242297081585555117872758 06,
401.88173903873748253098703786510211487 26,
389.59001515346352598170862248224308494 05,
328.46939892899444062461790293932060747 18,
401.50757157581046018634408794600264120 93,
358.97362823423642453287471498799122021 12,
398.33147103866708723784217184115732423 98,
371.48387393644888951920034988249049517 42,
336.61215840740259626167314013926736673 54,
361.50888346586056177383659174673221952 58,
324.67144991927168153008281051354578387 10,
302.31384314383024345613938143415190035 95,
328.46938512972342675374086857432547606 85,
343.81340624095415843119838705783370350 74,
375.73285289976932967218496500260112577 81,
328.11709293778041450278874643169064390 73,
292.99969137635449773224173436893004550 23,
358.64341560231699284724003731201087233 82,
299.89866203902928717875916833929678031 20,
360.06173466425700536920317984602776101 19,
336.59441031791861986611284878000267708 99,
256.10753185244322569495677539392151476 28,
324.65521222913533921748679365998117220 18,
331.93806790540937809731979611878200648 50,
304.79958325023452407947201359325745475 27,
323.46169175842214790023689796387340753 30,
289.54595771661288680200210406352343133 52]

Cascade time 147.941
counts: 28, 28

 Iteration 48

Start Generation 1
1 --> 0  target = [12.00000000011075704099382772067088919900,
6.21701250297571058669614940908308093551 5,
485.54908089837605848176424360032270627 50]
one interval r = 23.40850301653418509783226844596165987319 ..
27.67578046425955547648930790403050954344
Time Approximations 0.041.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
```

```
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=7.7e-38
Equations at solution: [.2e-37, -.77e-37, -.16e-35]Solution in 1s

Time Plot 0 s.
Exiting SolveHard() after 4.655r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.000000000110757040993827720670889199 00,
6.217012502975710586696149409083080935515,
485.549080898376058481764243600322706275 0]
```

"Imaginary part neglected: ", $3.183223432210324649316045983684746581724 \times 10^{-17}$

```
one interval r = 32.628147792138946775014107035487529077 55 ..
36.10248388937061193998225738285437754848
Time Approximations 0.024.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [.6e-37, -.4e-37, -.196e-36]Solution in 0.563s

Time Plot 0 s.
Exiting SolveHard() after 0.993r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684477973156438476640867180412516,
6.58343472162889011251375481092663765 0114,
467.78730595927005224444486100723732755 80]
```

"Imaginary part neglected: ", $3.18322343221032464931604598368474658172 4 \times 10^{-17}$

```
one interval r = 32.41978955664014930322144550549083446996 ..
35.85152417367543047105514044240507493745
Time Approximations 2.398.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, .9932e-35]Solution in 0.599s

Time Plot 0 s.
Exiting SolveHard() after 3.352r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20, none, none,
401.88173904292682140637411963730317590 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684477973156438476640867180412516,
6.58343472162889011251375481092663765 0114,
467.78730595927005224444486100723732755 80]
two intervals r = 12.92327160844349326489810147396523192399 ..
38000000000383014034271779384722936 37/20000000000000000000000000000000000
000 or r = 18.39424858047203690833727188583053599117 ..
38000000000383014034271779384722936 37/20000000000000000000000000000000000
000
Time Approximations 0.042.
```

```
hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=0
Equations at solution: [0., 0., -.626e-35]Solution in 38.857s

Time Plot 0 s.
Exiting SolveHard() after 40.208r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882 44929,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89, none,
401.88173904292682140637411963730317590 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962820707204938755614540895854624,
4.12565179681317311173162814394638281843 1,
440.67123064992011465672802355764218385 24]
two intervals r = 14.356597051409120129634162687318943254 96 ..
38000000000383014034271779384722936 37/2000000000000000000000000000000000
000 or r = 17.703526138197276581002925696194915144 16 ..
38000000000383014034271779384722936 37/2000000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [.27e-37, .2e-37, .1230e-34]Solution in 1.322s

Time Plot 0 s.
Exiting SolveHard() after 4.657r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89, none,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962820707204938755614540895854624,
4.1256517968131731117316281439463828184 31,
440.67123064992011465672802355764218385 24]
one interval r = 22.397611543629625144911659376295935377 70 ..
27.237223515952610257333684575228301273 80
Time Approximations 0.036.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.214 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643811467283174600078231133064 93, rm =
14.378187703341716144601286832088782320 48}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., -.231e-34]Solution in 9.527s

Time Plot 0 s.
Exiting SolveHard() after 12.77r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887954630509783212228804339568 44,
4.004869081792999543495790108984643491045,
404.86224501354202129784354441701068768 05]
two intervals r = 16.08011007780131435860001052135747067508 ..
38000000000383014034271779384722936 37/20000000000000000000000000000000000
000 or r = 16.41579812704201183946224448946585393978 ..
38000000000383014034271779384722936 37/20000000000000000000000000000000000
000
Time Approximations 0.053.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=2e-38
Equations at solution: [-.68e-37, -.2e-37, -.2878e-34]Solution in
3.655s

Time Plot 0 s.
Exiting SolveHard() after 4.738r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882 44929,
441.64295973231409571670665285656018 49999,
436.91748165232850439846694983139425 80520,
422.98493397659364928140463050541262 09489,
361.52580256016552246186767750551237 13653,
401.88173904292682140637411963730317 59073,
389.59001515707444899952894553093035 79910, none, none,
358.97362823877544233246921465336875 01896, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887954630509783212228804339568 44,
4.004869081792999543495790108984643491045,
404.86224501354202129784354441701068768 05]
one interval r = 21.64194399409813000401427206686775174623 ..
26.76330660040109195138417946853054482409
Time Approximations 0.053.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.5e-38
Equations at solution: [.1e-37, .25e-37, -.209e-34]Solution in 3.417s

Time Plot 0 s.
Exiting SolveHard() after 4.467r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77, none,
358.97362823877544233246921465336875018 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941796225817671936681572791099260,
5.5896371830105447733774010225535942783 07,
443.83065884716883141892112225286035628 16]
one interval r = 22.46725374478374560699525060683376108944 ..
27.27388428353684839973452636964681133740
Time Approximations 0.038.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.4e-38
Equations at solution: [0., .54e-37, -.72e-35]Solution in 3.491s

Time Plot 0 s.
Exiting SolveHard() after 4.474r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77, none,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941796225817671936681572791099260,
5.5896371830105447733774010225535942783 07,
443.83065884716883141892112225286035628 16]
```

"Imaginary part neglected: ", $3.18322343221032464931604598368474658172 4 \times 10^{-17}$

```
one interval r = 32.15575279503935074389018525521651197956 ..
35.50872228735402517925193264579123064501
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=4e-38
Equations at solution: [.6e-37, -.4e-37, -.6076e-35]Solution in 0.48s

Time Plot 0 s.
Exiting SolveHard() after 0.848r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
```

```
398.331471043312678843706818689168806336 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136539166089060679792395024858338,
5.187783578539733044674440845222715096759,
408.65773862557153589815747433928389397 91]
one interval r = 21.71840114645437102848003855299442839545 ..
26.81849303507573421959207714962132172846
Time Approximations 0.054.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.32e-37
Equations at solution: [-.1e-37, -.132e-36, -.223e-34]Solution in
0.912s

Time Plot 0 s.
Exiting SolveHard() after 4.479r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.331471043312678843706818689168806336 3, none, none,
361.50888347055252565398447161490898465 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136539166089060679792395024858338,
5.187783578539733044674440845222715096759,
408.65773862557153589815747433928389397 91]
```

"Imaginary part neglected: ", $3.183223432210324649316045983684746581724 \times 10^{-17}$

```
one interval r = 31.80828598751741794089766571701574416492 ..
35.00011460039880627334084931647367601668
```

```
Time Approximations 0.02.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, .5488e-35]Solution in 0.393s

Time Plot 0 s.
Exiting SolveHard() after 0.697r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96, none,
361.50888347055252565398447161490898465 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110537089482893483132087346694697,
6.19626256535409989680690749405192141921 7,
385.44474379270362574394190498554601190 69]
```

"Imaginary part neglected: ", $3.1832234322103246493160459836847465817 24 \times 10^{-17}$

```
one interval r = 31.60836097538115893558999151839706346758 ..
34.66372795606052527453063346478876259281
Time Approximations 0.02.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=9e-38
Equations at solution: [-.5e-37, .9e-37, .15612e-34]Solution in 2.72s

Time Plot 0 s.
Exiting SolveHard() after 3.003r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96, none,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110537089482893483132087346694697,
6.19626256535409989680690749405192141921 7,
385.44474379270362574394190498554601190 69]
two intervals r = 16.875634087780650401822570452172614103 10 ..
38000000000383014034271779384722936 37/20000000000000000000000000000000000
000 or r = 15.556404938071032326719325472340175040 23 ..
38000000000383014034271779384722936 37/20000000000000000000000000000000000
000
Time Approximations 0.058.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.179e-37, 0., .1174e-34]Solution in 3.545s

Time Plot 0 s.
Exiting SolveHard() after 4.689r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
```

```
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874760270192799918808465080615320,
4.8838107798854300159668988649496081440 39,
376.61967855733596863080164228117899025 18]
one interval r = 21.11001304874633529070342958650362608854 ..
26.31784243471613989758213899148192500810
Time Approximations 0.033.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .200e-34]Solution in 0.81s

Time Plot 0 s.
Exiting SolveHard() after 1.487r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
```

```
389.590015157074448999528945530930357991 0,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49, none,
328.46938513540745973834520380000655821 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747602701927999188084650806153 20,
4.8838107798854300159668988649496081440 39,
376.61967855733596863080164228117899025 18]
```

"Imaginary part neglected: ", $3.18322343221032464931604598368474658172 4 \times 10^{-17}$

```
one interval r = 31.53899497712299604192238622188334630855 ..
34.53618386089295871551842222064405252049
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.073e-35
Equations at solution: [-.824e-35, .1073e-34, -.10434e-34]Solution in
3.449s

Time Plot 0 s.
Exiting SolveHard() after 3.72r=34.0898 in [32.52213872 .. 34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.590015157074448999528945530930357991 0,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
```

```
361.5088834705525256539844716149089846562,
324.6714499254345022740086132514367698649, none,
328.4693851354074597383452038000065582116,
343.8134062465985072140518929043094456863, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017539536900951230934995924649100,
6.025813549314261196523299111511881359778,
351.4270294843352091179913069277506119710]
```

"Imaginary part neglected: ", $3.183223432210324649316045983684746581724 \times 10^{-17}$

```
one interval r = 31.36230206115930466971618021817204664381 ..
34.17446640612662729460194553645734422115
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=9e-38
Equations at solution: [-.4e-37, .9e-37, .1790e-35]Solution in 0.533s

Time Plot 0 s.
Exiting SolveHard() after 0.797r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349367735282526591092289288244929,
441.6429597323140957167066528565601849999,
436.9174816523285043984669498313942580520,
422.9849339765936492814046305054126209489,
361.5258025601655224618676775055123713653,
401.8817390429268214063741196373031759073,
389.5900151570744489995289455309303579910,
328.4693989346803911033599556162507154877,
401.5075715804173768799697767419226269477,
358.9736282387754423324692146533687501896,
398.3314710433126788437068186891688063363,
371.4838739411646569452549464779322365596,
336.6121584131722657603968549728057035652,
361.5088834705525256539844716149089846562,
324.6714499254345022740086132514367698649, none,
328.4693851354074597383452038000065582116,
343.8134062465985072140518929043094456863, none, none,
292.9996913834298655249889903821918723229, none, none, none, none,
none, none, none, none, none, none]
```

```
0 --> 1  target = [25.872050175395369009512309349959246491 00,
6.0258135493142611965232991115118813597 78,
351.42702948433520911799130692775061197 10]
two intervals r = 17.981355144634550985731101989067291455 52 ..
38000000000383014034271779384722936 37/20000000000000000000000000000000000
000 or r = 13.846080154244623009654430423956616352 41 ..
38000000000383014034271779384722936 37/20000000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=8e-38
Equations at solution: [-.248e-36, .8e-37, .52e-36]Solution in 3.421s

Time Plot 0 s.
Exiting SolveHard() after 4.484r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63, none, none,
292.99969138342986552498899038219187232 29, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418740200524755723955505487713 08,
6.3779438739124060826339592985454760479 13,
423.28832784215829991372848406504658458 73]
```

"Imaginary part neglected: ", $3.183223432210324649316045983684746581724 \times 10^{-17}$

```
one interval r = 31.946618176006501442577527439969166704 15 ..
35.212123086510718686571626216623209353 54
Time Approximations 0.02.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=1.0e-37
Equations at solution: [-.8e-37, .10e-36, -.6824e-35]Solution in 0.605s

Time Plot 0 s.
Exiting SolveHard() after 0.952r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63, none, none,
292.99969138342986552498899038219187232 29, none, none,
360.06173467027827102168546772820812545 45, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418740200524755723955505487713 08,
6.3779438739124060826339592985454760479 13,
423.28832784215829991372848406504658458 73]
two intervals r = 15.228867024455235305070604472778546030 62 ..
38000000000383014034271779384722936 37/2000000000000000000000000000000000
000 or r = 17.129657770934624848623572272087584023 97 ..
38000000000383014034271779384722936 37/2000000000000000000000000000000000
```

```
000
Time Approximations 0.058.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.16e-37, -.1e-37, -.3960e-34]Solution in
1.244s

Time Plot 0 s.
Exiting SolveHard() after 5.063r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63,
375.73285290550849707622864286383298121 42, none,
292.99969138342986552498899038219187232 29, none, none,
360.06173467027827102168546772820812545 45, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343392865348972152965727338741 88,
4.0035598154928714903210586445528825919 07,
404.47973594054709053633084856189054800 54]
two intervals r = 16.096839663912620198681765625637876443 87 ..
380000000003830140342717793847229363 7/2000000000000000000000000000000000
000 or r = 16.399886491238532868254959327849989886 43 ..
380000000003830140342717793847229363 7/2000000000000000000000000000000000
000
Time Approximations 0.052.
```

```
hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.3300e-34]Solution in 1.492s

Time Plot 0 s.
Exiting SolveHard() after 4.978r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63,
375.73285290550849707622864286383298121 42, none,
292.99969138342986552498899038219187232 29,
358.64341560722510322939921675183203014 24, none,
360.06173467027827102168546772820812545 45, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234339286534897215296572733874188,
4.0035598154928714903210586445528825919 07,
404.47973594054709053633084856189054800 54]
one interval r = 21.63429629986973844832517253698007613016 ..
26.75768169889848064609704073528130169847
Time Approximations 0.047.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
```

```
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, -.234e-34]Solution in 1.016s

Time Plot 0 s.
Exiting SolveHard() after 4.529r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63,
375.73285290550849707622864286383298121 42,
328.11709294385934592450104553920768703 78,
292.99969138342986552498899038219187232 29,
358.64341560722510322939921675183203014 24, none,
360.06173467027827102168546772820812545 45, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954455295649966529679607998135050,
6.19617723020766028066681152309588527569 9,
385.42734025646303402274607377620288848 62]
```

"Imaginary part neglected: ", $3.18322343221032464931604598368474658172 4 \times 10^{-17}$

```
one interval r = 31.60822049093487510696489366889369166352 ..
34.66347615043725866201002370265720551934
Time Approximations 0.019.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
```

```
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, .19910e-34]Solution in 3.025s

Time Plot 0 s.
Exiting SolveHard() after 3.327r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63,
375.73285290550849707622864286383298121 42,
328.11709294385934592450104553920768703 78,
292.99969138342986552498899038219187232 29,
358.64341560722510322939921675183203014 24, none,
360.06173467027827102168546772820812545 45, none, none,
324.65521223522990333798204501212101359 30, none, none, none, none]

0 --> 1  target = [26.46318954455295649966529679607998135050,
6.19617723020766028066681152309588527569 9,
385.42734025646303402274607377620288848 62]
two intervals r = 16.87629600316672319294620861354416940096 ..
38000000000383014034271779384722936 37/20000000000000000000000000000000000
000 or r = 15.55559000655694769737795470595921110545 ..
38000000000383014034271779384722936 37/20000000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
```

```
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.179e-37, 0., -.3009e-34]Solution in 3.517s

Time Plot 0 s.
Exiting SolveHard() after 4.679r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63,
375.73285290550849707622864286383298121 42,
328.11709294385934592450104553920768703 78,
292.99969138342986552498899038219187232 29,
358.64341560722510322939921675183203014 24, none,
360.06173467027827102168546772820812545 45,
336.59441032361361846929716095576452627 08, none,
324.65521223522990333798204501212101359 30, none, none, none, none]

0 --> 2  target = [34.49522661158274404647227252890637496455,
3.89713131589986061874429931013017507559 8,
373.78018184174320893775304761642212372 49]
two intervals r = 17.29769086256152838970120858394724309093 ..
38000000000383014034271779384722936 37/20000000000000000000000000000000000
000 or r = 14.99436407423251307070172530145052318780 ..
38000000000383014034271779384722936 37/20000000000000000000000000000000000
000
Time Approximations 0.087.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
```

```
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.72e-37, 0., -.173e-35]Solution in 3.972s

Time Plot 0 s.
Exiting SolveHard() after 5.639r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63,
375.73285290550849707622864286383298121 42,
328.11709294385934592450104553920768703 78,
292.99969138342986552498899038219187232 29,
358.64341560722510322939921675183203014 24, none,
360.06173467027827102168546772820812545 45,
336.59441032361361846929716095576452627 08, none,
324.65521223522990333798204501212101359 30,
331.93806791045010616982396416465264478 83, none, none, none]

1 --> 2  target = [34.49522661158274404647227252890637496455,
3.8971313158998606187442993101301750755 98,
373.78018841743208937753047616422123724 9]
one interval r = 21.06068473196819182435747710727243719556 ..
26.26979834276315074125882616898564688841
Time Approximations 0.035.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=4e-38
Equations at solution: [-.2e-37, -.4e-37, .147e-34]Solution in 3.147s

Time Plot 0 s.
Exiting SolveHard() after 3.873r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63,
375.73285290550849707622864286383298121 42,
328.11709294385934592450104553920768703 78,
292.99969138342986552498899038219187232 29,
358.64341560722510322939921675183203014 24,
299.89866204519381651445427913044724684 75,
360.06173467027827102168546772820812545 45,
336.59441032361361846929716095576452627 08, none,
324.65521223522990333798204501212101359 30,
331.93806791045010616982396416465264478 83, none, none, none]

0 --> 2  target = [33.81362495406463541889722654706255407715,
3.72564899353977585474389490077623571976 9,
325.89209972768169604496141105373042872 39]
two intervals r = 18.552270490272141861288870659468618296 6 ..
38000000000383014034271779384729363 7/2000000000000000000000000000000000
000 or r = 12.49196935783921539528263079866876556694 ..
38000000000383014034271779384729363 7/2000000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
```

```
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=2e-38
Equations at solution: [.53e-37, -.2e-37, -.74e-36]Solution in 3.653s

Time Plot 0 s.
Exiting SolveHard() after 5.027r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63,
375.73285290550849707622864286383298121 42,
328.11709294385934592450104553920768703 78,
292.99969138342986552498899038219187232 29,
358.64341560722510322939921675183203014 24,
299.89866204519381651445427913044724684 75,
360.06173467027827102168546772820812545 45,
336.59441032361361846929716095576452627 08, none,
324.65521223522990333798204501212101359 30,
331.93806791045010616982396416465264478 83, none, none,
289.54595772297827528179158759967352709 20]

1 --> 2  target = [33.81362495406463541889722654706255407715,
3.72564899353977585474389490077623571976 9,
325.89209972768169604496141105373042872 39]
one interval r = 20.37468935104452527624200855864649677911 ..
25.37892165296239619053007180264525999861
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, -.410e-34]Solution in 0.573s

Time Plot 0 s.
Exiting SolveHard() after 1.116r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63,
375.73285290550849707622864286383298121 42,
328.11709294385934592450104553920768703 78,
292.99969138342986552498899038219187232 29,
358.64341560722510322939921675183203014 24,
299.89866204519381651445427913044724684 75,
360.06173467027827102168546772820812545 45,
336.59441032361361846929716095576452627 08,
256.10753185993384896373100115063331766 87,
324.65521223522990333798204501212101359 30,
331.93806791045010616982396416465264478 83, none, none,
289.54595772297827528179158759967352709 20]

1 --> 0  target = [17.93041369728457101344675533606692636371,
4.68650870204238758152464867608141560378 6,
353.30541095101512368479065907676277595 86]
one interval r = 20.73150479090570289843394898948093964862 ..
25.90675353521406050453921245996579043239
Time Approximations 2.484.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
```

```
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.8e-38
Equations at solution: [.2e-37, .48e-37, -.414e-34]Solution in 0.631s

Time Plot 0 s.
Exiting SolveHard() after 3.795r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63,
375.73285290550849707622864286383298121 42,
328.11709294385934592450104553920768703 78,
292.99969138342986552498899038219187232 29,
358.64341560722510322939921675183203014 24,
299.89866204519381651445427913044724684 75,
360.06173467027827102168546772820812545 45,
336.59441032361361846929716095576452627 08,
256.10753185993384896373100115063331766 87,
324.65521223522990333798204501212101359 30,
331.93806791045010616982396416465264478 83,
304.79958325718046671958386657082729419 76, none,
289.54595772297827528179158759967352709 20]

2 --> 0  target = [17.93041369728457101344675533606692636371,
4.68650870204238758152464867608141560378 6,
353.30541095101512368479065907676277595 86]
```

"Imaginary part neglected: ", $3.18322343221032464931604598368474658172 4 \times 10^{-17}$

```
one interval r = 31.37435486995428309800903595995094234438 ..
34.20127520025898703854950266099231645869
Time Approximations 0.018.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.2791e-35]Solution in 0.33s

Time Plot 0 s.
Exiting SolveHard() after 0.61r=33.7963 in [32.25770943 .. 34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493677352825265910922892882449 29,
441.64295973231409571670665285656018499 99,
436.91748165232850439846694983139425805 20,
422.98493397659364928140463050541262094 89,
361.52580256016552246186767750551237136 53,
401.88173904292682140637411963730317590 73,
389.59001515707444899952894553093035799 10,
328.46939893468039110335995561625071548 77,
401.50757158041737687996977674192262694 77,
358.97362823877544233246921465336875018 96,
398.33147104331267884370681868916880633 63,
371.48387394116465694525494647793223655 96,
336.61215841317226576039685497280570356 52,
361.50888347055252565398447161490898465 62,
324.67144992543450227400861325143676986 49,
302.31384315044861384273296396486038881 47,
328.46938513540745973834520380000655821 16,
343.81340624659850721405189290430944568 63,
375.73285290550849707622864286383298121 42,
328.11709294385934592450104553920768703 78,
292.99969138342986552498899038219187232 29,
358.64341560722510322939921675183203014 24,
299.89866204519381651445427913044724684 75,
360.06173467027827102168546772820812545 45,
336.59441032361361846929716095576452627 08,
256.10753185993384896373100115063331766 87,
324.65521223522990333798204501212101359 30,
331.93806791045010616982396416465264478 83,
304.79958325718046671958386657082729419 76,
323.46169176523405228850972301078111057 66,
289.54595772297827528179158759967352709 20]
```

```
Cascade time 148.17
counts: 28, 28

 Iteration 49

Start Generation 1
1 --> 0  target = [12.00000000001433598798099789680107017600,
6.217012503037691156482224493747215108853,
485.549080905937753708530923715616533076 2]
one interval r = 23.40850301671876959062198630804390715439 ..
27.67578046431193484206159444699681195837
Time Approximations 0.042.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.5e-38
Equations at solution: [.1e-37, -.25e-37, .4e-36]Solution in 3.189s

Time Plot 0 s.
Exiting SolveHard() after 4.331r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000001433598798099789680107017600,
6.217012503037691156482224493747215108853,
485.549080905937753708530923715616533076 2]
one interval r = 32.62814779217538744912760746900751496968 ..
36.10248388945398530607250236537391458368
Time Approximations 0.021.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
```

```
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .127e-34]Solution in 0.583s

Time Plot 0 s.
Exiting SolveHard() after 0.982r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684484192672471728809402434661628,
6.58343472170456515626361286339485922951 6,
467.78730596658570379112497551778789528 74]
one interval r = 32.41978955666723189748623817976713954389 ..
35.85152417375476408916622603975239864714
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, .14e-35]Solution in 3.009s

Time Plot 0 s.
Exiting SolveHard() after 3.379r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65, none, none,
401.88173904907749550879156653892391357 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684484192672471728809402434661628,
```

```
6.583434721704565156263612863394859229516,
467.787305966585703791124975517787895287 4]
two intervals r = 12.92327160827931015001511730603535232713 ..
237500000000140525077982632864120851 3/125000000000000000000000000000000000
000 or r = 18.39424858043752592791198069289422140532 ..
237500000000140525077982632864120851 3/125000000000000000000000000000000000
000
Time Approximations 0.042.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.3e-38
Equations at solution: [-.2e-37, -.23e-37, -.453e-35]Solution in
39.923s

Time Plot 0 s.
Exiting SolveHard() after 41.305r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31, none,
401.88173904907749550879156653892391357 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962828042186808487370722049088062,
4.125651796822417304556548264439672576282,
440.67123065693502349445177580567263827 12]
two intervals r = 14.35659705117760636676284955304407022316 ..
237500000000140525077982632864120851 3/125000000000000000000000000000000000
000 or r = 17.70352613823862590943918315349579443597 ..
237500000000140525077982632864120851 3/125000000000000000000000000000000000
000
Time Approximations 0.042.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [.29e-37, .2e-37, -.2223e-34]Solution in 1.306s

Time Plot 0 s.
Exiting SolveHard() after 4.682r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31, none,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962828042186808487370722049088062,
4.12565179682241730455654826443967257628 2,
440.67123065693502349445177805672638271 2]
one interval r = 22.39761154380328540517209545699592491875 ..
27.23722351602901820014985118004781211165
Time Approximations 0.037.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.228 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064394239997176554332663310641670, rm =
14.37818770443861847225285069216922926190}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, .383e-34]Solution in 9.292s

Time Plot 0 s.
Exiting SolveHard() after 12.515r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437757,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888015667804939997121742859398 11,
4.0048690818007690896636050790253292587 55,
404.86224501996020607357088262406011576 74]
two intervals r = 16.080110077542847855097977866455435191 32 ..
23750000000014052507798263286412085 13/125000000000000000000000000000000
000 or r = 16.415798127183192653764249846216236595 00 ..
23750000000014052507798263286412085 13/125000000000000000000000000000000
000
Time Approximations 0.053.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., .1126e-34]Solution in 3.921s

Time Plot 0 s.
Exiting SolveHard() after 5.018r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437757,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81, none, none,
358.97362824492445287528067985802632110 91, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888015667804939997121742859398 11,
4.0048690818007690896636050790253292587 55,
```

```
404.86224501996020607357088262406011576 74]
one interval r = 21.641943994251825681303159767990787119 94 ..
26.763306600490663324950925781158296877 39
Time Approximations 0.054.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=1.01e-37
Equations at solution: [.3e-37, .101e-36, -.208e-34]Solution in 3.635s

Time Plot 0 s.
Exiting SolveHard() after 4.681r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36, none,
358.97362824492445287528067985802632110 91, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941776283346792222391756651985700,
5.5896371830386083530053270389191729705 01,
443.83065885371288254200430143091904409 72]
one interval r = 22.467253744947286764620330866862610147 36 ..
27.273884283605885876240868580245999100 04
Time Approximations 0.039.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.4e-38
```

```
Equations at solution: [-.1e-37, .54e-37, .68e-35]Solution in 3.222s

Time Plot 0 s.
Exiting SolveHard() after 4.168r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36, none,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941776283346792222391756651985700,
5.58963718303860835300532703891917297050 1,
443.83065885371288254200430143091904409 72]
one interval r = 32.15575279504908148430040364502036399882 ..
35.50872228742084469857070200563376842977
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, .33e-35]Solution in 0.524s

Time Plot 0 s.
Exiting SolveHard() after 0.91r=34.9395 in [33.37332721 .. 35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
```

```
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136511932187037770844957803073883,
5.18778357855569264321769884845844988517 5,
408.65773863238308525525540288795118810 63]
one interval r = 21.71840114661716875862676411766818249219 ..
26.81849303516886487817267737545811406696
Time Approximations 2.408.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [0., .79e-37, -.160e-34]Solution in 0.94s

Time Plot 0 s.
Exiting SolveHard() after 4.482r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55, none, none,
361.50888347639503071344308886240692785 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136511932187037770844957803073883,
5.18778357855569264321769884845844988517 5,
408.65773863238308525525540288795118810 63]
one interval r = 31.80828598751621807303703093455068379442 ..
```

```
35.00011460046601293621543562801138571292
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, .169e-34]Solution in 0.397s

Time Plot 0 s.
Exiting SolveHard() after 0.699r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92, none,
361.50888347639503071344308886240692785 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110546147064568094713825167239809,
6.196262565424794661489990331176583555242,
385.44474379853099793108213474780045905 82]
one interval r = 31.60836097536245090750002958808262282717 ..
34.66372795611036135171052167535009869103
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
```

```
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=5e-38
Equations at solution: [-.3e-37, .5e-37, .157e-34]Solution in 2.903s

Time Plot 0 s.
Exiting SolveHard() after 3.191r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92, none,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110546147064568094713825167239809,
6.1962625654247946614899903311765835552 42,
385.44474379853099793108213474780045905 82]
two intervals r = 16.875634087532642215119358306094320437 68 ..
23750000000014052507798263286412085 13/12500000000000000000000000000000000
000 or r = 15.556404938255461597922356976979034694 00 ..
23750000000014052507798263286412085 13/12500000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [.896e-37, -.1e-37, .1461e-34]Solution in 3.722s

Time Plot 0 s.
Exiting SolveHard() after 4.874r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874733005718824000389517906284093,
4.8838107798858255755616998834688183410 92,
376.61967856376234520156895082796833285 20]
one interval r = 21.11001304888862839100748251229319977282 ..
26.31784243482109158211651266368463450062
Time Approximations 0.034.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=4.9e-38
Equations at solution: [.1e-37, .49e-37, -.123e-34]Solution in 0.844s

Time Plot 0 s.
Exiting SolveHard() after 1.528r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
```

```
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66, none,
328.46938514064880481297797486641049877 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874733005718824000389517906284093,
4.8838107798858255755616998834688183410 92,
376.61967856376234520156895082796833285 20]
one interval r = 31.53899497710557368153160873732091587827 ..
34.53618386095010507359560998876418939022
Time Approximations 0.018.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=4.59e-36
Equations at solution: [.354e-35, -.459e-35, -.74e-35]Solution in 0.5s

Time Plot 0 s.
Exiting SolveHard() after 0.781r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66, none,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
2 --> 1  target = [25.87205017549103010091211067942276860816,
6.025813549383081542656198736877319413471,
351.42702948956721670808330778320901918 74]
one interval r = 31.362302061123991196236096087498517446 08 ..
34.174466406162343340110204436430114653 81
Time Approximations 0.014.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, .281e-34]Solution in 0.528s

Time Plot 0 s.
Exiting SolveHard() after 3.079r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66, none,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96, none, none,
292.99969138748336085542025594329683762 22, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017549103010091211067942276860816,
6.025813549383081542656198736877319413471,
351.42702948956721670808330778320901918 74]
two intervals r = 17.981355144399049079449141360943558057 15 ..
23750000000014052507798263286412085 13/1250000000000000000000000000000000
000 or r = 13.846080154523159383519850077026111594 46 ..
23750000000014052507798263286412085 13/1250000000000000000000000000000000
000
```

```
Time Approximations 0.056.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=5e-38
Equations at solution: [-.161e-36, .5e-37, -.145e-34]Solution in 3.515s

Time Plot 0 s.
Exiting SolveHard() after 4.61r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66,
302.31384315390519768243366538499230649 64,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96, none, none,
292.99969138748336085542025594329683762 22, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418811734009489535833463638776 29,
6.3779438739821278308837520201033244478 37,
423.28832784798957292813304554475853894 32]
one interval r = 31.946618176001572000440480291111693643 74 ..
35.212123086565360046889927869815351604 19
Time Approximations 0.02.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
```

```
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.29e-35]Solution in 0.618s

Time Plot 0 s.
Exiting SolveHard() after 0.962r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437757,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66,
302.31384315390519768243366538499230649 64,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96, none, none,
292.99969138748336085542025594329683762 22, none, none,
360.06173467497873415454092453930596230 98, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418811734009489535833463638776 29,
6.3779438739821278308837520201033244478 37,
423.28832784798957292813304554475853894 32]
two intervals r = 15.22886702424838143353437389782465415611 ..
23750000000014052507798263286412085 13/12500000000000000000000000000000000
000 or r = 17.12965777099182880276683447349268712800 ..
23750000000014052507798263286412085 13/12500000000000000000000000000000000
000
Time Approximations 0.059.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [-.15e-37, 0., -.49e-36]Solution in 1.289s

Time Plot 0.001 s.
Exiting SolveHard() after 4.972r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66,
302.31384315390519768243366538499230649 64,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96,
375.73285290981129239521054816903786256 41, none,
292.99969138748336085542025594329683762 22, none, none,
360.06173467497873415454092453930596230 98, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343451635814811565302587576787 01,
4.0035598155001225760180116427610536855 34,
404.47973594681224900208846680709021484 60]
two intervals r = 16.096839663660486774286507588980127663 59 ..
237500000000140525077982632864120851 3/125000000000000000000000000000000
000 or r = 16.399886491374694466854483488787378420 88 ..
237500000000140525077982632864120851 3/125000000000000000000000000000000
000
Time Approximations 0.052.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.15e-37, 0., .2521e-34]Solution in 1.431s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.873r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66,
302.31384315390519768243366538499230649 64,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96,
375.73285290981129239521054816903786256 41, none,
292.99969138748336085542025594329683762 22,
358.64341561324383145323877823069866798 46, none,
360.06173467497873415454092453930596230 98, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234345163581481156530258757678701,
4.0035598155001225760180116427610536855 34,
404.47973594681224900208846680709021484 60]
one interval r = 21.634296300020257861729232836173460593 66 ..
26.757681698986010556504373600213338812 98
Time Approximations 0.046.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=4.9e-38
Equations at solution: [-.2e-37, -.49e-37, -.43e-35]Solution in 1.02s

Time Plot 0 s.
Exiting SolveHard() after 4.997r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66,
302.31384315390519768243366538499230649 64,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96,
375.73285290981129239521054816903786256 41,
328.11709294895906738149848683775753784 43,
292.99969138748336085542025594329683762 22,
358.64341561324383145323877823069866798 46, none,
360.06173467497873415454092453930596230 98, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954464384922362561885764300302099,
6.19617723027844903027642645509006776810 6,
385.42734026230946546140987133392370605 22]
one interval r = 31.60822049091631445139979417460283739024 ..
34.66347615048736795133300551486783223482
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.249e-34]Solution in 2.83s

Time Plot 0 s.
Exiting SolveHard() after 3.127r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66,
302.31384315390519768243366538499230649 64,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96,
375.73285290981129239521054816903786256 41,
328.11709294895906738149848683775753784 43,
292.99969138748336085542025594329683762 22,
358.64341561324383145323877823069866798 46, none,
360.06173467497873415454092453930596230 98, none, none,
324.65521223989853924360672106470462504 18, none, none, none, none]

0 --> 1  target = [26.46318954464384922362561885764300302099,
6.19617723027844903027642645509006776810 6,
385.42734026230946546140987133392370605 22]
two intervals r = 16.87629600291798119459289958553974228357 ..
23750000000014052507798263286412085 13/12500000000000000000000000000000000
000 or r = 15.55559000674233219070728782107279981096 ..
23750000000014052507798263286412085 13/12500000000000000000000000000000000
000
Time Approximations 0.059.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.179e-37, 0., .1122e-34]Solution in 3.774s

Time Plot 0 s.
Exiting SolveHard() after 4.929r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
```

```
441.642959739594472521137015629050644375 7,
436.917481659065267730920028353804717286 5,
422.984933982781771213830335834659554483 1,
361.525802565989500956254204246033012526 8,
401.881739049077495508791566538923913576 9,
389.590015163556753206822685492793145228 1,
328.469398939921388024520883783318415883 6,
401.507571586418339792097112452348838090 2,
358.973628244924452875280679858026321109 1,
398.331471049144513634729150186737386345 5,
371.483873947573712016670451812951506489 2,
336.612158417321864101569884718356623224 0,
361.508883476395030713443088862406927859 2,
324.671449930085377520841519721113622086 6,
302.313843153905197682433665384992306496 4,
328.469385140648804812977974866410498775 1,
343.813406252857086489616092577155358529 6,
375.732852909811292395210548169037862564 1,
328.117092948959067381498486837757537844 3,
292.999691387483360855420255943296837622 2,
358.643415613243831453238778230698667984 6, none,
360.061734674978734154540924539305962309 8,
336.594410327782595903192486603099040195 3, none,
324.655212239898539243606721064704625041 8, none, none, none, none]

0 --> 2  target = [34.495226611643152550648300768209896485 88,
3.897131315908998439766698194797675717018,
373.780818848427458991819935363044661440 4]
two intervals r = 17.297690862279870887771051003746785432 36 ..
237500000000140525077982632864120851 3/125000000000000000000000000000000000
000 or r = 14.994364074501956293832836982169020979 98 ..
237500000000140525077982632864120851 3/125000000000000000000000000000000000
000
Time Approximations 2.407.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., -.1041e-34]Solution in 1.197s

Time Plot 0 s.
Exiting SolveHard() after 5.179r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434943876004438342641126682992334 1,
```

```
441.642959739594472521137015629050644375 7,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66,
302.31384315390519768243366538499230649 64,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96,
375.73285290981129239521054816903786256 41,
328.11709294895906738149848683775753784 43,
292.99969138748336085542025594329683762 22,
358.64341561324383145323877823069866798 46, none,
360.06173467497873415454092453930596230 98,
336.59441032778259590319248660309904019 53, none,
324.65521223989853924360672106470462504 18,
331.93806791697744690135534876053734792 02, none, none, none]

1 --> 2  target = [34.49522661164315255064830076820989648588,
3.897131315908998439766698194797675717018,
373.78081884842745899181993536304466144 04]
one interval r = 21.06068473211344887749926015707061532786 ..
26.26979834287398141565251940480715307381
Time Approximations 0.033.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., -.252e-34]Solution in 3.393s

Time Plot 0 s.
Exiting SolveHard() after 4.097r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.642959739594472521137015629050644375 7,
```

```
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66,
302.31384315390519768243366538499230649 64,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96,
375.73285290981129239521054816903786256 41,
328.11709294895906738149848683775753784 43,
292.99969138748336085542025594329683762 22,
358.64341561324383145323877823069866798 46,
299.89866205065107088204026672955494645 30,
360.06173467497873415454092453930596230 98,
336.59441032778259590319248660309904019 53, none,
324.65521223989853924360672106470462504 18,
331.93806791697744690135534876053734792 02, none, none, none]

0 --> 2  target = [33.81362495409063850875916831871726914245,
3.72564899354302557007798655421118859443 0,
325.89209973256956829316057301151586088 93]
two intervals r = 18.552270490050777264869733107203028102 58 ..
237500000000140525077982632864120851 3/125000000000000000000000000000000000
000 or r = 12.491969358186693010826182983930406947 35 ..
237500000000140525077982632864120851 3/125000000000000000000000000000000000
000
Time Approximations 0.042.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=7e-38
Equations at solution: [.174e-36, -.7e-37, .115e-34]Solution in 3.672s

Time Plot 0 s.
Exiting SolveHard() after 5.076r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66,
302.31384315390519768243366538499230649 64,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96,
375.73285290981129239521054816903786256 41,
328.11709294895906738149848683775753784 43,
292.99969138748336085542025594329683762 22,
358.64341561324383145323877823069866798 46,
299.89866205065107088204026672955494645 30,
360.06173467497873415454092453930596230 98,
336.59441032778259590319248660309904019 53, none,
324.65521223989853924360672106470462504 18,
331.93806791697744690135534876053734792 02, none, none,
289.54595772809648501614579311046511747 92]

1 --> 2  target = [33.81362495409063850875916831871726914245,
3.72564899354302557007798655421118859443 0,
325.89209973256956829316057301151586088 93]
one interval r = 20.37468935113475110128366677915336038331 ..
25.37892165306136018844536900456940200693
Time Approximations 0.025.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, .8e-36]Solution in 0.561s

Time Plot 0 s.
Exiting SolveHard() after 1.078r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66,
302.31384315390519768243366538499230649 64,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96,
375.73285290981129239521054816903786256 41,
328.11709294895906738149848683775753784 43,
292.99969138748336085542025594329683762 22,
358.64341561324383145323877823069866798 46,
299.89866205065107088204026672955494645 30,
360.06173467497873415454092453930596230 98,
336.59441032778259590319248660309904019 53,
256.10753186370638443952526510731491065 47,
324.65521223989853924360672106470462504 18,
331.93806791697744690135534876053734792 02, none, none,
289.54595772809648501614579311046511747 92]

1 --> 0  target = [17.93041369707397433181646341060893716838,
4.68650870201832725544432175533051190886 9,
353.30541095533597915252923727109405716 68]
one interval r = 20.73150479100285041348695337378605869337 ..
25.90675353529215996841091027623827517136
Time Approximations 0.029.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, -.133e-34]Solution in 0.667s

Time Plot 0 s.
Exiting SolveHard() after 3.808r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66,
302.31384315390519768243366538499230649 64,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96,
375.73285290981129239521054816903786256 41,
328.11709294895906738149848683775753784 43,
292.99969138748336085542025594329683762 22,
358.64341561324383145323877823069866798 46,
299.89866205065107088204026672955494645 30,
360.06173467497873415454092453930596230 98,
336.59441032778259590319248660309904019 53,
256.10753186370638443952526510731491065 47,
324.65521223989853924360672106470462504 18,
331.93806791697744690135534876053734792 02,
304.79958326017061284106306240288850395 28, none,
289.54595772809648501614579311046511747 92]

2 --> 0  target = [17.93041369707397433181646341060893716838,
4.68650870201832725544432175533051190886 9,
353.30541095533597915252923727109405716 68]
one interval r = 31.37435486991373408536830057259575907823 ..
34.20127520028202610891676779920748486198
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, -.187e-34]Solution in 0.336s

Time Plot 0 s.
Exiting SolveHard() after 0.616r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
```

```
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494387600443834264112668299233 41,
441.64295973959447252113701562905064437 57,
436.91748165906526773092002835380471728 65,
422.98493398278177121383033583465955448 31,
361.52580256598950095625420424603301252 68,
401.88173904907749550879156653892391357 69,
389.59001516355675320682268549279314522 81,
328.46939893992138802452088378331841588 36,
401.50757158641833979209711245234883809 02,
358.97362824492445287528067985802632110 91,
398.33147104914451363472915018673738634 55,
371.48387394757371201667045181295150648 92,
336.61215841732186410156988471835662322 40,
361.50888347639503071344308886240692785 92,
324.67144993008537752084151972111362208 66,
302.31384315390519768243366538499230649 64,
328.46938514064880481297797486641049877 51,
343.81340625285708648961609257715535852 96,
375.73285290981129239521054816903786256 41,
328.11709294895906738149848683775753784 43,
292.99969138748336085542025594329683762 22,
358.64341561324383145323877823069866798 46,
299.89866205065107088204026672955494645 30,
360.06173467497873415454092453930596230 98,
336.59441032778259590319248660309904019 53,
256.10753186370638443952526510731491065 47,
324.65521223989853924360672106470462504 18,
331.93806791697744690135534876053734792 02,
304.79958326017061284106306240288850395 28,
323.46169176977517510009687568871665804 33,
289.54595772809648501614579311046511747 92]

Cascade time 148.99
counts: 28, 28

 Iteration 50

Start Generation 1
1 --> 0  target = [12.00000000000550568218533974658426748000,
6.21701250288251885920966447384460140365 0,
485.54908089611844809241172474767324493 16]
one interval r = 23.40850301649163218918952797207208144180 ..
27.67578046417984449721011290794586716904
Time Approximations 0.039.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
```

```
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.84e-37
Equations at solution: [-.6e-37, .184e-36, .12e-35]Solution in 3.374s

Time Plot 0 s.
Exiting SolveHard() after 4.519r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000000550568218533974658426748000,
6.21701250288251885920966447384460140365 0,
485.54908089611844809241172474767324493 16]
one interval r = 32.62814779202161044315352088588833577064 ..
36.10248388935547806684374855327571578657
Time Approximations 0.025.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=1.0e-37
Equations at solution: [-.15e-36, .10e-36, -.44e-35]Solution in 0.591s

Time Plot 0 s.
Exiting SolveHard() after 1.011r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
```

```
2 --> 1  target = [27.52359684467800534879327846650417792626,
6.58343472183441184199871328863693372974 5,
467.78730595491126904780928299387756038 68]
one interval r = 32.41978955649290832120856303856004857900 ..
35.85152417362305185143402840711393484649
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, -.54e-35]Solution in 3.309s

Time Plot 0 s.
Exiting SolveHard() after 3.668r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41, none, none,
401.88173903584947393120552301595436300 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684467800534879327846650417792626,
6.58343472183441184199871328863693372974 5,
467.78730595491126904780928299387756038 68]
```

"Imaginary part neglected: ", $1.10311211489721680011018212216172588 6513 \times 10^{-17}$

```
two intervals r = 12.92327160843015949785312471457627762813 ..
475000000000457971927658285324295862 3/2500000000000000000000000000000000
000 or r = 18.39424858024986947486857526683608501827 ..
475000000000457971927658285324295862 3/2500000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=6.7e-38
Equations at solution: [-.5e-37, -.67e-37, .271e-35]Solution in 41.166s

Time Plot 0 s.
Exiting SolveHard() after 42.502r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97, none,
401.88173903584947393120552301595436300 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962818335072405706850028235920527,
4.12565179678092930450716663863226188500 9,
440.67123064837251688871053907664132096 98]
```

"Imaginary part neglected: ", $1.10311211489721680011018212216172588651 3 \times 10^{-17}$

```
two intervals r = 14.35659705122805748777790365873814757898 ..
47500000000045797192765828532429586 23/2500000000000000000000000000000000
000 or r = 17.70352613803817689612907487266949281890 ..
47500000000045797192765828532429586 23/2500000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [.57e-37, .3e-37, -.1206e-34]Solution in 1.319s

Time Plot 0 s.
Exiting SolveHard() after 5.243r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97, none,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962818335072405706850028235920527,
4.1256517967809293045071666386322618850 09,
440.67123064837251688871053907664132096 98]
one interval r = 22.397611543628018889352114501628155915 57 ..
27.237223515876570303214535430157397286 59
Time Approximations 0.039.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.46 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643788365440027739773934972750 03, rm =
14.378187705595845933447740224542693490 48}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.3e-38
Equations at solution: [0., .53e-37, .650e-34]Solution in 9.729s

Time Plot 0 s.
Exiting SolveHard() after 10.658r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888783524245906349695142398297302,
4.0048690817416508935587258223767213978 37,
```

```
404.862245006448607911479537161769418518 3]
```

"Imaginary part neglected: ", $1.10311211489721680011018212216172588651 3 \times 10^{-17}$

```
two intervals r = 16.08011007785528174671481255903068376300 ..
475000000000457971927658285324295862 3/2500000000000000000000000000000000000
000 or r = 16.41579812666055910904053402385104701704 ..
475000000000457971927658285324295862 3/2500000000000000000000000000000000000
000
Time Approximations 0.053.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.33e-37, -.1e-37, .296e-35]Solution in 1.482s

Time Plot 0 s.
Exiting SolveHard() after 4.952r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24, none, none,
358.97362823338714300591521670296877734 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888783524245906349695142398297302,
4.00486908174165089355872582237672139783 7,
404.86224500644860791147953716176941851 83]
one interval r = 21.64194399400801714061805871840833184909 ..
26.76330660024504649116873582685832821347
Time Approximations 0.05.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, .196e-34]Solution in 1.066s

Time Plot 0 s.
Exiting SolveHard() after 5.582r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44, none,
358.97362823338714300591521670296877734 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941823854702728695028652411882990,
5.589637182803492703315458655064319784313,
443.83065883687402312567297062904948217 28]
one interval r = 22.46725374458695136876327790528909042863 ..
27.27388428336049557296626865552884071509
Time Approximations 0.035.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [0., .81e-37, .128e-34]Solution in 0.961s

Time Plot 0 s.
Exiting SolveHard() after 4.338r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
```

```
441.642959728029705445107331818589741433 6,
436.917481650675273171176543130300208034 1,
422.984933966566305071106359725930381809 7,
361.525802554947663284664730902533062849 9,
401.881739035849473931205523015954363004 9,
389.590015156305361976752715060265031452 4,
328.469398924351290524267508685973663504 4, none,
358.973628233387143005915216702968777345 8,
398.331471028617060106092848425803658519 1, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589418238547027286950286524118829 90,
5.589637182803492703315458655064319784313,
443.830658836874023125672970629049482172 8]
one interval r = 32.155752794823844623947581498905141579 41 ..
35.508722287205758764248709026694021018 77
Time Approximations 0.019.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, .53e-35]Solution in 0.458s

Time Plot 0 s.
Exiting SolveHard() after 0.831r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434934316179989087221044626963226 1,
441.642959728029705445107331818589741433 6,
436.917481650675273171176543130300208034 1,
422.984933966566305071106359725930381809 7,
361.525802554947663284664730902533062849 9,
401.881739035849473931205523015954363004 9,
389.590015156305361976752715060265031452 4,
328.469398924351290524267508685973663504 4,
401.507571572320904907284921762702798323 2,
358.973628233387143005915216702968777345 8,
398.331471028617060106092848425803658519 1, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365157751745473936889117989779 06,
5.187783578445723317744386115162094918365,
```

```
408.65773862503424891408425689356098565 95]
one interval r = 21.71840114649324675661397425181820196373 ..
26.81849303501541038037051020064807288453
Time Approximations 0.058.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [-.1e-37, -.79e-37, .168e-34]Solution in 1.002s

Time Plot 0 s.
Exiting SolveHard() after 4.517r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91, none, none,
361.50888346545067855760129700912891897 14, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136515775174547393688911798977906,
5.18778357844572331774438611516209491836 5,
408.65773862503424891408425689356098565 95]
one interval r = 31.80828598739365561582393973535318820495 ..
35.00011460037611283319229282282277566852
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.592e-34]Solution in 2.8s

Time Plot 0 s.
Exiting SolveHard() after 3.114r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37, none,
361.50888346545067855760129700912891897 14, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110523704314420550301405303616818,
6.19626256555134873826115939032747806 7207,
385.44474378737088269777950142626782775 25]
one interval r = 31.60836097521206940696362374354261003155 ..
34.66372795595843127501709588675298668969
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=1.4e-37
Equations at solution: [-.9e-37, .14e-36, -.327e-34]Solution in 0.576s

Time Plot 0 s.
Exiting SolveHard() after 0.863r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37, none,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110523704314420550301405303616818,
6.19626256555134873826115939032747806720 7,
385.44474378737088269777950142626782775 25]
```

"Imaginary part neglected: ", $1.10311211489721680011018212216172588651 3 \times 10^{-17}$

```
two intervals r = 16.875634087735763386981880729454302485 76 ..
475000000000457971927658285324295862 3/2500000000000000000000000000000000000
000 or r = 15.556404937748936081037748714966779669 17 ..
475000000000457971927658285324295862 3/2500000000000000000000000000000000000
000
Time Approximations 0.064.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.359e-37, 0., -.1989e-34]Solution in 3.446s

Time Plot 0 s.
Exiting SolveHard() after 4.611r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
```

```
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874754962254107538409100758167139,
4.8838107797407140173932588209206371416 76,
376.61967855195059662740874228992826520 19]
one interval r = 21.11001304871847387968292246985502987349 ..
26.31784243458013735961231583779757157927
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., .81e-35]Solution in 0.869s

Time Plot 0 s.
Exiting SolveHard() after 1.575r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08, none,
328.46938512508095071726328321949272918 26, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874754962254107538409100758167139,
4.8838107797407140173932588209206371416 76,
376.61967855195059662740874228992826520 19]
one interval r = 31.53899497695326842711742047513715421684 ..
34.53618386078643064957468485637569859517
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=8.30e-36
Equations at solution: [.638e-35, -.830e-35, .202e-34]Solution in
0.507s

Time Plot 0 s.
Exiting SolveHard() after 3.265r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08, none,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017516045209917254491452314453781,
6.0258135494826060991069343436672185450 20,
351.42702947373283795968887679080543878 42]
one interval r = 31.36230206095680393924132392938605411231 ..
34.17446640593549308748375740090641961916
Time Approximations 0.016.
```

```
hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .134e-34]Solution in 0.508s

Time Plot 0 s.
Exiting SolveHard() after 0.752r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08, none,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80, none, none,
292.99969137033136301688301197515331060 83, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017516045209917254491452314453781,
6.0258135494826060991069343436672185450 20,
351.42702947373283795968887679080543878 42]

"Imaginary part neglected: ", 1.10311211489721680011018212216172588 6513 × 10^-17
two intervals r = 17.98135514469178162088764442993576521152 ..
475000000000457971927658285324295862 3/2500000000000000000000000000000000
000 or r = 13.84608015363864915258915281246861646632 ..
475000000000457971927658285324295862 3/2500000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.54e-37, -.2e-37, -.2596e-34]Solution in 1.127s

Time Plot 0 s.
Exiting SolveHard() after 4.596r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80, none, none,
292.99969137033136301688301197515331060 83, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941848606192538061674306632047189,
6.37794387406574174214739452191427809725 0,
423.28832782709177853493154808781528332 98]
one interval r = 31.94661817574544454044021104223944123762 ..
35.21212308628392473414691034141827619231
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
```

```
Accepted {r=34.3272, rm=11.3958} with Delta=7e-38
Equations at solution: [-.7e-37, .7e-37, -.90e-35]Solution in 0.595s

Time Plot 0 s.
Exiting SolveHard() after 0.942r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80, none, none,
292.99969137033136301688301197515331060 83, none, none,
360.06173465308582925089655064179910426 48, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941848606192538061674306632047189,
6.37794387406574174214739452191427809725 0,
423.28832782709177853493154808781528332 98]

"Imaginary part neglected: ", 1.10311211489721680011018212216172588651 3 × 10^-17
two intervals r = 15.22886702492357888396232453088691514594 ..
47500000000045797192765828532429586 23/2500000000000000000000000000000000
000 or r = 17.12965777029428283511098629295925334274 ..
47500000000045797192765828532429586 23/2500000000000000000000000000000000
000
Time Approximations 0.063.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=16.5334, rm=15.6907} with Delta=3e-38
Equations at solution: [-.61e-37, -.3e-37, .1212e-34]Solution in 1.314s

Time Plot 0 s.
Exiting SolveHard() after 5.067r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80,
375.73285288372153946915421622139469329 21, none,
292.99969137033136301688301197515331060 83, none, none,
360.06173465308582925089655064179910426 48, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343258205611099031672011303149 73,
4.0035598154379482027038448978102988598 20,
404.47973593241179308305592986809744721 47]

"Imaginary part neglected: ", 1.1031121148972168001101821221617258865 13 × 10^-17
two intervals r = 16.0968396640114908076206142122883932761 7 ..
4750000000004579719276582853242958623/2500000000000000000000000000000000
000 or r = 16.3998864908131649573056352224809278827 1 ..
4750000000004579719276582853242958623/2500000000000000000000000000000000
000
Time Approximations 0.055.

hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.7615110884558244986626642365610012490 6, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., .160e-35]Solution in 4.124s

Time Plot 0 s.
Exiting SolveHard() after 7.727r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80,
375.73285288372153946915421622139469329 21, none,
292.99969137033136301688301197515331060 83,
358.64341560093790110313168014428458890 56, none,
360.06173465308582925089655064179910426 48, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234325820561109903167201130314973,
4.0035598154379482027038448978102988598 20,
404.47973593241179308305592986809744721 47]
one interval r = 21.63429629975921361779951504182218007796 ..
26.75768169872696154688196837916820204912
Time Approximations 0.053.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.103e-34]Solution in 1.067s

Time Plot 0 s.
Exiting SolveHard() after 2.08r=25.8653 in [23.83864811 .. 26.75768170]
```

```
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80,
375.73285288372153946915421622139469329 21,
328.11709293257088052008430100493040784 21,
292.99969137033136301688301197515331060 83,
358.64341560093790110313168014428458890 56, none,
360.06173465308582925089655064179910426 48, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954442103271147787875907189678744,
6.1961772304054933167208328120187148146 64,
385.42734025124962126550303964705382950 29]
one interval r = 31.608220490766748134472975572833859745 68 ..
34.663476150336883915121696810981239410 23
Time Approximations 0.016.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=9e-38
Equations at solution: [-.5e-37, .9e-37, -.60e-35]Solution in 0.523s

Time Plot 0 s.
Exiting SolveHard() after 3.142r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80,
375.73285288372153946915421622139469329 21,
328.11709293257088052008430100493040784 21,
292.99969137033136301688301197515331060 83,
358.64341560093790110313168014428458890 56, none,
360.06173465308582925089655064179910426 48, none, none,
324.65521222720587177745575038409206002 92, none, none, none, none]

0 --> 1  target = [26.46318954442103271147787875907189678744,
6.19617723040549331672083281201871481466 4,
385.42734025124962126550303964705382950 29]
```

"Imaginary part neglected: ", $1.10311211489721680011018212216172588651 3 \times 10^{-17}$

```
two intervals r = 16.87629600311727823790121183359882286712 ..
47500000000045797192765828532429586 23/2500000000000000000000000000000000
000 or r = 15.55559000624042963948414136525231876866 ..
47500000000045797192765828532429586 23/2500000000000000000000000000000000
000
Time Approximations 0.063.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [-.1253e-36, .1e-37, -.2886e-34]Solution in
3.824s

Time Plot 0 s.
Exiting SolveHard() after 4.986r=17.9309 in [16.87629601 .. 19]
```

```
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80,
375.73285288372153946915421622139469329 21,
328.11709293257088052008430100493040784 21,
292.99969137033136301688301197515331060 83,
358.64341560093790110313168014428458890 56, none,
360.06173465308582925089655064179910426 48,
336.59441031231764656782435675421724708 54, none,
324.65521222720587177745575038409206002 92, none, none, none, none]

0 --> 2  target = [34.49522661156194734645432124511974487764,
3.8971313158749731196012860918272356417 84,
373.78081884238420969376780290198949114 80]
```

"Imaginary part neglected: ", $1.10311211489721680011018212216172588651 3 \times 10^{-17}$

```
two intervals r = 17.29769086229840986090913596589314230906 ..
47500000000045797192765828532429586 23/2500000000000000000000000000000000
000 or r = 14.99436407420069364817302028972780327121 ..
47500000000045797192765828532429586 23/2500000000000000000000000000000000
000
Time Approximations 0.09.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.36e-37, 0., -.548e-35]Solution in 3.689s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.358r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80,
375.73285288372153946915421622139469329 21,
328.11709293257088052008430100493040784 21,
292.99969137033136301688301197515331060 83,
358.64341560093790110313168014428458890 56, none,
360.06173465308582925089655064179910426 48,
336.59441031231764656782435675421724708 54, none,
324.65521222720587177745575038409206002 92,
331.93806791187549218711581016697693032 53, none, none, none]

1 --> 2  target = [34.49522661156194734645432124511974487764,
3.8971313158749731196012860918272356417 84,
373.78018842384209693767802901989491148 0]
one interval r = 21.06068473204715410841325070556316339884 ..
26.26979834272938459615994008656413031014
Time Approximations 0.035.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [.2e-37, .5e-37, -.27e-35]Solution in 0.813s

Time Plot 0 s.
Exiting SolveHard() after 1.546r=25.3005 in [23.14060343 ..
```

```
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80,
375.73285288372153946915421622139469329 21,
328.11709293257088052008430100493040784 21,
292.99969137033136301688301197515331060 83,
358.64341560093790110313168014428458890 56,
299.89866204197902058715127720457641770 29,
360.06173465308582925089655064179910426 48,
336.59441031231764656782435675421724708 54, none,
324.65521222720587177745575038409206002 92,
331.93806791187549218711581016697693032 53, none, none, none]

0 --> 2  target = [33.81362495389898611165930779256065468471,
3.72564899348247006708368586008065570878 4,
325.89209971949662263807604186891262072 98]

"Imaginary part neglected: ", 1.1031121148972168001101821221617258865 13 × 10^-17
two intervals r = 18.55227049020928541220343084187794935982 ..
47500000000045797192765828532429586 23/2500000000000000000000000000000000
000 or r = 12.49196935738655559798774514185195204987 ..
47500000000045797192765828532429586 23/2500000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [-.139e-36, .6e-37, -.2732e-34]Solution in
3.543s

Time Plot 0 s.
Exiting SolveHard() after 7.299r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80,
375.73285288372153946915421622139469329 21,
328.11709293257088052008430100493040784 21,
292.99969137033136301688301197515331060 83,
358.64341560093790110313168014428458890 56,
299.89866204197902058715127720457641770 29,
360.06173465308582925089655064179910426 48,
336.59441031231764656782435675421724708 54, none,
324.65521222720587177745575038409206002 92,
331.93806791187549218711581016697693032 53, none, none,
289.54595771663755800446691681709501561 60]

1 --> 2  target = [33.81362495389898611165930779256065468471,
3.72564899348247006708368586008065570878 4,
325.89209971949662263807604186891262072 98]
one interval r = 20.37468935104257276334963960418602280393 ..
25.37892165276825232654491710479484814388
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.2e-37, -.2e-37, -.575e-34]Solution in 0.567s

Time Plot 0 s.
Exiting SolveHard() after 1.097r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80,
375.73285288372153946915421622139469329 21,
328.11709293257088052008430100493040784 21,
292.99969137033136301688301197515331060 83,
358.64341560093790110313168014428458890 56,
299.89866204197902058715127720457641770 29,
360.06173465308582925089655064179910426 48,
336.59441031231764656782435675421724708 54,
256.10753184865600690743284540450757110 56,
324.65521222720587177745575038409206002 92,
331.93806791187549218711581016697693032 53, none, none,
289.54595771663755800446691681709501561 60]

1 --> 0  target = [17.93041369737661128279501576531541879714,
4.68650870184523958434666344725266315710 1,
353.30541093934753846216834985393822232 61]
one interval r = 20.73150479080994788542834535657751047970 ..
25.90675353496024309181304363027417201958
Time Approximations 0.034.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
```

```
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., -.133e-34]Solution in 3.203s

Time Plot 0 s.
Exiting SolveHard() after 3.896r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80,
375.73285288372153946915421622139469329 21,
328.11709293257088052008430100493040784 21,
292.99969137033136301688301197515331060 83,
358.64341560093790110313168014428458890 56,
299.89866204197902058715127720457641770 29,
360.06173465308582925089655064179910426 48,
336.59441031231764656782435675421724708 54,
256.10753184865600690743284540450757110 56,
324.65521222720587177745575038409206002 92,
331.93806791187549218711581016697693032 53,
304.79958324049144008417367459571366057 22, none,
289.54595771663755800446691681709501561 60]

2 --> 0  target = [17.93041369737661128279501576531541879714,
4.68650870184523958434666344725266315710 1,
353.30541093934753846216834985393822232 61]
one interval r = 31.37435486974429503267551968961732940163 ..
34.20127520005329860012426177594370627456
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
```

```
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [-.6e-37, .8e-37, -.112e-34]Solution in 0.377s

Time Plot 0 s.
Exiting SolveHard() after 0.646r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493431617998908722104462696322 61,
441.64295972802970544510733181858974143 36,
436.91748165067527317117654313030020803 41,
422.98493396656630507110635972593038180 97,
361.52580255494766328466473090253306284 99,
401.88173903584947393120552301595436300 49,
389.59001515630536197675271506026503145 24,
328.46939892435129052426750868597366350 44,
401.50757157232090490728492176270279832 32,
358.97362823338714300591521670296877734 58,
398.33147102861706010609284842580365851 91,
371.48387394165331328980789265205333138 37,
336.61215840175448150161880540291419648 59,
361.50888346545067855760129700912891897 14,
324.67144991729916662627830887868893820 08,
302.31384313404287965021534843658575566 19,
328.46938512508095071726328321949272918 26,
343.81340624309632901461017396681077065 80,
375.73285288372153946915421622139469329 21,
328.11709293257088052008430100493040784 21,
292.99969137033136301688301197515331060 83,
358.64341560093790110313168014428458890 56,
299.89866204197902058715127720457641770 29,
360.06173465308582925089655064179910426 48,
336.59441031231764656782435675421724708 54,
256.10753184865600690743284540457557110 56,
324.65521222720587177745575038409206002 92,
331.93806791187549218711581016697693032 53,
304.79958324049144008417367459571366057 22,
323.46169175631354246909569945345560444 40,
289.54595771663755800446691681709501561 60]

Cascade time 150.445
counts: 28, 28

 Iteration 51

Start Generation 1
```

```
1 --> 0  target = [11.999999999846358668196611907685305489 00,
6.217012503002117773378910365954114545349,
485.549080896225298240408515671233841258 2]
one interval r = 23.40850301637051340469997548551487007405 ..
27.67578046409498634614372866329597489949
Time Approximations 0.043.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=7.7e-38
Equations at solution: [-.3e-37, .77e-37, .17e-35]Solution in 3.756s

Time Plot 0 s.
Exiting SolveHard() after 4.925r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999846358668196611907685305489 00,
6.217012503002117773378910365954114545349,
485.549080896225298240408515671233841258 2]
one interval r = 32.62814779215290155618104831390746648566 ..
36.10248388940112105640510747639633016827
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [.7e-37, -.4e-37, -.17688e-34]Solution in 0.615s

Time Plot 0 s.
Exiting SolveHard() after 1.041r=35.4632 in [33.94922194 ..
```

```
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684460963743981203458356931867370,
6.58343472176764978430553015970882638 7889,
467.78730595625873313384105765211977828 64]
one interval r = 32.41978955664434705750607524327295029457 ..
35.85152417369161163378680797748825776477
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., -.18907e-34]Solution in 0.629s

Time Plot 0 s.
Exiting SolveHard() after 0.99r=34.9451 in [33.70078237 .. 35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95, none, none,
401.88173903836494479969050053046950820 41, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684460963743981203458356931867370,
6.58343472176764978430553015970882638 7889,
467.78730595625873313384105765211977828 64]
```

"Imaginary part neglected: ", $1.10311211489534305821720227840064208072 3 \times 10^{-17}$

```
two intervals r = 12.92327160820671597842323687983103510301 ..
94999999999945628439439981589443040 77/50000000000000000000000000000000000
```

```
000 or r = 18.39424858026971761497791767872265095910 ..
94999999999945628439439981589443040 77/500000000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.3e-38
Equations at solution: [.3e-37, .33e-37, -.900e-35]Solution in 41.139s

Time Plot 0 s.
Exiting SolveHard() after 45.189r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35, none,
401.88173903836494479969050053046950820 41, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962823059883123266939605117699913,
4.12565179681219369305475181642177656434 5,
440.67123064763662056750634367544071225 27]

"Imaginary part neglected: ", 1.1031121148953430582172022784006420807 23 × 10^-17
two intervals r = 14.35659705112782871523595930129212529686 ..
94999999999945628439439981589443040 77/500000000000000000000000000000000000
000 or r = 17.70352613801350945004885096914292102759 ..
94999999999945628439439981589443040 77/500000000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
```

```
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [.55e-37, .3e-37, .493e-35]Solution in 1.309s

Time Plot 0 s.
Exiting SolveHard() after 4.833r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35, none,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962823059883123266939605117699913,
4.12565179681219369305475181642177656434 5,
440.67123064763662056750634367544071225 27]
one interval r = 22.39761154345972398823311852761713554904 ..
27.23722351579204556198376348652442804146
Time Approximations 0.04.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.573 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064369116922993044390834246362318, rm =
14.37818770501183238061823968641519845786}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., .500e-34]Solution in 9.979s

Time Plot 0 s.
Exiting SolveHard() after 10.912r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888794210334952338420609624466464,
4.0048690817843804387734211390257865192 30,
404.86224500901298347406950018462960585 92]
```

"Imaginary part neglected: ", $1.10311211489534305821720227840064208072 3 \times 10^{-17}$

```
two intervals r = 16.08011007763077303779471697658138015288 ..
94999999999945628439439981589443040 77/50000000000000000000000000000000000
000 or r = 16.41579812676716284103030577750784668360 ..
94999999999945628439439981589443040 77/50000000000000000000000000000000000
000
Time Approximations 0.054.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.50e-37, -.1e-37, -.3767e-34]Solution in
4.205s

Time Plot 0 s.
Exiting SolveHard() after 7.657r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80, none, none,
358.97362823485132092219324598913795006 38, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]
```

```
1 --> 2  target = [34.945078887942103349523384206096244 66464,
4.004869081784380438773421139025786519230,
404.86224500901298347406950018462960585 92]
one interval r = 21.641943993882596847546264608812108018 77 ..
26.763306600209603025234329772488660665 92
Time Approximations 0.052.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=1.24e-37
Equations at solution: [-.3e-37, -.124e-36, -.5e-36]Solution in 1.083s

Time Plot 0 s.
Exiting SolveHard() after 2.1r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544 08158,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94, none,
358.97362823485132092219324598913795006 38, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589417848634231182936159579813403 93,
5.589637182969727748979338400792648927554,
443.83065884166364582383929182089529280 82]
one interval r = 22.467253744542810197521102334383784578 81 ..
27.273884283339073186084266323617237687 56
Time Approximations 0.038.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
```

```
Accepted {r=27.0204, rm=13.5759} with Delta=5.3e-38
Equations at solution: [0., -.53e-37, -.159e-34]Solution in 0.976s

Time Plot 0 s.
Exiting SolveHard() after 4.618r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94, none,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941784863423118293615957981340393,
5.589637182969727748979338400792648927554,
443.83065884166364582383929182089529280 82]
one interval r = 32.15575279501831493932763679001872954604 ..
35.50872228733152526215889584325862668845
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=9e-38
Equations at solution: [-.10e-36, .9e-37, -.10203e-34]Solution in
3.006s

Time Plot 0 s.
Exiting SolveHard() after 3.386r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
```

```
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050050346950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136511528010669762729531604926797,
5.18778357852814573324751477091839775649 5,
408.65773862341352483224109904417716386 13]
one interval r = 21.71840114628626842731747181519866195837 ..
26.81849303491889973122546473185280243431
Time Approximations 0.054.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.84e-37
Equations at solution: [-.2e-37, -.184e-36, .171e-34]Solution in 1.024s

Time Plot 0 s.
Exiting SolveHard() after 2.063r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050050346950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99, none, none,
361.50888346592651382373018092817641073 38, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136511528010669762729531604926797,
```

```
5.18778357852814573324751477091839775649 5,
408.65773862341352483224109904417716386 13]
one interval r = 31.80828598753306391716555049048953826551 ..
35.00011460042100334645222335361415197998
Time Approximations 0.016.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=8e-38
Equations at solution: [-.7e-37, .8e-37, -.26235e-34]Solution in 0.414s

Time Plot 0 s.
Exiting SolveHard() after 3.309r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51, none,
361.50888346592651382373018092817641073 38, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110517922063457501655354095163454,
6.19626256548266743995891080873140837604 0,
385.44474378834349303401610383348570176 99]
one interval r = 31.60836097538026160696333561429817507320 ..
34.66372795604886125847897324390714101707
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
```

```
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=8e-38
Equations at solution: [.6e-37, -.8e-37, .5465e-35]Solution in 0.545s

Time Plot 0 s.
Exiting SolveHard() after 0.813r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51, none,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 13, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110517922063457501655354095163454,
6.19626256548266743995891080873140837604 0,
385.44474378834349303401610383348570176 99]
```

"Imaginary part neglected: ", $1.10311211489534305821720227840064208072 3 \times 10^{-17}$

```
two intervals r = 16.875634087601383343920089599223883045 54 ..
94999999999945628439439981589443040 77/50000000000000000000000000000000000
000 or r = 15.556404937793236480076560704182709944 44 ..
94999999999945628439439981589443040 77/50000000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 6.571 s
```

```
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175124859803206185030566114069887, rm
= 2.33653277396870218955502409055771817231 4}});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [-.179e-37, .1e-37, -.2450e-34]Solution in
29.434s

Time Plot 0 s.
Exiting SolveHard() after 33.059r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 13, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874741248957877382153514696755741,
4.88381077984826006774945368021316754914 0,
376.61967855326661290342615873382126956 98]
one interval r = 21.11001304854585693328041181265206161991 ..
26.31784243452805669706362446517795050720
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [0., .26e-37, -.162e-34]Solution in 0.847s

Time Plot 0 s.
Exiting SolveHard() after 1.541r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 13, none,
328.46938512866717242619965090818856389 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874741248957877382153514696755741,
4.8838107798482600677494536802131675491 40,
376.61967855326661290342615873382126956 98]
one interval r = 31.53899497712581319425790351250746997695 ..
34.53618386088487213565324063878771543293
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.127e-35
Equations at solution: [.866e-35, -.1127e-34, -.26025e-34]Solution in
0.489s

Time Plot 0 s.
Exiting SolveHard() after 0.764r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
```

```
401.881739038364944799690500530469508204 1,
389.590015154953566187189791299798689538 0,
328.469398927938702383206624976651022289 4,
401.507571575373515213603625173339630616 7,
358.973628234851320922193245989137950063 8,
398.331471035522446042075814932910213849 9,
371.483873939129614432839310216212838675 1,
336.612158406129808826402632933191734495 5,
361.508883465926513823730180928176410733 8,
324.671449919529398598349458271429093111 3, none,
328.469385128667172426199650908188563896 0,
343.813406243041575654703912028744617985 7, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.872050175159090610470626670306754739 79,
6.025813549429984743938498760482462617009,
351.427029477851851932856989817349399891 4]
one interval r = 31.362302061151009161221624555306643984 67 ..
34.174466406082651380422512201566638777 25
Time Approximations 0.014.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=1.5e-37
Equations at solution: [.8e-37, -.15e-36, -.15973e-34]Solution in
0.526s

Time Plot 0 s.
Exiting SolveHard() after 3.26r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434934554538419254752973005440815 8,
441.642959728924542403029819772097552676 3,
436.917481649963349312468927969882819499 5,
422.984933971268315084854989140476624183 5,
361.525802555478580437362285942936135097 6,
401.881739038364944799690500530469508204 1,
389.590015154953566187189791299798689538 0,
328.469398927938702383206624976651022289 4,
401.507571575373515213603625173339630616 7,
358.973628234851320922193245989137950063 8,
398.331471035522446042075814932910213849 9,
371.483873939129614432839310216212838675 1,
336.612158406129808826402632933191734495 5,
361.508883465926513823730180928176410733 8,
```

```
324.67144991952939859834945827142909311 13, none,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57, none, none,
292.99969137552416330291881559183366525 38, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175159090610470626670306754739 79,
6.0258135494299847439384987604824626170 09,
351.42702947785185193285698981734939989 14]
```

"Imaginary part neglected: ", $1.10311211489534305821720227840064208072 3 \times 10^{-17}$

```
two intervals r = 17.981355144508668981258636719311656391 80 ..
949999999999456284394399815894430407 7/500000000000000000000000000000000
000 or r = 13.846080153847146219844379592765253688 46 ..
949999999999456284394399815894430407 7/500000000000000000000000000000000
000
Time Approximations 0.051.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 5.626 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713709517864482988732575504508 42, rm
= 2.7345009929931799817908678418809206918 81}});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [-.18e-37, 0., -.304e-34]Solution in 18.988s

Time Plot 0 s.
Exiting SolveHard() after 20.052r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
```

```
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 3,
302.31384314139753375983939879382279052 88,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57, none, none,
292.99969137552416330291881559183366525 38, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941851143173881308807117816324250,
6.37794387402796377850394412393793197853 3,
423.28832783461644388568906028214073305 79]
one interval r = 31.94661817596895842164953050493595981537 ..
35.21212308645635814521362612522145099166
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, .11701e-34]Solution in 0.628s

Time Plot 0 s.
Exiting SolveHard() after 0.978r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 3,
302.31384314139753375983939879382279052 88,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57, none, none,
292.99969137552416330291881559183366525 38, none, none,
```

```
360.06173466142260842919282120112077155 14, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941851143173881308807117816324250,
6.3779438740279637785039441239379319785 33,
423.28832783461644388568906028214073305 79]
```

"Imaginary part neglected: ", $1.10311211489534305821720227840064208072 3 \times 10^{-17}$

```
two intervals r = 15.22886702443293882492092576361082899794 ..
949999999999456284394399815894430407 7/50000000000000000000000000000000000
000 or r = 17.12965777056359464553343401377586867609 ..
949999999999456284394399815894430407 7/50000000000000000000000000000000000
000
Time Approximations 0.064.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 8.541 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541351457913727220977313079381 03, rm
= 2.0640682986476835043725869592467731481 68}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.47e-37, .1e-37, -.295e-35]Solution in 28.976s

Time Plot 0 s.
Exiting SolveHard() after 32.948r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 13,
```

```
302.31384314139753375983939879382279052 88,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57,
375.73285289509341803480835857799178146 11, none,
292.99969137552416330291881559183366525 38, none, none,
360.06173466142260842919282120112077155 14, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343373159125837173330818789110 03,
4.0035598154825635220952577065342288404 87,
404.47973593552531183692553163063048619 50]
```

"Imaginary part neglected: ", $1.10311211489534305821720227840064208072 3 \times 10^{-17}$

```
two intervals r = 16.09683966376355925445121305697792758648 ..
949999999999456284394399815894430407 7/500000000000000000000000000000000
000 or r = 16.39988649094292438057950941989323478096 ..
949999999999456284394399815894430407 7/500000000000000000000000000000000
000
Time Approximations 0.054.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.17e-37, 0., -.1089e-34]Solution in 1.696s

Time Plot 0 s.
Exiting SolveHard() after 5.15r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 13,
```

```
302.31384314139753375983939879382279052 88,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57,
375.73285289509341803480835857799178146 11, none,
292.99969137552416330291881559183366525 38,
358.64341560287649969038976906129757620 60, none,
360.06173466142260842919282120112077155 14, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343373159125837173330818789110 03,
4.0035598154825635220952577065342288404 87,
404.47973593552531183692553163063048619 50]
one interval r = 21.634296299644429745898351441678784742 45 ..
26.757681698699683631107713351884812730 11
Time Approximations 0.047.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, -.198e-34]Solution in 0.997s

Time Plot 0 s.
Exiting SolveHard() after 4.345r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 13,
302.31384314139753375983939879382279052 88,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57,
375.73285289509341803480835857799178146 11,
```

```
328.11709293666411060713612495278133803 54,
292.99969137552416330291881559183366525 38,
358.64341560287649969038976906129757620 60, none,
360.06173466142260842919282120112077155 14, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954436229374686620301657919179774,
6.19617723033653417117253571130081371698 2,
385.42734025216557236339252557426273164 68]
one interval r = 31.60822049093448651603122341628142282682 ..
34.66347615042650012428687673612234051077
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=8e-38
Equations at solution: [.6e-37, -.8e-37, -.3503e-35]Solution in 2.851s

Time Plot 0 s.
Exiting SolveHard() after 3.127r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 13,
302.31384314139753375983939879382279052 88,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57,
375.73285289509341803480835857799178146 11,
328.11709293666411060713612495278133803 54,
292.99969137552416330291881559183366525 38,
358.64341560287649969038976906129757620 60, none,
```

```
360.06173466142260842919282120112077155 14, none, none,
324.65521222938325805881835236892279328 90, none, none, none, none]

0 --> 1  target = [26.46318954436229374686620301657919179774,
6.19617723033653417117253571130081371698 2,
385.42734025216557236339252557426273164 68]
```

"Imaginary part neglected: ", $1.10311211489534305821720227840064208072 3 \times 10^{-17}$

```
two intervals r = 16.87629600298507142705795393771198611287 ..
94999999999945628439439981589443040 77/50000000000000000000000000000000000
000 or r = 15.55559000628207884586018154249988328651 ..
94999999999945628439439981589443040 77/50000000000000000000000000000000000
000
Time Approximations 0.063.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 9.742 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852536455521685981093218517939421, rm
= 2.33669042803338588552584509774189582073 4}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.179e-37, 0., .1482e-34]Solution in 30.299s

Time Plot 0 s.
Exiting SolveHard() after 31.449r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 13,
```

```
302.31384314139753375983939879382279052 88,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57,
375.73285289509341803480835857799178146 11,
328.11709293666411060713612495278133803 54,
292.99969137552416330291881559183366525 38,
358.64341560287649969038976906129757620 60, none,
360.06173466142260842919282120112077155 14,
336.59441031663511634515133490571979131 36, none,
324.65521222938325805881835236892279328 90, none, none, none, none]

0 --> 2  target = [34.49522661160509192081744230437912808863,
3.8971313159001361573827024771312265959 54,
373.78081883979752412110971654224262246 37]
```

"Imaginary part neglected: ", $1.10311211489534305821720227840064208072 3 \times 10^{-17}$

```
two intervals r = 17.29769086229847827591650850739656356575 ..
94999999999945628439439981589443040 77/50000000000000000000000000000000000
000 or r = 14.99436407406964839647882488215195317973 ..
94999999999945628439439981589443040 77/50000000000000000000000000000000000
000
Time Approximations 0.085.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.18e-37, 0., -.2647e-34]Solution in 1.182s

Time Plot 0 s.
Exiting SolveHard() after 5.213r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050050346950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
```

```
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 3,
302.31384314139753375983939879382279052 88,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57,
375.73285289509341803480835857799178146 11,
328.11709293666411060713612495278133803 54,
292.99969137552416330291881559183366525 38,
358.64341560287649969038976906129757620 60, none,
360.06173466142260842919282120112077155 14,
336.59441031663511634515133490571979131 36, none,
324.65521222938325805881835236892279328 90,
331.93806790885681908865142893142838725 12, none, none, none]

1 --> 2  target = [34.49522661160509192081744230437912808863,
3.8971313159001361573827024771312265959 54,
373.78081883979752412110971654224262246 37]
one interval r = 21.06068473180509013689273697644641237459 ..
26.26979834261085309482158788006711881901
Time Approximations 0.032.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [.3e-37, .7e-37, .153e-34]Solution in 0.753s

Time Plot 0 s.
Exiting SolveHard() after 3.919r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050050346950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
336.61215840612980882640263293319173449 55,
```

```
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 13,
302.31384314139753375983939879382279052 88,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57,
375.73285289509341803480835857799178146 11,
328.11709293666411060713612495278133803 54,
292.99969137552416330291881559183366525 38,
358.64341560287649969038976906129757620 60,
299.89866204084647846321341839118805870 05,
360.06173466142260842919282120112077155 14,
336.59441031663511634515133490571979131 36, none,
324.65521222938325805881835236892279328 90,
331.93806790885681908865142893142838725 12, none, none, none]

0 --> 2  target = [33.81362495402848483578948103816190442081,
3.7256489935256513065563332667514048635 40,
325.89209972177598277928192316169376599 73]
```

"Imaginary part neglected: ", $1.10311211489534305821720227840064208072 3 \times 10^{-17}$

```
two intervals r = 18.552270490112437867420835219657718259 06 ..
94999999999945628439439981589443040 77/50000000000000000000000000000000
000 or r = 12.491969357485565683939038496357281213 89 ..
94999999999945628439439981589443040 77/50000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [.121e-36, -.6e-37, .12e-35]Solution in 1.079s

Time Plot 0 s.
Exiting SolveHard() after 4.91r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
```

```
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 3,
302.31384314139753375983939879382279052 88,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57,
375.73285289509341803480835857799178146 11,
328.11709293666411060713612495278133803 54,
292.99969137552416330291881559183366525 38,
358.64341560287649969038976906129757620 60,
299.89866204084647846321341839118805870 05,
360.06173466142260842919282120112077155 14,
336.59441031663511634515133490571979131 36, none,
324.65521222938325805881835236892279328 90,
331.93806790885681908865142893142838725 12, none, none,
289.54595771795338504193704729642645626 28]

1 --> 2  target = [33.81362495402848483578948103816190442081,
3.72564899352565130655633326675140486354 0,
325.89209972177598277928192316169376599 73]
one interval r = 20.37468935084387833682573876779846073075 ..
25.37892165273026896394529293767332866893
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, .80e-35]Solution in 0.554s

Time Plot 0 s.
Exiting SolveHard() after 1.1r=24.3395 in [22.07732228 .. 25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
```

```
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 13,
302.31384314139753375983939879382279052 88,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57,
375.73285289509341803480835857799178146 11,
328.11709293666411060713612495278133803 54,
292.99969137552416330291881559183366525 38,
358.64341560287649969038976906129757620 60,
299.89866204084647846321341839118805870 05,
360.06173466142260842919282120112077155 14,
336.59441031663511634515133490571979131 36,
256.10753185200646421366264691062604503 51,
324.65521222938325805881835236892279328 90,
331.93806790885681908865142893142838725 12, none, none,
289.54595771795338504193704729642645626 28]

1 --> 0  target = [17.93041369718351726187998558895243886513,
4.68650870197505612978341310755624604 8704,
353.30541094368639815211508512891602190 64]
one interval r = 20.73150479066446201214656571698000864762 ..
25.90675353496277526517580493831340708333
Time Approximations 0.033.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.6e-38
Equations at solution: [-.2e-37, -.46e-37, .407e-34]Solution in 0.667s

Time Plot 0 s.
Exiting SolveHard() after 4.588r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.64295972892454240302981977209755267 63,
436.91748164996334931246892796988281949 95,
422.98493397126831508485498914047662418 35,
361.52580255547858043736228592942936135 0976,
401.88173903836494479969050050469508204 1,
```

```
389.5900151549535661871897912997986895380,
328.4693989279387023832066249766510222894,
401.5075715753735152136036251733396306167,
358.9736282348513209221932459891379500638,
398.3314710355224460420758149329102138499,
371.4838739391296144328393102162128386751,
336.6121584061298088264026329331917344955,
361.5088834659265138237301809281764107338,
324.6714499195293985983494582714290931113,
302.3138431413975337598393987938227905288,
328.4693851286671724261996509081885638960,
343.8134062430415756547039120287446179857,
375.7328528950934180348083585779917814611,
328.1170929366641106071361249527813380354,
292.9996913755241633029188155918336652538,
358.6434156028764996903897690612975762060,
299.8986620408464784632134183911880587005,
360.0617346614226084291928212011207715514,
336.5944103166351163451513349057197913136,
256.1075318520064642136626469106260450351,
324.6552122293832580588183523689227932890,
331.9380679088568190886514289314283872512,
304.7995832471187914554635060674923357804, none,
289.5459577179533850419370472964264562628]

2 --> 0  target = [17.93041369718351726187998558895243886513,
4.686508701975056129783413107556246048704,
353.3054109436863981521150851289160219064]
one interval r = 31.37435486993994970383043921305943122223 ..
34.20127520020298992996260733204589641411
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.16507e-34]Solution in 0.358s

Time Plot 0 s.
Exiting SolveHard() after 0.638r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493455453841925475297300544081 58,
441.6429597289245424030298197720975526763,
436.9174816499633493124689279698828194995,
```

```
422.98493397126831508485498914047662418 35,
361.52580255547858043736228594293613509 76,
401.88173903836494479969050053046950820 41,
389.59001515495356618718979129979868953 80,
328.46939892793870238320662497665102228 94,
401.50757157537351521360362517333963061 67,
358.97362823485132092219324598913795006 38,
398.33147103552244604207581493291021384 99,
371.48387393912961443283931021621283867 51,
336.61215840612980882640263293319173449 55,
361.50888346592651382373018092817641073 38,
324.67144991952939859834945827142909311 13,
302.31384314139753375983939879382279052 88,
328.46938512866717242619965090818856389 60,
343.81340624304157565470391202874461798 57,
375.73285289509341803480835857799178146 11,
328.11709293666411060713612495278133803 54,
292.99969137552416330291881559183366525 38,
358.64341560287649969038976906129757620 60,
299.89866204084647846321341839118805870 05,
360.06173466142260842919282120112077155 14,
336.59441031663511634515133490571979131 36,
256.10753185200646421366264691062604503 51,
324.65521222938325805881835236892279328 90,
331.93806790885681908865142893142838725 12,
304.79958324711879145546350606749233578 04,
323.46169175889190593180252765711685830 52,
289.54595771795338504193704729642645626 28]

Cascade time 248.943
counts: 28, 28

 Iteration 52

Start Generation 1
1 --> 0  target = [12.00000000004201750274104223658428172100,
6.21701250301346793033111514981629796855 9,
485.54908089173361649541678602506776800 05]
```

"Imaginary part neglected: ", $1.8899423791508501959972168187594385022 97 \times 10^{-17}$

```
one interval r = 23.40850301639793464734139742201056876331 ..
27.67578046423447325861566789956310537754
Time Approximations 0.06.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.8e-38
Equations at solution: [-.1e-37, .28e-37, -.12e-35]Solution in 1.013s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.179r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000004201750274104223658428172100,
6.217012503013467930331115149816297968559,
485.54908089173361649541678602506776800 05]
one interval r = 32.628147791999167895844237214816694310 51 ..
36.102483889263405913466683352177974161 19
Time Approximations 0.021.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [.6e-37, -.4e-37, .13863e-34]Solution in 0.557s

Time Plot 0 s.
Exiting SolveHard() after 3.716r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684475100189533007624021309925322,
6.583434721534594989567008020516606669981,
467.78730595352658196573660561998414155 45]
one interval r = 32.419789556511295922603948941857240800 39 ..
35.851524173576899873967744871655272848 06
Time Approximations 0.021.
```

```
hint used Hint := [34.9450788879772991463756999080445721474 1, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, -.17919e-34]Solution in 0.619s

Time Plot 0 s.
Exiting SolveHard() after 0.97r=34.9451 in [33.70078237 .. 35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04, none, none,
401.88173903819378366070893680361422931 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844751001895330076240213099253 22,
6.5834347215345949895670080205166066699 81,
467.78730595352658196573660561998414155 45]
two intervals r = 12.923271608311232592605551095126204056 39 ..
9499999999967137474285708976456988999/5000000000000000000000000000000000
000 or r = 18.394248580167013505971283795200127907 27 ..
9499999999967137474285708976456988999/5000000000000000000000000000000000
000
Time Approximations 0.039.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=6.7e-38
Equations at solution: [-.5e-37, -.67e-37, .579e-35]Solution in 41.539s

Time Plot 0 s.
Exiting SolveHard() after 45.605r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52, none,
401.88173903819378366070893680361422931 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962808377735650731674075778324805,
4.12565179679321588531880318846549126635 1,
440.67123064285672837708548893707961755 09]
two intervals r = 14.35659705132352859369602701468576166254 ..
94999999999671374742857089764569889 99/50000000000000000000000000000000000
000 or r = 17.70352613782907165385254015972458519523 ..
94999999999671374742857089764569889 99/50000000000000000000000000000000000
000
Time Approximations 0.059.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.28e-37, .1e-37, -.1604e-34]Solution in 3.637s

Time Plot 0 s.
Exiting SolveHard() after 4.722r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52, none,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962808377735650731674075778324805,
4.12565179679321588531880318846549126635 1,
440.67123064285672837708548893707961755 09]
```

"Imaginary part neglected: ", $1.889942379150850195997216818759438502297 \times 10^{-17}$

```
one interval r = 22.39761154348958590666369169481434297637 ..
27.23722351589273542793940554209298097756
Time Approximations 0.044.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.863 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064368667699740736069413617202761, rm =
14.37818770179022430491769201515035513046}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, .14e-35]Solution in 10.615s

Time Plot 0 s.
Exiting SolveHard() after 11.518r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888785872226379050278790348803324,
4.00486908178040022111263216240871314923 5,
404.86224500885652322142342564295276456 40]
two intervals r = 16.08011007757905883689294562099019778503 ..
9499999999967137474285708976456988999/50000000000000000000000000000000000
000 or r = 16.41579812672930504662918386883208492075 ..
9499999999967137474285708976456988999/50000000000000000000000000000000000
000
Time Approximations 0.052.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.35e-37, .1e-37, .3103e-34]Solution in 3.884s

Time Plot 0 s.
Exiting SolveHard() after 7.646r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71, none, none,
358.97362823332736723865686644143412812 40, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888785872226379050278790348803324,
4.0048690817804002211126321624087131492 35,
404.86224500885652322142342564295276456 40]

"Imaginary part neglected: ", 1.8899423791508501959972168187594385022 97 × 10^-17
one interval r = 21.641943994020267502288153603490994431 00 ..
26.763306600352935913325332850380700527 64
Time Approximations 0.053.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, .43e-35]Solution in 1.061s

Time Plot 0 s.
Exiting SolveHard() after 2.066r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
```

```
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95, none,
358.97362823332736723865686644143412812 40, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941768168298416377593830092171171,
5.58963718308497897832730021970735770803 4,
443.83065884389985236622266204444541781 03]
```

"Imaginary part neglected: ", $1.8899423791508501959972168187594385022 97 \times 10^{-17}$

```
one interval r = 22.46725374472671508770192346207658895702 ..
27.27388428352262848032981138362736899637
Time Approximations 0.045.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., .67e-35]Solution in 0.977s

Time Plot 0 s.
Exiting SolveHard() after 4.382r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95, none,
```

```
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941768168298416377593830092171171,
5.589637183084978978327300219707357708034,
443.83065884389985236622266204444541781 03]
one interval r = 32.15575279493759244882228016720372502139 ..
35.50872228728600664490792517285915726852
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., .5759e-35]Solution in 2.771s

Time Plot 0 s.
Exiting SolveHard() after 3.154r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136529930136377993627838488989976,
5.187783578556425293505529373150038575032,
408.65773861801229027265760314025979852 20]

"Imaginary part neglected: ", 1.8899423791508501959972168187594385022 97 × 10^-17
one interval r = 21.71840114631687993944457496588090105885 ..
26.81849303498787081165360416864793057839
Time Approximations 0.059.
```

```
hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.06e-37
Equations at solution: [-.1e-37, -.106e-36, .30e-35]Solution in 0.985s

Time Plot 0 s.
Exiting SolveHard() after 2.09r=26.4632 in [23.93303356 .. 26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99, none, none,
361.50888346504916849370810887715941133 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136529930136377993627838488989976,
5.18778357855642529350552937315003857503 2,
408.65773861801229027265760314025979852 20]
one interval r = 31.80828598737579756296310463224595763218 ..
35.00011460026192948472833134547858781292
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=6e-38
Equations at solution: [-.5e-37, .6e-37, .45732e-34]Solution in 0.393s

Time Plot 0 s.
```

```
Exiting SolveHard() after 3.21r=34.4952 in [32.91337941 .. 35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68, none,
361.50888346504916849370810887715941133 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110529969556533840695191046685260,
6.19626256525880191410505999194412312 7074,
385.44474378709434636390590537260406360 61]
one interval r = 31.60836097526020822593616803675671008399 ..
34.66372795594779094941495195395933140303
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .18971e-34]Solution in 0.542s

Time Plot 0 s.
Exiting SolveHard() after 0.809r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
```

```
401.881739038193783660708936803614229318 9,
389.5900151495727507684418861950523251271,
328.4693989314491371178897945080370074895,
401.5075715760754264567954380740423022254,
358.9736282333273672386568664414341281240,
398.3314710420731947324576396372036118299,
371.4838739331055627411479137856306271468, none,
361.5088834650491684937081088771594113333,
324.6714499209570724215025089445142380488, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110529969556533840695191046685260,
6.196262565258801914105059991944123127074,
385.4447437870943463639059053726040636061]
two intervals r = 16.875634087588685507226158539301163436 02 ..
9499999999967137474285708976456988999/500000000000000000000000000000000000
000 or r = 15.556404937709405685530222169875295826 09 ..
9499999999967137474285708976456988999/500000000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 6.424 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175120555232348433535695303158287, rm
= 2.336532774079363907239247604587633677077}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.358e-37, 0., .2790e-34]Solution in 29.492s

Time Plot 0 s.
Exiting SolveHard() after 33.155r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.6429597267162146664325105344958409774,
436.9174816452659749582132251023813562104,
422.9849339731411463623997868587095251952,
361.5258025547049454088272293216364457015,
401.881739038193783660708936803614229318 9,
389.5900151495727507684418861950523251271,
328.4693989314491371178897945080370074895,
401.5075715760754264567954380740423022254,
358.9736282333273672386568664414341281240,
```

```
398.331471042073194732457639637203611829 9,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874739962794145982378627354605661,
4.8838107799177854247537092063124850151 22,
376.61967855191200780459907051669026335 87]
```

"Imaginary part neglected: ", $1.8899423791508501959972168187594385022 97 \times 10^{-17}$

```
one interval r = 21.11001304866791114335948757044914761721 ..
26.31784243464396805684813214358527356740
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.105e-34]Solution in 0.852s

Time Plot 0 s.
Exiting SolveHard() after 1.544r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.331471042073194732457639637203611829 9,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88, none,
328.46938513217521806616331076528363244 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
2 --> 0  target = [17.19898874739962794145982378627354605661,
4.883810779917785424753709206312485015122,
376.61967855191200780459907051669026335 87]
one interval r = 31.53899497700476709603828797435785948392 ..
34.53618386078143074327261454602541796358
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=9.09e-36
Equations at solution: [-.698e-35, .909e-35, -.37747e-34]Solution in
0.485s

Time Plot 0 s.
Exiting SolveHard() after 0.766r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88, none,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017535170879722319133826280836586,
6.025813549229611292825280130114569244967,
351.42702948101667854508619998084592948 10]
one interval r = 31.36230206105863855524062353985327361878 ..
34.17446640604125230394971217406811040041
Time Approximations 0.015.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
```

```
12.1428029945613367595242565268234247214 4, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .13822e-34]Solution in 0.503s

Time Plot 0 s.
Exiting SolveHard() after 3.332r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88, none,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73, none, none,
292.99969138113475212686190488135542690 06, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017535170879722319133826280836586,
6.0258135492296112928252801301145692449 67,
351.42702948101667854508619998084592948 10]
two intervals r = 17.98135514436321828016555059669008635261 ..
94999999999671374742857089764569889 99/50000000000000000000000000000000000
000 or r = 13.84608015400181772034419120257630882345 ..
94999999999671374742857089764569889 99/50000000000000000000000000000000000
000
Time Approximations 0.055.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
```

```
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 5.685 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.9135707136255511859701397563400059731 0, rm
= 2.7345009931157792565074235478626583464 92}});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [-.107e-36, .3e-37, -.532e-35]Solution in 19.49s

Time Plot 0 s.
Exiting SolveHard() after 20.549r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73, none, none,
292.99969138113475212686190488135542690 06, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418745322503301016529541840 97023,
6.3779438738384637812589847662514533209 62,
423.28832784089130049951383083891336746 00]
one interval r = 31.946618175923740575945604298947868418 86 ..
35.212123086467762490922160324578117157 31
Time Approximations 0.02.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
```

```
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .9840e-35]Solution in 0.632s

Time Plot 0 s.
Exiting SolveHard() after 0.978r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73, none, none,
292.99969138113475212686190488135542690 06, none, none,
360.06173466981163010828029644938190190 26, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418745322503301016529541840 97023,
6.3779438738384637812589847662514533209 62,
423.28832784089130049951383083891336746 00]
two intervals r = 15.22886702407330215656025040566236671784 ..
9499999999967137474285708976456988999/50000000000000000000000000000000
000 or r = 17.12965777075226648959017897263768554584 ..
9499999999967137474285708976456988999/50000000000000000000000000000000
000
Time Approximations 0.062.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 8.622 s
```

```
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541082752580901684330053504218 18, rm
= 2.0640682986699376555969418295584062731 48}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.32e-37, -.1e-37, -.268e-35]Solution in
28.581s

Time Plot 0 s.
Exiting SolveHard() after 32.634r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73,
375.73285290693383611849070921651013685 45, none,
292.99969138113475212686190488135542690 06, none, none,
360.06173466981163010828029644938190190 26, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343302683914861584200776111912 70,
4.0035598154816345264253214545973394987 89,
404.47973593626137120896131323747554553 35]
two intervals r = 16.09683966367279525260882092871455201190 ..
9499999999967137474285708976456988999/500000000000000000000000000000000
000 or r = 16.39988649094233654998742343875783121535 ..
9499999999967137474285708976456988999/500000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.17e-37, 0., .486e-35]Solution in 1.666s

Time Plot 0 s.
Exiting SolveHard() after 5.578r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73,
375.73285290693383611849070921651013685 45, none,
292.99969138113475212686190488135542690 06,
358.64341560212271535917634871506886215 50, none,
360.06173466981163010828029644938190190 26, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234330268391486158420077611191270,
4.0035598154816345264253214545973394987 89,
404.47973593626137120896131323747554553 35]
```

"Imaginary part neglected: ", $1.8899423791508501959972168187594385022 97 \times 10^{-17}$

```
one interval r = 21.634296299800003790105003384253013514 73 ..
26.75768169885604488558902989292347476123
Time Approximations 0.052.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
```

```
Equations at solution: [-.1e-37, -.26e-37, .302e-34]Solution in 1.046s

Time Plot 0 s.
Exiting SolveHard() after 4.663r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73,
375.73285290693383611849070921651013685 45,
328.11709294099659833330344379618420795 31,
292.99969138113475212686190488135542690 06,
358.64341560212271535917634871506886215 50, none,
360.06173466981163010828029644938190190 26, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954448103704020852417379461472598,
6.19617723011214602197592570269691217609 2,
385.42734025080974755884959083340053604 80]
one interval r = 31.60822049081357163574286201019639935935 ..
34.66347615032388466114799962864050076402
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, -.9323e-35]Solution in 0.564s

Time Plot 0 s.
```

```
Exiting SolveHard() after 0.871r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73,
375.73285290693383611849070921651013685 45,
328.11709294099659833330344379618420795 31,
292.99969138113475212686190488135542690 06,
358.64341560212271535917634871506886215 50, none,
360.06173466981163010828029644938190190 26, none, none,
324.65521223071144185047254867479422076 26, none, none, none, none]

0 --> 1  target = [26.46318954448103704020852417379461472598,
6.19617723011214602197592570269691217609 2,
385.42734025080974755884959083340053604 80]
two intervals r = 16.87629600297642289575858874900554171175 ..
949999999996713747428570897645698899 9/500000000000000000000000000000000000
000 or r = 15.55559000619325298392967108811483528324 ..
949999999996713747428570897645698899 9/500000000000000000000000000000000000
000
Time Approximations 0.059.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 9.596 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852532350748548279618502549223749, rm
```

```
= 2.336690428145042543689536942629964483552}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.179e-37, 0., -.411e-35]Solution in 29.728s

Time Plot 0 s.
Exiting SolveHard() after 33.639r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73,
375.73285290693383611849070921651013685 45,
328.11709294099659833330344379618420795 31,
292.99969138113475212686190488135542690 06,
358.64341560212271535917634871506886215 50, none,
360.06173466981163010828029644938190190 26,
336.59441032036703525490916297089285727 69, none,
324.65521223071144185047254867479422076 26, none, none, none, none]

0 --> 2  target = [34.49522661143231719854346503385391887472,
3.897131315874777452424402714237098406291,
373.78081883365379734176265123508049369 12]
two intervals r = 17.29769086244858722511302757653987989108 ..
9499999999967137474285708976456988999/50000000000000000000000000000000
000 or r = 14.99436407374443925206073169273167367638 ..
9499999999967137474285708976456988999/50000000000000000000000000000000
000
Time Approximations 0.082.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [0., 0., -.3155e-34]Solution in 1.185s

Time Plot 0 s.
Exiting SolveHard() after 5.221r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73,
375.73285290693383611849070921651013685 45,
328.11709294099659833330344379618420795 31,
292.99969138113475212686190488135542690 06,
358.64341560212271535917634871506886215 50, none,
360.06173466981163010828029644938190190 26,
336.59441032036703525490916297089285727 69, none,
324.65521223071144185047254867479422076 26,
331.93806790206890159479045329345969471 30, none, none, none]

1 --> 2  target = [34.49522661143231719854346503385391887472,
3.89713131587477745242440271423709840629 1,
373.78081883365379734176265123508049369 12]
```

"Imaginary part neglected: ", $1.8899423791508501959972168187594385022 97 \times 10^{-17}$

```
one interval r = 21.06068473184553359112843751171478115673 ..
26.26979834264435262195572198260136238144
Time Approximations 0.034.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., -.91e-35]Solution in 0.755s

Time Plot 0 s.
Exiting SolveHard() after 3.87r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73,
375.73285290693383611849070921651013685 45,
328.11709294099659833330344379618420795 31,
292.99969138113475212686190488135542690 06,
358.64341560212271535917634871506886215 50,
299.89866203886394237798495947540297803 60,
360.06173466981163010828029644938190190 26,
336.59441032036703525490916297089285727 69, none,
324.65521223071144185047254867479422076 26,
331.93806790206890159479045329345969471 30, none, none, none]

0 --> 2  target = [33.81362495395974248754521866346792439776,
3.72564899352664648231158131074789484267 2,
325.89209972325123445856734076376064460 29]
two intervals r = 18.552270490026545754470110994604033785 81 ..
949999999996713747428570897645698899 9/500000000000000000000000000000000
000 or r = 12.491969357561807661976257006052550929 79 ..
949999999996713747428570897645698899 9/500000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
```

```
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [.70e-37, -.3e-37, -.3122e-34]Solution in 3.501s

Time Plot 0 s.
Exiting SolveHard() after 4.962r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73,
375.73285290693383611849070921651013685 45,
328.11709294099659833330344379618420795 31,
292.99969138113475212686190488135542690 06,
358.64341560212271535917634871506886215 50,
299.89866203886394237798495947540297803 60,
360.06173466981163010828029644938190190 26,
336.59441032036703525490916297089285727 69, none,
324.65521223071144185047254867479422076 26,
331.93806790206890159479045329345969471 30, none, none,
289.54595771790825766273556052630474102 12]

1 --> 2  target = [33.81362495395974248754521866346792439776,
3.72564899352664648231158131074789484 2672,
325.89209972325123445856734076376064460 29]

"Imaginary part neglected: ", 1.8899423791508501959972168187594385022 97 × 10^-17
one interval r = 20.37468935101374428182122127855738645534 ..
25.37892165289040469292731412369376672706
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .50e-35]Solution in 0.587s

Time Plot 0 s.
Exiting SolveHard() after 1.132r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73,
375.73285290693383611849070921651013685 45,
328.11709294099659833330344379618420795 31,
292.99969138113475212686190488135542690 06,
358.64341560212271535917634871506886215 50,
299.89866203886394237798495947540297803 60,
360.06173466981163010828029644938190190 26,
336.59441032036703525490916297089285727 69,
256.10753185699377519287252141902404706 84,
324.65521223071144185047254867479422076 26,
331.93806790206890159479045329345969471 30, none, none,
289.54595771790825766273556052630474102 12]

1 --> 0  target = [17.93041369700672849572934480847320318138,
4.68650870208959605442731749466478078 0043,
353.30541094791741035770445444212103525 21]
```

"Imaginary part neglected: ", $1.8899423791508501959972168187594385022 97 \times 10^{-17}$

```
one interval r = 20.73150479087702086663454241957163812425 ..
25.90675353517495102939687297887057869698
```

```
Time Approximations 0.033.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., .132e-34]Solution in 0.672s

Time Plot 0 s.
Exiting SolveHard() after 3.973r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73,
375.73285290693383611849070921651013685 45,
328.11709294099659833330344379618420795 31,
292.99969138113475212686190488135542690 06,
358.64341560212271535917634871506886215 50,
299.89866203886394237798495947540297803 60,
360.06173466981163010828029644938190190 26,
336.59441032036703525490916297089285727 69,
256.10753185699377519287252141902404706 84,
324.65521223071144185047254867479422076 26,
331.93806790206890159479045329345969471 30,
304.79958325629005922684570595567312899 70, none,
289.54595771790825766273556052630474102 12]

2 --> 0  target = [17.93041369700672849572934480847320318138,
4.68650870208959605442731749466478078 0043,
```

```
353.30541094791741035770445444212103525 21]
one interval r = 31.374354869854908091266402027623958696 61 ..
34.201275200177071245410997444196912424 45
Time Approximations 0.016.

hint used Hint := [33.796329522691901201472146203888695425 08, 2, 1, -1,
17.863534069716629640044658426895081929 62, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=5e-38
Equations at solution: [.3e-37, -.5e-37, .10161e-34]Solution in 0.341s

Time Plot 0 s.
Exiting SolveHard() after 0.624r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493011649311710427242553642340 87,
441.64295972671621466643251053449584097 74,
436.91748164526597495821322510238135621 04,
422.98493397314114636239978685870952519 52,
361.52580255470494540882722932163644570 15,
401.88173903819378366070893680361422931 89,
389.59001514957275076844188619505232512 71,
328.46939893144913711788979450803700748 95,
401.50757157607542645679543807404230222 54,
358.97362823332736723865686644143412812 40,
398.33147104207319473245763963720361182 99,
371.48387393310556274114791378563062714 68,
336.61215840997060980498245712125938729 26,
361.50888346504916849370810887715941133 33,
324.67144992095707242150250894451423804 88,
302.31384314942646132871832058280496523 09,
328.46938513217521806616331076528363244 75,
343.81340624036583625871186354539671678 73,
375.73285290693383611849070921651013685 45,
328.11709294099659833330344379618420795 31,
292.99969138113475212686190488135542690 06,
358.64341560212271535917634871506886215 50,
299.89866203886394237798495947540297803 60,
360.06173466981163010828029644938190190 26,
336.59441032036703525490916297089285727 69,
256.10753185699377519287252141902404706 84,
324.65521223071144185047254867479422076 26,
331.93806790206890159479045329345969471 30,
304.79958325629005922684570595567312899 70,
323.46169176105594738296505918436656454 56,
```

```
289.54595771790825766273556052630474102 12]

Cascade time 249.623
counts: 28, 28

 Iteration 53

Start Generation 1
1 --> 0  target = [11.999999999885323011153152264997700837 00,
6.2170125030688304395302867718942740243 25,
485.54908090170419237086694163197479655 13]
one interval r = 23.408503016531177861958686000675742481 29 ..
27.675780464092997716208099140373583509 73
Time Approximations 0.043.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.4921114030735554665825127038899393785 94, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=8.0e-38
Equations at solution: [.2e-37, -.80e-37, -.1e-36]Solution in 1.024s

Time Plot 0 s.
Exiting SolveHard() after 2.193r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999885323011153152264997700837 00,
6.2170125030688304395302867718942740243 25,
485.54908090170419237086694163197479655 13]
one interval r = 32.628147792128105459898362947666071316 32 ..
36.102483889376068717272617530598154628 54
Time Approximations 0.022.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .33e-35]Solution in 0.584s

Time Plot 0 s.
Exiting SolveHard() after 3.357r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684462759270521694205703342388263,
6.58343472168058068902386498471303002873 8,
467.78730596284018097791246436139388244 86]
one interval r = 32.41978955662932639863069321590351797181 ..
35.85152417368457258841187832792852301509
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=3e-38
Equations at solution: [.3e-37, -.3e-37, .206e-34]Solution in 0.615s

Time Plot 0 s.
Exiting SolveHard() after 0.969r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53, none, none,
401.88173904961606712384616872049163096 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
0 --> 1  target = [27.523596844627592705216942057033423882 63,
6.583434721680580689023864984713030028738,
467.787305962840180977912464361393892448 6]
two intervals r = 12.923271608183599054985559907584199621 65 ..
190000000001256645375447084275988264 77/100000000000000000000000000000000
00000 or r = 18.394248580495330233170836477053619307 84 ..
190000000001256645375447084275988264 77/100000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=8.7e-38
Equations at solution: [-.8e-37, -.87e-37, -.103e-35]Solution in 41.82s

Time Plot 0 s.
Exiting SolveHard() after 45.869r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52, none,
401.88173904961606712384616872049163096 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628248463245685680429629495163 67,
4.125651797140036161070269795220037730568,
440.671230655726792745395794737949022190 7]
two intervals r = 14.356597051028107970397963153851893305 78 ..
190000000001256645375447084275988264 77/100000000000000000000000000000000
00000 or r = 17.703526138329418384987395236158973190 94 ..
190000000001256645375447084275988264 77/100000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.27e-37, -.2e-37, -.160e-35]Solution in 3.822s

Time Plot 0 s.
Exiting SolveHard() after 4.901r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52, none,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962824846324568568042962949516367,
4.12565179714003616107026979522003773056 8,
440.67123065572679274539579473794902219 07]
one interval r = 22.39761154370148604887637573128754944107 ..
27.23722351584600688170253964764678820330
Time Approximations 0.038.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.635 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064385726998338062903515639329354, rm =
14.37818770812252767063514748979204218279}});
Accepted {r=26.4635, rm=16.5329} with Delta=7.9e-38
Equations at solution: [.1e-37, .79e-37, -.560e-34]Solution in 10.296s

Time Plot 0 s.
Exiting SolveHard() after 11.192r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888010779600251636981475966810 65,
4.004869082123851870506516799166955426202,
404.86224502040451099442137413190578106 38]
two intervals r = 16.080110077415780947035074901959439956 49 ..
19000000000125664537544708427598826477/1000000000000000000000000000000000
00000 or r = 16.415798127296318830603283256816957995 32 ..
19000000000125664537544708427598826477/1000000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., .53e-36]Solution in 1.416s

Time Plot 0 s.
Exiting SolveHard() after 5.079r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89, none, none,
358.97362824865376549957753765739534558 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888010779600251636981475966810 65,
4.004869082123851870506516799166955426202,
```

```
404.86224502040451099442137413190578106 38]
one interval r = 21.641943994197359406853184491161101532 99 ..
26.763306600342129129665698718467933467 69
Time Approximations 0.052.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .434e-34]Solution in 1.057s

Time Plot 0 s.
Exiting SolveHard() after 4.442r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57, none,
358.97362824865376549957753765739534558 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589417668741588201171729190701504 53,
5.5896371830907449879698989093021252604 05,
443.83065885126477766693580142848725428 45]
one interval r = 22.467253744816859259155781242748631576 6 ..
27.273884283407768249460776392701391692 05
Time Approximations 0.034.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.4e-38
```

```
Equations at solution: [-.1e-37, .54e-37, .110e-34]Solution in 0.933s

Time Plot 0 s.
Exiting SolveHard() after 4.771r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57, none,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589417668741588201171729190701504 53,
5.589637183090744987969898909302125260405,
443.83065885126477766693580142848725428 45]
one interval r = 32.155752795029893858123667124497173636 98 ..
35.508722287370783222664109630671759909 67
Time Approximations 0.019.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .200e-34]Solution in 0.448s

Time Plot 0 s.
Exiting SolveHard() after 0.807r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
```

```
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136489652543440623625119055731094,
5.1877835786570790526160248062284749348 11,
408.65773863472236447947156784254054085 19]
one interval r = 21.718401146600038575051732954242080834 18 ..
26.818493035046006210860283909102360109 07
Time Approximations 0.057.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, -.330e-34]Solution in 0.996s

Time Plot 0 s.
Exiting SolveHard() after 4.553r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38, none, none,
361.50888347980214158071819265547641697 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136489652543440623625119055731094,
5.1877835786570790526160248062284749348 11,
408.65773863472236447947156784254054085 19]
```

```
one interval r = 31.80828598754666817175591021316708370099 ..
35.00011460048825758777037883712417455191
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., .128e-34]Solution in 3.535s

Time Plot 0 s.
Exiting SolveHard() after 3.855r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16, none,
361.50888347980214158071819265547641697 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110538085973472002020267825232342,
6.19626256543428032233041208949258962270 4,
385.44474380267239935023991396911381222 88]
one interval r = 31.60836097540631541716832670710168594849 ..
34.66372795616071288674114929326102474516
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., -.364e-34]Solution in 0.598s

Time Plot 0 s.
Exiting SolveHard() after 0.892r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16, none,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110538085973472002020267825232342,
6.196262565434280322330412089492589622704,
385.44474380267239935023991396911381222 88]
two intervals r = 16.875634087317006614464239615339889675 53 ..
190000000001256645375447084275988264 77/100000000000000000000000000000000
00000 or r = 15.556404938514340968738667863635490734 55 ..
190000000001256645375447084275988264 77/100000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [.718e-37, -.1e-37, .1600e-34]Solution in 4.048s

Time Plot 0 s.
Exiting SolveHard() after 5.245r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874714999399254361977308640736796,
4.8838107799982575003072471465855416261 58,
376.61967856780827166938260799533420574 75]
one interval r = 21.11001304890471968883579324554714527605 ..
26.31784243474492516670961777313442017445
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.4e-38
Equations at solution: [0., -.24e-37, -.280e-34]Solution in 3.329s

Time Plot 0 s.
Exiting SolveHard() after 4.046r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
```

```
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42, none,
328.46938514581918691123659836637233335 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874714999399254361977308640736796,
4.8838107799982575003072471465855416261 58,
376.61967856780827166938260799533420574 75]
one interval r = 31.53899497714741306259628999328842281958 ..
34.53618386099973776391843598191154734603
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.79e-36
Equations at solution: [-.137e-35, .179e-35, .324e-34]Solution in
0.491s

Time Plot 0 s.
Exiting SolveHard() after 0.778r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42, none,
328.46938514581918691123659836637233335 89,
```

```
343.8134062609510569744135404117685631276, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.872050175463228965724328042280101108 74,
6.0258135494003188077901316781483183529 95,
351.42702949532895170694762934646674023 87]
one interval r = 31.362302061172787912605252920331144304 75 ..
34.174466406238050137100154209152429533 26
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, .156e-34]Solution in 0.523s

Time Plot 0 s.
Exiting SolveHard() after 0.78r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42, none,
328.46938514581918691123659836637233335 89,
343.8134062609510569744135404117685631276, none, none,
292.99969139734708363030864915382256367 64, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175463228965724328042280101108 74,
6.0258135494003188077901316781483183529 95,
351.42702949532895170694762934646674023 87]
two intervals r = 17.981355144259220894840461067391334910 46 ..
19000000000125664537544708427598826477/10000000000000000000000000000000
00000 or r = 13.846080154816249732013223304246815078 10 ..
```

```
190000000001256645375447084275988264 77/100000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [-.54e-37, .1e-37, -.430e-35]Solution in 1.124s

Time Plot 0 s.
Exiting SolveHard() after 4.779r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76, none, none,
292.99969139734708363030864915382256367 64, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418631609335449368705676772888 07,
6.3779438739666086931014108536839086279 24,
423.28832784662745708130981339193119825 94]
one interval r = 31.946618175996404642204772886587471683 11 ..
35.212123086532647472923599455513498240 85
Time Approximations 0.02.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=7e-38
Equations at solution: [-.7e-37, .7e-37, .129e-34]Solution in 0.595s

Time Plot 0 s.
Exiting SolveHard() after 0.937r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76, none, none,
292.99969139734708363030864915382256367 64, none, none,
360.06173467793538111999465530447354142 75, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418631609335449368705676772888 07,
6.3779438739666086931014108536839086279 24,
423.28832784662745708130981339193119825 94]
two intervals r = 15.22886702415481237460757217866774134328 ..
19000000000125664537544708427598826477/10000000000000000000000000000000
00000 or r = 17.12965771105771246800024217808117219178 ..
19000000000125664537544708427598826477/10000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., .613e-35]Solution in 1.255s

Time Plot 0 s.
Exiting SolveHard() after 4.941r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76,
375.73285291001973657219195470306932584 47, none,
292.99969139734708363030864915382256367 64, none, none,
360.06173467793538111999465530447354142 75, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343460742082802631218870789200 78,
4.0035598158265009086400589176201667330 69,
404.47973594821927657528680556570678046 32]
two intervals r = 16.096839663492441556259498886781570337 08 ..
190000000001256645375447084275988264 77/100000000000000000000000000000000
00000 or r = 16.399886491527474967204051897493832089 59 ..
190000000001256645375447084275988264 77/100000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.18e-37, 0., -.2219e-34]Solution in 3.8s

Time Plot 0 s.
Exiting SolveHard() after 7.501r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76,
375.73285291001973657219195470306932584 47, none,
292.99969139734708363030864915382256367 64,
358.64341561780623613896131992625674184 22, none,
360.06173467793538111999465530447354142 75, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343460742082802631218870789200 78,
4.0035598158265009086400589176201667330 69,
404.47973594821927657528680556570678046 32]
one interval r = 21.634296299985135075957904104618372467 47 ..
26.757681698851808292280529019286371245 82
Time Approximations 0.05.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=1.00e-37
Equations at solution: [.2e-37, .100e-36, .12e-35]Solution in 1.031s

Time Plot 0 s.
Exiting SolveHard() after 2.037r=25.8653 in [23.83864811 ..
```

```
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76,
375.73285291001973657219195470306932584 47,
328.11709295502217052206969440786272906 57,
292.99969139734708363030864915382256367 64,
358.64341561780623613896131992625674184 22, none,
360.06173467793538111999465530447354142 75, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954456003276838571691916153962801,
6.19617723028695919800189943484417136226 1,
385.42734026625199413383055833922896065 00]
one interval r = 31.60822049095857122509103646670279622294 ..
34.66347615053484342376356175413831441977
Time Approximations 0.016.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.82e-35]Solution in 0.538s

Time Plot 0 s.
Exiting SolveHard() after 3.633r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76,
375.73285291001973657219195470306932584 47,
328.11709295502217052206969440786272906 57,
292.99969139734708363030864915382256367 64,
358.64341561780623613896131992625674184 22, none,
360.06173467793538111999465530447354142 75, none, none,
324.65521224795299721365742386814322500 65, none, none, none, none]

0 --> 1  target = [26.46318954456003276838571691916153962801,
6.19617723028695919800189943484417136226 1,
385.42734026625199413383055833922896065 00]
two intervals r = 16.87629600270997315641946342302300640611 ..
19000000000125664537544708427598826477/10000000000000000000000000000000
00000 or r = 15.55559000699188567645354541740884867083 ..
19000000000125664537544708427598826477/10000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., .2091e-34]Solution in 3.716s

Time Plot 0 s.
Exiting SolveHard() after 4.87r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76,
375.73285291001973657219195470306932584 47,
328.11709295502217052206969440786272906 57,
292.99969139734708363030864915382256367 64,
358.64341561780623613896131992625674184 22, none,
360.06173467793538111999465530447354142 75,
336.59441033340431331923807866220797534 03, none,
324.65521224795299721365742386814322500 65, none, none, none, none]

0 --> 2  target = [34.49522661172461652226570459797042654554,
3.8971313162524175611013997629857784826 46,
373.78081885466675601916457004187525108 43]
two intervals r = 17.29769086203465636129697572798976524269 ..
19000000000125664537544708427598826477/10000000000000000000000000000000
00000 or r = 14.99436407485306141525508765064587037616 ..
19000000000125664537544708427598826477/10000000000000000000000000000000
00000
Time Approximations 0.086.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.72e-37, 0., .2786e-34]Solution in 3.622s

Time Plot 0 s.
Exiting SolveHard() after 5.317r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76,
375.73285291001973657219195470306932584 47,
328.11709295502217052206969440786272906 57,
292.99969139734708363030864915382256367 64,
358.64341561780623613896131992625674184 22, none,
360.06173467793538111999465530447354142 75,
336.59441033340431331923807866220797534 03, none,
324.65521224795299721365742386814322500 65,
331.93806792588496353939787371015509611 87, none, none, none]

1 --> 2  target = [34.49522661172461652226570459797042654554,
3.8971313162524175611013997629857784826 46,
373.78081885466675601916457004187525108 43]
one interval r = 21.06068473216693962665699782374349518444 ..
26.26979834283706413025643021201218843695
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=8e-38
Equations at solution: [-.3e-37, -.8e-37, .112e-34]Solution in 3.435s

Time Plot 0 s.
Exiting SolveHard() after 4.162r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76,
375.73285291001973657219195470306932584 47,
328.11709295502217052206969440786272906 57,
292.99969139734708363030864915382256367 64,
358.64341561780623613896131992625674184 22,
299.89866206117690247055760656398710077 27,
360.06173467793538111999465530447354142 75,
336.59441033340431331923807866220797534 03, none,
324.65521224795299721365742386814322500 65,
331.93806792588496353939787371015509611 87, none, none, none]

0 --> 2  target = [33.81362495420284522572325208344795443642,
3.72564899389489765446852417647658413879 3,
325.89209974089485124132296453370689913 52]
two intervals r = 18.552270489959506054536506015612962007 06 ..
19000000000125664537544708427598826477/10000000000000000000000000000000
00000 or r = 12.491969358532353278361751260571944181 03 ..
19000000000125664537544708427598826477/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1e-38
Equations at solution: [.36e-37, -.1e-37, -.839e-35]Solution in 3.705s

Time Plot 0 s.
Exiting SolveHard() after 5.109r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76,
375.73285291001973657219195470306932584 47,
328.11709295502217052206969440786272906 57,
292.99969139734708363030864915382256367 64,
358.64341561780623613896131992625674184 22,
299.89866206117690247055760656398710077 27,
360.06173467793538111999465530447354142 75,
336.59441033340431331923807866220797534 03, none,
324.65521224795299721365742386814322500 65,
331.93806792588496353939787371015509611 87, none, none,
289.54595773889594796457791482580361567 02]

1 --> 2  target = [33.81362495420284522572325208344795443642,
3.72564899389489765446852417647658413879 3,
325.89209974089485124132296453370689913 52]
one interval r = 20.37468935118466467393437246165288549796 ..
25.37892165310337580991237746504284719932
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, .11e-35]Solution in 0.568s

Time Plot 0 s.
Exiting SolveHard() after 1.096r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
```

```
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76,
375.73285291001973657219195470306932584 47,
328.11709295502217052206969440786272906 57,
292.99969139734708363030864915382256367 64,
358.64341561780623613896131992625674184 22,
299.89866206117690247055760656398710077 27,
360.06173467793538111999465530447354142 75,
336.59441033340431331923807866220797534 03,
256.10753187614430943302649096309364125 70,
324.65521224795299721365742386814322500 65,
331.93806792588496353939787371015509611 87, none, none,
289.54595773889594796457791482580361567 02]

1 --> 0  target = [17.93041369691433811432045183934140795794,
4.68650870214676279126684257063915216769 5,
353.30541096155491710871765856626521402 52]
one interval r = 20.73150479104673907308084069868930840333 ..
25.90675353527136602259900489501889286233
Time Approximations 0.035.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.8e-38
Equations at solution: [.2e-37, .48e-37, -.128e-34]Solution in 3.149s

Time Plot 0 s.
Exiting SolveHard() after 3.846r=25.4021 in [22.67806074 ..
```

```
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76,
375.73285291001973657219195470306932584 47,
328.11709295502217052206969440786272906 57,
292.99969139734708363030864915382256367 64,
358.64341561780623613896131992625674184 22,
299.89866206117690247055760656398710077 27,
360.06173467793538111999465530447354142 75,
336.59441033340431331923807866220797534 03,
256.10753187614430943302649096309364125 70,
324.65521224795299721365742386814322500 65,
331.93806792588496353939787371015509611 87,
304.79958326750524969965447790986370204 93, none,
289.54595773889594796457791482580361567 02]

2 --> 0  target = [17.93041369691433811432045183934140795794,
4.68650870214676279126684257063915216769 5,
353.30541096155491710871765856626521402 52]
one interval r = 31.37435486996597053772613544537375125686 ..
34.20127520036418629642186169039968527262
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=1.2e-37
Equations at solution: [.7e-37, -.12e-36, -.485e-34]Solution in 0.382s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.657r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493968517916439999884096361996 56,
441.64295973550770053704208446894886249 33,
436.91748165798656326925610564602059109 53,
422.98493398029134468444679207295818001 52,
361.52580256958994756171264394525220234 94,
401.88173904961606712384616872049163096 05,
389.59001516563388811214405216780163386 89,
328.46939894509735872897107810236179265 57,
401.50757158789863752747158901017955882 05,
358.97362824865376549957753765739534558 19,
398.33147104738724713718700492189339943 38,
371.48387395377873071710226789502208245 16,
336.61215842314646285927331011766262116 92,
361.50888347980214158071819265547641697 08,
324.67144993832532711593527389731040025 42,
302.31384316121001521723209933813246042 32,
328.46938514581918691123659836637233335 89,
343.81340626095105697441354041176856312 76,
375.73285291001973657219195470306932584 47,
328.11709295502217052206969440786272906 57,
292.99969139734708363030864915382256367 64,
358.64341561780623613896131992625674184 22,
299.89866206117690247055760656398710077 27,
360.06173467793538111999465530447354142 75,
336.59441033340431331923807866220797534 03,
256.10753187614430943302649096309364125 70,
324.65521224795299721365742386814322500 65,
331.93806792588496353939787371015509611 87,
304.79958326750524969965447790986370204 93,
323.46169177996979665165193410505456341 09,
289.54595773889594796457791482580361567 02]

Cascade time 152.677
counts: 28, 28

 Iteration 54

Start Generation 1
1 --> 0  target = [12.00000000005973463060207893160033659800,
6.21701250281559535877630329481048775932 9,
485.54908090192507352483467701920931272 69]
one interval r = 23.40850301665667774310655306178306876205 ..
27.67578046426943624268489986033432161067
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
```

```
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.12e-37
Equations at solution: [.7e-37, -.212e-36, -.3e-36]Solution in 3.537s

Time Plot 0 s.
Exiting SolveHard() after 4.712r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000005973463060207893160033659800,
6.21701250281559535877630329481048775932 9,
485.54908090192507352483467701920931272 69]
one interval r = 32.62814779227738642064718724874074472930 ..
36.10248388952318531957004447483894705690
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., -.75e-35]Solution in 0.617s

Time Plot 0 s.
Exiting SolveHard() after 1.048r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684479217354590842956617702126996,
6.58343472149783744956431069173119016799 1,
467.78730596255544031962006707634023271 85]
one interval r = 32.41978955677369837093596243397156617332 ..
35.85152417382372244964311628575407541547
Time Approximations 0.023.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.235e-34]Solution in 3.297s

Time Plot 0 s.
Exiting SolveHard() after 3.675r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55, none, none,
401.88173904467858886242489449719641679 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684479217354590842956617702126996,
6.58343472149783744956431069173119016799 1,
467.78730596255544031962006707634023271 85]
two intervals r = 12.92327160833428202333702437782110037530 ..
18999999999948609837976019770002036503/10000000000000000000000000000000
00000 or r = 18.39424858030896655960302991038775279109 ..
18999999999948609837976019770002036503/10000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.1e-38
Equations at solution: [.2e-37, .21e-37, -.530e-35]Solution in 42.685s

Time Plot 0 s.
Exiting SolveHard() after 44.043r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96, none,
401.88173904467858886242489449719641679 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962834639232434032284810941351095,
4.12565179658255131937420692613015637181 6,
440.67123065264764435752870066077846552 45]
two intervals r = 14.35659705124761317913628734752426136741 ..
189999999999486098379760197700020365 03/10000000000000000000000000000000
00000 or r = 17.70352613806939764444494479090034166997 ..
189999999999486098379760197700020365 03/10000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.15e-37, -.1e-37, .97e-36]Solution in 1.333s

Time Plot 0 s.
Exiting SolveHard() after 4.891r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
```

```
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96, none,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962834639232434032284810941351095,
4.1256517965825513193742069261301563718 16,
440.67123065264764435752870066077846552 45]
one interval r = 22.397611543756188466762674693194176441 67 ..
27.237223515966666926366444014944702568 77
Time Approximations 2.677.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.161 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643747389337325254842078751646 33, rm =
14.378187698846968010053148779815436343 96}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, -.196e-34]Solution in 8.085s

Time Plot 0 s.
Exiting SolveHard() after 11.663r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888808178732618693879926856168923,
4.0048690815605677947850407026902522965 99,
404.86224501552858163114397857529361453 73]
two intervals r = 16.080110077604908347888371406822442558 37 ..
```

```
1899999999994860983797601977000203650 3/1000000000000000000000000000000
00000 or r = 16.415798126961728867464922112607496736 86 ..
1899999999994860983797601977000203650 3/1000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=2e-38
Equations at solution: [-.69e-37, -.2e-37, -.479e-35]Solution in 4.381s

Time Plot 0 s.
Exiting SolveHard() after 8.151r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76, none, none,
358.97362824003359540543216630843319010 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888081787326186938799268561689 23,
4.0048690815605677947850407026902522965 99,
404.86224501552858163114397857529361453 73]
one interval r = 21.641943994225443714908213479272182671 02 ..
26.763306600415414194466600005995615488 13
Time Approximations 0.05.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.5e-38
Equations at solution: [.1e-37, .25e-37, -.210e-34]Solution in 1.079s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.102r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82, none,
358.97362824003359540543216630843319010 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941782577896726214293511747040356,
5.589637182820841422511412212743419254477,
443.83065884957047207973586654281755468 58]
one interval r = 22.46725374490158879300481704334731705925 ..
27.27388428354629880836059618740203395084
Time Approximations 0.036.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [0., .81e-37, .6e-36]Solution in 0.983s

Time Plot 0 s.
Exiting SolveHard() after 4.452r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
```

```
328.46939893518771923202255204446792638 82, none,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941782577896726214293511747040356,
5.589637182820841422511412212743419254477,
443.83065884957047207973586654281755468 58]
one interval r = 32.15575279516099754374045853285439555189 ..
35.50872228748877798473512874326795250112
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .152e-34]Solution in 3.083s

Time Plot 0 s.
Exiting SolveHard() after 3.472r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136519452681860515926274680136744,
5.187783578336187962385348758941931290586,
408.65773862770918724404007195986854945 31]
one interval r = 21.71840114658306972455881694811892480877 ..
26.81849303509116001355927526914450637982
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
```

```
15.9012851541434115876571010968637082152 1, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [-.1e-37, -.79e-37, -.32e-35]Solution in 1.027s

Time Plot 0 s.
Exiting SolveHard() after 2.08r=26.4632 in [23.93303356 .. 26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62, none, none,
361.50888347179146636032812517434787913 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136519452681860515926274680136744,
5.1877835783361879623853487589419312905 86,
408.65773862770918724404007195986854945 31]
one interval r = 31.80828598763370937902458754393765467058 ..
35.00011460052830878551840288718378403067
Time Approximations 0.016.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.231e-34]Solution in 0.411s

Time Plot 0 s.
Exiting SolveHard() after 3.488r=34.4952 in [32.91337941 ..
```

```
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88, none,
361.50888347179146636032812517434787913 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110537650665550445921945210747833,
6.19626256521226372417272687462668897 3127,
385.44474379378601351047246789372695165 02]
one interval r = 31.60836097548705842100337704844634682196 ..
34.66372795617381296386168054448986819122
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.61e-35]Solution in 0.54s

Time Plot 0 s.
Exiting SolveHard() after 0.81r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
```

```
389.590015158849549037811180658479408487 6,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88, none,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110537650665550445921945210747833,
6.19626256521226372417272687462668897312 7,
385.44474379378601351047246789372695165 02]
two intervals r = 16.87563408759313089652103450538182453097 ..
189999999999486098379760197700020365 03/1000000000000000000000000000000
00000 or r = 15.55640493799558672621920044630877884366 ..
189999999999486098379760197700020365 03/1000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 6.469 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175117370145562020351806591095944, rm
= 2.33653277429539204447098121869010800946 7}});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [-.897e-37, .1e-37, -.1199e-34]Solution in
29.954s

Time Plot 0 s.
Exiting SolveHard() after 34.525r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.590015158849549037811180658479408487 6,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
```

```
398.331471045052331886196141887855458706 2,
371.483873942508354329337342112020704348 8,
336.612158412530049329089880196314807353 0,
361.508883471791466360328125174347879137 6,
324.671449925034925402908120537004004322 6, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874738658188309978259401222402111,
4.883810779667505440276376946544352365377,
376.61967855890655070417371684585901835 90]
one interval r = 21.11001304887748276302853539497335229256 ..
26.31784243473231114756798149728984550572
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [0., .26e-37, .202e-34]Solution in 0.839s

Time Plot 0 s.
Exiting SolveHard() after 1.53r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.642959735655768544390102309531246328 8,
436.917481654808179068796712934922762835 5,
422.984933978592397869819167487639513119 6,
361.525802561364863277174874497935499727 2,
401.881739044678588862424894497196416794 8,
389.590015158849549037811180658479408487 6,
328.469398935187719232022552044467926388 2,
401.507571581909918217565470175754763862 9,
358.973628240033595405432166308433190104 8,
398.331471045052331886196141887855458706 2,
371.483873942508354329337342112020704348 8,
336.612158412530049329089880196314807353 0,
361.508883471791466360328125174347879137 6,
324.671449925034925402908120537004004322 6, none,
328.469385135915711879341586905740481947 1, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874738658188309978259401222402111,
4.883810779667505440276376946544352365377,
376.61967855890655070417371684585901835 90]
```

```
one interval r = 31.53899497723232755054477500247069517568 ..
34.53618386101296811519609276228380331886
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.127e-35
Equations at solution: [-.866e-35, .1127e-34, -.132e-34]Solution in
0.471s

Time Plot 0 s.
Exiting SolveHard() after 0.744r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26, none,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017539773062643079543475128238857,
6.02581354916858920636540395533962868045 1,
351.42702948470468726427248428466793339 42]
one interval r = 31.36230206125945218155402411625972745827 ..
34.17446640622855375426570818967273384886
Time Approximations 0.014.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.91e-35]Solution in 0.517s

Time Plot 0 s.
Exiting SolveHard() after 3.474r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26, none,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39, none, none,
292.99969138233057501453892140033102983 48, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175397730626430795434751282388 57,
6.0258135491685892063654039553396286804 51,
351.42702948470468726427248428466793339 42]
two intervals r = 17.981355144429881842140911211430016136 82 ..
189999999999486098379760197700020365 03/10000000000000000000000000000000
00000 or r = 13.846080154225053962242298355822425467 41 ..
189999999999486098379760197700020365 03/10000000000000000000000000000000
00000
Time Approximations 0.047.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 7.89 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713637866556590300904542831706 43, rm
= 2.7345009935319867906642848828383001785 38}});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [0., 0., .193e-35]Solution in 19.153s

Time Plot 0 s.
Exiting SolveHard() after 20.233r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39, none, none,
292.99969138233057501453892140033102983 48, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418745000123833248464940320607 15,
6.3779438737735071077000767837612123913 66,
423.28832784379704050258294740876344206 66]
one interval r = 31.946618176119026024266762554905447715 30 ..
35.212123086633597726851139763264346007 08
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.287e-34]Solution in 0.631s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.99r=34.3272 in [33.10127385 .. 35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39, none, none,
292.99969138233057501453892140033102983 48, none, none,
360.06173467043673413293277065321361778 81, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418745000123833248464940320607 15,
6.3779438737735071077000767837612123913 66,
423.28832784379704050258294740876344206 66]
two intervals r = 15.22886702430864322211352756529066459312 ..
189999999999486098379760197700020365 03/100000000000000000000000000000000
00000 or r = 17.12965777080326325573503073504352804913 ..
189999999999486098379760197700020365 03/100000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 8.761 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541167591600015751966640007381 53, rm
= 2.064068298817540849395943244130144427072}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., -.1150e-34]Solution in 29.458s
```

```
Time Plot 0 s.
Exiting SolveHard() after 33.35r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39,
375.73285290551611459718945725358708637 91, none,
292.99969138233057501453892140033102983 48, none, none,
360.06173467043673413293277065321361778 81, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343516164334145542190016367259 30,
4.0035598152595397294943611148954955405 49,
404.47973594226867576159843573260121165 08]
two intervals r = 16.096839663727200310749825297188517268 28 ..
189999999999486098379760197700020365 03/10000000000000000000000000000000
00000 or r = 16.399886491148076240099576265306091307 41 ..
189999999999486098379760197700020365 03/10000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.17e-37, .1e-37, -.327e-35]Solution in 1.649s

Time Plot 0 s.
Exiting SolveHard() after 5.448r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39,
375.73285290551611459718945725358708637 91, none,
292.99969138233057501453892140033102983 48,
358.64341560825577241646273648677822846 70, none,
360.06173467043673413293277065321361778 81, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234351616433414554219001636725930,
4.0035598152595397294943611148954955405 49,
404.47973594226867576159843573260121165 08]
one interval r = 21.63429629999192522100831055176206833713 ..
26.75768169890901182201098745986580502583
Time Approximations 0.051.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=4.9e-38
Equations at solution: [.2e-37, .49e-37, .365e-34]Solution in 1.051s

Time Plot 0 s.
Exiting SolveHard() after 4.852r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39,
375.73285290551611459718945725358708637 91,
328.11709294412255173695319907691911218 01,
292.99969138233057501453892140033102983 48,
358.64341560825577241646273648677822846 70, none,
360.06173467043673413293277065321361778 81, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954455923181582990853128710103082,
6.19617723006602371685356222805860615801 3,
385.42734025758615520104939852536410867 54]
one interval r = 31.60822049104110279920479201366155590554 ..
34.66347615055113505508930128073658082301
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., .31e-35]Solution in 3.289s

Time Plot 0 s.
Exiting SolveHard() after 3.581r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
```

```
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39,
375.73285290551611459718945725358708637 91,
328.11709294412255173695319907691911218 01,
292.99969138233057501453892140033102983 48,
358.64341560825577241646273648677822846 70, none,
360.06173467043673413293277065321361778 81, none, none,
324.65521223486831326803806615323982357 47, none, none, none, none]

0 --> 1  target = [26.46318954455923181582990853128710103082,
6.19617723006602371685356222805860615801 3,
385.42734025758615520104939852536410867 54]
two intervals r = 16.87629600297763058935557907194154456605 ..
189999999999486098379760197700020365 03/10000000000000000000000000000000
00000 or r = 15.55559000648345104365845314435400739280 ..
189999999999486098379760197700020365 03/10000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 9.556 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852529007249785420744451456760251, rm
= 2.33669042836035939732566153988288707355 8}});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [.717e-37, -.1e-37, .5558e-34]Solution in
30.517s

Time Plot 0.001 s.
Exiting SolveHard() after 31.635r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39,
375.73285290551611459718945725358708637 91,
328.11709294412255173695319907691911218 01,
292.99969138233057501453892140033102983 48,
358.64341560825577241646273648677822846 70, none,
360.06173467043673413293277065321361778 81,
336.59441032301292004996361042732618486 18, none,
324.65521223486831326803806615323982357 47, none, none, none, none]

0 --> 2  target = [34.49522661170266160681920583866655726700,
3.89713131566655762178715561202376653604 6,
373.78018843314969570436002345784341203 3]
two intervals r = 17.29769086234246389019179398592879710908 ..
189999999999486098379760197700020365 03/10000000000000000000000000000000
00000 or r = 14.99436407421053864122990876795979087633 ..
189999999999486098379760197700020365 03/10000000000000000000000000000000
00000
Time Approximations 0.087.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., -.572e-35]Solution in 1.199s

Time Plot 0 s.
Exiting SolveHard() after 5.351r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39,
375.73285290551611459718945725358708637 91,
328.11709294412255173695319907691911218 01,
292.99969138233057501453892140033102983 48,
358.64341560825577241646273648677822846 70, none,
360.06173467043673413293277065321361778 81,
336.59441032301292004996361042732618486 18, none,
324.65521223486831326803806615323982357 47,
331.93806791145337618749818893130773103 00, none, none, none]

1 --> 2  target = [34.49522661170266160681920583866655726700,
3.89713131566655762178715561202376653604 6,
373.78018843314969570436002345784341203 3]
one interval r = 21.06068473210044373191439694662045463362 ..
26.26979834278023159935092707752553401915
Time Approximations 0.031.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [.3e-37, .7e-37, .565e-34]Solution in 0.756s

Time Plot 0 s.
Exiting SolveHard() after 4.188r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39,
375.73285290551611459718945725358708637 91,
328.11709294412255173695319907691911218 01,
292.99969138233057501453892140033102983 48,
358.64341560825577241646273648677822846 70,
299.89866204531368842077822108201441018 75,
360.06173467043673413293277065321361778 81,
336.59441032301292004996361042732618486 18, none,
324.65521223486831326803806615323982357 47,
331.93806791145337618749818893130773103 00, none, none, none]

0 --> 2  target = [33.81362495415788592731163944643329458226,
3.72564899330084963385682471908401228435 0,
325.89209972747331665087075969043923588 13]
two intervals r = 18.552270490056454281407777835213335843 11 ..
18999999999948609837976019770002036503/10000000000000000000000000000000
00000 or r = 12.491969357874641690142622687242431596 77 ..
18999999999948609837976019770002036503/10000000000000000000000000000000
00000
Time Approximations 0.038.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [.104e-36, -.5e-37, .2132e-34]Solution in 1.076s

Time Plot 0 s.
Exiting SolveHard() after 5.174r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39,
375.73285290551611459718945725358708637 91,
328.11709294412255173695319907691911218 01,
292.99969138233057501453892140033102983 48,
358.64341560825577241646273648677822846 70,
299.89866204531368842077822108201441018 75,
360.06173467043673413293277065321361778 81,
336.59441032301292004996361042732618486 18, none,
324.65521223486831326803806615323982357 47,
331.93806791145337618749818893130773103 00, none, none,
289.54595772255192404749148436687373769 48]

1 --> 2  target = [33.81362495415788592731163944643329458226,
3.72564899330084963385682471908401228435 0,
325.89209972747331665087075969043923588 13]
one interval r = 20.37468935117013452761491850830675841391 ..
25.37892165296117481412716643943796650551
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [-.3e-37, -.4e-37, -.191e-34]Solution in 0.56s

Time Plot 0 s.
Exiting SolveHard() after 1.113r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39,
375.73285290551611459718945725358708637 91,
328.11709294412255173695319907691911218 01,
292.99969138233057501453892140033102983 48,
358.64341560825577241646273648677822846 70,
299.89866204531368842077822108201441018 75,
360.06173467043673413293277065321361778 81,
336.59441032301292004996361042732618486 18,
256.10753185841820136712237253260588455 94,
324.65521223486831326803806615323982357 47,
331.93806791145337618749818893130773103 00, none, none,
289.54595772255192404749148436687373769 48]

1 --> 0  target = [17.93041369710370052602595808233734530047,
4.68650870180220930944135818074047857402 0,
353.30541095058834445112386119189554574 11]
one interval r = 20.73150479101514752344783434170492996918 ..
25.90675353520123092594837926279864020553
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.9e-38
Equations at solution: [.2e-37, .49e-37, -.174e-34]Solution in 0.668s

Time Plot 0 s.
Exiting SolveHard() after 3.884r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
```

```
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39,
375.73285290551611459718945725358708637 91,
328.11709294412255173695319907691911218 01,
292.99969138233057501453892140033102983 48,
358.64341560825577241646273648677822846 70,
299.89866204531368842077822108201441018 75,
360.06173467043673413293277065321361778 81,
336.59441032301292004996361042732618486 18,
256.10753185841820136712237253260588455 94,
324.65521223486831326803806615323982357 47,
331.93806791145337618749818893130773103 00,
304.79958325558886822463967656568106583 34, none,
289.54595772255192404749148436687373769 48]

2 --> 0  target = [17.93041369710370052602595808233734530047,
4.68650870180220930944135818074047857402 0,
353.30541095058834445112386119189554574 11]
one interval r = 31.37435487004928317843240187995460507854 ..
34.20127520034959269630523190528696518082
Time Approximations 0.026.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, -.425e-34]Solution in 0.342s

Time Plot 0 s.
```

```
Exiting SolveHard() after 0.636r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493982492887714189220252355460 16,
441.64295973565576854439010230953124632 88,
436.91748165480817906879671293492276283 55,
422.98493397859239786981916748763951311 96,
361.52580256136486327717487449793549972 72,
401.88173904467858886242489449719641679 48,
389.59001515884954903781118065847940848 76,
328.46939893518771923202255204446792638 82,
401.50757158190991821756547017575476386 29,
358.97362824003359540543216630843319010 48,
398.33147104505233188619614188785545870 62,
371.48387394250835432933734211202070434 88,
336.61215841253004932908988019631480735 30,
361.50888347179146636032812517434787913 76,
324.67144992503492540290812053700400432 26,
302.31384314906088903675144004060674053 59,
328.46938513591571187934158690574048194 71,
343.81340624755929899802102171334480512 39,
375.73285290551611459718945725358708637 91,
328.11709294412255173695319907691911218 01,
292.99969138233057501453892140033102983 48,
358.64341560825577241646273648677822846 70,
299.89866204531368842077822108201441018 75,
360.06173467043673413293277065321361778 81,
336.59441032301292004996361042732618486 18,
256.10753185841820136712237253260588455 94,
324.65521223486831326803806615323982357 47,
331.93806791145337618749818893130773103 00,
304.79958325558886822463967656568106583 34,
323.46169176453194336128385569219473501 90,
289.54595772255192404749148436687373769 48]

Cascade time 255.356
counts: 28, 28

 Iteration 55

Start Generation 1
1 --> 0  target = [11.99999999995009414175470988030640811600,
6.21701250288808720972035690824082784749 4,
485.54908089414994565876630922245035744 64]
one interval r = 23.40850301636340720875563725737192367216 ..
27.67578046402464195663954344850831762965
Time Approximations 0.039.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
```

```
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.2e-38
Equations at solution: [.2e-37, -.52e-37, -.5e-36]Solution in 1.028s

Time Plot 0 s.
Exiting SolveHard() after 2.183r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999950094141754709880306408116 00,
6.2170125028880872097203569082408278474 94,
485.54908089414994565876630922245035744 64]
one interval r = 32.628147792215619434762286176882097974 65 ..
36.102483889471198564344809770242485787 88
Time Approximations 0.02.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=6e-38
Equations at solution: [.9e-37, -.6e-37, -.33e-35]Solution in 0.588s

Time Plot 0 s.
Exiting SolveHard() after 3.667r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684454801638481860308641012763015,
6.58343472150906630186243412214995746521 9,
467.78730595535909903838616079676290581 00]
one interval r = 32.41978955672137105487521468344686027158 ..
35.85152417377669486206267658425667142218
Time Approximations 0.024.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., .132e-34]Solution in 0.633s

Time Plot 0 s.
Exiting SolveHard() after 1.01r=34.9451 in [33.70078237 .. 35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84, none, none,
401.88173904126958628839093982925917188 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684454801638481860308641012763015,
6.58343472150906630186243412214995746521 9,
467.78730595535909903838616079676290581 00]
two intervals r = 12.92327160823985944652534330639135959633 ..
4749999999981193236333654157228126433/2500000000000000000000000000000000
000 or r = 18.39424858019254339930063045729288491340 ..
4749999999981193236333654157228126433/2500000000000000000000000000000000
000
Time Approximations 0.042.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
```

```
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=4.4e-38
Equations at solution: [-.4e-37, -.44e-37, -.1138e-34]Solution in
39.171s

Time Plot 0 s.
Exiting SolveHard() after 43.342r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79, none,
401.88173904126958628839093982925917188 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962831824951921727134818863546672,
4.12565179686040981161516687511489675103 3,
440.67123064705738336203852531368825322 26]
two intervals r = 14.35659705112573625120377883795308992011 ..
4749999999981193236333654157228126433/2500000000000000000000000000000
000 or r = 17.70352613794312837327111839425285358665 ..
4749999999981193236333654157228126433/2500000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [.57e-37, .3e-37, -.1310e-34]Solution in 1.321s

Time Plot 0 s.
Exiting SolveHard() after 4.867r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
```

```
422.98493397293481289493588242048344289 79, none,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962831824951921727134818863546672,
4.1256517968604098116151668751148967510 33,
440.67123064705738336203852531368825322 26]
one interval r = 22.397611543519935060927534268424700553 53 ..
27.237223515735260028570575983450924484 20
Time Approximations 0.051.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 3.679 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643589798890609851640367885947 68, rm =
14.378187702391457986177145531989323865 86}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.05e-37
Equations at solution: [-.1e-37, -.105e-36, .822e-34]Solution in
10.885s

Time Plot 0 s.
Exiting SolveHard() after 11.834r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888807481397033055821135801122462,
4.0048690818441512626349920660164147533 14,
404.86224501169511035297304774156755912 88]
two intervals r = 16.0801100774654259672276063318453649554 9 ..
```

```
474999999998119323633365415722812643 3/250000000000000000000000000000000
000 or r = 16.41579812683653490365088466531094940456 ..
474999999998119323633365415722812643 3/250000000000000000000000000000000
000
Time Approximations 0.053.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.17e-37, 0., -.60e-36]Solution in 1.692s

Time Plot 0 s.
Exiting SolveHard() after 5.641r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38, none, none,
358.97362823861982289495065051846187897 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888807481397033055821135801122462,
4.0048690818441512626349920660164147533 14,
404.86224501169511035297304774156755912 88]
one interval r = 21.641943994036047942839120772978801411 72 ..
26.76330660020634157388139419956773155441
Time Approximations 0.051.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., -.96e-35]Solution in 1.055s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.788r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95, none,
358.97362823861982289495065051846187897 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941770478058047406078070548918062,
5.5896371829182513215572035153978795057 76,
443.83065884384301027260119105489215159 93]
one interval r = 22.46725374466154563156888924759869766070 ..
27.27388428331360373881110651525421287684
Time Approximations 0.036.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.4e-38
Equations at solution: [.1e-37, -.54e-37, .14e-35]Solution in 0.97s

Time Plot 0 s.
Exiting SolveHard() after 4.738r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
```

```
328.46939893523660074899723975141521729 95, none,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941770478058047406078070548918062,
5.5896371829182513215572035153978795057 76,
443.83065884384301027260119105489215159 93]
one interval r = 32.15575279512833040734462817359966743340 ..
35.50872228745909141151170187684086833321
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., .110e-34]Solution in 0.462s

Time Plot 0 s.
Exiting SolveHard() after 0.835r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570 49419,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136501280977620395713267335686719,
5.1877835784592548787377389983251911566 29,
408.65773862466112169512363438923407984 73]
one interval r = 21.71840114640854676970305480526529358321 ..
26.81849303489324395235772214747966063233
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
```

```
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.38e-37
Equations at solution: [-.2e-37, -.238e-36, .166e-34]Solution in 0.999s

Time Plot 0 s.
Exiting SolveHard() after 4.774r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14, none, none,
361.50888346999853224689049585703229378 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136501280977620395713267335686719,
5.1877835784592548787377389983251911566 29,
408.65773862466112169512363438923407984 73]
one interval r = 31.80828598763099985801575146067545233527 ..
35.00011460053312666256900849715042558910
Time Approximations 0.02.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., -.385e-34]Solution in 0.412s

Time Plot 0 s.
```

```
Exiting SolveHard() after 0.738r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63, none,
361.50888346999853224689049585703229378 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110520943488556127620589214972547,
6.19626256524656901095026651236755850765 5,
385.44474379251001648671636846544037831 03]
one interval r = 31.60836097550022156738828260941088258825 ..
34.66372795620192207645010047451562438802
Time Approximations 0.016.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.351e-34]Solution in 0.554s

Time Plot 0 s.
Exiting SolveHard() after 3.709r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
```

```
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63, none,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110520943488556127620589214972547,
6.19626256524656901095026651236755850765 5,
385.44474379251001648671636846544037831 03]
two intervals r = 16.87563408738912392772181622088212683570 ..
47499999999811932363336541572281264 33/2500000000000000000000000000000000
000 or r = 15.55640493795427148764179630774960262849 ..
47499999999811932363336541572281264 33/2500000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 9.191 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175108472073708410606756729245956, rm
= 2.33653277417396992164846759703588696568 8}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.179e-37, 0., .1882e-34]Solution in 30.611s

Time Plot 0 s.
Exiting SolveHard() after 31.795r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
```

```
358.9736282386198228949506505184618789796,
398.3314710402756231687369206725964512614,
371.4838739426114568514696818639769625163,
336.6121584131953393839965034496651587161,
361.5088834699985322468904958570322937862,
324.6714499274705974553299258853360199605, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874720389657723551487442748104101,
4.883810779802723063051468379130690406742,
376.6196785575997663508691907310233348057]
one interval r = 21.11001304874171081139256591233876730128 ..
26.31784243456659943199049825181921704654
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, .224e-34]Solution in 3.86s

Time Plot 0 s.
Exiting SolveHard() after 4.602r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349322225378322277618071757049419,
441.6429597282346443545749196292572243165,
436.9174816496793979440504311157883887684,
422.9849339729348128949358824204834428979,
361.5258025597224192994671897230380309853,
401.8817390412695862883909398292591718888,
389.5900151557547199015689064996091496238,
328.4693989352366007489972397514152172995,
401.5075715792654747722848723914682281019,
358.9736282386198228949506505184618789796,
398.3314710402756231687369206725964512614,
371.4838739426114568514696818639769625163,
336.6121584131953393839965034496651587161,
361.5088834699985322468904958570322937862,
324.6714499274705974553299258853360199605, none,
328.4693851359602645925982249777573770711, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874720389657723551487442748104101,
4.883810779802723063051468379130690406742,
```

```
376.61967855759976635086919073102333348057]
one interval r = 31.53899497724519984985547496496407506414 ..
34.53618386103975225634751355787675009861
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.79e-36
Equations at solution: [.290e-35, -.379e-35, .251e-34]Solution in
0.508s

Time Plot 0 s.
Exiting SolveHard() after 0.785r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05, none,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017526960384850070573425471515557,
6.0258135492102179219594726425053585163 27,
351.42702948516399355837400914665017251 47]
one interval r = 31.36230206128365580649790540255647373486 ..
34.17446640627812976351951893877307284142
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .140e-34]Solution in 0.531s

Time Plot 0 s.
Exiting SolveHard() after 0.78r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05, none,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65, none, none,
292.99969138649667951150250073285985663 78, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175269603848500705734254715155 57,
6.0258135492102179219594726425053585163 27,
351.42702948516399355837400914665017251 47]
two intervals r = 17.981355144252110650726155608515944220 44 ..
4749999999981193236333654157228126433/2500000000000000000000000000000000
000 or r = 13.846080154217751878106236676704866672 28 ..
4749999999981193236333654157228126433/2500000000000000000000000000000000
000
Time Approximations 0.043.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 5.691 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713580403231993831118762613712 31, rm
= 2.7345009933015667395112793280710651836 79}});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [-.19e-37, 0., -.2363e-34]Solution in 17.088s

Time Plot 0 s.
Exiting SolveHard() after 20.793r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65, none, none,
292.99969138649667951150250073285985663 78, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418526470534992615312337771274 19,
6.3779438737944125968228168654088359795 56,
423.28832783930635010229592656337443351 64]
one interval r = 31.946618176101449040581447181392055663 90 ..
35.212123086619148112139407067431612597 40
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.39e-35]Solution in 3.222s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.577r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65, none, none,
292.99969138649667951150250073285985663 78, none, none,
360.06173466973517968391367831117097536 13, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418526470534992615312337771274 19,
6.3779438737944125968228168654088359795 56,
423.28832783930635010229592656337443351 64]
two intervals r = 15.22886702416510887365186641617026744794 ..
474999999998119323633365415722812643 3/2500000000000000000000000000000000
000 or r = 17.12965777068355134877788752686571703212 ..
474999999998119323633365415722812643 3/2500000000000000000000000000000000
000
Time Approximations 0.062.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 9.191 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541110251667691019967419928925 32, rm
= 2.0640682987623019967327309479626817637 39}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
```

```
Equations at solution: [.31e-37, .1e-37, -.2587e-34]Solution in 29.637s

Time Plot 0 s.
Exiting SolveHard() after 30.962r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65,
375.73285290290851148845989434393238022 80, none,
292.99969138649667951150250073285985663 78, none, none,
360.06173466973517968391367831117097536 13, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343520484347778787562346935184 30,
4.0035598155457975001488688619858557284 36,
404.47973593921680360882633446275082906 21]
two intervals r = 16.096839663554272437131506775699837549 22 ..
474999999998119323633365415722812643 3/2500000000000000000000000000000000
000 or r = 16.399886491054783390464166784874848950 78 ..
474999999998119323633365415722812643 3/2500000000000000000000000000000000
000
Time Approximations 2.856.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [-.34e-37, 0., -.4050e-34]Solution in 1.707s

Time Plot 0 s.
```

```
Exiting SolveHard() after 5.625r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65,
375.73285290290851148845989434393238022 80, none,
292.99969138649667951150250073285985663 78,
358.64341560751831143875829203376176450 04, none,
360.06173466973517968391367831117097536 13, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234352048434777878756234693518430,
4.0035598155457975001488688619858557284 36,
404.47973593921680360882633446275082906 21]
one interval r = 21.634296299818254119815353924293821823 63 ..
26.757681698711458778282918146116795123 25
Time Approximations 0.049.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., -.616e-34]Solution in 1.043s

Time Plot 0 s.
Exiting SolveHard() after 4.973r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65,
375.73285290290851148845989434393238022 80,
328.11709294489197526575053178752811619 82,
292.99969138649667951150250073285985663 78,
358.64341560751831143875829203376176450 04, none,
360.06173466973517968391367831117097536 13, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954438965928485765208035205196392,
6.19617723009956915305705484765959880382 2,
385.42734025615535593491501518889876897 21]
one interval r = 31.60822049105301627280333127030884762401 ..
34.66347615057700263785950336557900809759
Time Approximations 0.029.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, -.222e-34]Solution in 0.571s

Time Plot 0 s.
Exiting SolveHard() after 0.872r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65,
375.73285290290851148845989434393238022 80,
328.11709294489197526575053178752811619 82,
292.99969138649667951150250073285985663 78,
358.64341560751831143875829203376176450 04, none,
360.06173466973517968391367831117097536 13, none, none,
324.65521223715961178989629535632897430 52, none, none, none, none]

0 --> 1  target = [26.46318954438965928485765208035205196392,
6.19617723009956915305705484765959880382 2,
385.42734025615535593491501518889876897 21]
two intervals r = 16.87629600277955164387553662915042134394 ..
474999999998119323633365415722812643 3/2500000000000000000000000000000000
000 or r = 15.55559000643486331768329258771158771017 ..
474999999998119323633365415722812643 3/2500000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 6.702 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852520402507268937443815479024831, rm
= 2.33669042824030966842136655759992756798 6}});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [.895e-37, -.1e-37, .459e-35]Solution in 27.924s

Time Plot 0 s.
Exiting SolveHard() after 31.81r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65,
375.73285290290851148845989434393238022 80,
328.11709294489197526575053178752811619 82,
292.99969138649667951150250073285985663 78,
358.64341560751831143875829203376176450 04, none,
360.06173466973517968391367831117097536 13,
336.59441032352029549449972048902552240 52, none,
324.65521223715961178989629535632897430 52, none, none, none, none]

0 --> 2  target = [34.49522661174451737558574868665892782598,
3.8971313159626240488415008364043999524 53,
373.78081884307268166200551799328436061 42]
two intervals r = 17.297690862129544954708032014688874545 98 ..
474999999998119323633365415722812643 3/2500000000000000000000000000000000
000 or r = 14.994364074203251613679709658562707503 02 ..
474999999998119323633365415722812643 3/2500000000000000000000000000000000
000
Time Approximations 0.087.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., .1712e-34]Solution in 3.895s

Time Plot 0 s.
Exiting SolveHard() after 8.415r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65,
375.73285290290851148845989434393238022 80,
328.11709294489197526575053178752811619 82,
292.99969138649667951150250073285985663 78,
358.64341560751831143875829203376176450 04, none,
360.06173466973517968391367831117097536 13,
336.59441032352029549449972048902552240 52, none,
324.65521223715961178989629535632897430 52,
331.93806791326969924469443439643335567 06, none, none, none]

1 --> 2  target = [34.49522661174451737558574868665892782598,
3.8971313159626240488415008364043999524 53,
373.78081884307268166200551799328436061 42]
one interval r = 21.06068473198335680276376227320396834861 ..
26.26979834263322801975955990625717820 30
Time Approximations 0.034.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [.3e-37, .7e-37, .321e-34]Solution in 0.8s

Time Plot 0 s.
Exiting SolveHard() after 1.477r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65,
375.73285290290851148845989434393238022 80,
328.11709294489197526575053178752811619 82,
292.99969138649667951150250073285985663 78,
358.64341560751831143875829203376176450 04,
299.89866204871051883406840039517128381 05,
360.06173466973517968391367831117097536 13,
336.59441032352029549449972048902552240 52, none,
324.65521223715961178989629535632897430 52,
331.93806791326969924469443439643335567 06, none, none, none]

0 --> 2  target = [33.81362495422828379761819318085888126986,
3.7256489936059097046923919760871406715 69,
325.89209972961905155414016489076755051 88]
two intervals r = 18.552270489908812487356407494234696611 55 ..
47499999999811932363336541572281264 33/25000000000000000000000000000000000
000 or r = 12.491969357897541151230058212069838752 88 ..
47499999999811932363336541572281264 33/25000000000000000000000000000000000
000
Time Approximations 0.038.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=0
Equations at solution: [0., 0., -.1484e-34]Solution in 1.101s

Time Plot 0 s.
Exiting SolveHard() after 5.221r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65,
375.73285290290851148845989434393238022 80,
328.11709294489197526575053178752811619 82,
292.99969138649667951150250073285985663 78,
358.64341560751831143875829203376176450 04,
299.89866204871051883406840039517128381 05,
360.06173466973517968391367831117097536 13,
336.59441032352029549449972048902552240 52, none,
324.65521223715961178989629535632897430 52,
331.93806791326969924469443439643335567 06, none, none,
289.54595772653044828987394786632742278 68]

1 --> 2  target = [33.81362495422828379761819318085888126986,
3.7256489936059097046923919760871406715 69,
325.89209972961905155414016489076755051 88]
one interval r = 20.37468935108010069471202078064894339926 ..
25.37892165287377931983926709619152107663
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=3e-38
Equations at solution: [.2e-37, .3e-37, -.596e-34]Solution in 3.023s

Time Plot 0 s.
Exiting SolveHard() after 3.592r=24.3395 in [22.07732228 ..
25.37892164]
```

```
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65,
375.73285290290851148845989434393238022 80,
328.11709294489197526575053178752811619 82,
292.99969138649667951150250073285985663 78,
358.64341560751831143875829203376176450 04,
299.89866204871051883406840039517128381 05,
360.06173466973517968391367831117097536 13,
336.59441032352029549449972048902552240 52,
256.10753186405038886173621277314561328 17,
324.65521223715961178989629535632897430 52,
331.93806791326969924469443439643335567 06, none, none,
289.54595772653044828987394786632742278 68]

1 --> 0  target = [17.93041369691047468716392780282168572623,
4.6865087019529396614429501483860620128 92,
353.30541095143764137247604553287011353 78]
one interval r = 20.73150479091449792530837721751009253975 ..
25.90675353507973379530217787473824489676
Time Approximations 0.033.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., .135e-34]Solution in 0.705s

Time Plot 0 s.
```

```
Exiting SolveHard() after 1.477r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65,
375.73285290290851148845989434393238022 80,
328.11709294489197526575053178752811619 82,
292.99969138649667951150250073285985663 78,
358.64341560751831143875829203376176450 04,
299.89866204871051883406840039517128381 05,
360.06173466973517968391367831117097536 13,
336.59441032352029549449972048902552240 52,
256.10753186405038886173621277314561328 17,
324.65521223715961178989629535632897430 52,
331.93806791326969924469443439643335567 06,
304.79958325780700042393136018415125403 45, none,
289.54595772653044828987394786632742278 68]

2 --> 0  target = [17.93041369691047468716392780282168572623,
4.68650870195293966144295014838606201289 2,
353.30541095143764137247604553287011353 78]
one interval r = 31.37435487007616473721940876664968947510 ..
34.20127520040493994108667219570007432241
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
```

```
Equations at solution: [0., 0., -.150e-34]Solution in 0.355s

Time Plot 0 s.
Exiting SolveHard() after 0.623r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493222253783222776180717570494 19,
441.64295972823464435457491962925722431 65,
436.91748164967939794405043111578838876 84,
422.98493397293481289493588242048344289 79,
361.52580255972241929946718972303803098 53,
401.88173904126958628839093982925917188 88,
389.59001515575471990156890649960914962 38,
328.46939893523660074899723975141521729 95,
401.50757157926547477228487239146822810 19,
358.97362823861982289495065051846187897 96,
398.33147104027562316873692067259645126 14,
371.48387394261145685146968186397696251 63,
336.61215841319533938399650344966515871 61,
361.50888346999853224689049585703229378 62,
324.67144992747059745532992588533601996 05,
302.31384315136507068766902133082456116 63,
328.46938513596026459259822497775737707 11,
343.81340624946924011127136713045595635 65,
375.73285290290851148845989434393238022 80,
328.11709294489197526575053178752811619 82,
292.99969138649667951150250073285985663 78,
358.64341560751831143875829203376176450 04,
299.89866204871051883406840039517128381 05,
360.06173466973517968391367831117097536 13,
336.59441032352029549449972048902552240 52,
256.10753186405038886173621277314561328 17,
324.65521223715961178989629535632897430 52,
331.93806791326969924469443439643335567 06,
304.79958325780700042393136018415125403 45,
323.46169176848029005629892026232923687 44,
289.54595772653044828987394786632742278 68]

Cascade time 249.568
counts: 28, 28

 Iteration 56

Start Generation 1
1 --> 0  target = [11.99999999995275302137998558151385411200,
6.21701250296186439323289156838287500 1156,
485.54908089786984668946256992751831899 83]
one interval r = 23.40850301656553514291837565598225923738 ..
27.67578046433181703236057578817610263139
Time Approximations 0.042.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
```

```
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.7e-38
Equations at solution: [.1e-37, -.27e-37, -.17e-35]Solution in 3.751s

Time Plot 0 s.
Exiting SolveHard() after 4.934r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999995275302137998558151385411200,
6.217012502961864393232891568382875001156,
485.54908089786984668942569927518318 9983]
one interval r = 32.62814779220280117994482295770925721082 ..
36.10248389432066703871351193801978 38467
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, .41e-35]Solution in 0.629s

Time Plot 0 s.
Exiting SolveHard() after 1.058r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
```

```
441.642959731575242568013762720843369368 2,
436.91748165140533290942611753089910143 91, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844848159651042949785652936482 36,
6.583434721572226077600525195589903386174,
467.787305958854911350709399008666067259 2]
one interval r = 32.419789556705730185509705585427120681 72 ..
35.851524173737778293140515333912103170 65
Time Approximations 0.024.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., -.245e-34]Solution in 0.658s

Time Plot 0 s.
Exiting SolveHard() after 1.037r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936004566806601045024290505658 2,
441.642959731575242568013762720843369368 2,
436.91748165140533290942611753089910143 91, none, none,
401.881739042367018839276810255461789137 4, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844848159651042949785652936482 36,
6.583434721572226077600525195589903386174,
467.787305958854911350709399008666067259 2]
two intervals r = 12.923271608263072885448564591469534765 36 ..
38000000000119055729322086895575328 87/2000000000000000000000000000000000
000 or r = 18.394248580365198406922056141418999509 31 ..
38000000000119055729322086895575328 87/2000000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
```

```
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.1e-38
Equations at solution: [.2e-37, .21e-37, -.810e-35]Solution in 40.471s

Time Plot 0 s.
Exiting SolveHard() after 44.626r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910 14391,
422.98493397634270226507857975365929 32560, none,
401.88173904236701883927681025546178 91374, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962826198747558204267605009903142,
4.12565179675572544222103046447814300 9125,
440.67123064901746778830664186702677829 08]
two intervals r = 14.35659705124321943500024526878348840998 ..
38000000000119055729322086895575328 87/20000000000000000000000000000000000
000 or r = 17.70352613808689664977216828274399115986 ..
38000000000119055729322086895575328 87/20000000000000000000000000000000000
000
Time Approximations 0.046.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=4e-38
Equations at solution: [.86e-37, .4e-37, -.1858e-34]Solution in 1.328s

Time Plot 0 s.
Exiting SolveHard() after 4.905r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910 14391,
422.98493397634270226507857975365929 32560, none,
401.88173904236701883927681025546178 91374,
389.59001515574662283204482693377991 68211, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962826198747558204267605009903142,
4.12565179675572544222103046447814300 9125,
440.67123064901746778830664186702677829 08]
one interval r = 22.39761154365226810427278281845599631201 ..
27.23722351600878932000848042963185470147
Time Approximations 0.037.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.229 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064382174364822157642397722616233, rm =
14.37818770192724308899974491086283253208}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .41e-35]Solution in 7.553s

Time Plot 0 s.
Exiting SolveHard() after 11.085r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910 14391,
422.98493397634270226507857975365929 32560,
361.52580255888136852605200986282321 21850,
401.88173904236701883927681025546178 91374,
389.59001515574662283204482693377991 68211, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888801468626296174976616648401732,
4.00486908173705445236288917566826221 3195,
```

```
404.86224501301184081928288355493689306 25]
two intervals r = 16.08011007761894534671984215056217484718 ..
380000000001190557293220868955753288 7/2000000000000000000000000000000000
000 or r = 16.41579812695621889171385920451653971345 ..
380000000001190557293220868955753288 7/2000000000000000000000000000000000
000
Time Approximations 0.051.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.34e-37, 0., .308e-35]Solution in 1.498s

Time Plot 0 s.
Exiting SolveHard() after 5.41r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11, none, none,
358.97362823783728415603058668219542651 77, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888801468626296174976616648401732,
4.00486908173705445236288917566826221319 5,
404.86224501301184081928288355493689306 25]
one interval r = 21.64194399413087976656504114680067571917 ..
26.76330660045351146938803881830889989407
Time Approximations 0.051.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
```

```
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., .237e-34]Solution in 1.074s

Time Plot 0 s.
Exiting SolveHard() after 4.876r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15, none,
358.97362823783728415603058668219542651 77, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941775006242232519295069254386266,
5.58963718298753819727356088738995097653 2,
443.83065884716739510766263389675986759 88]
one interval r = 22.46725374482617252825913531587148402097 ..
27.27388428360422851264695655670321316261
Time Approximations 0.036.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .1e-36]Solution in 0.983s

Time Plot 0 s.
Exiting SolveHard() after 4.681r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
```

```
361.525802558881368526052009862823212185O,
401.881739042367018839276810255461789137 4,
389.590015155746622832044826933779916821 1,
328.469398933744520399517979798070008011 5, none,
358.973628237837284156030586682195426517 7,
398.331471043259178242449940831611136359 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941775006242232519295069254386266,
5.589637182987538197273560887389950976532,
443.83065884716739510766263389675986759 88]
one interval r = 32.15575279511020860888778573785898906139 ..
35.50872228742195424762600786968437763398
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.126e-34]Solution in 0.467s

Time Plot 0 s.
Exiting SolveHard() after 0.839r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050565 82,
441.642959731575242568013762720843369368 2,
436.917481651405332909426117530899101439 1,
422.984933976342702265078579753659293256 0,
361.525802558881368526052009862823212185O,
401.881739042367018839276810255461789137 4,
389.590015155746622832044826933779916821 1,
328.469398933744520399517979798070008011 5,
401.507571579942193514876642077846328244 0,
358.973628237837284156030586682195426517 7,
398.331471043259178242449940831611136359 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136523683553759320317807807749092,
5.187783578494254473900393756801955520880,
408.65773862440773275327197078690532220 08]
one interval r = 21.71840114647399170869054193152075945791 ..
26.81849303511984943544901439580276521776
```

```
Time Approximations 0.058.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.85e-37
Equations at solution: [-.2e-37, -.185e-36, .91e-35]Solution in 1.019s

Time Plot 0 s.
Exiting SolveHard() after 4.744r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910 14391,
422.98493397634270226507857975365929 32560,
361.52580255888136852605200986282321 21850,
401.88173904236701883927681025546178 91374,
389.59001515574662283204482693377991 68211,
328.46939893374452039951797979807000 80115,
401.50757157994219351487664207784632 82440,
358.97362823783728415603058668219542 65177,
398.33147104325917824244994083161113 63599, none, none,
361.50888346925933842282337511096521 52107, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136523683553759320317807807749092,
5.18778357849425447390039375680195552 0880,
408.65773862440773275327197078690532 22008]
one interval r = 31.80828598757842867400950248166843692870 ..
35.00011460044968048802859541675965558153
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
```

```
Equations at solution: [0., 0., .477e-34]Solution in 0.405s

Time Plot 0 s.
Exiting SolveHard() after 0.727r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336 93682,
436.91748165140533290942611753089910 14391,
422.98493397634270226507857975365929 32560,
361.52580255888136852605200986282321 21850,
401.88173904236701883927681025546178 91374,
389.59001515574662283204482693377991 68211,
328.46939893374452039951797979807000 80115,
401.50757157994219351487664207784632 82440,
358.97362823783728415603058668219542 65177,
398.33147104325917824244994083161113 63599,
371.48387393975185037929652156021717 15384, none,
361.50888346925933842282337511096521 52107, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110541220373767438821778606129186,
6.19626256529583696816157696638098387 3470,
385.44474379173186977122608795199483 37734]
one interval r = 31.60836097544543034995587662590202114932 ..
34.66372795611422783869966726779831940505
Time Approximations 0.016.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, -.212e-34]Solution in 0.561s

Time Plot 0 s.
Exiting SolveHard() after 3.753r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
```

```
441.642959731575242568013762720843369368 2,
436.917481651405332909426117530899101439 1,
422.984933976342702265078579753659293256 0,
361.525802558881368526052009862823212185 0,
401.881739042367018839276810255461789137 4,
389.590015155746622832044826933779916821 1,
328.469398933744520399517979798070008011 5,
401.507571579942193514876642077846328244 0,
358.973628237837284156030586682195426517 7,
398.331471043259178242449940831611136359 9,
371.483873939751850379296521560217171538 4, none,
361.508883469259338422823375110965215210 7,
324.671449924365548757911569094758747379 6, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.463471105412203737674388217786061291 86,
6.196262565295836968161576966380983873470,
385.444743791731869771226087951994833773 4]
two intervals r = 16.875634087620742994209425916114137946 68 ..
380000000001190557293220868955753288 7/2000000000000000000000000000000000
000 or r = 15.556404937972233371114726915501693267 06 ..
380000000001190557293220868955753288 7/2000000000000000000000000000000000
000
Time Approximations 0.06.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., .1915e-34]Solution in 3.764s

Time Plot 0 s.
Exiting SolveHard() after 4.938r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936004566806601045024290505658 2,
441.642959731575242568013762720843369368 2,
436.917481651405332909426117530899101439 1,
422.984933976342702265078579753659293256 0,
361.525802558881368526052009862823212185 0,
401.881739042367018839276810255461789137 4,
389.590015155746622832044826933779916821 1,
328.469398933744520399517979798070008011 5,
401.507571579942193514876642077846328244 0,
358.973628237837284156030586682195426517 7,
398.331471043259178242449940831611136359 9,
```

```
371.48387393975185037929652156021717153 84,
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874743767556413027060700881076273,
4.88381077983820086844786822007913646203 4,
376.61967855656225309441956020945792218 03]
one interval r = 21.11001304877743248050973651772415942780 ..
26.31784243475880303062064406411668494768
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=4.9e-38
Equations at solution: [-.2e-37, -.49e-37, .347e-34]Solution in 0.878s

Time Plot 0 s.
Exiting SolveHard() after 1.592r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
398.33147104325917824244994083161113635 99,
371.48387393975185037929652156021717153 84,
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96, none,
328.46938513447136013156457727807920596 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874743767556413027060700881076273,
4.88381077983820086844786822007913646203 4,
376.61967855656225309441956020945792218 03]
one interval r = 31.53899497718936240556191270883139475604 ..
```

```
34.53618386094958699229525304173004056196
Time Approximations 0.018.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.19e-36
Equations at solution: [.91e-36, -.119e-35, .23e-35]Solution in 4.023s

Time Plot 0 s.
Exiting SolveHard() after 4.308r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
398.33147104325917824244994083161113635 99,
371.48387393975185037929652156021717153 84,
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96, none,
328.46938513447136013156457727807920596 92,
343.81340624558028911820620513677264968 37, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017543617570023887278247841533933,
6.0258135492580911248266419436050886466 07,
351.42702948372290398919041366197208465 14]
one interval r = 31.36230206122811322569397733810801327079 ..
34.17446640618586592492422914456993585627
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
```

```
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .171e-34]Solution in 0.542s

Time Plot 0 s.
Exiting SolveHard() after 0.811r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336 93682,
436.91748165140533290942611753089910 14391,
422.98493397634270226507857975365929 32560,
361.52580255888136852605200986282321 21850,
401.88173904236701883927681025546178 91374,
389.59001515574662283204482693377991 68211,
328.46939893374452039951797979807000 80115,
401.50757157994219351487664207784632 82440,
358.97362823783728415603058668219542 65177,
398.33147104325917824244994083161113 63599,
371.48387393975185037929652156021717 15384,
336.61215841221333179247863775402154 67890,
361.50888346925933842282337511096521 52107,
324.67144992436554875791156909475874 73796, none,
328.46938513447136013156457727807920 59692,
343.81340624558028911820620513677264 96837, none, none,
292.99969138270050949661996367945626 21606, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017543617570023887278247841533933,
6.0258135492580911248266419436050886466 07,
351.42702948372290398919041366197208465 14]
two intervals r = 17.98135514447302117755404289038782886130 ..
38000000000119055729322086895575328 87/20000000000000000000000000000000000
000 or r = 13.84608015417745603197318719157797861463 ..
38000000000119055729322086895575328 87/20000000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
```

```
Equations at solution: [.36e-37, -.1e-37, -.13e-35]Solution in 3.739s

Time Plot 0 s.
Exiting SolveHard() after 4.825r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
398.33147104325917824244994083161113635 99,
371.48387393975185037929652156021717153 84,
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96,
302.31384314979533680169770675941312829 76,
328.46938513447136013156457727807920596 92,
343.81340624558028911820620513677264968 37, none, none,
292.99969138270050949661996367945626216 06, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418807528882580956164708230497 95,
6.3779438738597247483160297501263172367 25,
423.28832784245008140444391932257474599 34]
one interval r = 31.946618176080681828202727565967641500 34 ..
35.212123086582741712221053112216965990 54
Time Approximations 0.018.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.200e-34]Solution in 0.618s

Time Plot 0 s.
Exiting SolveHard() after 0.962r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910 14391,
422.98493397634270226507857975365929 32560,
361.52580255888136852605200986282321 21850,
401.88173904236701883927681025546178 91374,
389.59001515574662283204482693377991 68211,
328.46939893374452039951797979807000 80115,
401.50757157994219351487664207784632 82440,
358.97362823783728415603058668219542 65177,
398.33147104325917824244994083161113 63599,
371.48387393975185037929652156021717 15384,
336.61215841221333179247863775402154 67890,
361.50888346925933842282337511096521 52107,
324.67144992436554875791156909475874 73796,
302.31384314979533680169770675941312 82976,
328.46938513447136013156457727807920 59692,
343.81340624558028911820620513677264 96837, none, none,
292.99969138270050949661996367945626 21606, none, none,
360.06173467038687933028226657082061 23237, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941880752888258095616470823049795,
6.37794387385972474831602975012631723 6725,
423.28832784245008140444391932257474 59934]
two intervals r = 15.22886702422994561330356715689220363653 ..
38000000000119055729322086895575328 87/20000000000000000000000000000000000
000 or r = 17.12965777087136456943379148570281666231 ..
38000000000119055729322086895575328 87/20000000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [.63e-37, .2e-37, .3271e-34]Solution in 1.296s

Time Plot 0 s.
Exiting SolveHard() after 5.343r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
```

```
441.642959731575242568013762720843369368 2,
436.917481651405332909426117530899101439 1,
422.984933976342702265078579753659293256 0,
361.525802558881368526052009862823212185 0,
401.881739042367018839276810255461789137 4,
389.590015155746622832044826933779916821 1,
328.469398933744520399517979798070008011 5,
401.507571579942193514876642077846328244 0,
358.973628237837284156030586682195426517 7,
398.331471043259178242449940831611136359 9,
371.483873939751850379296521560217171538 4,
336.612158412213331792478637754021546789 0,
361.508883469259338422823375110965215210 7,
324.671449924365548757911569094758747379 6,
302.313843149795336801697706759413128297 6,
328.469385134471360131564577278079205969 2,
343.813406245580289118206205136772649683 7,
375.732852905940421246918867917056671121 8, none,
292.999691382700509496619963679456262160 6, none, none,
360.061734670386879330282266570820612323 7, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343454175303227749357707311664 52,
4.003559815437225475999431760106604899082,
404.479735940103428574360406852688347711 9]
two intervals r = 16.09683966372656360792044633586384716139 ..
380000000001190557293220868955753288 7/2000000000000000000000000000000000
000 or r = 16.39988649115649051104116343096351619299 ..
380000000001190557293220868955753288 7/2000000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.1261e-34]Solution in 1.496s

Time Plot 0 s.
Exiting SolveHard() after 5.325r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936004566806601045024290505658 2,
441.642959731575242568013762720843369368 2,
436.917481651405332909426117530899101439 1,
422.984933976342702265078579753659293256 0,
```

```
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
398.33147104325917824244994083161113635 99,
371.48387393975185037929652156021717153 84,
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96,
302.31384314979533680169770675941312829 76,
328.46938513447136013156457727807920596 92,
343.81340624558028911820620513677264968 37,
375.73285290594042124691886791705667112 18, none,
292.99969138270050949661996367945626216 06,
358.64341560636241895417367136542698918 59, none,
360.06173467038687933028226657082061232 37, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234345417530322774935770731166452,
4.0035598154372254759994317601066048990 82,
404.47973594010342857436040685268834771 19]
one interval r = 21.634296299904246020789319978283532410 49 ..
26.757681698952092475889397653643074775 94
Time Approximations 0.049.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., -.754e-34]Solution in 1.027s

Time Plot 0 s.
Exiting SolveHard() after 4.853r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
```

```
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
398.33147104325917824244994083161113635 99,
371.48387393975185037929652156021717153 84,
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96,
302.31384314979533680169770675941312829 76,
328.46938513447136013156457727807920596 92,
343.81340624558028911820620513677264968 37,
375.73285290594042124691886791705667112 18,
328.11709294300319915937274342827273118 54,
292.99969138270050949661996367945626216 06,
358.64341560636241895417367136542698918 59, none,
360.06173467038687933028226657082061232 37, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954459411110472561771068043574507,
6.19617723014935193905279680491296752195 2,
385.42734025548198662148722488161279718 18]
one interval r = 31.60822049099907266506688511220023573409 ..
34.66347615049082689729331222930749905667
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., .196e-34]Solution in 0.575s

Time Plot 0 s.
Exiting SolveHard() after 0.859r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
398.33147104325917824244994083161113635 99,
```

```
371.48387393975185037929652156021717153 84,
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96,
302.31384314979533680169770675941312829 76,
328.46938513447136013156457727807920596 92,
343.81340624558028911820620513677264968 37,
375.73285290594042124691886791705667112 18,
328.11709294300319915937274342827273118 54,
292.99969138270050949661996367945626216 06,
358.64341560636241895417367136542698918 59, none,
360.06173467038687933028226657082061232 37, none, none,
324.65521223415228382716890975700531142 17, none, none, none, none]

0 --> 1  target = [26.46318954459411110472561771068043574507,
6.19617723014935193905279680491296752195 2,
385.42734025548198662148722488161279718 18]
two intervals r = 16.87629600300717259008318731036656208016 ..
38000000000119055729322086895575328 87/20000000000000000000000000000000000
000 or r = 15.55559000645771887299261596755068470153 ..
38000000000119055729322086895575328 87/20000000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.359e-37, 0., .249e-35]Solution in 1.181s

Time Plot 0 s.
Exiting SolveHard() after 5.14r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
398.33147104325917824244994083161113635 99,
371.48387393975185037929652156021717153 84,
```

```
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96,
302.31384314979533680169770675941312829 76,
328.46938513447136013156457727807920596 92,
343.81340624558028911820620513677264968 37,
375.73285290594042124691886791705667112 18,
328.11709294300319915937274342827273118 54,
292.99969138270050949661996367945626216 06,
358.64341560636241895417367136542698918 59, none,
360.06173467038687933028226657082061232 37,
336.59441032264517993560916882462198261 06, none,
324.65521223415228382716890975700531142 17, none, none, none, none]

0 --> 2  target = [34.49522661163034437450392036751274509019,
3.89713131584109701599426101965867343522 2,
373.78081884034172255130791515451525460 97]
two intervals r = 17.29769086241886039142785973330173643902 ..
38000000000119055729322086895575328 87/20000000000000000000000000000000000
000 or r = 14.99436407411619430071123002237136412254 ..
38000000000119055729322086895575328 87/20000000000000000000000000000000000
000
Time Approximations 0.086.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.54e-37, 0., -.1874e-34]Solution in 1.242s

Time Plot 0 s.
Exiting SolveHard() after 5.597r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
398.33147104325917824244994083161113635 99,
371.48387393975185037929652156021717153 84,
```

```
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96,
302.31384314979533680169770675941312829 76,
328.46938513447136013156457727807920596 92,
343.81340624558028911820620513677264968 37,
375.73285290594042124691886791705667112 18,
328.11709294300319915937274342827273118 54,
292.99969138270050949661996367945626216 06,
358.64341560636241895417367136542698918 59, none,
360.06173467038687933028226657082061232 37,
336.59441032264517993560916882462198261 06, none,
324.65521223415228382716890975700531142 17,
331.93806790880953636722101556892164624 34, none, none, none]

1 --> 2  target = [34.49522661163034437450392036751274509019,
3.8971313158410970159942610196586734352 22,
373.78081884034172255130791515451525460 97]
one interval r = 21.06068473198885710691015186845335750234 ..
26.26979834279454740338080194329067255390
Time Approximations 0.031.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, -.179e-34]Solution in 0.725s

Time Plot 0 s.
Exiting SolveHard() after 4.088r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
398.33147104325917824244994083161113635 99,
371.48387393975185037929652156021717153 84,
336.61215841221333179247863775402154678 90,
```

```
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96,
302.31384314979533680169770675941312829 76,
328.46938513447136013156457727807920596 92,
343.81340624558028911820620513677264968 37,
375.73285290594042124691886791705667112 18,
328.11709294300319915937274342827273118 54,
292.99969138270050949661996367945626216 06,
358.64341560636241895417367136542698918 59,
299.89866204346173875091655543419994385 70,
360.06173467038687933028226657082061232 37,
336.59441032264517993560916882462198261 06, none,
324.65521223415228382716890975700531142 17,
331.93806790880953636722101556892164624 34, none, none, none]

0 --> 2  target = [33.81362495411824115213177267403506163211,
3.72564899348257482770653344697286915101 9,
325.89209972663326407089063277027868193 30]
two intervals r = 18.552270490128424923087899098081307310 52 ..
38000000000119055729322086895575328 87/2000000000000000000000000000000000
000 or r = 12.491969357763503369010146921861838871 01 ..
38000000000119055729322086895575328 87/2000000000000000000000000000000000
000
Time Approximations 0.039.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [.87e-37, -.3e-37, .55e-35]Solution in 1.206s

Time Plot 0 s.
Exiting SolveHard() after 5.317r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
398.33147104325917824244994083161113635 99,
```

```
371.48387393975185037929652156021717153 84,
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96,
302.31384314979533680169770675941312829 76,
328.46938513447136013156457727807920596 92,
343.81340624558028911820620513677264968 37,
375.73285290594042124691886791705667112 18,
328.11709294300319915937274342827273118 54,
292.99969138270050949661996367945626216 06,
358.64341560636241895417367136542698918 59,
299.89866204346173875091655543419994385 70,
360.06173467038687933028226657082061232 37,
336.59441032264517993560916882462198261 06, none,
324.65521223415228382716890975700531142 17,
331.93806790880953636722101556892164624 34, none, none,
289.54595772172033984718120165222468599 71]

1 --> 2  target = [33.81362495411824115213177267403506163211,
3.72564899348257482770653344697286915101 9,
325.89209972663326407089063277027868193 30]
one interval r = 20.37468935108076296504718514041095548872 ..
25.37892165299048860008154730843861616729
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., -.608e-34]Solution in 3.086s

Time Plot 0 s.
Exiting SolveHard() after 3.662r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
```

```
398.33147104325917824244994083161113635 99,
371.48387393975185037929652156021717153 84,
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96,
302.31384314979533680169770675941312829 76,
328.46938513447136013156457727807920596 92,
343.81340624558028911820620513677264968 37,
375.73285290594042124691886791705667112 18,
328.11709294300319915937274342827273118 54,
292.99969138270050949661996367945626216 06,
358.64341560636241895417367136542698918 59,
299.89866204346173875091655543419994385 70,
360.06173467038687933028226657082061232 37,
336.59441032264517993560916882462198261 06,
256.10753185853305795754181403475838035 67,
324.65521223415228382716890975700531142 17,
331.93806790880953636722101556892164624 34, none, none,
289.54595772172033984718120165222468599 71]

1 --> 0  target = [17.93041369712765413091321205496514240280,
4.68650870199154008581455338053219240868 4,
353.30541095020852542218058028581152785 94]
one interval r = 20.73150479093974515458917979404542037734 ..
25.90675353525160845941034413652265407992
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, .72e-35]Solution in 0.67s

Time Plot 0 s.
Exiting SolveHard() after 1.411r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
```

```
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
398.33147104325917824244994083161113635 99,
371.48387393975185037929652156021717153 84,
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96,
302.31384314979533680169770675941312829 76,
328.46938513447136013156457727807920596 92,
343.81340624558028911820620513677264968 37,
375.73285290594042124691886791705667112 18,
328.11709294300319915937274342827273118 54,
292.99969138270050949661996367945626216 06,
358.64341560636241895417367136542698918 59,
299.89866204346173875091655543419994385 70,
360.06173467038687933028226657082061232 37,
336.59441032264517993560916882462198261 06,
256.10753185853305795754181403475838035 67,
324.65521223415228382716890975700531142 17,
331.93806790880953636722101556892164624 34,
304.79958325617289018134418838347820322 67, none,
289.54595772172033984718120165222468599 71]

2 --> 0  target = [17.93041369712765413091321205496514240280,
4.68650870199154008581455338053219240868 4,
353.30541095020852542218058028581152785 94]
one interval r = 31.37435487002174634013892209979119314828 ..
34.20127520031542395345960686639648023315
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=1.1e-37
Equations at solution: [-.7e-37, .11e-36, -.178e-34]Solution in 0.35s

Time Plot 0 s.
Exiting SolveHard() after 0.625r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493600456680660104502429050 56582,
441.64295973157524256801376272084336936 82,
436.91748165140533290942611753089910143 91,
422.98493397634270226507857975365929325 60,
361.52580255888136852605200986282321218 50,
```

```
401.88173904236701883927681025546178913 74,
389.59001515574662283204482693377991682 11,
328.46939893374452039951797979807000801 15,
401.50757157994219351487664207784632824 40,
358.97362823783728415603058668219542651 77,
398.33147104325917824244994083161113635 99,
371.48387393975185037929652156021717153 84,
336.61215841221333179247863775402154678 90,
361.50888346925933842282337511096521521 07,
324.67144992436554875791156909475874737 96,
302.31384314979533680169770675941312829 76,
328.46938513447136013156457727807920596 92,
343.81340624558028911820620513677264968 37,
375.73285290594042124691886791705667112 18,
328.11709294300319915937274342827273118 54,
292.99969138270050949661996367945626216 06,
358.64341560636241895417367136542698918 59,
299.89866204346173875091655543419994385 70,
360.06173467038687933028226657082061232 37,
336.59441032264517993560916882462198261 06,
256.10753185853305795754181403475838035 67,
324.65521223415228382716890975700531142 17,
331.93806790880953636722101556892164624 34,
304.79958325617289018134418838347820322 67,
323.46169176423164765632748766002624538 86,
289.54595772172033984718120165222468599 71]

Cascade time 152.392
counts: 28, 28

 Iteration 57

Start Generation 1
1 --> 0  target = [12.00000000005127077736486088992468383700,
6.21701250296616907506859107544646392954 1,
485.54908089314374723805365555660035925 96]
one interval r = 23.40850301648603850364918171679055057931 ..
27.67578046438071939178449027425560323369
Time Approximations 0.045.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.8e-38
Equations at solution: [-.1e-37, .28e-37, .6e-36]Solution in 1.031s

Time Plot 0 s.
Exiting SolveHard() after 2.219r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000005127077736486088992468383700,
6.217012502966169075068591075446463929541,
485.54908089314374723805365555660035925 96]
```

"Imaginary part neglected: ", $3.18322343222853381992386065023548542934 8 \times 10^{-17}$

```
one interval r = 32.628147792052489025218690177858405582 10 ..
36.10248388936695566010245776742413846624
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [.6e-37, -.4e-37, -.109e-34]Solution in 0.594s

Time Plot 0 s.
Exiting SolveHard() after 3.937r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684488963466277817088050464514057,
6.583434721535175167082086602038459966221,
467.78730595446787634475663930427757777703]
```

"Imaginary part neglected: ", $3.18322343222853381992386065023548542934 8 \times 10^{-17}$

```
one interval r = 32.419789556555585841054157581990411082 33 ..
```

```
35.85152417367052640752754582207294028091
Time Approximations 0.024.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., .220e-34]Solution in 0.645s

Time Plot 0 s.
Exiting SolveHard() after 1.016r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95, none, none,
401.88173903802739648541971345578977941 90, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684488963466277817088050464514057,
6.58343472153517516708208660203845996622 1,
467.78730595446787634475663930427757777 03]
two intervals r = 12.92327160831060411392828649561268234203 ..
11874999999883059470214305050432565 27/6250000000000000000000000000000000
00 or r = 18.39424858006596438593377630071372940088 ..
11874999999883059470214305050432565 27/6250000000000000000000000000000000
00
Time Approximations 0.045.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.1e-38
Equations at solution: [.1e-37, .11e-37, -.58e-35]Solution in 40.984s

Time Plot 0 s.
Exiting SolveHard() after 45.188r=14.1926 in [12.92327158 ..
```

```
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11, none,
401.88173903802739648541971345578977941 90, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962817304951381460488997221356501,
4.12565179671473899859016426611864851573 0,
440.67123064385597676315323327860885320 10]
two intervals r = 14.35659705127722609333309643253799200403 ..
11874999999883059470214305050432565 27/62500000000000000000000000000000
00 or r = 17.70352613774713812471434106361626385830 ..
11874999999883059470214305050432565 27/62500000000000000000000000000000
00
Time Approximations 0.047.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.42e-37, -.2e-37, .76e-35]Solution in 1.326s

Time Plot 0 s.
Exiting SolveHard() after 4.974r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11, none,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962817304951381460488997221356501,
```

```
4.125651796714738998590164266118648515730,
440.671230643855976763153233278608853201 0]
one interval r = 22.39761154355414719665292085654755248607 ..
27.23722351602581510772067062237336783641
Time Approximations 0.039.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.389 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064378335983391890839804669718547, rm =
14.37818770090483607686596386554555527967}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.2e-38
Equations at solution: [-.1e-37, -.52e-37, .381e-34]Solution in 10.71s

Time Plot 0 s.
Exiting SolveHard() after 11.657r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888792358611001679564503105180750,
4.00486908169827550050176188923768093793 6,
404.86224500865444657962291235338482464 88]
two intervals r = 16.08011007754587675213608141058872150920 ..
11874999999883059470214305050432565 27/6250000000000000000000000000000000
00 or r = 16.41579812663021707468880023413437107740 ..
11874999999883059470214305050432565 27/6250000000000000000000000000000000
00
Time Approximations 0.052.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., .339e-34]Solution in 1.697s

Time Plot 0 s.
Exiting SolveHard() after 5.447r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85, none, none,
358.97362823269349663719882907007076575 57, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888792358611001679564503105180750,
4.00486908169827550050176188923768093 7936,
404.86224500865444657962291235338482464 88]
one interval r = 21.641943994049103355700031787976743286 29 ..
26.76330660045998911468712917089240568542
Time Approximations 0.051.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., -.374e-34]Solution in 1.063s

Time Plot 0 s.
Exiting SolveHard() after 4.902r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63, none,
358.97362823269349663719882907007076575 57, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941770516786135444024970109432031,
5.5896371830121574594118685966893545495 61,
443.83065884364409591119119419901744976 68]
one interval r = 22.46725374476456249770426924353185748806 ..
27.27388428364203692842748749858265002356
Time Approximations 0.038.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., -.40e-35]Solution in 0.975s

Time Plot 0 s.
Exiting SolveHard() after 4.742r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63, none,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941770516786135444024970109432031,
```

```
5.589637183012157459411868596689354549561,
443.830658843644095911191194199017449766 8]
```

"Imaginary part neglected: ", $3.183223432228533819923860650235485429348 \times 10^{-17}$

```
one interval r = 32.15575279496474803852342702968956965231 ..
35.50872228735782762138030676651373289653
Time Approximations 0.022.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.75e-35]Solution in 0.457s

Time Plot 0 s.
Exiting SolveHard() after 0.839r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136523118596068891686958933494017,
5.18778357849204497158304484570948524397 0,
408.65773861869242274460922763992225191 89]
one interval r = 21.71840114636452432490203015678086583578 ..
26.81849303510888642316708174487830561337
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
```

```
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.84e-37
Equations at solution: [.2e-37, .184e-36, .41e-35]Solution in 1.005s

Time Plot 0 s.
Exiting SolveHard() after 4.745r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58, none, none,
361.50888346465935455515489999718021309 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136523118596068891686958933494017,
5.18778357849204497158304484570948524397 0,
408.65773861869242274460922763992225191 89]
```

"Imaginary part neglected: ", $3.18322343222853381992386065023548542934 8 \times 10^{-17}$

```
one interval r = 31.80828598740700594459249480348919221888 ..
35.00011460034033394371052203995194938376
Time Approximations 0.02.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.266e-34]Solution in 0.409s

Time Plot 0 s.
Exiting SolveHard() after 0.738r=34.4952 in [32.91337941 ..
```

```
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50, none,
361.50888346465935455515489999718021309 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110539631660164260848818407679440,
6.19626256525571380729467119721793383 1922,
385.44474378664902354102680843170606550 75]

"Imaginary part neglected: ", 3.1832234322285338199238606502354854293 48 × 10^-17
one interval r = 31.60836097527876487207948954500892419849 ..
34.66372795600532392364852188061332066753
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, .132e-34]Solution in 0.567s

Time Plot 0 s.
Exiting SolveHard() after 3.803r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
```

```
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50, none,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110539631660164260848818407679440,
6.19626256525571380729467119721793383192 2,
385.44474378664902354102680843170606550 75]
two intervals r = 16.87563408754325087613394507193770004588 ..
11874999999883059470214305050432565 27/62500000000000000000000000000000000
00 or r = 15.55640493761529880965787917061298274625 ..
11874999999883059470214305050432565 27/62500000000000000000000000000000000
00
Time Approximations 0.06.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 9.359 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175110656308025235780206004250337, rm
= 2.33653277413974005576091610731804105360 9}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.358e-37, 0., -.91e-35]Solution in 31.315s

Time Plot 0 s.
Exiting SolveHard() after 32.496r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
```

```
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874734175624086897223466349238017,
4.8838107798404998969890777020262934949 54,
376.61967855155434908317034116611007009 61]
one interval r = 21.11001304868780958062071931714655589620 ..
26.31784243473895090841457219190497671289
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.5e-38
Equations at solution: [.1e-37, .25e-37, .260e-34]Solution in 3.454s

Time Plot 0 s.
Exiting SolveHard() after 4.181r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06, none,
328.46938513063214403766514244965064769 67, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none]
```

```
2 --> 0  target = [17.19898874734175624086897223466349238017,
4.883810779840499896989077702026293494954,
376.61967855155434908317034116611007009 61]
```

"Imaginary part neglected: ", $3.183223432228533819923860650235485429348 \times 10^{-17}$

```
one interval r = 31.53899497702318918026391326759946365532 ..
34.53618386083851789537174883805941166618
Time Approximations 0.018.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.90e-36
Equations at solution: [-.145e-35, .190e-35, -.162e-34]Solution in
0.513s

Time Plot 0 s.
Exiting SolveHard() after 0.802r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06, none,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017541510657396911934489063782795,
6.025813549221586527880219767046863985767,
351.42702947942457344623216568495491179 88]
```

"Imaginary part neglected: ", $3.183223432228533819923860650235485429348 \times 10^{-17}$

```
one interval r = 31.362302061067094686073386501371475666 98 ..
34.174466406075904272853261979510977771 32
Time Approximations 0.017.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.160e-34]Solution in 0.552s

Time Plot 0 s.
Exiting SolveHard() after 0.801r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06, none,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92, none, none,
292.99969137851817262148897302828765341 77, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175415106573969119344890637827 95,
6.025813549221586527880219767046863985767,
351.42702947942457344623216568495491179 88]
two intervals r = 17.981355144315887891840642377508676633 55 ..
11874999999883059470214305050432565 27/6250000000000000000000000000000000
00 or r = 13.846080153887370647029848920098437257 37 ..
11874999999883059470214305050432565 27/6250000000000000000000000000000000
00
Time Approximations 0.042.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
```

```
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 5.787 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.91357071351897106246651979471960383318, rm
= 2.7345009932384113402171298428067609553 14}});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [.54e-37, -.1e-37, -.130e-34]Solution in 17.139s

Time Plot 0 s.
Exiting SolveHard() after 20.893r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92, none, none,
292.99969137851817262148897302828765341 77, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941884467511959550767094091471913,
6.37794387383043706683331667527121306857 0,
423.28832783964021312301396278506494889 17]

"Imaginary part neglected: ", 3.18322343222853381992386065023548542934 8 × 10^-17
one interval r = 31.94661817593846371424876581161125216057 ..
35.21212308652108504959011731239093191295
Time Approximations 0.021.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
```

```
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .229e-34]Solution in 3.352s

Time Plot 0 s.
Exiting SolveHard() after 3.705r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92, none, none,
292.99969137851817262148897302828765341 77, none, none,
360.06173466755443706172155310604235050 31, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941884467511959550767094091471913,
6.37794387383043706683331667527121306857 0,
423.28832783964021312301396278506494889 17]
two intervals r = 15.22886702411461329643241508342615968392 ..
11874999999883059470214305050432565 27/62500000000000000000000000000000
00 or r = 17.12965777060414749496507612587620391030 ..
11874999999883059470214305050432565 27/62500000000000000000000000000000
00
Time Approximations 0.074.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 8.921 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541037913578182158407391723467 57, rm
= 2.06406829870937421268042628413443947984 8}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., .35e-35]Solution in 30.194s

Time Plot 0 s.
Exiting SolveHard() after 31.552r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92,
375.73285290392522202133637825357691160 39, none,
292.99969137851817262148897302828765341 77, none, none,
360.06173466755443706172155310604235050 31, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343363676128710179029518128958 24,
4.0035598153986170849562098745307862769 30,
404.47973593579749286793932625326301221 52]
two intervals r = 16.096839663650592329179001699120428139 27 ..
118749999998830594702143050504325652 7/625000000000000000000000000000000
00 or r = 16.399886490832620664598157670900625010 06 ..
118749999998830594702143050504325652 7/625000000000000000000000000000000
00
Time Approximations 0.048.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.33e-37, 0., .189e-34]Solution in 1.704s

Time Plot 0 s.
Exiting SolveHard() after 5.473r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92,
375.73285290392522202133637825357691160 39, none,
292.99969137851817262148897302828765341 77,
358.64341560126324523414149998660764683 07, none,
360.06173466755443706172155310604235050 31, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234336367612871017902951812895824,
4.0035598153986170849562098745307862769 30,
404.47973593579749286793932625326301221 52]
one interval r = 21.63429629982351741539588671843679027485 ..
26.75768169895911643657344471222927395007
Time Approximations 0.049.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.1e-38
Equations at solution: [-.1e-37, -.51e-37, -.259e-34]Solution in 1.049s

Time Plot 0 s.
Exiting SolveHard() after 5.135r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92,
375.73285290392522202133637825357691160 39,
328.11709293921143916491128246703496158 87,
292.99969137851817262148897302828765341 77,
358.64341560126324523414149998660764683 07, none,
360.06173466755443706172155310604235050 31, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954457826740437734999709860560789,
6.19617723010924494892776066941438295 3745,
385.42734025040246601542049265202715376 68]
```

"Imaginary part neglected: ", $3.18322343222853381992386065023548542934 8 \times 10^{-17}$

```
one interval r = 31.60822049083243370223467397260265543664 ..
34.66347615038196465404817924164551551673
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.234e-34]Solution in 0.569s

Time Plot 0 s.
Exiting SolveHard() after 0.866r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92,
375.73285290392522202133637825357691160 39,
328.11709293921143916491128246703496158 87,
292.99969137851817262148897302828765341 77,
358.64341560126324523414149998660764683 07, none,
360.06173466755443706172155310604235050 31, none, none,
324.65521222921992756002354654800134665 80, none, none, none, none]

0 --> 1  target = [26.46318954457826740437734999709860560789,
6.19617723010924494892776066941438295374 5,
385.42734025040246601542049265202715376 68]
two intervals r = 16.87629600292952245490980713594003202870 ..
11874999999883059470214305050432565 27/62500000000000000000000000000000000
00 or r = 15.55559000610094334379570833456060202369 ..
11874999999883059470214305050432565 27/62500000000000000000000000000000000
00
Time Approximations 0.057.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 6.72 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852522379359550055386395226896100, rm
= 2.33669042820509023879004680011093874209 2}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., -.40e-35]Solution in 31.158s

Time Plot 0 s.
Exiting SolveHard() after 35.275r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92,
375.73285290392522202133637825357691160 39,
328.11709293921143916491128246703496158 87,
292.99969137851817262148897302828765341 77,
358.64341560126324523414149998660764683 07, none,
360.06173466755443706172155310604235050 31,
336.59441031824249414716659457010987252 69, none,
324.65521222921992756002354654800134665 80, none, none, none, none]

0 --> 2  target = [34.49522661149945237325638827378536643140,
3.89713131579496044122949506367681280817 4,
373.78081883403100960844145700423639958 19]
two intervals r = 17.29769086236271416651564024376409377248 ..
11874999999883059470214305050432565 27/6250000000000000000000000000000000
00 or r = 14.99436407370266649833669943930130167431 ..
11874999999883059470214305050432565 27/6250000000000000000000000000000000
00
Time Approximations 0.089.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., -.450e-34]Solution in 3.838s

Time Plot 0 s.
Exiting SolveHard() after 5.518r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92,
375.73285290392522202133637825357691160 39,
328.11709293921143916491128246703496158 87,
292.99969137851817262148897302828765341 77,
358.64341560126324523414149998660764683 07, none,
360.06173466755443706172155310604235050 31,
336.59441031824249414716659457010987252 69, none,
324.65521222921992756002354654800134665 80,
331.93806790198151934196342392272220720 08, none, none, none]

1 --> 2  target = [34.49522661149945237325638827378536643140,
3.89713131579496044122949506367681280817 4,
373.78081883403100960844145700423639958 19]
one interval r = 21.06068473187758122425306675816638386570 ..
26.26979834275086393946921254971119841717
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [-.1e-37, -.3e-37, .71e-35]Solution in 0.805s

Time Plot 0 s.
Exiting SolveHard() after 1.529r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92,
375.73285290392522202133637825357691160 39,
328.11709293921143916491128246703496158 87,
292.99969137851817262148897302828765341 77,
358.64341560126324523414149998660764683 07,
299.89866203784825207810931295049386316 63,
360.06173466755443706172155310604235050 31,
336.59441031824249414716659457010987252 69, none,
324.65521222921992756002354654800134665 80,
331.93806790198151934196342392272220720 08, none, none, none]

0 --> 2  target = [33.81362495398997976461125821788772798213,
3.72564899344009116140293963074429234949 5,
325.89209972165939989433449870484470996 92]
two intervals r = 18.552270489950136268302001943359606075 11 ..
11874999999883059470214305050432565 27/6250000000000000000000000000000000
00 or r = 12.491969357491849494484850181488053291 8 ..
11874999999883059470214305050432565 27/6250000000000000000000000000000000
00
Time Approximations 0.037.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
```

```
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [-.140e-36, .6e-37, -.248e-34]Solution in 1.086s

Time Plot 0 s.
Exiting SolveHard() after 5.16r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92,
375.73285290392522202133637825357691160 39,
328.11709293921143916491128246703496158 87,
292.99969137851817262148897302828765341 77,
358.64341560126324523414149998660764683 07,
299.89866203784825207810931295049386316 63,
360.06173466755443706172155310604235050 31,
336.59441031824249414716659457010987252 69, none,
324.65521222921992756002354654800134665 80,
331.93806790198151934196342392272220720 08, none, none,
289.54595771613890146756335103598820007 66]

1 --> 2  target = [33.81362495398997976461125821788772798213,
3.72564899344009116140293963074429234949 5,
325.89209972165939989433449870484470996 92]
one interval r = 20.37468935101329021344580480080750796535 ..
25.37892165294318096978716107382003473872
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
```

```
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .587e-34]Solution in 0.552s

Time Plot 0 s.
Exiting SolveHard() after 3.705r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92,
375.73285290392522202133637825357691160 39,
328.11709293921143916491128246703496158 87,
292.99969137851817262148897302828765341 77,
358.64341560126324523414149998660764683 07,
299.89866203784825207810931295049386316 63,
360.06173466755443706172155310604235050 31,
336.59441031824249414716659457010987252 69,
256.10753185417722374399671650326498910 81,
324.65521222921992756002354654800134665 80,
331.93806790198151934196342392272220720 08, none, none,
289.54595771613890146756335103598820007 66]

1 --> 0  target = [17.93041369697169404525379766446253307186,
4.68650870199725804545296676527103557748 2,
353.30541094596134591994434921966172207 62]
one interval r = 20.73150479086943029881472353624583244151 ..
25.90675353523244298908577509118267938381
Time Approximations 0.034.
```

```
hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., -.299e-34]Solution in 0.669s

Time Plot 0 s.
Exiting SolveHard() after 1.388r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92,
375.73285290392522202133637825357691160 39,
328.11709293921143916491128246703496158 87,
292.99969137851817262148897302828765341 77,
358.64341560126324523414149998660764683 07,
299.89866203784825207810931295049386316 63,
360.06173466755443706172155310604235050 31,
336.59441031824249414716659457010987252 69,
256.10753185417722374399671650326498910 81,
324.65521222921992756002354654800134665 80,
331.93806790198151934196342392272220720 08,
304.79958325310013198781633082691466716 08, none,
289.54595771613890146756335103598820007 66]

2 --> 0  target = [17.93041369697169404525379766446253307186,
4.68650870199725804545296676527103557748 2,
353.30541094596134591994434921966172207 62]
```

"Imaginary part neglected: ", $3.183223432228533819923860650235485429348 \times 10^{-17}$

```
one interval r = 31.374354869861031098894384534504242874 70 ..
34.201275200206861556566772961562383178 72
Time Approximations 0.017.

hint used Hint := [33.796329522691901201472146203888695425 08, 2, 1, -1,
17.863534069716629640044658426895081929 62, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=5e-38
Equations at solution: [-.3e-37, .5e-37, -.26e-35]Solution in 3.191s

Time Plot 0 s.
Exiting SolveHard() after 3.458r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493114194675754571798780893997 42,
441.64295972764917271392261701123607388 10,
436.91748164627660084373509508194435711 95,
422.98493397259896077143376124595225369 11,
361.52580255427815092533049934515185319 13,
401.88173903802739648541971345578977941 90,
389.59001514992299876053548765126518896 85,
328.46939892990514954258849775740629132 63,
401.50757157565293354138540333081589317 85,
358.97362823269349663719882907007076575 57,
398.33147104086659937135793443812344627 58,
371.48387393350454189609051671846339048 50,
336.61215840780726590941740731767462725 45,
361.50888346465935455515489999718021309 08,
324.67144991943007709360687114262855732 06,
302.31384314614284644723697590859805436 40,
328.46938513063214403766514244965064769 67,
343.81340623990944815100206336814113526 92,
375.73285290392522202133637825357691160 39,
328.11709293921143916491128246703496158 87,
292.99969137851817262148897302828765341 77,
358.64341560126324523414149998660764683 07,
299.89866203784825207810931295049386316 63,
360.06173466755443706172155310604235050 31,
336.59441031824249414716659457010987252 69,
256.10753185417722374399671650326498910 81,
324.65521222921992756002354654800134665 80,
331.93806790198151934196342392272220720 08,
304.79958325310013198781633082691466716 08,
```

```
323.46169175923200714971890241726436749 89,
289.54595771613890146756335103598820007 66]

Cascade time 256.221
counts: 28, 28

 Iteration 58

Start Generation 1
1 --> 0  target = [12.00000000006078361914159511643825135600,
6.21701250298265233060860377181985201 2044,
485.54908089615432495048162167429636697 10]
```

"Imaginary part neglected: ", $1.8899423791448596009005596140357906587 38 \times 10^{-17}$

```
one interval r = 23.40850301646890906243757963500345739532 ..
27.67578046421895819838876128432779061981
Time Approximations 0.049.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=7.8e-38
Equations at solution: [.2e-37, -.78e-37, .6e-36]Solution in 3.55s

Time Plot 0 s.
Exiting SolveHard() after 4.742r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536 32073,
441.64295972994238481318332487219079171 54, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000006078361914159511643825135600,
6.21701250298265233060860377181985201 2044,
485.54908089615432495048162167429636697 10]
one interval r = 32.62814779208927775850433829781366576388 ..
36.10248389389924417720472609658913503 06
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.185e-34]Solution in 0.609s

Time Plot 0 s.
Exiting SolveHard() after 1.034r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684473151516864590895139474568081,
6.58343472171360796941515392326733718388 4,
467.78730595650271331345236597378569035 58]
one interval r = 32.41978955658073657115118172057252361391 ..
35.85152417368157835509248219067862841015
Time Approximations 0.023.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .210e-34]Solution in 0.645s

Time Plot 0 s.
Exiting SolveHard() after 1.022r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
```

436.91748164961882095020640908847672025 75, none, none,
401.88173903871583252986857572202353756 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1 target = [27.52359684473151516864590895139474568081,
6.58343472171360796941515392326733718388 4,
467.78730595650271331345236597378569035 58]
two intervals r = 12.92327160836302618394648325024696292590 ..
2374999999982557008647167462794974849/1250000000000000000000000000000000
000 or r = 18.39424858013769930659434228114175515314 ..
2374999999982557008647167462794974849/1250000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139 k=217.686
scos=281.304
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=4.4e-38
Equations at solution: [.4e-37, .44e-37, .14e-35]Solution in 41.611s

Time Plot 0 s.
Exiting SolveHard() after 45.786r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04, none,
401.88173903871583252986857572202353756 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2 target = [35.46322962820405881001713221797268828413,
4.12565179675749448066184149882048199596 6,
440.67123064709686879642375431312758768 24]
two intervals r = 14.35659705125845332957361350937943559118 ..
2374999999982557008647167462794974849/1250000000000000000000000000000000
000 or r = 17.70352613787000995234571842182100200397 ..
2374999999982557008647167462794974849/1250000000000000000000000000000000
000
Time Approximations 0.046.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,

```
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., .30e-35]Solution in 1.36s

Time Plot 0 s.
Exiting SolveHard() after 4.975r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04, none,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962820405881001713221797268828413,
4.1256517967574944806618414988204819959 66,
440.67123064709686879642375431312758768 24]
```

"Imaginary part neglected: ", $1.8899423791448596009005596140357906587 38 \times 10^{-17}$

```
one interval r = 22.39761154355295556219347958543779487269 ..
27.23722351589866288665152515658780740816
Time Approximations 0.041.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.244 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064375518099504144446605501801000, rm =
14.37818770366433308785278152368942256880}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., .102e-34]Solution in 10.479s
```

```
Time Plot 0 s.
Exiting SolveHard() after 14.085r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888791850288799836274912031340482,
4.0048690817319896880210690550382352343 65,
404.86224500918349769088819892619774080 75]
two intervals r = 16.08011007764734715948792227561518590774 ..
23749999999825570086471674627949748 49/12500000000000000000000000000000000
000 or r = 16.41579812667640786466110944953426357391 ..
23749999999825570086471674627949748 49/12500000000000000000000000000000000
000
Time Approximations 0.054.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.34e-37, 0., -.3e-36]Solution in 4.417s

Time Plot 0 s.
Exiting SolveHard() after 5.509r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
```

```
389.590015153652670808875502108984656042 6, none, none,
358.973628234127080061245398706972826315 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887918502887998362749120313404 82,
4.004869081731989688021069055038235234365,
404.862245009183497690888198926197740807 5]
```

"Imaginary part neglected: ", $1.8899423791448596009005596140357906587 38 \times 10^{-17}$

```
one interval r = 21.641943993996119332078331815921099079 14 ..
26.763306600318005607871415430408212765 15
Time Approximations 0.055.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, .160e-34]Solution in 3.964s

Time Plot 0 s.
Exiting SolveHard() after 5.018r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434934322166551741438530715363 2073,
441.642959729942384813183324872190791715 4,
436.917481649618820950206409088476720257 5,
422.984933972046308163179582167069285330 4,
361.525802555695716848830258545442966274 8,
401.881739038715832529868575722023537569 9,
389.590015153652670808875502108984656042 6,
328.469398928805742037296085825970749458 8, none,
358.973628234127080061245398706972826315 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589417904746975591411130099004027 88,
5.589637182976955720786892566474161281763,
443.830658842703635246725708550791765211 6]
```

"Imaginary part neglected: ", $1.8899423791448596009005596140357906587 38 \times 10^{-17}$

```
one interval r = 22.467253744670476149603934364136418409 84 ..
27.273884283464670264312235308496782116 09
```

```
Time Approximations 0.043.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., .32e-35]Solution in 1.021s

Time Plot 0 s.
Exiting SolveHard() after 1.995r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88, none,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941790474697559141113009900402788,
5.589637182976955720786892566474161281763,
443.83065884270363524672570855079176521 16]
one interval r = 32.15575279495823671691194758602435903493 ..
35.50872228732835358795372803272938959838
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.12e-35]Solution in 3.11s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.47r=34.9395 in [33.37332721 .. 35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136519800448307680487544751657561,
5.18778357851206634780924890337329134649 6,
408.65773862227385832739492017815191452 80]
```

"Imaginary part neglected: ", $1.8899423791448596009005596140357906587 38 \times 10^{-17}$

```
one interval r = 21.71840114637272297100397896149586971601 ..
26.81849303500905171612628189133069608787
Time Approximations 0.062.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.11e-37
Equations at solution: [.2e-37, .211e-36, .109e-34]Solution in 1.01s

Time Plot 0 s.
Exiting SolveHard() after 2.137r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
```

```
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99, none, none,
361.50888346614782456401072306566180637 72, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136519800448307680487544751657561,
5.18778357851206634780924890337329134649 6,
408.65773862227385832739492017815191452 80]
one interval r = 31.80828598744525864522436000378417771971 ..
35.00011460037919736051421392371953801477
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, .21e-35]Solution in 0.423s

Time Plot 0 s.
Exiting SolveHard() after 3.62r=34.4952 in [32.91337941 .. 35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04, none,
361.50888346614782456401072306566180637 72, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110527289431141321834833256307947,
```

```
6.196262565426795593902973993447998091442,
385.4447437877723623366943761704550303493]
one interval r = 31.608360975294557152155303235997520572 14 ..
34.663727956010419249540176389209010831 33
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., .55e-35]Solution in 0.557s

Time Plot 0 s.
Exiting SolveHard() after 0.832r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04, none,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110527289431141321834833256307947,
6.196262565426795593902973993447998091442,
385.4447437877723623366943761704550303493]
two intervals r = 16.875634087611240234441745665624186260 72 ..
23749999999825570086471674627949748 49/12500000000000000000000000000000
000 or r = 15.556404937693933926149376700377000150 98 ..
23749999999825570086471674627949748 49/12500000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 6.707 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175116813643658960185987955043418, rm
= 2.33653277401867495251333573685943624979 9}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.358e-37, 0., .204e-34]Solution in 28.056s

Time Plot 0 s.
Exiting SolveHard() after 32.049r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874740472407976281883436701209703,
4.88381077983772546153098534411705599399 1,
376.61967855273329940225025241837871929 29]
```

"Imaginary part neglected: ", $1.8899423791448596009005596140357906587 38 \times 10^{-17}$

```
one interval r = 21.11001304865059698755299652077886035084 ..
26.31784243462084147340216044794630683991
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
```

```
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, .142e-34]Solution in 0.811s

Time Plot 0 s.
Exiting SolveHard() after 4.304r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92, none,
328.46938512953439610290496257445085014 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747404724079762818834367012097 03,
4.8838107798377254615309853441170559939 91,
376.61967855273329940225025241837871929 29]
one interval r = 31.538994977039343416663366282403653331 06 ..
34.536183860845102965694077799698889555 00
Time Approximations 0.016.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.71e-36
Equations at solution: [-.284e-35, .371e-35, -.131e-34]Solution in
0.476s

Time Plot 0 s.
Exiting SolveHard() after 0.768r=34.0898 in [32.52213872 ..
34.53618387]
```

```
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92, none,
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017526010323964977290310657904279,
6.02581354937826848622640335763036970358 4,
351.42702947795256544620622044784723330 27]
one interval r = 31.36230206106591935801491981457584904776 ..
34.17446640604654154320270374293748152486
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .40e-35]Solution in 0.521s

Time Plot 0 s.
Exiting SolveHard() after 0.745r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
```

```
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92, none,
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64, none, none,
292.99969137574453273682639562856808854 71, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017526010323964977290310657904279,
6.02581354937826848622640335763036970358 4,
351.42702947795256544620622044784723330 27]
two intervals r = 17.98135514444291558113356551639556392626 ..
237499999998255700864716746279497484 9/125000000000000000000000000000000
000 or r = 13.84608015384315651772550500988990588060 ..
237499999998255700864716746279497484 9/125000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.019 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.91357071359087075071541120885880092505, rm
= 2.73450099307115170154614733616716011882 8}});
Accepted {r=18.6878, rm=15.3648} with Delta=4e-38
Equations at solution: [-.126e-36, .4e-37, -.394e-34]Solution in
20.105s

Time Plot 0 s.
Exiting SolveHard() after 23.858r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
```

```
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92,
302.31384314198791124372030605395880036 68,
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64, none, none,
292.99969137574453273682639562856808854 71, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941864303961433671048150480835675,
6.37794387398192113975954541612237200865 0,
423.28832783630773331666991996410129642 68]
one interval r = 31.94661817591061427065869209025082399892 ..
35.21212308645857689713674157229798859008
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.130e-34]Solution in 0.595s

Time Plot 0 s.
Exiting SolveHard() after 0.932r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
```

```
398.3314710379438895877290975719173740999,
371.4838739375504258575233091298603686704,
336.6121584061031603314138370408527726939,
361.5088834661478245640107230656618063772,
324.6714499191249625073868693452918743892,
302.3138431419879112437203060539588003668,
328.4693851295343961029049625744508501410,
343.8134062419337001881053532053050344764, none, none,
292.9996913757445327368263956285680885471, none, none,
360.0617346631346181741569235029962958255, none, none, none, none,
none, none, none]

0 --> 1  target = [27.0203794186430396143367104815048083567 5,
6.3779438739819211397595454161223720086 50,
423.2883278363077333166699199641012964268]
two intervals r = 15.2288670244136610295066114321770482927 7 ..
2374999999982557008647167462794974849/1250000000000000000000000000000000
000 or r = 17.1296577705087165265065482757037313425 5 ..
2374999999982557008647167462794974849/1250000000000000000000000000000000
000
Time Approximations 0.067.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 9.015 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541253611823862416257710840852 90, rm
= 2.0640682986636184170865438829651417229 24}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.14e-37, -.1e-37, .80e-35]Solution in 30.005s

Time Plot 0 s.
Exiting SolveHard() after 31.346r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349343221665517414385307153632073,
441.6429597299423848131833248721907917154,
436.9174816496188209502064090884767202575,
422.9849339720463081631795821670692853304,
361.5258025556957168488302585454429662748,
401.8817390387158325298685757220235375699,
389.5900151536526708088755021089846560426,
328.4693989288057420372960858259707494588,
401.5075715757647520975039802753277910505,
```

```
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92,
302.31384314198791124372030605395880036 68,
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64,
375.73285289746023878320067316411960093 22, none,
292.99969137574453273682639562856808854 71, none, none,
360.06173466313461817415692350299629582 55, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343350076697930251489724465081 30,
4.0035598154303131085446746382510493482 49,
404.47973593573706985169198355851704169 4]
two intervals r = 16.096839663777594538421235815603565057 59 ..
237499999998255700864716746279497484 9/125000000000000000000000000000000
000 or r = 16.399886490854335868017152605161674790 65 ..
237499999998255700864716746279497484 9/125000000000000000000000000000000
000
Time Approximations 0.052.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.18e-37, -.1e-37, .189e-34]Solution in 4.445s

Time Plot 0 s.
Exiting SolveHard() after 8.14r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
```

```
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92,
302.31384314198791124372030605395880036 68,
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64,
375.73285289746023878320067316411960093 22, none,
292.99969137574453273682639562856808854 71,
358.64341560218783986127138207977585110 97, none,
360.06173466313461817415692350299629582 55, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234335007669793025148972446508130,
4.0035598154303131085446746382510493482 49,
404.47973593573706985169199835585170416 94]
```

"Imaginary part neglected: ", $1.8899423791448596009005596140357906587 38 \times 10^{-17}$

```
one interval r = 21.63429629975882647212422558364802691200 ..
26.75768169880863390485386242461018214812
Time Approximations 0.054.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=4.9e-38
Equations at solution: [.2e-37, .49e-37, .218e-34]Solution in 1.078s

Time Plot 0 s.
Exiting SolveHard() after 2.099r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
336.61215840610316033141383704085277269 39,
```

```
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92,
302.31384314198791124372030605395880036 68,
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64,
375.73285289746023878320067316411960093 22,
328.11709293756904592811144160892014793 90,
292.99969137574453273682639562856808854 71,
358.64341560218783986127138207977585110 97, none,
360.06173466313461817415692350299629582 55, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954445603377447156158008775876743,
6.19617723028068375408692048310941613374 1,
385.42734025159873693426685944877865840 20]
one interval r = 31.60822049084881458359400069936357211581 ..
34.66347615038811657633778431410835677023
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, .84e-35]Solution in 3.15s

Time Plot 0 s.
Exiting SolveHard() after 3.429r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92,
302.31384314198791124372030605395880036 68,
```

```
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64,
375.73285289746023878320067316411960093 22,
328.11709293756904592811144160892014793 90,
292.99969137574453273682639562856808854 71,
358.64341560218783986127138207977585110 97, none,
360.06173466313461817415692350299629582 55, none, none,
324.65521222898283062809856790810334806 78, none, none, none, none]

0 --> 1  target = [26.46318954445603377447156158008775876743,
6.19617723028068375408692048310941613374 1,
385.42734025159873693426685944877865840 20]
two intervals r = 16.87629600299473190082769482551616701979 ..
237499999998255700864716746279497484 9/125000000000000000000000000000000
000 or r = 15.55559000618300095705511825862045813784 ..
237499999998255700864716746279497484 9/125000000000000000000000000000000
000
Time Approximations 0.062.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 9.54 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852528399244553138668064379620201, rm
= 2.33669042808333328280901691008168769837 3}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.358e-37, 0., .241e-34]Solution in 31.438s

Time Plot 0 s.
Exiting SolveHard() after 32.598r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
```

```
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92,
302.31384314198791124372030605395880036 68,
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64,
375.73285289746023878320067316411960093 22,
328.11709293756904592811144160892014793 90,
292.99969137574453273682639562856808854 71,
358.64341560218783986127138207977585110 97, none,
360.06173466313461817415692350299629582 55,
336.59441031661282705456492586535121270 57, none,
324.65521222898283062809856790810334806 78, none, none, none, none]

0 --> 2  target = [34.49522661154639660773732885768870617 60,
3.89713131584072249688679715330058686177 1,
373.78018883799367939884596213189905949 52]
two intervals r = 17.29769086232929713710120511796260348330 ..
23749999999825570086471674627949748 49/125000000000000000000000000000000
000 or r = 14.99436407392668340726065906487134749826 ..
23749999999825570086471674627949748 49/125000000000000000000000000000000
000
Time Approximations 0.084.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.36e-37, 0., -.306e-34]Solution in 1.169s

Time Plot 0.001 s.
Exiting SolveHard() after 5.499r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
```

```
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92,
302.31384314198791124372030605395880036 68,
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64,
375.73285289746023878320067316411960093 22,
328.11709293756904592811144160892014793 90,
292.99969137574453273682639562856808854 71,
358.64341560218783986127138207977585110 97, none,
360.06173466313461817415692350299629582 55,
336.59441031661282705456492586535121270 57, none,
324.65521222898283062809856790810334806 78,
331.93806790641493661978816233562574417 03, none, none, none]

1 --> 2  target = [34.49522661154639666077373288576887061760,
3.8971313158407224968867971533005868617 71,
373.78081883799367939884596213189905949 52]
```

"Imaginary part neglected: ", $1.8899423791448596009005596140357906587 38 \times 10^{-17}$

```
one interval r = 21.06068473188847926005860228219308480296 ..
26.26979834268155357513531049397849292341
Time Approximations 0.037.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., .85e-35]Solution in 3.516s

Time Plot 0 s.
Exiting SolveHard() after 4.258r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
```

```
398.3314710379438895877290975719173740999,
371.4838739375504258575233091298603686704,
336.6121584061031603314138370408527726939,
361.5088834661478245640107230656618063772,
324.6714499191249625073868693452918743892,
302.3138431419879112437203060539588003668,
328.4693851295343961029049625744508501410,
343.8134062419337001881053532053050344764,
375.7328528974602387832006731641196009322,
328.1170929375690459281114416089201479390,
292.9996913757445327368263956285680885471,
358.6434156021878398612713820797758511097,
299.8986620398938742840551109631191149943,
360.0617346631346181741569235029962958255,
336.5944103166128270545649258653512127057, none,
324.6552122289828306280985679081033480678,
331.9380679064149366197881623356257441703, none, none, none]

0 --> 2  target = [33.8136249539768730001924306566544341776 2,
3.7256489934701058249207854698456447706 23,
325.8920997211719010758479590923236856576]
two intervals r = 18.552270490030944960578730425645240407 34 ..
2374999999982557008647167462794974849/1250000000000000000000000000000000
000 or r = 12.491969357508787186640385520724231498 17 ..
2374999999982557008647167462794974849/1250000000000000000000000000000000
000
Time Approximations 0.052.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=7e-38
Equations at solution: [-.155e-36, .7e-37, .122e-34]Solution in 3.76s

Time Plot 0 s.
Exiting SolveHard() after 5.19r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349343221665517414385307153632073,
441.6429597299423848131833248721907917154,
436.9174816496188209502064090884767202575,
422.9849339720463081631795821670692853304,
361.5258025556957168488302585454429662748,
401.8817390387158325298685757220235375699,
389.5900151536526708088755021089846560426,
328.4693989288057420372960858259707494588,
```

```
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92,
302.31384314198791124372030605395880036 68,
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64,
375.73285289746023878320067316411960093 22,
328.11709293756904592811144160892014793 90,
292.99969137574453273682639562856808854 71,
358.64341560218783986127138207977585110 97,
299.89866203989387428405511096311911499 43,
360.06173466313461817415692350299629582 55,
336.59441031661282705456492586535121270 57, none,
324.65521222898283062809856790810334806 78,
331.93806790641493661978816233562574417 03, none, none,
289.54595771662069276547283036123647382 97]

1 --> 2  target = [33.81362495397687300019243065665443417762,
3.72564899347010582492078546984564477062 3,
325.89209972117190107584795909232368565 76]
```

"Imaginary part neglected: ", $1.8899423791448596009005596140357906587 38 \times 10^{-17}$

```
one interval r = 20.37468935095680919754322029995879699090 ..
25.37892165281570933850480626234026612179
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, .573e-34]Solution in 0.577s

Time Plot 0 s.
Exiting SolveHard() after 1.127r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
```

```
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92,
302.31384314198791124372030605395880036 68,
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64,
375.73285289746023878320067316411960093 22,
328.11709293756904592811144160892014793 90,
292.99969137574453273682639562856808854 71,
358.64341560218783986127138207977585110 97,
299.89866203989387428405511096311911499 43,
360.06173466313461817415692350299629582 55,
336.59441031661282705456492586535121270 57,
256.10753185214102033015599004732393459 61,
324.65521222898283062809856790810334806 78,
331.93806790641493661978816233562574417 03, none, none,
289.54595771662069276547283036123647382 97]

1 --> 0  target = [17.93041369711770700922907241153945985775,
4.68650870196882622198419626663691348640 6,
353.30541094387617948300258196797265750 05]

"Imaginary part neglected: ", 1.8899423791448596009005596140357906587 38 × 10^-17
one interval r = 20.73150479078428780630688855925972772514 ..
25.90675353506557502199926056307025062840
Time Approximations 0.037.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.9e-38
Equations at solution: [-.2e-37, -.49e-37, -.279e-34]Solution in 3.342s

Time Plot 0 s.
Exiting SolveHard() after 4.06r=25.4021 in [22.67806074 .. 25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92,
302.31384314198791124372030605395880036 68,
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64,
375.73285289746023878320067316411960093 22,
328.11709293756904592811144160892014793 90,
292.99969137574453273682639562856808854 71,
358.64341560218783986127138207977585110 97,
299.89866203989387428405511096311911499 43,
360.06173466313461817415692350299629582 55,
336.59441031661282705456492586535121270 57,
256.10753185214102033015599004732393459 61,
324.65521222898283062809856790810334806 78,
331.93806790641493661978816233562574417 03,
304.79958324870196055386414781143149990 74, none,
289.54595771662069276547283036123647382 97]

2 --> 0  target = [17.93041369711770700922907241153945985775,
4.68650870196882622198419626663691348640 6,
353.30541094387617948300258196797265750 05]
one interval r = 31.37435486985567491505774784276529023369 ..
34.20127520016856492590863296925708042108
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [-.5e-37, .8e-37, .141e-34]Solution in 0.376s

Time Plot 0 s.
Exiting SolveHard() after 0.64r=33.7963 in [32.25770943 .. 34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493432216655174143853071536320 73,
441.64295972994238481318332487219079171 54,
436.91748164961882095020640908847672025 75,
422.98493397204630816317958216706928533 04,
361.52580255569571684883025854544296627 48,
401.88173903871583252986857572202353756 99,
389.59001515365267080887550210898465604 26,
328.46939892880574203729608582597074945 88,
401.50757157576475209750398027532779105 05,
358.97362823412708006124539870697282631 50,
398.33147103794388958772909757191737409 99,
371.48387393755042585752330912986036867 04,
336.61215840610316033141383704085277269 39,
361.50888346614782456401072306566180637 72,
324.67144991912496250738686934529187438 92,
302.31384314198791124372030605395880036 68,
328.46938512953439610290496257445085014 10,
343.81340624193370018810535320530503447 64,
375.73285289746023878320067316411960093 22,
328.11709293756904592811144160892014793 90,
292.99969137574453273682639562856808854 71,
358.64341560218783986127138207977585110 97,
299.89866203989387428405511096311911499 43,
360.06173466313461817415692350299629582 55,
336.59441031661282705456492586535121270 57,
256.10753185214102033015599004732393459 61,
324.65521222898283062809856790810334806 78,
331.93806790641493661978816233562574417 03,
304.79958324870196055386414781143149990 74,
323.46169175841117929412292045709241633 98,
289.54595771662069276547283036123647382 97]

Cascade time 255.327
counts: 28, 28

 Iteration 59

Start Generation 1
1 --> 0  target = [12.00000000002304172467958335968367910700,
6.21701250301622693164471068975445138305 3,
485.54908090189473521140891888296398998 61]
one interval r = 23.40850301661074568637372100469234367475 ..
27.67578046427616971921919294459010159937
Time Approximations 0.047.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
```

```
Accepted {r=27.5236, rm=6.49211} with Delta=1.32e-37
Equations at solution: [.4e-37, -.132e-36, -.15e-35]Solution in 3.702s

Time Plot 0 s.
Exiting SolveHard() after 4.869r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000002304172467958335968367910700,
6.21701250301622693164471068975445138305 3,
485.54908090189473521140891888296398998 61]
one interval r = 32.628147792187467139041600430008822303 66 ..
36.10248388948778098557615993122710481248
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., -.16987e-34]Solution in 0.611s

Time Plot 0 s.
Exiting SolveHard() after 1.035r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684480480805403135176546582626644,
6.58343472161065613722887531615467014616 0,
467.78730596309915868032707029317222467 81]
```

```
one interval r = 32.41978955668635606723339779389212507980 ..
35.85152417379329218277169073371035290357
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=5e-38
Equations at solution: [.6e-37, -.5e-37, .544e-36]Solution in 0.658s

Time Plot 0 s.
Exiting SolveHard() after 1.033r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71, none, none,
401.88173904649178903352408830441699061 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684480480805403135176546582626644,
6.58343472161065613722887531615467014616 0,
467.78730596309915868032707029317222467 81]
two intervals r = 12.92327160829442313754533222952960920541 ..
19000000000057573273541488652226934719/10000000000000000000000000000000
00000 or r = 18.39424858042065239431024742320618955875 ..
19000000000057573273541488652226934719/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.1e-38
Equations at solution: [-.1e-37, -.11e-37, .573e-35]Solution in 44.226s

Time Plot 0 s.
```

```
Exiting SolveHard() after 48.529r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26, none,
401.88173904649178903352408830441699061 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962830252829865353980012902431364,
4.1256517967759112323686262730984762216 55,
440.67123065261013195110332009200461651 42]
two intervals r = 14.35659705127480283659549730468258742467 ..
190000000000575732735414886522269347 19/100000000000000000000000000000000
00000 or r = 17.70352613815814076554290193600043798026 ..
190000000000575732735414886522269347 19/100000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=5e-38
Equations at solution: [-.84e-37, -.5e-37, -.871e-35]Solution in 1.358s

Time Plot 0 s.
Exiting SolveHard() after 2.416r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26, none,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
1 --> 2  target = [35.46322962830252829865353980012902431364,
4.12565179677591123236862627309847622165 5,
440.67123065261013195110332009200461651 42]
one interval r = 22.39761154368849972551285963223513277499 ..
27.23722351597342667213346882193065285397
Time Approximations 0.037.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.243 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064382788147280310824758585473395, rm =
14.37818770261482693707300124399545234491}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.05e-37
Equations at solution: [.2e-37, .105e-36, .47e-35]Solution in 10.111s

Time Plot 0 s.
Exiting SolveHard() after 13.781r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888805350817233158425712969882252,
4.00486908175915921269466866549866659962 2,
404.86224501707200907653912811507064353 31]
two intervals r = 16.08011007759858262090828851099047128062 ..
19000000000057573273541488652226934719/10000000000000000000000000000000
00000 or r = 16.41579812709392177776773429207149955435 ..
19000000000057573273541488652226934719/10000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
```

```
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.17e-37, -.1e-37, -.98e-36]Solution in 4.424s

Time Plot 0 s.
Exiting SolveHard() after 5.499r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32, none, none,
358.97362824106433223048951241295954739 03, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888805350817233158425712969882252,
4.00486908175915921269466866549866659962 2,
404.86224501707200907653912811507064353 31]
one interval r = 21.641943994171458534495897742753213973 37 ..
26.763306600442520089112992502650530654 59
Time Approximations 0.054.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=1.24e-37
Equations at solution: [.3e-37, .124e-36, .671e-34]Solution in 3.882s

Time Plot 0 s.
Exiting SolveHard() after 4.89r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51, none,
358.97362824106433223048951241295954739 03, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941773000081171891775958993997066,
5.5896371830512665524847353985980728031 81,
443.83065885181919931962710697632360705 80]
one interval r = 22.467253744885998718328971997706327850 04 ..
27.273884283579403733452782704795959609 35
Time Approximations 0.038.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.4e-38
Equations at solution: [0., -.54e-37, .62e-35]Solution in 1.007s

Time Plot 0 s.
Exiting SolveHard() after 1.971r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51, none,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941773000081171891775958993997066,
```

```
5.589637183051266552484735398598072803181,
443.830658851819199319627106976323607058 0]
one interval r = 32.155752795086017100906358653505329943 48 ..
35.508722287478434296520600485588854009 61
Time Approximations 0.02.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [-.8e-37, .7e-37, -.9351e-35]Solution in 3.104s

Time Plot 0 s.
Exiting SolveHard() after 3.462r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434939660010103841165589706136411 1,
441.642959736028917046329493101282772226 1,
436.917481655074030904700094865848942147 1,
422.984933980583736529889413534967132632 6,
361.525802562448914042266110987650332957 0,
401.881739046491789033524088304416990616 4,
389.590015158565965931070818945199556523 2,
328.469398937739255927891784403643007535 1,
401.507571584046944743191903198554077398 6,
358.973628241064332230489512412959547390 3,
398.331471048414650090803251889824498426 5, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365259291601467087245423277208 69,
5.187783578539243472385022371212152260734,
408.657738627559388170274165543613659983 6]
one interval r = 21.718401146497060472812986261620387873 25 ..
26.818493035093857765152717817959562665 71
Time Approximations 0.055.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.11e-37
Equations at solution: [.2e-37, .211e-36, -.142e-34]Solution in 1.039s

Time Plot 0 s.
Exiting SolveHard() after 2.082r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65, none, none,
361.50888347283680814866376780176142986 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136525929160146708724542327720869,
5.1877835785392434723850223712121522607 34,
408.65773862755938817027416554361365998 36]
one interval r = 31.80828598752644157317439727496117185260 ..
35.00011460047628010557887080298351352852
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .1705e-35]Solution in 0.421s

Time Plot 0 s.
Exiting SolveHard() after 4.281r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12, none,
361.50888347283680814866376780176142986 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110539899613754745480786553546063,
6.19626256532972023995543839004718027236 9,
385.44474379506480340863326407488560386 39]
one interval r = 31.60836097538723670401528166430587639124 ..
34.66372795613746403945289163648595035382
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .13778e-34]Solution in 0.561s

Time Plot 0 s.
Exiting SolveHard() after 0.831r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12, none,
```

```
361.5088834728368081486637678017614298629,
324.6714499276120146921621199019424403309, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110539899613754745480786553546063,
6.196262565329720239955438390047180272369,
385.44474379506480340863326407488560386 39]
two intervals r = 16.875634087620165269342742969014495867 27 ..
19000000000057573273541488652226934719/10000000000000000000000000000000
00000 or r = 15.556404938107183042845307431553316581 79 ..
19000000000057573273541488652226934719/10000000000000000000000000000000
00000
Time Approximations 0.08.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [.538e-37, -.1e-37, -.1383e-34]Solution in 1.13s

Time Plot 0 s.
Exiting SolveHard() after 5.137r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.5088834728368081486637678017614298629,
324.6714499276120146921621199019424403309, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874742438233325701190071084631660,
4.883810779886120741925578953299496864719,
376.61967856013833913035353122329358701 14]
one interval r = 21.11001304880057005135095349663644273757 ..
```

```
26.31784243475337141244317200537472511569
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=1.01e-37
Equations at solution: [.2e-37, .101e-36, -.39e-35]Solution in 3.472s

Time Plot 0 s.
Exiting SolveHard() after 4.2r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.50888347283680814866376780176142986 29,
324.67144992761201469216211990194244033 09, none,
328.46938513846639502762326470633491808 72, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874742438233325701190071084631660,
4.88381077988612074192557895329949686 4719,
376.61967856013833913035353122329358701 14]
one interval r = 31.53899497713009743086022032501428996998 ..
34.53618386097394852677323095506203260705
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.027e-35
Equations at solution: [.790e-35, -.1027e-34, -.18437e-34]Solution in
0.501s

Time Plot 0 s.
Exiting SolveHard() after 0.788r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.50888347283680814866376780176142986 29,
324.67144992761201469216211990194244033 09, none,
328.46938513846639502762326470633491808 72,
343.81340624872191109278589633602346618 55, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017544471403953854494230899390516,
6.0258135492943636717500021544730707837 26,
351.42702948749637461549906915462353017 18]
one interval r = 31.362302061161666616129413086124026596 29 ..
34.174466406205888161410488564226061152 60
Time Approximations 0.016.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=1.2e-37
Equations at solution: [-.6e-37, .12e-36, -.7945e-35]Solution in 0.526s

Time Plot 0 s.
Exiting SolveHard() after 0.764r=33.3686 in [32.23723258 ..
34.17446642]
```

```
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.50888347283680814866376780176142986 29,
324.67144992761201469216211990194244033 09, none,
328.46938513846639502762326470633491808 72,
343.81340624872191109278589633602346618 55, none, none,
292.99969138634717233083966632998918500 02, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175444714039538544942308993905 16,
6.0258135492943636717500021544730707837 26,
351.42702948749637461549906915462353017 18]
two intervals r = 17.981355144448894244952840548225887930 47 ..
19000000000057573273541488652226934719/10000000000000000000000000000000000
00000 or r = 13.846080154386343548547573861625436163 24 ..
19000000000057573273541488652226934719/10000000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [-.88e-37, .3e-37, .1533e-34]Solution in 1.127s

Time Plot 0 s.
Exiting SolveHard() after 4.914r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.50888347283680814866376780176142986 29,
324.67144992761201469216211990194244033 09,
302.31384315340643586714485687927251444 54,
328.46938513846639502762326470633491808 72,
343.81340624872191109278589633602346618 55, none, none,
292.99969138634717233083966632998918500 02, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941879895473872048705128057519079,
6.37794387390135154377056276474333000549 6,
423.28832784741510804671941630757911045 48]
one interval r = 31.94661817605133036104861790726024822252 ..
35.21212308663915823207577732808527191911
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., .10081e-34]Solution in 0.576s

Time Plot 0 s.
Exiting SolveHard() after 0.929r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
```

```
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.50888347283680814866376780176142986 29,
324.67144992761201469216211990194244033 09,
302.31384315340643586714485687927251444 54,
328.46938513846639502762326470633491808 72,
343.81340624872191109278589633602346618 55, none, none,
292.99969138634717233083966632998918500 02, none, none,
360.06173467517238877720507559950041700 69, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941879895473872048705128057519079,
6.37794387390135154377056276474333000549 6,
423.28832784741510804671941630757911045 48]
two intervals r = 15.22886702417775014298571117896519653101 ..
19000000000057573273541488652226934719/10000000000000000000000000000000
00000 or r = 17.12965777101352719838217565054613570536 ..
19000000000057573273541488652226934719/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.16e-37, -.1e-37, .3832e-34]Solution in 1.32s

Time Plot 0 s.
Exiting SolveHard() after 5.248r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
```

```
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.50888347283680814866376780176142986 29,
324.67144992761201469216211990194244033 09,
302.31384315340643586714485687927251444 54,
328.46938513846639502762326470633491808 72,
343.81340624872191109278589633602346618 55,
375.73285291090335899840336697540895326 23, none,
292.99969138634717233083966632998918500 02, none, none,
360.06173467517238877720507559950041700 69, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343492604124160129745592807219 74,
4.0035598154592638821659953351678519555 13,
404.47973594414314598752833587540826887 51]
two intervals r = 16.096839663706893591552247352454260666 44 ..
19000000000057573273541488652226934719/10000000000000000000000000000000
00000 or r = 16.399886491293942816378495916914385499 57 ..
19000000000057573273541488652226934719/10000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.18e-37, 0., -.1544e-34]Solution in 1.521s

Time Plot 0 s.
Exiting SolveHard() after 5.545r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.50888347283680814866376780176142986 29,
```

```
324.67144992761201469216211990194244033 09,
302.31384315340643586714485687927251444 54,
328.46938513846639502762326470633491808 72,
343.81340624872191109278589633602346618 55,
375.73285291090335899840336697540895326 23, none,
292.99969138634717233083966632998918500 02,
358.64341560957204371832412121963519830 92, none,
360.06173467517238877720507559950041700 69, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343492604124160129745592807219 74,
4.0035598154592638821659953351678519555 13,
404.47973594414314598752833587540826887 51]
one interval r = 21.634296299944320354846239925316176867 28 ..
26.757681698940993242774768255087475172 47
Time Approximations 0.047.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, .190e-34]Solution in 1.042s

Time Plot 0 s.
Exiting SolveHard() after 4.801r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.50888347283680814866376780176142986 29,
324.67144992761201469216211990194244033 09,
302.31384315340643586714485687927251444 54,
328.46938513846639502762326470633491808 72,
343.81340624872191109278589633602346618 55,
```

```
375.7328529109033589984033669754089532623,
328.1170929469790350761245094956950238841,
292.9996913863471723308396663299891850002,
358.6434156095720437183241212196351983092, none,
360.0617346751723887772050755995004170069, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954458107604155151044813912078843,
6.196177230183285289147076930367832126699,
385.42734025882512882368086909738643882 95]
one interval r = 31.608220490940955605070426608113693046 32 ..
34.66347615051420615641924784856101519257
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=9e-38
Equations at solution: [-.6e-37, .9e-37, .12971e-34]Solution in 3.262s

Time Plot 0.001 s.
Exiting SolveHard() after 3.561r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.6429597360289170463294931012827722261,
436.9174816550740309047000948658489421471,
422.9849339805837365298894135349671326326,
361.5258025624489140422661109876503329570,
401.8817390464917890335240883044169906164,
389.5900151585659659310708189451995565232,
328.4693989377392559278917844036430075351,
401.5075715840469447431919031985540773986,
358.9736282410643322304895124129595473903,
398.3314710484146500908032518898244984265,
371.4838739425378412003462657952359324012,
336.6121584154280510612051590558230482804,
361.5088834728368081486637678017614298629,
324.6714499276120146921621199019424403309,
302.3138431534064358671448568792725144454,
328.4693851384663950276232647063349180872,
343.8134062487219110927858963360234661855,
375.7328529109033589984033669754089532623,
328.1170929469790350761245094956950238841,
292.9996913863471723308396663299891850002,
```

```
358.6434156095720437183241212196351983092, none,
360.0617346751723887772050755995004170069, none, none,
324.6552122374082700254753015004612115172, none, none, none, none]

0 --> 1  target = [26.46318954458107604155151044813912078843,
6.196177230183285289147076930367832126699,
385.42734025882512882368086909738643882 95]
two intervals r = 16.87629600300619764607265979242605799269 ..
19000000000057573273541488652226934719/10000000000000000000000000000000000
00000 or r = 15.55559000659317205827896881401527440731 ..
19000000000057573273541488652226934719/10000000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.180e-37, 0., -.197e-35]Solution in 3.882s

Time Plot 0 s.
Exiting SolveHard() after 5.034r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.50888347283680814866376780176142986 29,
324.67144992761201469216211990194244033 09,
302.31384315340643586714485687927251444 54,
328.46938513846639502762326470633491808 72,
343.81340624872191109278589633602346618 55,
375.73285291090335899840336697540895326 23,
328.11709294697903507612450949569502388 41,
292.99969138634717233083966632998918500 02,
358.6434156095720437183241212196351983092, none,
```

```
360.06173467517238877720507559950041700 69,
336.59441032587028942263737477218787661 89, none,
324.65521223740827002547530150046121151 72, none, none, none, none]

0 --> 2  target = [34.49522661164117607765939946547316536235,
3.8971313158586764766039093876832974156 35,
373.78081884303382430332727361424630479 05]
two intervals r = 17.297690862434952295922654065945909582 88 ..
19000000000057573273541488652226934719/10000000000000000000000000000000000
00000 or r = 14.994364074236746913169694180899631350 89 ..
19000000000057573273541488652226934719/10000000000000000000000000000000000
00000
Time Approximations 0.083.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.36e-37, 0., -.2306e-34]Solution in 4.116s

Time Plot 0 s.
Exiting SolveHard() after 5.732r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.50888347283680814866376780176142986 29,
324.67144992761201469216211990194244033 09,
302.31384315340643586714485687927251444 54,
328.46938513846639502762326470633491808 72,
343.81340624872191109278589633602346618 55,
375.73285291090335899840336697540895326 23,
328.11709294697903507612450949569502388 41,
292.99969138634717233083966632998918500 02,
358.64341560957204371832412121963519830 92, none,
```

```
360.06173467517238877720507559950041700 69,
336.59441032587028942263737477218787661 89, none,
324.65521223740827002547530150046121151 72,
331.93806791086298738010739094145371349 99, none, none, none]

1 --> 2  target = [34.49522661164117607765939946547316536235,
3.89713131585867647660390938768329741563 5,
373.78081884303382430332727361424630479 05]
one interval r = 21.06068473199579804960923941769338932181 ..
26.26979834277527786490214731108261088622
Time Approximations 0.033.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [-.2e-37, -.5e-37, -.190e-34]Solution in 0.798s

Time Plot 0 s.
Exiting SolveHard() after 1.5r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.50888347283680814866376780176142986 29,
324.67144992761201469216211990194244033 09,
302.31384315340643586714485687927251444 54,
328.46938513846639502762326470633491808 72,
343.81340624872191109278589633602346618 55,
375.73285291090335899840336697540895326 23,
328.11709294697903507612450949569502388 41,
292.99969138634717233083966632998918500 02,
358.64341560957204371832412121963519830 92,
299.89866204619572914791638306624809645 63,
360.06173467517238877720507559950041700 69,
```

```
336.5944103258702894226373747721878766189, none,
324.6552122374082700254753015004612115172,
331.9380679108629873801073909414537134999, none, none, none]

0 --> 2  target = [33.8136249541219705992875343449365876649 8,
3.7256489935020202075095240762341568481 03,
325.8920997297969870500760883080968661690]
two intervals r = 18.552270490117365522339791072803799419 33 ..
190000000000575732735414886522269347 19/100000000000000000000000000000000
00000 or r = 12.491969357973489922999763914758848777 93 ..
190000000000575732735414886522269347 19/100000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [-.139e-36, .6e-37, -.635e-35]Solution in 1.186s

Time Plot 0 s.
Exiting SolveHard() after 5.642r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349396600101038411655897061364111,
441.6429597360289170463294931012827722261,
436.9174816550740309047000948658489421471,
422.9849339805837365298894135349671326326,
361.5258025624489140422661109876503329570,
401.8817390464917890335240883044169906164,
389.5900151585659659310708189451995565232,
328.4693989377392559278917844036430075351,
401.5075715840469447431919031985540773986,
358.9736282410643322304895124129595473903,
398.3314710484146500908032518898244984265,
371.4838739425378412003462657952359324012,
336.6121584154280510612051590558230482804,
361.5088834728368081486637678017614298629,
324.6714499276120146921621199019424403309,
302.3138431534064358671448568792725144454,
328.4693851384663950276232647063349180872,
343.8134062487219110927858963360234661855,
375.7328529109033589984033669754089532623,
328.1170929469790350761245094956950238841,
292.9996913863471723308396663299891850002,
358.6434156095720437183241212196351983092,
```

```
299.8986620461957291479163830662480964563,
360.0617346751723887772050755995004170069,
336.5944103258702894226373747721878766189, none,
324.6552122374082700254753015004612115172,
331.9380679108629873801073909414537134999, none, none,
289.5459577242321965808166241578053554508]

1 --> 2  target = [33.81362495412197059928753434493658766498,
3.7256489935020202075095240762341568481 03,
325.8920997297969870500760883080968661690]
one interval r = 20.37468935107720817135676945916787464442 ..
25.37892165299642548827032418989404009102
Time Approximations 0.025.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.2e-37, -.2e-37, -.669e-34]Solution in 0.551s

Time Plot 0 s.
Exiting SolveHard() after 3.794r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349396600101038411655897061364111,
441.6429597360289170463294931012827722261,
436.9174816550740309047000948658489421471,
422.9849339805837365298894135349671326326,
361.5258025624489140422661109876503329570,
401.8817390464917890335240883044169906164,
389.5900151585659659310708189451995565232,
328.4693989377392559278917844036430075351,
401.5075715840469447431919031985540773986,
358.9736282410643322304895124129595473903,
398.3314710484146500908032518898244984265,
371.4838739425378412003462657952359324012,
336.6121584154280510612051590558230482804,
361.5088834728368081486637678017614298629,
324.6714499276120146921621199019424403309,
302.3138431534064358671448568792725144454,
328.4693851384663950276232647063349180872,
343.8134062487219110927858963360234661855,
375.7328529109033589984033669754089532623,
328.1170929469790350761245094956950238841,
292.9996913863471723308396663299891850002,
```

```
358.6434156095720437183241212196351983092,
299.8986620461957291479163830662480964563,
360.0617346751723887772050755995004170069,
336.5944103258702894226373747721878766189,
256.1075318616939367532590498395721455539,
324.6552122374082700254753015004612115172,
331.9380679108629873801073909414537134999, none, none,
289.5459577242321965808166241578053554508]

1 --> 0  target = [17.9304136971094591727925902304465035251 2,
4.6865087020381773875716440619028526848 14,
353.3054109537685499815426233475074495347]
one interval r = 20.7315047909537105425063663952565371432 1 ..
25.9067535352554595594809739771519861044 6
Time Approximations 0.033.

hint used Hint := [25.4020715378188039710536173439704738275 4, 3, -1, 1,
17.0062444547662134530206245122467435705 3, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.8e-38
Equations at solution: [-.2e-37, -.48e-37, -.127e-34]Solution in 0.668s

Time Plot 0 s.
Exiting SolveHard() after 1.382r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349396600101038411655897061364111,
441.6429597360289170463294931012827722261,
436.9174816550740309047000948658489421471,
422.9849339805837365298894135349671326326,
361.5258025624489140422661109876503329570,
401.8817390464917890335240883044169906164,
389.5900151585659659310708189451995565232,
328.4693989377392559278917844036430075351,
401.5075715840469447431919031985540773986,
358.9736282410643322304895124129595473903,
398.3314710484146500908032518898244984265,
371.4838739425378412003462657952359324012,
336.6121584154280510612051590558230482804,
361.5088834728368081486637678017614298629,
324.6714499276120146921621199019424403309,
302.3138431534064358671448568792725144454,
328.4693851384663950276232647063349180872,
343.8134062487219110927858963360234661855,
375.7328529109033589984033669754089532623,
```

```
328.11709294697903507612450949569502388 41,
292.99969138634717233083966632998918500 02,
358.64341560957204371832412121963519830 92,
299.89866204619572914791638306624809645 63,
360.06173467517238877720507559950041700 69,
336.59441032587028942263737477218787661 89,
256.10753186169393675325904983957214555 39,
324.65521223740827002547530150046121151 72,
331.93806791086298738010739094145371349 99,
304.79958326012755453066351435362184937 84, none,
289.54595772423219658081662415780535545 08]

2 --> 0  target = [17.93041369710945917279259023044650352512,
4.68650870203817738757164406190285268481 4,
353.30541095376854998154262334750744953 47]
one interval r = 31.37435486995455508115702670350119772638 ..
34.20127520033295086448036931984810468874
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., .11458e-34]Solution in 3.136s

Time Plot 0 s.
Exiting SolveHard() after 3.4r=33.7963 in [32.25770943 .. 34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493966001010384116558970613641 11,
441.64295973602891704632949310128277222 61,
436.91748165507403090470009486584894214 71,
422.98493398058373652988941353496713263 26,
361.52580256244891404226611098765033295 70,
401.88173904649178903352408830441699061 64,
389.59001515856596593107081894519955652 32,
328.46939893773925592789178440364300753 51,
401.50757158404694474319190319855407739 86,
358.97362824106433223048951241295954739 03,
398.33147104841465009080325188982449842 65,
371.48387394253784120034626579523593240 12,
336.61215841542805106120515905582304828 04,
361.50888347283680814866376780176142986 29,
324.67144992761201469216211990194244033 09,
302.31384315340643586714485687927251444 54,
328.46938513846639502762326470633491808 72,
```

```
343.8134062487219110927858963360234661855,
375.7328529109033589984033669754089532623,
328.1170929469790350761245094956950238841,
292.9996913863471723308396663299891850002,
358.6434156095720437183241212196351983092,
299.8986620461957291479163830662480964563,
360.0617346751723887772050755995004170069,
336.5944103258702894226373747721878766189,
256.1075318616939367532590498395721455539,
324.6552122374082700254753015004612115172,
331.9380679108629873801073909414537134999,
304.7995832601275545306635143536218493784,
323.4616917673662876124567694744247455366,
289.5459577242321965808166241578053554508]

Cascade time 157.112
counts: 28, 28

 Iteration 60

Start Generation 1
1 --> 0  target = [11.999999999791209486486931595038868682 00,
6.217012503020175512453249811819090825814,
485.5490809088045157268282057583028363416]
```

"Imaginary part neglected: ", $1.8899423791430885538076363092229225465 07 \times 10^{-17}$

```
one interval r = 23.40850301681371610771041710448615241050 ..
27.67578046426775042459498962605212793225
Time Approximations 0.049.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.85e-37
Equations at solution: [.6e-37, -.185e-36, -.8e-36]Solution in 3.749s

Time Plot 0 s.
Exiting SolveHard() after 4.919r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349465387998971402342433174744911,
441.6429597411940536319681452809379860550, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none]

2 --> 0  target = [11.999999999791209486486931595038868682 00,
6.217012503020175512453249811819090825814,
485.54908090880451572682820575830283634 16]
one interval r = 32.628147792340449551401983020195916104 51 ..
36.102483889633141997825518240070148461 54
Time Approximations 0.024.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., -.157e-34]Solution in 0.613s

Time Plot 0 s.
Exiting SolveHard() after 1.039r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844793845110644155133969939573 86,
6.583434721812115276756791666002979439464,
467.78730596841684140857523804179164574 91]
one interval r = 32.419789556818894293160582213072967518 24 ..
35.851524173918351583395115712109323490 36
Time Approximations 0.022.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., .26e-35]Solution in 3.466s
```

```
Time Plot 0 s.
Exiting SolveHard() after 3.842r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56, none, none,
401.88173904966662023561607728312183273 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844793845110644155133969939573 86,
6.5834347218121152767567916660029794394 64,
467.78730596841684140857523804179164574 91]
two intervals r = 12.923271608158328734827452615157543164 56 ..
190000000002303144162240124508981216 91/100000000000000000000000000000000
00000 or r = 18.394248580676400745144273181888401917 98 ..
190000000002303144162240124508981216 91/100000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.1e-38
Equations at solution: [.1e-37, .21e-37, .3e-37]Solution in 44.567s

Time Plot 0 s.
Exiting SolveHard() after 45.945r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93, none,
401.88173904966662023561607728312183273 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628458151267291742731884024022 84,
```

```
4.125651796812831706604121552987519042561,
440.671230659849400301900870085681503517 8]
two intervals r = 14.35659705107686660463840226110812413570 ..
190000000002303144162240124508981216 91/100000000000000000000000000000000
00000 or r = 17.70352613850544379827449176105040817702 ..
190000000002303144162240124508981216 91/100000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [-.57e-37, -.3e-37, .2460e-34]Solution in 4.144s

Time Plot 0 s.
Exiting SolveHard() after 5.2r=15.9119 in [14.35659706 .. 18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93, none,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962845815126729174273188402402284,
4.125651796812831706604121552987519042561,
440.671230659849400301900870085681503517 8]
```

"Imaginary part neglected: ", $1.8899423791430885538076363092229225465 07 \times 10^{-17}$

```
one interval r = 22.39761154393020059288305941032489502861 ..
27.23722351600144635586944240962766964528
Time Approximations 0.041.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.272 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643966050247065960093473827239 25, rm =
14.37818770571349381047043345151022555038}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., -.920e-34]Solution in 10.342s

Time Plot 0 s.
Exiting SolveHard() after 11.199r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888815391353693870645228615495525,
4.00486908178317655556260519179252573917 9,
404.86224502026636565954872980588541157 62]
two intervals r = 16.08011007765249041117047732470091245286 ..
19000000000230314416224012450898121691/10000000000000000000000000000000
00000 or r = 16.41579812733003768978564804283489577551 ..
19000000000230314416224012450898121691/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.18e-37, 0., -.163e-35]Solution in 4.075s

Time Plot 0 s.
Exiting SolveHard() after 5.192r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48, none, none,
358.97362824560416242639371122666463988 09, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888815391353693870645228615495525,
4.0048690817831765555626051917925257391 79,
404.86224502026636565954872980588541157 62]
```

"Imaginary part neglected: ", $1.8899423791430885538076363092229225465 07 \times 10^{-17}$

```
one interval r = 21.641943994348104656065400366347269466 66 ..
26.763306600440974164014035263108049684 47
Time Approximations 0.055.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., .77e-35]Solution in 3.681s

Time Plot 0 s.
Exiting SolveHard() after 4.706r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25, none,
358.97362824560416242639371122666463988 09, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941784870148943098098597628646481,
5.58963718296067102721301928474340136037 7,
443.83065885287382838396029971070721380 30]
```

"Imaginary part neglected: ", $1.8899423791430885538076363092229225465 07 \times 10^{-17}$

```
one interval r = 22.46725374498918155700463907010306469184 ..
27.27388428353397700252818965164742977574
Time Approximations 0.042.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [-.1e-37, .81e-37, -.41e-35]Solution in 1.02s

Time Plot 0 s.
Exiting SolveHard() after 2.005r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25, none,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941784870148943098098597628646481,
5.58963718296067102721301928474340136037 7,
443.83065885287382838396029971070721380 30]
one interval r = 32.15575279517108176917753026576512919323 ..
35.50872228754470679450615123747089979852
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
```

```
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, -.326e-34]Solution in 0.469s

Time Plot 0 s.
Exiting SolveHard() after 3.691r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136509784666094336872751089806334,
5.18778357852598555026412245614392985426 2,
408.65773863546192406454035403953868493 23]

"Imaginary part neglected: ", 1.8899423791430885538076363092229225465 07 × 10^-17
one interval r = 21.71840114676689004052705621892059142112 ..
26.81849303515800817713420060053155383248
Time Approximations 0.061.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.05e-37
Equations at solution: [0., -.105e-36, -.102e-34]Solution in 3.968s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.078r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18, none, none,
361.50888347714368560191887822332822392 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136509784666094336872751089806334,
5.18778357852598555026412245614392985426 2,
408.65773863546192406454035403953868493 23]
one interval r = 31.80828598767556877307468683712851144871 ..
35.00011460064464222184334960718588360371
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=8e-38
Equations at solution: [.7e-37, -.8e-37, .232e-34]Solution in 0.415s

Time Plot 0 s.
Exiting SolveHard() after 0.711r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
```

```
361.525802566679161137684824622539829671 1,
401.881739049666620235616077283121832730 8,
389.590015166473353434674550197563348004 8,
328.469398938202521944820234981549539642 5,
401.507571586539763846355099207885824995 9,
358.973628245604162426393711226664639880 9,
398.331471046040577355593737245032571381 8,
371.483873951083600270903709825351646208 6, none,
361.508883477143685601918878223328223927 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.463471105427274700698360409075984508 68,
6.196262565526101737738243966795471349206,
385.444743799420411370272600972955195928 6]
one interval r = 31.608360975501086501478832278311552845 05 ..
34.663727956256199326131574959362124326 97
Time Approximations 0.018.

hint used Hint := [33.813624954087722337763841029414448686 70, 3, 1, 1,
11.783029068742676904624149068588691721 32, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=1.1e-37
Equations at solution: [-.8e-37, .11e-36, .293e-34]Solution in 0.562s

Time Plot 0 s.
Exiting SolveHard() after 0.83r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434946538799897140234243317474491 1,
441.642959741194053631968145280937986055 0,
436.917481662209657974276459196899923325 6,
422.984933981983566425029605043754792569 3,
361.525802566679161137684824622539829671 1,
401.881739049666620235616077283121832730 8,
389.590015166473353434674550197563348004 8,
328.469398938202521944820234981549539642 5,
401.507571586539763846355099207885824995 9,
358.973628245604162426393711226664639880 9,
398.331471046040577355593737245032571381 8,
371.483873951083600270903709825351646208 6, none,
361.508883477143685601918878223328223927 8,
324.671449929698363246923520221860940741 5, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.463471105427274700698360409075984508 68,
```

```
6.196262565526101737738243966795471349206,
385.44474379942041137027260097295519592 86]
two intervals r = 16.875634087651096641808191490992259132 52 ..
190000000002303144162240124508981216 91/100000000000000000000000000000000
00000 or r = 15.556404938397683910620642874465509827 20 ..
190000000002303144162240124508981216 91/100000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.717e-37, 0., -.990e-35]Solution in 1.143s

Time Plot 0 s.
Exiting SolveHard() after 5.047r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18,
371.48387395108360027090370982535164620 86,
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747467253286592988607435956258 73,
4.883810779837474999649777450565231083012,
376.61967856450806254372797213276690257 34]
```

"Imaginary part neglected: ", $1.8899423791430885538076363092229225465 07 \times 10^{-17}$

```
one interval r = 21.110013049012467972700048139576740902 97 ..
26.317842434787947406488888793643425353 08
Time Approximations 0.037.
```

```
hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.761063912186698964426645072343113601 29, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [0., .26e-37, -.148e-34]Solution in 3.79s

Time Plot 0 s.
Exiting SolveHard() after 4.531r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122664639880 9,
398.33147104604057735559373724503257138 18,
371.48387395108360027090370982535164620 86,
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15, none,
328.46938513893140859765771130558834952 14, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747467253286592988607435956258 73,
4.8838107798374749996497774505652310830 12,
376.61967856450806254372797213276690257 34]
one interval r = 31.538994977242821186784194829949449988 99 ..
34.536183861093504996440809227740607517 96
Time Approximations 0.015.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=9.8e-37
```

```
Equations at solution: [-.75e-36, .98e-36, .559e-34]Solution in 0.494s

Time Plot 0 s.
Exiting SolveHard() after 0.771r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18,
371.48387395108360027090370982535164620 86,
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15, none,
328.46938513893140859765771130558834952 14,
343.81340625435259985390409802044535778 08, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017542530435622952292083414632305,
6.02581354947103425242946929310404514628 0,
351.42702948797481455453217203226800347 72]
one interval r = 31.36230206124563695285985449765710720561 ..
34.17446640627146915345579318387710974944
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., -.177e-34]Solution in 0.536s

Time Plot 0 s.
Exiting SolveHard() after 0.782r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18,
371.48387395108360027090370982535164620 86,
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15, none,
328.46938513893140859765771130558834952 14,
343.81340625435259985390409802044535778 08, none, none,
292.99969138474690122239969470429520469 67, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017542530435622952292083414632305,
6.0258135494710342524294692931040451462 80,
351.42702948797481455453217203226800347 72]
two intervals r = 17.98135514463071520832579350223321850584 ..
19000000000230314416224012450898121691/10000000000000000000000000000000
00000 or r = 13.84608015446224134912803818851373896946 ..
19000000000230314416224012450898121691/10000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [-.35e-37, .2e-37, .1818e-34]Solution in 1.134s

Time Plot 0 s.
Exiting SolveHard() after 4.786r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
```

```
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18,
371.48387395108360027090370982535164620 86,
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15,
302.31384314970035062547247980179479706 22,
328.46938513893140859765771130558834952 14,
343.81340625435259985390409802044535778 08, none, none,
292.99969138474690122239969470429520469 67, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941871350767610645713311980654941,
6.37794387406594540063761159038888147625 0,
423.28832784498293780194376393807649986 95]
one interval r = 31.94661817610327833316973501109519523271 ..
35.21212308665657161636862818202266452045
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .96e-35]Solution in 0.59s

Time Plot 0 s.
Exiting SolveHard() after 0.954r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18,
```

```
371.48387395108360027090370982535164620 86,
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15,
302.31384314970035062547247980179479706 22,
328.46938513893140859765771130558834952 14,
343.81340625435259985390409802044535778 08, none, none,
292.99969138474690122239969470429520469 67, none, none,
360.06173467098273946294678693176903868 45, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941871350767610645713311980654941,
6.37794387406594540063761159038888147625 0,
423.28832784498293780194376393807649986 95]
two intervals r = 15.22886702448278303106979950948471813928 ..
19000000000230314416224012450898121691/10000000000000000000000000000000
00000 or r = 17.12965771104376511490145561286872802383 ..
19000000000230314416224012450898121691/10000000000000000000000000000000
00000
Time Approximations 0.063.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., -.1657e-34]Solution in 1.256s

Time Plot 0 s.
Exiting SolveHard() after 5.33r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18,
371.48387395108360027090370982535164620 86,
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15,
```

```
302.31384314970035062547247980179479706 22,
328.46938513893140859765771130558834952 14,
343.81340625435259985390409802044535778 08,
375.73285290372600271239859592705217919 65, none,
292.99969138474690122239969470429520469 67, none, none,
360.06173467098273946294678693176903868 45, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343582931034747631537638242920 62,
4.0035598154809005407831042664090949032 78,
404.47973594664031117644510032212664505 05]
two intervals r = 16.096839663791688215899367296076434824 38 ..
19000000000230314416224012450898121691/10000000000000000000000000000000
00000 or r = 16.399886491501065897189615745601108823 97 ..
19000000000230314416224012450898121691/10000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [-.34e-37, 0., .3301e-34]Solution in 1.521s

Time Plot 0 s.
Exiting SolveHard() after 5.51r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18,
371.48387395108360027090370982535164620 86,
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15,
302.31384314970035062547247980179479706 22,
328.46938513893140859765771130558834952 14,
343.81340625435259985390409802044535778 08,
```

```
375.7328529037260027123985959270521791965, none,
292.9996913847469012223996947042952046967,
358.6434156135102882942626868256965187640, none,
360.0617346709827394629467869317690386845, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234358293103474763153763824292062,
4.0035598154809005407831042664090949032 78,
404.4797359466403111764451003221266450505]
```

"Imaginary part neglected: ", $1.8899423791430885538076363092229225465 07 \times 10^{-17}$

```
one interval r = 21.63429630010726410602681424655947913120 ..
26.75768169892939306322906243649980000482
Time Approximations 0.053.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.226e-34]Solution in 1.052s

Time Plot 0 s.
Exiting SolveHard() after 4.9r=25.8653 in [23.83864811 .. 26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349465387998971402342433174744911,
441.6429597411940536319681452809379860550,
436.9174816622096579742764591968999233256,
422.9849339819835664250296050437547925693,
361.5258025666791611376848246225398296711,
401.8817390496666202356160772831218327308,
389.5900151664733534346745501975633480048,
328.4693989382025219448202349815495396425,
401.5075715865397638463550992078858249959,
358.9736282456041624263937112266646398809,
398.3314710460405773555937372450325713818,
371.4838739510836002709037098253516462086,
336.6121584154350138025879452887861894545,
361.5088834771436856019188782233282239278,
324.6714499296983632469235202218609407415,
302.3138431497003506254724798017947970622,
328.4693851389314085976577113055883495214,
343.8134062543525998539040980204453577808,
375.7328529037260027123985959270521791965,
328.1170929467999081874651000739132697488,
```

```
292.99969138474690122239969470429520469 67,
358.6434156135102882942626868256965187640, none,
360.0617346709827394629467869317690386845, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954461064170816417208241686901877,
6.196177230380053296135614292217123694922,
385.42734026325956158416024689394416900 41]
one interval r = 31.608220491055439302740881351180048618 09 ..
34.66347615063408318529849911963184893158
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.12e-35]Solution in 3.3s

Time Plot 0 s.
Exiting SolveHard() after 3.593r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18,
371.48387395108360027090370982535164620 86,
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15,
302.31384314970035062547247980179479706 22,
328.46938513893140859765771130558834952 14,
343.81340625435259985390409802044535778 08,
375.73285290372600271239859592705217919 65,
328.11709294679990818746510007391326974 88,
292.99969138474690122239969470429520469 67,
358.6434156135102882942626868256965187640, none,
360.0617346709827394629467869317690386845, none, none,
```

```
324.655212239568121596468994738440764518 9, none, none, none, none]

0 --> 1  target = [26.46318954461064170816417208241686901877,
6.196177230380053296135614292217123694922,
385.42734026325956158416024689394416900 41]
two intervals r = 16.87629600303415594253479622620532277991 ..
190000000002303144162240124508981216 91/100000000000000000000000000000000
00000 or r = 15.55559000688736660948500972251116656357 ..
190000000002303144162240124508981216 91/100000000000000000000000000000000
00000
Time Approximations 0.062.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., .1512e-34]Solution in 3.879s

Time Plot 0 s.
Exiting SolveHard() after 5.04r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18,
371.48387395108360027090370982535164620 86,
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15,
302.31384314970035062547247980179479706 22,
328.46938513893140859765771130558834952 14,
343.81340625435259985390409802044535778 08,
375.73285290372600271239859592705217919 65,
328.11709294679990818746510007391326974 88,
292.99969138474690122239969470429520469 67,
358.64341561351028829426268682569651876 40, none,
360.06173467098273946294678693176903868 45,
336.59441032595770230112342065143102243 58, none,
```

```
324.65521223956812159646899473844076451 89, none, none, none, none]

0 --> 2  target = [34.49522661182321331430540628549219593567,
3.89713131590249580391833554822034240970 6,
373.78081885172320284726957668316687035 60]
two intervals r = 17.29769086233361035834553046106009135688 ..
19000000000230314416224012450898121691/10000000000000000000000000000000
00000 or r = 14.99436407474182620929018144391851328133 ..
19000000000230314416224012450898121691/10000000000000000000000000000000
00000
Time Approximations 0.085.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., .1899e-34]Solution in 3.939s

Time Plot 0 s.
Exiting SolveHard() after 5.58r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18,
371.48387395108360027090370982535164620 86,
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15,
302.31384314970035062547247980179479706 22,
328.46938513893140859765771130558834952 14,
343.81340625435259985390409802044535778 08,
375.73285290372600271239859592705217919 65,
328.11709294679990818746510007391326974 88,
292.99969138474690122239969470429520469 67,
358.64341561351028829426268682569651876 40, none,
360.06173467098273946294678693176903868 45,
336.59441032595770230112342065143102243 58, none,
```

```
324.65521223956812159646899473844076451 89,
331.93806792022806873046928986470643169 76, none, none, none]

1 --> 2  target = [34.49522661182321331430540628549219593567,
3.89713131590249580391833554822034240970 6,
373.78081885172320284726957668316687035 60]
```

"Imaginary part neglected: ", $1.8899423791430885538076363092229225465 07 \times 10^{-17}$

```
one interval r = 21.06068473228297380332916944608796557232 ..
26.26979834288538062399467013368499653564
Time Approximations 0.037.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [-.1e-37, -.3e-37, .264e-34]Solution in 0.805s

Time Plot 0 s.
Exiting SolveHard() after 1.53r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
358.97362824560416242639371122666463988 09,
398.33147104604057735559373724503257138 18,
371.48387395108360027090370982535164620 86,
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15,
302.31384314970035062547247980179479706 22,
328.46938513893140859765771130558834952 14,
343.81340625435259985390409802044535778 08,
375.73285290372600271239859592705217919 65,
328.11709294679990818746510007391326974 88,
292.99969138474690122239969470429520469 67,
358.64341561351028829426268682569651876 40,
299.89866205167740940756329232743082824 42,
```

```
360.06173467098273946294678693176903868 45,
336.594410325957702301123420651431022 4358, none,
324.655212239568121596468994738440764 5189,
331.938067920228068730469289864706431 6976, none, none, none]

0 --> 2  target = [33.81362495421230099669817455938714604477,
3.725648993523357744406070352050621492146,
325.892099731883497551738358655531768 8460]
two intervals r = 18.552270490269380894599557396395039527 79 ..
190000000002303144162240124508981216 91/100000000000000000000000000000000000
00000 or r = 12.491969358100136235051016284797040918 86 ..
190000000002303144162240124508981216 91/100000000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1e-38
Equations at solution: [-.36e-37, .1e-37, .509e-35]Solution in 1.2s

Time Plot 0 s.
Exiting SolveHard() after 5.462r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.642959741194053631968145280937986055 0,
436.917481662209657974276459196899923325 6,
422.984933981983566425029605043754792569 3,
361.525802566679161137684824622539829671 1,
401.881739049666620235616077283121832730 8,
389.590015166473353434674550197563348004 8,
328.469398938202521944820234981549539642 5,
401.507571586539763846355099207885824995 9,
358.973628245604162426393711226664639880 9,
398.331471046040577355593737245032571381 8,
371.483873951083600270903709825351646208 6,
336.612158415435013802587945288786189454 5,
361.508883477143685601918878223328223927 8,
324.671449929698363246923520221860940741 5,
302.313843149700350625472479801794797062 2,
328.469385138931408597657711305588349521 4,
343.813406254352599853904098020445357780 8,
375.732852903726002712398595927052179196 5,
328.117092946799908187465100073913269748 8,
292.999691384746901222399694704295204696 7,
```

```
358.6434156135102882942626868256965187640,
299.8986620516774094075632923274308282442,
360.0617346709827394629467869317690386845,
336.5944103259577023011234206514310224358, none,
324.6552122395681215964689947384407645189,
331.9380679202280687304692898647064316976, none, none,
289.5459577277355148526038521030449612766]

1 --> 2  target = [33.81362495421230099669817455938714604477,
3.7256489935233577444060703520506214921 46,
325.8920997318834975517383586555317688460]
```

"Imaginary part neglected: ", $1.8899423791430885538076363092229225465 07 \times 10^{-17}$

```
one interval r = 20.37468935126859296143461338105285275080 ..
25.37892165302283849524513469136655432274
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, -.342e-34]Solution in 0.558s

Time Plot 0 s.
Exiting SolveHard() after 4.108r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.6429597411940536319681452809379860550,
436.9174816622096579742764591968999233256,
422.9849339819835664250296050437547925693,
361.5258025666791611376848246225398296711,
401.8817390496666202356160772831218327308,
389.5900151664733534346745501975633480048,
328.4693989382025219448202349815495396425,
401.5075715865397638463550992078858249959,
358.9736282456041624263937112266646398809,
398.3314710460405773555937372450325713818,
371.4838739510836002709037098253516462086,
336.6121584154350138025879452887861894545,
361.5088834771436856019188782233282239278,
324.6714499296983632469235202218609407415,
302.3138431497003506254724798017947970622,
328.4693851389314085976577113055883495214,
```

```
343.8134062543525998539040980204453577808,
375.7328529037260027123985959270521791965,
328.1170929467999081874651000739132697488,
292.9996913847469012223996947042952046967,
358.6434156135102882942626868256965187640,
299.8986620516774094075632923274308282442,
360.0617346709827394629467869317690386845,
336.5944103259577023011234206514310224358,
256.1075318610846535671772454970840699045,
324.6552122395681215964689947384407645189,
331.9380679202280687304692898647064316976, none, none,
289.5459577277355148526038521030449612766]

1 --> 0  target = [17.9304136973139140153682746208132630131 3,
4.6865087019525234527901645458026417858 95,
353.3054109534212116370327018543739576084]
```

"Imaginary part neglected: ", $1.889942379143088553807636309222922546507 \times 10^{-17}$

```
one interval r = 20.7315047911006171841921452399173983313 5 ..
25.9067535352199213088349503682503790073 6
Time Approximations 0.035.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., -.40e-35]Solution in 0.654s

Time Plot 0 s.
Exiting SolveHard() after 1.358r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349465387998971402342433174744911,
441.6429597411940536319681452809379860550,
436.9174816622096579742764591968999233256,
422.9849339819835664250296050437547925693,
361.5258025666791611376848246225398296711,
401.8817390496666202356160772831218327308,
389.5900151664733534346745501975633480048,
328.4693989382025219448202349815495396425,
401.5075715865397638463550992078858249959,
358.9736282456041624263937112266646398809,
398.3314710460405773555937372450325713818,
371.4838739510836002709037098253516462086,
```

```
336.61215841543501380258794528878618945 45,
361.50888347714368560191887822332822392 78,
324.67144992969836324692352022186094074 15,
302.31384314970035062547247980179479706 22,
328.46938513893140859765771130558834952 14,
343.81340625435259985390409802044535778 08,
375.73285290372600271239859592705217919 65,
328.11709294679990818746510007391326974 88,
292.99969138474690122239969470429520469 67,
358.64341561351028829426268682569651876 40,
299.89866205167740940756329232743082824 42,
360.06173467098273946294678693176903868 45,
336.59441032595770230112342065143102243 58,
256.10753186108465356717724549708406990 45,
324.65521223956812159646899473844076451 89,
331.93806792022806873046928986470643169 76,
304.79958325579364727716383549872923751 10, none,
289.54595772773551485260385210304496127 66]

2 --> 0  target = [17.93041369731391401536827462081326301313,
4.68650870195252345279016454580264178589 5,
353.30541095342121163703270185437395760 84]
one interval r = 31.37435487003312267255150690169763519005 ..
34.20127520038657394113024742047566236699
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2e-38
Equations at solution: [-.1e-37, .2e-37, .360e-34]Solution in 2.995s

Time Plot 0 s.
Exiting SolveHard() after 3.257r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494653879989714023424331747449 11,
441.64295974119405363196814528093798605 50,
436.91748166220965797427645919689992332 56,
422.98493398198356642502960504375479256 93,
361.52580256667916113768482462253982967 11,
401.88173904966662023561607728312183273 08,
389.59001516647335343467455019756334800 48,
328.46939893820252194482023498154953964 25,
401.50757158653976384635509920788582499 59,
```

```
358.9736282456041624263937112266646398809,
398.3314710460405773555937372450325713818,
371.4838739510836002709037098253516462086,
336.6121584154350138025879452887861894545,
361.5088834771436856019188782233282239278,
324.6714499296983632469235202218609407415,
302.3138431497003506254724798017947970622,
328.4693851389314085976577113055883495214,
343.8134062543525998539040980204453577808,
375.7328529037260027123985959270521791965,
328.1170929467999081874651000739132697488,
292.9996913847469012223996947042952046967,
358.6434156135102882942626868256965187640,
299.8986620516774094075632923274308282442,
360.0617346709827394629467869317690386845,
336.5944103259577023011234206514310224358,
256.1075318610846535671772454970840699045,
324.6552122395681215964689947384407645189,
331.9380679202280687304692898647064316976,
304.7995832557936472771638354987292375110,
323.4616917689079141038540086054994556788,
289.5459577277355148526038521030449612766]

Cascade time 156.96
counts: 28, 28

 Iteration 61

Start Generation 1
1 --> 0  target = [12.00000000000054662322746973425975334300,
6.217012502888617625663639331028120576838,
485.5490809041917958410450882181754706487]
one interval r = 23.40850301667255464519173588221928108827 ..
27.67578046434401343296916192404616082197
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.4e-38
Equations at solution: [.1e-37, -.24e-37, 0.]Solution in 3.715s

Time Plot 0 s.
Exiting SolveHard() after 4.881r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000000054662322746973425975334300,
6.21701250288861762566363933102812057683 8,
485.54908090419179584104508821817547064 87]
one interval r = 32.62814779215020971401518777482019906612 ..
36.10248388945306689595514300254277800668
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.58e-35]Solution in 0.588s

Time Plot 0 s.
Exiting SolveHard() after 1.005r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684486002875632283846206786687598,
6.58343472169120948466413486826050815451 9,
467.78730596385117814954186469557668285 16]
one interval r = 32.41978955662997611944509913930142723359 ..
35.85152417373754048686600461495071759275
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.32e-35]Solution in 3.418s

Time Plot 0 s.
Exiting SolveHard() after 3.776r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88, none, none,
401.88173904595759595849521813226714246 95, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684486002875632283846206786687598,
6.5834347216912094846641348682650815451 9,
467.78730596385117814954186469557668285 16]
```

"Imaginary part neglected: ", $1.10311211490568056360070138858916566919 1 \times 10^{-17}$

```
two intervals r = 12.92327160835862516669200252964007116622 ..
11875000000084288626170543958747475 73/62500000000000000000000000000000
00 or r = 18.39424858049739249985232414212578242624 ..
11875000000084288626170543958747475 73/62500000000000000000000000000000
00
Time Approximations 0.047.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.2e-38
Equations at solution: [-.2e-37, -.22e-37, .898e-35]Solution in 44.688s

Time Plot 0 s.
Exiting SolveHard() after 46.055r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
```

```
441.642959736534281292267688099962072009 0,
436.917481658384247149753515836988145988 8,
422.984933977613861837657649737894102518 0, none,
401.881739045957595958495218132267142469 5, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628289177712065162776276675904 20,
4.125651796777658713081093804099728743868,
440.671230656249350722098212783785205439 1]
```

"Imaginary part neglected: ", $1.10311211490568056360070138858916566919 1 \times 10^{-17}$

```
two intervals r = 14.356597051198052197706374064381878432 83 ..
118750000000842886261705439587474757 3/625000000000000000000000000000000
00 or r = 17.703526138321441255107965046112403311 23 ..
118750000000842886261705439587474757 3/625000000000000000000000000000000
00
Time Approximations 0.054.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.29e-37, -.2e-37, -.3452e-34]Solution in
4.253s

Time Plot 0 s.
Exiting SolveHard() after 5.336r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434942092032199036554752372952692 7,
441.642959736534281292267688099962072009 0,
436.917481658384247149753515836988145988 8,
422.984933977613861837657649737894102518 0, none,
401.881739045957595958495218132267142469 5,
389.590015163855494205264833335516329609 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.463229628289177712065162776276675904 20,
4.125651796777658713081093804099728743868,
440.671230656249350722098212783785205439 1]
one interval r = 22.397611543768191552562887242479001139 89 ..
27.237223516061210825794290060191310276 84
```

```
Time Approximations 0.038.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.241 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064394885826602985959483212651346, rm =
14.37818770385116463307414078378638139961}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, -.950e-34]Solution in 11.405s

Time Plot 0 s.
Exiting SolveHard() after 12.296r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888798415599127225492287436499727,
4.00486908174750535043931217139679176576 6,
404.86224501678134432491715117292035467 62]
```

"Imaginary part neglected: ", $1.10311211490568056360070138858916566919 1 \times 10^{-17}$

```
two intervals r = 16.08011007772135474768808370387485289247 ..
11875000000084288626170543958747475 73/62500000000000000000000000000000
00 or r = 16.41579812712547487131792666601410826598 ..
11875000000084288626170543958747475 73/62500000000000000000000000000000
00
Time Approximations 0.054.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.17e-37, 0., -.893e-35]Solution in 1.459s

Time Plot 0 s.
Exiting SolveHard() after 5.42r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99, none, none,
358.97362824309975894777085114858179627 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888798415599127225492287436499727,
4.0048690817475053504393121713967917657 66,
404.86224501678134432491715117292035467 62]
one interval r = 21.641943994157681026202364662477411281 52 ..
26.763306600478612088895594043295153162 95
Time Approximations 0.048.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., .236e-34]Solution in 1.015s

Time Plot 0 s.
Exiting SolveHard() after 5.156r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39, none,
358.97362824309975894777085114858179627 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941801941574842982274412349920150,
5.589637182848672704180172752109570595753,
443.83065884839591964771536846412210726 01]
one interval r = 22.46725374481072249289388577077057003697 ..
27.27388428358559161045643007645616823750
Time Approximations 0.043.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.4e-38
Equations at solution: [0., -.54e-37, .73e-35]Solution in 3.948s

Time Plot 0 s.
Exiting SolveHard() after 4.912r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39, none,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

2 --> 0  target = [14.19258941801941574842982274412349920150,
5.589637182848672704180172752109570595753,
443.83065884839591964771536846412210726 01]
```

```
one interval r = 32.15575279498459527431508473576164960039 ..
35.50872228736332058437886331279597240680
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.105e-34]Solution in 0.484s

Time Plot 0 s.
Exiting SolveHard() after 0.852r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136513282279855904838437069277651,
5.18778357844029344244417302975609239 9616,
408.65773863284705969992338024670389166 82]
one interval r = 21.71840114659711075371423395749051661525 ..
26.81849303521083553458980031089786046619
Time Approximations 0.054.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
```

```
Accepted {r=26.4632, rm=15.9013} with Delta=1.85e-37
Equations at solution: [.1e-37, .185e-36, -.90e-35]Solution in 0.987s

Time Plot 0 s.
Exiting SolveHard() after 4.947r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92, none, none,
361.50888347469371148159910858360176649 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136513282279855904838437069277651,
5.18778357844029344244417302975609239961 6,
408.65773863284705969992338024670389166 82]
one interval r = 31.80828598750925788265358914376518792322 ..
35.00011460048775584166656817715943237279
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., .36e-35]Solution in 0.407s

Time Plot 0 s.
Exiting SolveHard() after 0.722r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
```

```
441.642959736534281292267688099962072009 0,
436.917481658384247149753515836988145988 8,
422.984933977613861837657649737894102518 0,
361.525802564269761676565155593858138560 7,
401.881739045957595958495218132267142469 5,
389.590015163855494205264833335516329609 9,
328.469398935906071720297462943950469463 9,
401.507571583009337429755954201773056023 2,
358.973628243099758947770851148581796274 2,
398.331471041572909494244518458863076179 2,
371.483873949243042824789071869097625251 1, none,
361.508883474693711481599108583601766492 7, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110547067558797777755558706461874,
6.196262565418400287891383552686519368977,
385.44474379717560933723556704904183582 46]
one interval r = 31.608360975338576684264620360753679657 17 ..
34.663727956103115658041107771501065978 03
Time Approximations 0.018.

hint used Hint := [33.813624954087722337763841029414448686 70, 3, 1, 1,
11.783029068742676904624149068588691721 32, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=1.2e-37
Equations at solution: [-.7e-37, .12e-36, -.148e-34]Solution in 3.787s

Time Plot 0 s.
Exiting SolveHard() after 4.048r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434942092032199036554752372952692 7,
441.642959736534281292267688099962072009 0,
436.917481658384247149753515836988145988 8,
422.984933977613861837657649737894102518 0,
361.525802564269761676565155593858138560 7,
401.881739045957595958495218132267142469 5,
389.590015163855494205264833335516329609 9,
328.469398935906071720297462943950469463 9,
401.507571583009337429755954201773056023 2,
358.973628243099758947770851148581796274 2,
398.331471041572909494244518458863076179 2,
371.483873949243042824789071869097625251 1, none,
361.508883474693711481599108583601766492 7,
```

```
324.67144992824717693654284654619102968 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110547067558797777755558706461874,
6.196262565418400287891383552686519368977,
385.44474379717560933723556704904183582 46]
```

"Imaginary part neglected: ", $1.10311211490568056360070138858916566919 1 \times 10^{-17}$

```
two intervals r = 16.875634087646475193056805615332390505 09 ..
11875000000084288626170543958747475 73/62500000000000000000000000000000000
00 or r = 15.556404938243922916235845218286232194 45 ..
11875000000084288626170543958747475 73/62500000000000000000000000000000000
00
Time Approximations 0.062.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.2562e-34]Solution in 1.197s

Time Plot 0 s.
Exiting SolveHard() after 2.356r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747462958698484527379487776100 07,
4.883810779753198243681137037601000263099,
```

```
376.619678562014460678338928294993871811 3]
one interval r = 21.11001304881992525956531676192246884420 ..
26.31784243482111154493618535151445971061
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, .224e-34]Solution in 0.834s

Time Plot 0 s.
Exiting SolveHard() after 4.299r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76, none,
328.46938513663375994981953077170056083 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874746295869848452737948777610007,
4.8838107797531982436811370376010002630 99,
376.619678562014460678338928294993871811 3]
one interval r = 31.53899497707867395982166242537907959422 ..
34.53618386093623045240503179068662534551
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
```

```
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=8.87e-36
Equations at solution: [-.681e-35, .887e-35, .81e-35]Solution in 0.497s

Time Plot 0 s.
Exiting SolveHard() after 0.8r=34.0898 in [32.52213872 .. 34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76, none,
328.46938513663375994981953077170056083 10,
343.81340625268796570999901899485040352 30, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017544627017309013995978316432528,
6.0258135493646849418950362076292793645 80,
351.42702948584861747075975869079987857 56]
one interval r = 31.362302061085237575299030501367083894 91 ..
34.174466406117772513089365810045575236 90
Time Approximations 0.017.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, .156e-34]Solution in 3.535s

Time Plot 0 s.
Exiting SolveHard() after 3.764r=33.3686 in [32.23723258 ..
34.17446642]
```

```
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76, none,
328.46938513663375994981953077170056083 10,
343.81340625268796570999901899485040352 30, none, none,
292.99969138344319518565363420506345560 21, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017544627017309013995978316432528,
6.0258135493646849418950362076292793645 80,
351.42702948584861747075975869079987857 56]
```

"Imaginary part neglected: ", $1.10311211490568056360070138858916566919 1 \times 10^{-17}$

```
two intervals r = 17.98135514459145627622916508721161007402 ..
11875000000084288626170543958747475 73/62500000000000000000000000000000000
00 or r = 13.84608015432629650755351993150912527477 ..
11875000000084288626170543958747475 73/62500000000000000000000000000000000
00
Time Approximations 0.05.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [-.19e-37, .1e-37, .50e-36]Solution in 3.932s

Time Plot 0 s.
Exiting SolveHard() after 5.009r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76,
302.31384314797749241690330097409365230 68,
328.46938513663375994981953077170056083 10,
343.81340625268796570999901899485040352 30, none, none,
292.99969138344319518565363420506345560 21, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941875209871146367371362779524421,
6.37794387394716422402459490558266179615 1,
423.28832784061882986918752359588644194 37]
one interval r = 31.94661817591958590636866559031325555170 ..
35.21212308647533429035825723010048767240
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=1.5e-37
Equations at solution: [.13e-36, -.15e-36, -.54e-35]Solution in 0.6s

Time Plot 0 s.
Exiting SolveHard() after 0.932r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
```

```
361.525802564269761676565155593858138560 7,
401.881739045957595958495218132267142469 5,
389.590015163855494205264833335516329609 9,
328.469398935906071720297462943950469463 9,
401.507571583009337429755954201773056023 2,
358.973628243099758947770851148581796274 2,
398.331471041572909494244518458863076179 2,
371.483873949243042824789071869097625251 1,
336.612158413603860663648708608678256881 7,
361.508883474693711481599108583601766492 7,
324.671449928247176936542846546191029687 6,
302.313843147977492416903300974093652306 8,
328.469385136633759949819530771700560831 0,
343.813406252687965709999018994850403523 0, none, none,
292.999691383443195185653634205063455602 1, none, none,
360.061734667509813613313827244355409922 6, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418752098711463673713627795244 21,
6.377943873947164224024594905582661796151,
423.288327840618829869187523595886441943 7]
```

"Imaginary part neglected: ", $1.10311211490568056360070138858916566919 1 \times 10^{-17}$

```
two intervals r = 15.228867024618775046463479588988957193 11 ..
118750000000842886261705439587474757 3/625000000000000000000000000000000000
00 or r = 17.129657770818684135799618454253084917 76 ..
118750000000842886261705439587474757 3/625000000000000000000000000000000000
00
Time Approximations 0.068.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.14e-37, .1e-37, -.1875e-34]Solution in 4.166s

Time Plot 0.001 s.
Exiting SolveHard() after 5.509r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434942092032199036554752372952692 7,
441.642959736534281292267688099962072009 0,
436.917481658384247149753515836988145988 8,
422.984933977613861837657649737894102518 0,
361.525802564269761676565155593858138560 7,
```

```
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76,
302.31384314797749241690330097409365230 68,
328.46938513663375994981953077170056083 10,
343.81340625268796570999901899485040352 30,
375.73285289967549660576334257170825856 90, none,
292.99969138344319518565363420506345560 21, none, none,
360.06173466750981361331382724435540992 26, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234341579482773188761307855370528,
4.00355981544584450159722466724869108453 4,
404.47973594333782685842368808464785613 94]
```

"Imaginary part neglected: ", $1.10311211490568056360070138858916566919 1 \times 10^{-17}$

```
two intervals r = 16.09683966385206206474813554140920565671 ..
11875000000084288626170543958747475 73/62500000000000000000000000000000000
00 or r = 16.39988649130391556029262295047977309550 ..
11875000000084288626170543958747475 73/62500000000000000000000000000000000
00
Time Approximations 0.058.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.18e-37, -.1e-37, .871e-35]Solution in 4.443s

Time Plot 0 s.
Exiting SolveHard() after 5.549r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
```

```
401.881739045957595958495218132267142469 5,
389.590015163855494205264833335516329609 9,
328.469398935906071720297462943950469463 9,
401.507571583009337429755954201773056023 2,
358.973628243099758947770851148581796274 2,
398.331471041572909494244518458863076179 2,
371.483873949243042824789071869097625251 1,
336.612158413603860663648708608678256881 7,
361.508883474693711481599108583601766492 7,
324.671449928247176936542846546191029687 6,
302.313843147977492416903300974093652306 8,
328.469385136633759949819530771700560831 0,
343.813406252687965709999018994850403523 0,
375.732852899675496605763342571708258569 0, none,
292.999691383443195185653634205063455602 1,
358.643415611163593369684104966331219398 6, none,
360.061734667509813613313827244355409922 6, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343415794827731887613078553705 28,
4.003559815445844501597224667248691084534,
404.479735943337826858423688084647856139 4]
one interval r = 21.634296299920189636807119643455993796 80 ..
26.757681698969446736495394658818473563 34
Time Approximations 0.052.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.0e-38
Equations at solution: [-.1e-37, -.50e-37, .516e-34]Solution in 3.788s

Time Plot 0 s.
Exiting SolveHard() after 4.818r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434942092032199036554752372952692 7,
441.642959736534281292267688099962072009 0,
436.917481658384247149753515836988145988 8,
422.984933977613861837657649737894102518 0,
361.525802564269761676565155593858138560 7,
401.881739045957595958495218132267142469 5,
389.590015163855494205264833335516329609 9,
328.469398935906071720297462943950469463 9,
401.507571583009337429755954201773056023 2,
```

```
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76,
302.31384314797749241690330097409365230 68,
328.46938513663375994981953077170056083 10,
343.81340625268796570999901899485040352 30,
375.73285289967549660576334257170825856 90,
328.11709294467166891711564533561993530 41,
292.99969138344319518565363420506345560 21,
358.64341561116359336968410496633121939 86, none,
360.06173466750981361331382724435540992 26, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954465335566925029447239774129060,
6.19617723027214765145041923682726806027 6,
385.42734026097302458274532559272860368 87]
one interval r = 31.60822049089259307467182323946209492844 ..
34.66347615048039448858011726026491519625
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.1e-37, -.3e-37, -.96e-35]Solution in 0.56s

Time Plot 0 s.
Exiting SolveHard() after 0.856r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
```

```
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76,
302.31384314797749241690330097409365230 68,
328.46938513663375994981953077170056083 10,
343.81340625268796570999901899485040352 30,
375.73285289967549660576334257170825856 90,
328.11709294467166891711564533561993530 41,
292.99969138344319518565363420506345560 21,
358.64341561116359336968410496633121939 86, none,
360.06173466750981361331382724435540992 26, none, none,
324.65521223807801372859569451874045154 93, none, none, none, none]

0 --> 1  target = [26.46318954465335566925029447239774129060,
6.19617723027214765145041923682726806027 6,
385.42734026097302458274532559272860368 87]
```

"Imaginary part neglected: ", $1.10311211490568056360070138858916566919 1 \times 10^{-17}$

```
two intervals r = 16.87629600303110488017039766246200458054 ..
11875000000084288626170543958747475 73/625000000000000000000000000000000000
00 or r = 15.55559000673165255821342001706647086757 ..
11875000000084288626170543958747475 73/625000000000000000000000000000000000
00
Time Approximations 0.058.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.358e-37, 0., -.2980e-34]Solution in 1.134s

Time Plot 0 s.
Exiting SolveHard() after 5.103r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
```

```
398.3314710415729094942445184588630761792,
371.4838739492430428247890718690976252511,
336.6121584136038606636487086086782568817,
361.5088834746937114815991085836017664927,
324.6714499282471769365428465461910296876,
302.3138431479774924169033009740936523068,
328.4693851366337599498195307717005608310,
343.8134062526879657099990189948504035230,
375.7328528996754966057633425717082585690,
328.1170929446716689171156453356199353041,
292.9996913834431951856536342050634556021,
358.6434156111635933696841049663312193986, none,
360.0617346675098136133138272443554099226,
336.5944103240839839762887895884949624081, none,
324.6552122380780137285956945187404515493, none, none, none, none]

0 --> 2  target = [34.4952266116789562364682860552678157042 1,
3.8971313158727075960421470580099877503 45,
373.7808188501454621676193893635185192786]
```

"Imaginary part neglected: ", $1.10311211490568056360070138858916566919 1 \times 10^{-17}$

```
two intervals r = 17.29769086229499231456152820185360137549 ..
11875000000084288626170543958747475 73/6250000000000000000000000000000000
00 or r = 14.99436407462286137663985696605311116655 ..
11875000000084288626170543958747475 73/6250000000000000000000000000000000
00
Time Approximations 0.081.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., .620e-35]Solution in 1.199s

Time Plot 0 s.
Exiting SolveHard() after 5.638r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349420920321990365547523729526927,
441.6429597365342812922676880999620720090,
436.9174816583842471497535158369881459888,
422.9849339776138618376576497378941025180,
361.5258025642697616765651555938581385607,
401.8817390459575959584952181322671424695,
389.5900151638554942052648333355163296099,
```

```
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76,
302.31384314797749241690330097409365230 68,
328.46938513663375994981953077170056083 10,
343.81340625268796570999901899485040352 30,
375.73285289967549660576334257170825856 90,
328.11709294467166891711564533561993530 41,
292.99969138344319518565363420506345560 21,
358.64341561116359336968410496633121939 86, none,
360.06173466750981361331382724435540992 26,
336.59441032408398397628878958849496240 81, none,
324.65521223807801372859569451874045154 93,
331.93806791939902397124369358086898967 18, none, none, none]

1 --> 2  target = [34.49522661167895623646828605526781570421,
3.8971313158727075960421470580099877503 45,
373.78081885014546216761938936351851927 86]
one interval r = 21.06068473210427235537325091835156669611 ..
26.26979834293208898719153960098315377704
Time Approximations 0.035.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=8e-38
Equations at solution: [.3e-37, .8e-37, -.116e-34]Solution in 3.73s

Time Plot 0 s.
Exiting SolveHard() after 4.484r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
```

```
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76,
302.31384314797749241690330097409365230 68,
328.46938513663375994981953077170056083 10,
343.81340625268796570999901899485040352 30,
375.73285289967549660576334257170825856 90,
328.11709294467166891711564533561993530 41,
292.99969138344319518565363420506345560 21,
358.64341561116359336968410496633121939 86,
299.89866205113988388557556581515760510 09,
360.06173466750981361331382724435540992 26,
336.59441032408398397628878958849496240 81, none,
324.65521223807801372859569451874045154 93,
331.93806791939902397124369358086898967 18, none, none, none]

0 --> 2  target = [33.81362495407006276056470411393484102870,
3.72564899349393093613855767606307621 2494,
325.89209973069950785505342480204019415 81]
```

"Imaginary part neglected: ", $1.10311211490568056360070138858916566919 1 \times 10^{-17}$

```
two intervals r = 18.552270490192654037344741119815535513 46 ..
11875000000084288626170543958747475 73/62500000000000000000000000000000000
00 or r = 12.491969358038413243007186104693916223 70 ..
11875000000084288626170543958747475 73/62500000000000000000000000000000000
00
Time Approximations 0.046.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [.68e-37, -.3e-37, -.206e-35]Solution in 3.937s

Time Plot 0 s.
Exiting SolveHard() after 5.397r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
```

```
422.984933977613861837657649737894102518 0,
361.525802564269761676565155593858138560 7,
401.881739045957595958495218132267142469 5,
389.590015163855494205264833335516329609 9,
328.469398935906071720297462943950469463 9,
401.507571583009337429755954201773056023 2,
358.973628243099758947770851148581796274 2,
398.331471041572909494244518458863076179 2,
371.483873949243042824789071869097625251 1,
336.612158413603860663648708608678256881 7,
361.508883474693711481599108583601766492 7,
324.671449928247176936542846546191029687 6,
302.313843147977492416903300974093652306 8,
328.469385136633759949819530771700560831 0,
343.813406252687965709999018994850403523 0,
375.732852899675496605763342571708258569 0,
328.117092944671668917115645335619935304 1,
292.999691383443195185653634205063455602 1,
358.643415611163593369684104966331219398 6,
299.898662051139883885575565815157605100 9,
360.061734667509813613313827244355409922 6,
336.594410324083983976288789588494962408 1, none,
324.655212238078013728595694518740451549 3,
331.938067919399023971243693580868989671 8, none, none,
289.545957727270689651434595761826555136 7]

1 --> 2  target = [33.813624954070062760564704113934841028 70,
3.725648993493930936138557676063076212494,
325.892099730699507855053424802040194158 1]
one interval r = 20.374689351065795670371053631759113088 25 ..
25.378921653042147758238771949065713372 26
Time Approximations 0.026.

hint used Hint := [24.339499030253814217461077660681155334 23, 3, -1, 1,
17.272158782512944212267074944284355815 74, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [-.3e-37, -.4e-37, .283e-34]Solution in 0.576s

Time Plot 0 s.
Exiting SolveHard() after 1.111r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434942092032199036554752372952692 7,
441.642959736534281292267688099962072009 0,
```

```
436.91748165838424714975351583698814598888,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76,
302.31384314797749241690330097409365230 68,
328.46938513663375994981953077170056083 10,
343.81340625268796570999901899485040352 30,
375.73285289967549660576334257170825856 90,
328.11709294467166891711564533561993530 41,
292.99969138344319518565363420506345560 21,
358.64341561116359336968410496633121939 86,
299.89866205113988388557556581515760510 09,
360.06173466750981361331382724435540992 26,
336.59441032408398397628878958849496240 81,
256.10753186091804860400869793530864765 94,
324.65521223807801372859569451874045154 93,
331.93806791939902397124369358086898967 18, none, none,
289.54595772727068965143459576182655513 67]

1 --> 0  target = [17.93041369726710464231484703450048799088,
4.68650870187341203350179681163843921710 0,
353.30541095162788770407892210616056526 53]
one interval r = 20.73150479090435061368854154407538423589 ..
25.90675353524845349920639894389793887099
Time Approximations 0.029.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, -.322e-34]Solution in 0.632s

Time Plot 0 s.
Exiting SolveHard() after 4.191r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76,
302.31384314797749241690330097409365230 68,
328.46938513663375994981953077170056083 10,
343.81340625268796570999901899485040352 30,
375.73285289967549660576334257170825856 90,
328.11709294467166891711564533561993530 41,
292.99969138344319518565363420506345560 21,
358.64341561116359336968410496633121939 86,
299.89866205113988388557556581515760510 09,
360.06173466750981361331382724435540992 26,
336.59441032408398397628878958849496240 81,
256.10753186091804860400869793530864765 94,
324.65521223807801372859569451874045154 93,
331.93806791939902397124369358086898967 18,
304.79958325421281440217287860099301215 56, none,
289.54595772727068965143459576182655513 67]

2 --> 0  target = [17.93041369726710464231484703450048799088,
4.68650870187341203350179681163843921710 0,
353.30541095162788770407892210616056526 53]
one interval r = 31.37435486987478846064975869783443549959 ..
34.20127520023775891965860794178572449435
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.131e-34]Solution in 0.337s

Time Plot 0 s.
Exiting SolveHard() after 0.621r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494209203219903655475237295269 27,
441.64295973653428129226768809996207200 90,
436.91748165838424714975351583698814598 88,
422.98493397761386183765764973789410251 80,
361.52580256426976167656515559385813856 07,
401.88173904595759595849521813226714246 95,
389.59001516385549420526483333551632960 99,
328.46939893590607172029746294395046946 39,
401.50757158300933742975595420177305602 32,
358.97362824309975894777085114858179627 42,
398.33147104157290949424451845886307617 92,
371.48387394924304282478907186909762525 11,
336.61215841360386066364870860867825688 17,
361.50888347469371148159910858360176649 27,
324.67144992824717693654284654619102968 76,
302.31384314797749241690330097409365230 68,
328.46938513663375994981953077170056083 10,
343.81340625268796570999901899485040352 30,
375.73285289967549660576334257170825856 90,
328.11709294467166891711564533561993530 41,
292.99969138344319518565363420506345560 21,
358.64341561116359336968410496633121939 86,
299.89866205113988388557556581515760510 09,
360.06173466750981361331382724435540992 26,
336.59441032408398397628878958849496240 81,
256.10753186091804860400869793530864765 94,
324.65521223807801372859569451874045154 93,
331.93806791939902397124369358086898967 18,
304.79958325421281440217287860099301215 56,
323.46169176789455505412326612524664474 79,
289.54595772727068965143459576182655513 67]

Cascade time 159.904
counts: 28, 28

 Iteration 62

Start Generation 1
1 --> 0  target = [11.99999999995200664502224863037304089900,
6.21701250299170215502124266586980013187 6,
485.54908090202859663959980809393438006 64]
one interval r = 23.40850301663752093724797490092841971001 ..
27.67578046424204674589907588023018052612
Time Approximations 0.044.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
```

```
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.85e-37
Equations at solution: [-.5e-37, .185e-36, .13e-35]Solution in 1.024s

Time Plot 0 s.
Exiting SolveHard() after 2.19r=27.5236 in [25.56992694 .. 27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999995200664502224863037304089900,
6.2170125029917021550212426658698001318 76,
485.54908090202859663959980809393438006 64]
one interval r = 32.628147792124843185142672509709858608 33 ..
36.10248388941499040540605201338751741334
Time Approximations 0.02.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., -.149e-34]Solution in 0.565s

Time Plot 0 s.
Exiting SolveHard() after 3.756r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684476531009961314314035731271978,
6.58343472167326682396864666891771787677,
467.78730596259361002950383501399493104 11]
```

```
one interval r = 32.41978955661669430791505119125611417957 ..
35.85152417371255944173314046360088360774
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, .47e-35]Solution in 0.63s

Time Plot 0 s.
Exiting SolveHard() after 0.992r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75, none, none,
401.88173904636444520303892510630217157 90, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684476531009961314314035731271978,
6.58343472167326682396864666891771787677,
467.78730596259361002950383501399493104 11]
two intervals r = 12.92327160825505475769136699273976236247 ..
19000000000038662146631529337348808497/10000000000000000000000000000000
00000 or r = 18.39424858040388968861133400988067887638 ..
19000000000038662146631529337348808497/10000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.3e-38
Equations at solution: [-.2e-37, -.33e-37, .183e-35]Solution in 41.78s

Time Plot 0 s.
```

```
Exiting SolveHard() after 46.124r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36, none,
401.88173904636444520303892510630217157 90, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962825270483084495128663472158493,
4.1256517968800326429888616566124498156 91,
440.67123065414592392645720192374245017 34]
two intervals r = 14.35659705113184627651267130329022165481 ..
190000000000386621466315293373488084 97/100000000000000000000000000000000
00000 or r = 17.70352613820289634255211862720978665942 ..
190000000000386621466315293373488084 97/100000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=5e-38
Equations at solution: [.100e-36, .5e-37, -.1022e-34]Solution in 4.119s

Time Plot 0 s.
Exiting SolveHard() after 5.212r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36, none,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
1 --> 2  target = [35.46322962825270483084495128663472158493,
4.12565179688003264298886165661244981569 1,
440.67123065414592392645720192374245017 34]
one interval r = 22.39761154376741971313532674081170719841 ..
27.23722351595837673672700504225227442991
Time Approximations 0.038.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.281 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064387445465127394054478735391777, rm =
14.37818770478120910100902791798366063974}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.3e-38
Equations at solution: [0., .53e-37, .24e-35]Solution in 11.085s

Time Plot 0 s.
Exiting SolveHard() after 11.984r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888798365419104679634663863295002,
4.00486908185788644243363656808453836436 1,
404.86224501708628946890705496814819817 74]
two intervals r = 16.08011007754296480389771651634545151882 ..
19000000000038662146631529337348808497/10000000000000000000000000000000
00000 or r = 16.41579812709789288196099944339688163860 ..
19000000000038662146631529337348808497/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
```

```
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.17e-37, 0., .1068e-34]Solution in 4.491s

Time Plot 0 s.
Exiting SolveHard() after 5.615r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31, none, none,
358.97362824326284955304335689589245909 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887983654191046796346638632950 02,
4.0048690818578864424336365680845383643 61,
404.86224501708628946890705496814819817 74]
one interval r = 21.641943994234644168336615946075882723 46 ..
26.763306600413918927760657546316379330 64
Time Approximations 0.051.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [.2e-37, .49e-37, .240e-34]Solution in 3.972s

Time Plot 0 s.
Exiting SolveHard() after 5.002r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53, none,
358.97362824326284955304335689589245909 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941778537556185056769402692128062,
5.5896371829893101555176647948905467816 74,
443.83065884952660509078903389850383514 20]
one interval r = 22.46725374487893171511836146255099745954 ..
27.27388428351976979086052295031840228630
Time Approximations 0.041.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .114e-34]Solution in 1.029s

Time Plot 0 s.
Exiting SolveHard() after 2.007r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53, none,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]
```

```
2 --> 0  target = [14.19258941778537556185056769402692128062,
5.589637182989310155517664794890546781674,
443.83065884952660509078903389850383514 20]
one interval r = 32.15575279499829179279392210167691684260 ..
35.50872228737331421240240005988960976434
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, .63e-35]Solution in 0.452s

Time Plot 0 s.
Exiting SolveHard() after 3.673r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098 6144,
441.64295973554354919583361616267865 36377,
436.91748165643424769405028557103174 68275,
422.98493397860754834922718900474850 11936,
361.52580256449364521506735964033089 75572,
401.88173904636444520303892510630217 15790,
389.59001516190623160492151050695055 88631,
328.46939893837601329798440059935328 69853,
401.50757158390501349258285401527573 99649,
358.97362824326284955304335689589245 90919,
398.33147104479061026405370370990058 44837, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136505556114039788748583245132305,
5.187783578537498750800708267880851800780,
408.65773863085422421409539112331359119 43]
one interval r = 21.71840114662490802375954090371332236994 ..
26.81849303511189244710111749846777063270
Time Approximations 0.061.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.85e-37
Equations at solution: [.2e-37, .185e-36, -.53e-35]Solution in 3.888s

Time Plot 0 s.
Exiting SolveHard() after 5.006r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37, none, none,
361.50888347484907027661383803506330160 00, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136505556114039788748583245132305,
5.18778357853749875080070826788085180 0780,
408.65773863085422421409539112331359119 43]
one interval r = 31.80828598749358485068121373911476650189 ..
35.00011460045380468482645129080483453471
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., -.129e-34]Solution in 0.43s

Time Plot 0 s.
Exiting SolveHard() after 0.735r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37,
371.48387394752994199679025362024562137 14, none,
361.50888347484907027661383803506330160 00, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110540732813727123931653319241538,
6.19626256540353069216335221405948005082 2,
385.44474379715603472124777250667065701 82]
one interval r = 31.60836097534141241923752202165174414008 ..
34.66372795609858269897928415197510370143
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.1e-37, -.3e-37, .34e-35]Solution in 0.57s

Time Plot 0 s.
Exiting SolveHard() after 0.843r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
```

```
398.331471044790610264053703709900584483 7,
371.483873947529941996790253620245621371 4, none,
361.508883474849070276613838035063301600 0,
324.671449929940619206282743092812226618 2, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.463471105407328137271239316533192415 38,
6.196262565403530692163352214059480050822,
385.444743797156034721247772506670657018 2]
two intervals r = 16.875634087491219241314733363375716264 27 ..
190000000000386621466315293373488084 97/100000000000000000000000000000000
00000 or r = 15.556404938210276195080941662837906580 77 ..
190000000000386621466315293373488084 97/100000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., .3420e-34]Solution in 1.133s

Time Plot 0 s.
Exiting SolveHard() after 5.329r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434939960250622363997694120988614 4,
441.642959735543549195833616162678653637 7,
436.917481656434247694050285571031746827 5,
422.984933978607548349227189004748501193 6,
361.525802564493645215067359640330897557 2,
401.881739046364445203038925106302171579 0,
389.590015161906231604921510506950558863 1,
328.469398938376013297984400599353286985 3,
401.507571583905013492582854015275739964 9,
358.973628243262849553043356895892459091 9,
398.331471044790610264053703709900584483 7,
371.483873947529941996790253620245621371 4,
336.612158415961438344596922734926331176 2,
361.508883474849070276613838035063301600 0,
324.671449929940619206282743092812226618 2, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747304290870042982458749369581 07,
4.883810779867252953511509487123698109622,
```

```
376.6196785622189333038813757873473793965]
one interval r = 21.11001304891338290520972830860188540105 ..
26.31784243476461286243224387832555158847
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=5.2e-38
Equations at solution: [.2e-37, .52e-37, -.273e-34]Solution in 3.72s

Time Plot 0 s.
Exiting SolveHard() after 4.441r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349399602506223639976941209886144,
441.6429597355435491958336161626786536377,
436.9174816564342476940502855710317468275,
422.9849339786075483492271890047485011936,
361.5258025644936452150673596403308975572,
401.8817390463644452030389251063021715790,
389.5900151619062316049215105069505588631,
328.4693989383760132979844005993532869853,
401.5075715839050134925828540152757399649,
358.9736282432628495530433568958924590919,
398.3314710447906102640537037099005844837,
371.4838739475299419967902536202456213714,
336.6121584159614383445969227349263311762,
361.5088834748490702766138380350633016000,
324.6714499299406192062827430928122266182, none,
328.4693851391019254525379497011740920041, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874730429087004298245874936958107,
4.883810779867252953511509487123698109622,
376.6196785622189333038813757873473793965]
one interval r = 31.53899497708338613955925044421876373403 ..
34.53618386093523737256270038172891105455
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
```

```
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=5.49e-36
Equations at solution: [-.422e-35, .549e-35, .334e-34]Solution in
0.514s

Time Plot 0 s.
Exiting SolveHard() after 0.79r=34.0898 in [32.52213872 .. 34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37,
371.48387394752994199679025362024562137 14,
336.61215841596143834459692273492633117 62,
361.50888347484907027661383803506330160 00,
324.67144992994061920628274309281222661 82, none,
328.46938513910192545253794970117409200 41,
343.81340625292330048890472716698025422 21, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017543863834137076765559338160942,
6.0258135493607210161136283017993773164 39,
351.42702948813820436388461418764759383 30]
one interval r = 31.362302061103349882374500512535421371 95 ..
34.174466406147352041588673511931627197 93
Time Approximations 0.017.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [-.3e-37, .6e-37, .42e-35]Solution in 0.533s

Time Plot 0 s.
Exiting SolveHard() after 0.778r=33.3686 in [32.23723258 ..
```

```
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37,
371.48387394752994199679025362024562137 14,
336.61215841596143834459692273492633117 62,
361.50888347484907027661383803506330160 00,
324.67144992994061920628274309281222661 82, none,
328.46938513910192545253794970117409200 41,
343.81340625292330048890472716698025422 21, none, none,
292.99969138732962592334283426508933314 91, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017543863834137076765559338160942,
6.02581354936072101611362830179937731643 9,
351.42702948813820436388461418764759383 30]
two intervals r = 17.98135514439235197741341684775474456160 ..
19000000000038662146631529337348808497/10000000000000000000000000000000
00000 or r = 13.84608015442654591684148871238372788612 ..
19000000000038662146631529337348808497/10000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.54e-37, -.2e-37, .1198e-34]Solution in 1.125s

Time Plot 0 s.
Exiting SolveHard() after 4.855r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37,
371.48387394752994199679025362024562137 14,
336.61215841596143834459692273492633117 62,
361.50888347484907027661383803506330160 00,
324.67144992994061920628274309281222661 82,
302.31384315250391670367755113611073684 85,
328.46938513910192545253794970117409200 41,
343.81340625292330048890472716698025422 21, none, none,
292.99969138732962592334283426508933314 91, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941871835033322695440450551307549,
6.37794387394793328932812226611768759310 4,
423.28832784368047313844611110616535675 82]
one interval r = 31.94661817595170290958932194181552921255 ..
35.21212308651409167463718605641648554775
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.328e-34]Solution in 0.585s

Time Plot 0 s.
Exiting SolveHard() after 0.942r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
```

```
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37,
371.48387394752994199679025362024562137 14,
336.61215841596143834459692273492633117 62,
361.50888347484907027661383803506330160 00,
324.67144992994061920628274309281222661 82,
302.31384315250391670367755113611073684 85,
328.46938513910192545253794970117409200 41,
343.81340625292330048890472716698025422 21, none, none,
292.99969138732962592334283426508933314 91, none, none,
360.06173467203756869686638987479223126 89, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941871835033322695440450551307549,
6.37794387394793328932812226611768759310 4,
423.28832784368047313844611110616535675 82]
two intervals r = 15.22886702429882397920091544241040656221 ..
19000000000038662146631529337348808497/10000000000000000000000000000000
00000 or r = 17.12965777088070126025412020610372123366 ..
19000000000038662146631529337348808497/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.32e-37, .1e-37, .1212e-34]Solution in 1.265s

Time Plot 0 s.
Exiting SolveHard() after 5.47r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37,
```

```
371.48387394752994199679025362024562137 14,
336.61215841596143834459692273492633117 62,
361.50888347484907027661383803506330160 00,
324.67144992994061920628274309281222661 82,
302.31384315250391670367755113611073684 85,
328.46938513910192545253794970117409200 41,
343.81340625292330048890472716698025422 21,
375.73285290536972827240645414378544666 37, none,
292.99969138732962592334283426508933314 91, none, none,
360.06173467203756869686638987479223126 89, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343422552534861131887289124658 70,
4.0035598155579363957143813311146185544 64,
404.47973594414249823003181881323086003 33]
two intervals r = 16.096839663652036034770184672521091975 27 ..
19000000000038662146631529337348808497/10000000000000000000000000000000
00000 or r = 16.399886491297335607597395170649975816 15 ..
19000000000038662146631529337348808497/10000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.16e-37, 0., .1625e-34]Solution in 1.479s

Time Plot 0 s.
Exiting SolveHard() after 5.268r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37,
371.48387394752994199679025362024562137 14,
336.61215841596143834459692273492633117 62,
361.50888347484907027661383803506330160 00,
```

```
324.67144992994061920628274309281222661 82,
302.31384315250391670367755113611073684 85,
328.46938513910192545253794970117409200 41,
343.81340625292330048890472716698025422 21,
375.73285290536972827240645414378544666 37, none,
292.99969138732962592334283426508933314 91,
358.64341561175886806185334304495866949 44, none,
360.06173467203756869686638987479223126 89, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343422552534861131887289124658 70,
4.0035598155579363957143813311146185544 64,
404.47973594414249823003181881323086003 33]
one interval r = 21.634296300007394913935786582283440409 59 ..
26.757681698912231160397008553184392662 28
Time Approximations 0.047.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., -.314e-34]Solution in 1.015s

Time Plot 0 s.
Exiting SolveHard() after 5.774r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37,
371.48387394752994199679025362024562137 14,
336.61215841596143834459692273492633117 62,
361.50888347484907027661383803506330160 00,
324.67144992994061920628274309281222661 82,
302.31384315250391670367755113611073684 85,
328.46938513910192545253794970117409200 41,
343.81340625292330048890472716698025422 21,
```

```
375.7328529053697282724064541437854466637,
328.1170929476020211779215737075536565874,
292.9996913873296259233428342650893331491,
358.6434156117588680618533430449586694944, none,
360.0617346720375686968663898747922312689, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954458887366935618678246161703523,
6.196177230256931922889446752332211956 43,
385.4273402608829613226129491252408360454]
one interval r = 31.60822049089486017412492403663886752168 ..
34.66347615047484226004954392222373349842
Time Approximations 0.019.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.85e-35]Solution in 0.562s

Time Plot 0 s.
Exiting SolveHard() after 0.851r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349399602506223639976941209886144,
441.6429597355435491958336161626786536377,
436.9174816564342476940502855710317468275,
422.9849339786075483492271890047485011936,
361.5258025644936452150673596403308975572,
401.8817390463644452030389251063021715790,
389.5900151619062316049215105069505588631,
328.4693989383760132979844005993532869853,
401.5075715839050134925828540152757399649,
358.9736282432628495530433568958924590919,
398.3314710447906102640537037099005844837,
371.4838739475299419967902536202456213714,
336.6121584159614383445969227349263311762,
361.5088834748490702766138380350633016000,
324.6714499299406192062827430928122266182,
302.3138431525039167036775511361107368485,
328.4693851391019254525379497011740920041,
343.8134062529233004889047271669802542221,
375.7328529053697282724064541437854466637,
328.1170929476020211779215737075536565874,
292.9996913873296259233428342650893331491,
```

```
358.6434156117588680618533430449586694944, none,
360.0617346720375686968663898747922312689, none, none,
324.6552122397057138209518720398942407800, none, none, none, none]

0 --> 1  target = [26.46318954458887366935618678246161703523,
6.196177230256931922288944675233221195643,
385.42734026088296132261294912524083604 54]
two intervals r = 16.87629600287853828904305164305823809712 ..
19000000000038662146631529337348808497/10000000000000000000000000000000000
00000 or r = 15.55559000669471147422542863553221205208 ..
19000000000038662146631529337348808497/10000000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.179e-37, 0., .1996e-34]Solution in 1.164s

Time Plot 0.001 s.
Exiting SolveHard() after 5.268r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.6429597355435491958336161626786536377,
436.9174816564342476940502855710317468275,
422.9849339786075483492271890047485011936,
361.5258025644936452150673596403308975572,
401.8817390463644452030389251063021715790,
389.5900151619062316049215105069505588631,
328.4693989383760132979844005993532869853,
401.5075715839050134925828540152757399649,
358.9736282432628495530433568958924590919,
398.3314710447906102640537037099005844837,
371.4838739475299419967902536202456213714,
336.6121584159614383445969227349263311762,
361.5088834748490702766138380350633016000,
324.6714499299406192062827430928122266182,
302.3138431525039167036775511361107368485,
328.4693851391019254525379497011740920041,
343.8134062529233004889047271669802542221,
375.7328529053697282724064541437854466637,
328.1170929476020211779215737075536565874,
292.9996913873296259233428342650893331491,
358.6434156117588680618533430449586694944, none,
```

```
360.06173467203756869686638987479223126 89,
336.59441032636961421880516875399306092 27, none,
324.65521223970571382095187203989424078 00, none, none, none, none]

0 --> 2  target = [34.49522661164824404069163961525270453305,
3.8971313159754848911696825003688068788 34,
373.78081884828241714558776746712901069 48]
two intervals r = 17.297690862208940128555842705356679699 68 ..
19000000000038662146631529337348808497/10000000000000000000000000000000000
00000 or r = 14.994364074502274839664721846895829419 05 ..
19000000000038662146631529337348808497/10000000000000000000000000000000000
00000
Time Approximations 0.081.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [-.108e-36, .1e-37, -.71e-36]Solution in 1.227s

Time Plot 0 s.
Exiting SolveHard() after 5.875r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37,
371.48387394752994199679025362024562137 14,
336.61215841596143834459692273492633117 62,
361.50888347484907027661383803506330160 00,
324.67144992994061920628274309281222661 82,
302.31384315250391670367755113611073684 85,
328.46938513910192545253794970117409200 41,
343.81340625292330048890472716698025422 21,
375.73285290536972827240645414378544666 37,
328.11709294760202117792157370755365658 74,
292.99969138732962592334283426508933314 91,
358.64341561175886806185334304495866949 44, none,
```

```
360.06173467203756869686638987479223126 89,
336.59441032636961421880516875399306092 27, none,
324.65521223970571382095187203989424078 00,
331.93806791771849430937092258726916786 63, none, none, none]

1 --> 2  target = [34.49522661164824404069163961525270453305,
3.89713131597548489116968250036880687883 4,
373.78081884828241714558776746712901069 48]
one interval r = 21.06068473216386470900702148611470609622 ..
26.26979834284146739276904205612283952023
Time Approximations 0.034.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [.1e-37, .3e-37, -.30e-35]Solution in 0.761s

Time Plot 0 s.
Exiting SolveHard() after 4.291r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37,
371.48387394752994199679025362024562137 14,
336.61215841596143834459692273492633117 62,
361.50888347484907027661383803506330160 00,
324.67144992994061920628274309281222661 82,
302.31384315250391670367755113611073684 85,
328.46938513910192545253794970117409200 41,
343.81340625292330048890472716698025422 21,
375.73285290536972827240645414378544666 37,
328.11709294760202117792157370755365658 74,
292.99969138732962592334283426508933314 91,
358.64341561175886806185334304495866949 44,
299.89866205163109880360002054791744172 52,
```

```
360.06173467203756869686638987479223126 89,
336.5944103263696142188051687539930609227, none,
324.6552122397057138209518720398942407800,
331.9380679177184943093709225872691678663, none, none, none]

0 --> 2  target = [33.81362495409051657748340597789634610753,
3.7256489936091525963809366908473385739 83,
325.8920997323212040263457127423613077939]
two intervals r = 18.552270490040226978686070231284354420 93 ..
19000000000038662146631529337348808497/10000000000000000000000000000000000
00000 or r = 12.49196935811017529228576429188537198496 ..
19000000000038662146631529337348808497/10000000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=4e-38
Equations at solution: [.87e-37, -.4e-37, .1477e-34]Solution in 1.152s

Time Plot 0 s.
Exiting SolveHard() after 5.572r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37,
371.48387394752994199679025362024562137 14,
336.61215841596143834459692273492633117 62,
361.50888347484907027661383803506330160 00,
324.67144992994061920628274309281222661 82,
302.31384315250391670367755113611073684 85,
328.46938513910192545253794970117409200 41,
343.81340625292330048890472716698025422 21,
375.73285290536972827240645414378544666 37,
328.11709294760202117792157370755365658 74,
292.99969138732962592334283426508933314 91,
```

```
358.6434156117588680618533430449586694944,
299.8986620516310988036000205479174417252,
360.0617346720375686968663898747922312689,
336.5944103263696142188051687539930609227, none,
324.6552122397057138209518720398942407800,
331.9380679177184943093709225872691678663, none, none,
289.5459577287353386113199743267669402289]

1 --> 2  target = [33.81362495409051657748340597789634610753,
3.7256489936091525963809366908473385739 83,
325.8920997323212040263457127423613077939]
one interval r = 20.37468935120161065169621885350176183096 ..
25.37892165303615937758154342384939641473
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, .410e-34]Solution in 0.552s

Time Plot 0 s.
Exiting SolveHard() after 1.084r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.6429597355435491958336161626786536377,
436.9174816564342476940502855710317468275,
422.9849339786075483492271890047485011936,
361.5258025644936452150673596403308975572,
401.8817390463644452030389251063021715790,
389.5900151619062316049215105069505588631,
328.4693989383760132979844005993532869853,
401.5075715839050134925828540152757399649,
358.9736282432628495530433568958924590919,
398.3314710447906102640537037099005844837,
371.4838739475299419967902536202456213714,
336.6121584159614383445969227349263311762,
361.5088834748490702766138380350633016000,
324.6714499299406192062827430928122266182,
302.3138431525039167036775511361107368485,
328.4693851391019254525379497011740920041,
343.8134062529233004889047271669802542221,
375.7328529053697282724064541437854466637,
328.1170929476020211779215737075536565874,
```

```
292.99969138732962592334283426508933314 91,
358.6434156117588680618533430449586694944,
299.8986620516310988036000205479174417252,
360.0617346720375686968663898747922312689,
336.5944103263696142188051687539930609227,
256.1075318646047550579226947019470132769,
324.6552122397057138209518720398942407800,
331.9380679177184943093709225872691678663, none, none,
289.5459577287353386113199743267669402289]

1 --> 0  target = [17.9304136970602816872735968893017477454 1,
4.6865087020028551967726448666608200081539,
353.3054109541068550568834715659238824060]
one interval r = 20.7315047910452013983672538656084679120 3 ..
25.9067535352432437569210866086627571016 4
Time Approximations 0.029.

hint used Hint := [25.4020715378188039710536173439704738275 4, 3, -1, 1,
17.0062444547662134530206245122467435705 3, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, .183e-34]Solution in 0.63s

Time Plot 0 s.
Exiting SolveHard() after 4.34r=25.4021 in [22.67806074 .. 25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349399602506223639976941209886144,
441.6429597355435491958336161626786536377,
436.9174816564342476940502855710317468275,
422.9849339786075483492271890047485011936,
361.5258025644936452150673596403308975572,
401.8817390463644452030389251063021715790,
389.5900151619062316049215105069505588631,
328.4693989383760132979844005993532869853,
401.5075715839050134925828540152757399649,
358.9736282432628495530433568958924590919,
398.3314710447906102640537037099005844837,
371.4838739475299419967902536202456213714,
336.6121584159614383445969227349263311762,
361.5088834748490702766138380350633016000,
324.6714499299406192062827430928122266182,
302.3138431525039167036775511361107368485,
328.4693851391019254525379497011740920041,
343.8134062529233004889047271669802542221,
375.7328529053697282724064541437854466637,
```

```
328.11709294760202117792157370755365658 74,
292.99969138732962592334283426508933314 91,
358.64341561175886806185334304495866949 44,
299.89866205163109880360002054791744172 52,
360.06173467203756869686638987479223126 89,
336.59441032636961421880516875399306092 27,
256.10753186460475505792269470194701327 69,
324.65521223970571382095187203989424078 00,
331.93806791771849430937092258726916786 63,
304.79958325895425509763166637728732778 24, none,
289.54595772873533861131997432676694022 89]

2 --> 0  target = [17.93041369706028168727359688930174774541,
4.6865087020028551967726448666082000815 39,
353.30541095410685505688347156592388240 60]
one interval r = 31.374354869894324397971577855996770414 90 ..
34.201275200270016369440034229231936348 81
Time Approximations 0.016.

hint used Hint := [33.796329522691901201472146203888695425 08, 2, 1, -1,
17.863534069716629640044658426895081929 62, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, .128e-34]Solution in 0.348s

Time Plot 0 s.
Exiting SolveHard() after 0.619r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493996025062236399769412098861 44,
441.64295973554354919583361616267865363 77,
436.91748165643424769405028557103174682 75,
422.98493397860754834922718900474850119 36,
361.52580256449364521506735964033089755 72,
401.88173904636444520303892510630217157 90,
389.59001516190623160492151050695055886 31,
328.46939893837601329798440059935328698 53,
401.50757158390501349258285401527573996 49,
358.97362824326284955304335689589245909 19,
398.33147104479061026405370370990058448 37,
371.48387394752994199679025362024562137 14,
336.61215841596143834459692273492633117 62,
361.50888347484907027661383803506330160 00,
324.67144992994061920628274309281222661 82,
302.31384315250391670367755113611073684 85,
```

```
328.46938513910192545253794970117409200 41,
343.81340625292330048890472716698025422 21,
375.73285290536972827240645414378544666 37,
328.11709294760202117792157370755365658 74,
292.99969138732962592334283426508933314 91,
358.64341561175886806185334304495866949 44,
299.89866205163109880360002054791744172 52,
360.06173467203756869686638987479223126 89,
336.59441032636961421880516875399306092 27,
256.10753186460475505792269470194701327 69,
324.65521223970571382095187203989424078 00,
331.93806791771849430937092258726916786 63,
304.79958325895425509763166637728732778 24,
323.46169177016655793836919361166578149 09,
289.54595772873533861131997432676694022 89]

Cascade time 154.749
counts: 28, 28

 Iteration 63

Start Generation 1
1 --> 0  target = [11.99999999985070991739542999552318793900,
6.21701250309451291045103493820038194005 2,
485.54908090817851287651719976531348228 72]
one interval r = 23.40850301676203328528971790493635684399 ..
27.67578046437399203215438479684348055086
Time Approximations 0.045.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=7.7e-38
Equations at solution: [.2e-37, -.77e-37, -.9e-36]Solution in 1.019s

Time Plot 0 s.
Exiting SolveHard() after 2.181r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0  target = [11.999999999850709917395429995523187939 00,
6.2170125030945129104510349382003819400 52,
485.5490809081785128765171997653134822872]
one interval r = 32.628147792250147329389488570013485667 75 ..
36.102483889570241604324328012953754735 47
Time Approximations 0.021.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., -.135e-34]Solution in 0.593s

Time Plot 0 s.
Exiting SolveHard() after 4.07r=35.4632 in [33.94922194 .. 36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844914941694849306177459042387 82,
6.5834347215999338357066565205596536688 29,
467.7873059697452399707191093743255067183]
one interval r = 32.419789556749106656129171319550248987 54 ..
35.851524173881610062508182534649137554 98
Time Approximations 0.022.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=7e-38
Equations at solution: [-.8e-37, .7e-37, -.94e-35]Solution in 0.63s

Time Plot 0 s.
Exiting SolveHard() after 1.01r=34.9451 in [33.70078237 .. 35.85152418]
```

```
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15, none, none,
401.88173905543314875306153704945201393 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684491494169484930617745904238782,
6.58343472159993383570665652055965366882 9,
467.78730596974523997071910937432550671 83]
two intervals r = 12.92327160809620761017756857766744367981 ..
38000000000300834777992134882482414 27/2000000000000000000000000000000000
000 or r = 18.39424858062947671259369493654610303901 ..
38000000000300834777992134882482414 27/2000000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=0
Equations at solution: [0., 0., .605e-35]Solution in 41.425s

Time Plot 0 s.
Exiting SolveHard() after 45.842r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60, none,
401.88173905543314875306153704945201393 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962840669904060074406505279269705,
4.12565179699328649080554759124528571871 1,
440.67123066031463802618777490168966382 46]
two intervals r = 14.35659705103499485658728880016518672283 ..
```

```
380000000003008347779921348824824142 7/2000000000000000000000000000000000
000 or r = 17.703526138452172499773735768893941886 67 ..
380000000003008347779921348824824142 7/2000000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.29e-37, -.1e-37, -.3673e-34]Solution in
4.221s

Time Plot 0 s.
Exiting SolveHard() after 5.27r=15.9119 in [14.35659706 .. 18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60, none,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962840669904060074406505279269705,
4.12565179699328649080554759124528571871 1,
440.67123066031463802618777490168966382 46]
one interval r = 22.397611543853576763191591900636130506 29 ..
27.237223516109504065722761890112568697 09
Time Approximations 0.037.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.158 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
```

```
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064404479085269328227984411098831, rm =
14.37818770533646499343715556391454049316}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., -.756e-34]Solution in 11.241s

Time Plot 0 s.
Exiting SolveHard() after 12.132r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888177691341927813884735673775 70,
4.0048690819820385140503585662143523608 10,
404.86224502617311159962820101562262843 76]
two intervals r = 16.080110077349782534405282963695917908 94 ..
380000000003008347779921348824824142 7/2000000000000000000000000000000000
000 or r = 16.415798127523382699751320379979342779 18 ..
380000000003008347779921348824824142 7/2000000000000000000000000000000000
000
Time Approximations 0.054.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.84e-37, -.1e-37, .1223e-34]Solution in 4.615s

Time Plot 0 s.
Exiting SolveHard() after 5.76r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95, none, none,
358.97362825221633804427833032084868766 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888817769134192781388473567377570,
4.0048690819820385140503585662143523608 10,
404.86224502617311159962820101562262843 76]
one interval r = 21.641943994338973072224286298785979277 30 ..
26.763306600617939708214490995839276476 49
Time Approximations 0.057.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, -.374e-34]Solution in 4.13s

Time Plot 0 s.
Exiting SolveHard() after 5.202r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62, none,
358.97362825221633804427833032084868766 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941747774497006596139352342380058,
5.5896371831331058294207125415547582646 81,
443.83065885970238975029610251011137215 16]
one interval r = 22.467253745056807130968709869335080287 09 ..
27.273884283715734678122358800367974903 46
```

```
Time Approximations 0.04.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.4e-38
Equations at solution: [.1e-37, -.54e-37, -.48e-35]Solution in 3.78s

Time Plot 0 s.
Exiting SolveHard() after 4.755r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62, none,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941747774497006596139352342380058,
5.58963718313310582942071254155475826468 1,
443.83065885970238975029610251011137215 16]
one interval r = 32.15575279515550006632019714836767424889 ..
35.50872228758512481897133887126306846933
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, -.94e-35]Solution in 0.477s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.828r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136497569384668924562258969683574,
5.18778357862738909355727728651791333903 4,
408.65773863725659842724557288479056074 84]
one interval r = 21.71840114668020981736473643115504078838 ..
26.81849303527558234212393269540295361780
Time Approximations 0.061.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.06e-37
Equations at solution: [.1e-37, .106e-36, .83e-35]Solution in 3.781s

Time Plot 0 s.
Exiting SolveHard() after 4.903r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
```

```
401.881739055433148753061537049452013931 6,
389.590015168260714587612950189584056709 5,
328.469398949681948291509364866272063886 2,
401.507571593718296120996012705078960360 7,
358.973628252216338044278330320848687667 0,
398.331471057041911358443575318175059773 2, none, none,
361.508883483364076213527142243302890032 4, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.911931364975693846689245622589696835 74,
5.187783578627389093557277286517913339034,
408.657738637256598427245572884790560748 4]
one interval r = 31.808285987600224487102166669556293909 63 ..
35.000114600609204534005663126300240089 82
Time Approximations 0.018.

hint used Hint := [34.495226611650080820729149248019775312 21, 3, 1, 1,
15.763855890238725343023875204542137417 16, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=1.3e-37
Equations at solution: [.12e-36, -.13e-36, -.327e-34]Solution in 0.424s

Time Plot 0 s.
Exiting SolveHard() after 0.725r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434946029328625162957808444777870 7,
441.642959742568199539335227020809978597 0,
436.917481662652350493193866423484146481 5,
422.984933988511231355812427599822181536 0,
361.525802573115965031355380911276409958 3,
401.881739055433148753061537049452013931 6,
389.590015168260714587612950189584056709 5,
328.469398949681948291509364866272063886 2,
401.507571593718296120996012705078960360 7,
358.973628252216338044278330320848687667 0,
398.331471057041911358443575318175059773 2,
371.483873954270541375838939591423097973 4, none,
361.508883483364076213527142243302890032 4, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.463471105614590221795038987555113973 10,
6.196262565343625273246759053419949597553,
```

```
385.4447438059718898290972900624416441683]
one interval r = 31.60836097545992749817339913465894116072 ..
34.66372795628694400233783729614411071876
Time Approximations 0.029.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, .26e-35]Solution in 0.551s

Time Plot 0 s.
Exiting SolveHard() after 0.834r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34, none,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110561459022179503898755511397310,
6.196262565343625273246759053419949597553,
385.4447438059718898290972900624416441683]
two intervals r = 16.87563408734727586495782411420437649749 ..
380000000003008347779921348824824 1427/2000000000000000000000000000000000
000 or r = 15.55640493866399556304174986588420194496 ..
380000000003008347779921348824824 1427/2000000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.538e-37, 0., .1314e-34]Solution in 1.15s

Time Plot 0 s.
Exiting SolveHard() after 5.191r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874716841865108159416951327452870,
4.88381077997836254781321952361182929137 9,
376.61967857132461152593548352563781647 29]
one interval r = 21.11001304897417405806879148445332241756 ..
26.31784243497940951223727868997001099954
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, .235e-34]Solution in 3.628s

Time Plot 0 s.
Exiting SolveHard() after 4.355r=25.872 in [23.20517308 .. 26.31784245]
```

```
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10, none,
328.46938515040511386034476925427212070 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874716841865108159416951327452870,
4.8838107799783625478132195236118292913 79,
376.61967857132461152593548352563781647 29]
one interval r = 31.53899497720008368198476932929924675213 ..
34.53618386112682933637022215745452415287
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=2.78e-36
Equations at solution: [-.214e-35, .278e-35, -.53e-35]Solution in
0.506s

Time Plot 0 s.
Exiting SolveHard() after 0.797r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
```

```
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10, none,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017570526302402294845212378668456,
6.0258135493179861582424268760029825887 06,
351.42702949972843314929454401699472213 22]
one interval r = 31.36230206122354147412420574320551024472 ..
34.17446640637104710120050932487838138208
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.181e-34]Solution in 0.536s

Time Plot 0 s.
Exiting SolveHard() after 0.78r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
```

```
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10, none,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74, none, none,
292.99969140068872520560704863736957961 51, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175705263024022948452123786684 56,
6.0258135493179861582424268760029825887 06,
351.42702949972843314929454401699472213 22]
two intervals r = 17.981355144247679699233399608470636383 65 ..
380000000003008347779921348824824142 7/2000000000000000000000000000000000
000 or r = 13.846080155069736453270004948286468667 71 ..
380000000003008347779921348824824142 7/2000000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [.125e-36, -.3e-37, .1648e-34]Solution in 1.125s

Time Plot 0 s.
Exiting SolveHard() after 5.138r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74, none, none,
292.99969140068872520560704863736957961 51, none, none, none, none,
```

```
none, none, none, none, none, none]

2 --> 1  target = [27.02037941895899274746522087537485043931,
6.37794387390260246684394762744091471214 0,
423.28832785621465997616146644317991658 02]
one interval r = 31.94661817612278266300661961742391348726 ..
35.21212308675925717964980140051885844636
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.28e-35]Solution in 0.581s

Time Plot 0 s.
Exiting SolveHard() after 0.939r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74, none, none,
292.99969140068872520560704863736957961 51, none, none,
360.06173468621037886932037366597724398 76, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941895899274746522087537485043931,
6.37794387390260246684394762744091471214 0,
423.28832785621465997616146644317991658 02]
two intervals r = 15.22886702390697720936075270840214076675 ..
```

```
380000000003008347779921348824824142 7/2000000000000000000000000000000000
000 or r = 17.12965777138572481972629512634741746396 ..
380000000003008347779921348824824142 7/2000000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.32e-37, .1e-37, -.944e-35]Solution in 1.245s

Time Plot 0 s.
Exiting SolveHard() after 5.312r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74,
375.73285292105369163954184079116981039 80, none,
292.99969140068872520560704863736957961 51, none, none,
360.06173468621037886932037366597724398 76, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343627598037928452144179218136 66,
4.0035598156847074614052632488438814600 08,
404.47973595399055702635636973319129753 46]
two intervals r = 16.096839663426319712074007478680526243 58 ..
380000000003008347779921348824824142 7/2000000000000000000000000000000000
000 or r = 16.399886491755404942700155528090369678 56 ..
380000000003008347779921348824824142 7/2000000000000000000000000000000000
```

```
000
Time Approximations 0.051.

hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.34e-37, -.1e-37, -.409e-35]Solution in 1.487s

Time Plot 0 s.
Exiting SolveHard() after 5.401r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74,
375.73285292105369163954184079116981039 80, none,
292.99969140068872520560704863736957961 51,
358.64341562137079227579008814137551644 10, none,
360.06173468621037886932037366597724398 76, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234362759803792845214417921813666,
4.0035598156847074614052632488438814600 08,
404.47973595399055702635636973319129753 46]
one interval r = 21.63429630012638101341635793305137239540 ..
26.75768169912762554338146251746056388630
Time Approximations 0.049.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
```

```
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.2e-38
Equations at solution: [-.1e-37, -.52e-37, -.318e-34]Solution in 1.041s

Time Plot 0 s.
Exiting SolveHard() after 4.971r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74,
375.73285292105369163954184079116981039 80,
328.11709295960888005715333551744798361 43,
292.99969140068872520560704863736957961 51,
358.64341562137079227579008814137551644 10, none,
360.06173468621037886932037366597724398 76, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954479435553545769808197503536396,
6.19617723019648669796777971758346588 1799,
385.42734026958843540273576788527008870 23]
one interval r = 31.60822049101247641425296893481981668578 ..
34.66347615066160472270687246906398450343
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
```

```
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, .214e-34]Solution in 0.561s

Time Plot 0 s.
Exiting SolveHard() after 0.851r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74,
375.73285292105369163954184079116981039 80,
328.11709295960888005715333551744798361 43,
292.99969140068872520560704863736957961 51,
358.64341562137079227579008814137551644 10, none,
360.06173468621037886932037366597724398 76, none, none,
324.65521225033666569984371992695372026 72, none, none, none, none]

0 --> 1  target = [26.46318954479435553545769808197503536396,
6.19617723019648669796777971758346588 1799,
385.42734026958843540273576785270088 7023]
two intervals r = 16.87629600273882813467171882910705880200 ..
38000000000300834777992134882482414 27/20000000000000000000000000000000000
000 or r = 15.55559000714329385566395375201009337816 ..
38000000000300834777992134882482414 27/20000000000000000000000000000000000
000
Time Approximations 0.053.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
```

```
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.538e-37, 0., -.2859e-34]Solution in 1.138s

Time Plot 0 s.
Exiting SolveHard() after 5.243r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74,
375.73285292105369163954184079116981039 80,
328.11709295960888005715333551744798361 43,
292.99969140068872520560704863736957961 51,
358.64341562137079227579008814137551644 10, none,
360.06173468621037886932037366597724398 76,
336.59441033781895045380601606440146952 17, none,
324.65521225033666569984371992695372026 72, none, none, none, none]

0 --> 2  target = [34.49522661180496485144622382294889240470,
3.89713131609236259949913096279939078718 3,
373.78081885499292991424972091067040798 89]
two intervals r = 17.29769086216152412739193519889336374767 ..
38000000000300834777992134882482414 27/2000000000000000000000000000000000
000 or r = 14.99436407487138701403364647083272743868 ..
38000000000300834777992134882482414 27/2000000000000000000000000000000000
000
Time Approximations 0.084.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
```

```
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.18e-37, 0., -.2573e-34]Solution in 1.215s

Time Plot 0 s.
Exiting SolveHard() after 5.701r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74,
375.73285292105369163954184079116981039 80,
328.11709295960888005715333551744798361 43,
292.99969140068872520560704863736957961 51,
358.64341562137079227579008814137551644 10, none,
360.06173468621037886932037366597724398 76,
336.59441033781895045380601606440146952 17, none,
324.65521225033666569984371992695372026 72,
331.93806792467598041477410023312741314 52, none, none, none]

1 --> 2  target = [34.49522661180496485144622382294889240470,
3.89713131609236259949913096279939078718 3,
373.78081885499292991424972091067040798 89]
one interval r = 21.06068473217865404466052975621153275232 ..
26.26979834301631423718182670525367116229
Time Approximations 0.031.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, -.595e-34]Solution in 0.767s

Time Plot 0 s.
Exiting SolveHard() after 4.269r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74,
375.73285292105369163954184079116981039 80,
328.11709295960888005715333551744798361 43,
292.99969140068872520560704863736957961 51,
358.64341562137079227579008814137551644 10,
299.89866206074138669229536039523055652 48,
360.06173468621037886932037366597724398 76,
336.59441033781895045380601606440146952 17, none,
324.65521225033666569984371992695372026 72,
331.93806792467598041477410023312741314 52, none, none, none]

0 --> 2  target = [33.81362495429818844209197400929587710192,
3.72564899374231973195755580052220690 0104,
325.89209974312380065344683596411445731 81]
two intervals r = 18.55227048999386736764129065042846309273 ..
38000000000300834777992134882482414 27/20000000000000000000000000000000
000 or r = 12.49196935870036995700632142734893511633 ..
38000000000300834777992134882482414 27/20000000000000000000000000000000
000
Time Approximations 0.037.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
```

```
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [.122e-36, -.5e-37, .1345e-34]Solution in 1.156s

Time Plot 0 s.
Exiting SolveHard() after 5.44r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74,
375.73285292105369163954184079116981039 80,
328.11709295960888005715333551744798361 43,
292.99969140068872520560704863736957961 51,
358.64341562137079227579008814137551644 10,
299.89866206074138669229536039523055652 48,
360.06173468621037886932037366597724398 76,
336.59441033781895045380601606440146952 17, none,
324.65521225033666569984371992695372026 72,
331.93806792467598041477410023312741314 52, none, none,
289.54595773939109081140669138595975378 48]

1 --> 2  target = [33.81362495429818844209197400929587710192,
3.72564899374231973195755580052220690 0104,
325.89209974312380065344683596411445731 81]
one interval r = 20.37468935119022248161283439973015294026 ..
25.37892165329273947866085356499865268073
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.2e-37, -.2e-37, .209e-34]Solution in 0.547s

Time Plot 0 s.
Exiting SolveHard() after 1.085r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74,
375.73285292105369163954184079116981039 80,
328.11709295960888005715333551744798361 43,
292.99969140068872520560704863736957961 51,
358.64341562137079227579008814137551644 10,
299.89866206074138669229536039523055652 48,
360.06173468621037886932037366597724398 76,
336.59441033781895045380601606440146952 17,
256.10753187743634210235571972765039241 48,
324.65521225033666569984371992695372026 72,
331.93806792467598041477410023312741314 52, none, none,
289.54595773939109081140669138595975378 48]

1 --> 0  target = [17.93041369690348926132394081161401731687,
4.68650870213330449374791013895292487790 0,
353.30541096591842690451693970635318568 00]
one interval r = 20.73150479110582620889764407232954537932 ..
25.90675353551333952116100675798267384294
Time Approximations 0.03.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
```

```
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.6e-38
Equations at solution: [.2e-37, .46e-37, .275e-34]Solution in 0.634s

Time Plot 0 s.
Exiting SolveHard() after 4.923r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74,
375.73285292105369163954184079116981039 80,
328.11709295960888005715333551744798361 43,
292.99969140068872520560704863736957961 51,
358.64341562137079227579008814137551644 10,
299.89866206074138669229536039523055652 48,
360.06173468621037886932037366597724398 76,
336.59441033781895045380601606440146952 17,
256.10753187743634210235571972765039241 48,
324.65521225033666569984371992695372026 72,
331.93806792467598041477410023312741314 52,
304.79958327292129671341328470334277643 52, none,
289.54595773939109081140669138595975378 48]

2 --> 0  target = [17.93041369690348926132394081161401731687,
4.68650870213330449374791013895292487790 0,
353.30541096591842690451693970635318568 00]
one interval r = 31.37435487001713179334534577987736310988 ..
34.20127520049719259820766487136804605312
Time Approximations 0.016.
```

```
hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .58e-35]Solution in 0.344s

Time Plot 0 s.
Exiting SolveHard() after 0.61r=33.7963 in [32.25770943 .. 34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494602932862516295780844477787 07,
441.64295974256819953933522702080997859 70,
436.91748166265235049319386642348414648 15,
422.98493398851123135581242759982218153 60,
361.52580257311596503135538091127640995 83,
401.88173905543314875306153704945201393 16,
389.59001516826071458761295018958405670 95,
328.46939894968194829150936486627206388 62,
401.50757159371829612099601270507896036 07,
358.97362825221633804427833032084868766 70,
398.33147105704191135844357531817505977 32,
371.48387395427054137583893959142309797 34,
336.61215842752345960195303348330760689 96,
361.50888348336407621352714224330289003 24,
324.67144994067453504143531498868049235 10,
302.31384316659452252669041222783141998 16,
328.46938515040511386034476925427212070 23,
343.81340626210806749248687868676427042 74,
375.73285292105369163954184079116981039 80,
328.11709295960888005715333551744798361 43,
292.99969140068872520560704863736957961 51,
358.64341562137079227579008814137551644 10,
299.89866206074138669229536039523055652 48,
360.06173468621037886932037366597724398 76,
336.59441033781895045380601606440146952 17,
256.10753187743634210235571972765039241 48,
324.65521225033666569984371992695372026 72,
331.93806792467598041477410023312741314 52,
304.79958327292129671341328470334277643 52,
323.46169178179511559532820304907596878 32,
289.54595773939109081140669138595975378 48]

Cascade time 154.59
counts: 28, 28

 Iteration 64
```

```
Start Generation 1
1 --> 0  target = [11.999999999980219804216022476154847965 00,
6.217012503099440926733407653239227755734,
485.549080903692615702644579786216714 7014]
one interval r = 23.40850301668285219384407395977302251753 ..
27.67578046438359578399207055935226615736
Time Approximations 0.044.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.89e-37
Equations at solution: [-.9e-37, .289e-36, .13e-35]Solution in 3.86s

Time Plot 0 s.
Exiting SolveHard() after 5.005r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999980219804216022476154847965 00,
6.217012503099440926733407653239227755734,
485.549080903692615702644579786216714 7014]
one interval r = 32.62814779211760390041942696767325896439 ..
36.10248388952021092171262760760292210876
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .87e-35]Solution in 0.618s

Time Plot 0 s.
```

```
Exiting SolveHard() after 1.031r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684490251042575723221496804448977,
6.58343472181600007245010297821274373119 8,
467.78730596389826802156430514405789404 95]
one interval r = 32.41978955659741896218572025876105748864 ..
35.85152417380544154966459716714294684190
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, -.152e-34]Solution in 0.643s

Time Plot 0 s.
Exiting SolveHard() after 1.011r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55, none, none,
401.88173904643463763117525772745196630 86, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684490251042575723221496804448977,
6.58343472181600007245010297821274373119 8,
467.78730596389826802156430514405789404 95]
two intervals r = 12.92327160826950945769551553219215761617 ..
189999999999498712234450213758052507 09/10000000000000000000000000000000
00000 or r = 18.39424858034359774230835784387549393412 ..
```

```
189999999999498712234450213758052507 09/1000000000000000000000000000000000
00000
Time Approximations 0.043.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.2e-38
Equations at solution: [-.3e-37, -.32e-37, -.177e-35]Solution in
43.457s

Time Plot 0 s.
Exiting SolveHard() after 47.827r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84, none,
401.88173904634637631175257727451966308 6, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962833209464112014496442149098844,
4.12565179694808656091739855378380298492 4,
440.67123065531025760633794869482626188 98]
two intervals r = 14.35659705111297975457411393485467269002 ..
189999999999498712234450213758052507 09/1000000000000000000000000000000000
00000 or r = 17.70352613815986563661411591246085651736 ..
189999999999498712234450213758052507 09/1000000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.43e-37, -.2e-37, .2967e-34]Solution in 4.16s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.254r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84, none,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962833209464112014496442149098844,
4.12565179694808656091739855378380298492 4,
440.67123065531025760633794869482626188 98]
one interval r = 22.39761154376497398827598265290317386478 ..
27.23722351609002333309510114751534062962
Time Approximations 0.038.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.159 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064404310035558493399837146017460, rm =
14.37818770746951702961736282005578679021}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.32e-37
Equations at solution: [-.1e-37, -.132e-36, -.2e-36]Solution in 11.231s

Time Plot 0 s.
Exiting SolveHard() after 12.107r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
```

```
361.525802564058201784379689806020400134 6,
401.881739046434637631175257727451966308 6,
389.590015162765087253938849328682826938 1, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888033901235084996196770599036 35,
4.004869081923205999908896619345091296171,
404.862245017323879925386041820753909319 6]
two intervals r = 16.080110077526235708871539909277357321 28 ..
189999999999498712234450213758052507 09/100000000000000000000000000000000
00000 or r = 16.415798127052061783464770035302965204 87 ..
189999999999498712234450213758052507 09/100000000000000000000000000000000
00000
Time Approximations 2.93.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.17e-37, 0., -.416e-35]Solution in 1.65s

Time Plot 0 s.
Exiting SolveHard() after 5.676r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434941714620986409443078132412 5777,
441.642959736584901219562978309850822688 1,
436.917481657412612562760192321633796445 5,
422.984933979167833633577483021927974008 4,
361.525802564058201784379689806020400134 6,
401.881739046434637631175257727451966308 6,
389.590015162765087253938849328682826938 1, none, none,
358.973628243358587643529694794917620704 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888033901235084996196770599036 35,
4.004869081923205999908896619345091296171,
404.862245017323879925386041820753909319 6]
one interval r = 21.641943994182643620331141006311963718 87 ..
26.763306600523436928063345083749181511 51
Time Approximations 0.05.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
```

```
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, .480e-34]Solution in 1.028s

Time Plot 0 s.
Exiting SolveHard() after 5.094r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20, none,
358.97362824335858764352969479491762070 43, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941781068274475740105360036452995,
5.58963718307833872682520866441728708612 0,
443.83065884992454607613715732509013216 51]
one interval r = 22.46725374486222919976000855858534147370 ..
27.27388428364321755616878047220199313836
Time Approximations 0.043.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.61e-37
Equations at solution: [-.1e-37, .161e-36, -.47e-35]Solution in 3.933s

Time Plot 0 s.
Exiting SolveHard() after 4.926r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
```

```
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20, none,
358.97362824335858764352969479491762070 43,
398.33147104419746226562957149171127659 7, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941781068274475740105360036452995,
5.5896371830783387268252086644172870861 20,
443.83065884992454607613715732509013216 51]
one interval r = 32.15575279495937009506497395180496379583 ..
35.50872228744303390274371254887805878135
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., .234e-34]Solution in 0.494s

Time Plot 0 s.
Exiting SolveHard() after 0.862r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
```

```
398.331471044197462265662957149171127659 7, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136502187772027522893229182995983,
5.1877835786286256215453687267653369777 73,
408.65773863156487071071060853006142229 52]
one interval r = 21.71840114658556342337469522525767210857 ..
26.81849303522956049086166734454914680975
Time Approximations 0.061.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.06e-37
Equations at solution: [.1e-37, .106e-36, .348e-34]Solution in 3.958s

Time Plot 0 s.
Exiting SolveHard() after 5.078r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.331471044197462265662957149171127659 7, none, none,
361.50888347445062541981315035412787109 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136502187772027522893229182995983,
5.1877835786286256215453687267653369777 73,
408.65773863156487071071060853006142229 52]
one interval r = 31.80828598744504625740337054824746647581 ..
35.00011460051260070766473434590244215852
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
```

```
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, .312e-34]Solution in 0.438s

Time Plot 0 s.
Exiting SolveHard() after 0.749r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226562957149171127659 7,
371.48387394762806071580805732282914046 65, none,
361.50888347445062541981315035412787109 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110550233801679457719032765070935,
6.19626255542833697263259678313969611745,
385.44474379695665781723403466088570483 86]
one interval r = 31.60836097527784853797033626188652972216 ..
34.66372795613381843172056553777555469994
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, -.115e-34]Solution in 0.57s

Time Plot 0 s.
```

```
Exiting SolveHard() after 0.848r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65, none,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110550233801679457719032765070935,
6.19626256554283369726325967831396961174 5,
385.44474379695665781723403466088570483 86]
two intervals r = 16.87563408747154453347949455094835123495 ..
189999999999498712234450213758052507 09/1000000000000000000000000000000000
00000 or r = 15.55640493816308780940526343033012556680 ..
189999999999498712234450213758052507 09/1000000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.07 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175114834522194932592367896270437, rm
= 2.33653277386350255007141927368192211711 7}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.717e-37, 0., -.343e-35]Solution in 29.982s

Time Plot 0 s.
Exiting SolveHard() after 34.254r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874727190524440593871983806609303,
4.8838107799486412212129827688310410359 83,
376.61967856212606772502187466880349803 79]
one interval r = 21.11001304883349619143193317260040914178 ..
26.31784243485743566576198790500743324359
Time Approximations 0.033.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, -.216e-34]Solution in 0.812s

Time Plot 0 s.
Exiting SolveHard() after 4.629r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
```

```
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27, none,
328.46938513780791974131071806258964415 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874727190524440593871983806609303,
4.8838107799486412212129827688310410359 83,
376.61967856212606772502187466880349803 79]
one interval r = 31.53899497701830809656790711272959429875 ..
34.53618386096805762466466110813220154795
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=4.30e-36
Equations at solution: [-.330e-35, .430e-35, .105e-34]Solution in
0.485s

Time Plot 0 s.
Exiting SolveHard() after 0.776r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27, none,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.872050175500948221443153292902944429 17,
6.025813549496754820686337979211883690192,
351.42702948705832899734135556474081570 87]
one interval r = 31.362302061025932309144936215992584189 00 ..
34.174466406154791535711697387018145491 57
Time Approximations 0.016.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, .273e-34]Solution in 0.527s

Time Plot 0 s.
Exiting SolveHard() after 0.779r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27, none,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14, none, none,
292.99969138510612695623147122393602316 64, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175500948221443153292902944429 17,
6.025813549496754820686337979211883690192,
351.42702948705832899734135556474081570 87]
two intervals r = 17.981355144365263314671172120191852668 48 ..
189999999999498712234450213758052507 09/10000000000000000000000000000000
00000 or r = 13.846080154374184984858665083482891980 21 ..
```

```
189999999999498712234450213758052507 09/100000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.094 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713635821505263469838048007590 09, rm
= 2.7345009928067474051287642898872679550 31}});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.53e-37, -.2e-37, -.2629e-34]Solution in
18.273s

Time Plot 0 s.
Exiting SolveHard() after 22.292r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14, none, none,
292.99969138510612695623147122393602316 64, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418826070574079208608330768948 45,
6.3779438740849370317832205204309518254 86,
423.28832784331342852739557927688831988 24]
one interval r = 31.946618175897843135688132727218356619 88 ..
35.212123086563756323077180820269488106 60
```

```
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=7e-38
Equations at solution: [.7e-37, -.7e-37, .120e-34]Solution in 3.46s

Time Plot 0 s.
Exiting SolveHard() after 3.818r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14, none, none,
292.99969138510612695623147122393602316 64, none, none,
360.06173467050953935294115106628324734 04, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941882607057407920860833076894845,
6.37794387408493703178322052043095182548 6,
423.28832784331342852739557927688831988 24]
two intervals r = 15.22886702433335753464219508553267528150 ..
18999999999949871223445021375805250709/10000000000000000000000000000000
00000 or r = 17.12965777079873847416693973620670855569 ..
18999999999949871223445021375805250709/10000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
```

```
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 9.818 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.51537054123711759343009629485720043889, rm
= 2.06406829858398164997755113209435356727 2}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.31e-37, -.1e-37, .3705e-34]Solution in
34.045s

Time Plot 0 s.
Exiting SolveHard() after 35.373r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14,
375.73285290403221683700020460118551747 02, none,
292.99969138510612695623147122393602316 64, none, none,
360.06173467050953935294115106628324734 04, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234346922380814029849179906405003,
4.00355981562245520335881262855737184185 0,
404.47973594414524703950155701509966140 63]
two intervals r = 16.09683966364514511145983555334631276921 ..
18999999999949871223445021375805250709/10000000000000000000000000000000
00000 or r = 16.39988649124205905681357128844806993019 ..
18999999999949871223445021375805250709/10000000000000000000000000000000
00000
```

```
Time Approximations 0.058.

hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [-.18e-37, 0., -.2674e-34]Solution in 4.69s

Time Plot 0 s.
Exiting SolveHard() after 5.799r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14,
375.73285290403221683700020460118551747 02, none,
292.99969138510612695623147122393602316 64,
358.64341561165218660153065531957780529 46, none,
360.06173467050953935294115106628324734 04, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234346922380814029849179906405003,
4.00355981562245520335881262855737184185 0,
404.47973594414524703950155701509966140 63]
one interval r = 21.63429629995039450372048594416776360288 ..
26.75768169901815484737969784129652086689
Time Approximations 0.057.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, -.270e-34]Solution in 4.236s

Time Plot 0 s.
Exiting SolveHard() after 5.301r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14,
375.73285290403221683700020460118551747 02,
328.11709294609075395108979434673530073 00,
292.99969138510612695623147122393602316 64,
358.64341561165218660153065531957780529 46, none,
360.06173467050953935294115106628324734 04, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954468449166837009195819559131665,
6.19617723039642221920390981239405986 6444,
385.42734026072164481098721471567508123 26]
one interval r = 31.60822049083159879079415359155391593498 ..
34.66347615051062088285438997495005072010
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
```

```
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., .11e-35]Solution in 0.593s

Time Plot 0 s.
Exiting SolveHard() after 0.891r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14,
375.73285290403221683700020460118551747 02,
328.11709294609075395108979434673530073 00,
292.99969138510612695623147122393602316 64,
358.64341561165218660153065531957780529 46, none,
360.06173467050953935294115106628324734 04, none, none,
324.65521223835190440696234901363177247 67, none, none, none, none]

0 --> 1  target = [26.46318954468449166837009195819559131665,
6.19617723039642221920390981239405986644 4,
385.42734026072164481098721471567508123 26]
two intervals r = 16.87629600285739779957771678019479413898 ..
189999999999498712234450213758052507 09/1000000000000000000000000000000
00000 or r = 15.55559000664932251516775209686719940454 ..
189999999999498712234450213758052507 09/1000000000000000000000000000000
00000
Time Approximations 0.085.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
```

```
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 6.851 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852526538556359210253672334224437, rm
= 2.33669042792867722077615394868696026392 5}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.716e-37, 0., .565e-35]Solution in 29.713s

Time Plot 0 s.
Exiting SolveHard() after 33.94r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14,
375.73285290403221683700020460118551747 02,
328.11709294609075395108979434673530073 00,
292.99969138510612695623147122393602316 64,
358.64341561165218660153065531957780529 46, none,
360.06173467050953935294115106628324734 04,
336.59441032524438814212397373686764548 23, none,
324.65521223835190440696234901363177247 67, none, none, none, none]

0 --> 2  target = [34.49522661168497867648853237741440428219,
3.89713131604113039661675038511952109477 7,
373.78081884854932168704509350998434466 93]
two intervals r = 17.297690862161567944875455497535393642 99 ..
189999999999498712234450213758052507 09/10000000000000000000000000000000
00000 or r = 14.994364074490533796463680981341525334 37 ..
189999999999498712234450213758052507 09/10000000000000000000000000000000
00000
Time Approximations 0.081.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
```

```
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [.36e-37, -.1e-37, .2549e-34]Solution in 1.202s

Time Plot 0 s.
Exiting SolveHard() after 5.698r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14,
375.73285290403221683700020460118551747 02,
328.11709294609075395108979434673530073 00,
292.99969138510612695623147122393602316 64,
358.64341561165218660153065531957780529 46, none,
360.06173467050953935294115106628324734 04,
336.59441032524438814212397373686764548 23, none,
324.65521223835190440696234901363177247 67,
331.93806791787297710395025113667005323 49, none, none, none]

1 --> 2  target = [34.49522661168497867648853237741440428219,
3.89713131604113039661675038511952109477 7,
373.78081884854932168704509350998434466 93]
one interval r = 21.06068473208814884502413887350183364417 ..
26.26979834293911664251386839687792664954
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, -.454e-34]Solution in 3.676s

Time Plot 0 s.
Exiting SolveHard() after 4.443r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14,
375.73285290403221683700020460118551747 02,
328.11709294609075395108979434673530073 00,
292.99969138510612695623147122393602316 64,
358.64341561165218660153065531957780529 46,
299.89866205036459467497581704483300911 51,
360.06173467050953935294115106628324734 04,
336.59441032524438814212397373686764548 23, none,
324.65521223835190440696234901363177247 67,
331.93806791787297710395025113667005323 49, none, none, none]

0 --> 2  target = [33.81362495408439118013829256251900047353,
3.72564899366943147840669496124874204130 8,
325.89209973106650712457028885202336212 21]
two intervals r = 18.552270489991043892791877420108283082 25 ..
18999999999949871223445021375805250709/10000000000000000000000000000000
00000 or r = 12.491969358090564671228043586387003300 89 ..
18999999999949871223445021375805250709/10000000000000000000000000000000
00000
Time Approximations 0.043.
```

```
hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=8e-38
Equations at solution: [.173e-36, -.8e-37, .2639e-34]Solution in 4.174s

Time Plot 0 s.
Exiting SolveHard() after 5.635r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226562957149171127659 7,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14,
375.73285290403221683700020460118551747 02,
328.11709294609075395108979434673530073 00,
292.99969138510612695623147122393602316 64,
358.64341561165218660153065531957780529 46,
299.89866205036459467497581704483300911 51,
360.06173467050953935294115106628324734 04,
336.59441032524438814212397373686764548 23, none,
324.65521223835190440696234901363177247 67,
331.93806791787297710395025113667005323 49, none, none,
289.54595772759918872952188039439088329 94]

1 --> 2  target = [33.81362495408439118013829256251900047353,
3.72564899366943147840669496124874204 1308,
325.89209973106650712457028885202336212 21]
one interval r = 20.37468935107757088999522518834009419404 ..
25.37892165307949757887702648116896104542
Time Approximations 0.028.
```

```
hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, -.519e-34]Solution in 0.593s

Time Plot 0 s.
Exiting SolveHard() after 1.142r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14,
375.73285290403221683700020460118551747 02,
328.11709294609075395108979434673530073 00,
292.99969138510612695623147122393602316 64,
358.64341561165218660153065531957780529 46,
299.89866205036459467497581704483300911 51,
360.06173467050953935294115106628324734 04,
336.59441032524438814212397373686764548 23,
256.10753186195934380180505851502822833 04,
324.65521223835190440696234901363177247 67,
331.93806791787297710395025113667005323 49, none, none,
289.54595772759918872952188039439088329 94]

1 --> 0  target = [17.93041369704425799707219757931992706882,
4.68650870207198486262191541159939380255 5,
353.30541095269802745085642859427422536 49]
one interval r = 20.73150479092995520148469269465776497180 ..
25.90675353530058016842228570830083172218
```

```
Time Approximations 0.036.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=9.4e-38
Equations at solution: [-.4e-37, -.94e-37, .157e-34]Solution in 3.663s

Time Plot 0 s.
Exiting SolveHard() after 4.385r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14,
375.73285290403221683700020460118551747 02,
328.11709294609075395108979434673530073 00,
292.99969138510612695623147122393606316 64,
358.64341561165218660153065531957780529 46,
299.89866205036459467497581704483300911 51,
360.06173467050953935294115106628324734 04,
336.59441032524438814212397373686764548 23,
256.10753186195934380180505851502822833 04,
324.65521223835190440696234901363177247 67,
331.93806791787297710395025113667005323 49,
304.79958325630529381330068920329187569 11, none,
289.54595772759918872952188039439088329 94]

2 --> 0  target = [17.93041369704425799707219757931992706882,
4.68650870207198486262191541159939380255 5,
```

```
353.30541095269802745085642859427422536 49]
one interval r = 31.37435486981509607330246054759998184608 ..
34.20127520027358399247132324231406015028
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, -.235e-34]Solution in 0.383s

Time Plot 0 s.
Exiting SolveHard() after 0.661r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494171462098640944307813241257 77,
441.64295973658490121956297830985082268 81,
436.91748165741261256276019232163379644 55,
422.98493397916783363357748302192797400 84,
361.52580256405820178437968980602040013 46,
401.88173904643463763117525772745196630 86,
389.59001516276508725393884932868282693 81,
328.46939893708103865307902769150156771 20,
401.50757158374547506995821301813084938 44,
358.97362824335858764352969479491762070 43,
398.33147104419746226566295714917112765 97,
371.48387394762806071580805732282914046 65,
336.61215841479739295845394187836945974 39,
361.50888347445062541981315035412787109 17,
324.67144992855130734914323193898415051 27,
302.31384315047890646752261470079605968 02,
328.46938513780791974131071806258964415 99,
343.81340625234370342531798721020635709 14,
375.73285290403221683700020460118551747 02,
328.11709294609075395108979434673530073 00,
292.99969138510612695623147122393602316 64,
358.64341561165218660153065531957780529 46,
299.89866205036459467497581704483300911 51,
360.06173467050953935294115106628324734 04,
336.59441032524438814212397373686764548 23,
256.10753186195934380180505851502822833 04,
324.65521223835190440696234901363177247 67,
331.93806791787297710395025113667005323 49,
304.79958325630529381330068920329187569 11,
323.46169176845402154288629356730209140 35,
```

```
289.54595772759918872952188039439088329 94]

Cascade time 265.355
counts: 28, 28

 Iteration 65

Start Generation 1
1 --> 0  target = [12.00000000013814020933135625759707821500,
6.217012502829566603971700611474902455808,
485.54908089264182074688311287423507012 07]
one interval r = 23.40850301641618372729087488793407046864 ..
27.67578046424354954127303666847272241023
Time Approximations 0.041.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.05e-37
Equations at solution: [.4e-37, -.105e-36, -.15e-35]Solution in 1.046s

Time Plot 0 s.
Exiting SolveHard() after 2.231r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024 28750,
441.64295972576634789202408367067986741 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000013814020933135625759707821500,
6.217012502829566603971700611474902455808,
485.54908089264182074688311287423507012 07]
one interval r = 32.62814779209908109949498864352679324455 ..
36.10248388935865404361309100752240680793
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, -.144e-34]Solution in 0.595s

Time Plot 0 s.
Exiting SolveHard() after 4.095r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684474352700913185078426760201679,
6.58343472163882350971254425188052299268 6,
467.78730595249519489044803552401033059 58]
one interval r = 32.41978955658919015072866615598176611772 ..
35.85152417364476923949018049196111136696
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, -.147e-34]Solution in 0.648s

Time Plot 0 s.
Exiting SolveHard() after 1.037r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54, none, none,
401.88173903639853980254901696689739505 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
0 --> 1  target = [27.523596844743527009131850784267602016 79,
6.5834347216388235097125442518805229926 86,
467.78730595249519489044803552401033059 58]
two intervals r = 12.923271608517376234033014849335474130 35 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000 or r = 18.394248580195448022373162357845183333 40 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.5e-38
Equations at solution: [-.5e-37, -.55e-37, -.9299e-35]Solution in
42.994s

Time Plot 0 s.
Exiting SolveHard() after 47.457r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13, none,
401.88173903639853980254901696689739505 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628205526062587182455711221580 72,
4.1256517968041817468972461533711519772 05,
440.67123064556376353251529439157464952 72]
two intervals r = 14.356597051336326516050068462592408777 74 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000 or r = 17.703526137950030128918482850836714617 06 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
```

```
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [.27e-37, .2e-37, -.10050e-34]Solution in 4.284s

Time Plot 0 s.
Exiting SolveHard() after 5.375r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13, none,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962820552606258718245571122158072,
4.12565179680418174689724615337115197720 5,
440.67123064556376353251529439157464952 72]
one interval r = 22.39761154354434240647164572670626461951 ..
27.23722351592681475362398436804983568887
Time Approximations 0.04.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.298 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064376500734884397506001114718945, rm =
14.37818770346397202661235844539993868492}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, .177e-34]Solution in 11.581s

Time Plot 0 s.
Exiting SolveHard() after 12.48r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888790821743590915889795111091589,
4.0048690817747734153360852245925209862 15,
404.86224500652356069829207873320835240 64]
two intervals r = 16.08011007782237117554753758018023262487 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000 or r = 16.41579812665967031882941413589404923717 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000
Time Approximations 4.037.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.34e-37, 0., .33876e-34]Solution in 1.456s

Time Plot 0 s.
Exiting SolveHard() after 6.591r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99, none, none,
358.97362823365367479312987506788222833 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888790821743590915889795111091589,
```

```
4.0048690817747734153360852245925209862 15,
404.8622450065235606982920787332083524064]
one interval r = 21.641943993966801814428538379442269461 38 ..
26.763306600320868577145407685628154571 23
Time Approximations 0.055.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, -.496e-34]Solution in 3.845s

Time Plot 0 s.
Exiting SolveHard() after 4.938r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80, none,
358.97362823365367479312987506788222833 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589418171415122817888044681260535 34,
5.5896371828157599521779382657104618701 09,
443.83065883744251120429641282707158904 73]
one interval r = 22.467253744579431064783506553717648767 74 ..
27.273884283450872285020796648858496744 61
Time Approximations 0.038.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
```

```
Accepted {r=27.0204, rm=13.5759} with Delta=5.4e-38
Equations at solution: [-.1e-37, .54e-37, .99e-35]Solution in 3.792s

Time Plot 0 s.
Exiting SolveHard() after 4.796r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80, none,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941817141512281788804468126053534,
5.5896371828157599521779382657104618701 09,
443.83065883744251120429641282707158904 73]
one interval r = 32.15575279495983903493326988306812979772 ..
35.50872228727579525110450542046433856550
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, .130e-34]Solution in 0.491s

Time Plot 0 s.
Exiting SolveHard() after 0.851r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
```

```
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136522322638672951452483115337247,
5.18778357842722817027198466267580567551 9,
408.65773862289487798555792141264933037 47]
one interval r = 21.71840114640932063098385053472004191735 ..
26.81849303506035614209808228213536455153
Time Approximations 0.062.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.58e-37
Equations at solution: [-.2e-37, -.158e-36, .141e-34]Solution in 4.011s

Time Plot 0 s.
Exiting SolveHard() after 5.13r=26.4632 in [23.93303356 .. 26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85, none, none,
361.50888346596556202765396007029833534 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136522322638672951452483115337247,
5.18778357842722817027198466267580567551 9,
408.65773862289487798555792141264933037 47]
```

```
one interval r = 31.80828598751218621516337552367528065544 ..
35.00011460041597405209152864950946637372
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.293e-34]Solution in 0.408s

Time Plot 0 s.
Exiting SolveHard() after 0.695r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51, none,
361.50888346596556202765396007029833534 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110531532881713374857504480250977,
6.19626256537196457826546518117621783265 2,
385.44474378793533812171218119821684979 07]
one interval r = 31.60836097535942481657583869922697176389 ..
34.66372795604372144860067708966622132816
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=8e-38
Equations at solution: [.5e-37, -.8e-37, -.204e-34]Solution in 0.547s

Time Plot 0 s.
Exiting SolveHard() after 0.818r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51, none,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110531532881713374857504480250977,
6.19626256537196457826546518117621783265 2,
385.44474378793533812171218119821684979 07]
two intervals r = 16.875634087688664407416659089378729533 75 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000 or r = 15.556404937770360138736953301460924684 34 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000
Time Approximations 0.058.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.359e-37, 0., .12815e-34]Solution in 1.169s

Time Plot 0 s.
Exiting SolveHard() after 5.549r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874750966180883209232883452175779,
4.88381077974740334124413488471054249349 0,
376.61967855240724254772247117849415240 96]
one interval r = 21.11001304866818452920978741064357934378 ..
26.31784243465421709420472720074833050374
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, .168e-34]Solution in 3.668s

Time Plot 0 s.
Exiting SolveHard() after 4.396r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
```

```
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23, none,
328.46938512838804040179666642415235759 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874750966180883209232883452175779,
4.8838107797474033412441348847105424934 90,
376.61967855240724254772247117849415240 96]
one interval r = 31.53899497710120058486462992075414920827 ..
34.53618386087256705666642428826174934680
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.8e-37
Equations at solution: [.29e-36, -.38e-36, -.41e-35]Solution in 0.513s

Time Plot 0 s.
Exiting SolveHard() after 0.81r=34.0898 in [32.52213872 .. 34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23, none,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
2 --> 1  target = [25.87205017527990219495108865649729711736,
6.025813549318431128375475696680991334953,
351.42702947706168970978325106292586700 50]
one interval r = 31.362302061126621271320427948910872425 70 ..
34.17446640606955166621151876681941484901
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .14e-35]Solution in 0.528s

Time Plot 0 s.
Exiting SolveHard() after 0.782r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23, none,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08, none, none,
292.99969137650484406754519975593905793 56, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017527990219495108865649729711736,
6.025813549318431128375475696680991334953,
351.42702947706168970978325106292586700 50]
two intervals r = 17.98135514458340268039247845693213824843 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000 or r = 13.84608015380632868210210034974672753524 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000
```

```
Time Approximations 0.041.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [.18e-37, 0., -.2545e-35]Solution in 1.124s

Time Plot 0 s.
Exiting SolveHard() after 5.02r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08, none, none,
292.99969137650484406754519975593905793 56, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941860504848845602963913135258717,
6.37794387389661097897431586476300561934 0,
423.28832783009217285068980433518465072 74]
one interval r = 31.94661817590958296831473116844487050429 ..
35.21212308639441773874056711944091631211
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
```

```
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=8e-38
Equations at solution: [.7e-37, -.8e-37, -.277e-34]Solution in 0.59s

Time Plot 0 s.
Exiting SolveHard() after 0.95r=34.3272 in [33.10127385 .. 35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08, none, none,
292.99969137650484406754519975593905793 56, none, none,
360.06173465881570914189667626589090725 51, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941860504848845602963913135258717,
6.37794387389661097897431586476300561934 0,
423.28832783009217285068980433518465072 74]
two intervals r = 15.22886702474476502593339125734245750797 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000 or r = 17.12965777039831811689893389821826854395 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.30e-37, -.1e-37, -.676e-36]Solution in 1.328s

Time Plot 0 s.
Exiting SolveHard() after 5.464r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624448720320535898601087480200 8,
375.73285289009648490314344110254685601 57, none,
292.99969137650484406754519975593905793 56, none, none,
360.06173465881570914189667626589090725 51, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343342370256743726765206259551 87,
4.0035598154736919026577696718179764410 33,
404.47973593325140815792092972414706745 45]
two intervals r = 16.096839663945265151941249735330558222 41 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000 or r = 16.399886490844121508376082513462156857 92 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.52e-37, -.1e-37, -.7879e-35]Solution in
1.519s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.678r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08,
375.73285289009648490314344110254685601 57, none,
292.99969137650484406754519975593905793 56,
358.64341560186493294884429563122271704 74, none,
360.06173465881570914189667626589090725 51, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234334237025674372676520625955187,
4.0035598154736919026577696718179764410 33,
404.47973593325140815792092972414706745 45]
one interval r = 21.634296299733055574400516210432383056 74 ..
26.757681698813938896067167880069155889 17
Time Approximations 0.048.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=1.00e-37
Equations at solution: [-.3e-37, -.100e-36, -.445e-34]Solution in
1.045s

Time Plot 0 s.
Exiting SolveHard() after 5.218r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
```

```
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08,
375.73285289009648490314344110254685601 57,
328.11709293658536534704147910666561917 35,
292.99969137650484406754519975593905793 56,
358.64341560186493294884429563122271704 74, none,
360.06173465881570914189667626589090725 51, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954449741927051927683221828536932,
6.19617723022553552325266941019205531281 4,
385.42734025169697764786770764647204564 19]
one interval r = 31.60822049091316120340578639803253011125 ..
34.66347615042048454758952096217256399741
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., -.220e-34]Solution in 0.568s

Time Plot 0 s.
Exiting SolveHard() after 0.851r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08,
375.73285289009648490314344110254685601 57,
328.11709293658536534704147910666561917 35,
292.99969137650484406754519975593905793 56,
358.64341560186493294884429563122271704 74, none,
360.06173465881570914189667626589090725 51, none, none,
324.65521223062302954208557009562325014 64, none, none, none, none]

0 --> 1  target = [26.46318954449741927051927683221828536932,
6.19617723022553552325266941019205531281 4,
385.42734025169697764786770764647204564 19]
two intervals r = 16.87629600307463919056238247291862028041 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000 or r = 15.55559000625637147803263968831769147452 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.358e-37, 0., -.41024e-34]Solution in 1.19s

Time Plot 0 s.
Exiting SolveHard() after 6.245r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624448720320535898601087480200 8,
375.73285289009648490314344110254685601 57,
328.11709293658536534704147910666561917 35,
292.99969137650484406754519975593905793 56,
358.64341560186493294884429563122271704 74, none,
360.06173465881570914189667626589090725 51,
336.59441031591586823537805890201025631 90, none,
324.65521223062302954208557009562325014 64, none, none, none, none]

0 --> 2  target = [34.49522661162145449113359755023337100415,
3.89713131590307567283446051410032165321 5,
373.78081884094027350320223686290777922 21]
two intervals r = 17.29769086232462617571807884352064769709 ..
19000000000009771568580044401979484529/10000000000000000000000000000000000
00000 or r = 14.99436407412338086811823614968634700978 ..
19000000000009771568580044401979484529/10000000000000000000000000000000000
00000
Time Approximations 0.084.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.72e-37, 0., -.14779e-34]Solution in 1.227s

Time Plot 0 s.
Exiting SolveHard() after 6.209r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08,
375.73285289009648490314344110254685601 57,
328.11709293658536534704147910666561917 35,
292.99969137650484406754519975593905793 56,
358.64341560186493294884429563122271704 74, none,
360.06173465881570914189667626589090725 51,
336.59441031591586823537805890201025631 90, none,
324.65521223062302954208557009562325014 64,
331.93806791083046667813777122577684600 88, none, none, none]

1 --> 2  target = [34.49522661162145449113359755023337100415,
3.89713131590307567283446051410032165321 5,
373.78081884094027350320223686290777922 21]
one interval r = 21.06068473196249895141242573500124490910 ..
26.26979834277028294201986110448455873296
Time Approximations 0.034.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., -.444e-34]Solution in 0.761s

Time Plot 0 s.
Exiting SolveHard() after 4.71r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
```

```
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08,
375.73285289009648490314344110254685601 57,
328.11709293658536534704147910666561917 35,
292.99969137650484406754519975593905793 56,
358.64341560186493294884429563122271704 74,
299.89866204390666654227449515992716480 22,
360.06173465881570914189667626589090725 51,
336.59441031591586823537805890201025631 90, none,
324.65521223062302954208557009562325014 64,
331.93806791083046667813777122577684600 88, none, none, none]

0 --> 2  target = [33.81362495403650970544465465678881861035,
3.72564899352766460560471273023222727857 5,
325.89209972262261679973843014468018789 38]
two intervals r = 18.552270490138435222418626444275789826 75 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000 or r = 12.491969357538279667556850677862759259 86 ..
19000000000009771568580044401979484529/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1e-38
Equations at solution: [.34e-37, -.1e-37, -.13385e-34]Solution in
1.142s

Time Plot 0 s.
Exiting SolveHard() after 5.784r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08,
375.73285289009648490314344110254685601 57,
328.11709293658536534704147910666561917 35,
292.99969137650484406754519975593905793 56,
358.64341560186493294884429563122271704 74,
299.89866204390666654227449515992716480 22,
360.06173465881570914189667626589090725 51,
336.59441031591586823537805890201025631 90, none,
324.65521223062302954208557009562325014 64,
331.93806791083046667813777122577684600 88, none, none,
289.54595771965109441738319337024228166 44]

1 --> 2  target = [33.81362495403650970544465465678881861035,
3.72564899352766460560471273023222727857 5,
325.89209972262261679973843014468018789 38]
one interval r = 20.37468935099577333268094973773523087518 ..
25.37892165287872025078830594257514386527
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, -.394e-34]Solution in 0.514s

Time Plot 0 s.
Exiting SolveHard() after 1.038r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08,
375.73285289009648490314344110254685601 57,
328.11709293658536534704147910666561917 35,
292.99969137650484406754519975593905793 56,
358.64341560186493294884429563122271704 74,
299.89866204390666654227449515992716480 22,
360.06173465881570914189667626589090725 51,
336.59441031591586823537805890201025631 90,
256.10753185477822918381416293472261580 00,
324.65521223062302954208557009562325014 64,
331.93806791083046667813777122577684600 88, none, none,
289.54595771965109441738319337024228166 44]

1 --> 0  target = [17.93041369724601735553797840513441122961,
4.68650870188049358663108368944195922844 8,
353.30541094339561345276640320003978552 31]
one interval r = 20.73150479080016647060763552750735889595 ..
25.90675353509322062495362605992990093612
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.6e-38
Equations at solution: [.1e-37, .46e-37, .132e-34]Solution in 0.669s

Time Plot 0 s.
Exiting SolveHard() after 4.478r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08,
375.73285289009648490314344110254685601 57,
328.11709293658536534704147910666561917 35,
292.99969137650484406754519975593905793 56,
358.64341560186493294884429563122271704 74,
299.89866204390666654227449515992716480 22,
360.06173465881570914189667626589090725 51,
336.59441031591586823537805890201025631 90,
256.10753185477822918381416293472261580 00,
324.65521223062302954208557009562325014 64,
331.93806791083046667813777122577684600 88,
304.79958324742996542667977216242671140 61, none,
289.54595771965109441738319337024228166 44]

2 --> 0  target = [17.93041369724601735553797840513441122961,
4.68650870188049358663108368944195922844 8,
353.30541094339561345276640320003978552 31]
one interval r = 31.37435486991880492976723461303532300045 ..
34.20127520019715343876095949464385385591
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, -.8e-36]Solution in 0.347s

Time Plot 0 s.
Exiting SolveHard() after 0.661r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
```

```
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493088707508245766097589024287 50,
441.64295972576634789202408367067986741 27,
436.91748164840365760589823793093044951 54,
422.98493396692518525728120789066118445 13,
361.52580255557638815330989612199334616 42,
401.88173903639853980254901696689739505 49,
389.59001515409273989157484785048242459 99,
328.46939892766137413917128159447956076 80,
401.50757157361793953994035517373019431 47,
358.97362823365367479312987506788222833 55,
398.33147103162024796477428946596451869 85,
371.48387394071572277639101011725621186 51,
336.61215840547219009536062335048086040 58,
361.50888346596556202765396007029833534 24,
324.67144992082554070240144232760556760 23,
302.31384314036488505736041924005225108 46,
328.46938512838804040179666642415235759 31,
343.81340624444872032053589860108748020 08,
375.73285289009648490314344110254685601 57,
328.11709293658536534704147910666561917 35,
292.99969137650484406754519975593905793 56,
358.64341560186493294884429563122271704 74,
299.89866204390666654227449515992716480 22,
360.06173465881570914189667626589090725 51,
336.59441031591586823537805890201025631 90,
256.10753185477822918381416293472261580 00,
324.65521223062302954208557009562325014 64,
331.93806791083046667813777122577684600 88,
304.79958324742996542667977216242671140 61,
323.46169176084676107190456698635277188 59,
289.54595771965109441738319337024228166 44]

Cascade time 160.4
counts: 28, 28

 Iteration 66

Start Generation 1
1 --> 0  target = [11.99999999996333723143855431520131664300,
6.21701250299546764918169995338475212786 0,
485.54908090574976162967012584901219053 92]
```

"Imaginary part neglected: ", $1.8899423791567962262486435959637755993 62 \times 10^{-17}$

```
one interval r = 23.40850301672236217144337536758074059218 ..
27.67578046440638557073814736179048917245
Time Approximations 0.046.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
```

```
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.04e-37
Equations at solution: [-.3e-37, .104e-36, .4e-36]Solution in 4.023s

Time Plot 0 s.
Exiting SolveHard() after 5.201r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999963337231438554315201316643 00,
6.217012502995467649181699953384752127860,
485.549080905749761629670125849012190539 2]
one interval r = 32.628147792158974368760318140076348626 29 ..
36.102483889453035256033688198603602875 08
Time Approximations 0.023.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, .13e-35]Solution in 0.619s

Time Plot 0 s.
Exiting SolveHard() after 1.032r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684492267951321630820549354351062,
6.58343472179236766497885690904582286379 1,
467.78730596535285749018873070297708217 38]
one interval r = 32.41978955663783160945793409202990948753 ..
35.85152417373795713221989576917418612832
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, -.249e-34]Solution in 0.624s

Time Plot 0 s.
Exiting SolveHard() after 0.99r=34.9451 in [33.70078237 .. 35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82, none, none,
401.88173904810028807168816736877672679 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684492267951321630820549354351062,
6.58343472179236766497885690904582286379 1,
467.78730596535285749018873070297708217 38]
two intervals r = 12.92327160829607270986020045777879322383 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000 or r = 18.39424858044847386669688081010662970921 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
```

```
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.0e-38
Equations at solution: [-.1e-37, -.10e-37, -.8002e-35]Solution in
40.683s

Time Plot 0 s.
Exiting SolveHard() after 45.176r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22, none,
401.88173904810028807168816736877672679 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962829870622878631305726688127453,
4.12565179692783803357106634408984147 2034,
440.67123065826201183329774936000653030 03]
two intervals r = 14.35659705107909933134950763637991259343 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000 or r = 17.70352613831206196625290198081768700195 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000
Time Approximations 0.047.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [.13e-37, 0., -.9991e-35]Solution in 1.351s

Time Plot 0 s.
Exiting SolveHard() after 5.482r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
```

```
422.98493397895319933436813665512279927 22, none,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962829870622878631305726688127453,
4.12565179692783803357106634408984147203 4,
440.67123065826201183329774936000653030 03]
```

"Imaginary part neglected: ", $1.8899423791567962262486435959637755993 62 \times 10^{-17}$

```
one interval r = 22.39761154381809389437191112483048836675 ..
27.23722351613058546247425981849387894611
Time Approximations 0.04.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.224 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064408563531036736360906313143638, rm =
14.37818770703653303571864957537752240374}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.1044e-33]Solution in 7.894s

Time Plot 0 s.
Exiting SolveHard() after 11.917r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888799548866410270528895521194946,
4.00486908189772988964373825308730788751 4,
```

```
404.86224501877563503544313066044539494 79]
two intervals r = 16.08011007758698201614352632870715725546 ..
190000000000459615321163921881705491 03/100000000000000000000000000000000
00000 or r = 16.41579812715873658759660571835547076631 ..
190000000000459615321163921881705491 03/100000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.18e-37, 0., .7109e-35]Solution in 1.472s

Time Plot 0 s.
Exiting SolveHard() after 5.765r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77, none, none,
358.97362824584742246736066168013365884 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888799548866410270528895521194946,
4.00486908189772988964373825308730788751 4,
404.86224501877563503544313066044539494 79]
```

"Imaginary part neglected: ", $1.8899423791567962262486435959637755993 62 \times 10^{-17}$

```
one interval r = 21.64194399419639757575891665428432187745 ..
26.76330660054943029331925357919632859408
Time Approximations 0.05.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, .9e-36]Solution in 1.005s

Time Plot 0 s.
Exiting SolveHard() after 5.3r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814566706800085980158535 7,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23, none,
358.97362824584742246736066168013365884 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941793595978237042969908862691976,
5.5896371829458518231136524340825440618 13,
443.83065884977618443500445943359935765 33]
```

"Imaginary part neglected: ", $1.8899423791567962262486435959637755993 62 \times 10^{-17}$

```
one interval r = 22.46725374484749262598822676067989852393 ..
27.27388428364751493165519237012453581738
Time Approximations 0.044.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.5e-38
Equations at solution: [-.1e-37, .55e-37, .80e-35]Solution in 4.142s

Time Plot 0 s.
Exiting SolveHard() after 5.136r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23, none,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941793595978237042969908862691976,
5.5896371829458518231136524340825440618 13,
443.83065884977618443500445943359935765 33]
one interval r = 32.15575279499063860878296610531439546423 ..
35.50872228736354527042661672539004623060
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., -.209e-34]Solution in 0.48s

Time Plot 0 s.
Exiting SolveHard() after 0.819r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158298559727310851925953930268 8,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136496653604854463999835039181432,
```

```
5.187783578545494447849505510020111077040,
408.657738635571265506011657137277612569 7]
```

"Imaginary part neglected: ", $1.8899423791567962262486435959637755993 62 \times 10^{-17}$

```
one interval r = 21.718401146651877483698029122682218248 80 ..
26.818493035292030473846757090462598500 73
Time Approximations 0.062.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, -.340e-34]Solution in 4.058s

Time Plot 0 s.
Exiting SolveHard() after 5.159r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158298559727310851925953930268 8,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15, none, none,
361.50888347738063481312549166986090048 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.911931364966536048544639998350391814 32,
5.187783578545494447849505510020111077040,
408.657738635571265506011657137277612569 7]
one interval r = 31.808285987526538918553028322509500489 46 ..
35.000114600509473941156753691854623648 75
Time Approximations 0.018.

hint used Hint := [34.495226611650080820729149248019775312 21, 3, 1, 1,
15.763855890238725343023875204542137417 16, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, .27e-35]Solution in 0.45s

Time Plot 0 s.
Exiting SolveHard() after 0.763r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49, none,
361.50888347738063481312549166986090048 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110555252874737013855270290255005,
6.19626256552690852786497375532165930216 5,
385.44474379982450583016227878259380035 39]
one interval r = 31.60836097535285426645656921185477882539 ..
34.66372795612485027997230651569161653057
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .128e-34]Solution in 0.566s

Time Plot 0 s.
Exiting SolveHard() after 0.842r=33.8136 in [32.62689490 ..
34.66372796]
```

```
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49, none,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110555252874737013855270290255005,
6.196262565526908527864973755321659302165,
385.44474379982450583016227878259380035 39]
two intervals r = 16.87563408748579631193086683127866903586 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000 or r = 15.55640493833343780025384117251782997504 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., .10906e-34]Solution in 1.121s

Time Plot 0 s.
Exiting SolveHard() after 5.386r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
```

```
361.525802566992348306397761684970154520 0,
401.881739048100288071688167368776726798 9,
389.590015166491059329713814246215477397 7,
328.469398938601581845080008862629317582 3,
401.507571585298559727310851925953930268 8,
358.973628245847422467360661680133658847 8,
398.331471042937220923604111511794172721 5,
371.483873952896191866920357111405540864 9,
336.612158415947218981614941258385688639 5,
361.508883477380634813125491669860900481 2,
324.671449931481092266450874268434598564 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747298739319220852211969476894 75,
4.883810779852765406661202911902676498857,
376.619678564835086892742049395981283207 3]
```

"Imaginary part neglected: ", $1.8899423791567962262486435959637755993 62 \times 10^{-17}$

```
one interval r = 21.110013048863116789591986610093816936 56 ..
26.317842434906360888944167438768867861 73
Time Approximations 0.038.

hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.761063912186698964426645072343113601 29, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.3e-38
Equations at solution: [0., .23e-37, -.136e-34]Solution in 4.262s

Time Plot 0 s.
Exiting SolveHard() after 4.949r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434943582504182008772299467583651 4,
441.642959738148146566706800085980158535 7,
436.917481660545403531584013896549045808 2,
422.984933978953199334368136655122799272 2,
361.525802566992348306397761684970154520 0,
401.881739048100288071688167368776726798 9,
389.590015166491059329713814246215477397 7,
328.469398938601581845080008862629317582 3,
401.507571585298559727310851925953930268 8,
358.973628245847422467360661680133658847 8,
398.331471042937220923604111511794172721 5,
371.483873952896191866920357111405540864 9,
```

```
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43, none,
328.46938513932815852717513816546560549 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874729873931922085221196947689475,
4.8838107798527654066612029119026764988 57,
376.61967856483508689274204939598128320 73]
one interval r = 31.53899497709330080363584545911094563774 ..
34.53618386096079682915568368290500728980
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=9.68e-36
Equations at solution: [-.744e-35, .968e-35, .263e-34]Solution in
0.483s

Time Plot 0 s.
Exiting SolveHard() after 0.763r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43, none,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017552911649898706236892123690387,
6.0258135494741020688738742966371128147 62,
```

```
351.42702948847363224674209621411523501 37]
one interval r = 31.362302061095407159651342432472881960 67 ..
34.174466406140308778438319652913614764 03
Time Approximations 0.026.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, -.42e-35]Solution in 0.504s

Time Plot 0 s.
Exiting SolveHard() after 0.748r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43, none,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50, none, none,
292.99969138666060882885915824190345156 58, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017552911649898706236892123690387,
6.025813549474102068873874296637112814762,
351.42702948847363224674209621411523501 37]
two intervals r = 17.981355144445033958572844539804897648 88 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000 or r = 13.846080154464694429607509558779105731 10 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000
Time Approximations 3.19.
```

```
hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.88e-37, -.2e-37, .813e-36]Solution in 1.135s

Time Plot 0 s.
Exiting SolveHard() after 5.335r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158298559727310851925953930268 8,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43,
302.31384315022077824468231910529215075 53,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50, none, none,
292.99969138666060882885915824190345156 58, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941881234885615413951556221803751,
6.37794387404803665090281174726800514889 3,
423.28832784186758030396288719039097374 22]
one interval r = 31.94661817592379635943769648022945413775 ..
35.21212308647486449995771110724504609095
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., .99e-35]Solution in 0.584s

Time Plot 0 s.
Exiting SolveHard() after 0.95r=34.3272 in [33.10127385 .. 35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43,
302.31384315022077824468231910529215075 53,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50, none, none,
292.99969138666060882885915824190345156 58, none, none,
360.06173466942636292537277764214045041 47, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941881234885615413951556221803751,
6.37794387404803665090281174726800514889 3,
423.28832784186758030396288719039097374 22]
two intervals r = 15.22886702452632753131276072470050670365 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000 or r = 17.12965777080226630744536827116845795959 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.30e-37, .1e-37, .3343e-35]Solution in 1.267s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.593r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43,
302.31384315022077824468231910529215075 53,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50,
375.73285290065294676147090183971515000 17, none,
292.99969138666060882885915824190345156 58, none, none,
360.06173466942636292537277764214045041 47, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343429319383410122579483864227 20,
4.0035598155965831036400748196528493553 67,
404.47973594548192050369668831944527458 7]
two intervals r = 16.096839663711074776078630933948857908 19 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000 or r = 16.399886491343908618229650590921175208 59 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.18e-37, 0., .2955e-35]Solution in 1.49s

Time Plot 0 s.
Exiting SolveHard() after 5.866r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43,
302.31384315022077824468231910529215075 53,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50,
375.73285290065294676147090183971515000 17, none,
292.99969138666060882885915824190345156 58,
358.64341561404180176727364848496838625 39, none,
360.06173466942636292537277764214045041 47, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234342931938341012257948386422720,
4.0035598155965831036400748196528493553 67,
404.47973594548192050369668831944527458 7]

"Imaginary part neglected: ", 1.8899423791567962262486435959637755993 62 × 10^-17
one interval r = 21.63429629996177198656862750564194278500 ..
26.75768169904248212093349469469068880455
Time Approximations 0.052.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., .473e-34]Solution in 1.029s

Time Plot 0 s.
Exiting SolveHard() after 5.479r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43,
302.31384315022077824468231910529215075 53,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50,
375.73285290065294676147090183971515000 17,
328.11709294750503936011694810529659318 26,
292.99969138666060882885915824190345156 58,
358.64341561404180176727364848496838625 39, none,
360.06173466942636292537277764214045041 47, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954473461643012231224559870924806,
6.19617723038047652876470067678973048824 9,
385.42734026358523859014679784213258901 14]
one interval r = 31.60822049090657270849181731318840862381 ..
34.66347615050159909959585044506582986616
Time Approximations 0.028.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, .240e-34]Solution in 0.557s

Time Plot 0 s.
Exiting SolveHard() after 0.877r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43,
302.31384315022077824468231910529215075 53,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50,
375.73285290065294676147090183971515000 17,
328.11709294750503936011694810529659318 26,
292.99969138666060882885915824190345156 58,
358.64341561404180176727364848496838625 39, none,
360.06173466942636292537277764214045041 47, none, none,
324.65521224127770648500922492269333789 11, none, none, none, none]

0 --> 1  target = [26.46318954473461643012231224559870924806,
6.19617723038047652876470067678973048824 9,
385.42734026358523859014679784213258901 14]
two intervals r = 16.87629600287182519842990479349922842201 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000 or r = 15.55559000681947595482123390436315111854 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.358e-37, 0., -.4066e-35]Solution in 1.144s

Time Plot 0 s.
Exiting SolveHard() after 5.499r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
```

```
441.642959738148146566706800085980158535 7,
436.917481660545403531584013896549045808 2,
422.984933978953199334368136655122799272 2,
361.525802566992348306397761684970154520 0,
401.881739048100288071688167368776726798 9,
389.590015166491059329713814246215477397 7,
328.469398938601581845080008862629317582 3,
401.507571585298559727310851925953930268 8,
358.973628245847422467360661680133658847 8,
398.331471042937220923604111511794172721 5,
371.483873952896191866920357111405540864 9,
336.612158415947218981614941258385688639 5,
361.508883477380634813125491669860900481 2,
324.671449931481092266450874268434598564 3,
302.313843150220778244682319105292150755 3,
328.469385139328158527175138165465605497 8,
343.813406256597248704568962640045302085 0,
375.732852900652946761470901839715150001 7,
328.117092947505039360116948105296593182 6,
292.999691386660608828859158241903451565 8,
358.643415614041801767273648484968386253 9, none,
360.061734669426362925372777642140450414 7,
336.594410326389891148682505933834550954 9, none,
324.655212241277706485009224922693337891 1, none, none, none, none]

0 --> 2  target = [34.495226611713991866822571344328895187 52,
3.897131316028463317163645617847287472609,
373.780818853684959039146422465565979804 5]
two intervals r = 17.297690862107242966761368355692891904 32 ..
19000000000045961532116392188170549103/100000000000000000000000000000000
00000 or r = 14.994364074774022199813952975502030978 72 ..
19000000000045961532116392188170549103/100000000000000000000000000000000
00000
Time Approximations 0.081.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.18e-37, 0., .46931e-34]Solution in 1.2s

Time Plot 0 s.
Exiting SolveHard() after 5.722r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
```

```
441.642959738148146566706800085980158535 7,
436.917481660545403531584013896549045808 2,
422.984933978953199334368136655122799272 2,
361.525802566992348306397761684970154520 0,
401.881739048100288071688167368776726798 9,
389.590015166491059329713814246215477397 7,
328.469398938601581845080008862629317582 3,
401.507571585298559727310851925953930268 8,
358.973628245847422467360661680133658847 8,
398.331471042937220923604111511794172721 5,
371.483873952896191866920357111405540864 9,
336.612158415947218981614941258385688639 5,
361.508883477380634813125491669860900481 2,
324.671449931481092266450874268434598564 3,
302.313843150220778244682319105292150755 3,
328.469385139328158527175138165465605497 8,
343.813406256597248704568962640045302085 0,
375.732852900652946761470901839715150001 7,
328.117092947505039360116948105296593182 6,
292.999691386660608828859158241903451565 8,
358.643415614041801767273648484968386253 9, none,
360.061734669426362925372777642140450414 7,
336.594410326389891148682505933834550954 9, none,
324.655212241277706485009224922693337891 1,
331.938067923586409697562066751782526705 6, none, none, none]

1 --> 2  target = [34.495226611713991866822571344328895187 52,
3.897131316028463317163645617847287472609,
373.780818853684959039146422465565979804 5]
```

"Imaginary part neglected: ", $1.8899423791567962262486435959637755993 62 \times 10^{-17}$

```
one interval r = 21.060684732158535096548406732466490115 48 ..
26.269798343029755749892129910990571744 94
Time Approximations 0.037.

hint used Hint := [25.300466340350721366923008834208363590 84, 3, -1, 1,
16.974729527641787545085746138369074748 18, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, .580e-34]Solution in 3.814s

Time Plot 0 s.
Exiting SolveHard() after 4.551r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43,
302.31384315022077824468231910529215075 53,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50,
375.73285290065294676147090183971515000 17,
328.11709294750503936011694810529659318 26,
292.99969138666060882885915824190345156 58,
358.64341561404180176727364848496838625 39,
299.89866205524385328683048454738189654 59,
360.06173466942636292537277764214045041 47,
336.59441032638989114868250593383455095 49, none,
324.65521224127770648500922492269333789 11,
331.93806792358640969756206675178252670 56, none, none, none]

0 --> 2  target = [33.81362495409987069103772449301839367986,
3.72564899364852975398170128911175659161 3,
325.89209973381176079730985769934679869 93]
two intervals r = 18.552270490053372864412402836326808525 63 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000 or r = 12.491969358232818427602490580366834848 28 ..
19000000000045961532116392188170549103/10000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1.5e-37
Equations at solution: [.347e-36, -.15e-36, -.268e-36]Solution in
4.498s

Time Plot 0 s.
Exiting SolveHard() after 5.874r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43,
302.31384315022077824468231910529215075 53,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50,
375.73285290065294676147090183971515000 17,
328.11709294750503936011694810529659318 26,
292.99969138666060882885915824190345156 58,
358.64341561404180176727364848496838625 39,
299.89866205524385328683048454738189654 59,
360.06173466942636292537277764214045041 47,
336.59441032638989114868250593383455095 49, none,
324.65521224127770648500922492269333789 11,
331.93806792358640969756206675178252670 56, none, none,
289.54595773117417115023071158911373313 57]

1 --> 2  target = [33.81362495409987069103772449301839367986,
3.72564899364852975398170128911175659 1613,
325.89209973381176079730985769934679869 93]
```

"Imaginary part neglected: ", $1.8899423791567962262486435959637755993 62 \times 10^{-17}$

```
one interval r = 20.37468935108658055282727036267104020074 ..
25.37892165313479399107006426897265069424
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., -.183e-34]Solution in 0.571s

Time Plot 0 s.
Exiting SolveHard() after 1.104r=24.3395 in [22.07732228 ..
```

```
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814566706800085980158535 7,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158298559727310851925953930268 8,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43,
302.31384315022077824468231910529215075 53,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50,
375.73285290065294676147090183971515000 17,
328.11709294750503936011694810529659318 26,
292.99969138666060882885915824190345156 58,
358.64341561404180176727364848496838625 39,
299.89866205524385328683048454738189654 59,
360.06173466942636292537277764214045041 47,
336.59441032638989114868250593383455095 49,
256.10753186461007971023161702626901866 83,
324.65521224127770648500922492269333789 11,
331.93806792358640969756206675178252670 56, none, none,
289.54595773117417115023071158911373313 57]

1 --> 0  target = [17.93041369712623063670393068689891434330,
4.68650870196557399553138094986560206413 3,
353.30541095401462639047744051494493072 40]
```

"Imaginary part neglected: ", $1.8899423791567962262486435959637755993 62 \times 10^{-17}$

```
one interval r = 20.731504790929832159836012116767691935 98 ..
25.906753535326873794387513344396994144 48
Time Approximations 0.037.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
```

```
Accepted {r=25.4021, rm=17.0062} with Delta=4.6e-38
Equations at solution: [-.2e-37, -.46e-37, -.424e-34]Solution in 0.648s

Time Plot 0 s.
Exiting SolveHard() after 1.357r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43,
302.31384315022077824468231910529215075 53,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50,
375.73285290065294676147090183971515000 17,
328.11709294750503936011694810529659318 26,
292.99969138666060882885915824190345156 58,
358.64341561404180176727364848496838625 39,
299.89866205524385328683048454738189654 59,
360.06173466942636292537277764214045041 47,
336.59441032638989114868250593383455095 49,
256.10753186461007971023161702626901866 83,
324.65521224127770648500922492269333789 11,
331.93806792358640969756206675178252670 56,
304.79958325640497377386233132508908996 61, none,
289.54595773117417115023071158911373313 57]

2 --> 0  target = [17.93041369712623063670393068689891434330,
4.68650870196557399553138094986560206413 3,
353.30541095401462639047744051494493072 40]
one interval r = 31.37435486988365010046334707893646927838 ..
34.20127520025684471454581193349659881867
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
```

```
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, -.43e-35]Solution in 0.352s

Time Plot 0 s.
Exiting SolveHard() after 4.003r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494358250418200877229946758365 14,
441.64295973814814656670680008598015853 57,
436.91748166054540353158401389654904580 82,
422.98493397895319933436813665512279927 22,
361.52580256699234830639776168497015452 00,
401.88173904810028807168816736877672679 89,
389.59001516649105932971381424621547739 77,
328.46939893860158184508000886262931758 23,
401.50757158529855972731085192595393026 88,
358.97362824584742246736066168013365884 78,
398.33147104293722092360411151179417272 15,
371.48387395289619186692035711140554086 49,
336.61215841594721898161494125838568863 95,
361.50888347738063481312549166986090048 12,
324.67144993148109226645087426843459856 43,
302.31384315022077824468231910529215075 53,
328.46938513932815852717513816546560549 78,
343.81340625659724870456896264004530208 50,
375.73285290065294676147090183971515000 17,
328.11709294750503936011694810529659318 26,
292.99969138666060882885915824190345156 58,
358.64341561404180176727364848496838625 39,
299.89866205524385328683048454738189654 59,
360.06173466942636292537277764214045041 47,
336.59441032638989114868250593383455095 49,
256.10753186461007971023161702626901866 83,
324.65521224127770648500922492269333789 11,
331.93806792358640969756206675178252670 56,
304.79958325640497377386233132508908996 61,
323.46169177152551528101555808294386776 38,
289.54595773117417115023071158911373313 57]

Cascade time 157.704
counts: 28, 28

 Iteration 67

Start Generation 1
1 --> 0  target = [12.00000000010347620090844300791134904300,
6.21701250292724350058763936330411013 3,
485.54908089538374643291871033822688937 01]
```

"Imaginary part neglected: ", $1.8899423791329654370548526225297496108 52 \times 10^{-17}$

```
one interval r = 23.40850301644279140329776408554637336880 ..
27.67578046407608635602913028066501842504
Time Approximations 0.045.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.5e-38
Equations at solution: [-.1e-37, .25e-37, .4e-36]Solution in 1.028s

Time Plot 0 s.
Exiting SolveHard() after 2.203r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000010347620090844300791134904300,
6.217012502927243500587639363304110133013,
485.549080895383746432918710338226889370 1]
one interval r = 32.62814779201793875853707770189167351119 ..
36.10248389382450019589338073545325182 86
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .47e-35]Solution in 3.486s

Time Plot 0 s.
Exiting SolveHard() after 3.889r=35.4632 in [33.94922194 ..
36.10248389]
```

```
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844588851580576587501068708304 17,
6.5834347217144849293024863205384625642 55,
467.78730595551660470865365961618306401 63]
one interval r = 32.419789556502981870037381258886682913 97 ..
35.851524173666652976481784549069019608 84
Time Approximations 0.022.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.286e-34]Solution in 0.669s

Time Plot 0 s.
Exiting SolveHard() after 1.064r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69, none, none,
401.88173903851870682813869764938847537 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844588851580576587501068708304 17,
6.5834347217144849293024863205384625642 55,
467.78730595551660470865365961618306401 63]
two intervals r = 12.923271608446849372260380823318897601 74 ..
9499999999990948761555609165233626711/500000000000000000000000000000000
000 or r = 18.394248580224468762413681896063425456 47 ..
9499999999990948761555609165233626711/500000000000000000000000000000000
000
```

```
Time Approximations 0.041.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=8.7e-38
Equations at solution: [.7e-37, .87e-37, .695e-35]Solution in 42.039s

Time Plot 0 s.
Exiting SolveHard() after 46.532r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78, none,
401.88173903851870682813869764938847537 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962820376453589564247697908470399,
4.12565179683064003124079295732780347251 0,
440.67123064760578512948861339393253960 11]
two intervals r = 14.35659705129711321560124565114974878268 ..
9499999999990948761555609165233626711/50000000000000000000000000000000000
000 or r = 17.70352613798043784510270121501437269219 ..
9499999999990948761555609165233626711/50000000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., .122e-35]Solution in 1.331s

Time Plot 0 s.
Exiting SolveHard() after 5.31r=15.9119 in [14.35659706 .. 18.96093397]
```

```
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78, none,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962820376453589564247697908470399,
4.1256517968306400312407929573278034725 10,
440.67123064760578512948861339393253960 11]
```

"Imaginary part neglected: ", $1.8899423791329654370548526225297496108 52 \times 10^{-17}$

```
one interval r = 22.397611543594675336565836626910664760 66 ..
27.237223515778108350792276909833520083 62
Time Approximations 0.044.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.324 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643680076699756227850862815343 36, rm =
14.378187704596724882749300018351706343 96}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.3e-38
Equations at solution: [-.1e-37, -.53e-37, -.48e-35]Solution in 12.08s

Time Plot 0 s.
Exiting SolveHard() after 13.023r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
```

```
361.525802556892743666561624721208549660 2,
401.881739038518706828138697649388475372 3,
389.590015155708769266492804273110908043 5, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887901265526442127886366348141 43,
4.004869081803338708595867523657603993415,
404.862245009157829948520548604445467421 2]
two intervals r = 16.08011007774329753376368297961699989797 ..
949999999999094876155560916523362671 1/500000000000000000000000000000000000
000 or r = 16.41579812674763319603292891531534137961 ..
949999999999094876155560916523362671 1/500000000000000000000000000000000000
000
Time Approximations 0.064.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.17e-37, 0., -.1596e-34]Solution in 1.671s

Time Plot 0 s.
Exiting SolveHard() after 5.807r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434933844947574301701869595590066 7,
441.642959728556079059872631173978345738 9,
436.917481649944623140035598302438317276 9,
422.984933970636428953733151683947577047 8,
361.525802556892743666561624721208549660 2,
401.881739038518706828138697649388475372 3,
389.590015155708769266492804273110908043 5, none, none,
358.973628235779606560934233089767409411 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887901265526442127886366348141 43,
4.004869081803338708595867523657603993415,
404.862245009157829948520548604445467421 2]
```

"Imaginary part neglected: ", $1.8899423791329654370548526225297496108 52 \times 10^{-17}$

```
one interval r = 21.641943994058084915398279061643654571 21 ..
26.763306600203268821533404173927136876 59
```

```
Time Approximations 0.051.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .106e-34]Solution in 1.011s

Time Plot 0 s.
Exiting SolveHard() after 5.597r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64, none,
358.97362823577960656093423308976740941 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589418056937697596043426347236705 22,
5.5896371829169936566991623671402415258 79,
443.83065884096530068584955346291673520 63]

"Imaginary part neglected: ", 1.8899423791329654370548526225297496108 52 × 10^-17
one interval r = 22.467253744659771692718882170567517869 59 ..
27.273884283317301088704817375451400176 02
Time Approximations 0.045.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.3e-38
```

```
Equations at solution: [0., -.53e-37, -.28e-35]Solution in 3.904s

Time Plot 0 s.
Exiting SolveHard() after 4.851r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64, none,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589418056937697596043426347236705 22,
5.5896371829169936566991623671402415258 79,
443.83065884096530068584955346291673520 63]
one interval r = 32.155752794867731826647088512055336996 24 ..
35.508722287296508164628700410898763952 30
Time Approximations 0.02.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .6e-36]Solution in 0.488s

Time Plot 0 s.
Exiting SolveHard() after 0.853r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
```

```
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586285136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136520404976045370598319779180305,
5.18778357849877205779963747433000929451 4,
408.65773862419862656384413589894451768 78]
```

"Imaginary part neglected: ", $1.8899423791329654370548526225297496108 52 \times 10^{-17}$

```
one interval r = 21.71840114647088260389949940536253427582 ..
26.81849303492093039882489592529562883555
Time Approximations 0.063.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=0
Equations at solution: [0., 0., -.280e-34]Solution in 4.305s

Time Plot 0 s.
Exiting SolveHard() after 5.433r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586285136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45, none, none,
361.50888346730663125337563902122343223 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0  target = [15.91193136520404976045370598319779180305,
5.18778357849877205779963747433000929451 4,
408.65773862419862656384413589894451768 78]
one interval r = 31.80828598738319981278560893679424250929 ..
35.00011460039123697715367129637854880448
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., -.133e-34]Solution in 0.45s

Time Plot 0 s.
Exiting SolveHard() after 0.763r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586286136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45, none,
361.50888346730663125337563902122343223 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110519340446400962120528989468940,
6.19626256543649011054637021207160009257 0,
385.44474378942782245437552618520759478 27]
one interval r = 31.60836097522387386856608739315669361152 ..
34.66372795601235841042668548644082432513
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
```

```
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, .90e-35]Solution in 0.55s

Time Plot 0 s.
Exiting SolveHard() after 0.828r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586285136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45, none,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110519340446400962120528989468940,
6.19626256543649011054637021207160009257 0,
385.44474378942782245437552618520759478 27]
two intervals r = 16.87563408765385828155123990045666621923 ..
94999999999909487615556091652336267 11/50000000000000000000000000000000000
000 or r = 15.55640493782744428610444603008701702810 ..
94999999999909487615556091652336267 11/50000000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 6.98 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175125744733887821571822853403963, rm
```

```
= 2.336532774019836851054425922610447507666}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.179e-37, 0., .1436e-34]Solution in 30.14s

Time Plot 0 s.
Exiting SolveHard() after 35.312r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586285136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874746112611585323255445920885293,
4.8838107798220990311166525075394587723 34,
376.61967855423924267327206093535882196 12]

"Imaginary part neglected: ", 1.8899423791329654370548526225297496108 52 × 10^-17
one interval r = 21.11001304876360434357941655597363267929 ..
26.31784243454393095657168317334320209381
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .144e-34]Solution in 0.87s

Time Plot 0 s.
Exiting SolveHard() after 1.575r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586285136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25, none,
328.46938513026296619791196613289652524 52, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874746112611585323255445920885293,
4.8838107798220990311166525075394587723 34,
376.61967855423924267327206093535882196 12]
one interval r = 31.53899497696527522863729218919230363365 ..
34.53618386084246242889365273462410709046
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.54e-36
Equations at solution: [-.272e-35, .354e-35, -.356e-34]Solution in
0.505s

Time Plot 0 s.
Exiting SolveHard() after 3.805r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
```

```
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586285136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25, none,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017519227863000850601878247609559,
6.0258135493844305968800179425693402237 65,
351.42702947912340840127429584513818052 08]
one interval r = 31.362302060983866274130957980145728226 76 ..
34.174466406032054518062620975590292765 99
Time Approximations 0.016.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., -.151e-34]Solution in 0.529s

Time Plot 0 s.
Exiting SolveHard() after 0.771r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586285136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
```

```
324.67144992149988306552611252471238816 25, none,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61, none, none,
292.99969137769850102740206843656657696 48, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175192278630008506018782476095 59,
6.0258135493844305968800179425693402237 65,
351.42702947912340840127429584513818052 08]
two intervals r = 17.981355144529070336378242963688617066 25 ..
949999999999094876155560916523362671 1/500000000000000000000000000000000000
000 or r = 13.846080153924394554190693488423427574 65 ..
949999999999094876155560916523362671 1/500000000000000000000000000000000000
000
Time Approximations 0.048.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 9.509 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713704888331025491832409260577 19, rm
= 2.7345009930788704808378502159017853426 51}});
Accepted {r=18.6878, rm=15.3648} with Delta=7e-38
Equations at solution: [.196e-36, -.7e-37, -.4227e-34]Solution in
21.886s

Time Plot 0 s.
Exiting SolveHard() after 22.955r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586286136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
```

```
324.67144992149988306552611252471238816 25,
302.31384314295610348313231216271781589 69,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61, none, none,
292.99969137769850102740206843656657696 48, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941849166756364785525642383501759,
6.3779438739743837247671688293847623562 64,
423.28832783401176958856024396988543690 91]
one interval r = 31.946618175811563505033277031921837395 68 ..
35.21212308641327490334158483379033085168
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, -.203e-34]Solution in 0.626s

Time Plot 0 s.
Exiting SolveHard() after 0.976r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586286136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25,
302.31384314295610348313231216271781589 69,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61, none, none,
292.99969137769850102740206843656657696 48, none, none,
360.06173466176984907359098302341936213 01, none, none, none, none,
none, none, none]
```

```
0 --> 1  target = [27.020379418491667563647855256423835017 59,
6.3779438739743837247671688293847623562 64,
423.28832783401176958856024396988543690 91]
two intervals r = 15.22886702460616561100933864904511308235 ..
949999999999094876155560916523362671 1/500000000000000000000000000000000000
000 or r = 17.12965777051148284954829496548353449999 ..
949999999999094876155560916523362671 1/500000000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.03 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541436526200137271495199798522 72, rm
= 2.0640682986872109682167956646583938154 3}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [.16e-37, 0., .1251e-34]Solution in 30.325s

Time Plot 0 s.
Exiting SolveHard() after 34.882r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586285136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25,
302.31384314295610348313231216271781589 69,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61,
375.73285289470818986220613618896383873 45, none,
292.99969137769850102740206843656657696 48, none, none,
360.06173466176984907359098302341936213 01, none, none, none, none,
```

```
none, none, none]

0 --> 2  target = [34.93953234333404440322948193293352494115,
4.0035598155019698179942575001792314746 03,
404.47973593580144484367221288529585981 15]
two intervals r = 16.09683966386985340532344349901405883473 ..
9499999999990948761555609165233626711/50000000000000000000000000000000
000 or r = 16.39988649092901702195365745165046741723 ..
9499999999990948761555609165233626711/50000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.17e-37, .1e-37, -.147e-35]Solution in 4.697s

Time Plot 0 s.
Exiting SolveHard() after 5.805r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586286136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25,
302.31384314295610348313231216271781589 69,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61,
375.73285289470818986220613618896383873 45, none,
292.99969137769850102740206843656657696 48,
358.64341560391835271465342286339301347 00, none,
360.06173466176984907359098302341936213 01, none, none, none, none,
none, none, none]
```

```
1 --> 2  target = [34.93953234333404440322948193293352494115,
4.0035598155019698179942575001792314746 03,
404.47973593580144484367221288529585981 15]
```

"Imaginary part neglected: ", $1.8899423791329654370548526225297496108 52 \times 10^{-17}$

```
one interval r = 21.63429629982292848926414300008742479134 ..
26.75768169869537092886135747725299743264
Time Approximations 0.056.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=4.9e-38
Equations at solution: [.2e-37, .49e-37, -.680e-34]Solution in 4.167s

Time Plot 0 s.
Exiting SolveHard() after 5.23r=25.8653 in [23.83864811 .. 26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586285136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25,
302.31384314295610348313231216271781589 69,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61,
375.73285289470818986220613618896383873 45,
328.11709293838205079270139379222350844 63,
292.99969137769850102740206843656657696 48,
358.64341560391835271465342286339301347 00, none,
360.06173466176984907359098302341936213 01, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954437591768748782348237095090803,
```

```
6.196177230290184902244002215384196746277,
385.427340253214880490176797421957828213 5]
one interval r = 31.608220490777809466555491931034432367 35 ..
34.663476150389482139846348737053828514 51
Time Approximations 0.018.

hint used Hint := [33.813392675100157147469018161182119369 13, 3, 1, 1,
11.783209121413521870719438174471899656 27, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=5e-38
Equations at solution: [-.3e-37, .5e-37, -.95e-35]Solution in 0.601s

Time Plot 0 s.
Exiting SolveHard() after 0.912r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434933844947574301701869595590066 7,
441.642959728556079059872631173978345738 9,
436.917481649944623140035598302438317276 9,
422.984933970636428953733151683947577047 8,
361.525802556892743666561624721208549660 2,
401.881739038518706828138697649388475372 3,
389.590015155708769266492804273110908043 5,
328.469398929535561343945876096786825606 4,
401.507571575655709240037485862851369834 1,
358.973628235779606560934233089767409411 9,
398.331471035291567233720732291848937154 5,
371.483873940780759194185260233118330224 5,
336.612158407520824584655959320311554172 4,
361.508883467306631253375639021223432237 3,
324.671449921499883065526112524712388162 5,
302.313843142956103483132312162717815896 9,
328.469385130262966197911966132896525245 2,
343.813406244929942266660464814575447746 1,
375.732852894708189862206136188963838734 5,
328.117092938382050792701393792223508446 3,
292.999691377698501027402068436566576964 8,
358.643415603918352714653422863393013470 0, none,
360.061734661769849073590983023419362130 1, none, none,
324.655212231321084147344230802262230445 0, none, none, none, none]

0 --> 1  target = [26.463189544375917687487823482370950908 03,
6.196177230290184902244002215384196746277,
385.427340253214880490176797421957828213 5]
two intervals r = 16.876296003038862402889172311425423989 92 ..
```

```
9499999999990948761555609165233626711/500000000000000000000000000000000
000 or r = 15.555590006314661295899915090394001576 50 ..
9499999999990948761555609165233626711/500000000000000000000000000000000
000
Time Approximations 3.21.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.063 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852537405383165139241747176774813, rm
= 2.33669042808485130268003903219036057992 6}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.179e-37, 0., -.1629e-34]Solution in 29.73s

Time Plot 0 s.
Exiting SolveHard() after 34.051r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586285136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25,
302.31384314295610348313231216271781589 69,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61,
375.73285289470818986220613618896383873 45,
328.11709293838205079270139379222350844 63,
292.99969137769850102740206843656657696 48,
358.64341560391835271465342286339301347 00, none,
360.06173466176984907359098302341936213 01,
336.59441031799041320713303343833983144 56, none,
324.65521223132108414734423080226223044 50, none, none, none, none]
```

```
0 --> 2  target = [34.49522661157142552792386888468453564916,
3.897131315924304158651013524279516395177,
373.78081884147277889570483018771685259 62]
two intervals r = 17.29769086232051871234286939623380033789 ..
9499999999990948761555609165233626711/500000000000000000000000000000000
000 or r = 14.99436407414344788621339073445662648731 ..
9499999999990948761555609165233626711/500000000000000000000000000000000
000
Time Approximations 0.09.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.54e-37, 0., .1517e-34]Solution in 4.453s

Time Plot 0 s.
Exiting SolveHard() after 6.154r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559 00667,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586285136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25,
302.31384314295610348313231216271781589 69,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61,
375.73285289470818986220613618896383873 45,
328.11709293838205079270139379222350844 63,
292.99969137769850102740206843656657696 48,
358.64341560391835271465342286339301347 00, none,
360.06173466176984907359098302341936213 01,
336.59441031799041320713303343833983144 56, none,
324.65521223132108414734423080226223044 50,
331.93806791113976276419668333876533519 14, none, none, none]
```

```
1 --> 2  target = [34.49522661157142552792386888468453564916,
3.89713131592430415865101352427951639517 7,
373.78081884147277889570483018771685259 62]
```

"Imaginary part neglected: ", $1.8899423791329654370548526225297496108 52 \times 10^{-17}$

```
one interval r = 21.06068473203735255002769564929159686326 ..
26.26979834263988374350256408465777298415
Time Approximations 0.038.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [.2e-37, .5e-37, .572e-34]Solution in 0.818s

Time Plot 0 s.
Exiting SolveHard() after 1.555r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586286136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25,
302.31384314295610348313231216271781589 69,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61,
375.73285289470818986220613618896383873 45,
328.11709293838205079270139379222350844 63,
292.99969137769850102740206843656657696 48,
358.64341560391835271465342286339301347 00,
299.89866204385682887727349248779316194 41,
360.06173466176984907359098302341936213 01,
336.59441031799041320713303343833983144 56, none,
```

```
324.65521223132108414734423080226223044 50,
331.93806791113976276419668333876533519 14, none, none, none]

0 --> 2  target = [33.81362495397514736061772900557463704386,
3.7256489935507645624312861578583185243 22,
325.89209972377660625874580750026893731 30]
two intervals r = 18.552270490106290068182974178924408907 25 ..
949999999999094876155560916523362671 1/500000000000000000000000000000000
000 or r = 12.491969357629865899508356849463023713 13 ..
949999999999094876155560916523362671 1/500000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=2e-38
Equations at solution: [.53e-37, -.2e-37, -.297e-35]Solution in 1.125s

Time Plot 0 s.
Exiting SolveHard() after 5.88r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586285136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25,
302.31384314295610348313231216271781589 69,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61,
375.73285289470818986220613618896383873 45,
328.11709293838205079270139379222350844 63,
292.99969137769850102740206843656657696 48,
358.64341560391835271465342286339301347 00,
299.89866204385682887727349248779316194 41,
```

```
360.06173466176984907359098302341936213 01,
336.59441031799041320713303343833983144 56, none,
324.65521223132108414734423080226223044 50,
331.93806791113976276419668333876533519 14, none, none,
289.54595772055020390225392713528391008 87]

1 --> 2  target = [33.81362495397514736061772900557463704386,
3.72564899355076456243128615785831852432 2,
325.89209972377660625874580750026893731 30]
```

"Imaginary part neglected: ", $1.8899423791329654370548526225297496108 52 \times 10^{-17}$

```
one interval r = 20.37468935110859329560953734688493821128 ..
25.37892165279313653126898253265599071312
Time Approximations 0.03.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.2e-37, -.2e-37, -.124e-34]Solution in 3.728s

Time Plot 0 s.
Exiting SolveHard() after 4.293r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586286136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25,
302.31384314295610348313231216271781589 69,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61,
375.73285289470818986220613618896383873 45,
```

```
328.1170929383820507927013937922235084463,
292.9996913776985010274020684365665769648,
358.6434156039183527146534228633930134700,
299.8986620438568288772734924877931619441,
360.0617346617698490735909830234193621301,
336.5944103179904132071330334383398314456,
256.1075318553028086942676144703233370518,
324.6552122313210841473442308022622304450,
331.9380679111397627641966833387653351914, none, none,
289.5459577205502039022539271352839100887]

1 --> 0  target = [17.9304136972000227285090246785169053358 9,
4.6865087019530296970075447302999135787 56,
353.3054109451425047807443234909393847226]
```

"Imaginary part neglected: ", $1.8899423791329654370548526225297496108 52 \times 10^{-17}$

```
one interval r = 20.7315047909079030887764315286828679615 8 ..
25.9067535349983566901149004963485002049 3
Time Approximations 0.035.

hint used Hint := [25.4020715378188039710536173439704738275 4, 3, -1, 1,
17.0062444547662134530206245122467435705 3, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., -.19e-35]Solution in 0.699s

Time Plot 0 s.
Exiting SolveHard() after 1.452r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349338449475743017018695955900667,
441.6429597285560790598726311739783457389,
436.9174816499446231400355983024383172769,
422.9849339706364289537331516839475770478,
361.5258025568927436665616247212085496602,
401.8817390385187068281386976493884753723,
389.5900151557087692664928042731109080435,
328.4693989295355613439458760967868256064,
401.5075715756557092400374858628513698341,
358.9736282357796065609342330897674094119,
398.3314710352915672337207322918489371545,
371.4838739407807591941852602331183302245,
336.6121584075208245846559593203115541724,
361.5088834673066312533756390212234322373,
```

```
324.67144992149988306552611252471238816 25,
302.31384314295610348313231216271781589 69,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61,
375.73285289470818986220613618896383873 45,
328.11709293838205079270139379222350844 63,
292.99969137769850102740206843656657696 48,
358.64341560391835271465342286339301347 00,
299.89866204385682887727349248779316194 41,
360.06173466176984907359098302341936213 01,
336.59441031799041320713303343833983144 56,
256.10753185530280869426761447032333705 18,
324.65521223132108414734423080226223044 50,
331.93806791113976276419668333876533519 14,
304.79958324920257872278376373404058434 19, none,
289.54595772055020390225392713528391008 87]

2 --> 0  target = [17.93041369720002272850902467851690533589,
4.68650870195302969700754473029991357875 6,
353.30541094514250478074432349093938472 26]
one interval r = 31.37435486977462248324387748163918454946 ..
34.20127520015597075586808048913521660079
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [.5e-37, -.8e-37, -.433e-34]Solution in 0.352s

Time Plot 0 s.
Exiting SolveHard() after 0.616r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493384494757430170186959559006 67,
441.64295972855607905987263117397834573 89,
436.91748164994462314003559830243831727 69,
422.98493397063642895373315168394757704 78,
361.52580255689274366656162472120854966 02,
401.88173903851870682813869764938847537 23,
389.59001515570876926649280427311090804 35,
328.46939892953556134394587609678682560 64,
401.50757157565570924003748586286136983 41,
358.97362823577960656093423308976740941 19,
398.33147103529156723372073229184893715 45,
```

```
371.48387394078075919418526023311833022 45,
336.61215840752082458465595932031155417 24,
361.50888346730663125337563902122343223 73,
324.67144992149988306552611252471238816 25,
302.31384314295610348313231216271781589 69,
328.46938513026296619791196613289652524 52,
343.81340624492994226666046481457544774 61,
375.73285289470818986220613618896383873 45,
328.11709293838205079270139379222350844 63,
292.99969137769850102740206843656657696 48,
358.64341560391835271465342286339301347 00,
299.89866204385682887727349248779316194 41,
360.06173466176984907359098302341936213 01,
336.59441031799041320713303343833983144 56,
256.10753185530280869426761447032333705 18,
324.65521223132108414734423080226223044 50,
331.93806791113976276419668333876533519 14,
304.79958324920257872278376373404058434 19,
323.46169176124555323237531669723008896 34,
289.54595772055020390225392713528391008 87]

Cascade time 262.436
counts: 28, 28

 Iteration 68

Start Generation 1
1 --> 0  target = [11.99999999979339123293568057816518648500,
6.21701250313750650773459053032629448814 7,
485.54908090777251118586928099125958645 41]
```

"Imaginary part neglected: ", $1.8899423791527298913631508074755547675 11 \times 10^{-17}$

```
one interval r = 23.40850301672251882418578747171836473084 ..
27.67578046439949472021961855776535839293
Time Approximations 0.045.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.3e-38
Equations at solution: [-.1e-37, .53e-37, .3e-36]Solution in 1.034s

Time Plot 0 s.
Exiting SolveHard() after 2.225r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999979339123293568057816518648500,
6.2170125031375065077345905303262944881 47,
485.54908090777251118586928099125958645 41]
one interval r = 32.628147792311434874811073859411540444 38 ..
36.102483889560296830213305185001316822 12
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [.6e-37, -.4e-37, .2293e-35]Solution in 0.608s

Time Plot 0 s.
Exiting SolveHard() after 4.506r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684493891750302378376999575102650,
6.5834347216350615768788690019570575314 98,
467.78730596931900959542446518720493788 69]
one interval r = 32.419789556815200770804664910764868746 91 ..
35.851524173875811441755975089563076662 65
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
```

```
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=6e-38
Equations at solution: [.8e-37, -.6e-37, -.14014e-34]Solution in 0.633s

Time Plot 0 s.
Exiting SolveHard() after 1r=34.9451 in [33.70078237 .. 35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58, none, none,
401.88173905124986341206084488812084364 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844938917503023783769995751026 50,
6.5834347216350615768788690019570575314 98,
467.78730596931900959542446518720493788 69]
two intervals r = 12.923271608022908885949097812572638988 02 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000 or r = 18.394248580538968525965699960371989651 27 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.2e-38
Equations at solution: [-.3e-37, -.32e-37, .926e-36]Solution in 39.741s

Time Plot 0 s.
Exiting SolveHard() after 44.423r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
```

```
422.98493398773587870388865613029390181 06, none,
401.88173905124986341206084488812084364 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962835893462031004740952144563416,
4.12565179669119259024937010780878672095 7,
440.67123065648535262601266510677903041 78]
two intervals r = 14.35659705112022305688203121843251951233 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000 or r = 17.70352613826586262582247393175211510156 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [.58e-37, .3e-37, -.5564e-35]Solution in 1.332s

Time Plot 0 s.
Exiting SolveHard() after 5.447r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091 53135,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06, none,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962835893462031004740952144563416,
4.12565179669119259024937010780878672095 7,
440.67123065648535262601266510677903041 78]
```

"Imaginary part neglected: ", $1.8899423791527298913631508074755547675 11 \times 10^{-17}$

```
one interval r = 22.39761154371899864684597053052850871347 ..
27.23722351609045816545048375723144733800
Time Approximations 0.041.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
```

```
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.208 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643927263948122798736238270552 96, rm =
14.378187701818648747186295685196759191 08}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., .325e-34]Solution in 8.333s

Time Plot 0 s.
Exiting SolveHard() after 12.284r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888132656010350723532850404197 49,
4.0048690816779789657249102858893782295 94,
404.86224502187633213215142584435440761 29]
two intervals r = 16.080110077416870027677153591296365933 13 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000 or r = 16.415798127290416133130504000777332314 46 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., .22374e-34]Solution in 1.402s

Time Plot 0 s.
Exiting SolveHard() after 5.702r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.642959742129780532488160303153228452 4,
436.917481658924942586391478103481651775 8,
422.984933987735878703888656130293901810 6,
361.525802564574149270991661179432452074 6,
401.881739051249863412060844888120843646 8,
389.590015160357216251207301684815279994 4, none, none,
358.973628243592548147371515383946759489 2, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888132656010350723532850404197 49,
4.004869081677978965724910285889378229594,
404.862245021876332132151425844354407612 9]
```

"Imaginary part neglected: ", $1.889942379152729891363150807475554767511 \times 10^{-17}$

```
one interval r = 21.641943994186271346789399639932344445 00 ..
26.763306600576085650537691104802519196 76
Time Approximations 0.051.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.8e-38
Equations at solution: [.2e-37, .78e-37, -.100e-34]Solution in 1.032s

Time Plot 0 s.
Exiting SolveHard() after 5.245r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.642959742129780532488160303153228452 4,
```

```
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54, none,
358.97362824359254814737151538394675948 92, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941739000892997709388589575318064,
5.58963718316843577809113403848351498746 9,
443.83065885924988387866169926366643466 17]
```

"Imaginary part neglected: ", $1.8899423791527298913631508074755547675 11 \times 10^{-17}$

```
one interval r = 22.46725374499786550360086573319344132424 ..
27.27388428373675560953112484464718525138
Time Approximations 0.045.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.61e-37
Equations at solution: [.1e-37, -.161e-36, -.67e-35]Solution in 4.308s

Time Plot 0 s.
Exiting SolveHard() after 5.314r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54, none,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941739000892997709388589575318064,
5.58963718316843577809113403848351498746 9,
```

```
443.83065885924988387866169926366643466 17]
one interval r = 32.15575279522788133769695015835923222409 ..
35.50872228758517102813271127161134695760
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, .14741e-34]Solution in 0.488s

Time Plot 0 s.
Exiting SolveHard() after 0.848r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136521116914153176231080176861481,
5.18778357858083928356619939658292965677 6,
408.65773862931055195276509229257260383 16]

"Imaginary part neglected: ", 1.8899423791527298913631508074755547675 11 × 10^-17
one interval r = 21.71840114645404628383781654289940403796 ..
26.81849303518310771855698736775693216664
Time Approximations 0.063.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
```

```
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.59e-37
Equations at solution: [.1e-37, .159e-36, .145e-34]Solution in 4.271s

Time Plot 0 s.
Exiting SolveHard() after 5.404r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31, none, none,
361.50888347501642118669501806495932877 06, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.911931365211169141531762310801768614 81,
5.1877835785808392835661993965829296567 76,
408.65773862931055195276509229257260383 16]
one interval r = 31.808285987613130183897858176979855792 59 ..
35.000114600510193254686768411066194625 44
Time Approximations 0.018.

hint used Hint := [34.495226611650080820729149248019775312 21, 3, 1, 1,
15.763855890238725343023875204542137417 16, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, .10749e-34]Solution in 0.449s

Time Plot 0 s.
Exiting SolveHard() after 0.76r=34.4952 in [32.91337941 .. 35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133713734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
371.48387394205901953393780535435054086 57, none,
361.50888347501642118669501806495932877 06, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110549072455438393806265972146824,
6.19626256533888217832293502041687301951 9,
385.44474379721375750304028545171719762 66]
one interval r = 31.60836097547965925668175870116581943921 ..
34.66372795618300997236489542477827961355
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=1.7e-37
Equations at solution: [-.11e-36, .17e-36, -.4500e-35]Solution in
0.553s

Time Plot 0 s.
Exiting SolveHard() after 0.826r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133713734 62,
```

```
358.9736282435925481473715153839467594892,
398.3314710565340441468877197279737944331,
371.4838739420590195339378053543505408657, none,
361.5088834750164211866950180649593287706,
324.6714499283982469740916772328651998792, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110549072455438393806265972146824,
6.196262565338882178322935020416873019519,
385.44474379721375750304028545171719762 66]
two intervals r = 16.875634087558882464279812537513328402 11 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000 or r = 15.556404938209839130025321708747334340 19 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.179e-37, 0., .1686e-35]Solution in 1.151s

Time Plot 0 s.
Exiting SolveHard() after 5.373r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349452806482819322084202809153135,
441.6429597421297805324881603031532284524,
436.9174816589249425863914781034816517758,
422.9849339877358787038886561302939018106,
361.5258025645741492709916611794324520746,
401.8817390512498634120608448881208436468,
389.5900151603572162512073016848152799944,
328.4693989406799242377751625422422779254,
401.5075715886987531217333113313371173462,
358.9736282435925481473715153839467594892,
398.3314710565340441468877197279737944331,
371.4838739420590195339378053543505408657,
336.6121584181461246257876288848986937203,
361.5088834750164211866950180649593287706,
324.6714499283982469740916772328651998792, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874735284756899412831542358398070,
```

```
4.883810779932398256464459012618687360815,
376.619678562652352529999035181507375579 6]
```

"Imaginary part neglected: ", $1.889942379152729891363150807475554767511 \times 10^{-17}$

```
one interval r = 21.11001304874333308062214705910790529106 ..
26.31784243484955993972349906771152697304
Time Approximations 0.039.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=9.8e-38
Equations at solution: [-.2e-37, -.98e-37, .375e-34]Solution in 4.288s

Time Plot 0 s.
Exiting SolveHard() after 5.023r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
371.48387394205901953393780535435054086 57,
336.61215841814612462578762888489869372 03,
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92, none,
328.46938514140890006038904534811619245 22, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874735284756899412831542358398070,
4.883810779932398256464459012618687360815,
376.619678562652352529999035181507375579 6]
one interval r = 31.53899497722593457362627290866799133028 ..
34.53618386102697415948552681589338170491
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
```

```
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=4.68e-36
Equations at solution: [.359e-35, -.468e-35, -.14109e-34]Solution in
0.525s

Time Plot 0 s.
Exiting SolveHard() after 0.815r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
371.48387394205901953393780535435054086 57,
336.61215841814612462578762888489869372 03,
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92, none,
328.46938514140890006038904534811619245 22,
343.81340624872186345044076275205010132 60, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017554421379910040258323130873815,
6.0258135493092228300923412334030811896 43,
351.42702949048992006584549153497883608 64]
one interval r = 31.362302061261315128217284923159605642 31 ..
34.174466406271912990919956884894887993 22
Time Approximations 0.028.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
```

```
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .28806e-34]Solution in 0.524s

Time Plot 0 s.
Exiting SolveHard() after 0.786r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
371.48387394205901953393780535435054086 57,
336.61215841814612462578762888489869372 03,
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92, none,
328.46938514140890006038904534811619245 22,
343.81340624872186345044076275205010132 60, none, none,
292.99969138786573087856685343694344373 03, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017554421379910040258323130873815,
6.0258135493092228300923412334030811896 43,
351.42702949048992006584549153497883608 64]
two intervals r = 17.98135514437703539781684411453398816322 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000 or r = 13.84608015455901501736999241907782870410 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000
Time Approximations 3.07.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [-.71e-37, .3e-37, .13758e-34]Solution in 1.149s

Time Plot 0 s.
Exiting SolveHard() after 5.217r=18.6878 in [17.98135512 .. 19]
```

```
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
371.48387394205901953393780535435054086 57,
336.61215841814612462578762888489869372 03,
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92,
302.31384315689950311953530097442655119 62,
328.46938514140890006038904534811619245 22,
343.81340624872186345044076275205010132 60, none, none,
292.99969138786573087856685343694344373 03, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941897638038333364203171893032225,
6.3779438739379693826246796629352291198 96,
423.28832785571912234752694114999625360 14]
one interval r = 31.94661817620011283104998203845645529187 ..
35.21212308676422275710241018387484893796
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.3e-37, -.3e-37, -.32771e-34]Solution in 0.585s

Time Plot 0 s.
Exiting SolveHard() after 0.942r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
```

```
441.642959742129780532488160303153228452 4,
436.917481658924942586391478103481651775 8,
422.984933987735878703888656130293901810 6,
361.525802564574149270991661179432452074 6,
401.881739051249863412060844888120843646 8,
389.590015160357216251207301684815279994 4,
328.469398940679924237775162542242277925 4,
401.507571588698753121733311331337117346 2,
358.973628243592548147371515383946759489 2,
398.331471056534044146887719727973794433 1,
371.483873942059019533937805354350540865 7,
336.612158418146124625787628884898693720 3,
361.508883475016421186695018064959328770 6,
324.671449928398246974091677232865199879 2,
302.313843156899503119535300974426551196 2,
328.469385141408900060389045348116192452 2,
343.813406248721863450440762752050101326 0, none, none,
292.999691387865730878566853436943443730 3, none, none,
360.061734681788448454635500722320718773 3, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418976380383336420317189303222 5,
6.377943873937969382624679662935229119896,
423.288327855719122347526941149996253601 4]
two intervals r = 15.228867023812710475471071418134603685 64 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000 or r = 17.129657771304614732554129924347689697 49 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000
Time Approximations 0.062.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [-.17e-37, 0., .2610e-35]Solution in 1.291s

Time Plot 0 s.
Exiting SolveHard() after 5.934r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434945280648281932208420280915313 5,
441.642959742129780532488160303153228452 4,
436.917481658924942586391478103481651775 8,
422.984933987735878703888656130293901810 6,
361.525802564574149270991661179432452074 6,
```

```
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
371.48387394205901953393780535435054086 57,
336.61215841814612462578762888489869372 03,
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92,
302.31384315689950311953530097442655119 62,
328.46938514140890006038904534811619245 22,
343.81340624872186345044076275205010132 60,
375.73285292014338892601695469192594029 73, none,
292.99969138786573087856685343694344373 03, none, none,
360.06173468178844845463550072232071877 33, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234357027388828409493453127397336,
4.0035598153777170769449987496941864413 94,
404.47973594883885819609408135697055446 00]
two intervals r = 16.09683966353026557492416498145620028175 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000 or r = 16.39988649148653928382365846164236923122 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.35e-37, 0., -.37133e-34]Solution in 1.444s

Time Plot 0 s.
Exiting SolveHard() after 6.065r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
```

```
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
371.48387394205901953393780535435054086 57,
336.61215841814612462578762888489869372 03,
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92,
302.31384315689950311953530097442655119 62,
328.46938514140890006038904534811619245 22,
343.81340624872186345044076275205010132 60,
375.73285292014338892601695469192594029 73, none,
292.99969138786573087856685343694344373 03,
358.64341561200625495780091904488275336 67, none,
360.06173468178844845463550072232071877 33, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234357027388828409493453127397336,
4.0035598153777170769449987496941864413 94,
404.47973594883885819609408135697055446 00]
```

"Imaginary part neglected: ", $1.8899423791527298913631508074755547675 11 \times 10^{-17}$

```
one interval r = 21.63429629995654835490796966288725653511 ..
26.75768169907299568916391094962785217328
Time Approximations 0.05.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.1e-38
Equations at solution: [-.2e-37, -.51e-37, -.147e-34]Solution in 1.032s

Time Plot 0 s.
Exiting SolveHard() after 5.435r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
```

```
358.9736282435925481473715153839467594892,
398.3314710565340441468877197279737944331,
371.4838739420590195339378053543505408657,
336.6121584181461246257876288848986937203,
361.5088834750164211866950180649593287706,
324.6714499283982469740916772328651998792,
302.3138431568995031195353009744265511962,
328.4693851414089000603890453481161924522,
343.8134062487218634504407627520501013260,
375.7328529201433889260169546919259402973,
328.1170929498192344148293183507525039781,
292.9996913878657308785668534369434437303,
358.6434156120062549578009190448827533667, none,
360.0617346817884484546355007223207187733, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954467370640226556483528819683415,
6.196177230192722211911563075328452224047,
385.4273402610300203745827802134669852379]
one interval r = 31.60822049103383099773497504147268128997 ..
34.66347615056056579867450170193522048207
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=9e-38
Equations at solution: [.6e-37, -.9e-37, .22815e-34]Solution in 0.548s

Time Plot 0 s.
Exiting SolveHard() after 0.865r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349452806482819322084202809153135,
441.6429597421297805324881603031532284524,
436.9174816589249425863914781034816517758,
422.9849339877358787038886561302939018106,
361.5258025645741492709916611794324520746,
401.8817390512498634120608448881208436468,
389.5900151603572162512073016848152799944,
328.4693989406799242377751625422422779254,
401.5075715886987531217333113313371173462,
358.9736282435925481473715153839467594892,
398.3314710565340441468877197279737944331,
371.4838739420590195339378053543505408657,
```

```
336.61215841814612462578762888489869372 03,
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92,
302.31384315689950311953530097442655119 62,
328.46938514140890006038904534811619245 22,
343.81340624872186345044076275205010132 60,
375.73285292014338892601695469192594029 73,
328.11709294981923441482931835075250397 81,
292.99969138786573087856685343694344373 03,
358.64341561200625495780091904488275336 67, none,
360.06173468178844845463550072232071877 33, none, none,
324.65521223824666527419550661138584106 66, none, none, none, none]

0 --> 1  target = [26.46318954467370640226556483528819683415,
6.19617723019272221191156307532845222404 7,
385.42734026103002037458278021346698523 79]
two intervals r = 16.87629600294279223935791365701854436802 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000 or r = 15.55559000669846202902319541846329139266 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.358e-37, 0., -.25994e-34]Solution in 1.145s

Time Plot 0 s.
Exiting SolveHard() after 5.613r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
371.48387394205901953393780535435054086 57,
336.61215841814612462578762888489869372 03,
```

```
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92,
302.31384315689950311953530097442655119 62,
328.46938514140890006038904534811619245 22,
343.81340624872186345044076275205010132 60,
375.73285292014338892601695469192594029 73,
328.11709294981923441482931835075250397 81,
292.99969138786573087856685343694344373 03,
358.64341561200625495780091904488275336 67, none,
360.06173468178844845463550072232071877 33,
336.59441032864537230999157019880131426 25, none,
324.65521223824666527419550661138584106 66, none, none, none, none]

0 --> 2  target = [34.49522661165073145636029913610227776962,
3.8971313157591599774287990099793481898 30,
373.78081884248457400776646690863549905 84]
two intervals r = 17.29769086246976658497774482845773564580 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000 or r = 14.99436407421538947893299276109160757322 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000
Time Approximations 0.078.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.18e-37, 0., .16594e-34]Solution in 1.199s

Time Plot 0 s.
Exiting SolveHard() after 6.17r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
371.48387394205901953393780535435054086 57,
336.61215841814612462578762888489869372 03,
```

```
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92,
302.31384315689950311953530097442655119 62,
328.46938514140890006038904534811619245 22,
343.81340624872186345044076275205010132 60,
375.73285292014338892601695469192594029 73,
328.11709294981923441482931835075250397 81,
292.99969138786573087856685343694344373 03,
358.64341561200625495780091904488275336 67, none,
360.06173468178844845463550072232071877 33,
336.59441032864537230999157019880131426 25, none,
324.65521223824666527419550661138584106 66,
331.93806790870215343462416307794581021 32, none, none, none]

1 --> 2  target = [34.49522661165073145636029913610227776962,
3.89713131575915997742879900997934818983 0,
373.78081884248457400776646690863549905 84]
```

"Imaginary part neglected: ", $1.8899423791527298913631508074755547675 11 \times 10^{-17}$

```
one interval r = 21.06068473188313733592703462671170694213 ..
26.26979834281879438340658037206150424762
Time Approximations 0.038.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [-.1e-37, -.3e-37, -.192e-34]Solution in 3.736s

Time Plot 0 s.
Exiting SolveHard() after 4.483r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
```

```
371.48387394205901953393780535435054086 57,
336.61215841814612462578762888489869372 03,
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92,
302.31384315689950311953530097442655119 62,
328.46938514140890006038904534811619245 22,
343.81340624872186345044076275205010132 60,
375.73285292014338892601695469192594029 73,
328.11709294981923441482931835075250397 81,
292.99969138786573087856685343694344373 03,
358.64341561200625495780091904488275336 67,
299.89866204419221517242270053384235569 85,
360.06173468178844845463550072232071877 33,
336.59441032864537230999157019880131426 25, none,
324.65521223824666527419550661138584106 66,
331.93806790870215343462416307794581021 32, none, none, none]

0 --> 2  target = [33.81362495416245439476464541167449550011,
3.7256489934073136631699503941713285646 24,
325.89209973054280945117312162791963204 41]
two intervals r = 18.552270490105274896605457335218531596 72 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000 or r = 12.491969358039046918028815347975969746 92 ..
19000000000049611207539629586637290097/10000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=2e-38
Equations at solution: [-.53e-37, .2e-37, .8573e-35]Solution in 4.291s

Time Plot 0 s.
Exiting SolveHard() after 5.701r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
```

```
358.9736282435925481473715153839467594892,
398.3314710565340441468877197279737944331,
371.4838739420590195339378053543505408657,
336.6121584181461246257876288848986937203,
361.5088834750164211866950180649593287706,
324.6714499283982469740916772328651998792,
302.3138431568995031195353009744265511962,
328.4693851414089000603890453481161924522,
343.8134062487218634504407627520501013260,
375.7328529201433889260169546919259402973,
328.1170929498192344148293183507525039781,
292.9996913878657308785668534369434437303,
358.6434156120062549578009190448827533667,
299.8986620441922151724227005338423556985,
360.0617346817884484546355007223207187733,
336.5944103286453723099915701988013142625, none,
324.6552122382466652741955066113858410666,
331.9380679087021534346241630779458102132, none, none,
289.5459577232319382844175415810810639151]

1 --> 2  target = [33.8136249541624543947646454116744955001 1,
3.7256489934073136631699503941713285646 24,
325.8920997305428094511731216279196320441]
```

"Imaginary part neglected: ", $1.8899423791527298913631508074755547675 11 \times 10^{-17}$

```
one interval r = 20.3746893509546828954843632927770912628 6 ..
25.3789216530436620479821258123105037668 9
Time Approximations 0.028.

hint used Hint := [24.3394990302538142174610776606811553342 3, 3, -1, 1,
17.2721587825129442122670749442843558157 4, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.2e-37, .2e-37, .40e-35]Solution in 0.581s

Time Plot 0 s.
Exiting SolveHard() after 1.117r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349452806482819322084202809153135,
441.6429597421297805324881603031532284524,
436.9174816589249425863914781034816517758,
422.9849339877358787038886561302939018106,
361.5258025645741492709916611794324520746,
```

```
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
371.48387394205901953393780535435054086 57,
336.61215841814612462578762888489869372 03,
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92,
302.31384315689950311953530097442655119 62,
328.46938514140890006038904534811619245 22,
343.81340624872186345044076275205010132 60,
375.73285292014338892601695469192594029 73,
328.11709294981923441482931835075250397 81,
292.99969138786573087856685343694344373 03,
358.64341561200625495780091904488275336 67,
299.89866204419221517242270053384235569 85,
360.06173468178844845463550072232071877 33,
336.59441032864537230999157019880131426 25,
256.10753186086161728382598983997625322 49,
324.65521223824666527419550661138584106 66,
331.93806790870215343462416307794581021 32, none, none,
289.54595772323193828441754158108106391 51]

1 --> 0  target = [17.93041369704415372001001423476141366836,
4.68650870208432815074895324995809297675 5,
353.30541095647446471140456537751607711 34]

"Imaginary part neglected: ", $1.8899423791527298913631508074755547675 11 \times 10^{-17}$

one interval r = 20.73150479087772819359068253622206512372 ..
25.90675353535009630882448983419906326296
Time Approximations 0.036.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., .204e-34]Solution in 0.672s

Time Plot 0 s.
Exiting SolveHard() after 1.388r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
371.48387394205901953393780535435054086 57,
336.61215841814612462578762888489869372 03,
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92,
302.31384315689950311953530097442655119 62,
328.46938514140890006038904534811619245 22,
343.81340624872186345044076275205010132 60,
375.73285292014338892601695469192594029 73,
328.11709294981923441482931835075250397 81,
292.99969138786573087856685343694344373 03,
358.64341561200625495780091904488275336 67,
299.89866204419221517242270053384235569 85,
360.06173468178844845463550072232071877 33,
336.59441032864537230999157019880131426 25,
256.10753186086161728382598983997625322 49,
324.65521223824666527419550661138584106 66,
331.93806790870215343462416307794581021 32,
304.79958326323224101487808051854861752 75, none,
289.54595772323193828441754158108106391 51]

2 --> 0  target = [17.93041369704415372001001423476141366836,
4.68650870208432815074895324995809297675 5,
353.30541095647446471140456537751607711 34]
one interval r = 31.37435487005236450861500064324655780772 ..
34.20127520039442364244231954479588791807
Time Approximations 0.015.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., -.10719e-34]Solution in 0.351s

Time Plot 0 s.
Exiting SolveHard() after 4.048r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494528064828193220842028091531 35,
441.64295974212978053248816030315322845 24,
436.91748165892494258639147810348165177 58,
422.98493398773587870388865613029390181 06,
361.52580256457414927099166117943245207 46,
401.88173905124986341206084488812084364 68,
389.59001516035721625120730168481527999 44,
328.46939894067992423777516254224227792 54,
401.50757158869875312173331133133711734 62,
358.97362824359254814737151538394675948 92,
398.33147105653404414688771972797379443 31,
371.48387394205901953393780535435054086 57,
336.61215841814612462578762888489869372 03,
361.50888347501642118669501806495932877 06,
324.67144992839824697409167723286519987 92,
302.31384315689950311953530097442655119 62,
328.46938514140890006038904534811619245 22,
343.81340624872186345044076275205010132 60,
375.73285292014338892601695469192594029 73,
328.11709294981923441482931835075250397 81,
292.99969138786573087856685343694344373 03,
358.64341561200625495780091904488275336 67,
299.89866204419221517242270053384235569 85,
360.06173468178844845463550072232071877 33,
336.59441032864537230999157019880131426 25,
256.10753186086161728382598983997625322 49,
324.65521223824666527419550661138584106 66,
331.93806790870215343462416307794581021 32,
304.79958326323224101487808051854861752 75,
323.46169176745870505434321799752915464 22,
289.54595772323193828441754158108106391 51]

Cascade time 159.018
counts: 28, 28

 Iteration 69

Start Generation 1
1 --> 0  target = [11.99999999988462572398539040026997285400,
6.21701250296091946576665438086724985414 4,
485.54908090392032556813708700927689936 25]
one interval r = 23.40850301662666279608371980107383613834 ..
27.67578046420292346829879554571554861476
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
```

```
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=0
Equations at solution: [0., 0., .4e-36]Solution in 1.039s

Time Plot 0 s.
Exiting SolveHard() after 2.193r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999988462572398539040026997285400,
6.2170125029609194657666543808672498541 44,
485.54908090392032556813708700927689936 25]
one interval r = 32.628147792249412447249365349492056651 34 ..
36.102483889501415962201672711534521389 30
Time Approximations 0.022.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, -.113e-34]Solution in 0.588s

Time Plot 0 s.
Exiting SolveHard() after 4.68r=35.4632 in [33.94922194 .. 36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684474286198281535719920948550113,
6.5834347214714159877737270059044600472 48,
467.78730596565461716254409800664963510 00]
```

```
one interval r = 32.41978955675678762389581222627881063120 ..
35.85152417381820698599490584518470276204
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, -.224e-34]Solution in 0.648s

Time Plot 0 s.
Exiting SolveHard() after 1.029r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05, none, none,
401.88173905157630511568883665627220550 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684474286198281535719920948550113,
6.58343472147141598777372700590446004724 8,
467.78730596565461716254409800664963510 00]
two intervals r = 12.92327160811855627927964838529425673577 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0 or r = 18.39424858046321344166323446771208478829 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0
Time Approximations 0.039.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.2e-38
Equations at solution: [-.2e-37, -.22e-37, .1043e-34]Solution in
40.913s
```

```
Time Plot 0 s.
Exiting SolveHard() after 46.522r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28, none,
401.88173905157630511568883665627220550 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962834937790551836559576063854777,
4.12565179687849351348948341657017370893 3,
440.67123065628244200966148264678400005 49]
two intervals r = 14.35659705104338520384062752112297194206 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0 or r = 17.70352613826246904749365599500640422803 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0
Time Approximations 0.046.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.14e-37, .1e-37, -.3860e-34]Solution in 1.34s

Time Plot 0 s.
Exiting SolveHard() after 5.555r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28, none,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
1 --> 2  target = [35.46322962834937790551836559576063854777,
4.12565179687849351348948341657017370893 3,
440.67123065628244200966148264678400005 49]
one interval r = 22.39761154375218313857221944793603066531 ..
27.23722351593570354560553231458782065149
Time Approximations 0.044.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.206 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064381041857312760842377457587809, rm =
14.37818770221711574447127769671400189717}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., .139e-34]Solution in 7.893s

Time Plot 0 s.
Exiting SolveHard() after 11.895r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888813285484971647148740455273427,
4.00486908186790206894513434110096765708 1,
404.86224502245410085962431430813521129 34]
two intervals r = 16.08011007732371320065453559114893783602 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0 or r = 16.41579812731655770187992580423598150636 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0
Time Approximations 0.051.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
```

```
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., -.2498e-34]Solution in 1.455s

Time Plot 0 s.
Exiting SolveHard() after 6.047r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81, none, none,
358.97362824862023445484688758932171654 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888132854849716471487404552734 27,
4.0048690818679020689451343411009676570 81,
404.86224502245410085962431430813521129 34]
one interval r = 21.641943994272367566789477366501001488 79 ..
26.763306600448026317557571036179123361 57
Time Approximations 0.05.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, -.107e-34]Solution in 1.016s

Time Plot 0 s.
Exiting SolveHard() after 5.34r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68, none,
358.97362824862023445484688758932171654 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941748044618330751345966923993406,
5.5896371830076302084981136672413355741 82,
443.83065885580882055541669981216297765 84]
one interval r = 22.467253744956188000051937094656919195 14 ..
27.273884283543715160994963902248340271 62
Time Approximations 0.042.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [-.1e-37, .81e-37, .39e-35]Solution in 4.331s

Time Plot 0 s.
Exiting SolveHard() after 5.309r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68, none,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941748044618330751345966923993406,
```

```
5.589637183007630208498113667241335574182,
443.830658855808820555416698121629776584]
one interval r = 32.155752795174049512722905753416555947 70 ..
35.508722287529152663556695960768868531 31
Time Approximations 0.02.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=4e-38
Equations at solution: [.6e-37, -.4e-37, -.125e-34]Solution in 0.478s

Time Plot 0 s.
Exiting SolveHard() after 0.838r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434941760754170599830006281725839 4,
441.642959738448001083469824457483972684 1,
436.917481658487491324878176278649157530 5,
422.984933984577222751783909507012284012 8,
361.525802569516965074723003515355660069 6,
401.881739051576305115688836656272205501 0,
389.590015164394460153937006464133505718 1,
328.469398946394567778076445401974574316 8,
401.507571589887035605893062165264563147 5,
358.973628248620234454846887589321716541 7,
398.331471053321874116821737348924280581 1, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931364956307643510893954654799978 54,
5.187783578502793498017636818223524104248,
408.657738633435526422047494837065900571 4]
one interval r = 21.718401146608384160023437608546851645 48 ..
26.818493035104049459710152388068400524 97
Time Approximations 0.061.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.24e-35]Solution in 4.282s

Time Plot 0 s.
Exiting SolveHard() after 5.41r=26.4632 in [23.93303356 .. 26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11, none, none,
361.50888347975799947801268780966289065 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136495630764351089395465479997854,
5.1877835785027934980176368182352410424 8,
408.65773863343552642204749483706590057 14]
one interval r = 31.80828598763218749075984674755072481414 ..
35.00011460056199219810405359616154506294
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=8e-38
Equations at solution: [-.7e-37, .8e-37, .240e-34]Solution in 0.426s

Time Plot 0 s.
Exiting SolveHard() after 0.728r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70, none,
361.50888347975799947801268780966289065 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110544831559975034295586423734880,
6.19626256521282510274656996732069980326 4,
385.44474380239854153085566943065323962 89]
one interval r = 31.60836097550217285480699280535907927114 ..
34.66372795624898411336945892774085595770
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, .308e-34]Solution in 0.535s

Time Plot 0 s.
Exiting SolveHard() after 0.814r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70, none,
```

```
361.508883479757999478012687809662890657 8,
324.671449937343967200428592418134873194 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.463471105448315599750342955864237348 80,
6.196262565212825102746569967320699803264,
385.444743802398541530855669430653239628 9]
two intervals r = 16.875634087304385205957442539552388098 48 ..
380000000000756222400674747508061553/200000000000000000000000000000000000
0 or r = 15.556404938452358286077767705755787925 82 ..
380000000000756222400674747508061553/200000000000000000000000000000000000
0
Time Approximations 0.058.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.538e-37, 0., -.2181e-34]Solution in 1.164s

Time Plot 0 s.
Exiting SolveHard() after 5.475r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434941760754170599830006281725839 4,
441.642959738448001083469824457483972684 1,
436.917481658487491324878176278649157530 5,
422.984933984577222751783909507012284012 8,
361.525802569516965074723003515355660069 6,
401.881739051576305115688836656272205501 0,
389.590015164394460153937006464133505718 1,
328.469398946394567778076445401974574316 8,
401.507571589887035605893062165264563147 5,
358.973628248620234454846887589321716541 7,
398.331471053321874116821737348924280581 1,
371.483873950517126690084977835040900257 0,
336.612158424211333844921468632100700827 8,
361.508883479757999478012687809662890657 8,
324.671449937343967200428592418134873194 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747119404317989539672513954644 63,
4.883810779855553379152594217342690053133,
376.619678567784189451149373377881105782 2]
one interval r = 21.110013048935322014417603952631175305 07 ..
```

```
26.31784243481465909389300323799209934873
Time Approximations 0.038.

hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.761063912186698964426645072343113601 29, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [.1e-37, .75e-37, -.229e-34]Solution in 4.13s

Time Plot 0 s.
Exiting SolveHard() after 4.862r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70,
336.61215842421133384492146863210070082 78,
361.50888347975799947801268780966289065 78,
324.67144993734396720042859241813487319 48, none,
328.46938514711746837541995171589358129 67, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747119404317989539672513954644 63,
4.883810779855553379152594217342690053133,
376.61967856778418945114937337788110578 22]
one interval r = 31.538994977245588450644946863968922045 50 ..
34.536183861091608032544912110767022640 56
Time Approximations 0.016.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=5.98e-36
Equations at solution: [.459e-35, -.598e-35, -.216e-34]Solution in 0.5s

Time Plot 0 s.
Exiting SolveHard() after 0.779r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70,
336.61215842421133384492146863210070082 78,
361.50888347975799947801268780966289065 78,
324.67144993734396720042859241813487319 48, none,
328.46938514711746837541995171589358129 67,
343.81340625861253617810951582919808196 06, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017554959151984779578937942296867,
6.0258135491867335594835547064221033095 84,
351.42702949646809601444204321797623575 63]
one interval r = 31.362302061279191834586237051239161717 04 ..
34.174466406346601464153657688627310580 26
Time Approximations 0.027.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., -.3e-36]Solution in 0.517s

Time Plot 0 s.
Exiting SolveHard() after 0.767r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
```

```
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70,
336.61215842421133384492146863210070082 78,
361.50888347975799947801268780966289065 78,
324.67144993734396720042859241813487319 48, none,
328.46938514711746837541995171589358129 67,
343.81340625861253617810951582919808196 06, none, none,
292.99969139766455936320108938573683051 22, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175549591519847795789379422968 67,
6.0258135491867335594835547064221033095 84,
351.42702949646809601444204321797623575 63]
two intervals r = 17.981355144176591687017694511495587330 42 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0 or r = 13.846080154866209503118558273311955916 74 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0
Time Approximations 3.269.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [-.36e-37, .1e-37, .1442e-34]Solution in 1.147s

Time Plot 0 s.
Exiting SolveHard() after 5.435r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
```

```
441.642959738448001083469824457483972684 1,
436.917481658487491324878176278649157530 5,
422.984933984577222751783909507012284012 8,
361.525802569516965074723003515355660069 6,
401.881739051576305115688836656272205501 0,
389.590015164394460153937006464133505718 1,
328.469398946394567778076445401974574316 8,
401.507571589887035605893062165264563147 5,
358.973628248620234454846887589321716541 7,
398.331471053321874116821737348924280581 1,
371.483873950517126690084977835040900257 0,
336.612158424211333844921468632100700827 8,
361.508883479757999478012687809662890657 8,
324.671449937343967200428592418134873194 8,
302.313843163612330450107717905332643710 8,
328.469385147117468375419951715893581296 7,
343.813406258612536178109515829198081960 6, none, none,
292.999691397664559363201089385736830512 2, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418789030534784905641179833909 64,
6.377943873773448514181859851045314173103,
423.288327852525383201617469177298959801 2]
one interval r = 31.946618176150773500933544559769624669 80 ..
35.212123086710613861739165980945615836 13
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.274e-34]Solution in 0.599s

Time Plot 0 s.
Exiting SolveHard() after 0.953r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434941760754170599830006281725839 4,
441.642959738448001083469824457483972684 1,
436.917481658487491324878176278649157530 5,
422.984933984577222751783909507012284012 8,
361.525802569516965074723003515355660069 6,
401.881739051576305115688836656272205501 0,
389.590015164394460153937006464133505718 1,
328.469398946394567778076445401974574316 8,
```

```
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70,
336.61215842421133384492146863210070082 78,
361.50888347975799947801268780966289065 78,
324.67144993734396720042859241813487319 48,
302.31384316361233045010771790533264371 08,
328.46938514711746837541995171589358129 67,
343.81340625861253617810951582919808196 06, none, none,
292.99969139766455936320108938573683051 22, none, none,
360.06173468275358508959574745866557097 17, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418789030534784905641179833909 64,
6.3779438737734485141818598510453141731 03,
423.28832785252538320161746917729895980 12]
two intervals r = 15.228867023889355719809314002831099014 03 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0 or r = 17.129657771193108606900629148107047983 04 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0
Time Approximations 0.061.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.31e-37, -.1e-37, .1528e-34]Solution in 1.314s

Time Plot 0 s.
Exiting SolveHard() after 5.955r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70,
```

```
336.61215842421133384492146863210070082 78,
361.50888347975799947801268780966289065 78,
324.67144993734396720042859241813487319 48,
302.31384316361233045010771790533264371 08,
328.46938514711746837541995171589358129 67,
343.81340625861253617810951582919808196 06,
375.73285291759384640624415826839647584 19, none,
292.99969139766455936320108938573683051 22, none, none,
360.06173468275358508959574745866557097 17, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234358323151545784095418262429032,
4.0035598155706566026889639358478951735 70,
404.47973595029768422423185323150940022 77]
two intervals r = 16.09683966339889423172389568099035646307 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0 or r = 16.39988649154941336605049730222572402368 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0
Time Approximations 0.049.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.18e-37, 0., .2422e-34]Solution in 1.413s

Time Plot 0 s.
Exiting SolveHard() after 5.886r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70,
336.61215842421133384492146863210070082 78,
361.50888347975799947801268780966289065 78,
324.67144993734396720042859241813487319 48,
```

```
302.31384316361233045010771790533264371 08,
328.46938514711746837541995171589358129 67,
343.81340625861253617810951582919808196 06,
375.73285291759384640624415826839647584 19, none,
292.99969139766455936320108938573683051 22,
358.64341561779730985969149865825125326 13, none,
360.06173468275358508959574745866557097 17, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234358323151545784095418262429032,
4.0035598155706560268896393584789517357 0,
404.47973595029768422423185323150940022 77]
one interval r = 21.634296300060617555689878263677139768 85 ..
26.757681698958113887236242292281092617 60
Time Approximations 0.045.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, .89e-35]Solution in 0.968s

Time Plot 0 s.
Exiting SolveHard() after 5.653r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70,
336.61215842421133384492146863210070082 78,
361.50888347975799947801268780966289065 78,
324.67144993734396720042859241813487319 48,
302.31384316361233045010771790533264371 08,
328.46938514711746837541995171589358129 67,
343.81340625861253617810951582919808196 06,
375.73285291759384640624415826839647584 19,
```

```
328.1170929563457995541494604029514049426,
292.9996913976645593632010893857368305122,
358.6434156177973098596914986582512532613, none,
360.0617346827535850895957474586655709717, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954462796480719637677739075222834,
6.196177230065649782209346086060527849768,
385.42734026600780373569025116963175733 99]
one interval r = 31.608220491054668938846167118960511982 91 ..
34.66347615062354385729444986997488395531
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, .141e-34]Solution in 0.558s

Time Plot 0 s.
Exiting SolveHard() after 0.866r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349417607541705998300062817258394,
441.6429597384480010834698244574839726841,
436.9174816584874913248781762786491575305,
422.9849339845772227517839095070122840128,
361.5258025695169650747230035153556600696,
401.8817390515763051156888366562722055010,
389.5900151643944601539370064641335057181,
328.4693989463945677780764454019745743168,
401.5075715898870356058930621652645631475,
358.9736282486202344548468875893217165417,
398.3314710533218741168217373489242805811,
371.4838739505171266900849778350409002570,
336.6121584242113338449214686321007008278,
361.5088834797579994780126878096628906578,
324.6714499373439672004285924181348731948,
302.3138431636123304501077179053326437108,
328.4693851471174683754199517158935812967,
343.8134062586125361781095158291980819606,
375.7328529175938464062441582683964758419,
328.1170929563457995541494604029514049426,
292.9996913976645593632010893857368305122,
358.6434156177973098596914986582512532613, none,
```

```
360.06173468275358508959574745866557097 17, none, none,
324.65521224699931040835471667548088935 02, none, none, none, none]

0 --> 1  target = [26.46318954462796480719637677739075222834,
6.19617723006564978220934608606052784976 8,
385.42734026600780373569025116963175733 99]
two intervals r = 16.87629600269620480606394706642352288269 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0 or r = 15.55559000693130717559239941540591763816 ..
38000000000075622240067474750806155 3/20000000000000000000000000000000000
0
Time Approximations 0.059.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.538e-37, 0., .590e-35]Solution in 1.157s

Time Plot 0 s.
Exiting SolveHard() after 5.319r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70,
336.61215842421133384492146863210070082 78,
361.50888347975799947801268780966289065 78,
324.67144993734396720042859241813487319 48,
302.31384316361233045010771790533264371 08,
328.46938514711746837541995171589358129 67,
343.81340625861253617810951582919808196 06,
375.73285291759384640624415826839647584 19,
328.11709295634579955414946040295140494 26,
292.99969139766455936320108938573683051 22,
358.64341561779730985969149865825125326 13, none,
360.06173468275358508959574745866557097 17,
```

```
336.594410334499407688123535101521253121 6, none,
324.655212246999310408354716675480889350 2, none, none, none, none]

0 --> 2  target = [34.49522661176941689536732995635431960191,
3.897131315978294502872718325180563043513,
373.78081885137851272974099519870500667 78]
two intervals r = 17.29769086211345492476546245602892523976 ..
38000000000007562224006747475080615 53/20000000000000000000000000000000000
0 or r = 14.99436407465287545431783765300643822138 ..
38000000000007562224006747475080615 53/20000000000000000000000000000000000
0
Time Approximations 0.083.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [0., 0., -.2469e-34]Solution in 1.186s

Time Plot 0 s.
Exiting SolveHard() after 6.055r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70,
336.61215842421133384492146863210070082 78,
361.50888347975799947801268780966289065 78,
324.67144993734396720042859241813487319 48,
302.31384316361233045010771790533264371 08,
328.46938514711746837541995171589358129 67,
343.81340625861253617810951582919808196 06,
375.73285291759384640624415826839647584 19,
328.11709295634579955414946040295140494 26,
292.99969139766455936320108938573683051 22,
358.64341561779730985969149865825125326 13, none,
360.06173468275358508959574745866557097 17,
```

```
336.594410334499407688123535101521253121 6, none,
324.655212246999310408354716675480889350 2,
331.938067921175725476874415268994754447 5, none, none, none]

1 --> 2  target = [34.495226611769416895367329956354319601 91,
3.897131315978294502872718325180563043513,
373.780818851378512729740995198705006677 8]
one interval r = 21.060684732141046451642664044350314059 46 ..
26.269798342850662747703666660890580555 63
Time Approximations 0.037.

hint used Hint := [25.300466340350721366923008834208363590 84, 3, -1, 1,
16.974729527641787545085746138369074748 18, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, .270e-34]Solution in 4.038s

Time Plot 0 s.
Exiting SolveHard() after 4.767r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434941760754170599830006281725839 4,
441.642959738448001083469824457483972684 1,
436.917481658487491324878176278649157530 5,
422.984933984577222751783909507012284012 8,
361.525802569516965074723003515355660069 6,
401.881739051576305115688836656272205501 0,
389.590015164394460153937006464133505718 1,
328.469398946394567778076445401974574316 8,
401.507571589887035605893062165264563147 5,
358.973628248620234454846887589321716541 7,
398.331471053321874116821737348924280581 1,
371.483873950517126690084977835040900257 0,
336.612158424211333844921468632100700827 8,
361.508883479757999478012687809662890657 8,
324.671449937343967200428592418134873194 8,
302.313843163612330450107717905332643710 8,
328.469385147117468375419951715893581296 7,
343.813406258612536178109515829198081960 6,
375.732852917593846406244158268396475841 9,
328.117092956345799554149460402951404942 6,
292.999691397664559363201089385736830512 2,
358.643415617797309859691498658251253261 3,
299.898662057567837036096934034153413489 0,
360.061734682753585089595747458665570971 7,
```

```
336.594410334499407688123535101521253121 6, none,
324.655212246999310408354716675480889350 2,
331.938067921175725476874415268994754447 5, none, none, none]

0 --> 2  target = [33.813624954282078334025829567117080391 27,
3.725648993629463838866643043805407726706,
325.892099739940714953310970705793279669 8]
two intervals r = 18.552270489908187263013849598617992445 55 ..
380000000000756222400674747508061553/200000000000000000000000000000000000
0 or r = 12.491969358511546095283142568191204642 76 ..
380000000000756222400674747508061553/200000000000000000000000000000000000
0
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=4e-38
Equations at solution: [.87e-37, -.4e-37, .332e-35]Solution in 4.333s

Time Plot 0 s.
Exiting SolveHard() after 5.782r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434941760754170599830006281725839 4,
441.642959738448001083469824457483972684 1,
436.917481658487491324878176278649157530 5,
422.984933984577222751783909507012284012 8,
361.525802569516965074723003515355660069 6,
401.881739051576305115688836656272205501 0,
389.590015164394460153937006464133505718 1,
328.469398946394567778076445401974574316 8,
401.507571589887035605893062165264563147 5,
358.973628248620234454846887589321716541 7,
398.331471053321874116821737348924280581 1,
371.483873950517126690084977835040900257 0,
336.612158424211333844921468632100700827 8,
361.508883479757999478012687809662890657 8,
324.671449937343967200428592418134873194 8,
302.313843163612330450107717905332643710 8,
328.469385147117468375419951715893581296 7,
343.813406258612536178109515829198081960 6,
375.732852917593846406244158268396475841 9,
328.117092956345799554149460402951404942 6,
292.999691397664559363201089385736830512 2,
358.643415617797309859691498658251253261 3,
```

```
299.89866205756783703609693403415341348 90,
360.06173468275358508959574745866557097 17,
336.59441033449940768812353510152125312 16, none,
324.65521224699931040835471667548088935 02,
331.93806792117572547687441526899475444 75, none, none,
289.54595773628066906839339915284314080 74]

1 --> 2  target = [33.81362495428207833402582956711708039127,
3.7256489936294638388666430438054077267 06,
325.89209973994071495331097070579327966 98]
one interval r = 20.37468935120128582387768938327104967150 ..
25.37892165314469685229322848103987367845
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, -.473e-34]Solution in 0.564s

Time Plot 0 s.
Exiting SolveHard() after 1.095r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70,
336.61215842421133384492146863210070082 78,
361.50888347975799947801268780966289065 78,
324.67144993734396720042859241813487319 48,
302.31384316361233045010771790533264371 08,
328.46938514711746837541995171589358129 67,
343.81340625861253617810951582919808196 06,
375.73285291759384640624415826839647584 19,
328.11709295634579955414946040295140494 26,
292.99969139766455936320108938573683051 22,
```

```
358.6434156177973098596914986582512532613,
299.8986620575678370360969340341534134890,
360.0617346827535850895957474586655709717,
336.5944103344994076881235351015212531216,
256.1075318746805320090438222221096730860,
324.6552122469993104083547166754808893502,
331.9380679211757254768744152689947544475, none, none,
289.5459577362806690683933991528431408074]

1 --> 0  target = [17.9304136968329885199277931238896657667 6,
4.6865087020118838847193192839409167105 09,
353.3054109626708802335601315927164852930]
one interval r = 20.7315047910923167919496425978353536929 9 ..
25.9067535353575237510599844107220930240 5
Time Approximations 0.033.

hint used Hint := [25.4020715378188039710536173439704738275 4, 3, -1, 1,
17.0062444547662134530206245122467435705 3, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.100e-34]Solution in 0.649s

Time Plot 0 s.
Exiting SolveHard() after 1.328r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349417607541705998300062817258394,
441.6429597384480010834698244574839726841,
436.9174816584874913248781762786491575305,
422.9849339845772227517839095070122840128,
361.5258025695169650747230035153556600696,
401.8817390515763051156888366562722055010,
389.5900151643944601539370064641335057181,
328.4693989463945677780764454019745743168,
401.5075715898870356058930621652645631475,
358.9736282486202344548468875893217165417,
398.3314710533218741168217373489242805811,
371.4838739505171266900849778350409002570,
336.6121584242113338449214686321007008278,
361.5088834797579994780126878096628906578,
324.6714499373439672004285924181348731948,
302.3138431636123304501077179053326437108,
328.4693851471174683754199517158935812967,
343.8134062586125361781095158291980819606,
375.7328529175938464062441582683964758419,
```

```
328.11709295634579955414946040295140494 26,
292.99969139766455936320108938573683051 22,
358.64341561779730985969149865825125326 13,
299.89866205756783703609693403415341348 90,
360.06173468275358508959574745866557097 17,
336.59441033449940768812353510152125312 16,
256.10753187468053200904382222210967308 60,
324.65521224699931040835471667548088935 02,
331.93806792117572547687441526899475444 75,
304.79958326994473059690994121406755542 06, none,
289.54595773628066906839339915284314080 74]

2 --> 0  target = [17.93041369683298851992779312388966576676,
4.68650870201188388471931928394091671050 9,
353.30541096267088023356013159271648529 30]
one interval r = 31.37435487007227991643237343470258284746 ..
34.20127520047241258384809507638932148401
Time Approximations 0.014.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [.6e-37, -.8e-37, .296e-34]Solution in 0.356s

Time Plot 0 s.
Exiting SolveHard() after 3.801r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494176075417059983000628172583 94,
441.64295973844800108346982445748397268 41,
436.91748165848749132487817627864915753 05,
422.98493398457722275178390950701228401 28,
361.52580256951696507472300351535566006 96,
401.88173905157630511568883665627220550 10,
389.59001516439446015393700646413350571 81,
328.46939894639456777807644540197457431 68,
401.50757158988703560589306216526456314 75,
358.97362824862023445484688758932171654 17,
398.33147105332187411682173734892428058 11,
371.48387395051712669008497783504090025 70,
336.61215842421133384492146863210070082 78,
361.50888347975799947801268780966289065 78,
324.67144993734396720042859241813487319 48,
302.31384316361233045010771790533264371 08,
```

```
328.46938514711746837541995171589358129 67,
343.81340625861253617810951582919808196 06,
375.73285291759384640624415826839647584 19,
328.11709295634579955414946040295140494 26,
292.99969139766455936320108938573683051 22,
358.64341561779730985969149865825125326 13,
299.89866205756783703609693403415341348 90,
360.06173468275358508959574745866557097 17,
336.59441033449940768812353510152125312 16,
256.10753187468053200904382222210967308 60,
324.65521224699931040835471667548088935 02,
331.93806792117572547687441526899475444 75,
304.79958326994473059690994121406755542 06,
323.46169177856823909151583803136993470 04,
289.54595773628066906839339915284314080 74]

Cascade time 161.201
counts: 28, 28

 Iteration 70

Start Generation 1
1 --> 0  target = [11.99999999992095478521603431326819396900,
6.21701250303736983222184974715959719589 0,
485.54908089771648727168106545329272387 44]

"Imaginary part neglected: ", $1.8899423791479898976257487132086681862 72 \times 10^{-17}$
one interval r = 23.40850301651087360572178561104503293105 ..
27.67578046425695199415195087699439283904
Time Approximations 0.045.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.05e-37
Equations at solution: [.4e-37, -.105e-36, .13e-35]Solution in 1.023s

Time Plot 0 s.
Exiting SolveHard() after 2.187r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999920954785216034313268193969 00,
6.2170125030373698322218497471595971958 90,
485.54908089771648727168106545329272387 44]
```

"Imaginary part neglected: ", $3.18322343222383739143407519736489181682 2 \times 10^{-17}$

```
one interval r = 32.628147792149080792921911073590572091 05 ..
36.102483889421544785706643778027720477 35
Time Approximations 3.44.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .45e-35]Solution in 0.573s

Time Plot 0 s.
Exiting SolveHard() after 4.386r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844780565797807743572181895987 91,
6.5834347215888380269837793234829405727 68,
467.78730595915646670410571980750929208 17]
```

"Imaginary part neglected: ", $3.18322343222383739143407519736489181682 2 \times 10^{-17}$

```
one interval r = 32.419789556654350557301133293987121269 81 ..
35.851524173730921059856166844055229235 11
Time Approximations 0.023.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
```

```
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=4e-38
Equations at solution: [.5e-37, -.4e-37, .18e-35]Solution in 0.641s

Time Plot 0 s.
Exiting SolveHard() after 1.025r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49, none, none,
401.88173904362313218841909937577335640 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844780565797807743572181895987 91,
6.5834347215888380269837793234829405727 68,
467.78730595915646670410571980750929208 17]
two intervals r = 12.923271608209086390464884564555198637 53 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000 or r = 18.394248580370394598603265361717359142 85 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.1e-38
Equations at solution: [-.2e-37, -.21e-37, .539e-35]Solution in 41.384s

Time Plot 0 s.
Exiting SolveHard() after 45.879r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
```

```
441.642959732041106370987736444343735591 5,
436.917481651441080699699253106419297984 9,
422.984933977577283054128730274548747714 5, none,
401.881739043623132188419099375773356407 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628248765132392470252846250653 88,
4.125651796858955178158034376147410778933,
440.671230649179216915099340966647873838 3]
two intervals r = 14.356597051201569156549414342467141222 31 ..
190000000000545712452484193291121902 89/100000000000000000000000000000000
00000 or r = 17.703526138091199251309783423112280418 69 ..
190000000000545712452484193291121902 89/100000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.29e-37, .1e-37, .1814e-34]Solution in 1.324s

Time Plot 0 s.
Exiting SolveHard() after 5.311r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434935680580494054545633103479499 6,
441.642959732041106370987736444343735591 5,
436.917481651441080699699253106419297984 9,
422.984933977577283054128730274548747714 5, none,
401.881739043623132188419099375773356407 0,
389.590015156110341985636348877773597409 6, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.463229628248765132392470252846250653 88,
4.125651796858955178158034376147410778933,
440.671230649179216915099340966647873838 3]
```

"Imaginary part neglected: ", $1.8899423791479898976257487132086681862 72 \times 10^{-17}$

```
one interval r = 22.397611543603315067644363333833063854 11 ..
27.237223515944797580581012981828937964 68
Time Approximations 0.044.
```

```
hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.356 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064379799534529255715556856057426, rm =
14.37818770337484522337046134603743418554}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.211e-34]Solution in 11.402s

Time Plot 0 s.
Exiting SolveHard() after 12.334r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888801346625688901306555051028615,
4.00486908184450772723647362703679130 1632,
404.86224501442086613791615872711075948 17]
two intervals r = 16.08011007753154017312516752179593981477 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000 or r = 16.41579812701334601339096501776737570439 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.69e-37, .1e-37, .5208e-34]Solution in 1.46s

Time Plot 0 s.
Exiting SolveHard() after 5.745r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96, none, none,
358.97362823932129770165106263055753586 93, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888801346625688901306555051028615,
4.00486908184450772723647362703679130163 2,
404.86224501442086613791615872711075948 17]
```

"Imaginary part neglected: ", $1.8899423791479898976257487132086681862 72 \times 10^{-17}$

```
one interval r = 21.641943994104693237005370344234590521 39 ..
26.763306600413434154652203248428751011 15
Time Approximations 0.052.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, -.194e-34]Solution in 1s

Time Plot 0 s.
Exiting SolveHard() after 5.903r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
```

```
441.642959732041106370987736444343735591 5,
436.917481651441080699699253106419297984 9,
422.984933977577283054128730274548747714 5,
361.525802560603209661357481382722849861 3,
401.881739043623132188419099375773356407 0,
389.590015156110341985636348877773597409 6,
328.469398936633528047641796903919212060 0, none,
358.973628239321297701651062630557535869 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589417638581090770957890454715071 15,
5.589637183081672022334699541183491727439,
443.830658848655545562639060822779956561 8]
```

"Imaginary part neglected: ", $1.8899423791479898976257487132086681 86272 \times 10^{-17}$

```
one interval r = 22.467253744806686113782749537120761979 82 ..
27.273884283554904780178924863955044756 24
Time Approximations 0.046.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [0., -.81e-37, -.47e-35]Solution in 4.497s

Time Plot 0 s.
Exiting SolveHard() after 5.486r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434935680580494054545633103479499 6,
441.642959732041106370987736444343735591 5,
436.917481651441080699699253106419297984 9,
422.984933977577283054128730274548747714 5,
361.525802560603209661357481382722849861 3,
401.881739043623132188419099375773356407 0,
389.590015156110341985636348877773597409 6,
328.469398936633528047641796903919212060 0, none,
358.973628239321297701651062630557535869 3,
398.331471045809955753809462374053841211 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589417638581090770957890454715071 15,
```

```
5.589637183081672022334699541183491727439,
443.830658848655545562639060822779956561 8]
```

"Imaginary part neglected: ", $3.183223432223837391434075197364891816822 \times 10^{-17}$

```
one interval r = 32.155752795067336028065018096588770285 08 ..
35.50872228742834019086170934116842486523
Time Approximations 0.022.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [-.6e-37, .5e-37, -.188e-34]Solution in 0.486s

Time Plot 0 s.
Exiting SolveHard() after 0.847r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577337540 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136518531914508243793041900247007,
5.18778357857372960232677796719684050361 8,
408.65773862496549542720929855084770286 23]

"Imaginary part neglected: ", 1.889942379147989897625748713208668186272 × 10^-17
one interval r = 21.71840114643114577235555769720148088936 ..
26.81849303506660524393003184942198887755
Time Approximations 0.063.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
```

```
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [-.1e-37, -.79e-37, -.53e-35]Solution in 4.185s

Time Plot 0 s.
Exiting SolveHard() after 5.289r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18, none, none,
361.50888347093237422772815653426449549 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136518531914508243793041900247007,
5.18778357857372960232677796719684050361 8,
408.65773862496549542720929855084770286 23]
```

"Imaginary part neglected: ", $3.18322343222383739143407519736489181682 2 \times 10^{-17}$

```
one interval r = 31.80828598751982916631161691203507950517 ..
35.00011460043677576500139412810501376725
Time Approximations 0.02.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .301e-34]Solution in 0.434s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.743r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96, none,
361.50888347093237422772815653426449549 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110537947814612627735260580451957,
6.19626256531763033530545530256346576262 4,
385.44474379332099366649588644758124066 41]
```

"Imaginary part neglected: ", $3.18322343222383739143407519736489181682 2 \times 10^{-17}$

```
one interval r = 31.60836097539160548242128461311967697941 ..
34.66372795611215523670601039957063350162
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.3e-37, -.3e-37, .29e-35]Solution in 0.541s

Time Plot 0 s.
Exiting SolveHard() after 0.831r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96, none,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110537947814612627735260580451957,
6.1962625653176303353054553025634657626 24,
385.44474379332099366649588644758124066 41]
two intervals r = 16.875634087538636430114957858302462817 95 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000 or r = 15.556404938043976602410322338472541857 97 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.1434e-36, 0., .4146e-34]Solution in 1.145s

Time Plot 0 s.
Exiting SolveHard() after 5.54r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
```

```
398.331471045809955753809462374053841211 8,
371.483873940617778409960220352189536249 6,
336.612158414805944847325810747922738641 1,
361.508883470932374227728156534264495497 0,
324.671449926914525442868596891723385900 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747357102530099135159071839615 44,
4.883810779926496864697078149795828944465,
376.619678558284366748244385488783803273 2]
```

"Imaginary part neglected: ", $1.889942379147989897625748713208668186272 \times 10^{-17}$

```
one interval r = 21.110013048751496205823618422596429498 16 ..
26.317842434730299954013871504251992635 40
Time Approximations 0.037.

hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.761063912186698964426645072343113601 29, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [.1e-37, .75e-37, -.124e-34]Solution in 3.896s

Time Plot 0 s.
Exiting SolveHard() after 4.625r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434935680580494054545633103479499 6,
441.642959732041106370987736444343735591 5,
436.917481651441080699699253106419297984 9,
422.984933977577283054128730274548747714 5,
361.525802560603209661357481382722849861 3,
401.881739043623132188419099375773356407 0,
389.590015156110341985636348877773597409 6,
328.469398936633528047641796903919212060 0,
401.507571581506383602121993695482275606 8,
358.973628239321297701651062630557535869 3,
398.331471045809955753809462374053841211 8,
371.483873940617778409960220352189536249 6,
336.612158414805944847325810747922738641 1,
361.508883470932374227728156534264495497 0,
324.671449926914525442868596891723385900 0, none,
328.469385137359055623314129889380229535 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
2 --> 0  target = [17.19898874735710253009913515907183961544,
4.883810779926496864697078149795828944465,
376.61967855828436674824438548878380327 32]
```

"Imaginary part neglected: ", $3.183223432223837391434075197364891816822 \times 10^{-17}$

```
one interval r = 31.53899497713484536806066673988163656853 ..
34.53618386094780812969199887451600080842
Time Approximations 0.018.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=4.49e-36
Equations at solution: [.345e-35, -.449e-35, .37e-35]Solution in 0.517s

Time Plot 0 s.
Exiting SolveHard() after 0.811r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577337640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00, none,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017543280532605805887126322927902,
6.025813549285914483980078483610187141967,
351.42702948651328965014575178746197812 54]
```

"Imaginary part neglected: ", $3.183223432223837391434075197364891816822 \times 10^{-17}$

```
one interval r = 31.3623020611755659520520685668564015339 3 ..
34.17446640619449687329022937783147530157
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, .134e-34]Solution in 0.533s

Time Plot 0 s.
Exiting SolveHard() after 0.783r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00, none,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25, none, none,
292.99969138640072848338354700719501823 28, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017543280532605805887126322927902,
6.025813549285914483980078483610187141967,
351.42702948651328965014575178746197812 54]
two intervals r = 17.98135514438300102346078106489139672736 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000 or r = 13.84608015431845013167495573318509865689 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
```

```
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [.36e-37, -.1e-37, .2388e-34]Solution in 4.404s

Time Plot 0 s.
Exiting SolveHard() after 5.45r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25, none, none,
292.99969138640072848338354700719501823 28, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941877648450057954726669617017762,
6.37794387388577005297314887642378329 1798,
423.28832784489201899043227232681587294 50]
```

"Imaginary part neglected: ", $3.1832234322238373914340751973648918168 22 \times 10^{-17}$

```
one interval r = 31.94661817604286260479678776105265403187 ..
35.21212308659961441482964906158264464286
Time Approximations 0.021.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
```

```
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .104e-34]Solution in 0.62s

Time Plot 0 s.
Exiting SolveHard() after 0.967r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25, none, none,
292.99969138640072848338354700719501823 28, none, none,
360.06173467369818091696582997295722214 18, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418776484500579547266696170177 62,
6.3779438738857700529731488764237832917 98,
423.28832784489201899043227232681587294 50]
two intervals r = 15.22886702408199982613106004292560440072 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000 or r = 17.12965777095817642703418948792246176339 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [.15e-37, 0., .156e-35]Solution in 1.319s

Time Plot 0 s.
Exiting SolveHard() after 5.917r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25,
375.73285290949859447737410799253714083 43, none,
292.99969138640072848338354700719501823 28, none, none,
360.06173467369818091696582997295722214 18, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343457455481296706990090402985 71,
4.0035598155457575441396115127994371343 98,
404.47973594182740119498235174398326991 46]
two intervals r = 16.096839663625631571617459812688823663 78 ..
190000000000545712452484193291121902 89/100000000000000000000000000000000
00000 or r = 16.399886491226875540546538789212093301 37 ..
190000000000545712452484193291121902 89/100000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.51e-37, -.1e-37, .2653e-34]Solution in 1.429s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.773r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577337640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25,
375.73285290949859447737410799253714083 43, none,
292.99969138640072848338354700719501823 28,
358.64341560811892899806496473939958642 77, none,
360.06173467369818091696582997295722214 18, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234345745548129670699009040298571,
4.0035598155457575441396115127994371343 98,
404.47973594182740119498235174398326991 46]

      "Imaginary part neglected: ", 1.8899423791479898976257487132086681862 72 × 10^-17
one interval r = 21.634296299884289819560967809199622676 19 ..
26.757681698916742902709326391217552971 10
Time Approximations 0.057.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, .341e-34]Solution in 4.242s

Time Plot 0 s.
Exiting SolveHard() after 5.299r=25.8653 in [23.83864811 ..
```

```
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25,
375.73285290949859447737410799253714083 43,
328.11709294618240624171426439604763782 76,
292.99969138640072848338354700719501823 28,
358.64341560811892899806496473939958642 77, none,
360.06173467369818091696582997295722214 18, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954456057728879791121887656121759,
6.19617723017089905676148015312692908586 3,
385.42734025702090744164743716902374878 95]
```

"Imaginary part neglected: ", $3.18322343222383739143407519736489181682 2 \times 10^{-17}$

```
one interval r = 31.60822049094483916529064727326676479264 ..
34.66347615048802475070856537474733038981
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=4e-38
Equations at solution: [.2e-37, -.4e-37, .158e-34]Solution in 0.598s

Time Plot 0 s.
Exiting SolveHard() after 0.911r=33.8134 in [32.62668594 ..
```

```
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25,
375.73285290949859447737410799253714083 43,
328.11709294618240624171426439604763782 76,
292.99969138640072848338354700719501823 28,
358.64341560811892899806496473939958642 77, none,
360.06173467369818091696582997295722214 18, none, none,
324.65521223665443798214276144953413578 54, none, none, none, none]

0 --> 1  target = [26.46318954456057728879791121887656121759,
6.19617723017089905676148015312692908586 3,
385.42734025702090744164743716902374878 95]
two intervals r = 16.87629600292698744465946312554372270 55 ..
19000000000054571245248419329112190289/10000000000000000000000000000000000
00000 or r = 15.55559000652711661953110674659216159280 ..
19000000000054571245248419329112190289/10000000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.179e-37, 0., .754e-35]Solution in 1.098s

Time Plot 0 s.
Exiting SolveHard() after 5.426r=17.9309 in [16.87629601 .. 19]
```

```
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25,
375.73285290949859447737410799253714083 43,
328.11709294618240624171426439604763782 76,
292.99969138640072848338354700719501823 28,
358.64341560811892899806496473939958642 77, none,
360.06173467369818091696582997295722214 18,
336.59441032518655753755114300538606399 86, none,
324.65521223665443798214276144953413578 54, none, none, none, none]

0 --> 2  target = [34.49522661161776587728309670730570215811,
3.8971313159471829656925358687354483394 65,
373.78081884135177044933005565773540804 81]
two intervals r = 17.297690862364856709453045707536523308 91 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000 or r = 14.994364074162439318829075445875199618 55 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000
Time Approximations 0.079.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [0., 0., -.2125e-34]Solution in 1.186s

Time Plot 0 s.
Exiting SolveHard() after 6.286r=18.0599 in [17.29769086 .. 19]
```

```
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25,
375.73285290949859447737410799253714083 43,
328.11709294618240624171426439604763782 76,
292.99969138640072848338354700719501823 28,
358.64341560811892899806496473939958642 77, none,
360.06173467369818091696582997295722214 18,
336.59441032518655753755114300538606399 86, none,
324.65521223665443798214276144953413578 54,
331.93806790998359889340456744234754697 81, none, none, none]

1 --> 2  target = [34.49522661161776587728309670730570215811,
3.89713131594718296569253586873544833946 5,
373.78081884135177044933005565773540804 81]

"Imaginary part neglected: ", 1.8899423791479898976257487132086681862 72 × 10^-17
one interval r = 21.06068473194996659418097006851241558149 ..
26.26979834275456590338677744141915997948
Time Approximations 0.037.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, -.154e-34]Solution in 4.161s

Time Plot 0 s.
```

```
Exiting SolveHard() after 4.916r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577337640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25,
375.73285290949859447737410799253714083 43,
328.11709294618240624171426439604763782 76,
292.99969138640072848338354700719501823 28,
358.64341560811892899806496473939958642 77,
299.89866204599437419311802103466180871 34,
360.06173467369818091696582997295722214 18,
336.59441032518655753755114300538606399 86, none,
324.65521223665443798214276144953413578 54,
331.93806790998359889340456744234754697 81, none, none, none]

0 --> 2  target = [33.81362495412061811572495779995345321797,
3.72564899359487071412008063718489657454 5,
325.89209972936510618137155736136301998 91]
two intervals r = 18.552270490068463174347109098925955644 84 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000 or r = 12.491969357895798428355742292854888751 14 ..
19000000000054571245248419329112190289/10000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=9e-38
```

```
Equations at solution: [-.209e-36, .9e-37, -.1213e-34]Solution in
4.363s

Time Plot 0 s.
Exiting SolveHard() after 5.782r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25,
375.73285290949859447737410799253714083 43,
328.11709294618240624171426439604763782 76,
292.99969138640072848338354700719501823 28,
358.64341560811892899806496473939958642 77,
299.89866204599437419311802103466180871 34,
360.06173467369818091696582997295722214 18,
336.59441032518655753755114300538606399 86, none,
324.65521223665443798214276144953413578 54,
331.93806790998359889340456744234754697 81, none, none,
289.54595772445529666735247316369991791 71]

1 --> 2  target = [33.81362495412061811572495779995345321797,
3.72564899359487071412008063718489657454 5,
325.89209972936510618137155736136301998 91]
```

"Imaginary part neglected: ", $1.8899423791479898976257487132086681862 72 \times 10^{-17}$

```
one interval r = 20.37468935105191943231797502758257277022 ..
25.37892165299201887332526277886528483499
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., -.440e-34]Solution in 0.562s

Time Plot 0 s.
Exiting SolveHard() after 1.088r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25,
375.73285290949859447737410799253714083 43,
328.11709294618240624171426439604763782 76,
292.99969138640072848338354700719501823 28,
358.64341560811892899806496473939958642 77,
299.89866204599437419311802103466180871 34,
360.06173467369818091696582997295722214 18,
336.59441032518655753755114300538606399 86,
256.10753186265039244159070824275641958 10,
324.65521223665443798214276144953413578 54,
331.93806790998359889340456744234754697 81, none, none,
289.54595772445529666735247316369991791 71]

1 --> 0  target = [17.93041369703329132338700948871801107223,
4.68650870208773604069188331791512618188 2,
353.30541095307967792834640252365708565 94]
```

"Imaginary part neglected: ", $1.8899423791479898976257487132086681862 72 \times 10^{-17}$

```
one interval r = 20.73150479092484466928717205390248308947 ..
25.90675353524922977698393451857005725537
Time Approximations 0.037.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
```

```
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., .206e-34]Solution in 0.666s

Time Plot 0 s.
Exiting SolveHard() after 1.366r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25,
375.73285290949859447737410799253714083 43,
328.11709294618240624171426439604763782 76,
292.99969138640072848338354700719501823 28,
358.64341560811892899806496473939958642 77,
299.89866204599437419311802103466180871 34,
360.06173467369818091696582997295722214 18,
336.59441032518655753755114300538606399 86,
256.10753186265039244159070824275641958 10,
324.65521223665443798214276144953413578 54,
331.93806790998359889340456744234754697 81,
304.79958326020425045115486574671795057 74, none,
289.54595772445529666735247316369991791 71]

2 --> 0  target = [17.93041369703329132338700948871801107223,
4.68650870208773604069188331791512618188 2,
353.30541095307967792834640252365708565 94]
```

"Imaginary part neglected: ", $3.18322343222383739143407519736489181682 2 \times 10^{-17}$

```
one interval r = 31.37435486997017474471666859521592001820 ..
34.20127520032559426125378243547701707813
Time Approximations 0.018.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=9e-38
Equations at solution: [.5e-37, -.9e-37, .33e-35]Solution in 3.697s

Time Plot 0 s.
Exiting SolveHard() after 3.947r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493568058049405454563310347949 96,
441.64295973204110637098773644434373559 15,
436.91748165144108069969925310641929798 49,
422.98493397757728305412873027454874771 45,
361.52580256060320966135748138272284986 13,
401.88173904362313218841909937577335640 70,
389.59001515611034198563634887777359740 96,
328.46939893663352804764179690391921206 00,
401.50757158150638360212199369548227560 68,
358.97362823932129770165106263055753586 93,
398.33147104580995575380946237405384121 18,
371.48387394061777840996022035218953624 96,
336.61215841480594484732581074792273864 11,
361.50888347093237422772815653426449549 70,
324.67144992691452544286859689172338590 00,
302.31384315350819440282245635913937396 49,
328.46938513735905562331412988938022953 50,
343.81340624752048133171114199552440611 25,
375.73285290949859447737410799253714083 43,
328.11709294618240624171426439604763782 76,
292.99969138640072848338354700719501823 28,
358.64341560811892899806496473939958642 77,
299.89866204599437419311802103466180871 34,
360.06173467369818091696582997295722214 18,
336.59441032518655753755114300538606399 86,
256.10753186265039244159070824275641958 10,
324.65521223665443798214276144953413578 54,
331.93806790998359889340456744234754697 81,
304.79958326020425045115486574671795057 74,
323.46169176722017590529933750789384753 44,
289.54595772445529666735247316369991791 71]
```

```
Cascade time 160.915
counts: 28, 28

 Iteration 71

Start Generation 1
1 --> 0  target = [12.00000000006475421943239362614702054700,
6.217012502857025129644554011188539554262,
485.549080898465355628019385822680141715 8]
one interval r = 23.40850301647440486828154702548806920862 ..
27.67578046426605879618446265374120047492
Time Approximations 0.042.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=0
Equations at solution: [0., 0., -.2e-36]Solution in 1.014s

Time Plot 0 s.
Exiting SolveHard() after 2.139r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000006475421943239362614702054700,
6.217012502857025129644554011188539554262,
485.549080898465355628019385822680141715 8]
one interval r = 32.62814779212309738704405079666812897186 ..
36.10248388940927400564064462469295035957
Time Approximations 4.379.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
```

```
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, -.111e-34]Solution in 0.571s

Time Plot 0 s.
Exiting SolveHard() after 5.324r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684478172159299629592231061669863,
6.58343472158205265154967266363535261308 5,
467.78730595880874533699993527286793956 43]
one interval r = 32.41978955661436750995592885851740161987 ..
35.85152417370262411848136099319662973256
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=4e-38
Equations at solution: [.5e-37, -.4e-37, -.287e-34]Solution in 0.637s

Time Plot 0 s.
Exiting SolveHard() after 1.028r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19, none, none,
401.88173904491049090164595076149363254 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684478172159299629592231061669863,
```

```
6.583434721582052651549672663635352613085,
467.787305958808745336999935272867939564 3]
two intervals r = 12.92327160836193481251094485441911539451 ..
474999999997486483321043647339672890 7/2500000000000000000000000000000000
000 or r = 18.39424858020881825297179596937483590595 ..
474999999997486483321043647339672890 7/2500000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.2e-38
Equations at solution: [.2e-37, .22e-37, -.489e-35]Solution in 42.274s

Time Plot 0 s.
Exiting SolveHard() after 46.948r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40, none,
401.88173904491049090164595076149363254 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962827768051406999016521711396627,
4.12565179698048373292103428687297826809 0,
440.67123065287558467799837925003561148 96]
two intervals r = 14.35659705107404045461765223259908618902 ..
474999999997486483321043647339672890 7/2500000000000000000000000000000000
000 or r = 17.70352613806275102706766658038879268477 ..
474999999997486483321043647339672890 7/2500000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.41e-37, -.2e-37, -.953e-35]Solution in 1.339s

Time Plot 0 s.
Exiting SolveHard() after 5.765r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40, none,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962827768051406999016521711396627,
4.12565179698048373292103428687297826809 0,
440.67123065287558467799837925003561148 96]
one interval r = 22.39761154360580367904929615272023354122 ..
27.23722351599285982766153872654376359313
Time Approximations 0.038.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.278 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064389148788889090096584003825714, rm =
14.37818770506993563576667014825328345999}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., .26e-37, .103e-34]Solution in 11.1s

Time Plot 0 s.
Exiting SolveHard() after 12.02r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888800259750176980487316982132716,
4.0048690819571773770782997002218169781 61,
404.86224501545322218217612910819441253 03]
two intervals r = 16.08011007747033980086487310686484477603 ..
47499999999748648332104364733967289 07/2500000000000000000000000000000000
000 or r = 16.41579812695256158928268076889921994332 ..
47499999999748648332104364733967289 07/2500000000000000000000000000000000
000
Time Approximations 0.051.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.15e-37, 0., -.3706e-34]Solution in 1.708s

Time Plot 0 s.
Exiting SolveHard() after 6.212r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26, none, none,
358.97362824479627787904326322042839125 98, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888800259750176980487316982132716,
4.0048690819571773770782997002218169781 61,
404.86224501545322218217612910819441253 03]
one interval r = 21.64194399402532709124632130119671204639 ..
```

```
26.763306600430837077571791865435977379 24
Time Approximations 0.05.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=5.2e-38
Equations at solution: [-.1e-37, -.52e-37, .233e-34]Solution in 1.043s

Time Plot 0 s.
Exiting SolveHard() after 5.129r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301 78781,
441.64295973160816150932269775235843 65823,
436.91748165533544995983566656836125 81719,
422.98493397450185126847160223607081 79040,
361.52580256566189892662402486430823 87661,
401.88173904491049090164595076149363 25489,
389.59001516350980637858050937549900 03626,
328.46939893923429718221758097287693 47174, none,
358.97362824479627787904326322042839 12598, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941789827959563208425873309458176,
5.58963718285077176716685136519484843 2795,
443.83065884504345570188042977748209 94249]
one interval r = 22.46725374464994178932097741433451826574 ..
27.27388428351821385524484481675846438532
Time Approximations 0.043.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.35e-37
Equations at solution: [-.1e-37, .135e-36, .72e-35]Solution in 4.115s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.076r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74, none,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941789827959563208425873309458176,
5.589637182850771767166851365194848432795,
443.83065884504345570188042977748209942 49]
one interval r = 32.15575279499174328610603568911895333471 ..
35.50872228735214281511937178968905841691
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.261e-34]Solution in 0.465s

Time Plot 0 s.
Exiting SolveHard() after 0.808r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
```

```
389.590015163509806378580509375499000362 6,
328.469398939234297182217580972876934717 4,
401.507571582873264056908269046456275403 0,
358.973628244796277879043263220428391259 8,
398.331471039643951582742511107750715666 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931364844610278688839633319753462 82,
5.187783578464678629700225947701253578825,
408.657738632377100494608799430178629502 9]
one interval r = 21.718401146483664957437340611630507315 99 ..
26.818493035175900750211374752072629967 60
Time Approximations 0.058.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.06e-37
Equations at solution: [-.1e-37, -.106e-36, .84e-35]Solution in 4.416s

Time Plot 0 s.
Exiting SolveHard() after 5.48r=26.4632 in [23.93303356 .. 26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936748500366028565078743017878 1,
441.642959731608161509322697752358436582 3,
436.917481655335449959835666568361258171 9,
422.984933974501851268471602236070817904 0,
361.525802565661898926624024864308238766 1,
401.881739044910490901645950761493632548 9,
389.590015163509806378580509375499000362 6,
328.469398939234297182217580972876934717 4,
401.507571582873264056908269046456275403 0,
358.973628244796277879043263220428391259 8,
398.331471039643951582742511107750715666 9, none, none,
361.508883475904290155531748712758339659 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.911931364844610278688839633319753462 82,
5.187783578464678629700225947701253578825,
408.657738632377100494608799430178629502 9]
one interval r = 31.808285987548295211256175434368198372 33 ..
35.000114600518572473962665902791507098 85
Time Approximations 0.017.
```

```
hint used Hint := [34.4952266116500808207291492480197753122 1, 3, 1, 1,
15.763855890238725343023875204542137417 16, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.134e-34]Solution in 0.444s

Time Plot 0 s.
Exiting SolveHard() after 0.737r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16, none,
361.50888347590429015553174871275833965 98, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.463471105464844149984933993568049389 94,
6.1962625653393470099024323828830671362 27,
385.44474379862490063414868420822601158 09]
one interval r = 31.608360975393992603105578028583752215 20 ..
34.663727956162159404351606411954874389 24
Time Approximations 0.018.

hint used Hint := [33.8136249540877223377638410294144486867 0, 3, 1, 1,
11.783029068742676904624149068588691721 32, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=1.2e-37
Equations at solution: [-.8e-37, .12e-36, .68e-35]Solution in 0.555s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.831r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16, none,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110546484414998493399356804938994,
6.1962625653393470099024323828830671362 27,
385.44474379862490063414868420822601158 09]
two intervals r = 16.875634087285913282370929190750605538 17 ..
47499999999748648332104364733967289 07/2500000000000000000000000000000000
000 or r = 15.556404938217801407474782246936304913 85 ..
47499999999748648332104364733967289 07/2500000000000000000000000000000000
000
Time Approximations 0.059.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.403 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175100917199306491228827817287981, rm
= 2.33653277412508882102445009170602317876 0}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.897e-37, 0., -.518e-35]Solution in 31.015s

Time Plot 0 s.
Exiting SolveHard() after 35.49r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874710325371542182176636671514929,
4.88381077978583037908938980663584814589 7,
376.61967856364384217491403275661443619 50]
one interval r = 21.11001304872915107181937579136810747016 ..
26.31784243481848122402923742214189756050
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=4.9e-38
Equations at solution: [.1e-37, .49e-37, -.136e-34]Solution in 0.866s

Time Plot 0 s.
Exiting SolveHard() after 1.571r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
```

```
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63, none,
328.46938513995666860617714289024359545 34, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874710325371542182176636671514929,
4.8838107797858303790893898066358481458 97,
376.61967856364384217491403275661443619 50]
one interval r = 31.53899497713497458257479731246969765720 ..
34.53618386099800506967510549174884036860
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=5.81e-36
Equations at solution: [-.447e-35, .581e-35, -.59e-35]Solution in 0.5s

Time Plot 0 s.
Exiting SolveHard() after 4.626r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63, none,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.872050175476835088718002796647959464 49,
6.025813549296183147856962931711494971860,
351.42702948929239156074985467730900239 34]
one interval r = 31.362302061151302817204369645656752760 70 ..
34.174466406205701965802181268064233520 45
Time Approximations 0.017.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., -.107e-34]Solution in 0.543s

Time Plot 0 s.
Exiting SolveHard() after 0.788r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63, none,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40, none, none,
292.99969139074478348936600145652483920 17, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175476835088718002796647959464 49,
6.025813549296183147856962931711494971860,
351.42702948929239156074985467730900239 34]
two intervals r = 17.981355144209308102375521890816725990 51 ..
474999999997486483321043647339672890 7/250000000000000000000000000000000
000 or r = 13.846080154460962193969645358025905113 97 ..
```

```
4749999999974864833210436473396728907/2500000000000000000000000000000000
000
Time Approximations 0.048.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 9.816 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713546001027956093140684320503 94, rm
= 2.7345009932481210051190623534551683002 76}});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [.71e-37, -.3e-37, -.174e-35]Solution in 22.796s

Time Plot 0 s.
Exiting SolveHard() after 23.88r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301 78781,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40, none, none,
292.99969139074478348936600145652483920 17, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418698015998277751838090209608 77,
6.3779438738517020938234005814497996385 06,
423.28832783865345023276692939296850490 66]
one interval r = 31.946618175943448458279815858966305475 35 ..
35.212123086484261689361442940902235545 05
Time Approximations 0.018.
```

```
hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .239e-34]Solution in 0.604s

Time Plot 0 s.
Exiting SolveHard() after 0.925r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995668680617714289024359545 34,
343.81340625790275519572601740824701770 40, none, none,
292.99969139074478348936600145652483920 17, none, none,
360.06173466951732751173986067805001513 01, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941869801599827775183809020960877,
6.37794387385170209382340058144979963850 6,
423.28832783865345023276692939296850490 66]
two intervals r = 15.22886702440674267027504152000021291858 ..
4749999999974864833210436473396728907/2500000000000000000000000000000000
000 or r = 17.12965777060959435073512993804418027071 ..
4749999999974864833210436473396728907/2500000000000000000000000000000000
000
Time Approximations 0.059.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 6.778 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541234193080167349596651547578 92, rm
= 2.0640682987717390442357678920923357771 48}});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [-.46e-37, -.2e-37, .1341e-34]Solution in
29.432s

Time Plot 0 s.
Exiting SolveHard() after 34.069r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301 78781,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40,
375.73285289992834408911726374660849805 91, none,
292.99969139074478348936600145652483920 17, none, none,
360.06173466951732751173986067805001513 01, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343447748334387879728561765128 60,
4.0035598156587054068058343660073216570 85,
404.47973594294105136163574487700385078 94]
two intervals r = 16.096839663560245162101686465835912400 36 ..
474999999997486483321043647339672890 7/2500000000000000000000000000000000
000 or r = 16.399886491170097316686219495184952942 24 ..
474999999997486483321043647339672890 7/2500000000000000000000000000000000
000
Time Approximations 0.049.
```

```
hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.4662e-34]Solution in 1.681s

Time Plot 0 s.
Exiting SolveHard() after 6.185r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40,
375.73285289992834408911726374660849805 91, none,
292.99969139074478348936600145652483920 17,
358.64341561366698840922812965829457647 49, none,
360.06173466951732751173986067805001513 01, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234344774833438787972856176512860,
4.0035598156587054068058343660073216570 85,
404.47973594294105136163574487700385078 94]
one interval r = 21.63429629980629309128790577156501422244 ..
26.75768169893533022426740297894834524340
Time Approximations 0.047.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
```

```
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, .37e-35]Solution in 0.961s

Time Plot 0 s.
Exiting SolveHard() after 5.173r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40,
375.73285289992834408911726374660849805 91,
328.11709294885812131830494629717570211 84,
292.99969139074478348936600145652483920 17,
358.64341561366698840922812965829457647 49, none,
360.06173466951732751173986067805001513 01, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954464450324836374233069455551258,
6.19617723019217880125733616447684049460 5,
385.42734026223555927434794964814176080 33]
one interval r = 31.60822049094650059539168173734189116360 ..
34.66347615053673639539560672133449645373
Time Approximations 0.016.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=2e-38
Equations at solution: [.1e-37, -.2e-37, -.165e-34]Solution in 0.546s

Time Plot 0 s.
Exiting SolveHard() after 0.847r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40,
375.73285289992834408911726374660849805 91,
328.11709294885812131830494629717570211 84,
292.99969139074478348936600145652483920 17,
358.64341561366698840922812965829457647 49, none,
360.06173466951732751173986067805001513 01, none, none,
324.65521224323403849321270915627207201 74, none, none, none, none]

0 --> 1  target = [26.46318954464450324836374233069455551258,
6.19617723019217880125733616447684049460 5,
385.42734026223555927434794964814176080 33]
two intervals r = 16.87629600267765873319755188330454310702 ..
47499999999748648332104364733967289 07/2500000000000000000000000000000000
000 or r = 15.55559000669681406455227880524704396806 ..
47499999999748648332104364733967289 07/2500000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.516 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852512913789352045694371658609169, rm
= 2.33669042819173037776063011027204897513 2}});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [-.897e-37, .1e-37, .622e-35]Solution in 30.763s

Time Plot 0 s.
Exiting SolveHard() after 35.576r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40,
375.73285289992834408911726374660849805 91,
328.11709294885812131830494629717570211 84,
292.99969139074478348936600145652483920 17,
358.64341561366698840922812965829457647 49, none,
360.06173466951732751173986067805001513 01,
336.59441032750652038306511853647452521 38, none,
324.65521224323403849321270915627207201 74, none, none, none, none]

0 --> 2  target = [34.49522661175696133170623618662256470011,
3.8971313160967517849100233331237897211 90,
373.78081885289779014889870930848559833 77]
two intervals r = 17.29769086189942854634691795426095611359 ..
4749999999974864833210436473396728907/2500000000000000000000000000000000
000 or r = 14.99436407468029383988387919349059537103 ..
4749999999974864833210436473396728907/2500000000000000000000000000000000
000
Time Approximations 0.08.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.54e-37, 0., -.1890e-34]Solution in 1.17s

Time Plot 0 s.
Exiting SolveHard() after 6.147r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40,
375.73285289992834408911726374660849805 91,
328.11709294885812131830494629717570211 84,
292.99969139074478348936600145652483920 17,
358.64341561366698840922812965829457647 49, none,
360.06173466951732751173986067805001513 01,
336.59441032750652038306511853647452521 38, none,
324.65521224323403849321270915627207201 74,
331.93806792485647752895127187011807021 01, none, none, none]

1 --> 2  target = [34.49522661175696133170623618662256470011,
3.89713131609675178491002333312378972119 0,
373.78081885289779014889870930848559833 77]
one interval r = 21.06068473203105569971397003656998261822 ..
26.26979834294862135996349622975235214361
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
```

```
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [.3e-37, .7e-37, -.405e-34]Solution in 3.8s

Time Plot 0 s.
Exiting SolveHard() after 4.55r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40,
375.73285289992834408911726374660849805 91,
328.11709294885812131830494629717570211 84,
292.99969139074478348936600145652483920 17,
358.64341561366698840922812965829457647 49,
299.89866205824634812958910681660146472 41,
360.06173466951732751173986067805001513 01,
336.59441032750652038306511853647452521 38, none,
324.65521224323403849321270915627207201 74,
331.93806792485647752895127187011807021 01, none, none, none]

0 --> 2  target = [33.81362495418214081328861306272634709131,
3.72564899372757020208432572535905262769 4,
325.89209973589791312395793564025546686 74]
two intervals r = 18.552270489826154756253081760920703444 66 ..
474999999997486483321043647339672890 7/2500000000000000000000000000000000
000 or r = 12.491969358301861867942003028857713641 44 ..
474999999997486483321043647339672890 7/2500000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1e-38
Equations at solution: [-.17e-37, .1e-37, -.486e-35]Solution in 1.142s

Time Plot 0 s.
Exiting SolveHard() after 2.535r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40,
375.73285289992834408911726374660849805 91,
328.11709294885812131830494629717570211 84,
292.99969139074478348936600145652483920 17,
358.64341561366698840922812965829457647 49,
299.89866205824634812958910681660146472 41,
360.06173466951732751173986067805001513 01,
336.59441032750652038306511853647452521 38, none,
324.65521224323403849321270915627207201 74,
331.93806792485647752895127187011807021 01, none, none,
289.54595773508204033226884937656222344 70]

1 --> 2  target = [33.81362495418214081328861306272634709131,
3.72564899372757020208432572535905262769 4,
325.89209973589791312395793564025546686 74]
one interval r = 20.37468935098357465492347436058379972321 ..
25.37892165310822521177183987344341408245
Time Approximations 0.025.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.2e-37, .2e-37, .135e-34]Solution in 0.567s

Time Plot 0 s.
Exiting SolveHard() after 4.126r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40,
375.73285289992834408911726374660849805 91,
328.11709294885812131830494629717570211 84,
292.99969139074478348936600145652483920 17,
358.64341561366698840922812965829457647 49,
299.89866205824634812958910681660146472 41,
360.06173466951732751173986067805001513 01,
336.59441032750652038306511853647452521 38,
256.10753187028460757567366281974480048 38,
324.65521224323403849321270915627207201 74,
331.93806792485647752895127187011807021 01, none, none,
289.54595773508204033226884937656222344 70]

1 --> 0  target = [17.93041369687738820579586973230054804237,
4.68650870191446839559199435639320022396 0,
353.30541095522747304290585368920176413 31]
one interval r = 20.73150479082824585526625521345404235770 ..
25.90675353528176653795675843044137287286
Time Approximations 0.034.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
```

```
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, .214e-34]Solution in 0.648s

Time Plot 0 s.
Exiting SolveHard() after 1.374r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40,
375.73285289992834408911726374660849805 91,
328.11709294885812131830494629717570211 84,
292.99969139074478348936600145652483920 17,
358.64341561366698840922812965829457647 49,
299.89866205824634812958910681660146472 41,
360.06173466951732751173986067805001513 01,
336.59441032750652038306511853647452521 38,
256.10753187028460757567366281974480048 38,
324.65521224323403849321270915627207201 74,
331.93806792485647752895127187011807021 01,
304.79958325943973236364871241091988138 72, none,
289.54595773508204033226884937656222344 70]

2 --> 0  target = [17.93041369687738820579586973230054804237,
4.68650870191446839559199435639320022396 0,
353.30541095522747304290585368920176413 31]
one interval r = 31.37435486994218085005579283514593028024 ..
34.20127520032792730340766015466618094718
```

```
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, .229e-34]Solution in 3.801s

Time Plot 0 s.
Exiting SolveHard() after 4.064r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493674850036602856507874301787 81,
441.64295973160816150932269775235843658 23,
436.91748165533544995983566656836125817 19,
422.98493397450185126847160223607081790 40,
361.52580256566189892662402486430823876 61,
401.88173904491049090164595076149363254 89,
389.59001516350980637858050937549900036 26,
328.46939893923429718221758097287693471 74,
401.50757158287326405690826904645627540 30,
358.97362824479627787904326322042839125 98,
398.33147103964395158274251110775071566 69,
371.48387395218567044621947809225128985 16,
336.61215841721695697271555205115724914 24,
361.50888347590429015553174871275833965 98,
324.67144993357738943663714931969563358 63,
302.31384315337381153131428391592189133 70,
328.46938513995666860617714289024359545 34,
343.81340625790275519572601740824701770 40,
375.73285289992834408911726374660849805 91,
328.11709294885812131830494629717570211 84,
292.99969139074478348936600145652483920 17,
358.64341561366698840922812965829457647 49,
299.89866205824634812958910681660146472 41,
360.06173466951732751173986067805001513 01,
336.59441032750652038306511853647452521 38,
256.10753187028460757567366281974480048 38,
324.65521224323403849321270915627207201 74,
331.93806792485647752895127187011807021 01,
304.79958325943973236364871241091989138 72,
323.46169177509633295467164809448718143 61,
289.54595773508204033226884937656222344 70]

Cascade time 269.492
```

```
counts: 28, 28

 Iteration 72

Start Generation 1
1 --> 0  target = [11.999999999881563939225953996049891363 00,
6.217012502922478020397304476829527455797,
485.549080899805300889286013304056016051 7]
one interval r = 23.408503016534074236532281101097056447 57 ..
27.675780464182874509894651791993777513 17
Time Approximations 0.039.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=7.7e-38
Equations at solution: [.3e-37, -.77e-37, -.17e-35]Solution in 1.02s

Time Plot 0 s.
Exiting SolveHard() after 2.154r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.642959733169627217649219783369339441 4, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999881563939225953996049891363 00,
6.217012502922478020397304476829527455797,
485.549080899805300889286013304056016051 7]
```

"Imaginary part neglected: ", $3.183223432232215032872361768244415693897 \times 10^{-17}$

```
one interval r = 32.628147792222418222746331873064837524 12 ..
36.102483889497944733221100306874533245 65
Time Approximations 0.023.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=1e-38
Equations at solution: [-.3e-37, .1e-37, .111e-34]Solution in 0.602s

Time Plot 0 s.
Exiting SolveHard() after 4.214r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684470240275821646426621810605586,
6.58343472164424089282948276077994482031 2,
467.78730596008625205004036264637805577 22]
```

"Imaginary part neglected: ", $3.18322343223221503287236176824441569389 7 \times 10^{-17}$

```
one interval r = 32.41978955671352314863644803480997418004 ..
35.85152417379102379244903713060349813578
Time Approximations 0.024.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=3e-38
Equations at solution: [.3e-37, -.3e-37, -.186e-34]Solution in 0.623s

Time Plot 0 s.
Exiting SolveHard() after 0.991r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80, none, none,
```

```
401.88173904203494163377004860739735626 66, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684470240275821646426621810605586,
6.5834347216442408928294827607799448203 12,
467.78730596008625205004036264637805577 22]
two intervals r = 12.92327160820285197866594973031952942010 ..
18999999999977579452611635173995068601/10000000000000000000000000000000
00000 or r = 18.39424858031705612464570110093121245590 ..
18999999999977579452611635173995068601/10000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.1e-38
Equations at solution: [.1e-37, .11e-37, -.3584e-35]Solution in 41.597s

Time Plot 0 s.
Exiting SolveHard() after 46.344r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90, none,
401.88173904203494163377004860739735626 66, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962831455899972446316439340342194,
4.1256517966729271383282765313129252268 17,
440.67123065050529764064543395472567018 84]
two intervals r = 14.35659705114403071750622814923134607315 ..
18999999999977579452611635173995068601/10000000000000000000000000000000
00000 or r = 17.70352613806681569722320190573821806267 ..
18999999999977579452611635173995068601/10000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
```

```
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [-.71e-37, -.3e-37, .20833e-34]Solution in
1.343s

Time Plot 0 s.
Exiting SolveHard() after 5.656r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90, none,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962831455899972446316439340342194,
4.12565179667292713832827653131292522681 7,
440.67123065050529764064543395472567018 84]
one interval r = 22.39761154362269470223547882070783225930 ..
27.23722351587911318856932017464711900326
Time Approximations 0.035.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.217 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064371787268509219364287288327612, rm =
14.37818770175357777048023371831773552223}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., .26e-37, .988e-34]Solution in 8.312s

Time Plot 0 s.
Exiting SolveHard() after 12.827r=26.4635 in [24.64256576 ..
```

```
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888032407028376904537494316422 67,
4.0048690816477599238453976032781790072 93,
404.86224501251312151205722625150763735 25]
two intervals r = 16.0801100775917043274752686272454952188 0 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000 or r = 16.4157981268881780244963359345956536195 1 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.17e-37, -.1e-37, -.7882e-35]Solution in
1.665s

Time Plot 0 s.
Exiting SolveHard() after 5.864r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61, none, none,
```

```
358.97362823715374244011678577400252205 77, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888803240702837690453749431642267,
4.0048690816477599238453976032781790072 93,
404.86224501251312151205722625150763735 25]
one interval r = 21.641943994067970177087422989509799145 02 ..
26.763306600312302723794599792549437733 45
Time Approximations 0.051.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.4e-38
Equations at solution: [0., .24e-37, .258e-34]Solution in 1.038s

Time Plot 0 s.
Exiting SolveHard() after 5.577r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80, none,
358.97362823715374244011678577400252205 77, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941778081038067241044486209437309,
5.5896371829010140237261549841715404147 30,
443.83065884620513570568230363694661214 45]
one interval r = 22.467253744741740969372832657687481421 64 ..
27.273884283444873141332113800200608539 68
Time Approximations 0.042.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
```

```
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.34e-37
Equations at solution: [-.1e-37, .134e-36, .69e-35]Solution in 4.431s

Time Plot 0 s.
Exiting SolveHard() after 5.372r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80, none,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941778081038067241044486209437309,
5.589637182901014023726154984171540414730,
443.83065884620513570568230363694661214 45]

"Imaginary part neglected: ", 3.183223432232215032872361768244415693897 × 10^-17
one interval r = 32.15575279509032004284281058098729149439 ..
35.50872228743978331950764456478377993856
Time Approximations 0.022.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.113e-34]Solution in 0.501s

Time Plot 0 s.
Exiting SolveHard() after 0.88r=34.9395 in [33.37332721 .. 35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136513912251765169489465897473164,
5.1877835784293271513729485116424669251 07,
408.65773862554543997432973759348167938 64]
one interval r = 21.71840114644320348641478950005736499558 ..
26.81849303500094184766561567052469737894
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.11e-37
Equations at solution: [.2e-37, .211e-36, -.151e-34]Solution in 0.975s

Time Plot 0 s.
Exiting SolveHard() after 5.09r=26.4632 in [23.93303356 .. 26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97, none, none,
```

```
361.50888346890493040720700018383506533 41, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136513912251765169489465897473164,
5.1877835784293271513729485116424669251 07,
408.65773862554543997432973759348167938 64]
```

"Imaginary part neglected: ", $3.18322343223221503287236176824441569389 7 \times 10^{-17}$

```
one interval r = 31.80828598757459013835207696413321509567 ..
35.00011460049178861888533700803297494984
Time Approximations 0.032.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .6e-36]Solution in 0.395s

Time Plot 0 s.
Exiting SolveHard() after 0.723r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41, none,
361.50888346890493040720700018383506533 41, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110527145331535804857773806772767,
6.19626256535473825181702036460686032503 4,
385.44474379086845801144510654531667618 08]
```

"Imaginary part neglected: ", $3.18322343223221503287236176824441569389 7 \times 10^{-17}$

```
one interval r = 31.6083609754215956284217062347018020568 6 ..
34.663727956123306697007291673242258357 57
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=8e-38
Equations at solution: [-.6e-37, .8e-37, .82e-35]Solution in 0.573s

Time Plot 0 s.
Exiting SolveHard() after 0.85r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41, none,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110527145331535804857773806772767,
6.196262565354738251817020364606860325034,
385.44474379086845801144510654531667618 08]
two intervals r = 16.87563408759397270461905876395606606417 ..
189999999999775794526116351739950686 01/10000000000000000000000000000000
00000 or r = 15.55640493789738963443417867389004781477 ..
189999999999775794526116351739950686 01/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
```

```
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.133 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175121577789517865997690027185317, rm
= 2.33653277413002093487699550469687956057 7}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.11820e-34]Solution in 30.644s

Time Plot 0 s.
Exiting SolveHard() after 35.061r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874739550816965546148100699382215,
4.8838107797537509838171601916509786326 36,
376.61967855595394324459677097548684594 09]
one interval r = 21.11001304872018434091113667044212002500 ..
26.31784243462498977377344481036910991363
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=5.2e-38
Equations at solution: [.1e-37, .52e-37, .64e-35]Solution in 4.072s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.779r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63, none,
328.46938513220871314705476821333429343 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874739550816965546148100699382215,
4.8838107797537509838171601916509786326 36,
376.61967855595394324459677097548684594 09]
```

"Imaginary part neglected: ", $3.18322343223221503287236176824441569389 7 \times 10^{-17}$

```
one interval r = 31.53899497716693110328595253894841923784 ..
34.53618386096083858178237523301276383588
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.59e-36
Equations at solution: [-.122e-35, .159e-35, .146e-34]Solution in
0.527s

Time Plot 0 s.
Exiting SolveHard() after 0.808r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63, none,
328.46938513220871314705476821333429343 99,
343.81340624504150478788624637705777849 57, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017527055941137264145882607269625,
6.02581354930586547917539819154378125786 4,
351.42702948096701718110937734843092200 80]
```

"Imaginary part neglected: ", $3.18322343223221503287236176824441569389 7 \times 10^{-17}$

```
one interval r = 31.36230206119044662686183007728867662479 ..
34.17446640616219804928730663271929461978
Time Approximations 0.018.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., -.65e-35]Solution in 0.522s

Time Plot 0 s.
Exiting SolveHard() after 0.77r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
```

```
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63, none,
328.46938513220871314705476821333429343 99,
343.81340624504150478788624637705777849 57, none, none,
292.99969137848104172986611033273633257 57, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175270559411372641458826072696 25,
6.0258135493058654791753981915437812578 64,
351.42702948096701718110937734843092200 80]
two intervals r = 17.981355144474314757971951443122549593 81 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000 or r = 13.846080154026816697094962105165854656 69 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.294 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713682055744319072785581015550 59, rm
= 2.7345009932607037716080145574501199577 73}});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.71e-37, -.2e-37, .15712e-34]Solution in
18.827s

Time Plot 0 s.
Exiting SolveHard() after 23.25r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
```

```
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63,
302.31384314456387599005889399286876796 24,
328.46938513220871314705476821333429343 99,
343.81340624504150478788624637705777849 57, none, none,
292.99969137848104172986611033273633257 57, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941863061803213678476205293753267,
6.3779438739113981127342332356460747877 48,
423.28832783970573396507174575357129860 13]

"Imaginary part neglected: ", 3.183223432232215032872361768244415693897 × 10^-17
one interval r = 31.94661817604145645099703436303113318875 ..
35.21212308657115880799836393880091435632
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .168e-34]Solution in 0.619s

Time Plot 0 s.
Exiting SolveHard() after 0.967r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
```

```
358.9736282371537424401167857740025220577,
398.3314710409914606524998100270439614297,
371.4838739406912467840727511716054450941,
336.6121584087687229350185677142813239671,
361.5088834689049304072070001838350653341,
324.6714499219517913865425883978293900663,
302.3138431445638759900588939928687679624,
328.4693851322087131470547682133342934399,
343.8134062450415047878862463770577784957, none, none,
292.9996913784810417298661103327363325757, none, none,
360.0617346662599047830393767109374746429, none, none, none, none,
none, none, none]

0 --> 1  target = [27.0203794186306180321367847620529375326 7,
6.3779438739113981127342332356460747877 48,
423.2883278397057339650717457535712986013]
two intervals r = 15.22886702432630463048558499344612882094 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000 or r = 17.12965777071566599417626886382379875513 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000
Time Approximations 0.058.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 6.957 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541237299306244550683481399763 11, rm
= 2.06406829872745587840089989631436588783 4}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., -.1327e-35]Solution in 30.764s

Time Plot 0 s.
Exiting SolveHard() after 35.275r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349376969220161494578079908362417,
441.6429597331696272176492197833693394414,
436.9174816530176928144782537138922902080,
422.9849339753202486318646986185161132990,
361.5258025584497428819089327134513609755,
401.8817390420349416337700486073973562666,
389.5900151567234578017724970435722440161,
328.4693989314799483804305707008459689980,
```

```
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63,
302.31384314456387599005889399286876796 24,
328.46938513220871314705476821333429343 99,
343.81340624504150478788624637705777849 57,
375.73285290046649648143019214475837137 47, none,
292.99969137848104172986611033273633257 57, none, none,
360.06173466625990478303937671093747464 29, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343463882735050523387254022029 20,
4.0035598153460557334849909107356587714 76,
404.47973593905666350299277476995968415 02]
two intervals r = 16.096839663722977584045698250550768019 41 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000 or r = 16.399886491065794845586341926616731018 08 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.35e-37, 0., .11111e-34]Solution in 1.707s

Time Plot 0 s.
Exiting SolveHard() after 6.75r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
```

```
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63,
302.31384314456387599005889399286876796 24,
328.46938513220871314705476821333429343 99,
343.81340624504150478788624637705777849 57,
375.73285290046649648143019214475837137 47, none,
292.99969137848104172986611033273633257 57,
358.64341560520599316187996198498057020 40, none,
360.06173466625990478303937671093747464 29, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234346388273505052338725402202920,
4.00355981534605573348499091073565877147 6,
404.47973593905666350299277476995968415 02]
one interval r = 21.63429629983049338051089800120518431515 ..
26.75768169880294180516642723144076267667
Time Approximations 0.051.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, -.484e-34]Solution in 1.058s

Time Plot 0 s.
Exiting SolveHard() after 5.359r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63,
```

```
302.31384314456387599005889399286876796 24,
328.46938513220871314705476821333429343 99,
343.81340624504150478788624637705777849 57,
375.73285290046649648143019214475837137 47,
328.11709294023393505218853805438103548 39,
292.99969137848104172986611033273633257 57,
358.64341560520599316187996198498057020 40, none,
360.06173466625990478303937671093747464 29, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954445465092997822695781119015205,
6.19617723020864193947021506873309257643 1,
385.42734025469800070008674660014518543 57]
```

"Imaginary part neglected: ", $3.18322343223221503287236176824441569389 7 \times 10^{-17}$

```
one interval r = 31.60822049097587791444386862594299400668 ..
34.66347615050105195104875554841951859200
Time Approximations 0.02.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, .15e-35]Solution in 0.573s

Time Plot 0 s.
Exiting SolveHard() after 0.872r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63,
```

```
302.31384314456387599005889399286876796 24,
328.46938513220871314705476821333429343 99,
343.81340624504150478788624637705777849 57,
375.73285290046649648143019214475837137 47,
328.11709294023393505218853805438103548 39,
292.99969137848104172986611033273633257 57,
358.64341560520599316187996198498057020 40, none,
360.06173466625990478303937671093747464 29, none, none,
324.65521223181260810695792745734000304 03, none, none, none, none]

0 --> 1  target = [26.46318954445465092997822695781119015205,
6.19617723020864193947021506873309257643 1,
385.42734025469800070008674660014518543 57]
two intervals r = 16.87629600297736865844192239402767362199 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000 or r = 15.55559000638660352978110778273674528995 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000
Time Approximations 0.058.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.139 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852533159780320780649172357198860, rm
= 2.33669042819467891459823790903100994610 5}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.180e-37, 0., -.25515e-34]Solution in 30.625s

Time Plot 0 s.
Exiting SolveHard() after 34.779r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
```

```
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63,
302.31384314456387599005889399286876796 24,
328.46938513220871314705476821333429343 99,
343.81340624504150478788624637705777849 57,
375.73285290046649648143019214475837137 47,
328.11709294023393505218853805438103548 39,
292.99969137848104172986611033273633257 57,
358.64341560520599316187996198498057020 40, none,
360.06173466625990478303937671093747464 29,
336.59441031928161355631303573446941712 62, none,
324.65521223181260810695792745734000304 03, none, none, none, none]

0 --> 2  target = [34.49522661166144321434376965488530975614,
3.8971313157564308544135874359942872298 21,
373.78081884114350786898098222850220369 43]
two intervals r = 17.29769086232657286677362454775181661404 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000 or r = 14.99436407413050067153243019431074445605 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000
Time Approximations 0.082.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.90e-37, 0., -.5461e-35]Solution in 1.219s

Time Plot 0 s.
Exiting SolveHard() after 6.017r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
```

```
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63,
302.31384314456387599005889399286876796 24,
328.46938513220871314705476821333429343 99,
343.81340624504150478788624637705777849 57,
375.73285290046649648143019214475837137 47,
328.11709294023393505218853805438103548 39,
292.99969137848104172986611033273633257 57,
358.64341560520599316187996198498057020 40, none,
360.06173466625990478303937671093747464 29,
336.59441031928161355631303573446941712 62, none,
324.65521223181260810695792745734000304 03,
331.93806790929082674860703902328419337 36, none, none, none]

1 --> 2  target = [34.49522661166144321434376965488530975614,
3.8971313157564308544135874359942872298 21,
373.78081884114350786898098222850220369 43]
one interval r = 21.06068473195665946078674095402720010405 ..
26.26979834268564412581110007651509901389
Time Approximations 0.033.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=1.2e-37
Equations at solution: [-.5e-37, -.12e-36, .591e-34]Solution in 0.763s

Time Plot 0 s.
Exiting SolveHard() after 4.727r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41,
```

```
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63,
302.31384314456387599005889399286876796 24,
328.46938513220871314705476821333429343 99,
343.81340624504150478788624637705777849 57,
375.73285290046649648143019214475837137 47,
328.11709294023393505218853805438103548 39,
292.99969137848104172986611033273633257 57,
358.64341560520599316187996198498057020 40,
299.89866204239631230218039658809058381 29,
360.06173466625990478303937671093747464 29,
336.59441031928161355631303573446941712 62, none,
324.65521223181260810695792745734000304 03,
331.93806790929082674860703902328419337 36, none, none, none]

0 --> 2  target = [33.81362495409271685456144465395162859332,
3.7256489933856000660079073738293314951 79,
325.89209972400457478307773197057586968 22]
two intervals r = 18.552270490096991254927134193771513050 64 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000 or r = 12.491969357648694187906984848605271019 14 ..
189999999999775794526116351739950686 01/100000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=8e-38
Equations at solution: [.190e-36, -.8e-37, .11748e-34]Solution in
4.512s

Time Plot 0 s.
Exiting SolveHard() after 5.977r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
```

```
358.9736282371537424401167857740025220577,
398.3314710409914606524998100270439614297,
371.4838739406912467840727511716054450941,
336.6121584087687229350185677142813239671,
361.5088834689049304072070001838350653341,
324.6714499219517913865425883978293900663,
302.3138431445638759900588939928687679624,
328.4693851322087131470547682133342934399,
343.8134062450415047878862463770577784957,
375.7328529004664964814301921447583713747,
328.1170929402339350521885380543810354839,
292.9996913784810417298661103327363325757,
358.6434156052059931618799619849805702040,
299.8986620423963123021803965880905838129,
360.0617346662599047830393767109374746429,
336.5944103192816135563130357344694171262, none,
324.6552122318126081069579274573400030403,
331.9380679092908267486070390232841933736, none, none,
289.5459577192031792319923059974725280662]

1 --> 2  target = [33.8136249540927168545614446539516285933 2,
3.7256489933856000660079073738293314951 79,
325.8920997240045747830777319705758696822]
one interval r = 20.3746893510134671760932163029308332921 0 ..
25.3789216528322596269935008024671354147 6
Time Approximations 0.027.

hint used Hint := [24.3394990302538142174610776606811553342 3, 3, -1, 1,
17.2721587825129442122670749442843558157 4, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, .119e-34]Solution in 0.585s

Time Plot 0 s.
Exiting SolveHard() after 1.124r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349376969220161494578079908362417,
441.6429597331696272176492197833693394414,
436.9174816530176928144782537138922902080,
422.9849339753202486318646986185161132990,
361.5258025584497428819089327134513609755,
401.8817390420349416337700486073973562666,
389.5900151567234578017724970435722440161,
328.4693989314799483804305707008459689980,
```

```
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63,
302.31384314456387599005889399286876796 24,
328.46938513220871314705476821333429343 99,
343.81340624504150478788624637705777849 57,
375.73285290046649648143019214475837137 47,
328.11709294023393505218853805438103548 39,
292.99969137848104172986611033273633257 57,
358.64341560520599316187996198498057020 40,
299.89866204239631230218039658809058381 29,
360.06173466625990478303937671093747464 29,
336.59441031928161355631303573446941712 62,
256.10753185434334004901729885234105396 49,
324.65521223181260810695792745734000304 03,
331.93806790929082674860703902328419337 36, none, none,
289.54595771920317923199230599747252806 62]

1 --> 0  target = [17.93041369715135521727293995491809604521,
4.68650870188225427583749641356192388 5027,
353.30541094672098633824430634026741099 08]
one interval r = 20.73150479084572972636705135160133134271 ..
25.90675353507218244015590407612847535119
Time Approximations 0.036.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.44e-35]Solution in 3.841s

Time Plot 0 s.
Exiting SolveHard() after 4.547r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
```

```
389.5900151567234578017724970435722440161,
328.4693989314799483804305707008459689980,
401.5075715790740203198509653614767500255,
358.9736282371537424401167857740025220577,
398.3314710409914606524998100270439614297,
371.4838739406912467840727511716054450941,
336.6121584087687229350185677142813239671,
361.5088834689049304072070001838350653341,
324.6714499219517913865425883978293900663,
302.3138431445638759900588939928687679624,
328.4693851322087131470547682133342934399,
343.8134062450415047878862463770577784957,
375.7328529004664964814301921447583713747,
328.1170929402339350521885380543810354839,
292.9996913784810417298661103327363325757,
358.6434156052059931618799619849805702040,
299.8986620423963123021803965880905838129,
360.0617346662599047830393767109374746429,
336.5944103192816135563130357344694171262,
256.1075318543433400490172988523410539649,
324.6552122318126081069579274573400030403,
331.9380679092908267486070390232841933736,
304.7995832509840750555065625830936651483, none,
289.5459577192031792319923059974725280662]

2 --> 0  target = [17.93041369715135521727293995491809604521,
4.686508701882254275837496413561923885027,
353.3054109467209863382443063402674109908]
```

"Imaginary part neglected: ", $3.183223432232215032872361768244415693897 \times 10^{-17}$

```
one interval r = 31.37435486997923829191607376973910869852 ..
34.20127520028158917768927160670570281960
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., .87e-35]Solution in 0.377s

Time Plot 0 s.
Exiting SolveHard() after 0.655r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493769692201614945780799083624 17,
441.64295973316962721764921978336933944 14,
436.91748165301769281447825371389229020 80,
422.98493397532024863186469861851611329 90,
361.52580255844974288190893271345136097 55,
401.88173904203494163377004860739735626 66,
389.59001515672345780177249704357224401 61,
328.46939893147994838043057070084596899 80,
401.50757157907402031985096536147675002 55,
358.97362823715374244011678577400252205 77,
398.33147104099146065249981002704396142 97,
371.48387394069124678407275117160544509 41,
336.61215840876872293501856771428132396 71,
361.50888346890493040720700018383506533 41,
324.67144992195179138654258839782939006 63,
302.31384314456387599005889399286876796 24,
328.46938513220871314705476821333429343 99,
343.81340624504150478788624637705777849 57,
375.73285290046649648143019214475837137 47,
328.11709294023393505218853805438103548 39,
292.99969137848104172986611033273633257 57,
358.64341560520599316187996198498057020 40,
299.89866204239631230218039658809058381 29,
360.06173466625990478303937671093747464 29,
336.59441031928161355631303573446941712 62,
256.10753185434334004901729885234105396 49,
324.65521223181260810695792745734000304 03,
331.93806790929082674860703902328419337 36,
304.79958325098407505550656258309366514 83,
323.46169176118899331836521678731866503 62,
289.54595771920317923199230599747252806 62]

Cascade time 268.324
counts: 28, 28

 Iteration 73

Start Generation 1
1 --> 0  target = [12.00000000009910341335493477560908661700,
6.21701250305168435402452680205002133473 7,
485.54908089882280346698676578977698924 82]
one interval r = 23.40850301664489004402408859359396824528 ..
27.67578046441856155269711780298392684423
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.32e-37
Equations at solution: [-.4e-37, .132e-36, 0.]Solution in 4.467s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.624r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000009910341335493477560908661700,
6.217012503051684354024526802050021334737,
485.54908089882280346698676578977698924 82]
one interval r = 32.628147792261529002647793715537752926 61 ..
36.10248388945152638171912706356267841962
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .84e-35]Solution in 0.605s

Time Plot 0 s.
Exiting SolveHard() after 1.037r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684493278073431177928153631348013,
6.583434721671250756381781081706590099280,
467.78730595978984406125415187347134178 37]
one interval r = 32.419789556766552318779030023693845235 53 ..
35.85152417375974643155063484400526323943
Time Approximations 0.022.
```

```
hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .190e-34]Solution in 0.625s

Time Plot 0 s.
Exiting SolveHard() after 0.989r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11, none, none,
401.88173904268288511617944622082367526 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844932780734311779281536313480 13,
6.5834347216712507563817810817065900992 80,
467.78730595978984406125415187347134178 37]
```

"Imaginary part neglected: ", $1.10311211490428060629848648125643749542 2 \times 10^{-17}$

```
two intervals r = 12.923271608406938790546870746559799663 19 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000 or r = 18.394248580410645335131071364665764228 24 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.1e-38
Equations at solution: [.2e-37, .21e-37, .167e-35]Solution in 41.906s

Time Plot 0 s.
Exiting SolveHard() after 46.364r=14.1926 in [12.92327158 ..
```

```
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15, none,
401.88173904268288511617944622082367526 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962828078906757245603961850619874,
4.1256517968037175724348256646862826245 12,
440.67123064943333965726247990508546652 47]
```

"Imaginary part neglected: ", $1.10311211490428060629848648125643749542 2 \times 10^{-17}$

```
two intervals r = 14.35659705140036940518410122806663819001 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000 or r = 17.70352613811915164574119671831738798977 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [-.56e-37, -.3e-37, -.336e-35]Solution in 4.578s

Time Plot 0 s.
Exiting SolveHard() after 5.666r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15, none,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962828078906757245603961850619874,
4.12565179680371757243482566468628262451 2,
440.67123064943333965726247990508546652 47]
one interval r = 22.39761154372134511332832148754108687387 ..
27.23722351608474408181451037445073525912
Time Approximations 0.039.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.567 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064392823766727204143060560275006, rm =
14.37818770375670053041566459813440939088}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., .527e-34]Solution in 12.128s

Time Plot 0 s.
Exiting SolveHard() after 13.044r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888803673891404476678647360150652,
4.00486908178453740406668327813487162108 3,
404.86224501325379891782294197396325487 72]
```

"Imaginary part neglected: ", $1.10311211490428060629848648125643749542 2 \times 10^{-17}$

```
two intervals r = 16.08011007775675009304364658416548411680 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000 or r = 16.41579812698332396972834414291437060761 ..
```

```
190000000001099257259852633195279030 19/100000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.52e-37, .1e-37, -.271e-35]Solution in 5.046s

Time Plot 0 s.
Exiting SolveHard() after 6.184r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336 12478,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97, none, none,
358.97362823711010360426979136770961331 28, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888803673891404476678647360150652,
4.00486908178453740406668327813487162108 3,
404.86224501325379891782294197396325487 72]
one interval r = 21.641943994198326696553668173425876547 92 ..
26.763306600522830382681404472228631206 35
Time Approximations 0.052.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [.1e-37, .49e-37, .577e-34]Solution in 4.643s

Time Plot 0 s.
```

```
Exiting SolveHard() after 5.69r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15, none,
358.97362823711010360426979136770961331 28, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941788418771327514461897707557271,
5.589637183088300953910180249312534613303,
443.83065884806526975008791011623745912 90]
one interval r = 22.46725374490574264156105074290071446756 ..
27.27388428368612288343271735420968427692
Time Approximations 0.038.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.08e-37
Equations at solution: [.1e-37, -.108e-36, .112e-34]Solution in 1.007s

Time Plot 0 s.
Exiting SolveHard() after 1.957r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15, none,
358.97362823711010360426979136770961331 28,
```

```
398.331471044361388399053491947388688234 7, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941788418771327514461897707557271,
5.589637183088300953910180249312534613303,
443.83065884806526975008791011623745912 90]
one interval r = 32.15575279517350174021562487055543121886 ..
35.50872228744719339783354586085159895252
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .27e-35]Solution in 3.677s

Time Plot 0 s.
Exiting SolveHard() after 4.032r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.331471044361388399053491947388688234 7, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136540352644974489722804384632875,
5.187783578587860493869646240611838569999,
408.65773862407573553990233063264905141 79]
one interval r = 21.71840114652955962366740707701324846496 ..
26.81849303518134265219560352502527052539
Time Approximations 0.058.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [0., .79e-37, -.120e-34]Solution in 0.988s

Time Plot 0 s.
Exiting SolveHard() after 2.046r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47, none, none,
361.50888346911648931794955054613483709 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136540352644974489722804384632875,
5.18778357858786049386964624061183856999 9,
408.65773862407573553990233063264905141 79]
one interval r = 31.80828598763412264052723505549744625241 ..
35.00011460046278981076387155632420723707
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=0
Equations at solution: [0., 0., -.561e-34]Solution in 0.414s

Time Plot 0 s.
Exiting SolveHard() after 0.709r=34.4952 in [32.91337941 ..
35.00011460]
```

```
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99, none,
361.50888346911648931794955054613483709 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110546862274802906917523605395268,
6.19626256538981943826145541928847978133 6,
385.44474379130083059643672412634665010 46]
one interval r = 31.60836097550358050723023830984468853256 ..
34.66372795612975393662043175703210382517
Time Approximations 0.016.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, -.208e-34]Solution in 0.579s

Time Plot 0 s.
Exiting SolveHard() after 4.625r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
```

```
389.590015155276225702752767826830113989 7,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99, none,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110546862274802906917523605395268,
6.1962625653898194382614554192884797813 36,
385.44474379130083059643672412634665010 46]
```

"Imaginary part neglected: ", $1.10311211490428060629848648125643749542 2 \times 10^{-17}$

```
two intervals r = 16.875634087768062549928191304704870870 43 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000 or r = 15.556404937970704305827781833368994235 78 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000
Time Approximations 0.062.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.496e-35]Solution in 4.629s

Time Plot 0 s.
Exiting SolveHard() after 5.802r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.590015155276225702752767826830113989 7,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
```

```
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874758133741390383326348237844268,
4.8838107799342729570891851219376103581 45,
376.61967855597874802944360058587182754 43]
one interval r = 21.11001304883215140322414615080488044528 ..
26.31784243481142224629053529122515101799
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., .307e-34]Solution in 0.87s

Time Plot 0 s.
Exiting SolveHard() after 1.565r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10, none,
328.46938513411695930114506451168731696 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874758133741390383326348237844268,
4.8838107799342729570891851219376103581 45,
376.61967855597874802944360058587182754 43]
one interval r = 31.53899497724754854620125172560835289360 ..
34.53618386096441060530319295770649946134
Time Approximations 0.017.
```

```
hint used Hint := [34.0897679959070796784441725040016442615 2, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=7.73e-36
Equations at solution: [.594e-35, -.773e-35, -.119e-34]Solution in
0.492s

Time Plot 0 s.
Exiting SolveHard() after 0.775r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336 12478,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10, none,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017548496157258372676574463969958,
6.0258135493515641461999609497312634654 65,
351.42702948311108811331597185822240366 24]
one interval r = 31.362302061289484098434943669893458795 69 ..
34.174466406204674590642305679959306154 48
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.216e-34]Solution in 3.944s

Time Plot 0 s.
Exiting SolveHard() after 4.174r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10, none,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49, none, none,
292.99969138148230707412396735122128742 54, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017548496157258372676574463969958,
6.0258135493515641461999609497312634654 65,
351.42702948311108811331597185822240366 24]
```

"Imaginary part neglected: ", $1.10311211490428060629848648125643749542 2 \times 10^{-17}$

```
two intervals r = 17.98135514459116755407948148905091417 97 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000 or r = 13.84608015417245816732696648358907033511 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=18.6878, rm=15.3648} with Delta=4e-38
Equations at solution: [-.123e-36, .4e-37, .717e-35]Solution in 1.151s

Time Plot 0 s.
Exiting SolveHard() after 2.21r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10,
302.31384314904887260577118696743031147 41,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49, none, none,
292.99969138148230707412396735122128742 54, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941888753619551473960999966811803,
6.37794387395939472642651681022495433792 1,
423.28832784333329874595506470347235708 42]
one interval r = 31.94661817614605332306905272123031345187 ..
35.21212308661108506604109509294969870965
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, .29e-35]Solution in 4.033s

Time Plot 0 s.
Exiting SolveHard() after 4.366r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10,
302.31384314904887260577118696743031147 41,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49, none, none,
292.99969138148230707412396735122128742 54, none, none,
360.06173467061869720076986074692398177 87, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418887536195514739609999668118 03,
6.3779438739593947264265168102249543379 21,
423.28832784333329874595506470347235708 42]
```

"Imaginary part neglected: ", $1.10311211490428060629848648125643749542 2 \times 10^{-17}$

```
two intervals r = 15.22886702435183526626509809451228078525 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000 or r = 17.12965777092060698588498667623379136407 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000
Time Approximations 0.067.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.16e-37, -.1e-37, -.3794e-34]Solution in
4.382s

Time Plot 0 s.
Exiting SolveHard() after 5.745r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10,
302.31384314904887260577118696743031147 41,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49,
375.73285290665448879610648423957515900 90, none,
292.99969138148230707412396735122128742 54, none, none,
360.06173467061869720076986074692398177 87, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234347396575269425694918908843856,
4.0035598154841605130845104998017889684 68,
404.47973594018505134941633848647908726 66]

"Imaginary part neglected: ", 1.1031121149042806062984864812564374954 22 × 10^-17
two intervals r = 16.09683966387105367269420976666378805568 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000 or r = 16.39988649117695371470985866187960636661 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.32e-37, .1e-37, -.347e-35]Solution in 1.474s

Time Plot 0 s.
Exiting SolveHard() after 2.551r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10,
302.31384314904887260577118696743031147 41,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49,
375.73285290665448879610648423957515900 90, none,
292.99969138148230707412396735122128742 54,
358.64341560549585568976228999927709640 09, none,
360.06173467061869720076986074692398177 87, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234347396575269425694918908843856,
4.00355981548416051308451049980178896846 8,
404.47973594018505134941633848647908726 66]
one interval r = 21.63429629996851374166729102967227745526 ..
26.75768169901900826688102804723535706021
Time Approximations 0.052.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., .124e-34]Solution in 1.071s

Time Plot 0 s.
Exiting SolveHard() after 5.697r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10,
302.31384314904887260577118696743031147 41,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49,
375.73285290665448879610648423957515900 90,
328.11709294249988869562788187174483909 83,
292.99969138148230707412396735122128742 54,
358.64341560549585568976228999927709640 09, none,
360.06173467061869720076986074692398177 87, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954465111845139076845837592109458,
6.19617723024351369918272760141819430043 7,
385.42734025508746732203743691602690547 15]
one interval r = 31.60822049105752001541777155642088537930 ..
34.66347615050688432490728754239477470480
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=5e-38
Equations at solution: [.3e-37, -.5e-37, .57e-35]Solution in 0.58s

Time Plot 0 s.
Exiting SolveHard() after 0.87r=33.8134 in [32.62668594 .. 34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
```

```
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10,
302.31384314904887260577118696743031147 41,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49,
375.73285290665448879610648423957515900 90,
328.11709294249988869562788187174483909 83,
292.99969138148230707412396735122128742 54,
358.64341560549585568976228999927709640 09, none,
360.06173467061869720076986074692398177 87, none, none,
324.65521223316070364136661819779664592 61, none, none, none, none]

0 --> 1  target = [26.46318954465111845139076845837592109458,
6.19617723024351369918272760141819430043 7,
385.42734025508746732203743691602690547 15]
```

"Imaginary part neglected: ", $1.10311211490428060629848648125643749542 2 \times 10^{-17}$

```
two intervals r = 16.87629600315308538731915228654103667821 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000 or r = 15.55559000645790058486261841574955175679 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.717e-37, 0., .52e-36]Solution in 1.186s

Time Plot 0 s.
Exiting SolveHard() after 5.557r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
```

```
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10,
302.31384314904887260577118696743031147 41,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49,
375.73285290665448879610648423957515900 90,
328.11709294249988869562788187174483909 83,
292.99969138148230707412396735122128742 54,
358.64341560549585568976228999927709640 09, none,
360.06173467061869720076986074692398177 87,
336.59441032208001579477032147039522748 40, none,
324.65521223316070364136661819779664592 61, none, none, none, none]

0 --> 2  target = [34.49522661163685849280314551848357200287,
3.89713131588363671581429323763718040404 7,
373.78081883914816748967707869909783727 57]

"Imaginary part neglected: ", 1.10311211490428060629848648125643749542 2 × 10^-17
two intervals r = 17.29769086258042795468352929429234137607 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000 or r = 14.99436407407790709131012183387525622309 ..
19000000000109925725985263319527903019/10000000000000000000000000000000
00000
Time Approximations 0.084.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.36e-37, 0., .1825e-34]Solution in 1.238s

Time Plot 0 s.
Exiting SolveHard() after 6.077r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10,
302.31384314904887260577118696743031147 41,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49,
375.73285290665448879610648423957515900 90,
328.11709294249988869562788187174483909 83,
292.99969138148230707412396735122128742 54,
358.64341560549585568976228999927709640 09, none,
360.06173467061869720076986074692398177 87,
336.59441032208001579477032147039522748 40, none,
324.65521223316070364136661819779664592 61,
331.93806790676225663401531886149287355 96, none, none, none]

1 --> 2  target = [34.49522661163685849280314551848357200287,
3.89713131588363671581429323763718040404 7,
373.78081883914816748967707869909783727 57]
one interval r = 21.06068473203347368606676837777176119571 ..
26.26979834283637063031139670077723000932
Time Approximations 0.032.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=4e-38
Equations at solution: [-.2e-37, -.4e-37, .177e-34]Solution in 0.775s

Time Plot 0 s.
Exiting SolveHard() after 4.833r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
```

```
441.642959732731200532648395382619095850 4,
436.917481651891778341313532311462634981 1,
422.984933977061712579171267886768928351 5,
361.525802558703016934516041011230295373 3,
401.881739042682885116179446220823675267 4,
389.590015155276225702752767826830113989 7,
328.469398933389049482405246183388578121 5,
401.507571580101216250154733225985187097 5,
358.973628237110103604269791367709613312 8,
398.331471044361388399053491947388688234 7,
371.483873938663555547602341629865299339 9,
336.612158411610902139584497245813059644 7,
361.508883469116489317949550546134837096 5,
324.671449923339907299712209320689439241 0,
302.313843149048872605771186967430311474 1,
328.469385134116959301145064511687316964 9,
343.813406244132413637949975127347527294 9,
375.732852906654488796106484239575159009 0,
328.117092942499888695627881871744839098 3,
292.999691381482307074123967351221287425 4,
358.643415605495855689762289999277096400 9,
299.898662041775970234817763064277787781 8,
360.061734670618697200769860746923981778 7,
336.594410322080015794770321470395227484 0, none,
324.655212233160703641366618197796645926 1,
331.938067906762256634015318861492873559 6, none, none, none]

0 --> 2  target = [33.813624954134392979348738060650598688 84,
3.725648993525370853825299118964577088800,
325.892099725504044611217836650774539198 2]
```

"Imaginary part neglected: ", $1.10311211490428060629848648125643749542 2 \times 10^{-17}$

```
two intervals r = 18.552270490230678826076505874801802684 96 ..
19000000000109925725985263319527903019/100000000000000000000000000000000
00000 or r = 12.491969357743287967887535358628529267 96 ..
19000000000109925725985263319527903019/100000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1e-38
Equations at solution: [-.36e-37, .1e-37, .789e-35]Solution in 4.418s

Time Plot 0 s.
Exiting SolveHard() after 5.905r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10,
302.31384314904887260577118696743031147 41,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49,
375.73285290665448879610648423957515900 90,
328.11709294249988869562788187174483909 83,
292.99969138148230707412396735122128742 54,
358.64341560549585568976228999927709640 09,
299.89866204177597023481776306427778778 18,
360.06173467061869720076986074692398177 87,
336.59441032208001579477032147039522748 40, none,
324.65521223316070364136661819779664592 61,
331.93806790676225663401531886149287355 96, none, none,
289.54595771967461320747151316506448819 19]

1 --> 2  target = [33.81362495413439297934873806065059868884,
3.72564899352537085382529911896457708880 0,
325.89209972550404461121783665077453919 82]
one interval r = 20.37468935113700612478063187630362678523 ..
25.37892165302628646330398265181184010683
Time Approximations 0.025.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=6e-38
Equations at solution: [.5e-37, .6e-37, -.22e-35]Solution in 0.583s

Time Plot 0 s.
Exiting SolveHard() after 1.091r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
```

```
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10,
302.31384314904887260577118696743031147 41,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49,
375.73285290665448879610648423957515900 90,
328.11709294249988869562788187174483909 83,
292.99969138148230707412396735122128742 54,
358.64341560549585568976228999927709640 09,
299.89866204177597023481776306427778778 18,
360.06173467061869720076986074692398177 87,
336.59441032208001579477032147039522748 40,
256.10753185689405415437908508806305165 91,
324.65521223316070364136661819779664592 61,
331.93806790676225663401531886149287355 96, none, none,
289.54595771967461320747151316506448819 19]

1 --> 0  target = [17.93041369724313932539019137523270376934,
4.68650870209115976989033297161121094927 2,
353.30541094976387546950004572462101971 67]
one interval r = 20.73150479099996181931732179711281732128 ..
25.90675353530369679405155991930848223471
Time Approximations 0.037.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=1.42e-37
Equations at solution: [.5e-37, .142e-36, -.78e-35]Solution in 4.261s

Time Plot 0 s.
Exiting SolveHard() after 4.975r=25.4021 in [22.67806074 ..
```

```
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10,
302.31384314904887260577118696743031147 41,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49,
375.73285290665448879610648423957515900 90,
328.11709294249988869562788187174483909 83,
292.99969138148230707412396735122128742 54,
358.64341560549585568976228999927709640 09,
299.89866204177597023481776306427778778 18,
360.06173467061869720076986074692398177 87,
336.59441032208001579477032147039522748 40,
256.10753185689405415437908508806305165 91,
324.65521223316070364136661819779664592 61,
331.93806790676225663401531886149287355 96,
304.79958325598875497175434640061024989 15, none,
289.54595771967461320747151316506448819 19]

2 --> 0  target = [17.93041369724313932539019137523270376934,
4.68650870209115976989033297161121094927 2,
353.30541094976387546950004572462101971 67]
one interval r = 31.37435487008395804878344331857795727432 ..
34.20127520033628763921695377870719452325
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.277e-34]Solution in 0.383s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.655r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493680821495625821068247336124 78,
441.64295973273120053264839538261909585 04,
436.91748165189177834131353231146263498 11,
422.98493397706171257917126788676892835 15,
361.52580255870301693451604101123029537 33,
401.88173904268288511617944622082367526 74,
389.59001515527622570275276782683011398 97,
328.46939893338904948240524618338857812 15,
401.50757158010121625015473322598518709 75,
358.97362823711010360426979136770961331 28,
398.33147104436138839905349194738868823 47,
371.48387393866355554760234162986529933 99,
336.61215841161090213958449724581305964 47,
361.50888346911648931794955054613483709 65,
324.67144992333990729971220932068943924 10,
302.31384314904887260577118696743031147 41,
328.46938513411695930114506451168731696 49,
343.81340624413241363794997512734752729 49,
375.73285290665448879610648423957515900 90,
328.11709294249988869562788187174483909 83,
292.99969138148230707412396735122128742 54,
358.64341560549585568976228999927709640 09,
299.89866204177597023481776306427778778 18,
360.06173467061869720076986074692398177 87,
336.59441032208001579477032147039522748 40,
256.10753185689405415437908508806305165 91,
324.65521223316070364136661819779664592 61,
331.93806790676225663401531886149287355 96,
304.79958325598875497175434640061024989 15,
323.46169176282957771562984884781129947 94,
289.54595771967461320747151316506448819 19]

Cascade time 160.89
counts: 28, 28

 Iteration 74

Start Generation 1
1 --> 0  target = [12.00000000010220734875342518196421627900,
6.21701250288063831025628342388884168924 1,
485.54908089429767654444530612470389018 99]
one interval r = 23.40850301639096786429616728760774064158 ..
27.67578046414352832529245307052974481661
Time Approximations 0.042.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
```

```
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.5e-38
Equations at solution: [-.1e-37, .25e-37, .4e-36]Solution in 1.032s

Time Plot 0 s.
Exiting SolveHard() after 2.2r=27.5236 in [25.56992694 .. 27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000010220734875342518196421627900,
6.21701250288063831025628342388884168924 1,
485.54908089429767654444530612470389018 99]
```

"Imaginary part neglected: ", $3.18322343222461444594354849137167035251 2 \times 10^{-17}$

```
one interval r = 32.62814779207472346724616320736099214393 ..
36.10248389936800886015031132447742061001
Time Approximations 0.025.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., -.146e-34]Solution in 0.618s

Time Plot 0 s.
Exiting SolveHard() after 4.332r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
```

```
441.642959727397654168274135977354640688 1,
436.91748164988140567010524716600322474 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844652700929141146667476036114 10,
6.583434721663218423698601411702371041908,
467.787305954398169986632151728251806 1058]
```

"Imaginary part neglected: ", $3.18322343222461444594354849137167035251 2 \times 10^{-17}$

```
one interval r = 32.419789556564728076803627657319436739 77 ..
35.851524173655962080396035614164264759 26
Time Approximations 0.023.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., .75e-35]Solution in 0.63s

Time Plot 0 s.
Exiting SolveHard() after 0.995r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434932694278631931157309593717389 8,
441.642959727397654168274135977354640688 1,
436.91748164988140567010524716600322474 68, none, none,
401.881739038544998929111063260916031630 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844652700929141146667476036114 10,
6.583434721663218423698601411702371041908,
467.787305954398169986632151728251806 1058]
two intervals r = 12.923271608442028147203070141176222885 14 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000 or r = 18.394248580163401205336542064313762395 86 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
```

```
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=4.4e-38
Equations at solution: [.4e-37, .44e-37, .1494e-34]Solution in 41.965s

Time Plot 0 s.
Exiting SolveHard() after 46.573r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80, none,
401.88173903854499892911106326091603163 00, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628215631759833139131714367736 74,
4.125651796889710070680730135325141746264,
440.671230647582759299631579183840211935 2]
two intervals r = 14.356597051227916956046591364453005105 21 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000 or r = 17.703526137947934956005931126251748685 82 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000
Time Approximations 0.051.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., -.80e-36]Solution in 4.658s

Time Plot 0 s.
Exiting SolveHard() after 5.75r=15.9119 in [14.35659706 .. 18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80, none,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962821563175983313913171436773674,
4.1256517968897100706807301353251417462 64,
440.67123064758275929963157918384021193 52]
one interval r = 22.397611543532304298287774237214489091 60 ..
27.237223515848150554239994475615926419 38
Time Approximations 0.039.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.557 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643737444016583148765469855009 30, rm =
14.378187704800979872227211657562647473 19}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, -.156e-34]Solution in 12.156s

Time Plot 0 s.
Exiting SolveHard() after 13.078r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
```

```
0 --> 2  target = [34.945078887924123030922095228811198770 35,
4.004869081862753624645617229751688774812,
404.862245009226595275281440378684487254 8]
two intervals r = 16.08011007766934283379687956239510330041 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000 or r = 16.41579812672715930191512810946357073999 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., .1593e-34]Solution in 4.859s

Time Plot 0 s.
Exiting SolveHard() after 5.997r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20, none, none,
358.97362823711011747988344278654282059 03, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887924123030922095228811198770 35,
4.004869081862753624645617229751688774812,
404.862245009226595275281440378684487254 8]
one interval r = 21.64194399396905125454271188391963633265 ..
26.76330660026616643825476599066688805954
Time Approximations 0.054.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=1.01e-37
Equations at solution: [-.2e-37, -.101e-36, -.777e-34]Solution in
4.628s

Time Plot 0 s.
Exiting SolveHard() after 5.68r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01, none,
358.97362823711011747988344278654282059 03, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941804549510010328285811259100458,
5.5896371828693181281700117506847020827 45,
443.83065883984201969553639116602127709 66]
one interval r = 22.46725374457560122508939248289247576662 ..
27.27388428337530320548597549353704530142
Time Approximations 0.038.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.6e-38
Equations at solution: [0., .26e-37, -.3e-36]Solution in 1.042s

Time Plot 0 s.
Exiting SolveHard() after 2.032r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
```

```
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01, none,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941804549510010328285811259100458,
5.5896371828693181281700117506847020827 45,
443.83065883984201969553639116602127709 66]
```

"Imaginary part neglected: ", $3.18322343222461444594354849137167035251 2 \times 10^{-17}$

```
one interval r = 32.15575279493646387519249566977910707088 ..
35.50872228729171265950747419921534108454
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., -.255e-34]Solution in 4.174s

Time Plot 0 s.
Exiting SolveHard() after 4.538r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]
```

```
1 --> 0  target = [15.91193136507435583081697397358593091423,
5.18778357847522823530998989925477964379 3,
408.65773862550461057523562147122182088 50]
one interval r = 21.71840114640994946884531814933484671157 ..
26.81849303500267446431129839863062024648
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=3.16e-37
Equations at solution: [.2e-37, .316e-36, -.7e-36]Solution in 1.007s

Time Plot 0 s.
Exiting SolveHard() after 2.117r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06, none, none,
361.50888346864622872471780285212605981 09, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136507435583081697397358593091423,
5.18778357847522823530998989925477964379 3,
408.65773862550461057523562147122182088 50]
```

"Imaginary part neglected: ", $3.18322343222461444594354849137167035251 2 \times 10^{-17}$

```
one interval r = 31.80828598748393200278123326362604056156 ..
35.00011460043098953396642048558485917913
Time Approximations 0.02.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.204e-34]Solution in 3.892s

Time Plot 0 s.
Exiting SolveHard() after 4.185r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48, none,
361.50888346864622872471780285212605981 09, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110527389393069217350338385476067,
6.19626256540029399779941370187699254 2348,
385.44474379095417231358785768947492218 25]

"Imaginary part neglected: ", 3.1832234322246144459435484913716703525 12 × 10^-17
one interval r = 31.60836097532979461847000852671242239903 ..
34.66372795606170864524271642042217259009
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, .273e-34]Solution in 0.591s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.885r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48, none,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110527389393069217350338385476067,
6.19626256540029399779941370187699254234 8,
385.44474379095417231358785768947492218 25]
two intervals r = 16.875634087526597349334322269915341582 32 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000 or r = 15.556404937880506819771200690833757343 82 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000
Time Approximations 0.063.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 10.869 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175116285089579734018883307094359, rm
= 2.33653277406674944961212816800949118781 0}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.538e-37, 0., .196e-35]Solution in 31.396s

Time Plot 0.001 s.
Exiting SolveHard() after 32.554r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874734020962647439160741829809068,
4.88381077979591986213568426767668632657 0,
376.61967855573785686689974202559919881 06]
one interval r = 21.11001304867793691808689373878989768043 ..
26.31784243462181027201392530247869281704
Time Approximations 0.033.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=1.25e-37
Equations at solution: [.4e-37, .125e-36, -.13e-35]Solution in 0.843s

Time Plot 0 s.
Exiting SolveHard() after 4.945r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
```

```
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06, none,
328.46938513174877423684540376438592315 57, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874734020962647439160741829809068,
4.88381077979591986213568426767668632657 0,
376.61967855573785686689974202559919881 06]
```

"Imaginary part neglected: ", $3.18322343222461444594354849137167035251 2 \times 10^{-17}$

```
one interval r = 31.53899497707201576731804714129522842224 ..
34.53618386089384735533522206078908867666
Time Approximations 0.019.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=8.95e-36
Equations at solution: [-.687e-35, .895e-35, -.98e-35]Solution in
0.476s

Time Plot 0 s.
Exiting SolveHard() after 0.783r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
```

```
361.5088834686462287247178028521260598109,
324.6714499240688463178361253062965128406, none,
328.4693851317487742368454037643859231557,
343.8134062479586944225384110181481181134, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017526472717528737553522617845779,
6.0258135493499446741709282688523245208 38,
351.4270294807392375851751451429337044051]
```

"Imaginary part neglected: ", $3.183223432224614445943548491371670352512 \times 10^{-17}$

```
one interval r = 31.36230206109376862320289022996483153184 ..
34.17446640609203882935009507981621258110
Time Approximations 0.018.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=9e-38
Equations at solution: [.5e-37, -.9e-37, -.53e-35]Solution in 0.524s

Time Plot 0 s.
Exiting SolveHard() after 0.744r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349326942786319311573095937173898,
441.6429597273976541682741359773546406881,
436.9174816498814056701052471660032247468,
422.9849339694514839186567922884896461580,
361.5258025582980430949101375845653559270,
401.8817390385449989291110632609160316300,
389.5900151568408979121427129888366243120,
328.4693989310233373215471038673034860901,
401.5075715759720992469294825294100696523,
358.9736282371101174798834427865428205903,
398.3314710341098971178141209867258961906,
371.4838739434453264083410690391478654948,
336.6121584090134530852071524157583150158,
361.5088834686462287247178028521260598109,
324.6714499240688463178361253062965128406, none,
328.4693851317487742368454037643859231557,
343.8134062479586944225384110181481181134, none, none,
292.9996913803799914517772908722786545371, none, none, none, none,
none, none, none, none, none, none]
```

```
0 --> 1  target = [25.872050175264727175287375535226178457 79,
6.025813549349944674170928268852324520838,
351.42702948073923758517514514293370440 51]
two intervals r = 17.981355144421417721518940475899850085 22 ..
37999999999863614470811855234152218 13/20000000000000000000000000000000000
000 or r = 13.846080154000632556409317432075574476 20 ..
37999999999863614470811855234152218 13/20000000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.278 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713635948444331759644660239912 5, rm
= 2.734500993158898013867921664601150623646}});
Accepted {r=18.6878, rm=15.3648} with Delta=6e-38
Equations at solution: [.196e-36, -.6e-37, -.1109e-34]Solution in
18.58s

Time Plot 0 s.
Exiting SolveHard() after 22.821r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06,
302.31384314447465593598594727635409944 35,
328.46938513174877423684540376438592315 57,
343.81340624795869442253841101814811811 34, none, none,
292.99969138037999145177729087227865453 71, none, none, none, none,
none, none, none, none, none, none]
```

```
2 --> 1  target = [27.02037941854331892072298813443387851513,
6.37794387392422699292654933855669925471 0,
423.28832783287390096710113364121695081 32]
```

"Imaginary part neglected: ", $3.18322343222461444594354849137167035251 2 \times 10^{-17}$

```
one interval r = 31.94661817588596862748844846332839834656 ..
35.21212308641363628602332178869318708242
Time Approximations 0.022.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.352e-34]Solution in 0.613s

Time Plot 0 s.
Exiting SolveHard() after 0.961r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06,
302.31384314447465593598594727635409944 35,
328.46938513174877423684540376438592315 57,
343.81340624795869442253841101814811811 34, none, none,
292.99969138037999145177729087227865453 71, none, none,
360.06173466181580360910518490326046954 28, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941854331892072298813443387851513,
6.37794387392422699292654933855669925471 0,
423.28832783287390096710113364121695081 32]
```

```
two intervals r = 15.22886702458984506359926895695182961129 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000 or r = 17.12965777044290329448922389421068410318 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000
Time Approximations 0.062.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.396 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541397674971791576835433736353 09, rm
= 2.0640682987290125503395008427369722087 49}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.31e-37, -.1e-37, .6e-37]Solution in 30.74s

Time Plot 0 s.
Exiting SolveHard() after 35.295r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06,
302.31384314447465593598594727635409944 35,
328.46938513174877423684540376438592315 57,
343.81340624795869442253841101814811811 34,
375.73285289349315326344784171515271647 24, none,
292.99969138037999145177729087227865453 71, none, none,
360.06173466181580360910518490326046954 28, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343361307781577434020902993030 42,
```

```
4.0035598155624008086268654858845509639 92,
404.4797359361667812978367449846296776521]
two intervals r = 16.09683966378297464568701999051236821461 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000 or r = 16.39988649092099877353845595438604853003 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.7615110884558244986626642365610012490 6, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.17e-37, 0., -.1785e-34]Solution in 5.16s

Time Plot 0 s.
Exiting SolveHard() after 6.275r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349326942786319311573095937173898,
441.6429597273976541682741359773546406881,
436.9174816498814056701052471660032247468,
422.9849339694514839186567922884896461580,
361.5258025582980430949101375845653559270,
401.8817390385449989291110632609160316300,
389.5900151568408979121427129888366243120,
328.4693989310233373215471038673034860901,
401.5075715759720992469294825294100696523,
358.9736282371101174798834427865428205903,
398.3314710341098971178141209867258961906,
371.4838739434453264083410690391478654948,
336.6121584090134530852071524157583150158,
361.5088834686462287247178028521260598109,
324.6714499240688463178361253062965128406,
302.3138431444746559359859472763540994435,
328.4693851317487742368454037643859231557,
343.8134062479586944225384110181481181134,
375.7328528934931532634478417151527164724, none,
292.9996913803799914517772908722786545371,
358.6434156055059426573625386662526766084, none,
360.0617346618158036091051849032604695428, none, none, none, none,
none, none, none]

1 --> 2  target = [34.9395323433613077815774340209029930304 2,
4.0035598155624008086268654858845509639 92,
404.4797359361667812978367449846296776521]
```

```
one interval r = 21.6342962997395174821093455038823900257 5 ..
26.75768169876252669494746391493598083880
Time Approximations 0.054.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.2e-38
Equations at solution: [-.2e-37, -.52e-37, -.181e-34]Solution in 4.548s

Time Plot 0 s.
Exiting SolveHard() after 5.615r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06,
302.31384314447465593598594727635409944 35,
328.46938513174877423684540376438592315 57,
343.81340624795869442253841101814811811 34,
375.73285289349315326344784171515271647 24,
328.11709294014296100771883659321347730 45,
292.99969138037999145177729087227865453 71,
358.64341560550594265736253866625267660 84, none,
360.06173466181580360910518490326046954 28, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954445530978885433479877319179452,
6.19617723025365806581048285889783175619 5,
385.42734025467364953255928447000386544 70]
```

"Imaginary part neglected: ", $3.18322343222461444594354849137167035251 2 \times 10^{-17}$

```
one interval r = 31.6082204908831868525929199555426064586 1 ..
34.663476150437859377251854318407649224 21
Time Approximations 0.019.

hint used Hint := [33.813392675100157147469018161182119369 13, 3, 1, 1,
11.783209121413521870719438174471899656 27, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., .85e-35]Solution in 0.585s

Time Plot 0 s.
Exiting SolveHard() after 0.887r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06,
302.31384314447465593598594727635409944 35,
328.46938513174877423684540376438592315 57,
343.81340624795869442253841101814811811 34,
375.73285289349315326344784171515271647 24,
328.11709294014296100771883659321347730 45,
292.99969138037999145177729087227865453 71,
358.64341560550594265736253866625267660 84, none,
360.06173466181580360910518490326046954 28, none, none,
324.65521223382700714718784894351211918 13, none, none, none, none]

0 --> 1  target = [26.46318954445530978885433479877319179452,
6.19617723025365806581048285889783175619 5,
385.42734025467364953255928447000386544 70]
two intervals r = 16.87629600291418189311712440889519907143 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000 or r = 15.55559000636457025219878291915564723380 ..
```

```
37999999999863614470811855234152218 13/20000000000000000000000000000000
000
Time Approximations 0.063.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 11.025 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852528073947282797379665025666699, rm
= 2.33669042813238880798886419478492305 6323}});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [-.896e-37, .1e-37, -.2631e-34]Solution in
35.011s

Time Plot 0 s.
Exiting SolveHard() after 36.194r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371 73898,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06,
302.31384314447465593598594727635409944 35,
328.46938513174877423684540376438592315 57,
343.81340624795869442253841101814811811 34,
375.73285289349315326344784171515271647 24,
328.11709294014296100771883659321347730 45,
292.99969138037999145177729087227865453 71,
358.64341560550594265736253866625267660 84, none,
360.06173466181580360910518490326046954 28,
336.59441031941408725877175346273559214 30, none,
324.65521223382700714718784894351211918 13, none, none, none, none]
```

```
0 --> 2  target = [34.49522661164189025169000029091250424412,
3.897131315993286633818305597921296318877,
373.78081884424020685257302540811774894 28]
two intervals r = 17.29769086215783508837930650827188598685 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000 or r = 14.99436407426503184338396992209374495412 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000
Time Approximations 0.087.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.18e-37, 0., -.3151e-34]Solution in 4.585s

Time Plot 0 s.
Exiting SolveHard() after 6.179r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371 73898,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06,
302.31384314447465593598594727635409944 35,
328.46938513174877423684540376438592315 57,
343.81340624795869442253841101814811811 34,
375.73285289349315326344784171515271647 24,
328.11709294014296100771883659321347730 45,
292.99969138037999145177729087227865453 71,
358.64341560550594265736253866625267660 84, none,
360.06173466181580360910518490326046954 28,
336.59441031941408725877175346273559214 30, none,
324.65521223382700714718784894351211918 13,
331.93806791489933793636255912664282171 38, none, none, none]
```

```
1 --> 2  target = [34.49522661164189025169000029091250424412,
3.897131315993286633818305597921296318877,
373.78081884424020685257302540811774894 28]
one interval r = 21.06068473197104178871515686645740838690 ..
26.26979834273861436917711743099421272626
Time Approximations 0.032.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [.3e-37, .7e-37, -.13e-35]Solution in 0.755s

Time Plot 0 s.
Exiting SolveHard() after 1.428r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06,
302.31384314447465593598594727635409944 35,
328.46938513174877423684540376438592315 57,
343.81340624795869442253841101814811811 34,
375.73285289349315326344784171515271647 24,
328.11709294014296100771883659321347730 45,
292.99969138037999145177729087227865453 71,
358.64341560550594265736253866625267660 84,
299.89866204782062163941429802252029525 59,
360.06173466181580360910518490326046954 28,
336.59441031941408725877175346273559214 30, none,
324.65521223382700714718784894351211918 13,
331.93806791489933793636255912664282171 38, none, none, none]
```

```
0 --> 2  target = [33.81362495405679853063519167127447794109,
3.72564899362004422158141611869609109173 2,
325.89209972644917283834045535832936851 45]
two intervals r = 18.552270489998654477532610619448595402 55 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000 or r = 12.49196935776778133560098024018002877117 ..
37999999999863614470811855234152218 13/2000000000000000000000000000000000
000
Time Approximations 0.038.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=2e-38
Equations at solution: [-.52e-37, .2e-37, .450e-35]Solution in 1.09s

Time Plot 0 s.
Exiting SolveHard() after 6.258r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371 73898,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06,
302.31384314447465593598594727635409944 35,
328.46938513174877423684540376438592315 57,
343.81340624795869442253841101814811811 34,
375.73285289349315326344784171515271647 24,
328.11709294014296100771883659321347730 45,
292.99969138037999145177729087227865453 71,
358.64341560550594265736253866625267660 84,
299.89866204782062163941429802252029525 59,
360.06173466181580360910518490326046954 28,
336.59441031941408725877175346273559214 30, none,
324.65521223382700714718784894351211918 13,
```

```
331.938067914899337936362559126642821713 8, none, none,
289.5459577242836813581562142059231966463]

1 --> 2  target = [33.81362495405679853063519167127447794109,
3.7256489936200442215814161186960910917 32,
325.8920997264491728383404553583293685145]
one interval r = 20.37468935099428815303161631392106060608 ..
25.37892165287702147795698038561157797135
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=8e-38
Equations at solution: [-.6e-37, -.8e-37, -.32e-35]Solution in 0.548s

Time Plot 0 s.
Exiting SolveHard() after 1.118r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06,
302.31384314447465593598594727635409944 35,
328.46938513174877423684540376438592315 57,
343.81340624795869442253841101814811811 34,
375.73285289349315326344784171515271647 24,
328.11709294014296100771883659321347730 45,
292.99969138037999145177729087227865453 71,
358.64341560550594265736253866625267660 84,
299.89866204782062163941429802252029525 59,
360.06173466181580360910518490326046954 28,
336.59441031941408725877175346273559214 30,
```

```
256.10753185917276939706980470116011829 34,
324.65521223382700714718784894351211918 13,
331.93806791489933793636255912664282171 38, none, none,
289.54595772428368135815621420592319664 63]

1 --> 0  target = [17.93041369708983909000907306454479494307,
4.6865087019256950027386112080041016749 63,
353.30541094680137738622643328664930307 43]
one interval r = 20.73150479080452187642303106639489081688 ..
25.90675353507222489155998695693015567213
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=9.4e-38
Equations at solution: [-.4e-37, -.94e-37, .93e-35]Solution in 0.677s

Time Plot 0 s.
Exiting SolveHard() after 4.913r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06,
302.31384314447465593598594727635409944 35,
328.46938513174877423684540376438592315 57,
343.81340624795869442253841101814811811 34,
375.73285289349315326344784171515271647 24,
328.11709294014296100771883659321347730 45,
292.99969138037999145177729087227865453 71,
358.64341560550594265736253866625267660 84,
299.89866204782062163941429802252029525 59,
```

```
360.06173466181580360910518490326046954 28,
336.59441031941408725877175346273559214 30,
256.10753185917276939706980470116011829 34,
324.65521223382700714718784894351211918 13,
331.93806791489933793636255912664282171 38,
304.79958325081092440132303327030403917 83, none,
289.54595772428368135815621420592319664 63]

2 --> 0  target = [17.930413697089839090009073064544794943 07,
4.6865087019256950027386112080041016749 63,
353.30541094680137738622643328664930307 43]
```

"Imaginary part neglected: ", $3.18322343222461444594354849137167035251 2 \times 10^{-17}$

```
one interval r = 31.374354869884664270099786865587304978 20 ..
34.201275200216057466274769511647891249 07
Time Approximations 0.018.

hint used Hint := [33.796329522691901201472146203888695425 08, 2, 1, -1,
17.863534069716629640044658426895081929 62, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., -.22e-35]Solution in 0.34s

Time Plot 0 s.
Exiting SolveHard() after 0.64r=33.7963 in [32.25770943 .. 34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493269427863193115730959371738 98,
441.64295972739765416827413597735464068 81,
436.91748164988140567010524716600322474 68,
422.98493396945148391865679228848964615 80,
361.52580255829804309491013758456535592 70,
401.88173903854499892911106326091603163 00,
389.59001515684089791214271298883662431 20,
328.46939893102333732154710386730348609 01,
401.50757157597209924692948252941006965 23,
358.97362823711011747988344278654282059 03,
398.33147103410989711781412098672589619 06,
371.48387394344532640834106903914786549 48,
336.61215840901345308520715241575831501 58,
361.50888346864622872471780285212605981 09,
324.67144992406884631783612530629651284 06,
302.31384314447465593598594727635409944 35,
328.46938513174877423684540376438592315 57,
343.81340624795869442253841101814811811 34,
```

```
375.7328528934931532634478417151527164724,
328.1170929401429610077188365932134773045,
292.9996913803799914517772908722786545371,
358.6434156055059426573625386662526766084,
299.8986620478206216394142980225202952559,
360.0617346618158036091051849032604695428,
336.5944103194140872587717534627355921430,
256.1075318591727693970698047011601182934,
324.6552122338270071471878489435121191813,
331.9380679148993379363625591266428217138,
304.7995832508109244013230332703040391783,
323.4616917645194404152188409360552437496,
289.5459577242836813581562142059231966463]

Cascade time 266.033
counts: 28, 28

 Iteration 75

Start Generation 1
1 --> 0  target = [12.00000000014845770827660920081453230400,
6.217012502833652019114262821458206817326,
485.5490808990240980431337490302675485325]
```

"Imaginary part neglected: ", $1.889942379153796472084452949303110763108 \times 10^{-17}$

```
one interval r = 23.40850301655084800342475845586888035733 ..
27.67578046432991434601333456899494948750
Time Approximations 0.047.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.2e-38
Equations at solution: [-.2e-37, .52e-37, -.3e-36]Solution in 4.163s

Time Plot 0 s.
Exiting SolveHard() after 5.329r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349369758539438465132589602708147,
441.6429597310427997278383327669071477704, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0  target = [12.000000000148457708276609200814532304 00,
6.2170125028336520191142628214582068173 26,
485.54908089902409804313374903026754853 25]
one interval r = 32.628147792179587653859304791256340427 36 ..
36.102483889429959015777147857268996254 37
Time Approximations 0.021.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., .37e-35]Solution in 0.587s

Time Plot 0 s.
Exiting SolveHard() after 1.015r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844831491514361598415920957452 96,
6.5834347217444476996316185537033248197 09,
467.78730595808172578964263408845238027 70]
one interval r = 32.419789556658451292084495254412767891 08 ..
35.851524173707381368237265032378290086 37
Time Approximations 0.021.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.64e-35]Solution in 0.62s

Time Plot 0 s.
```

```
Exiting SolveHard() after 0.975r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74, none, none,
401.88173904038994673918878256852374975 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684483149151436159841592095745296,
6.58343472174444769963161855370332481 9709,
467.78730595808172578964263408845238027 70]
two intervals r = 12.92327160853504598414667610935233984156 ..
95000000000123037131993754497603680 87/50000000000000000000000000000000
000 or r = 18.39424858029076951840534105493298370831 ..
95000000000123037131993754497603680 87/50000000000000000000000000000000
000
Time Approximations 0.042.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=0
Equations at solution: [0., 0., .64e-36]Solution in 41.925s

Time Plot 0 s.
Exiting SolveHard() after 46.933r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64, none,
401.88173904038994673918878256852374975 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962828642273449204388150340839726,
4.12565179684020242524166189468779696083 6,
```

```
440.671230652184149370668744266767307512 2]
two intervals r = 14.35659705126393363760790008738145585122 ..
95000000000123037131993754497603680 87/500000000000000000000000000000000000
000 or r = 17.70352613812396248982798569222743099790 ..
95000000000123037131993754497603680 87/500000000000000000000000000000000000
000
Time Approximations 3.593.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.29e-37, -.1e-37, .1689e-34]Solution in 1.26s

Time Plot 0 s.
Exiting SolveHard() after 5.896r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394846513258960270 8147,
441.64295973104279972783832766907147 7704,
436.91748165439606268338727923501133 75774,
422.98493397019458957204138692768123 34864, none,
401.88173904038994673918878256852374 97504,
389.59001516090304991289835876071424 06127, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962828642273449204388150340839726,
4.12565179684020242524166189468779696 0836,
440.671230652184149370668744266767307512 2]
```

"Imaginary part neglected: ", $1.8899423791537964720844529493031107631 08 \times 10^{-17}$

```
one interval r = 22.39761154366479783013330645869322865807 ..
27.23722351603773615563524859780906381625
Time Approximations 0.044.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.54 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643934482412035620141270301864 31, rm =
14.378187705361379507880117052819126479 49}});
Accepted {r=26.4635, rm=16.5329} with Delta=7.9e-38
Equations at solution: [.1e-37, .79e-37, -.444e-34]Solution in 12.093s

Time Plot 0 s.
Exiting SolveHard() after 13.029r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887963253616412807973670100222 00,
4.0048690818043714803305096566662009624 51,
404.86224501110089777667308715879383818 35]
two intervals r = 16.080110077822644500066051557908499030 19 ..
95000000000123037131993754497603680 87/500000000000000000000000000000000
000 or r = 16.415798126831489418870311738805380231 41 ..
95000000000123037131993754497603680 87/500000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.17e-37, .1e-37, -.1772e-34]Solution in 4.848s

Time Plot 0 s.
Exiting SolveHard() after 5.992r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27, none, none,
358.97362823881405225237889990999512071 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888796325361641280797367010022200,
4.00486908180437148033050965666620096 2451,
404.86224501110089777667308715879383818 35]
```

"Imaginary part neglected: ", $1.8899423791537964720844529493031107631 08 \times 10^{-17}$

```
one interval r = 21.64194399402175170950331365830042507782 ..
26.76330660041688578306268432322072086167
Time Approximations 0.059.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.225e-34]Solution in 5.299s

Time Plot 0 s.
Exiting SolveHard() after 6.353r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75, none,
358.97362823881405225237889990999512071 16, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941827044543384562473227894747332,
5.589637182776224650578318712670642772645,
443.83065884083123660924191139509264151 98]
```

"Imaginary part neglected: ", $1.889942379153796472084452949303110763108 \times 10^{-17}$

```
one interval r = 22.46725374463025582327958470100373187443 ..
27.27388428352326240720141475667426196845
Time Approximations 0.043.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.3e-38
Equations at solution: [0., -.53e-37, .50e-35]Solution in 1.026s

Time Plot 0 s.
Exiting SolveHard() after 2.021r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75, none,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941827044543384562473227894747332,
5.589637182776224650578318712670642772645,
443.83065884083123660924191139509264151 98]
one interval r = 32.15575279500296268441407403911705851052 ..
35.50872228730980043999985140314778872242
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
```

```
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .73e-35]Solution in 0.462s

Time Plot 0 s.
Exiting SolveHard() after 4.144r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136511240793923753457382235861216,
5.18778357842797132233729141600896871 9493,
408.65773863000795377124392822247329912 70]
```

"Imaginary part neglected: ", $1.8899423791537964720844529493031107631 08 \times 10^{-17}$

```
one interval r = 21.71840114651812875368653762607085392970 ..
26.81849303519201898321943467509551590529
Time Approximations 0.062.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.84e-37
Equations at solution: [.2e-37, .184e-36, -.180e-34]Solution in 1.011s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.154r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45, none, none,
361.50888347092832599329303031889012952 57, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136511240793923753457382235861216,
5.18778357842797132233729141600896871 9493,
408.65773863000795377124392822247329912 70]
one interval r = 31.80828598758705011763658926954562442887 ..
35.00011460050743649297233706301604190456
Time Approximations 0.021.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, .121e-34]Solution in 0.409s

Time Plot 0 s.
Exiting SolveHard() after 4.295r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
```

```
361.525802560509442311329852410653899992 5,
401.881739040389946739188782568523749750 4,
389.590015160903049912898358760714240612 7,
328.469398930671764601902748591276183577 5,
401.507571577297195538960826675884865257 0,
358.973628238814052252378899909995120711 6,
398.331471032982875473614424005239499244 5,
371.483873947748555087326592280640988145 8, none,
361.508883470928325993293030318890129525 7, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110542598141182281079805836550971,
6.196262565479050799831709283251292945345,
385.44474379318118342245212040871288474 45]
one interval r = 31.60836097541318056912638954001325721089 ..
34.66372795611002645743697254404897855987
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=2e-38
Equations at solution: [-.1e-37, .2e-37, .356e-34]Solution in 0.536s

Time Plot 0 s.
Exiting SolveHard() after 0.818r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.642959731042799727838332766907147770 4,
436.917481654396062683387279235011337577 4,
422.984933970194589572041386927681233486 4,
361.525802560509442311329852410653899992 5,
401.881739040389946739188782568523749750 4,
389.590015160903049912898358760714240612 7,
328.469398930671764601902748591276183577 5,
401.507571577297195538960826675884865257 0,
358.973628238814052252378899909995120711 6,
398.331471032982875473614424005239499244 5,
371.483873947748555087326592280640988145 8, none,
361.508883470928325993293030318890129525 7,
324.671449924374814133782948611236801487 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]
```

```
0 --> 1  target = [26.463471105425981411822810798058365509 71,
6.1962625654790507998317092832512929453 45,
385.44474379318118342245212040871288474 45]
two intervals r = 16.875634087653503358346582961435204490 69 ..
95000000000123037131993754497603680 87/500000000000000000000000000000000000
000 or r = 15.556404938009050325674490560926274217 48 ..
95000000000123037131993754497603680 87/500000000000000000000000000000000000
000
Time Approximations 0.063.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [-.717e-37, .1e-37, .93e-36]Solution in 4.703s

Time Plot 0 s.
Exiting SolveHard() after 5.877r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747468973894556465046912613384 89,
4.8838107797274865867236045643102045173 94,
376.61967855778104245694763156806914764 13]

"Imaginary part neglected: ", 1.8899423791537964720844529493031107631 08 × 10^-17
one interval r = 21.110013048717202711794151400851932060 94 ..
26.317842434768907191070482349768536967 25
```

```
Time Approximations 0.037.

hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.761063912186698964426645072343113601 29, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., .102e-34]Solution in 0.86s

Time Plot 0 s.
Exiting SolveHard() after 1.585r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027 08147,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73, none,
328.46938513139899932549767469013313508 36, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747468973894556465046912613384 89,
4.8838107797274865867236045643102045173 94,
376.61967855778104245694763156806914764 13]
one interval r = 31.538994977154243318744326456064827603 12 ..
34.536183860941335530236380985324805742 24
Time Approximations 0.017.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
```

```
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=6.98e-36
Equations at solution: [-.536e-35, .698e-35, .454e-34]Solution in
0.494s

Time Plot 0 s.
Exiting SolveHard() after 0.777r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73, none,
328.46938513139899932549767469013313508 36,
343.81340625012039542944862128517678837 35, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017536010612005282435587108594515,
6.0258135494169568742860688876601342330 15,
351.42702948033411085451691715330609473 68]
one interval r = 31.362302061160844182417464973120007282 92 ..
34.174466406109722485317355935031956366 50
Time Approximations 0.015.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .35e-35]Solution in 0.505s

Time Plot 0 s.
Exiting SolveHard() after 4.141r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
```

```
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73, none,
328.46938513139899932549767469013313508 36,
343.81340625012039542944862128517678837 35, none, none,
292.99969137817105798775589120341647622 17, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017536010612005282435587108594515,
6.0258135494169568742860688876601342330 15,
351.42702948033411085451691715330609473 68]
two intervals r = 17.98135514460266489613730269993138912028 ..
95000000000123037131993754497603680 87/500000000000000000000000000000000000
000 or r = 13.84608015400712699711988005154640698324 ..
95000000000123037131993754497603680 87/500000000000000000000000000000000000
000
Time Approximations 0.046.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [0., 0., .145e-34]Solution in 4.431s

Time Plot 0 s.
Exiting SolveHard() after 5.482r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
```

```
441.642959731042799727838332766907147770 4,
436.917481654396062683387279235011337577 4,
422.984933970194589572041386927681233486 4,
361.525802560509442311329852410653899992 5,
401.881739040389946739188782568523749750 4,
389.590015160903049912898358760714240612 7,
328.469398930671764601902748591276183577 5,
401.507571577297195538960826675884865257 0,
358.973628238814052252378899909995120711 6,
398.331471032982875473614424005239499244 5,
371.483873947748555087326592280640988145 8,
336.612158408061895941925570126625010794 8,
361.508883470928325993293030318890129525 7,
324.671449924374814133782948611236801487 3,
302.313843141007966172838454241811528780 3,
328.469385131398999325497674690133135083 6,
343.813406250120395429448621285176788373 5, none, none,
292.999691378171057987755891203416476221 7, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418657005224568648296811579401 58,
6.377943873984879976783738901805045590699,
423.288327831662316377435660332562156315 0]
one interval r = 31.946618175933729234094900521186778261 32 ..
35.212123086404178412259639339596297373 32
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.338e-34]Solution in 0.6s

Time Plot 0 s.
Exiting SolveHard() after 0.932r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936975853943846513258960270814 7,
441.642959731042799727838332766907147770 4,
436.917481654396062683387279235011337577 4,
422.984933970194589572041386927681233486 4,
361.525802560509442311329852410653899992 5,
401.881739040389946739188782568523749750 4,
389.590015160903049912898358760714240612 7,
328.469398930671764601902748591276183577 5,
```

```
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73,
302.31384314100796617283845424181152878 03,
328.46938513139899932549767469013313508 36,
343.81340625012039542944862128517678837 35, none, none,
292.99969137817105798775589120341647622 17, none, none,
360.06173465897484918215102785146654533 82, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418657005224568648296811579401 58,
6.3779438739848799767837389018050455906 99,
423.28832783166231637743566033256215631 50]
two intervals r = 15.228867024905198783726250116146237887 33 ..
95000000000123037131993754497603680 87/50000000000000000000000000000000000
000 or r = 17.129657770430522451686418797221812007 73 ..
95000000000123037131993754497603680 87/50000000000000000000000000000000000
000
Time Approximations 0.065.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., -.470e-35]Solution in 4.538s

Time Plot 0 s.
Exiting SolveHard() after 5.886r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
```

```
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73,
302.31384314100796617283845424181152878 03,
328.46938513139899932549767469013313508 36,
343.81340625012039542944862128517678837 35,
375.73285288886279504829721172560000550 98, none,
292.99969137817105798775589120341647622 17, none, none,
360.06173465897484918215102785146654533 82, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343392813351979395520935650410 08,
4.0035598155022017573454838606696892028 34,
404.47973593750965323022844143853950049 96]
two intervals r = 16.096839663959229748262673859563632402 68 ..
95000000000123037131993754497603680 87/500000000000000000000000000000000000
000 or r = 16.399886491003332409380346459746639830 21 ..
95000000000123037131993754497603680 87/500000000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.17e-37, 0., -.759e-35]Solution in 5.006s

Time Plot 0 s.
Exiting SolveHard() after 6.103r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73,
```

```
302.31384314100796617283845424181152878 03,
328.46938513139899932549767469013313508 36,
343.81340625012039542944862128517678837 35,
375.73285288886279504829721172560000550 98, none,
292.99969137817105798775589120341647622 17,
358.64341560675057208391172116975961855 27, none,
360.06173465897484918215102785146654533 82, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343392813351979395520935650410 08,
4.0035598155022017573454838606696892028 34,
404.47973593750965323022844143853950049 96]
```

"Imaginary part neglected: ", $1.8899423791537964720844529493031107631 08 \times 10^{-17}$

```
one interval r = 21.634296299781381294276462181486141774 34 ..
26.757681698905342115803193874779969132 76
Time Approximations 0.054.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, -.52e-35]Solution in 1.085s

Time Plot 0 s.
Exiting SolveHard() after 2.118r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73,
302.31384314100796617283845424181152878 03,
```

```
328.46938513139899932549767469013313508 36,
343.81340625012039542944862128517678837 35,
375.73285288886279504829721172560000550 98,
328.11709293930153594850439764726171512 93,
292.99969137817105798775589120341647622 17,
358.64341560675057208391172116975961855 27, none,
360.06173465897484918215102785146654533 82, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954460857026144180869687679741611,
6.19617723033277235778866750512824438737 2,
385.42734025697338639720167957876845307 25]
one interval r = 31.60822049096716040367648713697986904435 ..
34.66347615048723304561432765848054520047
Time Approximations 0.019.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .71e-35]Solution in 3.881s

Time Plot 0 s.
Exiting SolveHard() after 4.164r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73,
302.31384314100796617283845424181152878 03,
328.46938513139899932549767469013313508 36,
343.81340625012039542944862128517678837 35,
375.73285288886279504829721172560000550 98,
```

```
328.1170929393015359485043976472617151293,
292.9996913781710579877558912034164762217,
358.6434156067505720839117211697596185527, none,
360.0617346589748491821510278514665453382, none, none,
324.6552122342007936555427983378653382534, none, none, none, none]

0 --> 1  target = [26.46318954460857026144180869687679741611,
6.196177230332772357788667505128244387372,
385.42734025697338639720167957876845307 25]
two intervals r = 16.87629600303831273478141583711880000191 ..
9500000000012303713199375449760368087/500000000000000000000000000000000
000 or r = 15.555590006496528231072460944220791779 80 ..
9500000000012303713199375449760368087/500000000000000000000000000000000
000
Time Approximations 0.059.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.180e-37, 0., -.3135e-34]Solution in 1.159s

Time Plot 0 s.
Exiting SolveHard() after 2.329r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027 08147,
441.6429597310427997278383327669071477704,
436.9174816543960626833872792350113375774,
422.9849339701945895720413869276812334864,
361.5258025605094423113298524106538999925,
401.8817390403899467391887825685237497504,
389.5900151609030499128983587607142406127,
328.4693989306717646019027485912761835775,
401.5075715772971955389608266758848652570,
358.9736282388140522523788999099951207116,
398.3314710329828754736144240052394992445,
371.4838739477485550873265922806409881458,
336.6121584080618959419255701266250107948,
361.5088834709283259932930303188901295257,
324.6714499243748141337829486112368014873,
302.3138431410079661728384542418115287803,
328.4693851313989993254976746901331350836,
343.8134062501203954294486212851767883735,
375.7328528888627950482972117256000055098,
328.1170929393015359485043976472617151293,
```

```
292.99969137817105798775589120341647622 17,
358.6434156067505720839117211697596185527, none,
360.0617346589748491821510278514665453382,
336.5944103185367458198274371680849793844, none,
324.6552122342007936555427983378653382534, none, none, none, none]

0 --> 2  target = [34.49522661172379655385728551731146243581,
3.8971313159439514970798057430472591983 92,
373.7808188486296010348932647136425883123]
two intervals r = 17.29769086220451218918077567259903239172 ..
95000000000123037131993754497603680 87/50000000000000000000000000000000000
000 or r = 14.99436407450654658804411856269405452361 ..
95000000000123037131993754497603680 87/50000000000000000000000000000000000
000
Time Approximations 0.097.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [.54e-37, -.1e-37, .2540e-34]Solution in 4.77s

Time Plot 0 s.
Exiting SolveHard() after 9.845r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.6429597310427997278383327669071477704,
436.9174816543960626833872792350113375774,
422.9849339701945895720413869276812334864,
361.5258025605094423113298524106538999925,
401.8817390403899467391887825685237497504,
389.5900151609030499128983587607142406127,
328.4693989306717646019027485912761835775,
401.5075715772971955389608266758848652570,
358.9736282388140522523788999099951207116,
398.3314710329828754736144240052394992445,
371.4838739477485550873265922806409881458,
336.6121584080618959419255701266250107948,
361.5088834709283259932930303188901295257,
324.6714499243748141337829486112368014873,
302.3138431410079661728384542418115287803,
328.4693851313989993254976746901331350836,
343.8134062501203954294486212851767883735,
375.7328528888627950482972117256000055098,
328.1170929393015359485043976472617151293,
```

```
292.9996913781710579877558912034164762217,
358.6434156067505720839117211697596185527, none,
360.0617346589748491821510278514665453382,
336.5944103185367458198274371680849793844, none,
324.6552122342007936555427983378653382534,
331.9380679187703655471080709699044883002, none, none, none]

1 --> 2  target = [34.4952266117237965538572855173114624358 1,
3.8971313159439514970798057430472591983 92,
373.7808188486296010348932647136425883123]
```

"Imaginary part neglected: ", $1.8899423791537964720844529493031107631 08 \times 10^{-17}$

```
one interval r = 21.06068473204901583038699768362170089895 ..
26.26979834292498463236087490558759428442
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, -.143e-34]Solution in 0.797s

Time Plot 0 s.
Exiting SolveHard() after 1.514r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349369758539438465132589602708147,
441.6429597310427997278383327669071477704,
436.9174816543960626833872792350113375774,
422.9849339701945895720413869276812334864,
361.5258025605094423113298524106538999925,
401.8817390403899467391887825685237497504,
389.5900151609030499128983587607142406127,
328.4693989306717646019027485912761835775,
401.5075715772971955389608266758848652570,
358.9736282388140522523788999099951207116,
398.3314710329828754736144240052394992445,
371.4838739477485550873265922806409881458,
336.6121584080618959419255701266250107948,
361.5088834709283259932930303188901295257,
324.6714499243748141337829486112368014873,
302.3138431410079661728384542418115287803,
328.4693851313989993254976746901331350836,
343.8134062501203954294486212851767883735,
```

```
375.7328528888627950482972117256000055098,
328.1170929393015359485043976472617151293,
292.9996913781710579877558912034164762217,
358.6434156067505720839117211697596185527,
299.8986620497461115356382731255788465348,
360.0617346589748491821510278514665453382,
336.5944103185367458198274371680849793844, none,
324.6552122342007936555427983378653382534,
331.9380679187703655471080709699044883002, none, none, none]

0 --> 2  target = [33.81362495409005496087762619714639296940,
3.7256489935565721975995337602377858333303,
325.8920997267516987541739899938943161430]
two intervals r = 18.552270490134786976308183725187503514 60 ..
9500000000012303713199375449760368087/500000000000000000000000000000000
000 or r = 12.49196935782111186342284741417319798682 ..
9500000000012303713199375449760368087/500000000000000000000000000000000
000
Time Approximations 0.048.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=4e-38
Equations at solution: [-.86e-37, .4e-37, .248e-34]Solution in 1.192s

Time Plot 0 s.
Exiting SolveHard() after 6.729r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73,
302.31384314100796617283845424181152878 03,
```

```
328.46938513139899932549767469013313508 36,
343.81340625012039542944862128517678837 35,
375.73285288886279504829721172560000550 98,
328.11709293930153594850439764726171512 93,
292.99969137817105798775589120341647622 17,
358.64341560675057208391172116975961855 27,
299.89866204974611153563827312557884653 48,
360.06173465897484918215102785146654533 82,
336.59441031853674581982743716808497938 44, none,
324.65521223420079365554279833786533825 34,
331.93806791877036554710807096990448830 02, none, none,
289.54595772449933312911124466770761176 78]

1 --> 2  target = [33.81362495409005496087762619714639296940,
3.72564899355657219759953376023778583330 3,
325.89209972675169875417398999389431614 30]
```

"Imaginary part neglected: ", $1.8899423791537964720844529493031107631 08 \times 10^{-17}$

```
one interval r = 20.37468935098413757803438751439612666954 ..
25.37892165297552685167783151276338223425
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, .530e-34]Solution in 0.536s

Time Plot 0 s.
Exiting SolveHard() after 1.102r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
```

```
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73,
302.31384314100796617283845424181152878 03,
328.46938513139899932549767469013313508 36,
343.81340625012039542944862128517678837 35,
375.73285288886279504829721172560000550 98,
328.11709293930153594850439764726171512 93,
292.99969137817105798775589120341647622 17,
358.64341560675057208391172116975961855 27,
299.89866204974611153563827312557884653 48,
360.06173465897484918215102785146654533 82,
336.59441031853674581982743716808497938 44,
256.10753185731883332380106245312377993 82,
324.65521223420079365554279833786533825 34,
331.93806791877036554710807096990448830 02, none, none,
289.54595772449933312911124466770761176 78]

1 --> 0  target = [17.93041369728204963276382863302634913190,
4.68650870183691871488693406219310473723 4,
353.30541094608675871290828376812234164 25]
```

"Imaginary part neglected: ", $1.8899423791537964720844529493031107631 08 \times 10^{-17}$

```
one interval r = 20.73150479078876698141393930697649525219 ..
25.90675353516270335248865830273311895523
Time Approximations 0.033.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, .205e-34]Solution in 0.659s

Time Plot 0.001 s.
Exiting SolveHard() after 4.976r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
```

```
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73,
302.31384314100796617283845424181152878 03,
328.46938513139899932549767469013313508 36,
343.81340625012039542944862128517678837 35,
375.73285288886279504829721172560000550 98,
328.11709293930153594850439764726171512 93,
292.99969137817105798775589120341647622 17,
358.64341560675057208391172116975961855 27,
299.89866204974611153563827312557884653 48,
360.06173465897484918215102785146654533 82,
336.59441031853674581982743716808497938 44,
256.10753185731883332380106245312377993 82,
324.65521223420079365554279833786533825 34,
331.93806791877036554710807096990448830 02,
304.79958324767016627237751929549432871 00, none,
289.54595772449933312911124466770761176 78]

2 --> 0  target = [17.93041369728204963276382863302634913190,
4.68650870183691871488693406219310473723 4,
353.30541094608675871290828376812234164 25]
one interval r = 31.37435486994949138363327919659696042387 ..
34.20127520022890582391130538309317682170
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=1.1e-37
Equations at solution: [-.7e-37, .11e-36, -.123e-34]Solution in 0.342s

Time Plot 0 s.
Exiting SolveHard() after 0.634r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493697585394384651325896027081 47,
441.64295973104279972783833276690714777 04,
436.91748165439606268338727923501133757 74,
422.98493397019458957204138692768123348 64,
```

```
361.52580256050944231132985241065389999 25,
401.88173904038994673918878256852374975 04,
389.59001516090304991289835876071424061 27,
328.46939893067176460190274859127618357 75,
401.50757157729719553896082667588486525 70,
358.97362823881405225237889990999512071 16,
398.33147103298287547361442400523949924 45,
371.48387394774855508732659228064098814 58,
336.61215840806189594192557012662501079 48,
361.50888347092832599329303031889012952 57,
324.67144992437481413378294861123680148 73,
302.31384314100796617283845424181152878 03,
328.46938513139899932549767469013313508 36,
343.81340625012039542944862128517678837 35,
375.73285288886279504829721172560000550 98,
328.11709293930153594850439764726171512 93,
292.99969137817105798775589120341647622 17,
358.64341560675057208391172116975961855 27,
299.89866204974611153563827312557884653 48,
360.06173465897484918215102785146654533 82,
336.59441031853674581982743716808497938 44,
256.10753185731883332380106245312377993 82,
324.65521223420079365554279833786533825 34,
331.93806791877036554710807096990448830 02,
304.79958324767016627237751929549432871 00,
323.46169176424248392773887704210274597 01,
289.54595772449933312911124466770761176 78]

Cascade time 163.208
counts: 28, 28

 Iteration 76

Start Generation 1
1 --> 0  target = [11.99999999999683953121394078439214926800,
6.21701250311028509841918372395278806126 4,
485.54908089377659821130009897736946571 86]
one interval r = 23.40850301647660034731786273248773170271 ..
27.67578046421557713328937262875593915621
Time Approximations 0.046.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.09e-37
Equations at solution: [.7e-37, -.209e-36, -.18e-35]Solution in 4.234s

Time Plot 0 s.
Exiting SolveHard() after 5.413r=27.5236 in [25.56992694 ..
27.67578046]
```

```
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999999683953121394078439214926800,
6.2170125031102850984191837239527880612 64,
485.54908089377659821130009897736946571 86]
one interval r = 32.628147792006310831447331420308839424 68 ..
36.102483889305174753079119031577572765 27
Time Approximations 0.022.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, .5e-36]Solution in 0.609s

Time Plot 0 s.
Exiting SolveHard() after 1.01r=35.4632 in [33.94922194 .. 36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684473814262057044684220942371438,
6.5834347215913460285390751173039891635 17,
467.78730595582455620730486670901322168 02]
one interval r = 32.419789556518120030716801578862729444 84 ..
35.851524173620677477891398735255223086 41
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, -.41e-35]Solution in 0.628s

Time Plot 0 s.
Exiting SolveHard() after 0.996r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29, none, none,
401.88173904136722262300990806890363510 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684473814262057044684220942371438,
6.58343472159134602853907511730398916 3517,
467.78730595582455620730486670901322168 02]
two intervals r = 12.92327160825557961413308871133623303813 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0 or r = 18.39424858025629025725043520112148361610 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0
Time Approximations 0.041.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=0
Equations at solution: [0., 0., -.7113e-35]Solution in 43.473s

Time Plot 0 s.
Exiting SolveHard() after 48.363r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13, none,
401.88173904136722262300990806890363510 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962812678144103025970199233933255,
4.1256517969295110513289664274936369225 01,
440.67123064528730969618676639859317812 62]
two intervals r = 14.35659705127101135579712355393857220590 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0 or r = 17.70352613793776558859226955817432015085 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0
Time Approximations 3.265.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.14e-37, .1e-37, .14636e-34]Solution in 1.313s

Time Plot 0 s.
Exiting SolveHard() after 5.633r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13, none,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962812678144103025970199233933255,
4.1256517969295110513289664274936369225 01,
440.67123064528730969618676639859317812 62]
one interval r = 22.39761154359960683375129823657433387833 ..
27.23722351588870504356514521734233065708
Time Approximations 0.038.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
```

```
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.515 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643752344980278835014372517916 51, rm =
14.378187704190499346457480930372434555 59}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, .43e-35]Solution in 12s

Time Plot 0 s.
Exiting SolveHard() after 12.929r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887908931031199617045954254585 19,
4.0048690819196438949831990370728457057 13,
404.86224501206425776118225009792744510 59]
two intervals r = 16.080110077512735161969549513884561435 52 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0 or r = 16.415798126889397048462413489315334350 67 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0
Time Approximations 0.059.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.32e-37, -.1e-37, .13737e-34]Solution in
5.245s

Time Plot 0 s.
Exiting SolveHard() after 6.406r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96, none, none,
358.97362823699573158197476070086089367 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888790893103119961704595425458519,
4.0048690819196438949831990370728457057 13,
404.86224501206425776118225009792744510 59]
one interval r = 21.641943994161220845282101129433051022 69 ..
26.763306600372268936985679106780313212 31
Time Approximations 0.057.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, .505e-34]Solution in 4.487s

Time Plot 0 s.
Exiting SolveHard() after 5.56r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
```

```
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82, none,
358.97362823699573158197476070086089367 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941759719685790508529318469836769,
5.5896371831903837008486129318770066419 59,
443.83065884691507266000331726670879935 91]
one interval r = 22.46725374484815681257110988587871713417 ..
27.27388428352435299399204310438824500960
Time Approximations 0.039.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.0e-38
Equations at solution: [.1e-37, -.80e-37, -.9e-36]Solution in 1.013s

Time Plot 0 s.
Exiting SolveHard() after 2.006r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82, none,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941759719685790508529318469836769,
5.5896371831903837008486129318770066419 59,
443.83065884691507266000331726670879935 91]
one interval r = 32.15575279494735055052917978401492494827 ..
35.50872228733775220061026619048159013423
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
```

```
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, -.19e-35]Solution in 0.476s

Time Plot 0 s.
Exiting SolveHard() after 4.187r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365240802115963482739017510855 77,
5.1877835786574485430343302253814883379 13,
408.65773862097522788373876332983316197 91]
one interval r = 21.718401146451617913901940935022135914 26 ..
26.818493035002079935778592228008182766 12
Time Approximations 0.059.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [0., -.79e-37, .288e-34]Solution in 4.863s

Time Plot 0 s.
Exiting SolveHard() after 5.987r=26.4632 in [23.93303356 ..
```

```
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66, none, none,
361.50888346866764020347848152034027864 21, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136524080211596348273901751085577,
5.1877835786574485430343302253814883379 13,
408.65773862097522788373876332983316197 91]
one interval r = 31.80828598737708489600430492252569738761 ..
35.00011460030904881920055170196598760473
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=1.0e-37
Equations at solution: [-.9e-37, .10e-36, -.119e-34]Solution in 0.414s

Time Plot 0 s.
Exiting SolveHard() after 0.7r=34.4952 in [32.91337941 .. 35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
```

```
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01, none,
361.50888346866764020347848152034027864 21, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110533421629032757317043464595214,
6.19626256532144178071741623145577914065 4,
385.44474379071449497356911937472589175 66]
one interval r = 31.60836097526148657243132501191335655839 ..
34.66372795600147581897557998645032533715
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=1.4e-37
Equations at solution: [.9e-37, -.14e-36, .440e-34]Solution in 0.545s

Time Plot 0 s.
Exiting SolveHard() after 0.826r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01, none,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110533421629032757317043464595214,
6.19626256532144178071741623145577914065 4,
385.44474379071449497356911937472589175 66]
```

```
two intervals r = 16.875634087523550310086314893830453028 10 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0 or r = 15.556404937900665145313128427333488687 00 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0
Time Approximations 0.065.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 10.993 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175120993060233369894753240246656, rm
= 2.33653277396115955091321661622040226263 1}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.180e-37, 0., -.7582e-35]Solution in 35.405s

Time Plot 0 s.
Exiting SolveHard() after 36.542r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747340042601520932174285522233 57,
4.8838107800238392005404672834298817944 19,
376.61967855563401476552056714822791854 68]
one interval r = 21.110013048825157136296951567459533565 65 ..
26.317842434684509637822541663838318293 79
Time Approximations 0.035.
```

```
hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.761063912186698964426645072343113601 29, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.186e-34]Solution in 0.853s

Time Plot 0 s.
Exiting SolveHard() after 1.536r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33, none,
328.46938513656173602332239074209549138 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747340042601520932174285522233 57,
4.8838107800238392005404672834298817944 19,
376.61967855563401476552056714822791854 68]
one interval r = 31.538994977004517488334078231058624505 16 ..
34.536183860835363615149048345384120349 57
Time Approximations 0.017.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.043e-35
```

```
Equations at solution: [.802e-35, -.1043e-34, .207e-34]Solution in
0.493s

Time Plot 0 s.
Exiting SolveHard() after 0.771r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33, none,
328.46938513656173602332239074209549138 97,
343.81340624464492052432493175337540450 76, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.872050175417131808665108001361168352 47,
6.0258135492963656563383908245307967451 11,
351.42702948538199272748633264191692915 50]
one interval r = 31.362302061055748744305856054710859048 45 ..
34.174466406100566136783383139876398066 44
Time Approximations 0.017.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., -.108e-34]Solution in 4.089s

Time Plot 0 s.
Exiting SolveHard() after 4.318r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33, none,
328.46938513656173602332239074209549138 97,
343.81340624464492052432493175337540450 76, none, none,
292.99969138632606096694543176868938223 86, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175417131808665108001361168352 47,
6.0258135492963656563383908245307967451 11,
351.42702948538199272748633264191692915 50]
two intervals r = 17.981355144314131873591416681853291355 48 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0 or r = 13.846080154244367328049561425303794074 03 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0
Time Approximations 0.05.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.526 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713654738913189694786137999406 21, rm
= 2.7345009929030762384892946902354598401 24}});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [.88e-37, -.3e-37, -.36766e-34]Solution in
19.264s

Time Plot 0 s.
Exiting SolveHard() after 20.326r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33,
302.31384315441107090994044596985952898 72,
328.46938513656173602332239074209549138 97,
343.81340624464492052432493175337540450 76, none, none,
292.99969138632606096694543176868938223 86, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418762011779708488584726518719 48,
6.3779438739008477919793076580728784746 82,
423.28832784447245602700286402764664848 04]
one interval r = 31.946618175934385549470717689734039625 30 ..
35.212123086525528452747944340647146317 35
Time Approximations 0.018.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.116e-34]Solution in 0.631s

Time Plot 0 s.
Exiting SolveHard() after 0.978r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
```

```
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33,
302.31384315441107090994044596985952898 72,
328.46938513656173602332239074209549138 97,
343.81340624464492052432493175337540450 76, none, none,
292.99969138632606096694531768689382238 6, none, none,
360.06173467422794262606669752130915501 83, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418762011779708488584726518719 48,
6.3779438739008477919793076580728784746 82,
423.28832784447245602700286402764664848 04]
two intervals r = 15.22886702397426021987777058829699915038 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0 or r = 17.12965777091206306109819546059020074785 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0
Time Approximations 0.06.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 6.991 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541052502430285782826487025812 92, rm
= 2.0640682985999566220715324433269106888 59}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.47e-37, .1e-37, -.654e-36]Solution in 31.915s

Time Plot 0 s.
Exiting SolveHard() after 36.514r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
```

```
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33,
302.31384315441107090994044596985952898 72,
328.46938513656173602332239074209549138 97,
343.81340624464492052432493175337540450 76,
375.73285291128635719187205745538955505 14, none,
292.99969138632606096694543176868938223 86, none, none,
360.06173467422794262606669752130915501 83, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234335677168007076920387136447806,
4.0035598156218088775546846824056421798 66,
404.47973593973984653401093977210618181 41]
two intervals r = 16.09683966359496527606443633144135780385 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0 or r = 16.39988649111396878915989093974735158936 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0
Time Approximations 0.047.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., .301e-36]Solution in 1.655s

Time Plot 0 s.
Exiting SolveHard() after 6.307r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
```

```
389.590015152507830037634040190206973359 6,
328.469398935836963214475382282097383268 2,
401.507571579513685718989145063733166051 7,
358.973628236995731581974760700860893675 8,
398.331471045701389899167342930836993106 6,
371.483873936634809092131814227373277450 1,
336.612158414288682349711549479808624767 9,
361.508883468667640203478481520340278642 1,
324.671449925435260464091591903806641573 3,
302.313843154411070909940445969859528987 2,
328.469385136561736023322390742095491389 7,
343.813406244644920524324931753375404507 6,
375.732852911286357191872057455389555051 4, none,
292.999691386326060966945431768689382238 6,
358.643415606025481069765369514281895777 4, none,
360.061734674227942626066697521309155018 3, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343356771680070769203871364478 06,
4.0035598156218088775546846824056421798 66,
404.479735939739846534010939772106181814 1]
one interval r = 21.634296299946489219833804450674459703 95 ..
26.757681698879525872609403644342823874 44
Time Approximations 0.047.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., .123e-34]Solution in 1.008s

Time Plot 0 s.
Exiting SolveHard() after 6.25r=25.8653 in [23.83864811 .. 26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434932158730182801838540357953548 7,
441.642959729094624526065652594036755076 2,
436.917481647680383608426210430152352972 9,
422.984933976113616843071684100562854841 3,
361.525802558370468896680716602756952258 5,
401.881739041367222623009908068903635102 9,
389.590015152507830037634040190206973359 6,
328.469398935836963214475382282097383268 2,
401.507571579513685718989145063733166051 7,
358.973628236995731581974760700860893675 8,
398.331471045701389899167342930836993106 6,
```

```
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33,
302.31384315441107090994044596985952898 72,
328.46938513656173602332239074209549138 97,
343.81340624464492052432493175337540450 76,
375.73285291128635719187205745538955505 14,
328.11709294563378318335904122356852934 15,
292.99969138632606096694543176868938223 86,
358.64341560602548106976536951428189577 74, none,
360.06173467422794262606669752130915501 83, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954451478312566059402407028104581,
6.19617723017454866358354861386515430237 2,
385.42734025438149766954658864804723950 37]
one interval r = 31.60822049081445478006944707023800355722 ..
34.66347615037686691269836098850382411034
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=9e-38
Equations at solution: [-.6e-37, .9e-37, .29e-35]Solution in 0.576s

Time Plot 0 s.
Exiting SolveHard() after 0.866r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
```

```
324.67144992543526046409159190380664157 33,
302.31384315441107090994044596985952898 72,
328.46938513656173602332239074209549138 97,
343.81340624464492052432493175337540450 76,
375.73285291128635719187205745538955505 14,
328.11709294563378318335904122356852934 15,
292.99969138632606096694543176868938223 86,
358.64341560602548106976536951428189577 74, none,
360.06173467422794262606669752130915501 83, none, none,
324.65521223514448468681939119334689406 24, none, none, none, none]

0 --> 1  target = [26.463189544514783125660594024070281045 81,
6.1961772301745486635835486138651543023 72,
385.42734025438149766954658864804723950 37]
two intervals r = 16.876296002913146150846035491324062745 29 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0 or r = 15.555590006382256870016155734576008488 79 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0
Time Approximations 0.057.

hint used Hint := [17.930920830435909701694400819009216331 81, 3, 1, 1,
15.700860627013397511253157405159644880 12, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.845 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852532881047785190252244521412937, rm
= 2.33669042802724648796668552138509247068 6}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., -.35625e-34]Solution in 32.983s

Time Plot 0 s.
Exiting SolveHard() after 37.692r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
```

```
398.331471045701389899167342930836993106 6,
371.483873936634809092131814227373277450 1,
336.612158414288682349711549479808624767 9,
361.508883468667640203478481520340278642 1,
324.671449925435260464091591903806641573 3,
302.313843154411070909940445969859528987 2,
328.469385136561736023322390742095491389 7,
343.813406244644920524324931753375404507 6,
375.732852911286357191872057455389555051 4,
328.117092945633783183359041223568529341 5,
292.999691386326060966945431768689382238 6,
358.643415606025481069765369514281895777 4, none,
360.061734674227942626066697521309155018 3,
336.594410324635712987859414604151433361 4, none,
324.655212235144484686819391193346894062 4, none, none, none, none]

0 --> 2  target = [34.495226611483693828140425462602464132 99,
3.897131316015608063218794298528355899860,
373.780818837226675717795183199672337678 9]
two intervals r = 17.297690862397189204663275615728602971 54 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0 or r = 14.994364073939713420307962591863563455 98 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000
0
Time Approximations 0.087.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.36e-37, 0., -.32311e-34]Solution in 4.834s

Time Plot 0 s.
Exiting SolveHard() after 6.486r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434932158730182801838540357953548 7,
441.642959729094624526065652594036755076 2,
436.917481647680383608426210430152352972 9,
422.984933976113616843071684100562854841 3,
361.525802558370468896680716602756952258 5,
401.881739041367222623009908068903635102 9,
389.590015152507830037634040190206973359 6,
328.469398935836963214475382282097383268 2,
401.507571579513685718989145063733166051 7,
358.973628236995731581974760700860893675 8,
```

```
398.331471045701389899167342930836993106 6,
371.483873936634809092131814227373277450 1,
336.612158414288682349711549479808624767 9,
361.508883468667640203478481520340278642 1,
324.671449925435260464091591903806641573 3,
302.313843154411070909940445969859528987 2,
328.469385136561736023322390742095491389 7,
343.813406244644920524324931753375404507 6,
375.732852911286357191872057455389555051 4,
328.117092945633783183359041223568529341 5,
292.999691386326060966945431768689382238 6,
358.643415606025481069765369514281895777 4, none,
360.061734674227942626066697521309155018 3,
336.594410324635712987859414604151433361 4, none,
324.655212235144484686819391193346894062 4,
331.938067906099402037075282933404230936 5, none, none, none]

1 --> 2  target = [34.495226611483693828140425462602464132 99,
3.897131316015608063218794298528355899860,
373.780818837226675717795183199672337678 9]
one interval r = 21.060684732000570538445799666988869315 66 ..
26.269798342683781373974029217338357931 89
Time Approximations 0.035.

hint used Hint := [25.300466340350721366923008834208363590 84, 3, -1, 1,
16.974729527641787545085746138369074748 18, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., -.21e-35]Solution in 0.765s

Time Plot 0 s.
Exiting SolveHard() after 1.505r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434932158730182801838540357953548 7,
441.642959729094624526065652594036755076 2,
436.917481647680383608426210430152352972 9,
422.984933976113616843071684100562854841 3,
361.525802558370468896680716602756952258 5,
401.881739041367222623009908068903635102 9,
389.590015152507830037634040190206973359 6,
328.469398935836963214475382282097383268 2,
401.507571579513685718989145063733166051 7,
358.973628236995731581974760700860893675 8,
398.331471045701389899167342930836993106 6,
```

```
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33,
302.31384315441107090994044596985952898 72,
328.46938513656173602332239074209549138 97,
343.81340624464492052432493175337540450 76,
375.73285291128635719187205745538955505 14,
328.11709294563378318335904122356852934 15,
292.99969138632606096694543176868938223 86,
358.64341560602548106976536951428189577 74,
299.89866204358461212672268114570217763 99,
360.06173467422794262606669752130915501 83,
336.59441032463571298785941460415143336 14, none,
324.65521223514448468681939119334689406 24,
331.93806790609940203707528293340423093 65, none, none, none]

0 --> 2  target = [33.81362495401710762612244426377164439225,
3.72564899367151805037550784136918837879 1,
325.89209972779715932424293987798355787 95]
two intervals r = 18.55227049000876969110377803494795014819 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000000
0 or r = 12.49196935780324510714150828451049246368 ..
9499999999994041974926071064936641 51/50000000000000000000000000000000000000
0
Time Approximations 0.039.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=4e-38
Equations at solution: [-.104e-36, .4e-37, -.17566e-34]Solution in
1.113s

Time Plot 0 s.
Exiting SolveHard() after 5.781r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
```

```
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33,
302.31384315441107090994044596985952898 72,
328.46938513656173602332239074209549138 97,
343.81340624464492052432493175337540450 76,
375.73285291128635719187205745538955505 14,
328.11709294563378318335904122356852934 15,
292.99969138632606096694543176868938223 86,
358.64341560602548106976536951428189577 74,
299.89866204358461212672268114570217763 99,
360.06173467422794262606669752130915501 83,
336.59441032463571298785941460415143336 14, none,
324.65521223514448468681939119334689406 24,
331.93806790609940203707528293340423093 65, none, none,
289.54595772283402261394851755077245250 13]

1 --> 2  target = [33.81362495401710762612244426377164439225,
3.7256489936715180503755078413691883787 91,
325.89209972779715932424293987798355787 95]
one interval r = 20.37468935117867906084107480847666162150 ..
25.37892165297414622627322086449422273121
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .124e-34]Solution in 4.078s

Time Plot 0 s.
Exiting SolveHard() after 4.64r=24.3395 in [22.07732228 .. 25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
```

```
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33,
302.31384315441107090994044596985952898 72,
328.46938513656173602332239074209549138 97,
343.81340624464492052432493175337540450 76,
375.73285291128635719187205745538955505 14,
328.11709294563378318335904122356852934 15,
292.99969138632606096694543176868938223 86,
358.64341560602548106976536951428189577 74,
299.89866204358461212672268114570217763 99,
360.06173467422794262606669752130915501 83,
336.59441032463571298785941460415143336 14,
256.10753186258702977729880390195109535 55,
324.65521223514448468681939119334689406 24,
331.93806790609940203707528293340423093 65, none, none,
289.54595772283402261394851755077245250 13]

1 --> 0  target = [17.93041369695433383006937676011653312458,
4.6865087022000833224156420326023909297 73,
353.30541095231170060638434635490825735 58]
one interval r = 20.73150479104573440580906882064117874976 ..
25.906753535239841745145167906508828274 90
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., -.185e-34]Solution in 0.712s

Time Plot 0 s.
Exiting SolveHard() after 1.468r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.64295972909462452606565259403675507 62,
436.91748164768038360842621043015235297 29,
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
```

```
389.5900151525078300376340401902069733596,
328.4693989358369632144753822820973832682,
401.5075715795136857189891450637331660517,
358.9736282369957315819747607008608936758,
398.3314710457013898991673429308369931066,
371.4838739366348090921318142273732774501,
336.6121584142886823497115494798086247679,
361.5088834686676402034784815203402786421,
324.6714499254352604640915919038066415733,
302.3138431544110709099404459698595289872,
328.4693851365617360233223907420954913897,
343.8134062446449205243249317533754045076,
375.7328529112863571918720574553895550514,
328.1170929456337831833590412235685293415,
292.9996913863260609669454317686893822386,
358.6434156060254810697653695142818957774,
299.8986620435846121267226811457021776399,
360.0617346742279426260666975213091550183,
336.5944103246357129878594146041514333614,
256.1075318625870297772988039019510953555,
324.6552122351444846868193911933468940624,
331.9380679060994020370752829334042309365,
304.7995832613262212549534606521050633752, none,
289.5459577228340226139485175507724525013]

2 --> 0  target = [17.93041369695433383006937676011653312458,
4.6865087022000833224156420326023909297 73,
353.3054109523117006063843463549082573558]
one interval r = 31.37435486985277126290330627914726033767 ..
34.20127520023710748612327802467666431648
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .22e-35]Solution in 0.35s

Time Plot 0 s.
Exiting SolveHard() after 0.613r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493215873018280183854035795354 87,
441.6429597290946245260656525940367550762,
436.9174816476803836084262104301523529729,
```

```
422.98493397611361684307168410056285484 13,
361.52580255837046889668071660275695225 85,
401.88173904136722262300990806890363510 29,
389.59001515250783003763404019020697335 96,
328.46939893583696321447538228209738326 82,
401.50757157951368571898914506373316605 17,
358.97362823699573158197476070086089367 58,
398.33147104570138989916734293083699310 66,
371.48387393663480909213181422737327745 01,
336.61215841428868234971154947980862476 79,
361.50888346866764020347848152034027864 21,
324.67144992543526046409159190380664157 33,
302.31384315441107090994044596985952898 72,
328.46938513656173602332239074209549138 97,
343.81340624464492052432493175337540450 76,
375.73285291128635719187205745538955505 14,
328.11709294563378318335904122356852934 15,
292.99969138632606096694543176868938223 86,
358.64341560602548106976536951428189577 74,
299.89866204358461212672268114570217763 99,
360.06173467422794262606669752130915501 83,
336.59441032463571298785941460415143336 14,
256.10753186258702977729880390195109535 55,
324.65521223514448468681939119334689406 24,
331.93806790609940203707528293340423093 65,
304.79958326132622125495346065210506337 52,
323.46169176597503752641162517447543353 04,
289.54595772283402261394851755077245250 13]

Cascade time 272.674
counts: 28, 28

 Iteration 77

Start Generation 1
1 --> 0  target = [11.99999999997590198354041235473915511600,
6.21701250309322358407515968372525199018 4,
485.54908089859829878770087165142706158 17]
one interval r = 23.40850301664422419696207646631156896331 ..
27.67578046447797935860369969329458725216
Time Approximations 0.045.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=7.7e-38
Equations at solution: [-.3e-37, .77e-37, .1e-36]Solution in 1.032s

Time Plot 0 s.
Exiting SolveHard() after 2.18r=27.5236 in [25.56992694 .. 27.67578046]
```

```
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999975901983540412354739155116 00,
6.217012503093223584075159683725251990184,
485.549080898598298787700871651427061581 7]
one interval r = 32.628147792166964068463123098057211838 33 ..
36.102483889448785150015093172578199210 25
Time Approximations 0.023.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .67e-35]Solution in 3.845s

Time Plot 0 s.
Exiting SolveHard() after 4.245r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596845008808925219616182284547072 35,
6.583434721397783625420418463157659270089,
467.787305961601379844963311225405791986 0]
one interval r = 32.419789556689150440060117131578654597 97 ..
35.851524173779981949352712347852743244 31
Time Approximations 0.021.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
```

```
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, .58e-35]Solution in 0.669s

Time Plot 0 s.
Exiting SolveHard() after 1.06r=34.9451 in [33.70078237 .. 35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70, none, none,
401.88173904736973783044284549808940335 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684500880892521961618228454707235,
6.58343472139778362542041846315765927008 9,
467.78730596160137984496331122540579198 60]
two intervals r = 12.92327160813297474247234455527434652220 ..
18999999999884655860985209928569202961/10000000000000000000000000000000
00000 or r = 18.39424858026233904473371669970502204306 ..
18999999999884655860985209928569202961/10000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.1e-38
Equations at solution: [.1e-37, .21e-37, -.311e-35]Solution in 43.084s

Time Plot 0 s.
Exiting SolveHard() after 44.503r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29, none,
401.88173904736973783044284549808940335 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962825729785961304090054929288703,
4.12565179679871170240702237822745502659 0,
440.67123064883765935323028385316762356 31]
two intervals r = 14.35659705120990816836317257231320525421 ..
189999999998846558609852099285692029 61/100000000000000000000000000000000
00000 or r = 17.70352613793912540939344348355064254237 ..
189999999998846558609852099285692029 61/100000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.29e-37, .1e-37, .4229e-34]Solution in 1.334s

Time Plot 0 s.
Exiting SolveHard() after 5.93r=15.9119 in [14.35659706 .. 18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29, none,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962825729785961304090054929288703,
4.12565179679871170240702237822745502659 0,
440.67123064883765935323028385316762356 31]
one interval r = 22.39761154369754291198850951039156579196 ..
27.23722351613303431047170337927711643453
Time Approximations 0.039.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
```

```
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.735 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643888314184512493282604111341 43, rm =
14.37818770019427251996608047899666648307}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, -.734e-34]Solution in 12.003s

Time Plot 0 s.
Exiting SolveHard() after 12.938r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888081298473348860442278822736 54,
4.0048690817985656400379059780077622406 16,
404.86224501824856501163204461971870724 43]
two intervals r = 16.080110077289364606883054259540757858 91 ..
189999999998846558609852099285692029 61/100000000000000000000000000000000
00000 or r = 16.415798127067240296533686511964042389 95 ..
189999999998846558609852099285692029 61/100000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.17e-37, 0., .2145e-34]Solution in 5.403s

Time Plot 0 s.
Exiting SolveHard() after 6.543r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57, none, none,
358.97362824149971840463593438133979347 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888808129847334886044227882273654,
4.0048690817985656400379059780077622406 16,
404.86224501824856501163204461971870724 43]
one interval r = 21.641943994275550130884075066040050085 77 ..
26.763306600645428297339158842385236861 41
Time Approximations 0.055.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=1.01e-37
Equations at solution: [.3e-37, .101e-36, -.281e-34]Solution in 4.65s

Time Plot 0 s.
Exiting SolveHard() after 5.716r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
```

```
389.590015154728153389843930244121460275 7,
328.46939894204199179430888655188081588 83, none,
358.97362824149971840463593438133979347 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941727455607684476722482286221471,
5.589637183212816762747795829296285926380,
443.83065885559101349059913778657530487 97]
one interval r = 22.46725374506242378910539556956751643638 ..
27.27388428382818633615005435134604178458
Time Approximations 0.037.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., -.123e-34]Solution in 1.013s

Time Plot 0 s.
Exiting SolveHard() after 2.013r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.590015154728153389843930244121460275 7,
328.46939894204199179430888655188081588 83, none,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941727455607684476722482286221471,
5.589637183212816762747795829296285926380,
443.83065885559101349059913778657530487 97]
one interval r = 32.15575279514696289782422388443007104649 ..
35.50872228754169676359134643185526628013
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
```

```
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.67e-35]Solution in 3.908s

Time Plot 0 s.
Exiting SolveHard() after 4.269r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365192388597484253584888899801 98,
5.1877835785953474093926866605371749387 40,
408.65773862327674953481328692563866282 35]
one interval r = 21.718401146491890086545294749528380773 82 ..
26.818493035219770077051191837361994859 35
Time Approximations 0.06.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=3.70e-37
Equations at solution: [.3e-37, .370e-36, .103e-34]Solution in 1.031s

Time Plot 0 s.
Exiting SolveHard() after 2.155r=26.4632 in [23.93303356 ..
26.81849303]
```

```
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888651880815888 3,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75, none, none,
361.50888347242627552022557801888084293 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136519238859748425358488889980198,
5.1877835785953474093926866605371749387 40,
408.65773862327674953481328692563866282 35]
one interval r = 31.80828598750981340215119255969573853633 ..
35.00011460042473726503503512057241255606
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.543e-34]Solution in 4.164s

Time Plot 0 s.
Exiting SolveHard() after 4.469r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
```

```
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86, none,
361.50888347242627552022557801888084293 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110557425929006205670559850550430,
6.1962625651279404390717802250113726352 03,
385.44474379502281233206992454982782571 65]
one interval r = 31.608360975410281897791907404547453299 99 ..
34.663727956148499875895391341106066142 04
Time Approximations 0.016.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.338e-34]Solution in 0.57s

Time Plot 0 s.
Exiting SolveHard() after 0.863r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86, none,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110557425929006205670559850550430,
6.1962625651279404390717802250113726352 03,
385.44474379502281233206992454982782571 65]
```

```
two intervals r = 16.875634087372350859031910654389342885 61 ..
189999999998846558609852099285692029 61/100000000000000000000000000000000
00000 or r = 15.556404938046499618152126645218147840 29 ..
189999999998846558609852099285692029 61/100000000000000000000000000000000
00000
Time Approximations 0.062.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 11.652 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175105106288996241577096217182525, rm
= 2.33653277413930050355616354365022552388 8}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., .798e-35]Solution in 32.845s

Time Plot 0 s.
Exiting SolveHard() after 34r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747173687966749448947726652509 18,
4.883810779978841795237417420969370769395,
376.61967856030954808483098263119621357 53]
one interval r = 21.110013048876514347960712739372118035 44 ..
26.317842434927851537023290619853698808 41
Time Approximations 0.038.
```

```
hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.761063912186698964426645072343113601 29, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.25e-35]Solution in 4.398s

Time Plot 0 s.
Exiting SolveHard() after 5.128r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888651880815888 3,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68, none,
328.46938514276597425768705409321164176 03, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747173687966749448947726652509 18,
4.8838107799788417952374174209693707693 95,
376.61967856030954808483098263119621357 53]
one interval r = 31.538994977155098371141002282347297883 97 ..
34.536183860988486617601245298245725637 87
Time Approximations 0.017.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.035e-35
```

```
Equations at solution: [-.795e-35, .1035e-34, .142e-34]Solution in
0.513s

Time Plot 0 s.
Exiting SolveHard() after 0.806r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68, none,
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017569092963796929875203276772521,
6.0258135491184814469630509044527244230 53,
351.42702949217911732439308201828600241 62]
one interval r = 31.362302061215970004867893931724230782 61 ..
34.174466406285981799046199964046671705 48
Time Approximations 0.016.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., -.155e-34]Solution in 0.525s

Time Plot 0 s.
Exiting SolveHard() after 0.77r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68, none,
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15, none, none,
292.99969139328129365597893419317337060 39, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017569092963796929875203276772521,
6.0258135491184814469630509044527244230 53,
351.42702949217911732439308201828600241 62]
two intervals r = 17.98135514409451548997329596187853930832 ..
189999999998846558609852099285692029 61/10000000000000000000000000000000
00000 or r = 13.84608015460574589262116055184742897314 ..
189999999998846558609852099285692029 61/10000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.542 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.91357071350598030495602139624608929036, rm
= 2.7345009931723912530676913782573571317 62}});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [0., 0., .1092e-34]Solution in 23.205s

Time Plot 0 s.
Exiting SolveHard() after 24.26r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888651880815888 3,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68,
302.31384316328113952942039634737726159 93,
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15, none, none,
292.99969139328129365597893419317337060 39, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941909931450505369164582106501620,
6.37794387373536694166516231134672534432 5,
423.28832785539887098369793405661081916 18]
one interval r = 31.94661817615120854810894336252318815567 ..
35.21212308676462312535783925442206598523
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, -.92e-35]Solution in 0.599s

Time Plot 0 s.
Exiting SolveHard() after 0.923r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
```

```
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68,
302.31384316328113952942039634737726159 93,
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15, none, none,
292.99969139328129365597893419317337060 39, none, none,
360.06173468505593135549161615088699903 86, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941909931450505369164582106501620,
6.37794387373536694166516231134672534432 5,
423.28832785539887098369793405661081916 18]
two intervals r = 15.22886702352941942353020738196326431911 ..
189999999998846558609852099285692029 61/100000000000000000000000000000000
00000 or r = 17.12965777120872846794414522703716699072 ..
189999999998846558609852099285692029 61/100000000000000000000000000000000
00000
Time Approximations 0.064.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 11.04 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.51537054066938007037588411685257790905, rm
= 2.06406829866543209488907948793346806 1948}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., -.265e-35]Solution in 32.319s

Time Plot 0 s.
Exiting SolveHard() after 33.623r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
```

```
389.590015154728153389843930244121460275 7,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68,
302.31384316328113952942039634737726159 93,
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15,
375.73285292500844569961091156971164400 90, none,
292.99969139328129365597893419317337060 39, none, none,
360.06173468505593135549161615088699903 86, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234353567225816961690878509292845,
4.0035598155022747886407260127273326952 23,
404.47973594637209741763250613800118792 30]
two intervals r = 16.09683966335195612311366473425488102442 ..
189999999998846558609852099285692029 61/100000000000000000000000000000000
00000 or r = 16.39988649131155658852169479875348477681 ..
189999999998846558609852099285692029 61/100000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.17e-37, .1e-37, .1302e-34]Solution in 1.654s

Time Plot 0 s.
Exiting SolveHard() after 6.388r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.590015154728153389843930244121460275 7,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
```

```
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68,
302.31384316328113952942039634737726159 93,
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15,
375.73285292500844569961091156971164400 90, none,
292.99969139328129365597893419317337060 39,
358.64341561091699151647763804145835503 42, none,
360.06173468505593135549161615088699903 86, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234353567225816961690878509292845,
4.0035598155022747886407260127273326952 23,
404.47973594637209741763250613800118792 30]
one interval r = 21.63429630006929320937362872903439284907 ..
26.75768169915923621532158978378554825351
Time Approximations 0.046.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.5e-38
Equations at solution: [0., .25e-37, .678e-34]Solution in 1.008s

Time Plot 0 s.
Exiting SolveHard() after 5.669r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
```

```
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68,
302.31384316328113952942039634737726159 93,
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15,
375.73285292500844569961091156971164400 90,
328.11709295225106227184121179832964354 81,
292.99969139328129365597893419317337060 39,
358.64341561091699151647763804145835503 42, none,
360.06173468505593135549161615088699903 86, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954475416322841501872011228620826,
6.19617722998084952500666713505979598919 1,
385.42734025864917957113923326254731165 06]
one interval r = 31.60822049096292017616930460021026836608 ..
34.66347615052330463566684424118098961968
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .11e-35]Solution in 0.557s

Time Plot 0 s.
Exiting SolveHard() after 0.838r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68,
302.31384316328113952942039634737726159 93,
```

```
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15,
375.73285292500844569961091156971164400 90,
328.11709295225106227184121179832964354 81,
292.99969139328129365597893419317337060 39,
358.64341561091699151647763804145835503 42, none,
360.06173468505593135549161615088699903 86, none, none,
324.65521223973898780373619546701998485 59, none, none, none, none]

0 --> 1  target = [26.46318954475416322841501872011228620826,
6.1961772299808495250066671350597959891 91,
385.42734025864917957113923326254731165 06]
two intervals r = 16.876296002763486376628430774023254804 98 ..
189999999998846558609852099285692029 61/10000000000000000000000000000000
00000 or r = 15.555590006526230829851749895011595832 73 ..
189999999998846558609852099285692029 61/10000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.644 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852517071699380295490143990305199, rm
= 2.33669042820580243520325513926302061554 6}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., -.2828e-34]Solution in 28.66s

Time Plot 0 s.
Exiting SolveHard() after 33.426r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
```

```
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68,
302.31384316328113952942039634737726159 93,
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15,
375.73285292500844569961091156971164400 90,
328.11709295225106227184121179832964354 81,
292.99969139328129365597893419317337060 39,
358.64341561091699151647763804145835503 42, none,
360.06173468505593135549161615088699903 86,
336.59441033119366801067996927074234645 78, none,
324.65521223973898780373619546701998485 59, none, none, none, none]

0 --> 2  target = [34.49522661158367275504795965486903316592,
3.89713131587712948429307134444257216507 2,
373.78018188319986200324715416188043755 19]
two intervals r = 17.297690862357753559688810383679203716 48 ..
189999999998846558609852099285692029 61/100000000000000000000000000000000
00000 or r = 14.994364073949281434475671255042915849 42 ..
189999999998846558609852099285692029 61/100000000000000000000000000000000
00000
Time Approximations 0.086.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., .617e-35]Solution in 1.222s

Time Plot 0.001 s.
Exiting SolveHard() after 7.418r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
```

```
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68,
302.31384316328113952942039634737726159 93,
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15,
375.73285292500844569961091156971164400 90,
328.11709295225106227184121179832964354 81,
292.99969139328129365597893419317337060 39,
358.64341561091699151647763804145835503 42, none,
360.06173468505593135549161615088699903 86,
336.59441033119366801067996927074234645 78, none,
324.65521223973898780373619546701998485 59,
331.93806790611345974506417331851260922 40, none, none, none]

1 --> 2  target = [34.49522661158367275504795965486903316592,
3.8971313158771294842930713444425721650 72,
373.78018188319986200324715416188043755 19]
one interval r = 21.06068473198477748890768598415431649892 ..
26.26979834286319133195321297689673330578
Time Approximations 0.032.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [.2e-37, .5e-37, -.100e-34]Solution in 0.733s

Time Plot 0 s.
Exiting SolveHard() after 4.881r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
```

```
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68,
302.31384316328113952942039634737726159 93,
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15,
375.73285292500844569961091156971164400 90,
328.11709295225106227184121179832964354 81,
292.99969139328129365597893419317337060 39,
358.64341561091699151647763804145835503 42,
299.89866204498454243333818460319521040 38,
360.06173468505593135549161615088699903 86,
336.59441033119366801067996927074234645 78, none,
324.65521223973898780373619546701998485 59,
331.93806790611345974506417331851260922 40, none, none, none]

0 --> 2  target = [33.81362495417558620740411557610703182832,
3.72564899354773363908342419009324239515 5,
325.89209973258981770062097526419847396 77]
two intervals r = 18.552270489856603705644513521322707867 77 ..
189999999998846558609852099285692029 61/10000000000000000000000000000000
00000 or r = 12.491969358114915721104641204054935973 56 ..
189999999998846558609852099285692029 61/10000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [.121e-36, -.5e-37, -.2702e-34]Solution in
4.634s

Time Plot 0 s.
Exiting SolveHard() after 6.094r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
```

```
398.331471055950309151283064750108902237 5,
371.483873937537939409774967011059167028 6,
336.612158420888180458497068854424391947 4,
361.508883472426275520225578018880842936 5,
324.671449930067672477336608513428263216 8,
302.313843163281139529420396347377261599 3,
328.469385142765974257687054093211641760 3,
343.813406247567602808344432890814314481 5,
375.732852925008445699610911569711644009 0,
328.117092952251062271841211798329643548 1,
292.999691393281293655978934193173370603 9,
358.643415610916991516477638041458355034 2,
299.898662044984542433338184603195210403 8,
360.061734685055931355491616150886999038 6,
336.594410331193668010679969270742346457 8, none,
324.655212239738987803736195467019984855 9,
331.938067906113459745064173318512609224 0, none, none,
289.545957726333965530725381656192547491 2]

1 --> 2  target = [33.813624954175586207404115576107031828 32,
3.725648993547733639083424190093242395155,
325.892099732589817700620975264198473967 7]
one interval r = 20.374689351171248324504105490319589736 44 ..
25.378921653199840402778575218448699185 12
Time Approximations 0.039.

hint used Hint := [24.339499030253814217461077660681155334 23, 3, -1, 1,
17.272158782512944212267074944284355815 74, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, -.128e-34]Solution in 0.587s

Time Plot 0 s.
Exiting SolveHard() after 1.153r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936735562041884896250915438393 9,
441.642959734347907828398636536651692627 4,
436.917481651109465909332285510285106337 0,
422.984933984354740202698622099664458512 9,
361.525802562168613905189062310355055137 4,
401.881739047369737830442845498089403356 4,
389.590015154728153389843930244121460275 7,
328.469398942041991794308886551880815888 3,
401.507571585954328698110012419453616215 6,
```

```
358.9736282414997184046359343813397934769,
398.3314710559503091512830647501089022375,
371.4838739375379394097749670110591670286,
336.6121584208881804584970688544243919474,
361.5088834724262755202255780188808429365,
324.6714499300676724773366085134282632168,
302.3138431632811395294203963473772615993,
328.4693851427659742576870540932116417603,
343.8134062475676028083444328908143144815,
375.7328529250084456996109115697116440090,
328.1170929522510622718412117983296435481,
292.9996913932812936559789341931733706039,
358.6434156109169915164776380414583550342,
299.8986620449845424333381846031952104038,
360.0617346850559313554916161508869990386,
336.5944103311936680106799692707423464578,
256.1075318674663079353817043246252196833,
324.6552122397389878037361954670199848559,
331.9380679061134597450641733185126092240, none, none,
289.5459577263339655307253816561925474912]

1 --> 0  target = [17.93041369673149342227968969325319039106,
4.686508702168156673277309966235490704824,
353.3054109591548342363740706210353329181]
one interval r = 20.73150479110149920393375198257063727791 ..
25.90675353551496222690994513793258083934
Time Approximations 0.035.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=7.1e-38
Equations at solution: [-.3e-37, -.71e-37, .65e-35]Solution in 3.994s

Time Plot 0 s.
Exiting SolveHard() after 4.697r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349367355620418848962509154383939,
441.6429597343479078283986365366516926274,
436.9174816511094659093322855102851063370,
422.9849339843547402026986220996644585129,
361.5258025621686139051890623103550551374,
401.8817390473697378304428454980894033564,
389.5900151547281533898439302441214602757,
```

```
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68,
302.31384316328113952942039634737726159 93,
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15,
375.73285292500844569961091156971164400 90,
328.11709295225106227184121179832964354 81,
292.99969139328129365597893419317337060 39,
358.64341561091699151647763804145835503 42,
299.89866204498454243333818460319521040 38,
360.06173468505593135549161615088699903 86,
336.59441033119366801067996927074234645 78,
256.10753186746630793538170432462521968 33,
324.65521223973898780373619546701998485 59,
331.93806790611345974506417331851260922 40,
304.79958326964949862498751500522664948 01, none,
289.54595772633396553072538165619254749 12]

2 --> 0  target = [17.93041369673149342227968969325319039106,
4.68650870216815667327730996623549070482 4,
353.30541095915483423637407062103533291 81]
one interval r = 31.37435487001381234717627512599482047822 ..
34.20127520042307539262081130270184869385
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=9e-38
Equations at solution: [-.5e-37, .9e-37, .195e-34]Solution in 0.373s

Time Plot 0 s.
Exiting SolveHard() after 0.641r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493673556204188489625091543839 39,
441.64295973434790782839863653665169262 74,
436.91748165110946590933228551028510633 70,
422.98493398435474020269862209966445851 29,
```

```
361.52580256216861390518906231035505513 74,
401.88173904736973783044284549808940335 64,
389.59001515472815338984393024412146027 57,
328.46939894204199179430888655188081588 83,
401.50757158595432869811001241945361621 56,
358.97362824149971840463593438133979347 69,
398.33147105595030915128306475010890223 75,
371.48387393753793940977496701105916702 86,
336.61215842088818045849706885442439194 74,
361.50888347242627552022557801888084293 65,
324.67144993006767247733660851342826321 68,
302.31384316328113952942039634737726159 93,
328.46938514276597425768705409321164176 03,
343.81340624756760280834443289081431448 15,
375.73285292500844569961091156971164400 90,
328.11709295225106227184121179832964354 81,
292.99969139328129365597893419317337060 39,
358.64341561091699151647763804145835503 42,
299.89866204498454243333818460319521040 38,
360.06173468505593135549161615088699903 86,
336.59441033119366801067996927074234645 78,
256.10753186746630793538170432462521968 33,
324.65521223973898780373619546701998485 59,
331.93806790611345974506417331851260922 40,
304.79958326964949862498751500522664948 01,
323.46169177081853337992959062818326915 91,
289.54595772633396553072538165619254749 12]

Cascade time 267.662
counts: 28, 28

 Iteration 78

Start Generation 1
1 --> 0  target = [12.00000000001958753659533378379785281200,
6.21701250283457714791226588619193938465 3,
485.54908089875090571540973529596353816 30]
one interval r = 23.40850301651232752233078672220647609739 ..
27.67578046420391606339749128401606795392
Time Approximations 0.039.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.5e-38
Equations at solution: [.1e-37, -.25e-37, .3e-36]Solution in 1.018s

Time Plot 0 s.
Exiting SolveHard() after 2.164r=27.5236 in [25.56992694 ..
27.67578046]
```

```
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000001958753659533378379785281200,
6.217012502834577147912265886191939384653,
485.54908089875090571540973529596353816 30]
one interval r = 32.628147792073842254674722140725787803 60 ..
36.102483889436378398225880015948253962 35
Time Approximations 0.02.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, -.39e-35]Solution in 0.605s

Time Plot 0 s.
Exiting SolveHard() after 4.789r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684470557974593422428640173503813,
6.583434721802876988194870125647023007956,
467.78730595741292162295766060609475817 97]
one interval r = 32.419789556540740941965098621200584232 82 ..
35.851524173700824643926291331024037839 68
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
```

```
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=7e-38
Equations at solution: [-.8e-37, .7e-37, .47e-35]Solution in 0.642s

Time Plot 0 s.
Exiting SolveHard() after 1.011r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46, none, none,
401.88173903968383502728647319054150541 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684470557974593422428640173503813,
6.58343472180287698819487012564702300 7956,
467.78730595741292162295766060609475817 97]
two intervals r = 12.92327160841728644175611038977287105719 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000 or r = 18.39424858021478331397188412109703671713 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000
Time Approximations 0.04.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=8.7e-38
Equations at solution: [-.7e-37, -.87e-37, -.12952e-34]Solution in
44.515s

Time Plot 0.001 s.
Exiting SolveHard() after 49.194r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85, none,
401.88173903968383502728647319054150541 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962828052914650387844736327871048,
4.12565179687782733937697336315738545962 9,
440.67123065237202484301250835561278515 72]
two intervals r = 14.35659705109810334258618407587674657498 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000 or r = 17.70352613807712736160395876305814126074 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.29e-37, .1e-37, .23165e-34]Solution in 4.844s

Time Plot 0 s.
Exiting SolveHard() after 5.941r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85, none,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962828052914650387844736327871048,
4.12565179687782733937697336315738545962 9,
440.67123065237202484301250835561278515 72]
one interval r = 22.39761154365884397323075992358914484069 ..
27.23722351592743073316206605109405943219
```

```
Time Approximations 0.037.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.281 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064386983522164636392549396806871, rm =
14.37818770654103640106819293856772580501}});
Accepted {r=26.4635, rm=16.5329} with Delta=7.9e-38
Equations at solution: [.1e-37, .79e-37, -.934e-34]Solution in 8.746s

Time Plot 0 s.
Exiting SolveHard() after 9.654r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888792728511984715467942978554085,
4.00486908183865842703976770810717445001 2,
404.86224501030934634374871495222675028 66]
two intervals r = 16.08011007771101817810567650972816894393 ..
18999999999938877351707010998992451487/10000000000000000000000000000000
00000 or r = 16.41579812675744804851011525362709348682 ..
18999999999938877351707010998992451487/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
```

```
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.17e-37, .1e-37, .15e-37]Solution in 1.713s

Time Plot 0 s.
Exiting SolveHard() after 6.22r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50, none, none,
358.97362823884844931617240364582563394 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887927285119847154679429785540 85,
4.0048690818386584270397677081071744500 12,
404.86224501030934634374871495222675028 66]
one interval r = 21.641943994013324049466409392503554643 44 ..
26.763306600303437226031689296510312124 15
Time Approximations 0.048.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., .86e-35]Solution in 1.049s

Time Plot 0 s.
Exiting SolveHard() after 6.572r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
```

```
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82, none,
358.97362823884844931617240364582563394 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941821241419552815377700794185 66,
5.58963718274434927526276448882620551438 2,
443.83065883894395487211349374295828921 88]
one interval r = 22.46725374457689402341028858587883757880 ..
27.27388428338823442990733374633650728344
Time Approximations 0.042.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .58e-35]Solution in 5.829s

Time Plot 0 s.
Exiting SolveHard() after 6.794r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82, none,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941821241419552815377700794185 66,
5.58963718274434927526276448882620551438 2,
443.83065883894395487211349374295828921 88]
one interval r = 32.15575279486316360347838327894448875997 ..
35.50872228727541054257479172591598255283
```

```
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, .51e-35]Solution in 0.501s

Time Plot 0 s.
Exiting SolveHard() after 0.863r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136492471360082547076617564967777,
5.18778357842185946009862548488001969128 5,
408.65773863089912065359058826671742768 42]
one interval r = 21.71840114654179920629496993274675610667 ..
26.81849303510181994428788517947346069486
Time Approximations 0.059.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.37e-37
Equations at solution: [-.3e-37, -.237e-36, -.8e-36]Solution in 4.341s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.422r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65, none, none,
361.50888347070750583422195435457233527 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136492471360082547076617564967777,
5.18778357842185946009862548488001969128 5,
408.65773863089912065359058826671742768 42]
one interval r = 31.80828598746226797844299169243528330235 ..
35.00011460049758134333851054674073944377
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, .309e-34]Solution in 0.441s

Time Plot 0 s.
Exiting SolveHard() after 0.757r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
```

```
361.525802560287254996086943383118074237 1,
401.881739039683835027286473190541505415 3,
389.590015161650385533696935068130955125 0,
328.469398929566599073786753674076338828 2,
401.507571576472152248805235426043362296 6,
358.973628238848449316172403645825633943 0,
398.331471030032676937373622380869157096 5,
371.483873949509054616844866192881967555 5, none,
361.508883470707505834221954354572335277 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110532928516942690987672486704454,
6.196262565537205743145030981520030352353,
385.444743793102080806781321084299286786 8]
one interval r = 31.60836097527195005389030058167813016493 ..
34.66372795607553263791529878556849490484
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, -.160e-34]Solution in 0.56s

Time Plot 0 s.
Exiting SolveHard() after 0.85r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936560945012155344152425515797 5,
441.642959730235563236109523033494244808 9,
436.917481654647837775828383164187322544 6,
422.984933968286096917496725845912327508 5,
361.525802560287254996086943383118074237 1,
401.881739039683835027286473190541505415 3,
389.590015161650385533696935068130955125 0,
328.469398929566599073786753674076338828 2,
401.507571576472152248805235426043362296 6,
358.973628238848449316172403645825633943 0,
398.331471030032676937373622380869157096 5,
371.483873949509054616844866192881967555 5, none,
361.508883470707505834221954354572335277 0,
324.671449924246673972284871621296696099 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110532928516942690987672486704454,
```

```
6.196262565537205743145030981520030352353,
385.44474379310208080678132108429928678 68]
two intervals r = 16.875634087518555405243424795828014426 63 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000 or r = 15.556404937969714318846974336115513171 67 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.566 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.468341751160140805802326852529651255 97, rm
= 2.336532774032770736121644756662752257544}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.179e-37, 0., -.25631e-34]Solution in 33.602s

Time Plot 0 s.
Exiting SolveHard() after 38.42r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747330669268312344895566752970 63,
4.883810779708614022393536098827297486249,
376.61967855790769733444217111930788509 70]
one interval r = 21.110013048739710337393760933528603590 49 ..
```

```
26.31784243467867344950146709497433019537
Time Approximations 0.034.

hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.761063912186698964426645072343113601 29, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.5e-38
Equations at solution: [0., -.25e-37, -.238e-34]Solution in 0.856s

Time Plot 0 s.
Exiting SolveHard() after 1.538r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99, none,
328.46938513029365759151191176266064670 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747330669268312344895566752970 63,
4.883810779708614022393536098827297486249,
376.61967855790769733442171119307885097 0]
one interval r = 31.538994977011938453343711801283025269 66 ..
34.53618386090574453496862767325467221642
Time Approximations 0.017.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=6.73e-36
Equations at solution: [.517e-35, -.673e-35, .137e-34]Solution in
0.493s

Time Plot 0 s.
Exiting SolveHard() after 0.777r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99, none,
328.46938513029365759151191176266064670 44,
343.81340625138303504882262696874527219 90, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017525854981367453354736987861092,
6.02581354946932207107033038290581426498 8,
351.42702947934188850069205088104442912 67]
one interval r = 31.36230206100437446599230456407866904184 ..
34.17446640604642310074184263106801337883
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.105e-34]Solution in 0.537s

Time Plot 0 s.
Exiting SolveHard() after 0.769r=33.3686 in [32.23723258 ..
34.17446642]
```

```
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99, none,
328.46938513029365759151191176266064670 44,
343.81340625138303504882262696874527219 90, none, none,
292.99969137719933806538363011112949395 88, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175258549813674533547369878610 92,
6.0258135494693220710703303829058142649 88,
351.42702947934188850069205088104442912 67]
two intervals r = 17.981355144507668346804161338354972540 91 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000 or r = 13.846080153927760644619494616851876841 83 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.764 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713664416758848772149455900647 10, rm
= 2.7345009931542576636689212221623688361 08}});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [-.73e-37, .2e-37, .16390e-34]Solution in
19.956s

Time Plot 0 s.
Exiting SolveHard() after 24.587r=18.6878 in [17.98135512 .. 19]
```

```
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99,
302.31384313863690003281265048454961980 47,
328.46938513029365759151191176266064670 44,
343.81340625138303504882262696874527219 90, none, none,
292.99969137719933806538363011112949395 88, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941851150879530767531885936462737,
6.37794387403117663868182814959147642630 5,
423.28832782878428690768331034458595324 71]
one interval r = 31.94661817577794745673322877526001430375 ..
35.21212308634628206418105591382036261827
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.3e-37, -.3e-37, .165e-34]Solution in 0.628s

Time Plot 0 s.
Exiting SolveHard() after 0.982r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
```

```
441.642959730235563236109523033494244808 9,
436.917481654647837775828383164187322544 6,
422.984933968286096917496725845912327508 5,
361.525802560287254996086943383118074237 1,
401.881739039683835027286473190541505415 3,
389.590015161650385533696935068130955125 0,
328.469398929566599073786753674076338828 2,
401.507571576472152248805235426043362296 6,
358.973628238848449316172403645825633943 0,
398.331471030032676937373622380869157096 5,
371.483873949509054616844866192881967555 5,
336.612158406581116353443231156459664760 5,
361.508883470707505834221954354572335277 0,
324.671449924246673972284871621296696099 9,
302.313843138636900032812650484549619804 7,
328.469385130293657591511911762660646704 4,
343.813406251383035048822626968745272199 0, none, none,
292.999691377199338065383630111129493958 8, none, none,
360.061734656214404937083122954884193749 5, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418511508795307675318859364627 37,
6.377943874031176638681828149591476426305,
423.288327828784286907683310344585953247 1]
two intervals r = 15.22886702489200700554472209816595170357 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000 or r = 17.12965777028052082037509100536667987403 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.176 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541591354015795988366126014812 41, rm
= 2.064068298744563908172265827577780858016}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., .2353e-35]Solution in 31.888s

Time Plot 0 s.
Exiting SolveHard() after 36.607r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99,
302.31384313863690003281265048454961980 47,
328.46938513029365759151191176266064670 44,
343.81340625138303504882262696874527219 90,
375.73285288446742993479435267890110945 90, none,
292.99969137719933806538363011112949395 88, none, none,
360.06173465621440493708312295488419374 95, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234335490869194128584182174443247,
4.0035598155360709500693949626156948233 98,
404.47973593659650925659873368363667763 50]
two intervals r = 16.09683966385298175684441965222890808251 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000 or r = 16.39988649092425041113083739319192847496 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., .12543e-34]Solution in 5.36s

Time Plot 0 s.
Exiting SolveHard() after 6.435r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
```

```
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99,
302.31384313863690003281265048454961980 47,
328.46938513029365759151191176266064670 44,
343.81340625138303504882262696874527219 90,
375.73285288446742993479435267890110945 90, none,
292.99969137719933806538363011112949395 88,
358.64341560668104162321474687466077115 57, none,
360.06173465621440493708312295488419374 95, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234335490869194128584182174443247,
4.0035598155360709500693949626156948233 98,
404.47973593659650925659873368363667763 50]
one interval r = 21.634296299770728153582542460376028088 70 ..
26.757681698790201824780618892423804626 74
Time Approximations 0.051.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=1.01e-37
Equations at solution: [.2e-37, .101e-36, -.433e-34]Solution in 1.103s

Time Plot 0 s.
Exiting SolveHard() after 2.139r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
```

```
328.46939892956659907378675367407633882282,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99,
302.31384313863690003281265048454961980 47,
328.46938513029365759151191176266064670 44,
343.81340625138303504882262696874527219 90,
375.73285288446742993479435267890110945 90,
328.11709293808461243109460112329595398 04,
292.99969137719933806538363011112949395 88,
358.64341560668104162321474687466077115 57, none,
360.06173465621440493708312295488419374 95, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954451190256634840288779845350173,
6.19617723039093359898285983089395576612 5,
385.42734025689568791577229845248359151 86]
one interval r = 31.60822049082593586878692096330102993681 ..
34.66347615045275154471307040295804870665
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=8e-38
Equations at solution: [.6e-37, -.8e-37, -.185e-34]Solution in 0.55s

Time Plot 0 s.
Exiting SolveHard() after 4.615r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
```

```
398.331471030032676937373622380869157096 5,
371.483873949509054616844866192881967555 5,
336.612158406581116353443231156459664760 5,
361.508883470707505834221954354572335277 0,
324.671449924246673972284871621296696099 9,
302.313843138636900032812650484549619804 7,
328.469385130293657591511911762660646704 4,
343.813406251383035048822626968745272199 0,
375.732852884467429934794352678901109459 0,
328.117092938084612431094601123295953980 4,
292.999691377199338065383630111129493958 8,
358.643415606681041623214746874660771155 7, none,
360.061734656214404937083122954884193749 5, none, none,
324.655212234073966858187850026162989065 2, none, none, none, none]

0 --> 1  target = [26.463189544511902566348402887798453501 73,
6.196177230390933598982859830893955766125,
385.427340256895687915772298452483591518 6]
two intervals r = 16.876296002903318911305624597017792699 31 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000 or r = 15.555590006457262400750323651633509592 43 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000
Time Approximations 0.063.

hint used Hint := [17.930920830435909701694400819009216331 81, 3, 1, 1,
15.700860627013397511253157405159644880 12, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 8.093 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852527663918970827160677378970252, rm
= 2.33669042809772979993348765170298534872 4}});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [-.716e-37, .1e-37, .8282e-35]Solution in
33.392s

Time Plot 0 s.
Exiting SolveHard() after 34.601r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936560945012155344152425515797 5,
441.642959730235563236109523033494244808 9,
436.917481654647837775828383164187322544 6,
422.984933968286096917496725845912327508 5,
361.525802560287254996086943383118074237 1,
```

```
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99,
302.31384313863690003281265048454961980 47,
328.46938513029365759151191176266064670 44,
343.81340625138303504882262696874527219 90,
375.73285288446742993479435267890110945 90,
328.11709293808461243109460112329595398 04,
292.99969137719933806538363011112949395 88,
358.64341560668104162321474687466077115 57, none,
360.06173465621440493708312295488419374 95,
336.59441031705740233835487182846381959 78, none,
324.65521223407396685818785002616298906 52, none, none, none, none]

0 --> 2  target = [34.49522661171001139255923663572351803321,
3.8971313159869237210984601292133931105 12,
373.78018885035966434644395136320559431 29]
two intervals r = 17.297690862012668690829086045332474280 00 ..
18999999999938877351707010998992451487/1000000000000000000000000000000000
00000 or r = 14.994364074559615369541699984763344634 20 ..
18999999999938877351707010998992451487/1000000000000000000000000000000000
00000
Time Approximations 0.077.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [.18e-37, -.1e-37, -.27217e-34]Solution in
1.202s

Time Plot 0 s.
Exiting SolveHard() after 6.269r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
```

```
361.525802560287254996086943383118074237l,
401.881739039683835027286473190541505415 3,
389.590015161650385533696935068130955125 0,
328.469398929566599073786753674076338828 2,
401.507571576472152248805235426043362296 6,
358.973628238848449316172403645825633943 0,
398.331471030032676937373622380869157096 5,
371.483873949509054616844866192881967555 5,
336.612158406581116353443231156459664760 5,
361.508883470707505834221954354572335277 0,
324.671449924246673972284871621296696099 9,
302.313843138636900032812650484549619804 7,
328.469385130293657591511911762660646704 4,
343.813406251383035048822626968745272199 0,
375.732852884467429934794352678901109459 0,
328.117092938084612431094601123295953980 4,
292.999691377199338065383630111129493958 8,
358.643415606681041623214746874660771155 7, none,
360.061734656214404937083122954884193749 5,
336.594410317057402338354871828463819597 8, none,
324.655212234073966858187850026162989065 2,
331.938067921085541925774033871447798855 0, none, none, none]

1 --> 2  target = [34.495226611710011392559236635723518033 21,
3.897131315986923721098460129213393110512,
373.780818850359664346443951363205594312 9]
one interval r = 21.060684732100330420286406571917081144 30 ..
26.269798342863075085768784926881499593 38
Time Approximations 0.036.

hint used Hint := [25.300466340350721366923008834208363590 84, 3, -1, 1,
16.974729527641787545085746138369074748 18, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [.1e-37, .3e-37, .236e-34]Solution in 4.381s

Time Plot 0 s.
Exiting SolveHard() after 5.127r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936560945012155344152425515797 5,
441.642959730235563236109523033494244808 9,
436.917481654647837775828383164187322544 6,
422.984933968286096917496725845912327508 5,
361.525802560287254996086943383118074237l,
```

```
401.881739039683835027286473190541505415 3,
389.590015161650385533696935068130955125 0,
328.469398929566599073786753674076338828 2,
401.507571576472152248805235426043362296 6,
358.973628238848449316172403645825633943 0,
398.331471030032676937373622380869157096 5,
371.483873949509054616844866192881967555 5,
336.612158406581116353443231156459664760 5,
361.508883470707505834221954354572335277 0,
324.671449924246673972284871621296696099 9,
302.313843138636900032812650484549619804 7,
328.469385130293657591511911762660646704 4,
343.813406251383035048822626968745272199 0,
375.732852884467429934794352678901109459 0,
328.117092938084612431094601123295953980 4,
292.999691377199338065383630111129493958 8,
358.643415606681041623214746874660771155 7,
299.898662050971555279530134281891333009 3,
360.061734656214404937083122954884193749 5,
336.594410317057402338354871828463819597 8, none,
324.655212234073966858187850026162989065 2,
331.938067921085541925774033871447798855 0, none, none, none]

0 --> 2  target = [33.813624954026091152908903459601255919 38,
3.725648993592385796074354163030922384558,
325.892099726548342745267108080005192480 2]
two intervals r = 18.552270490029394474907479917412175604 33 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000 or r = 12.491969357789208627214614859996992085 56 ..
189999999999388773517070109989924514 87/100000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=2e-38
Equations at solution: [.51e-37, -.2e-37, -.6204e-35]Solution in 1.157s

Time Plot 0 s.
Exiting SolveHard() after 2.593r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936560945012155344152425515797 5,
441.642959730235563236109523033494244808 9,
436.917481654647837775828383164187322544 6,
```

```
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99,
302.31384313863690003281265048454961980 47,
328.46938513029365759151191176266064670 44,
343.81340625138303504882262696874527219 90,
375.73285288446742993479435267890110945 90,
328.11709293808461243109460112329595398 04,
292.99969137719933806538363011112949395 88,
358.64341560668104162321474687466077115 57,
299.89866205097155527953013428189133300 93,
360.06173465621440493708312295488419374 95,
336.59441031705740233835487182846381959 78, none,
324.65521223407396685818785002616298906 52,
331.93806792108554192577403387144779885 50, none, none,
289.54595772517697780907038411968503671 10]

1 --> 2  target = [33.81362495402609115290890345960125591938,
3.72564899359238579607435416303092238455 8,
325.89209972654834274526710808000519248 02]
one interval r = 20.37468935102095669254798227072116566195 ..
25.37892165289833653199762742933285607752
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, -.143e-34]Solution in 0.573s

Time Plot 0 s.
Exiting SolveHard() after 5.555r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
```

```
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99,
302.31384313863690003281265048454961980 47,
328.46938513029365759151191176266064670 44,
343.81340625138303504882262696874527219 90,
375.73285288446742993479435267890110945 90,
328.11709293808461243109460112329595398 04,
292.99969137719933806538363011112949395 88,
358.64341560668104162321474687466077115 57,
299.89866205097155527953013428189133300 93,
360.06173465621440493708312295488419374 95,
336.59441031705740233835487182846381959 78,
256.10753185678726060619508182085046060 42,
324.65521223407396685818785002616298906 52,
331.93806792108554192577403387144779885 50, none, none,
289.54595772517697780907038411968503671 10]

1 --> 0  target = [17.93041369719961685037736623903207332588,
4.68650870180242949690135506514709567582 4,
353.30541094462624822857781136533772946 95]
one interval r = 20.73150479079686067332533962293155826072 ..
25.90675353505193274386845913910726601147
Time Approximations 0.034.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.40e-35]Solution in 0.675s

Time Plot 0 s.
Exiting SolveHard() after 1.418r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99,
302.31384313863690003281265048454961980 47,
328.46938513029365759151191176266064670 44,
343.81340625138303504882262696874527219 90,
375.73285288446742993479435267890110945 90,
328.11709293808461243109460112329595398 04,
292.99969137719933806538363011112949395 88,
358.64341560668104162321474687466077115 57,
299.89866205097155527953013428189133300 93,
360.06173465621440493708312295488419374 95,
336.59441031705740233835487182846381959 78,
256.10753185678726060619508182085046060 42,
324.65521223407396685818785002616298906 52,
331.93806792108554192577403387144779885 50,
304.79958324492563750504883746974823411 07, none,
289.54595772517697780907038411968503671 10]

2 --> 0  target = [17.93041369719961685037736623903207332588,
4.68650870180242949690135506514709567582 4,
353.30541094462624822857781136533772946 95]
one interval r = 31.37435486979036783576722239574176532403 ..
34.20127520015978139200828843195302606570
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [.6e-37, -.8e-37, -.39e-35]Solution in 0.356s

Time Plot 0 s.
Exiting SolveHard() after 0.623r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493656094501215534415242551579 75,
441.64295973023556323610952303349424480 89,
436.91748165464783777582838316418732254 46,
422.98493396828609691749672584591232750 85,
361.52580256028725499608694338311807423 71,
401.88173903968383502728647319054150541 53,
389.59001516165038553369693506813095512 50,
328.46939892956659907378675367407633882 82,
401.50757157647215224880523542604336229 66,
358.97362823884844931617240364582563394 30,
398.33147103003267693737362238086915709 65,
371.48387394950905461684486619288196755 55,
336.61215840658111635344323115645966476 05,
361.50888347070750583422195435457233527 70,
324.67144992424667397228487162129669609 99,
302.31384313863690003281265048454961980 47,
328.46938513029365759151191176266064670 44,
343.81340625138303504882262696874527219 90,
375.73285288446742993479435267890110945 90,
328.11709293808461243109460112329595398 04,
292.99969137719933806538363011112949395 88,
358.64341560668104162321474687466077115 57,
299.89866205097155527953013428189133300 93,
360.06173465621440493708312295488419374 95,
336.59441031705740233835487182846381959 78,
256.10753185678726060619508182085046060 42,
324.65521223407396685818785002616298906 52,
331.93806792108554192577403387144779885 50,
304.79958324492563750504883746974823411 07,
323.46169176420116718710549499015813961 53,
289.54595772517697780907038411968503671 10]

Cascade time 273.354
counts: 28, 28

 Iteration 79

Start Generation 1
1 --> 0  target = [12.00000000020990605688369287671594569600,
6.21701250277334360430744527135909158830 6,
485.54908089517316834256498065703209465 97]
one interval r = 23.40850301639983465251206116763788319560 ..
27.67578046413050948085574043113269823731
Time Approximations 0.056.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
```

```
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.3e-38
Equations at solution: [-.2e-37, .53e-37, 0.]Solution in 1.03s

Time Plot 0 s.
Exiting SolveHard() after 2.173r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000020990605688369287671594569600,
6.21701250277334360430744527135909158830 6,
485.54908089517316834256498065703209465 97]
one interval r = 32.62814779214282465491322734246408432724 ..
36.10248388939401068016164846330255854420
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.28e-35]Solution in 3.971s

Time Plot 0 s.
Exiting SolveHard() after 4.381r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684463678699987689805133563862674,
6.58343472165219525866364077544421152928 2,
```

```
467.78730595470720307617315818188146824 17]
one interval r = 32.41978955662878749698957724611813602422 ..
35.85152417367688498777944093808021325011
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.321e-34]Solution in 0.672s

Time Plot 0 s.
Exiting SolveHard() after 1.071r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54, none, none,
401.88173903854736574114324284151111780 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684463678699987689805133563862674,
6.58343472165219525866364077544421152928 2,
467.78730595470720307617315818188146824 17]
two intervals r = 12.92327160862144669953236419061782753576 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000
000 or r = 18.39424858035136014849957126707289475301 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.0e-38
Equations at solution: [-.1e-37, -.10e-37, -.59e-36]Solution in 50.108s
```

```
Time Plot 0 s.
Exiting SolveHard() after 51.519r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49, none,
401.88173903854736574114324284151111178004, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962825489278456492146341217726832,
4.1256517968315465697007001070846337269 81,
440.67123064885004029420796985887850854 37]
two intervals r = 14.35659705140118296169658747942735202878 ..
190000000001586841467375069858233030 7/1000000000000000000000000000000000
000 or r = 17.70352613814119792808471022912719565874 ..
190000000001586841467375069858233030 7/1000000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.14e-37, -.1e-37, .1478e-34]Solution in 1.354s

Time Plot 0 s.
Exiting SolveHard() after 2.416r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49, none,
401.88173903854736574114324284151111178004,
389.59001515795846253491662257141080322 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
1 --> 2  target = [35.46322962825489278456492146341217726832,
4.12565179683154656970070010708463372698 1,
440.67123064885004029420796985887850854 37]
one interval r = 22.39761154354922228346388953580427387129 ..
27.23722351584566152446156256009088036584
Time Approximations 0.037.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.259 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064372598307095624912898196723704, rm =
14.37818770400422543181365795523810592776}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .138e-34]Solution in 8.425s

Time Plot 0 s.
Exiting SolveHard() after 12.91r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888794698308369970755895965932353,
4.00486908179944707469131123543466169720 1,
404.86224500892854786713818502365169952 18]
two intervals r = 16.08011007794601729499013090621829432441 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000000
000 or r = 16.41579812682823297597000242060438000386 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
```

```
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=2e-38
Equations at solution: [-.69e-37, -.2e-37, .1018e-34]Solution in 1.476s

Time Plot 0 s.
Exiting SolveHard() after 6.101r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44, none, none,
358.97362823678305098133273663329889185 79, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887946983083699707558959659323 53,
4.0048690817994470746913112354346616972 01,
404.86224500892854786713818502365169952 18]
one interval r = 21.641943993951942326371245271224388106 44 ..
26.763306600246135603627295594867171193 87
Time Approximations 0.056.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., .133e-34]Solution in 5.038s

Time Plot 0 s.
Exiting SolveHard() after 6.126r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60, none,
358.97362823678305098133273663329889185 79, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941834541768707821253302152256588,
5.5896371827495337442800649351090465141 17,
443.83065883859738025634594411220079074 66]
one interval r = 22.46725374453715845919149727371483695348 ..
27.27388428334350220595240696297057065772
Time Approximations 0.041.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [-.1e-37, .81e-37, -.71e-35]Solution in 1.032s

Time Plot 0 s.
Exiting SolveHard() after 2.066r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60, none,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]
```

```
2 --> 0  target = [14.19258941834541768707821253302152256588,
5.589637182749533744280064935109046514117,
443.83065883859738025634594411220079074 66]
one interval r = 32.15575279498757356793763821047119583336 ..
35.50872228729427349716419206824994080706
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., -.175e-34]Solution in 0.441s

Time Plot 0 s.
Exiting SolveHard() after 4.369r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136526833243517563917341285303388,
5.187783578400394664171631228268179553720,
408.65773862702747629540398226386618480 54]
one interval r = 21.71840114642971808327441732676975654777 ..
26.81849303500876423125677015782033052598
Time Approximations 0.061.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=0
Equations at solution: [0., 0., -.210e-34]Solution in 1.02s

Time Plot 0 s.
Exiting SolveHard() after 2.132r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08, none, none,
361.50888346917034449642478660786738840 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136526833243517563917341285303388,
5.1877835784003946641716312282681795537 20,
408.65773862702747629540398226386618480 54]
one interval r = 31.80828598756712507795367976808894358948 ..
35.00011460047958498676818956544595379970
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=7e-38
Equations at solution: [-.7e-37, .7e-37, -.29e-35]Solution in 0.423s

Time Plot 0 s.
Exiting SolveHard() after 4.769r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
371.48387394544851024354027229274911730 22, none,
361.50888346917034449642478660786738840 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110526599407184242639885891991729,
6.19626256538963171560908019362980874430 3,
385.44474379138202448202675879248874905 57]
one interval r = 31.60836097540540708819293521436681928568 ..
34.66372795609853488421345974777518222308
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.100e-34]Solution in 0.564s

Time Plot 0 s.
Exiting SolveHard() after 0.854r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
```

```
398.331471031791258205250892536921999140 8,
371.483873945448510243540272292749117302 2, none,
361.508883469170344496424786607867388408 9,
324.671449924122890159109086619187369407 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.463471105265994071842426398858919917 29,
6.196262565389631715609080193629808744303,
385.444743791382024482026758792488749055 7]
two intervals r = 16.875634087777602255758957033990538033 74 ..
190000000001586841467375069858233030 7/100000000000000000000000000000000
000 or r = 15.556404937987488737549016875746303542 01 ..
190000000001586841467375069858233030 7/100000000000000000000000000000000
000
Time Approximations 0.064.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [-.1255e-36, .1e-37, -.699e-35]Solution in
4.649s

Time Plot 0 s.
Exiting SolveHard() after 5.831r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434933260361745395837425337445123 1,
441.642959727769919998127036318466203203 0,
436.917481651415983017001042401077427055 4,
422.984933967978357094519903836423591274 9,
361.525802558786614718133119347609419129 7,
401.881739038547365741143242841511117800 4,
389.590015157958462534916622571410803224 4,
328.469398930057250538404163133428434916 0,
401.507571575690217769479533859341668903 5,
358.973628236783050981332736633298891857 9,
398.331471031791258205250892536921999140 8,
371.483873945448510243540272292749117302 2,
336.612158407803550884212046669783811174 2,
361.508883469170344496424786607867388408 9,
324.671449924122890159109086619187369407 9, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747606214527372286806394603672 28,
```

```
4.883810779715616552173334118351367251076,
376.619678555775720598578904006227936190 3]
one interval r = 21.110013048667134324520691072912921557 37 ..
26.317842434607979372132271102171976044 91
Time Approximations 0.037.

hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.761063912186698964426645072343113601 29, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.3e-38
Equations at solution: [0., .23e-37, .419e-34]Solution in 0.87s

Time Plot 0 s.
Exiting SolveHard() after 1.578r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
371.48387394548510243540272292749117302 2,
336.61215840780355088421204666978381117 42,
361.50888346917034449642478660786738840 89,
324.67144992412289015910908661918736940 79, none,
328.46938513078360159751918793068527767 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747606214527372286806394603672 28,
4.883810779715616552173334118351367251076,
376.619678555775720598578904006227936190 3]
one interval r = 31.538994977145461675379196991374238210 70 ..
34.536183860926617955056508025880236232 89
Time Approximations 0.017.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
```

```
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.03e-36
Equations at solution: [.80e-36, -.103e-35, .9e-36]Solution in 0.5s

Time Plot 0 s.
Exiting SolveHard() after 0.794r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
371.48387394544851024354027229274911730 22,
336.61215840780355088421204666978381117 42,
361.50888346917034449642478660786738840 89,
324.67144992412289015910908661918736940 79, none,
328.46938513078360159751918793068527767 60,
343.81340624858928413330822894532002997 44, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.872050175231352839478335281580398292 16,
6.0258135493315794782041835332428690646 16,
351.42702947966936538170221050261648744 01]
one interval r = 31.362302061162636152858282830668954472 40 ..
34.174466406113582150360373524245110610 36
Time Approximations 0.015.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, .56e-35]Solution in 0.532s

Time Plot 0 s.
```

```
Exiting SolveHard() after 4.23r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
371.48387394544851024354027229274911730 22,
336.61215840780355088421204666978381117 42,
361.50888346917034449642478660786738840 89,
324.67144992412289015910908661918736940 79, none,
328.46938513078360159751918793068527767 60,
343.81340624858928413330822894532002997 44, none, none,
292.99969137901795373042034910400371079 82, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017523135283947833528158039829216,
6.02581354933157947820418353324286906461 6,
351.42702947966936538170221050261648744 01]
two intervals r = 17.98135514471338779049504588146435902208 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000000
000 or r = 13.84608015397674556685593994414283364824 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=6e-38
Equations at solution: [-.196e-36, .6e-37, .73e-36]Solution in 1.164s

Time Plot 0 s.
Exiting SolveHard() after 2.263r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
371.48387394544851024354027229274911730 22,
336.61215840780355088421204666978381117 42,
361.50888346917034449642478660786738840 89,
324.67144992412289015910908661918736940 79,
302.31384314190437802474607812763068289 50,
328.46938513078360159751918793068527767 60,
343.81340624858928413330822894532002997 44, none, none,
292.99969137901795373042034910400371079 82, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941849437462447541216060756234549,
6.37794387390004583077542707619471859633 3,
423.28832783041670457566719572722456169 32]
one interval r = 31.94661817592925649754389595761736827951 ..
35.21212308640203246100962958292840496708
Time Approximations 0.018.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=1.0e-37
Equations at solution: [-.9e-37, .10e-36, .156e-34]Solution in 0.592s

Time Plot 0 s.
Exiting SolveHard() after 5.112r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
```

```
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
371.48387394544851024354027229274911730 22,
336.61215840780355088421204666978381117 42,
361.50888346917034449642478660786738840 89,
324.67144992412289015910908661918736940 79,
302.31384314190437802474607812763068289 50,
328.46938513078360159751918793068527767 60,
343.81340624858928413330822894532002997 44, none, none,
292.99969137901795373042034910400371079 82, none, none,
360.06173465919302070452688116429182245 22, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418494374624475412160607562345 49,
6.3779438739000458307754270761947185963 33,
423.28832783041670457566719572722456169 32]
two intervals r = 15.228867024963269383741433944866191815 81 ..
190000000001586841467375069858233030 7/1000000000000000000000000000000000
000 or r = 17.129657770490738227274536406938787020 33 ..
190000000001586841467375069858233030 7/1000000000000000000000000000000000
000
Time Approximations 0.064.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [-.17e-37, 0., -.69e-36]Solution in 4.917s

Time Plot 0 s.
Exiting SolveHard() after 6.309r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
```

```
371.48387394544851024354027229274911730 22,
336.61215840780355088421204666978381117 42,
361.50888346917034449642478660786738840 89,
324.67144992412289015910908661918736940 79,
302.31384314190437802474607812763068289 50,
328.46938513078360159751918793068527767 60,
343.81340624858928413330822894532002997 44,
375.73285288912335403967943222330366761 42, none,
292.99969137901795373042034910400371079 82, none, none,
360.06173465919302070452688116429182245 22, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343380021735308697292855521629 71,
4.0035598154981003916728441837946046787 30,
404.47973593557815293379695555348875844 58]
two intervals r = 16.096839664072437072112265256260835874 93 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000000
000 or r = 16.399886491009442024957767206122011879 76 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000000
000
Time Approximations 0.067.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=2e-38
Equations at solution: [-.86e-37, -.2e-37, .144e-35]Solution in 1.522s

Time Plot 0 s.
Exiting SolveHard() after 2.615r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
371.48387394544851024354027229274911730 22,
336.61215840780355088421204666978381117 42,
361.50888346917034449642478660786738840 89,
```

```
324.67144992412289015910908661918736940 79,
302.31384314190437802474607812763068289 50,
328.46938513078360159751918793068527767 60,
343.81340624858928413330822894532002997 44,
375.73285288912335403967943222330366761 42, none,
292.99969137901795373042034910400371079 82,
358.64341560492670674166738989084525537 16, none,
360.06173465919302070452688116429182245 22, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343380021735308697292855521629 71,
4.0035598154981003916728441837946046787 30,
404.47973593557815293379695555348875844 58]
one interval r = 21.634296299716607814625431239071435676 27 ..
26.757681698738226473560804764886977600 41
Time Approximations 0.053.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.0e-38
Equations at solution: [-.2e-37, -.50e-37, .677e-34]Solution in 1.082s

Time Plot 0 s.
Exiting SolveHard() after 6.051r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
371.48387394544851024354027229274911730 22,
336.61215840780355088421204666978381117 42,
361.50888346917034449642478660786738840 89,
324.67144992412289015910908661918736940 79,
302.31384314190437802474607812763068289 50,
328.46938513078360159751918793068527767 60,
343.81340624858928413330822894532002997 44,
```

```
375.7328528891233540396794322233036676142,
328.1170929389091866091305974678873492366,
292.9996913790179537304203491040037107982,
358.6434156049267067416673898908452553716, none,
360.0617346591930207045268811642918224522, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954444800272940755398616323629906,
6.196177230243175039647036425713174854863,
385.42734025513806353505183322924538260 78]
one interval r = 31.60822049095909610842370242158824665450 ..
34.66347615047521771361872276542884745658
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, 0.]Solution in 0.583s

Time Plot 0 s.
Exiting SolveHard() after 0.877r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349332603617453958374253374451231,
441.6429597277699199981270363184662032030,
436.9174816514159830170010424010774270554,
422.9849339679783570945199038364235912749,
361.5258025587866147181331193476094191297,
401.8817390385473657411432428415111178004,
389.5900151579584625349166225714108032244,
328.4693989300572505384041631334284349160,
401.5075715756902177694795338593416689035,
358.9736282367830509813327366332988918579,
398.3314710317912582052508925369219991408,
371.4838739454485102435402722927491173022,
336.6121584078035508842120466697838111742,
361.5088834691703444964247866078673884089,
324.6714499241228901591090866191873694079,
302.3138431419043780247460781276306828950,
328.4693851307836015975191879306852776760,
343.8134062485892841333082289453200299744,
375.7328528891233540396794322233036676142,
328.1170929389091866091305974678873492366,
292.9996913790179537304203491040037107982,
```

```
358.6434156049267067416673898908452553716, none,
360.0617346591930207045268811642918224522, none, none,
324.6552122339151509901544038289981870527, none, none, none, none]

0 --> 1  target = [26.46318954444800272940755398616323629906,
6.196177230243175039647036425713174854863,
385.42734025513806353505183322924538260 78]
two intervals r = 16.87629600316379966017585241163839146334 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000000
000 or r = 15.55559000647324095150930251820256864694 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000000
000
Time Approximations 0.06.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.359e-37, 0., .702e-35]Solution in 1.201s

Time Plot 0 s.
Exiting SolveHard() after 6.078r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
371.48387394544851024354027229274911730 22,
336.61215840780355088421204666978381117 42,
361.50888346917034449642478660786738840 89,
324.67144992412289015910908661918736940 79,
302.31384314190437802474607812763068289 50,
328.46938513078360159751918793068527767 60,
343.81340624858928413330822894532002997 44,
375.73285288912335403967943222330366761 42,
328.11709293890918660913059746788734923 66,
292.99969137901795373042034910400371079 82,
358.6434156049267067416673898908452553716, none,
```

```
360.06173465919302070452688116429182245 22,
336.59441031824153869411229191343829003 77, none,
324.65521223391515099015440382899818705 27, none, none, none, none]

0 --> 2  target = [34.49522661169984332631217417823655485573,
3.89713131593725738849374926928615582646 0,
373.78081884598884961785399867736174809 32]
two intervals r = 17.29769086236502313864677754763256823107 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000000
000 or r = 14.99436407442102314982624449473326271425 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000000
000
Time Approximations 0.077.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.54e-37, 0., -.943e-35]Solution in 1.224s

Time Plot 0 s.
Exiting SolveHard() after 6.52r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
371.48387394544851024354027229274911730 22,
336.61215840780355088421204666978381117 42,
361.50888346917034449642478660786738840 89,
324.67144992412289015910908661918736940 79,
302.31384314190437802474607812763068289 50,
328.46938513078360159751918793068527767 60,
343.81340624858928413330822894532002997 44,
375.73285288912335403967943222330366761 42,
328.11709293890918660913059746788734923 66,
292.99969137901795373042034910400371079 82,
358.64341560492670674166738989084525537 16, none,
```

```
360.06173465919302070452688116429182245 22,
336.59441031824153869411229191343829003 77, none,
324.65521223391515099015440382899818705 27,
331.93806791626160817966041070700096665 07, none, none, none]

1 --> 2  target = [34.49522661169984332631217417823655485573,
3.89713131593725738849374926928615582646 0,
373.78081884598884961785399867736174809 32]
one interval r = 21.06068473198966221074960565800321832091 ..
26.26979834275402750051596311424182624439
Time Approximations 0.038.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [-.2e-37, -.5e-37, .399e-34]Solution in 4.194s

Time Plot 0 s.
Exiting SolveHard() after 4.971r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
371.48387394544851024354027229274911730 22,
336.61215840780355088421204666978381117 42,
361.50888346917034449642478660786738840 89,
324.67144992412289015910908661918736940 79,
302.31384314190437802474607812763068289 50,
328.46938513078360159751918793068527767 60,
343.81340624858928413330822894532002997 44,
375.73285288912335403967943222330366761 42,
328.11709293890918660913059746788734923 66,
292.99969137901795373042034910400371079 82,
358.64341560492670674166738989084525537 16,
299.89866204872814509800070936912772493 83,
```

```
360.06173465919302070452688116429182245 22,
336.59441031824153869411229191343829003 77, none,
324.65521223391515099015440382899818705 27,
331.93806791626160817966041070700096665 07, none, none, none]

0 --> 2  target = [33.81362495409487585689960867548768284725,
3.72564899355725023910422376803129774890 6,
325.89209972620558394813973449637182585 68]
two intervals r = 18.552270490257347867323329308969769641 87 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000000
000 or r = 12.49196935774078748161967629437773882526 ..
19000000000158684146737506985823303 07/10000000000000000000000000000000000
000
Time Approximations 0.042.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [-.157e-36, .6e-37, -.1e-37]Solution in 4.854s

Time Plot 0 s.
Exiting SolveHard() after 6.298r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493326036174539583742533744512 31,
441.64295972776991999812703631846620320 30,
436.91748165141598301700104240107742705 54,
422.98493396797835709451990383642359127 49,
361.52580255878661471813311934760941912 97,
401.88173903854736574114324284151111780 04,
389.59001515795846253491662257141080322 44,
328.46939893005725053840416313342843491 60,
401.50757157569021776947953385934166890 35,
358.97362823678305098133273663329889185 79,
398.33147103179125820525089253692199914 08,
371.48387394544851024354027229274911730 22,
336.61215840780355088421204666978381117 42,
361.50888346917034449642478660786738840 89,
324.67144992412289015910908661918736940 79,
302.31384314190437802474607812763068289 50,
328.46938513078360159751918793068527767 60,
343.81340624858928413330822894532002997 44,
375.73285288912335403967943222330366761 42,
328.11709293890918660913059746788734923 66,
292.99969137901795373042034910400371079 82,
```

```
358.6434156049267067416673898908452553716,
299.8986620487281450980007093691277249383,
360.0617346591930207045268811642918224522,
336.5944103182415386941122919134382900377, none,
324.6552122339151509901544038289981870527,
331.9380679162616081796604107070009666507, none, none,
289.5459577237521508352062294585846412493]

1 --> 2  target = [33.8136249540948758568996086754876828472 5,
3.7256489935572502391042237680312977489 06,
325.8920997262055839481397344963718258568]
one interval r = 20.3746893509793219876428107514547259630 7 ..
25.3789216528593963392164114191652868361 9
Time Approximations 0.027.

hint used Hint := [24.3394990302538142174610776606811553342 3, 3, -1, 1,
17.2721587825129442122670749442843558157 4, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, -.135e-34]Solution in 0.59s

Time Plot 0 s.
Exiting SolveHard() after 1.135r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349332603617453958374253374451231,
441.6429597277699199981270363184662032030,
436.9174816514159830170010424010774270554,
422.9849339679783570945199038364235912749,
361.5258025587866147181331193476094191297,
401.8817390385473657411432428415111178004,
389.5900151579584625349166225714108032244,
328.4693989300572505384041631334284349160,
401.5075715756902177694795338593416689035,
358.9736282367830509813327366332988918579,
398.3314710317912582052508925369219991408,
371.4838739454485102435402722927491173022,
336.6121584078035508842120466697838111742,
361.5088834691703444964247866078673884089,
324.6714499241228901591090866191873694079,
302.3138431419043780247460781276306828950,
328.4693851307836015975191879306852776760,
343.8134062485892841333082289453200299744,
375.7328528891233540396794322233036676142,
328.1170929389091866091305974678873492366,
```

```
292.99969137901795373042034910400371079 82,
358.6434156049267067416673898908452553716,
299.8986620487281450980007093691277249383,
360.0617346591930207045268811642918224522,
336.5944103182415386941122919134382900377,
256.1075318582489131711797586865945835391,
324.6552122339151509901544038289981870527,
331.9380679162616081796604107070009666507, none, none,
289.5459577237521508352062294585846412493]

1 --> 0  target = [17.9304136973772769884814095987981692144 8,
4.6865087018440756779982119141894904987 40,
353.3054109459394740065852072620199512764]
one interval r = 20.7315047907799068981203444980573366908 9 ..
25.9067535350427449318997583605661830901 2
Time Approximations 0.036.

hint used Hint := [25.4020715378188039710536173439704738275 4, 3, -1, 1,
17.0062444547662134530206245122467435705 3, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.8e-38
Equations at solution: [-.2e-37, -.48e-37, -.28e-35]Solution in 0.654s

Time Plot 0 s.
Exiting SolveHard() after 1.368r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349332603617453958374253374451231,
441.6429597277699199981270363184662032030,
436.9174816514159830170010424010774270554,
422.9849339679783570945199038364235912749,
361.5258025587866147181331193476094191297,
401.8817390385473657411432428415111178004,
389.5900151579584625349166225714108032244,
328.4693989300572505384041631334284349160,
401.5075715756902177694795338593416689035,
358.9736282367830509813327366332988918579,
398.3314710317912582052508925369219991408,
371.4838739454485102435402722927491173022,
336.6121584078035508842120466697838111742,
361.5088834691703444964247866078673884089,
324.6714499241228901591090866191873694079,
302.3138431419043780247460781276306828950,
328.4693851307836015975191879306852776760,
343.8134062485892841333082289453200299744,
```

```
375.7328528891233540396794322233036676142,
328.1170929389091866091305974678873492366,
292.9996913790179537304203491040037107982,
358.6434156049267067416673898908452553716,
299.8986620487281450980007093691277249383,
360.0617346591930207045268811642918224522,
336.5944103182415386941122919134382900377,
256.1075318582489131711797586865945835391,
324.6552122339151509901544038289981870527,
331.9380679162616081796604107070009666507,
304.7995832490011797669710058182466040067, none,
289.5459577237521508352062294585846412493]

2 --> 0  target = [17.93041369737727698848140959879816921448,
4.686508701844075677998211914189490398740,
353.3054109459394740065852072620199512764]
one interval r = 31.37435486995460062607739031839964674193 ..
34.20127520024020985934437334617871461108
Time Approximations 0.019.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.134e-34]Solution in 0.384s

Time Plot 0 s.
Exiting SolveHard() after 4.534r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349332603617453958374253374451231,
441.6429597277699199981270363184662032030,
436.9174816514159830170010424010774270554,
422.9849339679783570945199038364235912749,
361.5258025587866147181331193476094191297,
401.8817390385473657411432428415111178004,
389.5900151579584625349166225714108032244,
328.4693989300572505384041631334284349160,
401.5075715756902177694795338593416689035,
358.9736282367830509813327366332988918579,
398.3314710317912582052508925369219991408,
371.4838739454485102435402722927491173022,
336.6121584078035508842120466697838111742,
361.5088834691703444964247866078673884089,
324.6714499241228901591090866191873694079,
```

```
302.31384314190437802474607812763068289 50,
328.46938513078360159751918793068527767 60,
343.81340624858928413330822894532002997 44,
375.73285288912335403967943222330366761 42,
328.11709293890918660913059746788734923 66,
292.99969137901795373042034910400371079 82,
358.64341560492670674166738989084525537 16,
299.89866204872814509800070936912772493 83,
360.06173465919302070452688116429182245 22,
336.59441031824153869411229191343829003 77,
256.10753185824891317117975868659458353 91,
324.65521223391515099015440382899818705 27,
331.93806791626160817966041070700096665 07,
304.79958324900117976697100581824660400 67,
323.46169176428438779910852697655707897 79,
289.54595772375215083520622945858464124 93]

Cascade time 167.511
counts: 28, 28

 Iteration 80

Start Generation 1
1 --> 0  target = [12.00000000007233783741013743832048571300,
6.21701250271989743205541921344932052609 7,
485.54908089233055284811137397160123224 66]
one interval r = 23.40850301634816349021121329811762547728 ..
27.67578046434718292132593742327523420096
Time Approximations 0.039.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.5e-38
Equations at solution: [.1e-37, -.25e-37, .10e-35]Solution in 1.041s

Time Plot 0 s.
Exiting SolveHard() after 2.157r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0  target = [12.000000000072337837410137438320485713 00,
6.2170125027198974320554192134493205260 97,
485.54908089233055284811137397160123224 66]
one interval r = 32.628147792089831583843230327291690774 16 ..
36.102483889317976034791311535303521857 42
Time Approximations 0.022.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.6540e-35]Solution in 0.581s

Time Plot 0 s.
Exiting SolveHard() after 4.562r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844830685984748688871961365125 72,
6.5834347217224108658533019349187242611 19,
467.78730595067298482681301908252115948 62]
one interval r = 32.419789556564847804983979184538428657 15 ..
35.851524173584674796781070234958528847 68
Time Approximations 0.022.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.6818e-35]Solution in 0.633s

Time Plot 0 s.
```

```
Exiting SolveHard() after 0.994r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841 64659,
436.91748165013345343471055618431781 09939, none, none,
401.88173903458998729582468306364764708 00, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684483068598474868887196136512572,
6.5834347217224108658533019349187242611 19,
467.78730595067298482681301908252115948 62]
two intervals r = 12.92327160851665897093047536701042314581 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000 or r = 18.39424858008456828444572330990342713649 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000
Time Approximations 0.039.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.2e-38
Equations at solution: [.1e-37, .12e-37, -.1756e-34]Solution in 44.273s

Time Plot 0 s.
Exiting SolveHard() after 49.291r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841 64659,
436.91748165013345343471055618431781 09939,
422.98493396123261536422247450579307 61665, none,
401.88173903458998729582468306364764708 00, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962820765013131112431434500422244,
4.125651796946749593928176326576890069167,
```

```
440.67123064785582711793823785770119832 47]
two intervals r = 14.35659705110656556573165278703476120898 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000 or r = 17.70352613795600855948726157334122485074 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000
Time Approximations 0.059.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [.27e-37, .2e-37, .1143e-34]Solution in 1.344s

Time Plot 0 s.
Exiting SolveHard() after 2.404r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65, none,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962820765013131112431434500422244,
4.12565179694674959392817632657689006916 7,
440.67123064785582711793823785770119832 47]
one interval r = 22.39761154347541299819535186947005905282 ..
27.23722351605229785516063513410732718712
Time Approximations 0.035.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
```

```
in partial time = 1.216 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643940103297239957645290958083 97, rm =
14.378187706584088761154994599577353731 47}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.3e-38
Equations at solution: [-.1e-37, -.53e-37, -.74e-35]Solution in 8.463s

Time Plot 0 s.
Exiting SolveHard() after 13.867r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887862545828103505672465729828 78,
4.0048690819050281118349025453013479827 03,
404.86224500519875728860912996842602435 31]
two intervals r = 16.080110077763790069586122956814114792 13 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000 or r = 16.415798126556467566430161324803951335 21 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000
Time Approximations 0.053.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.17e-37, 0., -.70e-36]Solution in 1.694s

Time Plot 0 s.
Exiting SolveHard() after 6.721r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21, none, none,
358.97362823607306945101368232295529152 82, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888786254582810350567246572982878,
4.0048690819050281118349025453013479827 03,
404.86224500519875728860912996842602435 31]
one interval r = 21.641943993772944800722836333366818925 02 ..
26.763306600387079194951752056704602943 76
Time Approximations 0.047.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [.2e-37, .49e-37, .625e-34]Solution in 1.023s

Time Plot 0 s.
Exiting SolveHard() after 5.951r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95, none,
358.97362823607306945101368232295529152 82, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941836712629139666293890432658509,
5.589637182627440710911568759601007440828,
443.83065883157304014703782854481506565 94]
```

```
one interval r = 22.46725374433463507101234475207606226714 ..
27.27388428348297711300519059411396373014
Time Approximations 0.041.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.07e-37
Equations at solution: [-.1e-37, .107e-36, .18e-35]Solution in 4.889s

Time Plot 0 s.
Exiting SolveHard() after 5.846r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841 64659,
436.91748165013345343471055618431781 09939,
422.98493396123261536422247450579307 61665,
361.52580255821065045165845891305501 06406,
401.88173903458998729582468306364764 70800,
389.59001515930182118931663897930510 81321,
328.46939892694385264775074244818492 59295, none,
358.97362823607306945101368232295529 15282,
398.33147102266519416792941392174309 73928, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941836712629139666293890432658509,
5.589637182627440710911568759601007440828,
443.83065883157304014703782854481506565 94]
one interval r = 32.15575279489683240449673364391019585969 ..
35.50872228716004594595353527372354962812
Time Approximations 0.019.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=3e-38
```

```
Equations at solution: [-.2e-37, .3e-37, -.5921e-35]Solution in 0.489s

Time Plot 0 s.
Exiting SolveHard() after 0.833r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136485574775612053690825462915112,
5.18778357836550716878170492157180396669 3,
408.65773862849377839813448136779450213 68]
one interval r = 21.71840114636054467137159100126378793740 ..
26.81849303522825882381371296459175328237
Time Approximations 0.062.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.05e-37
Equations at solution: [-.1e-37, -.105e-36, .4e-36]Solution in 4.742s

Time Plot 0 s.
Exiting SolveHard() after 5.879r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
```

```
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28, none, none,
361.50888346854988842769540753216913723 35, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136485574775612053690825462915112,
5.18778357836550716878170492157180396669 3,
408.65773862849377839813448136779450213 68]
one interval r = 31.80828598756599484162247973610748563241 ..
35.00011460047022105207126559474522699464
Time Approximations 0.017.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=3e-38
Equations at solution: [-.3e-37, .3e-37, -.36200e-34]Solution in 0.433s

Time Plot 0 s.
Exiting SolveHard() after 0.729r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
371.48387394927732814404566795498398979 54, none,
361.50888346854988842769540753216913723 35, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 1  target = [26.46347110543603000971898413231281100927,
6.196262565481189518906964528879125891956,
385.44474379069701317965491639287524730 04]
one interval r = 31.60836097538646028499903821629320264162 ..
34.66372795605965447227795868806786964926
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., .15230e-34]Solution in 0.544s

Time Plot 0 s.
Exiting SolveHard() after 0.82r=33.8136 in [32.62689490 .. 34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
371.48387394927732814404566795498398979 54, none,
361.50888346854988842769540753216913723 35,
324.67144992331839285042696618537527546 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110543603000971898413231281100927,
6.196262565481189518906964528879125891956,
385.44474379069701317965491639287524730 04]
two intervals r = 16.87563408746783023710171038611678981838 ..
189999999999179573735962742798561179 3/10000000000000000000000000000000
000 or r = 15.55640493786065509203835167291487332994 ..
189999999999179573735962742798561179 3/10000000000000000000000000000000
000
Time Approximations 0.066.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 11.269 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175113045121763042751246912391404, rm
= 2.33653277411398195208727401877568490971 6}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.896e-37, 0., .1311e-34]Solution in 32.82s

Time Plot 0 s.
Exiting SolveHard() after 33.999r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
371.48387394927732814404566795498398979 54,
336.61215840413307657019430208968030013 82,
361.50888346854988842769540753216913723 35,
324.67144992331839285042696618537527546 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874730160598469769371038286843941,
4.88381077964742259699658527910061143055 9,
376.61967855505250776940339949060824959 20]
one interval r = 21.11001304851096607666532252741357218895 ..
26.31784243476972925460436176555066626517
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
```

```
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=5.0e-38
Equations at solution: [-.2e-37, -.50e-37, -.86e-35]Solution in 0.836s

Time Plot 0 s.
Exiting SolveHard() after 5.092r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
371.48387394927732814404566795498398979 54,
336.61215840413307657019430208968030013 82,
361.50888346854988842769540753216913723 35,
324.67144992331839285042696618537527546 99, none,
328.46938512766846890182510074368061299 79, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874730160598469769371038286843941,
4.8838107796474225969965852791006114305 59,
376.61967855505250776940339949060824959 20]
one interval r = 31.53899497712691060214252217634622036180 ..
34.53618386088783642924560991122458745007
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=9.62e-36
Equations at solution: [-.739e-35, .962e-35, -.25922e-34]Solution in
0.469s

Time Plot 0 s.
Exiting SolveHard() after 0.772r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
371.48387394927732814404566795498398979 54,
336.61215840413307657019430208968030013 82,
361.50888346854988842769540753216913723 35,
324.67144992331839285042696618537527546 99, none,
328.46938512766846890182510074368061299 79,
343.81340625096132530463986586978139226 11, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017532180594629229847458436041292,
6.0258135494119958717303091853024534458 98,
351.42702947637975888551090258813608513 65]
one interval r = 31.36230206112970585153021672719675264036 ..
34.17446640604009326775089016365532469915
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .7887e-35]Solution in 0.528s

Time Plot 0 s.
Exiting SolveHard() after 0.752r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
```

```
389.590015159301821189316638979305108132 1,
328.469398926943852647750742448184925929 5,
401.507571571691443204488377096531177376 3,
358.973628236073069451013682322955291528 2,
398.331471022665194167929413921743097392 8,
371.483873949277328144045667954983989795 4,
336.612158404133076570194302089680300138 2,
361.508883468549888427695407532169137233 5,
324.671449923318392850426966185375275469 9, none,
328.469385127668468901825100743680612997 9,
343.813406250961325304639865869781392261 1, none, none,
292.999691375801251534718175037061641649 2, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175321805946292298474584360412 92,
6.025813549411995871730309185302453445898,
351.427029476379758885510902588136085136 5]
two intervals r = 17.981355144490830043541041429800478304 59 ..
189999999999179573735962742798561179 3/100000000000000000000000000000000
000 or r = 13.846080153749638189095841826154192621 30 ..
189999999999179573735962742798561179 3/100000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.526 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713646211244217491405935939168 28, rm
= 2.734500993298587358763252410988159876247}});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [-.36e-37, .1e-37, -.842e-35]Solution in 19.827s

Time Plot 0 s.
Exiting SolveHard() after 24.483r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434930429994165179508307602952148 4,
441.642959723955423611844312680088416465 9,
436.917481650133453434710556184317810993 9,
422.984933961232615364222474505793076166 5,
361.525802558210650451658458913055010640 6,
401.881739034589987295824683063647647080 0,
389.590015159301821189316638979305108132 1,
```

```
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
371.48387394927732814404566795498398979 54,
336.61215840413307657019430208968030013 82,
361.50888346854988842769540753216913723 35,
324.67144992331839285042696618537527546 99,
302.31384313563971195612591368752909440 04,
328.46938512766846890182510074368061299 79,
343.81340625096132530463986586978139226 11, none, none,
292.99969137580125153471817503706164164 92, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418577242067038308609128330920 59,
6.3779438739474797173353692684564156147 29,
423.28832782093390493573709613393651557 90]
one interval r = 31.946618175821370170076601678362525772 91 ..
35.212123086233083314771593582871383582 83
Time Approximations 0.02.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.41151e-34]Solution in 0.631s

Time Plot 0 s.
Exiting SolveHard() after 0.986r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
371.48387394927732814404566795498398979 54,
336.61215840413307657019430208968030013 82,
361.50888346854988842769540753216913723 35,
```

```
324.67144992331839285042696618537527546 99,
302.31384313563971195612591368752909440 04,
328.46938512766846890182510074368061299 79,
343.81340625096132530463986586978139226 11, none, none,
292.99969137580125153471817503706164164 92, none, none,
360.06173465014040438523508118208371515 18, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941857724206703830860912833092059,
6.3779438739474797173353692684564156147 29,
423.28832782093390493573709613393651557 90]
two intervals r = 15.22886702507316482524252115863995285195 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000 or r = 17.12965777001381173603246590778064880457 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000
Time Approximations 0.059.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.311 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541716307054832042023359021614 32, rm
= 2.0640682988243764077000234360260894380 86}});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [.15e-37, 0., -.3861e-34]Solution in 32.072s

Time Plot 0 s.
Exiting SolveHard() after 37.124r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
371.48387394927732814404566795498398979 54,
336.61215840413307657019430208968030013 82,
```

```
361.5088834685498884276954075321691372335,
324.6714499233183928504269661853752754699,
302.3138431356397119561259136875290944004,
328.4693851276684689018251007436806129979,
343.8134062509613253046398658697813922611,
375.7328528763260359184353959240615806844, none,
292.9996913758012515347181750370616416492, none, none,
360.0617346501404043852350811820837151518, none, none, none, none,
none, none, none]

0 --> 2  target = [34.9395323432949980072164445148452273134 2,
4.0035598156035307919816031133856218431 03,
404.4797359318060188770663580085982922157]
two intervals r = 16.0968396638918072573691691338674197567 2 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000 or r = 16.3998864907359193531797722441205769029 4 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.7615110884558244986626642365610012490 6, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.69e-37, -.1e-37, -.3288e-34]Solution in
5.222s

Time Plot 0 s.
Exiting SolveHard() after 6.313r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349304299941651795083076029521484,
441.6429597239554236118443126800884164659,
436.9174816501334534347105561843178109939,
422.9849339612326153642224745057930761665,
361.5258025582106504516584589130550106406,
401.8817390345899872958246830636476470800,
389.5900151593018211893166389793051081321,
328.4693989269438526477507424481849259295,
401.5075715716914432044883770965311773763,
358.9736282360730694510136823229552915282,
398.3314710226651941679294139217430973928,
371.4838739492773281440456679549839897954,
336.6121584041330765701943020896803001382,
361.5088834685498884276954075321691372335,
324.6714499233183928504269661853752754699,
```

```
302.31384313563971195612591368752909440 04,
328.46938512766846890182510074368061299 79,
343.81340625096132530463986586978139226 11,
375.73285287632603591843539592406158068 44, none,
292.99969137580125153471817503706164164 92,
358.64341560418247249846348204947915489 79, none,
360.06173465014040438523508118208371515 18, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234329499800721644451484522731342,
4.0035598156035307919816031133856218431 03,
404.47973593180601887706635800859829221 57]
one interval r = 21.634296299536320328604166581507385745 30 ..
26.75768169887816259419192703230511771204
Time Approximations 0.049.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [0., .26e-37, -.54e-35]Solution in 1.074s

Time Plot 0 s.
Exiting SolveHard() after 2.092r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
371.48387394927732814404566795498398979 54,
336.61215840413307657019430208968030013 82,
361.50888346854988842769540753216913723 35,
324.67144992331839285042696618537527546 99,
302.31384313563971195612591368752909440 04,
328.46938512766846890182510074368061299 79,
343.81340625096132530463986586978139226 11,
375.73285287632603591843539592406158068 44,
```

```
328.1170929357567593269177496201145358699,
292.9996913758012515347181750370616416492,
358.6434156041824724984634820494791548979, none,
360.0617346501404043852350811820837151518, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954461728354941415076686522494895,
6.196177230334509649765176435454759552858,
385.42734025440729079441173996784231364 41]
one interval r = 31.60822049093978129394864129163071300966 ..
34.66347615043567647247544092645599167090
Time Approximations 0.019.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=9e-38
Equations at solution: [-.6e-37, .9e-37, -.2239e-35]Solution in 4.407s

Time Plot 0 s.
Exiting SolveHard() after 4.697r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349304299941651795083076029521484,
441.6429597239554236118443126800884164659,
436.9174816501334534347105561843178109939,
422.9849339612326153642224745057930761665,
361.5258025582106504516584589130550106406,
401.8817390345899872958246830636476470800,
389.5900151593018211893166389793051081321,
328.4693989269438526477507424481849259295,
401.5075715716914432044883770965311773763,
358.9736282360730694510136823229552915282,
398.3314710226651941679294139217430973928,
371.4838739492773281440456679549839897954,
336.6121584041330765701943020896803001382,
361.5088834685498884276954075321691372335,
324.6714499233183928504269661853752754699,
302.3138431356397119561259136875290944004,
328.4693851276684689018251007436806129979,
343.8134062509613253046398658697813922611,
375.7328528763260359184353959240615806844,
328.1170929357567593269177496201145358699,
292.9996913758012515347181750370616416492,
358.6434156041824724984634820494791548979, none,
```

```
360.06173465014040438523508118208371515 18, none, none,
324.65521223306796012658482138898841561 59, none, none, none, none]

0 --> 1  target = [26.46318954461728354941415076686522494895,
6.19617723033450964976517643545475955285 8,
385.42734025440729079441173996784231364 41]
two intervals r = 16.87629600285577488413231906384410748072 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000 or r = 15.55559000634428171717344983192265231633 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000
Time Approximations 0.058.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.633 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852524851955433177900552067214799, rm
= 2.33669042817971609518590311787852598 3632}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.180e-37, 0., -.39e-36]Solution in 33.155s

Time Plot 0 s.
Exiting SolveHard() after 34.314r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
371.48387394927732814404566795498398979 54,
336.61215840413307657019430208968030013 82,
361.50888346854988842769540753216913723 35,
324.67144992331839285042696618537527546 99,
302.31384313563971195612591368752909440 04,
328.46938512766846890182510074368061299 79,
343.81340625096132530463986586978139226 11,
```

```
375.7328528763260359184353959240615806844,
328.1170929357567593269177496201145358699,
292.9996913758012515347181750370616416492,
358.6434156041824724984634820494791548979, none,
360.0617346501404043852350811820837151518,
336.5944103145243825092656075984841047336, none,
324.6552122330679601265848213889884156159, none, none, none, none]

0 --> 2  target = [34.4952266117327151140328661768949907544 9,
3.8971313160703240675518818144813349761 16,
373.7808188502199009939496251290682883586]
two intervals r = 17.29769086189176809955119582983054224218 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000 or r = 14.99436407454935339840010870627752532306 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000
Time Approximations 3.684.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.72e-37, 0., -.1199e-34]Solution in 1.156s

Time Plot 0 s.
Exiting SolveHard() after 6.459r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349304299941651795083076029521484,
441.6429597239554236118443126800884164659,
436.9174816501334534347105561843178109939,
422.9849339612326153642224745057930761665,
361.5258025582106504516584589130550106406,
401.8817390345899872958246830636476470800,
389.5900151593018211893166389793051081321,
328.4693989269438526477507424481849259295,
401.5075715716914432044883770965311773763,
358.9736282360730694510136823229552915282,
398.3314710226651941679294139217430973928,
371.4838739492773281440456679549839897954,
336.6121584041330765701943020896803001382,
361.5088834685498884276954075321691372335,
324.6714499233183928504269661853752754699,
302.3138431356397119561259136875290944004,
328.4693851276684689018251007436806129979,
343.8134062509613253046398658697813922611,
```

```
375.7328528763260359184353959240615806844,
328.1170929357567593269177496201145358699,
292.9996913758012515347181750370616416492,
358.6434156041824724984634820494791548979, none,
360.0617346501404043852350811820837151518,
336.5944103145243825092656075984841047336, none,
324.6552122330679601265848213889884156159,
331.9380679226767859415020918124693449286, none, none, none]

1 --> 2  target = [34.4952266117327151140328661768949907544 9,
3.8971313160703240675518818144813349761 16,
373.7808188502199009939496251290682883586]
one interval r = 21.06068473191532854989411760683495036761 ..
26.26979834299758539953034617502556630994
Time Approximations 0.037.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=4e-38
Equations at solution: [.2e-37, .4e-37, .487e-34]Solution in 0.77s

Time Plot 0 s.
Exiting SolveHard() after 1.517r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349304299941651795083076029521484,
441.6429597239554236118443126800884164659,
436.9174816501334534347105561843178109939,
422.9849339612326153642224745057930761665,
361.5258025582106504516584589130550106406,
401.8817390345899872958246830636476470800,
389.5900151593018211893166389793051081321,
328.4693989269438526477507424481849259295,
401.5075715716914432044883770965311773763,
358.9736282360730694510136823229552915282,
398.3314710226651941679294139217430973928,
371.4838739492773281440456679549839897954,
336.6121584041330765701943020896803001382,
361.5088834685498884276954075321691372335,
324.6714499233183928504269661853752754699,
302.3138431356397119561259136875290944004,
328.4693851276684689018251007436806129979,
343.8134062509613253046398658697813922611,
375.7328528763260359184353959240615806844,
```

```
328.11709293575675932691774962011453586 99,
292.99969137580125153471817503706164164 92,
358.64341560418247249846348204947915489 79,
299.89866205291836254587399640113730694 23,
360.06173465014040438523508118208371515 18,
336.59441031452438250926560759848410473 36, none,
324.65521223306796012658482138898841561 59,
331.93806792267678594150209181246934492 86, none, none, none]

0 --> 2  target = [33.81362495406297840307443237260255875416,
3.72564899367341699788610435047872198240 5,
325.89209972569714833071702671704914886 01]
two intervals r = 18.552270489978180963408025427428023001 06 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000 or r = 12.491969357693952797873612547953888039 56 ..
189999999999179573735962742798561179 3/1000000000000000000000000000000000
000
Time Approximations 0.04.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [-.121e-36, .6e-37, -.2684e-34]Solution in 1.14s

Time Plot 0 s.
Exiting SolveHard() after 6.439r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
371.48387394927732814404566795498398979 54,
336.61215840413307657019430208968030013 82,
361.50888346854988842769540753216913723 35,
324.67144992331839285042696618537527546 99,
302.31384313563971195612591368752909440 04,
328.46938512766846890182510074368061299 79,
```

```
343.8134062509613253046398658697813922611,
375.7328528763260359184353959240615806844,
328.1170929357567593269177496201145358699,
292.9996913758012515347181750370616416492,
358.6434156041824724984634820494791548979,
299.8986620529183625458739964011373069423,
360.0617346501404043852350811820837151518,
336.5944103145243825092656075984841047336, none,
324.6552122330679601265848213889884156159,
331.9380679226767859415020918124693449286, none, none,
289.5459577261261391525780488873221513734]

1 --> 2  target = [33.8136249540629784030744323726025587541 6,
3.7256489936734169978861043504787219824 05,
325.8920997256971483307170267170491488601]
one interval r = 20.37468935077254906405359172573872805545 ..
25.37892165297492908405753465914021397516
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=1e-38
Equations at solution: [.2e-37, .1e-37, -.124e-34]Solution in 4.211s

Time Plot 0 s.
Exiting SolveHard() after 4.763r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349304299941651795083076029521484,
441.6429597239554236118443126800884164659,
436.9174816501334534347105561843178109939,
422.9849339612326153642224745057930761665,
361.5258025582106504516584589130550106406,
401.8817390345899872958246830636476470800,
389.5900151593018211893166389793051081321,
328.4693989269438526477507424481849259295,
401.5075715716914432044883770965311773763,
358.9736282360730694510136823229552915282,
398.3314710226651941679294139217430973928,
371.4838739492773281440456679549839897954,
336.6121584041330765701943020896803001382,
361.5088834685498884276954075321691372335,
324.6714499233183928504269661853752754699,
302.3138431356397119561259136875290944004,
```

```
328.46938512766846890182510074368061299 79,
343.81340625096132530463986586978139226 11,
375.73285287632603591843539592406158068 44,
328.11709293575675932691774962011453586 99,
292.99969137580125153471817503706164164 92,
358.64341560418247249846348204947915489 79,
299.89866205291836254587399640113730694 23,
360.06173465014040438523508118208371515 18,
336.59441031452438250926560759848410473 36,
256.10753185808728329652497230429775511 25,
324.65521223306796012658482138898841561 59,
331.93806792267678594150209181246934492 86, none, none,
289.54595772612613915257804888732215137 34]

1 --> 0  target = [17.93041369717003788250445807764241854270,
4.68650870174422759756915074842655494891 2,
353.30541094211200374266766139029104936 31]
one interval r = 20.73150479055044992295381088735369975016 ..
25.90675353512546582540284092251577731720
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.5e-38
Equations at solution: [.1e-37, .25e-37, .134e-34]Solution in 0.67s

Time Plot 0 s.
Exiting SolveHard() after 1.416r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
371.48387394927732814404566795498398979 54,
336.61215840413307657019430208968030013 82,
361.50888346854988842769540753216913723 35,
```

```
324.67144992331839285042696618537527546 99,
302.31384313563971195612591368752909440 04,
328.46938512766846890182510074368061299 79,
343.81340625096132530463986586978139226 11,
375.73285287632603591843539592406158068 44,
328.11709293575675932691774962011453586 99,
292.99969137580125153471817503706164164 92,
358.64341560418247249846348204947915489 79,
299.89866205291836254587399640113730694 23,
360.06173465014040438523508118208371515 18,
336.59441031452438250926560759848410473 36,
256.10753185808728329652497230429775511 25,
324.65521223306796012658482138898841561 59,
331.93806792267678594150209181246934492 86,
304.79958324265215140624703516587230496 34, none,
289.54595772612613915257804888732215137 34]

2 --> 0  target = [17.93041369717003788250445807764241854270,
4.68650870174422759756915074842655494891 2,
353.30541094211200374266766139029104936 31]
one interval r = 31.37435486991779720686978347971265217579 ..
34.20127520015885428820480158724585529725
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, -.11225e-34]Solution in 0.348s

Time Plot 0 s.
Exiting SolveHard() after 0.606r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493042999416517950830760295214 84,
441.64295972395542361184431268008841646 59,
436.91748165013345343471055618431781099 39,
422.98493396123261536422247450579307616 65,
361.52580255821065045165845891305501064 06,
401.88173903458998729582468306364764708 00,
389.59001515930182118931663897930510813 21,
328.46939892694385264775074244818492592 95,
401.50757157169144320448837709653117737 63,
358.97362823607306945101368232295529152 82,
398.33147102266519416792941392174309739 28,
```

```
371.48387394927732814404566795498398979 54,
336.61215840413307657019430208968030013 82,
361.50888346854988842769540753216913723 35,
324.67144992331839285042696618537527546 99,
302.31384313563971195612591368752909440 04,
328.46938512766846890182510074368061299 79,
343.81340625096132530463986586978139226 11,
375.73285287632603591843539592406158068 44,
328.11709293575675932691774962011453586 99,
292.99969137580125153471817503706164164 92,
358.64341560418247249846348204947915489 79,
299.89866205291836254587399640113730694 23,
360.06173465014040438523508118208371515 18,
336.59441031452438250926560759848410473 36,
256.10753185808728329652497230429775511 25,
324.65521223306796012658482138898841561 59,
331.93806792267678594150209181246934492 86,
304.79958324265215140624703516587230496 34,
323.46169176416391579283837643420813352 87,
289.54595772612613915257804888732215137 34]

Cascade time 271.946
counts: 28, 28

 Iteration 81

Start Generation 1
1 --> 0  target = [11.99999999995062404080495224885573311300,
6.21701250308414475142788660428169190958 6,
485.54908089117433758797999366353028119 86]
```

"Imaginary part neglected: ", $1.8899423791497374626893977862580198137 04 \times 10^{-17}$

```
one interval r = 23.40850301640100724237907862112039332964 ..
27.67578046420581451482442232242682489405
Time Approximations 0.042.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.32e-37
Equations at solution: [.5e-37, -.132e-36, -.21e-35]Solution in 4.638s

Time Plot 0 s.
Exiting SolveHard() after 5.793r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999950624040804952248855733113 00,
6.2170125030841447514278866042816919095 86,
485.54908089117433758797999366353028119 86]
one interval r = 32.628147792023006773280863700911569432 92 ..
36.102483889229649402858618347197885462 03
Time Approximations 0.023.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., .11537e-34]Solution in 0.61s

Time Plot 0 s.
Exiting SolveHard() after 1.023r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844731561461278820354289778145 28,
6.5834347214423823244399214145547352468 81,
467.78730595405418549619846629663208489 11]
one interval r = 32.419789556551447535563480198108580493 98 ..
35.851524173562393957998832315516971943 50
Time Approximations 0.023.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
```

```
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., -.11120e-34]Solution in 0.637s

Time Plot 0 s.
Exiting SolveHard() after 1.009r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08, none, none,
401.88173903990702391684975357690343676 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844731561461278820354289778145 28,
6.5834347214423823244399214145547352468 81,
467.78730595405418549619846629663208489 11]
two intervals r = 12.923271608159537011311734071742141532 07 ..
189999999998743798972281744636191701 97/1000000000000000000000000000000
00000 or r = 18.394248580131135168353637223897958091 45 ..
189999999998743798972281744636191701 97/1000000000000000000000000000000
00000
Time Approximations 0.043.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=6.5e-38
Equations at solution: [-.5e-37, -.65e-37, .86e-36]Solution in 44.823s

Time Plot 0 s.
Exiting SolveHard() after 50.026r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
```

```
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70, none,
401.88173903990702391684975357690343676 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962804838348519240929005874401243,
4.12565179677677839323082224455963077772 7,
440.67123064150542069162186888020097263 97]
two intervals r = 14.35659705127272367035527006060406166605 ..
189999999998743798972281744636191701 97/10000000000000000000000000000000
00000 or r = 17.70352613774534762214564966293727305449 ..
189999999998743798972281744636191701 97/10000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [.56e-37, .3e-37, -.680e-35]Solution in 4.641s

Time Plot 0 s.
Exiting SolveHard() after 5.745r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70, none,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962804838348519240929005874401243,
4.12565179677677839323082224455963077772 7,
440.67123064150542069162186888020097263 97]
```

"Imaginary part neglected: ", $1.8899423791497374626893977862580198137 04 \times 10^{-17}$

```
one interval r = 22.39761154349157788986901688956054606782 ..
27.23722351585320479344479180482333179144
Time Approximations 0.043.
```

```
hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.29 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064361619727341781902195794162672, rm =
14.37818770030631275961788637594958798626}});
Accepted {r=26.4635, rm=16.5329} with Delta=7.9e-38
Equations at solution: [0., .79e-37, -.96e-35]Solution in 12.111s

Time Plot 0 s.
Exiting SolveHard() after 13.039r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888787292716196001949817934434360,
4.00486908177354523177650033914643651680 1,
404.86224501036806099647721734284159395 07]
two intervals r = 16.08011007740456280140363014364144119366 ..
189999999998743798972281744636191701 97/10000000000000000000000000000000
00000 or r = 16.41579812675187043643364110486392887189 ..
189999999998743798972281744636191701 97/10000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., -.5723e-34]Solution in 5.683s

Time Plot 0 s.
Exiting SolveHard() after 6.778r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65, none, none,
358.97362823314971229000555626066879372 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888787292716196001949817934434360,
4.00486908177354523177650033914643651680 1,
404.86224501036806099647721734284159395 07]
```

"Imaginary part neglected: ", $1.8899423791497374626893977862580198137 04 \times 10^{-17}$

```
one interval r = 21.641943994095592863071232457850997154 24 ..
26.763306600353974745817063443541753900 29
Time Approximations 0.055.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, -.360e-34]Solution in 1.09s

Time Plot 0 s.
Exiting SolveHard() after 2.1r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
```

```
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88, none,
358.97362823314971229000555626066879372 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941737450374383958450253674325550,
5.58963718320055109429856001604818584 4392,
443.83065884746412868007855692807082693 02]
```

"Imaginary part neglected: ", $1.8899423791497374626893977862580198137 04 \times 10^{-17}$

```
one interval r = 22.46725374483601632843538553167913662206 ..
27.27388428353970092541528369505512036540
Time Approximations 0.042.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.4e-38
Equations at solution: [.1e-37, -.54e-37, -.8e-36]Solution in 0.967s

Time Plot 0 s.
Exiting SolveHard() after 5.753r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88, none,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941737450374383958450253674325550,
5.58963718320055109429856001604818584 4392,
```

```
443.83065884746412868007855692807082693 02]
one interval r = 32.15575279501395670321445072327089549446 ..
35.50872228732065722041930149196466786617
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.3137e-35]Solution in 0.469s

Time Plot 0 s.
Exiting SolveHard() after 0.852r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984 44394,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136529095003201327063745184916085,
5.18778357859766438106623258878476202073 3,
408.65773861574845793292846754263145433 46]

"Imaginary part neglected: ", 1.8899423791497374626893977862580198137 04 × 10^-17
one interval r = 21.71840114631463835637195586493032024352 ..
26.81849303493377892526470339842957429925
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
```

```
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [0., .79e-37, .116e-34]Solution in 0.999s

Time Plot 0 s.
Exiting SolveHard() after 6.152r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99, none, none,
361.50888346472964994850524683500165645 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136529095003201327063745184916085,
5.18778357859766438106623258878476202073 3,
408.65773861574845793292846754263145433 46]
one interval r = 31.80828598739971562374884976216515413366 ..
35.00011460022051361156169931168581760373
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, -.30299e-34]Solution in 0.405s

Time Plot 0 s.
Exiting SolveHard() after 0.729r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89, none,
361.50888346472964994850524683500165645 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110528133612667904743831739601789,
6.1962625651634019934398219200622128372 51,
385.44474378713673568503541859062837171 54]
one interval r = 31.608360975308572088145467541571920247 94 ..
34.663727955945200491900884250795635335 45
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, -.11901e-34]Solution in 0.572s

Time Plot 0 s.
Exiting SolveHard() after 0.833r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
```

```
358.9736282331497122900055562606687937230,
398.3314710475836936104797146652696383799,
371.4838739301383356696019832074371262489, none,
361.5088834647296499485052468350016564544,
324.6714499222773866453145045246861869277, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110528133612667904743831739601789,
6.196262565163401993439821920062212837251,
385.44474378713673568503541859062837171 54]
two intervals r = 16.875634087486841087544124895207857421 30 ..
18999999999874379897228174463619170197/10000000000000000000000000000000
00000 or r = 15.556404937671657501506351920897342888 94 ..
18999999999874379897228174463619170197/10000000000000000000000000000000
00000
Time Approximations 0.069.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.621 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175112270014868334145587457001726, rm
= 2.33653277410455138112064863692066193893 7}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.178e-37, 0., -.323e-35]Solution in 32.676s

Time Plot 0 s.
Exiting SolveHard() after 37.171r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.9736282331497122900055562606687937230,
398.3314710475836936104797146652696383799,
371.4838739301383356696019832074371262489,
336.61215841248910403734180998441001406 99,
361.5088834647296499485052468350016564544,
```

```
324.67144992227738664531450452468618692 77, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874729399870351437956247356225363,
4.8838107799812356522454286120132310427 36,
376.61967855217107816639126366502997298 44]
```

"Imaginary part neglected: ", $1.8899423791497374626893977862580198137 04 \times 10^{-17}$

```
one interval r = 21.11001304872784190956886583294012862663 ..
26.31784243463042617788456346314219899582
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.26e-35]Solution in 0.86s

Time Plot 0 s.
Exiting SolveHard() after 1.59r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77, none,
328.46938513454283204925710074732747581 59, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874729399870351437956247356225363,
4.8838107799812356522454286120132310427 36,
376.61967855217107816639126366502997298 44]
one interval r = 31.53899497705594685826395877155441181486 ..
```

```
34.53618386078402284712799393726261469605
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=6.35e-36
Equations at solution: [-.487e-35, .635e-35, -.20118e-34]Solution in
0.5s

Time Plot 0 s.
Exiting SolveHard() after 0.783r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77, none,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017538898135768502755895092794253,
6.0258135491478062694939339766668521568 13,
351.42702948378283037123082511239184670 00]
one interval r = 31.36230206112841466100432229030223037869 ..
34.17446640608543605015598173205911995627
Time Approximations 0.015.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
```

```
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .7345e-35]Solution in 0.518s

Time Plot 0 s.
Exiting SolveHard() after 4.268r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77, none,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03, none, none,
292.99969138501226045485768239881018676 10, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017538898135768502755895092794253,
6.0258135491478062694939339766668521568 13,
351.42702948378283037123082511239184670 00]
two intervals r = 17.98135514420303605695763593896810254644 ..
189999999998743798972281744463619170197/10000000000000000000000000000000
00000 or r = 13.84608015410769477632841321932549558917 ..
189999999998743798972281744463619170197/10000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.717 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.9135707135367151861979416632392475273 3, rm
= 2.7345009931205179767482296287770044064 15}});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [-.71e-37, .2e-37, .1170e-34]Solution in 20.241s

Time Plot 0 s.
Exiting SolveHard() after 21.328r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03, none, none,
292.99969138501226045485768239881018676 10, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941880177128178983398016077102883,
6.37794387377152225030607312038660098243 2,
423.28832784688511012986236780877084115 60]
one interval r = 31.94661817602450588591448079993068844988 ..
35.21212308654257149897673567194338624501
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.37139e-34]Solution in 0.655s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.012r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03, none, none,
292.99969138501226045485768239881018676 10, none, none,
360.06173467668122368344928732028613806 98, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941880177128178983398016077102883,
6.37794387377152225030607312038660098243 2,
423.28832784688511012986236780877084115 60]
two intervals r = 15.22886702366761785596471506290203341611 ..
189999999998743798972281744463619170197/10000000000000000000000000000000
00000 or r = 17.12965777092584069009624195236768629457 ..
189999999998743798972281744463619170197/10000000000000000000000000000000
00000
Time Approximations 0.064.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.423 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370540793867316034074879635567742 37, rm
= 2.0640682986493356554669225288551780618 34}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
```

```
Equations at solution: [-.31e-37, -.1e-37, .983e-35]Solution in 32.889s

Time Plot 0 s.
Exiting SolveHard() after 37.765r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03,
375.73285291578657631710689331157525556 72, none,
292.99969138501226045485768239881018676 10, none, none,
360.06173467668122368344928732028613806 98, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343323471619176074994425049292 91,
4.0035598154763137937142352696514181228 49,
404.47973593822078220979987776161683783 59]
two intervals r = 16.096839663479049991043624505136354902 43 ..
189999999998743798972281744636191701 97/100000000000000000000000000000000
00000 or r = 16.399886490983743641896164932860269394 02 ..
189999999998743798972281744636191701 97/100000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.32e-37, -.1e-37, .1188e-34]Solution in 5.482s

Time Plot 0 s.
```

```
Exiting SolveHard() after 6.612r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03,
375.73285291578657631710689331157525556 72, none,
292.99969138501226045485768239881018676 10,
358.64341560233179403458049330643832054 26, none,
360.06173467668122368344928732028613806 98, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343323471619176074994425049292 91,
4.0035598154763137937142352696514181228 49,
404.47973593822078220979987776161683783 59]

"Imaginary part neglected: ", 1.8899423791497374626893977862580198137 04 × 10^-17
one interval r = 21.634296299884377568862923405349024111 15 ..
26.757681698863758671571265060185501845 70
Time Approximations 0.058.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=4.9e-38
Equations at solution: [.2e-37, .49e-37, .466e-34]Solution in 4.853s

Time Plot 0 s.
Exiting SolveHard() after 5.871r=25.8653 in [23.83864811 ..
26.75768170]
```

```
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03,
375.73285291578657631710689331157525556 72,
328.11709294377749729192848152201218807 16,
292.99969138501226045485768239881018676 10,
358.64341560233179403458049330643832054 26, none,
360.06173467668122368344928732028613806 98, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954446199489283521771291622989202,
6.19617723001653832831320461703151054739 3,
385.42734025080980366258028706941019485 86]
one interval r = 31.60822049086159614871383422135240978447 ..
34.66347615032068575038031932396500170005
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., .30477e-34]Solution in 0.6s

Time Plot 0 s.
Exiting SolveHard() after 0.906r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03,
375.73285291578657631710689331157525556 72,
328.11709294377749729192848152201218807 16,
292.99969138501226045485768239881018676 10,
358.64341560233179403458049330643832054 26, none,
360.06173467668122368344928732028613806 98, none, none,
324.65521223199227218316293250643556916 07, none, none, none, none]

0 --> 1  target = [26.46318954446199489283521771291622989202,
6.19617723001653832831320461703151054739 3,
385.42734025080980366258028706941019485 86]
two intervals r = 16.87629600287619601778305024781798088702 ..
189999999998743798972281744463619170197/10000000000000000000000000000000
00000 or r = 15.55559000615352275884181919723264325491 ..
189999999998743798972281744463619170197/10000000000000000000000000000000
00000
Time Approximations 0.064.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 11.211 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852524145791904227468934834288409, rm
= 2.33669042817062095617413710728414542053 2}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.179e-37, 0., -.1313e-34]Solution in 32.756s

Time Plot 0 s.
```

```
Exiting SolveHard() after 33.946r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03,
375.73285291578657631710689331157525556 72,
328.11709294377749729192848152201218807 16,
292.99969138501226045485768239881018676 10,
358.64341560233179403458049330643832054 26, none,
360.06173467668122368344928732028613806 98,
336.59441032284229504546876204816687108 18, none,
324.65521223199227218316293250643556916 07, none, none, none, none]

0 --> 2  target = [34.49522661138461203626353196155931314690,
3.897131315851225705877776451651978085489,
373.78081883037926779655355245579491876 21]
two intervals r = 17.29769086246561891676319542481257025367 ..
18999999999874379897228174463619170197/10000000000000000000000000000000
00000 or r = 14.99436407354271208785780302011336741704 ..
18999999999874379897228174463619170197/10000000000000000000000000000000
00000
Time Approximations 0.091.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.54e-37, 0., .1692e-34]Solution in 1.214s

Time Plot 0 s.
```

```
Exiting SolveHard() after 6.287r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03,
375.73285291578657631710689331157525556 72,
328.11709294377749729192848152201218807 16,
292.99969138501226045485768239881018676 10,
358.64341560233179403458049330643832054 26, none,
360.06173467668122368344928732028613806 98,
336.59441032284229504546876204816687108 18, none,
324.65521223199227218316293250643556916 07,
331.93806789771151057854427446846918674 53, none, none, none]

1 --> 2  target = [34.49522661138461203626353196155931314690,
3.89713131585122570587777645165197808548 9,
373.78081883037926779655355245579491876 21]
```

"Imaginary part neglected: ", $1.8899423791497374626893977862580198137 04 \times 10^{-17}$

```
one interval r = 21.06068473184551019619633516678803773747 ..
26.26979834257109676379125384386221486011
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [.1e-37, .3e-37, .10e-35]Solution in 0.778s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.338r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03,
375.73285291578657631710689331157525556 72,
328.11709294377749729192848152201218807 16,
292.99969138501226045485768239881018676 10,
358.64341560233179403458049330643832054 26,
299.89866203683756525966144412921142557 75,
360.06173467668122368344928732028613806 98,
336.59441032284229504546876204816687108 18, none,
324.65521223199227218316293250643556916 07,
331.93806789771151057854427446846918674 53, none, none, none]

0 --> 2  target = [33.81362495398592958959830793414530147104,
3.72564899351869564042150073024253578 5689,
325.89209972436254690430944048700004114 64]
two intervals r = 18.552270489930503501640928284301520391 01 ..
189999999998743798972281744463619170197/10000000000000000000000000000000
00000 or r = 12.491969357578884898240244998588065688 71 ..
189999999998743798972281744463619170197/10000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=18.8546, rm=16.5667} with Delta=1.2e-37
Equations at solution: [-.296e-36, .12e-36, -.718e-35]Solution in
4.632s

Time Plot 0 s.
Exiting SolveHard() after 6.138r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03,
375.73285291578657631710689331157525556 72,
328.11709294377749729192848152201218807 16,
292.99969138501226045485768239881018676 10,
358.64341560233179403458049330643832054 26,
299.89866203683756525966144412921142557 75,
360.06173467668122368344928732028613806 98,
336.59441032284229504546876204816687108 18, none,
324.65521223199227218316293250643556916 07,
331.93806789771151057854427446846918674 53, none, none,
289.54595771745244499924713460813413340 56]

1 --> 2  target = [33.81362495398592958959830793414530147104,
3.72564899351869564042150073024253578 5 89,
325.89209972436254690430944048700004114 64]
```

"Imaginary part neglected: ", $1.8899423791497374626893977862580198137 04 \times 10^{-17}$

```
one interval r = 20.37468935109727092718233895814507253735 ..
25.37892165290360042100329986037519966430
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
```

```
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=1e-38
Equations at solution: [.2e-37, .1e-37, .109e-34]Solution in 0.588s

Time Plot 0 s.
Exiting SolveHard() after 1.14r=24.3395 in [22.07732228 .. 25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03,
375.73285291578657631710689331157525556 72,
328.11709294377749729192848152201218807 16,
292.99969138501226045485768239881018676 10,
358.64341560233179403458049330643832054 26,
299.89866203683756525966144412921142557 75,
360.06173467668122368344928732028613806 98,
336.59441032284229504546876204816687108 18,
256.10753185898273928259024817634595751 30,
324.65521223199227218316293250643556916 07,
331.93806789771151057854427446846918674 53, none, none,
289.54595771745244499924713460813413340 56]

1 --> 0  target = [17.93041369683594120609137962435838195890,
4.68650870217391696444767918192894524442 3,
353.30541095097735510784274980040848807 58]
```

"Imaginary part neglected: ", $1.8899423791497374626893977862580198137 04 \times 10^{-17}$

```
one interval r = 20.73150479098574347703837868499651105755 ..
25.90675353521697654906619417615927268720
Time Approximations 0.037.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
```

```
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=7.1e-38
Equations at solution: [-.2e-37, -.71e-37, -.117e-34]Solution in 0.676s

Time Plot 0 s.
Exiting SolveHard() after 1.391r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03,
375.73285291578657631710689331157525556 72,
328.11709294377749729192848152201218807 16,
292.99969138501226045485768239881018676 10,
358.64341560233179403458049330643832054 26,
299.89866203683756525966144412921142557 75,
360.06173467668122368344928732028613806 98,
336.59441032284229504546876204816687108 18,
256.10753185898273928259024817634595751 30,
324.65521223199227218316293250643556916 07,
331.93806789771151057854427446846918674 53,
304.79958326147860323822630784749017458 05, none,
289.54595771745244499924713460813413340 56]

2 --> 0  target = [17.93041369683594120609137962435838195890,
4.68650870217391696444767918192894524442 3,
353.30541095097735510784274980040848807 58]
one interval r = 31.37435486992665542660750859729173894899 ..
34.20127520022505831766654971943924117680
Time Approximations 0.016.
```

```
hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
Equations at solution: [-.3e-37, .6e-37, .23006e-34]Solution in 0.377s

Time Plot 0 s.
Exiting SolveHard() after 4.589r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492911321458672191399518984443 94,
441.64295972690671599431735977001651730 91,
436.91748164417799587903719600740308183 08,
422.98493397605731356186688664885403543 70,
361.52580255442658099574241423770734628 94,
401.88173903990702391684975357690343676 33,
389.59001514697858942493827616033852499 65,
328.46939893381757536916462733569320880 88,
401.50757157822676615973616847252540680 50,
358.97362823314971229000555626066879372 30,
398.33147104758369361047971466526963837 99,
371.48387393013833566960198320743712624 89,
336.61215841248910403734180998441001406 99,
361.50888346472964994850524683500165645 44,
324.67144992227738664531450452468618692 77,
302.31384315452408900734773234516082302 74,
328.46938513454283204925710074732747581 59,
343.81340623948357125581343737428393144 03,
375.73285291578657631710689331157525556 72,
328.11709294377749729192848152201218807 16,
292.99969138501226045485768239881018676 10,
358.64341560233179403458049330643832054 26,
299.89866203683756525966144412921142557 75,
360.06173467668122368344928732028613806 98,
336.59441032284229504546876204816687108 18,
256.10753185898273928259024817634595751 30,
324.65521223199227218316293250643556916 07,
331.93806789771151057854427446846918674 53,
304.79958326147860323822630784749017458 05,
323.46169176259743258537014905726797475 37,
289.54595771745244499924713460813413340 56]

Cascade time 276.032
counts: 28, 28
```

```
 Iteration 82

Start Generation 1
1 --> 0  target = [12.00000000014814297579001719095612907900,
6.217012503047994505537967630849962592856,
485.549080902236278461020129950850006612 9]
one interval r = 23.40850301660156917551021179755082644744 ..
27.67578046431414281527553443540231126198
Time Approximations 0.041.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.59e-37
Equations at solution: [.5e-37, -.159e-36, -.20e-35]Solution in 1.022s

Time Plot 0 s.
Exiting SolveHard() after 2.226r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000014814297579001719095612907900,
6.217012503047994505537967630849962592856,
485.549080902236278461020129950850006612 9]
one interval r = 32.62814779224039354857678662528733076186 ..
36.10248388949454953910017987991623796185
Time Approximations 0.021.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=6e-38
Equations at solution: [.8e-37, -.6e-37, .62e-35]Solution in 0.6s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.023r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684483719215921053531635289519872,
6.58343472173427340080857704315788136 0312,
467.78730596287263770088231515859965326 12]
one interval r = 32.41978955673581035602277590888975037783 ..
35.85152417379501602443771122976479184133
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=4e-38
Equations at solution: [.5e-37, -.4e-37, -.258e-34]Solution in 0.642s

Time Plot 0 s.
Exiting SolveHard() after 1.008r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43, none, none,
401.88173904769091008426183167561263592 98, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684483719215921053531635289519872,
6.58343472173427340080857704315788136 0312,
467.78730596287263770088231515859965326 12]
```

"Imaginary part neglected: ", $1.10311211491137042826364785393053982259 0 \times 10^{-17}$

```
two intervals r = 12.92327160847159128239813328243549389511 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000 or r = 18.39424858053151374424982906308020365571 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000
Time Approximations 0.046.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.3e-38
Equations at solution: [.3e-37, .33e-37, -.8128e-35]Solution in 45.166s

Time Plot 0 s.
Exiting SolveHard() after 50.133r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39, none,
401.88173904769091008426183167561263592 98, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962835068082678816471942240200128,
4.12565179704947057096185177740382997964 0,
440.67123065528153236733038369516017397 13]
```

"Imaginary part neglected: ", $1.10311211491137042826364785393053982259 0 \times 10^{-17}$

```
two intervals r = 14.35659705131993294492349689604266784627 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000 or r = 17.70352613834025074809766878266155967797 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000
Time Approximations 0.054.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
```

```
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.15e-37, .1e-37, -.23660e-34]Solution in 5.11s

Time Plot 0 s.
Exiting SolveHard() after 6.231r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588199 0,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39, none,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962835068082678816471942240200128,
4.1256517970494705709618517774038299796 40,
440.67123065528153236733038369516017397 13]
one interval r = 22.397611543712839536047981704549952718 31 ..
27.237223516037213422532967122462398427 99
Time Approximations 0.037.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.28 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064400151531390502433325434960918, rm =
14.37818770774739072649780169793961628353}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., .26e-37, -.111e-34]Solution in 12.953s

Time Plot 0 s.
Exiting SolveHard() after 13.856r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888808972738935347564909770611148,
4.0048690820285902029422380739434650536 77,
404.86224501845256102258223304539253741 81]
```

"Imaginary part neglected: ", $1.10311211491137042826364785393053982259 0 \times 10^{-17}$

```
two intervals r = 16.08011007772563199060420118767885326425 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000 or r = 16.41579812723277463240116848674759150187 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000
Time Approximations 0.055.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., -.2173e-35]Solution in 1.476s

Time Plot 0 s.
Exiting SolveHard() after 2.579r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16, none, none,
358.97362824543815285135597998591097582 16, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888089727389353475649097706111 48,
4.0048690820285902029422380739434650536 77,
404.86224501845256102258223304539253741 81]
one interval r = 21.641943994150467043725786208093092037 94 ..
26.763306600490903001118842076403669291 52
Time Approximations 0.05.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, .54e-35]Solution in 1.052s

Time Plot 0 s.
Exiting SolveHard() after 5.902r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44, none,
358.97362824543815285135597998591097582 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589418004608427728100876272556033 35,
5.5896371830693430935499336365481978695 33,
443.83065885004120819137053124770553882 71]
one interval r = 22.467253744813575207197190635028343288 00 ..
27.273884283591735633407234095687042989 27
Time Approximations 0.043.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
```

```
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .115e-34]Solution in 4.672s

Time Plot 0 s.
Exiting SolveHard() after 5.651r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44, none,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941800460842772810087627255603335,
5.5896371830693430935499336365481978695 33,
443.83065885004120819137053124770553882 71]
one interval r = 32.15575279512340258676108945422415468211 ..
35.50872228746195187166882250004869886350
Time Approximations 0.022.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.76e-35]Solution in 0.517s

Time Plot 0 s.
Exiting SolveHard() after 0.895r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136520615987349626783184086325866,
5.1877835786447124826962058985933945897 81,
408.65773863293315084810897660747862301 85]
one interval r = 21.71840114655890551747458273144305920485 ..
26.81849303519982948205678203382842489259
Time Approximations 0.06.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=5.3e-38
Equations at solution: [.1e-37, .53e-37, -.84e-35]Solution in 4.581s

Time Plot 0 s.
Exiting SolveHard() after 5.695r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90, none, none,
361.50888347724208784220379005965801358 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0  target = [15.91193136520615987349626783184086325866,
5.1877835786447124826962058985933945897 81,
408.65773863293315084810897660747862301 85]
one interval r = 31.80828598763735596540864217147015040403 ..
35.00011460056969171703204318181402424589
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=1.0e-37
Equations at solution: [.9e-37, -.10e-36, -.44e-35]Solution in 0.455s

Time Plot 0 s.
Exiting SolveHard() after 0.765r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13, none,
361.50888347724208784220379005965801358 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110549962258132825358476699246211,
6.19626256547848417974166674607044118006 0,
385.44474379974016585458006753404979693 15]
one interval r = 31.60836097549008228251099755130983191464 ..
34.66372795622497371078398735736796505969
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
```

```
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, -.120e-34]Solution in 0.585s

Time Plot 0 s.
Exiting SolveHard() after 0.867r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13, none,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110549962258132825358476699246211,
6.19626256547848417974166674607044118006 0,
385.44474379974016585458006753404979693 15]

"Imaginary part neglected: ", 1.10311211491137042826364785393053982259 0 × 10^-17
two intervals r = 16.87563408762685316692244228009448951493 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000 or r = 15.55640493839543730923905342846793631221 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000
Time Approximations 0.06.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
```

```
Equations at solution: [-.538e-37, 0., .11295e-34]Solution in 1.16s

Time Plot 0 s.
Exiting SolveHard() after 6.476r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874745310563357535836493427515356,
4.88381077998149109039726007919929937349 8,
376.61967856450061418234213903437289951 01]
one interval r = 21.11001304881774842892295424315646413172 ..
26.31784243485014881394102541000834256558
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, -.143e-34]Solution in 4.562s

Time Plot 0 s.
Exiting SolveHard() after 5.284r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
```

```
441.642959735756012748170316682835889199 0,
436.917481657537198018304165726499479424 3,
422.984933979145722011193173207459971643 9,
361.525802566937473623975837930509210846 1,
401.881739047690910084261831675612635929 8,
389.590015163903061377408568296920914211 6,
328.469398941035954320028275908857427074 4,
401.507571585475211916778130692647054168 9,
358.973628245438152851355979985910975821 6,
398.331471045442124527698689069443564009 0,
371.483873950524155628241437734643121391 3,
336.612158419068405332419579851263872680 5,
361.508883477242087842203790059658013584 4,
324.671449933496761425861344807301591323 6, none,
328.469385141760417716660126107461939911 1, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747453105633575358364934275153 56,
4.883810779981491090397260079199299373498,
376.619678564500614182342139034372899510 1]
one interval r = 31.538994977229711065635736079640725928 45 ..
34.536183861058464608380152591584418759 89
Time Approximations 0.016.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=8.74e-36
Equations at solution: [.670e-35, -.874e-35, -.4e-36]Solution in 0.511s

Time Plot 0 s.
Exiting SolveHard() after 0.807r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434940259868772415031826044511726 8,
441.642959735756012748170316682835889199 0,
436.917481657537198018304165726499479424 3,
422.984933979145722011193173207459971643 9,
361.525802566937473623975837930509210846 1,
401.881739047690910084261831675612635929 8,
389.590015163903061377408568296920914211 6,
328.469398941035954320028275908857427074 4,
401.507571585475211916778130692647054168 9,
358.973628245438152851355979985910975821 6,
398.331471045442124527698689069443564009 0,
```

```
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36, none,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017552695030835999573030352955609,
6.0258135494383318856223682346608455079 74,
351.42702949095032518149837262924535651 91]
one interval r = 31.362302061252904108383332974272825065 42 ..
34.174466406281580369798800081864908514 51
Time Approximations 0.017.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .255e-34]Solution in 0.528s

Time Plot 0 s.
Exiting SolveHard() after 0.785r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36, none,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57, none, none,
292.99969139113049783511827686751008538 65, none, none, none, none,
none, none, none, none, none, none]
```

```
0 --> 1  target = [25.872050175526950308359995730303529556 09,
6.0258135494383318856223682346608455079 74,
351.42702949095032518149837262924535651 91]
```

"Imaginary part neglected: ", $1.10311211491137042826364785393053982259 0 \times 10^{-17}$

```
two intervals r = 17.981355144530898238789556900954387030 98 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000 or r = 13.846080154613765827113500199456460517 09 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000
Time Approximations 0.053.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [-.90e-37, .3e-37, .60e-37]Solution in 4.932s

Time Plot 0 s.
Exiting SolveHard() after 5.995r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57, none, none,
292.99969139113049783511827686751008538 65, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418789920285968018212312109206 90,
6.3779438740125446624221057620439027329 66,
```

```
423.28832784441625081155543799186973406 25]
one interval r = 31.9466181760816017382660891190368889078 0 ..
35.21212308660854367017906581008286253133
Time Approximations 0.031.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., .212e-34]Solution in 0.613s

Time Plot 0 s.
Exiting SolveHard() after 0.967r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57, none, none,
292.99969139113049783511827686751008538 65, none, none,
360.06173467383194238155966693436024305 67, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941878992028596801821231210920690,
6.37794387401254466242210576204390273296 6,
423.28832784441625081155543799186973406 25]
```

"Imaginary part neglected: ", $1.10311211491137042826364785393053982259 0 \times 10^{-17}$

```
two intervals r = 15.22886702451034724015821739180100872290 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
```

```
000 or r = 17.1296577709981684947676084500661678982 4 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.31e-37, -.1e-37, .3910e-35]Solution in 1.241s

Time Plot 0 s.
Exiting SolveHard() after 6.992r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57,
375.73285290652943811676802359964216819 31, none,
292.99969139113049783511827686751008538 65, none, none,
360.06173467383194238155966693436024305 67, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343532291641841880545320396013 10,
4.0035598157294988814863130153951256132 38,
404.47973594575796313110822153780529687 11]
```

"Imaginary part neglected: ", $1.10311211491137042826364785393053982259 0 \times 10^{-17}$

```
two intervals r = 16.0968396638238245835216181398449911921 4 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
```

```
000 or r = 16.39988649144247892701552587283949956471 ..
95000000001003869084736924233805117 47/500000000000000000000000000000000
000
Time Approximations 0.058.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.21559e-34]Solution in 5.305s

Time Plot 0 s.
Exiting SolveHard() after 6.389r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57,
375.73285290652943811676802359964216819 31, none,
292.99969139113049783511827686751008538 65,
358.64341561414901062345838135443062046 85, none,
360.06173467383194238155966693436024305 67, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234353229164184188054532039601310,
4.0035598157294988814863130153951256132 38,
404.47973594575796313110822153780529687 11]
one interval r = 21.63429629992782120500281356921066557004 ..
26.75768169899281018645587690483657699060
Time Approximations 0.053.
```

```
hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [0., .26e-37, .451e-34]Solution in 1.117s

Time Plot 0 s.
Exiting SolveHard() after 2.176r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57,
375.73285290652943811676802359964216819 31,
328.11709295049151568241772448658126528 02,
292.99969139113049783511827686751008538 65,
358.64341561414901062345838135443062046 85, none,
360.06173467383194238155966693436024305 67, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954468031956700557116015890470447,
6.19617723033162990843745882066692016827 8,
385.42734026341482957925242298414243231 01]
one interval r = 31.608220491043108197014426214225081340 47 ..
34.663476150600479475801773322482983000 05
Time Approximations 0.017.

hint used Hint := [33.813392675100157147469018161182119369 13, 3, 1, 1,
11.783209121413521870719438174471899656 27, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.160e-34]Solution in 0.56s

Time Plot 0 s.
Exiting SolveHard() after 4.637r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868907944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57,
375.73285290652943811676802359964216819 31,
328.11709295049151568241772448658126528 02,
292.99969139113049783511827686751008538 65,
358.64341561414901062345838135443062046 85, none,
360.06173467383194238155966693436024305 67, none, none,
324.65521224321310928007101886476044945 84, none, none, none, none]

0 --> 1  target = [26.46318954468031956700557116015890470447,
6.19617723033162990843745882066692016827 8,
385.42734026341482957925242298414243231 01]
```

"Imaginary part neglected: ", $1.10311211491137042826364785393053982259 0 \times 10^{-17}$

```
two intervals r = 16.87629600301616496094759515185148523171 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000 or r = 15.55559000687742363298440052251756001394 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000
Time Approximations 0.063.
```

```
hint used Hint := [17.9309208304359097016944008190092163318 1, 3, 1, 1,
15.7008606270133975112531574051596448801 2, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.180e-37, 0., -.3796e-35]Solution in 4.792s

Time Plot 0 s.
Exiting SolveHard() after 5.979r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57,
375.73285290652943811676802359964216819 31,
328.11709295049151568241772448658126528 02,
292.99969139113049783511827686751008538 65,
358.64341561414901062345838135443062046 85, none,
360.06173467383194238155966693436024305 67,
336.59441032942328284279687252751666035 61, none,
324.65521224321310928007101886476044945 84, none, none, none, none]

0 --> 2  target = [34.4952266117832863972951564071999378360 8,
3.8971313161526658156094379824761990423 22,
373.7808188513564330567475127676963819418]
```

"Imaginary part neglected: ", $1.10311211491137042826364785393053982259 0 \times 10^{-17}$

```
two intervals r = 17.2976908623310726856983575551247916138 6 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
000 or r = 14.9943640747099236956456541683343121932 7 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
```

```
000
Time Approximations 0.088.

hint used Hint := [18.0598902594553048959735086609147911477 6, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [.36e-37, -.1e-37, -.9225e-35]Solution in 5.448s

Time Plot 0 s.
Exiting SolveHard() after 7.098r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57,
375.73285290652943811676802359964216819 31,
328.11709295049151568241772448658126528 02,
292.99969139113049783511827686751008538 65,
358.64341561414901062345838135443062046 85, none,
360.06173467383194238155966693436024305 67,
336.59441032942328284279687252751666035 61, none,
324.65521224321310928007101886476044945 84,
331.93806792135688384866616935288908571 47, none, none, none]

1 --> 2  target = [34.49522661178328639729515640719993783608,
3.897131316152665815609437982476199042322,
373.78081885135643305674751276769638194 18]
one interval r = 21.06068473207890039855679773491655564633 ..
26.26979834293982660971542321557931428999
Time Approximations 0.035.
```

```
hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [.1e-37, .3e-37, .480e-34]Solution in 0.808s

Time Plot 0 s.
Exiting SolveHard() after 1.53r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57,
375.73285290652943811676802359964216819 31,
328.11709295049151568241772448658126528 02,
292.99969139113049783511827686751008538 65,
358.64341561414901062345838135443062046 85,
299.89866205586250984797949197497743459 31,
360.06173467383194238155966693436024305 67,
336.59441032942328284279687252751666035 61, none,
324.65521224321310928007101886476044945 84,
331.93806792135688384866616935288908571 47, none, none, none]

0 --> 2  target = [33.81362495423975762275723840187339541123,
3.72564899378953046320541411425474066475 4,
325.89209973597499276159595840739381979 05]

"Imaginary part neglected: ", 1.10311211491137042826364785393053982259 0 × 10^-17
two intervals r = 18.55227049017176807086235324046443895180 ..
95000000001003869084736924233805117 47/50000000000000000000000000000000000
```

```
000 or r = 12.4919693583199468977810975390830484880 0 ..
9500000000100386908473692423380511747/500000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1e-38
Equations at solution: [-.17e-37, .1e-37, -.52376e-34]Solution in
1.225s

Time Plot 0 s.
Exiting SolveHard() after 6.408r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57,
375.73285290652943811676802359964216819 31,
328.11709295049151568241772448658126528 02,
292.99969139113049783511827686751008538 65,
358.64341561414901062345838135443062046 85,
299.89866205586250984797949197497743459 31,
360.06173467383194238155966693436024305 67,
336.59441032942328284279687252751666035 61, none,
324.65521224321310928007101886476044945 84,
331.93806792135688384866616935288908571 47, none, none,
289.54595773283454095491092094531367876 47]

1 --> 2  target = [33.81362495423975762275723840187339541123,
```

```
3.7256489937895304632054141142547406647 54,
325.8920997359749927615959584073938197905]
one interval r = 20.37468935107071557079680704382245681619 ..
25.37892165313230000353212061844759412370
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., -.132e-34]Solution in 0.545s

Time Plot 0 s.
Exiting SolveHard() after 1.108r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57,
375.73285290652943811676802359964216819 31,
328.11709295049151568241772448658126528 02,
292.99969139113049783511827686751008538 65,
358.64341561414901062345838135443062046 85,
299.89866205586250984797949197497743459 31,
360.06173467383194238155966693436024305 67,
336.59441032942328284279687252751666035 61,
256.10753186937020314758887254623909675 82,
324.65521224321310928007101886476044945 84,
331.93806792135688384866616935288908571 47, none, none,
289.54595773283454095491092094531367876 47]
```

```
1 --> 0  target = [17.93041369718587598294359676860068541127,
4.686508702128213478106365810830136776815,
353.30541095736889855463754721830906111139]
one interval r = 20.731504790940031991877325234519287808 24 ..
25.906753535340392918076051586474917780 50
Time Approximations 0.03.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=6.8e-38
Equations at solution: [-.3e-37, -.68e-37, -.241e-34]Solution in 0.635s

Time Plot 0 s.
Exiting SolveHard() after 5.15r=25.4021 in [22.67806074 .. 25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868906944356400 90,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57,
375.73285290652943811676802359964216819 31,
328.11709295049151568241772448658126528 02,
292.99969139113049783511827686751008538 65,
358.64341561414901062345838135443062046 85,
299.89866205586250984797949197497743459 31,
360.06173467383194238155966693436024305 67,
336.59441032942328284279687252751666035 61,
256.10753186937020314758887254623909675 82,
324.65521224321310928007101886476044945 84,
331.93806792135688384866616935288908571 47,
304.79958326263065204057438466619440257 84, none,
```

```
289.54595773283454095491092094531367876 47]

2 --> 0  target = [17.93041369718587598294359676860068541127,
4.68650870212821347810636581083013677 6815,
353.30541095736889855463754721830906 11139]
one interval r = 31.37435487004687295667544548763263665493 ..
34.20127520041042451296769206963061808792
Time Approximations 0.018.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., -.252e-34]Solution in 0.349s

Time Plot 0 s.
Exiting SolveHard() after 0.642r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494025986877241503182604451172 68,
441.64295973575601274817031668283588919 90,
436.91748165753719801830416572649947942 43,
422.98493397914572201119317320745997164 39,
361.52580256693747362397583793050921084 61,
401.88173904769091008426183167561263592 98,
389.59001516390306137740856829692091421 16,
328.46939894103595432002827590885742707 44,
401.50757158547521191677813069264705416 89,
358.97362824543815285135597998591097582 16,
398.33147104544212452769868909694435640 090,
371.48387395052415562824143773464312139 13,
336.61215841906840533241957985126387268 05,
361.50888347724208784220379005965801358 44,
324.67144993349676142586134480730159132 36,
302.31384315583147640289494858915282341 69,
328.46938514176041771666012610746193991 11,
343.81340625613250038526106459739849029 57,
375.73285290652943811676802359964216819 31,
328.11709295049151568241772448658126528 02,
292.99969139113049783511827686751008538 65,
358.64341561414901062345838135443062046 85,
299.89866205586250984797949197497743459 31,
360.06173467383194238155966693436024305 67,
336.59441032942328284279687252751666035 61,
256.10753186937020314758887254623909675 82,
```

```
324.65521224321310928007101886476044945 84,
331.93806792135688384866616935288908571 47,
304.79958326263065204057438466619440257 84,
323.46169177424144842521053414769511913 60,
289.54595773283454095491092094531367876 47]

Cascade time 169.316
counts: 28, 28

 Iteration 83

Start Generation 1
1 --> 0  target = [11.999999999929323191733627389941047 84000,
6.217012502969206574226240423288796378764,
485.54908089378086817240894556186336126 01]
```

"Imaginary part neglected: ", $1.8899423791459196352184877777411626404 96 \times 10^{-17}$

```
one interval r = 23.408503016446777195248756514813795985 08 ..
27.675780464190230696709649946659862291 15
Time Approximations 0.045.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.5e-38
Equations at solution: [-.1e-37, .25e-37, -.6e-36]Solution in 1.027s

Time Plot 0 s.
Exiting SolveHard() after 2.216r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999929323191733627389941047 84000,
6.217012502969206574226240423288796378764,
485.54908089378086817240894556186336126 01]
one interval r = 32.628147792027250908627719245540139691 60 ..
36.102483889380446843928980783212959786 07
Time Approximations 0.024.
```

```
hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .62e-35]Solution in 4.104s

Time Plot 0 s.
Exiting SolveHard() after 4.524r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684470456052600480910153751835717,
6.58343472158590572308896306353780349454 9,
467.78730595484429271867315343247740809 02]
one interval r = 32.41978955652445910323821360562361967410 ..
35.85152417367801435722308847132287751380
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.103e-34]Solution in 0.657s

Time Plot 0 s.
Exiting SolveHard() after 1.034r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90, none, none,
401.88173903861525198759437783818797095 13, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844704560526004809101537518357 17,
6.5834347215859057230889630635378034945 49,
467.78730595484429271867315343247740809 02]
```

"Imaginary part neglected: ", $1.10311211488135410297829150845041648307 8 \times 10^{-17}$

```
two intervals r = 12.923271608209449802767500171147200924 11 ..
949999999991588350761788181217709385 9/500000000000000000000000000000000
000 or r = 18.394248580102315091572071846017208451 51 ..
949999999991588350761788181217709385 9/500000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.1e-38
Equations at solution: [-.1e-37, -.11e-37, .170e-34]Solution in 44.567s

Time Plot 0 s.
Exiting SolveHard() after 45.985r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11, none,
401.88173903861525198759437783818797095 13, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628189571081834892778996304856 39,
4.1256517967733405567669599797008491087 25,
440.67123064511933194929335101713962940 46]
```

"Imaginary part neglected: ", $1.10311211488135410297829150845041648307 8 \times 10^{-17}$

```
two intervals r = 14.356597051148269164184773506683457718 11 ..
```

```
9499999999915883507617881812177093859/500000000000000000000000000000000
000 or r = 17.70352613781424503755820115500198461828 ..
9499999999915883507617881812177093859/500000000000000000000000000000000
000
Time Approximations 3.502.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.15e-37, -.1e-37, .420e-34]Solution in 1.366s

Time Plot 0 s.
Exiting SolveHard() after 5.915r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11, none,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962818957108183489277899630485639,
4.12565179677334055676695997970084910 8725,
440.67123064511933194929335101713962940 46]

"Imaginary part neglected: ", 1.889942379145919635218487777741162640496 × 10^-17
one interval r = 22.39761154356154481251804383701866607785 ..
27.23722351586345455339509067538670599082
Time Approximations 0.043.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
```

```
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.893 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643685369714311045264228387301 51, rm =
14.37818770222828489493942565297811230045}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.32e-37
Equations at solution: [.2e-37, .132e-36, .124e-34]Solution in 12.908s

Time Plot 0 s.
Exiting SolveHard() after 13.84r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887925580216852294823074781934 70,
4.0048690817547911588330135103899078373 04,
404.86224500927053150967540759785177372 52]
```

"Imaginary part neglected: ", $1.10311211488135410297829150845041648307 8 \times 10^{-17}$

```
two intervals r = 16.08011007747929153450166972824386850208 ..
9499999999915883507617881812177093859/500000000000000000000000000000000
000 or r = 16.41579812668076175593665388471681197842 ..
9499999999915883507617881812177093859/500000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.17e-37, 0., .90e-35]Solution in 5.785s

Time Plot 0 s.
Exiting SolveHard() after 6.975r=17.199 in [16.08011004 .. 19]
```

```
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07, none, none,
358.97362823441274387077253548329026226 49, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888792558021685229482307478193470,
4.00486908175479115883301351038990783730 4,
404.86224500927053150967540759785177372 52]
```

"Imaginary part neglected: ", $1.8899423791459196352184877777411626404 96 \times 10^{-17}$

```
one interval r = 21.641943994067202553737424598167738784 73 ..
26.763306600308721214794685128300845519 26
Time Approximations 0.057.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., -.443e-34]Solution in 1.131s

Time Plot 0 s.
Exiting SolveHard() after 2.206r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69, none,
```

```
358.9736282344127438707725354832902622649, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941766645260116122756199003980986,
5.589637182990874844050044427566780590921,
443.83065884312441160922794200446559568 06]
```

"Imaginary part neglected: ", $1.889942379145919635218487777741162640496 \times 10^{-17}$

```
one interval r = 22.46725374473034710266442967916748775641 ..
27.27388428345749624753653286815954147836
Time Approximations 0.043.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .11e-35]Solution in 1.009s

Time Plot 0 s.
Exiting SolveHard() after 5.634r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69, none,
358.9736282344127438707725354832902622649,
398.33147103957869454782531089873984837 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

2 --> 0  target = [14.19258941766645260116122756199003980986,
5.589637182990874844050044427566780590921,
443.83065884312441160922794200446559568 06]
one interval r = 32.15575279492053548603802165548305832919 ..
35.50872228734912169299598279308841021173
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
```

```
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, .159e-34]Solution in 0.468s

Time Plot 0 s.
Exiting SolveHard() after 0.851r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136509433062814919446818896810279,
5.18778357849690794057873582709368457215 6,
408.65773862080715195590350472847757361 23]
```

"Imaginary part neglected: ", $1.8899423791459196352184877777411626404 96 \times 10^{-17}$

```
one interval r = 21.718401146410480312337283862254878269 14 ..
26.818493034977111360523766363725080464 37
Time Approximations 0.058.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=0
Equations at solution: [0., 0., -.120e-34]Solution in 0.999s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.818r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05, none, none,
361.50888346600101050801199030621309663 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136509433062814919446818896810279,
5.18778357849690794057873582709368457215 6,
408.65773862080715195590350472847757361 23]
one interval r = 31.80828598738320520510407833339140441843 ..
35.00011460036462520460175469625134993239
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=1.5e-37
Equations at solution: [.14e-36, -.15e-36, .188e-34]Solution in 0.393s

Time Plot 0 s.
Exiting SolveHard() after 0.704r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
```

```
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52, none,
361.50888346600101050801199030621309663 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110526976680495231391448456194514,
6.196262565307389338277672970311315336497,
385.44474378811744332580111240202013592 69]
one interval r = 31.60836097524489795554816081932262282987 ..
34.66372795601669476553023494424392236590
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, .206e-34]Solution in 0.572s

Time Plot 0 s.
Exiting SolveHard() after 0.833r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52, none,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]
```

```
0 --> 1  target = [26.46347110526976680495231391448456194514,
6.196262565307389338277672970311315336497,
385.44474378811744332580111240202013592 69]
```

"Imaginary part neglected: ", $1.10311211488135410297829150845041648307 8 \times 10^{-17}$

```
two intervals r = 16.875634087459785419763806816701766180 38 ..
949999999991588350761788181217709385 9/500000000000000000000000000000000000
000 or r = 15.556404937704033959300709152640759815 90 ..
949999999991588350761788181217709385 9/500000000000000000000000000000000000
000
Time Approximations 0.063.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.841 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175107794733998792095725555138382, rm
= 2.33653277410580266488590300326887541 6662}});
Accepted {r=17.9304, rm=15.701} with Delta=1e-38
Equations at solution: [-.179e-37, .1e-37, .348e-34]Solution in 33.862s

Time Plot 0 s.
Exiting SolveHard() after 38.765r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
```

```
1 --> 0  target = [17.19898874726194023902591668514165708100,
4.88381077983733249924662343572126120283 1,
376.61967855318103545668050678770764053 08]
```

"Imaginary part neglected: ", $1.8899423791459196352184877777411626404 96 \times 10^{-17}$

```
one interval r = 21.11001304874125463439862056847363917295 ..
26.31784243462076150652827512355659809307
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.9e-36]Solution in 0.832s

Time Plot 0 s.
Exiting SolveHard() after 1.541r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15, none,
328.46938513140420709223084841219737056 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874726194023902591668514165708100,
4.88381077983733249924662343572126120283 1,
376.61967855318103545668050678770764053 08]
one interval r = 31.53899497698899506301712170981390749800 ..
34.53618386085077277479520923005853440947
Time Approximations 0.019.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
```

```
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.6e-37
Equations at solution: [-.13e-36, .16e-36, -.117e-34]Solution in 0.494s

Time Plot 0 s.
Exiting SolveHard() after 0.78r=34.0898 in [32.52213872 .. 34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259659 66 15, none,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017529985172763146773567963621520,
6.0258135492688020971160625791329100201 49,
351.42702948029028570222020722716806157 71]
one interval r = 31.362302061023630144311413148039848637 28 ..
34.174466406073135221173457789881799495 53
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=9e-38
```

```
Equations at solution: [-.5e-37, .9e-37, -.138e-34]Solution in 4.371s

Time Plot 0 s.
Exiting SolveHard() after 4.601r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15, none,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32, none, none,
292.99969137959952262736374163643972285 49, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017529985172763146773567963621520,
6.0258135492688020971160625791329100201 49,
351.42702948029028570222020722716806157 71]
```

"Imaginary part neglected: ", $1.10311211488135410297829150845041648307 8 \times 10^{-17}$

```
two intervals r = 17.98135514428344831248743208507318467195 ..
94999999999158835076178818121770938 59/50000000000000000000000000000000000
000 or r = 13.84608015393861477862094176397758942119 ..
94999999999158835076178818121770938 59/50000000000000000000000000000000000
000
Time Approximations 0.048.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.596 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
```

```
= 3/2 .. 19}, avoid={{r = 18.91357071352474004071057411036749722320, rm
= 2.73450099319210122183239573711615181162 9}});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [-.73e-37, .3e-37, .329e-34]Solution in 20.281s

Time Plot 0 s.
Exiting SolveHard() after 21.388r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259659 66,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32, none, none,
292.99969137959952262736374163643972285 49, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941865904999130913587091339471460,
6.37794387387109851438829266904460162055 1,
423.28832783840041533920148547417364983 52]
one interval r = 31.94661817588500142727673643323535652982 ..
35.21212308649893203718384817902558179961
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.201e-34]Solution in 0.625s

Time Plot 0 s.
Exiting SolveHard() after 0.972r=34.3272 in [33.10127385 ..
35.21212310]
```

```
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32, none, none,
292.99969137959952262736374163643972285 49, none, none,
360.06173466664479270832718234284960721 79, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941865904999130913587091339471460,
6.3779438738710985143882926690446016205 51,
423.28832783840041533920148547417364983 52]
```

"Imaginary part neglected: ", $1.10311211488135410297829150845041648307 8 \times 10^{-17}$

```
two intervals r = 15.22886702412287636902688090782498289494 ..
94999999999158835076178818121770938 59/50000000000000000000000000000000000
000 or r = 17.12965770587500113570397274168948821 34 ..
94999999999158835076178818121770938 59/50000000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.336 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.51537054105614529953209774159535645529, rm
= 2.06406829870327489449446709820302120119 2}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
```

```
Equations at solution: [-.46e-37, -.1e-37, .312e-34]Solution in 33.348s

Time Plot 0 s.
Exiting SolveHard() after 38.57r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32,
375.73285290177412616747791105650814096 16, none,
292.99969137959952262736374163643972285 49, none, none,
360.06173466664479270832718234284960721 79, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234336505086256332390308019755937,
4.0035598154550005808906849332855138606 38,
404.47973593637481935484344793627790806 02]

"Imaginary part neglected: ", 1.10311211488135410297829150845041648307 8 × 10^-17
two intervals r = 16.09683966358601130115860131797716886467 ..
949999999991588350761788181217709385 9/500000000000000000000000000000000000
000 or r = 16.39988649088154239892391782240999278033 ..
949999999991588350761788181217709385 9/500000000000000000000000000000000000
000
Time Approximations 0.055.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
```

```
Equations at solution: [-.17e-37, 0., .97e-35]Solution in 1.651s

Time Plot 0 s.
Exiting SolveHard() after 2.736r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32,
375.73285290177412616747791105650814096 16, none,
292.99969137959952262736374163643972285 49,
358.64341560294979801406342314326599216 97, none,
360.06173466664479270832718234284960721 79, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234336505086256332390308019755937,
4.0035598154550005808906849332855138606 38,
404.47973593637481935484344793627790806 02]

"Imaginary part neglected: ", 1.8899423791459196352184877777411626404 96 × 10^-17
one interval r = 21.634296299841095722819529644595113217 53 ..
26.757681698807487949632630672344526179 42
Time Approximations 0.054.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, .205e-34]Solution in 1.051s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.754r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32,
375.73285290177412616747791105650814096 16,
328.11709293994848715700016094584087671 89,
292.99969137959952262736374163643972285 49,
358.64341560294979801406342314326599216 97, none,
360.06173466664479270832718234284960721 79, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954445153561572506334843819523726,
6.19617723016086119695735235380197164678 8,
385.42734025185894839795333354123066165 70]
one interval r = 31.608220490798467371970318710710653374 96 ..
34.663476150393160043547648499361925965 37
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.170e-34]Solution in 0.577s

Time Plot 0 s.
Exiting SolveHard() after 0.867r=33.8134 in [32.62668594 ..
34.66347615]
```

```
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32,
375.73285290177412616747791105650814096 16,
328.11709293994848715700016094584087671 89,
292.99969137959952262736374163643972285 49,
358.64341560294979801406342314326599216 97, none,
360.06173466664479270832718234284960721 79, none, none,
324.65521223084538364742644506009157438 99, none, none, none, none]

0 --> 1  target = [26.46318954445153561572506334843819523726,
6.19617723016086119695735235380197164678 8,
385.42734025185894839795333354123066165 70]
```

"Imaginary part neglected: ", $1.10311211488135410297829150845041648307 8 \times 10^{-17}$

```
two intervals r = 16.87629600284652925677793842972935545239 ..
94999999999158835076178818121770938 59/50000000000000000000000000000000000
000 or r = 15.55559000618912034316408047115279133991 ..
94999999999158835076178818121770938 59/50000000000000000000000000000000000
000
Time Approximations 0.066.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 7.976 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
```

```
= 3/2 .. 19}, avoid={{r = 18.46866852519541847561415405784227653067, rm
= 2.33669042817125322840432579002288420050 0}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.180e-37, 0., .322e-34]Solution in 29.813s

Time Plot 0 s.
Exiting SolveHard() after 34.966r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259659 15,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32,
375.73285290177412616747791105650814096 16,
328.11709293994848715700016094584087671 89,
292.99969137959952262736374163643972285 49,
358.64341560294979801406342314326599216 97, none,
360.06173466664479270832718234284960721 79,
336.59441031890357956001940601690217511 40, none,
324.65521223084538364742644506009157438 99, none, none, none, none]

0 --> 2  target = [34.49522661153231400426095745144589870165,
3.89713131586018132771804238801130483025 4,
373.78081883711849326497428797608481547 31]
```

"Imaginary part neglected: ", $1.10311211488135410297829150845041648307 8 \times 10^{-17}$

```
two intervals r = 17.29769086223587630091444115834092679079 ..
94999999999158835076178818121770938 59/50000000000000000000000000000000000
000 or r = 14.99436407387129929179199703433224235966 ..
94999999999158835076178818121770938 59/50000000000000000000000000000000000
000
Time Approximations 0.088.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
```

```
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.36e-37, 0., .34e-35]Solution in 1.233s

Time Plot 0 s.
Exiting SolveHard() after 6.788r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32,
375.73285290177412616747791105650814096 16,
328.11709293994848715700016094584087671 89,
292.99969137959952262736374163643972285 49,
358.64341560294979801406342314326599216 97, none,
360.06173466664479270832718234284960721 79,
336.59441031890357956001940601690217511 40, none,
324.65521223084538364742644506009157438 99,
331.93806790591234236876054505373944228 10, none, none, none]

1 --> 2  target = [34.49522661153231400426095745144589870165,
3.8971313158601813277180423880113048302 54,
373.78081883711849326497428797608481547 31]

"Imaginary part neglected: ", 1.8899423791459196352184877777411626404 96 × 10^-17
one interval r = 21.06068473195752555700992362545873280809 ..
26.26979834265942941920896968857565825831
Time Approximations 0.035.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [-.2e-37, -.5e-37, .172e-34]Solution in 0.796s

Time Plot 0 s.
Exiting SolveHard() after 5.783r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32,
375.73285290177412616747791105650814096 16,
328.11709293994848715700016094584087671 89,
292.99969137959952262736374163643972285 49,
358.64341560294979801406342314326599216 97,
299.89866204090030586635573406731999625 50,
360.06173466664479270832718234284960721 79,
336.59441031890357956001940601690217511 40, none,
324.65521223084538364742644506009157438 99,
331.93806790591234236876054505373944228 10, none, none, none]

0 --> 2  target = [33.81362495399449470798552727535683308531,
3.72564899350053744827670639081552028052 1,
325.89209972334852057837379964389027833 39]
```

"Imaginary part neglected: ", $1.10311211488135410297829150845041648307 8 \times 10^{-17}$

```
two intervals r = 18.552270489922152613119126428174522541 11 ..
94999999999158835076178818121770938 59/50000000000000000000000000000000000
000 or r = 12.491969357569393344919126513824930231 60 ..
94999999999158835076178818121770938 59/50000000000000000000000000000000000
000
```

```
Time Approximations 0.048.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [-.137e-36, .6e-37, .126e-34]Solution in 4.748s

Time Plot 0 s.
Exiting SolveHard() after 6.272r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32,
375.73285290177412616747791105650814096 16,
328.11709293994848715700016094584087671 89,
292.99969137959952262736374163643972285 49,
358.64341560294979801406342314326599216 97,
299.89866204090030586635573406731999625 50,
360.06173466664479270832718234284960721 79,
336.59441031890357956001940601690217511 40, none,
324.65521223084538364742644506009157438 99,
331.93806790591234236876054505373944228 10, none, none,
289.54595771886505861691001897591406748 65]

1 --> 2  target = [33.81362495399449470798552727535683308531,
3.72564899350053744827670639081552028052 1,
325.89209972334852057837379964389027833 39]
```

"Imaginary part neglected: ", $1.8899423791459196352184877777411626404 96 \times 10^{-17}$

```
one interval r = 20.37468935108470735976564282318433849703 ..
25.37892165286121347862376686481257214167
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, .100e-34]Solution in 0.59s

Time Plot 0 s.
Exiting SolveHard() after 1.14r=24.3395 in [22.07732228 .. 25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32,
375.73285290177412616747791105650814096 16,
328.11709293994848715700016094584087671 89,
292.99969137959952262736374163643972285 49,
358.64341560294979801406342314326599216 97,
299.89866204090030586635573406731999625 50,
360.06173466664479270832718234284960721 79,
336.59441031890357956001940601690217511 40,
256.10753185595191987290310461809660056 86,
324.65521223084538364742644506009157438 99,
331.93806790591234236876054505373944228 10, none, none,
289.54595771886505861691001897591406748 65]
```

```
1 --> 0  target = [17.93041369694691631619715468115910021861,
4.68650870198401357223069536083336214951 4,
353.30541094649392753680125931588917372 40]
```

"Imaginary part neglected: ", $1.8899423791459196352184877777411626404 96 \times 10^{-17}$

```
one interval r = 20.73150479091637376354112966567064656794 ..
25.906753535109851497600328240658596790 13
Time Approximations 0.038.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., .204e-34]Solution in 0.677s

Time Plot 0 s.
Exiting SolveHard() after 1.39r=25.4021 in [22.67806074 .. 25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32,
375.73285290177412616747791105650814096 16,
328.11709293994848715700016094584087671 89,
292.99969137959952262736374163643972285 49,
358.64341560294979801406342314326599216 97,
299.89866204090030586635573406731999625 50,
360.06173466664479270832718234284960721 79,
336.59441031890357956001940601690217511 40,
256.10753185595191987290310461809660056 86,
```

```
324.65521223084538364742644506009157438 99,
331.93806790591234236876054505373944228 10,
304.79958325274492656710614335323319717 65, none,
289.54595771886505861691001897591406748 65]

2 --> 0  target = [17.93041369694691631619715468115910021861,
4.6865087019840135722306953608333621495 14,
353.30541094649392753680125931588917372 40]
one interval r = 31.374354869815658955607460946924500421 42 ..
34.201275200199641284323945454282685964 77
Time Approximations 0.014.

hint used Hint := [33.796329522691901201472146203888695425 08, 2, 1, -1,
17.863534069716629640044658426895081929 62, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [.1e-37, -.3e-37, -.349e-34]Solution in 0.366s

Time Plot 0 s.
Exiting SolveHard() after 4.302r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493186719518445491868659699800 15,
441.64295972792460330039803513989993447 40,
436.91748164749808824671917981944183545 90,
422.98493397225789348978414549258607853 11,
361.52580255563140935639563587593618856 18,
401.88173903861525198759437783818797095 13,
389.59001515209649529395548825353378722 07,
328.46939893067772289587944152787324935 69,
401.50757157620287456243637698196893596 54,
358.97362823441274387077253548329026226 49,
398.33147103957869454782531089873984837 05,
371.48387393650892789545924568301900822 52,
336.61215840848052137322869475673879993 00,
361.50888346600101050801199030621309663 56,
324.67144992106666790471678750696259666 15,
302.31384314623146745722448785374623416 12,
328.46938513140420709223084841219737056 50,
343.81340624263515189772720016206737279 32,
375.73285290177412616747791105650814096 16,
328.11709293994848715700016094584087671 89,
292.99969137959952262736374163643972285 49,
358.64341560294979801406342314326599216 97,
299.89866204090030586635573406731999625 50,
```

```
360.06173466664479270832718234284960721 79,
336.59441031890357956001940601690217511 40,
256.10753185595191987290310461809660056 86,
324.65521223084538364742644506009157438 99,
331.93806790591234236876054505373944228 10,
304.79958325274492656710614335323319717 65,
323.46169176106754676438984780752288328 93,
289.54595771886505861691001897591406748 65]

Cascade time 273.207
counts: 28, 28

 Iteration 84

Start Generation 1
1 --> 0  target = [12.00000000008042158756545944234393434500,
6.21701250286867963478987756597503125 8695,
485.54908089945665534919366545521054529 12]
one interval r = 23.40850301663242993319104168772357367256 ..
27.67578046433381700704629387349710705337
Time Approximations 0.045.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.2e-38
Equations at solution: [.2e-37, -.52e-37, -.6e-36]Solution in 1.004s

Time Plot 0 s.
Exiting SolveHard() after 2.149r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000008042158756545944234393434500,
6.21701250286867963478987756597503125 8695,
485.54908089945665534919366545521054529 12]
one interval r = 32.62814779207514295001815397962667753373 ..
36.10248388944528399864728322980664244253
Time Approximations 0.024.
```

```
hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .136e-34]Solution in 4.203s

Time Plot 0 s.
Exiting SolveHard() after 4.605r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684483719569266842337632626597081,
6.58343472173838957662569944434355049202 4,
467.78730595873722366661237693665100749 19]
one interval r = 32.41978955654834368250912612324291995882 ..
35.85152417371821997353635490109175222591
Time Approximations 0.02.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.196e-34]Solution in 0.666s

Time Plot 0 s.
Exiting SolveHard() after 1.059r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39, none, none,
401.88173904097696995766157796374495702 91, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684483719569266842337632626597081,
6.5834347217383895766256994443435504920 24,
467.78730595873722366661237693665100749 19]
two intervals r = 12.92327160846235464063952480014040997434 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000000
000 or r = 18.39424858033098114487626852780297471858 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.4e-38
Equations at solution: [-.4e-37, -.54e-37, -.17e-35]Solution in 46.088s

Time Plot 0 s.
Exiting SolveHard() after 47.517r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02, none,
401.88173904097696995766157796374495702 91, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962827727592602852076013930720282,
4.1256517968381176975858872237183504987 26,
440.67123065229201245571165672329339956 84]
two intervals r = 14.35659705123418078416356988002399453735 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000000
000 or r = 17.70352613815184847881455123660339279826 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000000
000
Time Approximations 0.046.
```

```
hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.41e-37, -.2e-37, -.520e-34]Solution in 1.263s

Time Plot 0 s.
Exiting SolveHard() after 5.7r=15.9119 in [14.35659706 .. 18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02, none,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962827727592602852076013930720282,
4.12565179683811769758587223718350498726,
440.67123065229201245571165672329339956 84]
one interval r = 22.39761154376876845252748761281414310062 ..
27.23722351603879296106502332780934316251
Time Approximations 0.04.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 4.829 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064394079617293442897881813177928, rm =
14.37818770521944680366787306532176449635}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.3e-38
Equations at solution: [-.1e-37, -.53e-37, -.442e-34]Solution in
12.806s
```

```
Time Plot 0 s.
Exiting SolveHard() after 13.725r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887946438616936098564069109432 75,
4.004869081804434149870100962242184626311,
404.86224501183760149901763114249195668 05]
two intervals r = 16.080110077782629000246248337315756889 92 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000000
000 or r = 16.415798126880715141246067796710005531 18 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000000
000
Time Approximations 0.056.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.51e-37, -.1e-37, .36e-35]Solution in 5.563s

Time Plot 0 s.
Exiting SolveHard() after 6.718r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
```

```
389.5900151608478080677656958477447804027, none, none,
358.9736282392887226731963996929379872788, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888794643861693609856406910943275,
4.004869081804434149870100962242184626311,
404.8622450118376014990176311424919566805]
one interval r = 21.641943994162875223260672696022847639 80 ..
26.76330660043107314625394447411910612912
Time Approximations 0.055.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [.2e-37, .49e-37, .124e-34]Solution in 1.121s

Time Plot 0 s.
Exiting SolveHard() after 2.191r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.5900151608478080677656958477447804027,
328.46939893184120006622193579944396346 64, none,
358.9736282392887226731963996929379872788, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941817464130771332074945249446634,
5.589637182818053832672875296804144353757,
443.8306588421752857926564760936393697835]
one interval r = 22.46725374475937828780665101813729534429 ..
27.27388428353833144612108724426189202172
Time Approximations 0.038.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
```

```
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., -.63e-35]Solution in 0.998s

Time Plot 0 s.
Exiting SolveHard() after 5.895r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64, none,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589418174641307713320749452494466 34,
5.5896371828180538326728752968041443537 57,
443.83065884217528579265647609363936978 35]
one interval r = 32.155752794889343342559860990678444410 22 ..
35.508722287319888895976052691958222562 13
Time Approximations 0.02.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.236e-34]Solution in 0.455s

Time Plot 0 s.
Exiting SolveHard() after 0.834r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136511506833505181842840404839897,
5.18778357844937010609836747364975809060 7,
408.65773862974003460019382963036660947 40]
one interval r = 21.71840114663647754972337598138029090098 ..
26.81849303519097014253506898654832752274
Time Approximations 0.059.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [-.1e-37, -.79e-37, .184e-34]Solution in 1.004s

Time Plot 0 s.
Exiting SolveHard() after 5.603r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18, none, none,
```

```
361.50888347150330880578695354016464519 00, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136511506833505181842840404839897,
5.1877835784493701060983674736497580906 07,
408.65773862974003460019382963036660947 40]
one interval r = 31.80828598744305553654655542353760972572 ..
35.00011460047786490401106180508423034760
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, .109e-34]Solution in 0.398s

Time Plot 0 s.
Exiting SolveHard() after 0.718r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09, none,
361.50888347150330880578695354016464519 00, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110543650054089341393929436653247,
6.19626256547016351763738334339980064 3784,
385.44474379340489344237318343186581778 71]
one interval r = 31.60836097526544437620337718665422341772 ..
34.66372795607640955144198497937500089590
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
```

```
11.783029068742676904624149068588691721 32, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .126e-34]Solution in 0.581s

Time Plot 0 s.
Exiting SolveHard() after 0.842r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09, none,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110543650054089341393929436653247,
6.196262565470163517637383343399800643784,
385.44474379340489344237318343186581778 71]
two intervals r = 16.875634087644789194960452738788206987 27 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000
000 or r = 15.556404938032949411153723308707802059 00 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000
000
Time Approximations 0.06.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.895e-37, 0., .347e-34]Solution in 1.167s

Time Plot 0 s.
Exiting SolveHard() after 6.204r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874746176872974201759496068456933,
4.8838107797543737529002188699462550400 82,
376.61967855806767493370630293421442445 34]
one interval r = 21.11001304886728669500956019418934820260 ..
26.31784243478261208119016903997944759734
Time Approximations 0.038.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.104e-34]Solution in 4.496s

Time Plot 0 s.
Exiting SolveHard() after 5.233r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11, none,
328.46938513256847623907595972407594198 31, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874746176872974201759496068456933,
4.8838107797543737529002188699462550400 82,
376.61967855806767493370630293421442445 34]
one interval r = 31.53899497700410127436238449142671488797 ..
34.53618386090432261284337420729663479036
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=2.56e-36
Equations at solution: [.197e-35, -.256e-35, -.430e-34]Solution in
0.533s

Time Plot 0 s.
Exiting SolveHard() after 0.822r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
```

```
358.9736282392887226731963996929379872788,
398.3314710351812127057603198175116818218,
371.4838739469814996972645031751641835409,
336.6121584091101867651265010573727164268,
361.5088834715033088057869535401646451900,
324.6714499245977827006851284025362767811, none,
328.4693851325684762390759597240759419831,
343.8134062497939653491092124806431766233, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017539138215898885740770402894037,
6.025813549410942354425578029023353131916,
351.4270294811664959360307061049787804016]
one interval r = 31.36230206100666761917124918001592561613 ..
34.17446640606807179442420070698900712447
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, .49e-35]Solution in 0.52s

Time Plot 0 s.
Exiting SolveHard() after 0.764r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349375681039878103012176775382875,
441.6429597320378766953766280100380434666,
436.9174816543676801381963104394757522739,
422.9849339716628819057123228411920329202,
361.5258025610870196851952944692452163506,
401.8817390409769699576615779637449570291,
389.5900151608478080677656958477447804027,
328.4693989318412000662219357994439634664,
401.5075715779574153537046812917361274441,
358.9736282392887226731963996929379872788,
398.3314710351812127057603198175116818218,
371.4838739469814996972645031751641835409,
336.6121584091101867651265010573727164268,
361.5088834715033088057869535401646451900,
324.6714499245977827006851284025362767811, none,
328.4693851325684762390759597240759419831,
343.8134062497939653491092124806431766233, none, none,
292.9996913789689686675359757566095045256, none, none, none, none,
```

```
none, none, none, none, none, none]

0 --> 1  target = [25.872050175391382158988857407704028940 37,
6.025813549410942354425578029023353131916,
351.42702948116649593603070610497878040 16]
two intervals r = 17.981355144590974765384988777576689140 53 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000000
000 or r = 13.846080154051509864565371163857898281 36 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [-.35e-37, .1e-37, .110e-34]Solution in 1.148s

Time Plot 0 s.
Exiting SolveHard() after 6.495r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33, none, none,
292.99969137896896867535975756609504525 6, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418683019684163781022721058453 79,
6.377943873984214700253681108980433625738,
423.28832783355307788259512478938698736 82]
```

```
one interval r = 31.946618175816132937855857299648559227 19 ..
35.21212308641277893352747758407237373084
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., -.72e-35]Solution in 0.59s

Time Plot 0 s.
Exiting SolveHard() after 0.937r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33, none, none,
292.99969137896896867535975756609504525 6, none, none,
360.06173466074371573524085434554499960 59, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941868301968416378102272105845379,
6.37794387398421470025368110898043362573 8,
423.28832783355307788259512478938698736 82]
two intervals r = 15.22886702479777116061264828037751887714 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000000
000 or r = 17.12965777052056839510505278495119193121 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000000
000
Time Approximations 0.058.
```

```
hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., .107e-34]Solution in 1.311s

Time Plot 0 s.
Exiting SolveHard() after 5.93r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33,
375.73285289167338891889358295792299952 06, none,
292.99969137896896867535975756609504525 6, none, none,
360.06173466074371573524085434554499960 59, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343376900479986788307253661193 51,
4.0035598155025205699250522734201074153 54,
404.47973593832118264127274959655595735 44]
two intervals r = 16.096839663916182356760855106955064146 79 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000
000 or r = 16.399886491055669383104694872954597334 45 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000
000
Time Approximations 0.058.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.52e-37, .1e-37, .5e-36]Solution in 4.976s

Time Plot 0 s.
Exiting SolveHard() after 6.055r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33,
375.73285289167338891889358295792299952 06, none,
292.99969137896896866753597575660950452 56,
358.64341560728984631992682922158749708 15, none,
360.06173466074371573524085434554499960 59, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234337690047998678830725366119351,
4.0035598155025205699250522734201074153 54,
404.47973593832118264127274959655595735 44]
one interval r = 21.63429629992424050585328549979763395819 ..
26.75768169892071185046307218773038925169
Time Approximations 0.053.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, -.43e-35]Solution in 5.095s

Time Plot 0 s.
Exiting SolveHard() after 6.155r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33,
375.73285289167338891889358295792299952 06,
328.11709294053996115577700532405735206 65,
292.99969137896896867535975756609504525 6,
358.64341560728984631992682922158749708 15, none,
360.06173466074371573524085434554499960 59, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954461905017087579158146736768635,
6.19617723032387183377924459744957069728 7,
385.42734025719442709408134651154381829 54]
one interval r = 31.60822049081939683382483292226056043955 ..
34.66347615045356905728224389478802780977
Time Approximations 0.016.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
```

```
Equations at solution: [.1e-37, -.3e-37, -.22e-35]Solution in 0.576s

Time Plot 0 s.
Exiting SolveHard() after 0.873r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33,
375.73285289167338891889358295792299952 06,
328.11709294053996115577700532405735206 65,
292.99969137896896866753597575660950452 56,
358.64341560728984631992682922158749708 15, none,
360.06173466074371573524085434554499960 59, none, none,
324.65521223442127798843415146146238163 24, none, none, none, none]

0 --> 1  target = [26.46318954461905017087579158146736768635,
6.19617723032387183377924459744957069728 7,
385.42734025719442709408134651154381829 54]
two intervals r = 16.87629600302970761721414554492149956126 ..
9500000000024622091799268703103996699/5000000000000000000000000000000000
000 or r = 15.55559000652029793352190515153988348784 ..
9500000000024622091799268703103996699/5000000000000000000000000000000000
000
Time Approximations 0.062.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
```

```
Equations at solution: [0., 0., -.176e-34]Solution in 5.098s

Time Plot 0 s.
Exiting SolveHard() after 6.281r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33,
375.73285289167338891889358295792299952 06,
328.11709294053996115577700532405735206 65,
292.99969137896896866753597575660950452 56,
358.64341560728984631992682922158749708 15, none,
360.06173466074371573524085434554499960 59,
336.59441031958230263678762791328116632 15, none,
324.65521223442127798843415146146238163 24, none, none, none, none]

0 --> 2  target = [34.49522661167190725718504769179120131885,
3.8971313159391034924907235281877879046 33,
373.78018884798106947621085017675180887 12]
two intervals r = 17.29769086223060595420488616826059154820 ..
9500000000024622091799268703103996699/50000000000000000000000000000000
000 or r = 14.99436407448401497267038026450936154137 ..
9500000000024622091799268703103996699/50000000000000000000000000000000
000
Time Approximations 0.086.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
```

```
Equations at solution: [-.18e-37, 0., .218e-34]Solution in 1.262s

Time Plot 0 s.
Exiting SolveHard() after 2.916r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33,
375.73285289167338891889358295792299952 06,
328.11709294053996115577700532405735206 65,
292.99969137896896867535975756609504525 6,
358.64341560728984631992682922158749708 15, none,
360.06173466074371573524085434554499960 59,
336.59441031958230263678762791328116632 15, none,
324.65521223442127798843415146146238163 24,
331.93806791803393109992391804895857436 44, none, none, none]

1 --> 2  target = [34.49522661167190725718504769179120131885,
3.8971313159391034924907235281877879046 33,
373.78081884798106947621085017675180887 12]
one interval r = 21.06068473218468582468420413708541453709 ..
26.26979834292351474383420911152711662828
Time Approximations 0.034.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [.1e-37, .3e-37, -.49e-35]Solution in 0.775s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.171r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33,
375.73285289167338891889358295792299952 06,
328.11709294053996115577700532405735206 65,
292.99969137896896866753597575660950452 56,
358.64341560728984631992682922158749708 15,
299.89866204964819020489394532085602860 13,
360.06173466074371573524085434554499960 59,
336.59441031958230263678762791328116632 15, none,
324.65521223442127798843415146146238163 24,
331.93806791803393109992391804895857436 44, none, none, none]

0 --> 2  target = [33.81362495402896991547567876989873825816,
3.72564899355494690573836269025281817829 3,
325.89209972711388473723079925534012297 76]
two intervals r = 18.552270490145143924434334350344119546 31 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000
000 or r = 12.491969357827303260781105749229544576 30 ..
95000000000246220917992687031039966 99/50000000000000000000000000000000
000
Time Approximations 0.043.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [-.104e-36, .5e-37, -.126e-34]Solution in 4.741s

Time Plot 0 s.
Exiting SolveHard() after 6.225r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33,
375.73285289167338891889358295792299952 06,
328.11709294053996115577700532405735206 65,
292.99969137896896866753597575660950452 56,
358.64341560728984631992682922158749708 15,
299.89866204964819020489394532085602860 13,
360.06173466074371573524085434554499960 59,
336.59441031958230263678762791328116632 15, none,
324.65521223442127798843415146146238163 24,
331.93806791803393109992391804895857436 44, none, none,
289.54595772466131317975429850852544450 26]

1 --> 2  target = [33.81362495402896991547567876989873825816,
3.72564899355494690573836269025281817829 3,
325.89209972711388473723079925534012297 76]
one interval r = 20.37468935116048303819805382455499591177 ..
25.37892165300766327170317221074438068308
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
```

```
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., -.75e-35]Solution in 0.57s

Time Plot 0 s.
Exiting SolveHard() after 1.129r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33,
375.73285289167338891889358295792299952 06,
328.11709294053996115577700532405735206 65,
292.99969137896896866753597575660950452 56,
358.64341560728984631992682922158749708 15,
299.89866204964819020489394532085602860 13,
360.06173466074371573524085434554499960 59,
336.59441031958230263678762791328116632 15,
256.10753185814379970395623050234156940 87,
324.65521223442127798843415146146238163 24,
331.93806791803393109992391804895857436 44, none, none,
289.54595772466131317975429850852544450 26]

1 --> 0  target = [17.93041369726788473917701781487516581449,
4.68650870186882634352964427115008672138 7,
353.30541094697082445147154263594214134 18]
one interval r = 20.73150479096082871653523893822345550322 ..
25.90675353519424465132776455573213791832
Time Approximations 0.036.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.7e-36]Solution in 4.424s

Time Plot 0 s.
Exiting SolveHard() after 5.133r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33,
375.73285289167338891889358295792299952 06,
328.11709294053996115577700532405735206 65,
292.99969137896896867535975756609504525 6,
358.64341560728984631992682922158749708 15,
299.89866204964819020489394532085602860 13,
360.06173466074371573524085434554499960 59,
336.59441031958230263678762791328116632 15,
256.10753185814379970395623050234156940 87,
324.65521223442127798843415146146238163 24,
331.93806791803393109992391804895857436 44,
304.79958324944275304109650636929732552 69, none,
289.54595772466131317975429850852544450 26]

2 --> 0  target = [17.93041369726788473917701781487516581449,
4.68650870186882634352964427115008672138 7,
353.30541094697082445147154263594214134 18]
one interval r = 31.37435486979621914298570571219934025993 ..
34.20127520018893351967455457256088137426
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
```

```
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, -.367e-34]Solution in 0.392s

Time Plot 0 s.
Exiting SolveHard() after 0.67r=33.7963 in [32.25770943 .. 34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493756810398781030121767753828 75,
441.64295973203787669537662801003804346 66,
436.91748165436768013819631043947575227 39,
422.98493397166288190571232284119203292 02,
361.52580256108701968519529446924521635 06,
401.88173904097696995766157796374495702 91,
389.59001516084780806776569584774478040 27,
328.46939893184120006622193579944396346 64,
401.50757157795741535370468129173612744 41,
358.97362823928872267319639969293798727 88,
398.33147103518121270576031981751168182 18,
371.48387394698149969726450317516418354 09,
336.61215840911018676512650105737271642 68,
361.50888347150330880578695354016464519 00,
324.67144992459778270068512840253627678 11,
302.31384314263541509751312111484046591 54,
328.46938513256847623907595972407594198 31,
343.81340624979396534910921248064317662 33,
375.73285289167338891889358295792299952 06,
328.11709294053996115577700532405735206 65,
292.99969137896896866753597575660950452 56,
358.64341560728984631992682922158749708 15,
299.89866204964819020489394532085602860 13,
360.06173466074371573524085434554499960 59,
336.59441031958230263678762791328116632 15,
256.10753185814379970395623050234156940 87,
324.65521223442127798843415146146238163 24,
331.93806791803393109992391804895857436 44,
304.79958324944275304109650636929732552 69,
323.46169176442160906371564822289855602 35,
289.54595772466131317975429850852544450 26]

Cascade time 164.611
counts: 28, 28

 Iteration 85

Start Generation 1
1 --> 0  target = [11.99999999995609723126708575112312720300,
6.21701250277599829483726623396322760645 1,
485.54908089884822896116151836922292168 61]
```

```
one interval r = 23.40850301656576013838482462932342472885 ..
27.67578046429996766311865075126144697225
Time Approximations 0.044.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.29e-37
Equations at solution: [-.4e-37, .129e-36, .20e-35]Solution in 1.06s

Time Plot 0 s.
Exiting SolveHard() after 2.281r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999995609723126708575112312720300,
6.217012502775998294837266233963227606451,
485.549080898848228961161518369222921686 1]
one interval r = 32.62814779226054718666114565854737944691 ..
36.10248388938390819725715095423915465150
Time Approximations 0.024.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., .46e-35]Solution in 4.279s

Time Plot 0.001 s.
Exiting SolveHard() after 4.697r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844815555513583281241327564467 50,
6.583434721453845090718675738648306451278,
467.787305959392711810894558988284111869 2]
one interval r = 32.419789556764987914333204695343205768 34 ..
35.851524173690769161440735284277668051 43
Time Approximations 0.023.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., .45e-35]Solution in 0.682s

Time Plot 0 s.
Exiting SolveHard() after 1.083r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87, none, none,
401.88173904566935577687103630359420336 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844815555513583281241327564467 50,
6.583434721453845090718675738648306451278,
467.787305959392711810894558988284111869 2]
two intervals r = 12.923271608283269983953903567516725928 06 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000 or r = 18.394248580363320515235507566559451124 42 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000
Time Approximations 0.047.
```

```
hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.5e-38
Equations at solution: [-.5e-37, -.55e-37, .73e-36]Solution in 45.209s

Time Plot 0 s.
Exiting SolveHard() after 46.662r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29, none,
401.88173904566935577687103630359420336 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962828053974473947533848748192959,
4.12565179689812516594984875459138487751 2,
440.67123065331147648914921127958079176 98]
two intervals r = 14.35659705106141781833211554991082714220 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000 or r = 17.70352613820202130471417097870138172693 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [.58e-37, .3e-37, .1158e-34]Solution in 1.334s

Time Plot 0 s.
Exiting SolveHard() after 5.857r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29, none,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962828053974473947533848748192959,
4.1256517968981251659498487545913848775 12,
440.67123065331147648914921127958079176 98]
one interval r = 22.397611543725627712136898668241105541 81 ..
27.237223516024168715446045688744453652 89
Time Approximations 0.039.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 5.01 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643872088795573384555290923604 59, rm =
14.378187702458541740752593339275864755 36}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.7e-38
Equations at solution: [-.1e-37, -.27e-37, .225e-34]Solution in 13.389s

Time Plot 0 s.
Exiting SolveHard() after 14.313r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888037688466490826795931497037 48,
4.0048690818769495862373469152097282345 31,
404.86224501652147550320378119505978793 95]
two intervals r = 16.08011007749275439339066579845392927302 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000 or r = 16.41579812708869496988906585289434032235 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.34e-37, 0., .3050e-34]Solution in 5.499s

Time Plot 0 s.
Exiting SolveHard() after 6.646r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95, none, none,
358.97362824544638243848812743630541778 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888037688466490826795931497037 48,
4.0048690818769495862373469152097282345 31,
404.86224501652147550320378119505978793 95]
one interval r = 21.64194399418040840460573720980593785654 ..
26.76330660047320780382123964722672201564
Time Approximations 0.054.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
```

```
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=9.8e-38
Equations at solution: [.3e-37, .98e-37, .479e-34]Solution in 1.094s

Time Plot 0 s.
Exiting SolveHard() after 2.175r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53, none,
358.97362824544638243848812743630541778 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589417830521767321657994687651644 47,
5.5896371827787355519549741659396495544 81,
443.83065884641516881870244271530043697 39]
one interval r = 22.467253744788455959337984138421979266 65 ..
27.273884283560278587031520352636127854 45
Time Approximations 0.037.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.0e-38
Equations at solution: [.1e-37, -.80e-37, .12e-35]Solution in 0.982s

Time Plot 0 s.
Exiting SolveHard() after 6.134r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53, none,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941783052176732165799468765164447,
5.5896371827787355519549741659396495544 81,
443.83065884641516881870244271530043697 39]
one interval r = 32.15575279516449013185001389586078130025 ..
35.50872228736647787399120333467773866413
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., .203e-34]Solution in 0.463s

Time Plot 0 s.
Exiting SolveHard() after 0.837r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136488914343087185456102159896187,
```

```
5.18778357838276850146583241474648378997 6,
408.65773863280731886758301740701919987 18]
one interval r = 21.71840114662361146086157616799303921691 ..
26.81849303520853279408997208433298497217
Time Approximations 0.056.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=0
Equations at solution: [0., 0., .7e-36]Solution in 0.986s

Time Plot 0 s.
Exiting SolveHard() after 5.963r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19, none, none,
361.50888347692880397869805176381616727 34, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136488914343087185456102159896187,
5.18778357838276850146583241474648378997 6,
408.65773863280731886758301740701919987 18]
one interval r = 31.80828598772969834468486234477133396377 ..
35.00011460054192817523773862019103321173
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .91e-35]Solution in 0.41s

Time Plot 0 s.
Exiting SolveHard() after 0.733r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39, none,
361.50888347692880397869805176381616727 34, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110551178145671940164586048553130,
6.19626256521366725765118771286124517 6614,
385.44474379975932973126764980765296204 57]
one interval r = 31.60836097559193927905153363972913376590 ..
34.66372795621101092578537722537499922216
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, -.76e-35]Solution in 0.573s

Time Plot 0 s.
Exiting SolveHard() after 0.832r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39, none,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110551178145671940164586048553130,
6.19626256521366725765118771286124517 6614,
385.44474379975932973126764980765296204 57]
two intervals r = 16.87563408733634973981169350758720676631 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000 or r = 15.55640493833670477201073290516680124916 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.359e-37, 0., .515e-35]Solution in 1.159s

Time Plot 0 s.
Exiting SolveHard() after 6.339r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
```

```
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39,
336.61215841896814562480780852436779244 74,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874717307759812860936798675800693,
4.8838107797119618718477950919700896539 27,
376.61967856458715178100711699962357018 71]
one interval r = 21.11001304889739101340063289361252371248 ..
26.31784243486419036442081155947340519006
Time Approximations 0.038.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .41e-35]Solution in 4.615s

Time Plot 0 s.
Exiting SolveHard() after 5.358r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39,
336.61215841896814562480780852436779244 74,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47, none,
328.46938514156800214590184162368586747 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
2 --> 0  target = [17.19898874717307759812860936798675800693,
4.8838107797119618718477950919700896539 27,
376.61967856458715178100711699962357018 71]
one interval r = 31.53899497733494892387080550293703235100 ..
34.53618386104995753608797014481084725877
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.071e-35
Equations at solution: [-.822e-35, .1071e-34, .216e-34]Solution in
0.518s

Time Plot 0 s.
Exiting SolveHard() after 0.808r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39,
336.61215841896814562480780852436779244 74,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47, none,
328.46938514156800214590184162368586747 42,
343.81340625835421703979201086489793557 49, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017554342842513497569818675193696,
6.0258135491736396174825134381633462282 30,
351.42702949100887999972889064673395318 45]
one interval r = 31.36230206136523816915213622023809869051 ..
34.17446640628552538444416963847057572165
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
```

```
12.1428029945613367595242565268234247214 4, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., -.1e-36]Solution in 0.538s

Time Plot 0 s.
Exiting SolveHard() after 0.803r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39,
336.61215841896814562480780852436779244 74,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47, none,
328.46938514156800214590184162368586747 42,
343.81340625835421703979201086489793557 49, none, none,
292.99969139254502518039123300665093977 38, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017554342842513497569818675193696,
6.0258135491736396174825134381633462282 30,
351.42702949100887999972889064673395318 45]
two intervals r = 17.981355144288396237258308832906246466 22 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000 or r = 13.846080154557332058933460765986666219 98 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000
Time Approximations 0.048.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
```

```
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.35e-37, -.2e-37, -.264e-35]Solution in 1.155s

Time Plot 0 s.
Exiting SolveHard() after 5.897r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39,
336.61215841896814562480780852436779244 74,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47,
302.31384315568848376796265994291495826 87,
328.46938514156800214590184162368586747 42,
343.81340625835421703979201086489793557 49, none, none,
292.99969139254502518039123300665093977 38, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941875041149505451641485356715791,
6.37794387373006624692953398275309761454 4,
423.28832784065720149647929391503987044 66]
one interval r = 31.94661817613276485163501103311424716208 ..
35.21212308652090535781186559613487569199
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., .317e-34]Solution in 0.603s

Time Plot 0 s.
```

```
Exiting SolveHard() after 0.967r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39,
336.61215841896814562480780852436779244 74,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47,
302.31384315568848376796265994291495826 87,
328.46938514156800214590184162368586747 42,
343.81340625835421703979201086489793557 49, none, none,
292.99969139254502518039123300665093977 38, none, none,
360.06173467160513542876082998491529336 62, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418750411495054516414853567157 91,
6.3779438737300662469295339827530976145 44,
423.28832784065720149647929391503987044 66]
two intervals r = 15.22886702435079736334964046748766899400 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000 or r = 17.12965777079290595039036831733837014179 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000
Time Approximations 0.062.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [-.17e-37, 0., .1772e-34]Solution in 1.33s

Time Plot 0 s.
Exiting SolveHard() after 6.247r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39,
336.61215841896814562480780852436779244 74,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47,
302.31384315568848376796265994291495826 87,
328.46938514156800214590184162368586747 42,
343.81340625835421703979201086489793557 49,
375.73285290259096995720321354942573682 57, none,
292.99969139254502518039123300665093977 38, none, none,
360.06173467160513542876082998491529336 62, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343485770052873705688670174052 42,
4.0035598155791115270018615536945337510 76,
404.47973594419413062410454551571188885 46]
two intervals r = 16.096839663575229753358944968201995817 22 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000 or r = 16.399886491313591314227800506335814316 54 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.67e-37, .1e-37, .2645e-34]Solution in 5.21s

Time Plot 0 s.
Exiting SolveHard() after 6.305r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39,
336.61215841896814562480780852436779244 74,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47,
302.31384315568848376796265994291495826 87,
328.46938514156800214590184162368586747 42,
343.81340625835421703979201086489793557 49,
375.73285290259096995720321354942573682 57, none,
292.99969139254502518039123300665093977 38,
358.64341561447603893795491132280271468 75, none,
360.06173467160513542876082998491529336 62, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234348577005287370568867017405242,
4.0035598155791115270018615536945337510 76,
404.47973594419413062410454551571188885 46]
one interval r = 21.634296299965287123925732302265775750 13 ..
26.757681698980480145553672318626538224 13
Time Approximations 0.053.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, -.66e-35]Solution in 5.115s

Time Plot 0 s.
Exiting SolveHard() after 6.198r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
```

```
361.525802566709210316973263390473876750 5,
401.881739045669355776871036303594203361 0,
389.590015163676819321542797132277090299 5,
328.469398940846121594980291051775982405 3,
401.507571583812914954865218260972851172 7,
358.973628245446382438488127436305417787 3,
398.331471041537475692077785867228417021 9,
371.483873952263235487737247494832199713 9,
336.612158418968145624807808524367792447 4,
361.508883476928803978698051763816167273 4,
324.671449934843917249954477124621604144 7,
302.313843155688483767962659942914958268 7,
328.469385141568002145901841623685867474 2,
343.813406258354217039792010864897935574 9,
375.732852902590969957203213549425736825 7,
328.117092950639937258539899513715418100 4,
292.999691392545025180391233006650939773 8,
358.643415614476038937954911322802714687 5, none,
360.061734671605135428760829984915293366 2, none, none, none, none,
none, none, none]

2 --> 1  target = [26.463189544691064857766960527641101349 28,
6.196177230066384111140713857167121181943,
385.427340263346538335348612233776301624 1]
one interval r = 31.608220491144265067453086185155269463 90 ..
34.663476150585260160791879208526788203 89
Time Approximations 0.016.

hint used Hint := [33.813392675100157147469018161182119369 13, 3, 1, 1,
11.783209121413521870719438174471899656 27, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, .43e-35]Solution in 0.591s

Time Plot 0 s.
Exiting SolveHard() after 0.886r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434936857928363015953761862096089 3,
441.642959732120047975050826564240373644 8,
436.917481655475951922580557375199250318 7,
422.984933975561462282526120036515662092 9,
361.525802566709210316973263390473876750 5,
401.881739045669355776871036303594203361 0,
389.590015163676819321542797132277090299 5,
```

```
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39,
336.61215841896814562480780852436779244 74,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47,
302.31384315568848376796265994291495826 87,
328.46938514156800214590184162368586747 42,
343.81340625835421703979201086489793557 49,
375.73285290259096995720321354942573682 57,
328.11709295063993725853989951371541810 04,
292.99969139254502518039123300665093977 38,
358.64341561447603893795491132280271468 75, none,
360.06173467160513542876082998491529336 62, none, none,
324.65521224447868382413500029721146060 63, none, none, none, none]

0 --> 1  target = [26.46318954469106485776696052764110134928,
6.19617723006638411114071385716712118 1943,
385.42734026334653833534861223377630162 41]
two intervals r = 16.87629600272901281059783555048297512581 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000 or r = 15.55559000681459832038564876334231742346 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.359e-37, 0., -.118e-35]Solution in 5.079s

Time Plot 0 s.
Exiting SolveHard() after 6.258r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
```

```
401.5075715838129149548652182609728511727,
358.9736282454463824384881274363054177873,
398.3314710415374756920777858672284170219,
371.4838739522632354877372474948321997139,
336.6121584189681456248078085243677924474,
361.5088834769288039786980517638161672734,
324.6714499348439172499544771246216041447,
302.3138431556884837679626599429149582687,
328.4693851415680021459018416236858674742,
343.8134062583542170397920108648979355749,
375.7328529025909699572032135494257368257,
328.1170929506399372585398995137154181004,
292.9996913925450251803912330066509397738,
358.6434156144760389379549113228027146875, none,
360.0617346716051354287608299849152933662,
336.5944103292337887712588953275823119184, none,
324.6552122444786838241350002972114606063, none, none, none, none]

0 --> 2  target = [34.4952266118025745810911678677017362094 6,
3.8971313160142424195186912492883526020 87,
373.7808188532703089682458748384518939840]
two intervals r = 17.29769086199254867804352051749309236762 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000 or r = 14.99436407474607917246898873392892542655 ..
19000000000053373378762371264157325261/10000000000000000000000000000000
00000
Time Approximations 0.088.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.36e-37, 0., -.2315e-34]Solution in 5.085s

Time Plot 0 s.
Exiting SolveHard() after 6.788r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349368579283630159537618620960893,
441.6429597321200479750508265642403736448,
436.9174816554759519225805573751992503187,
422.9849339755614622825261200365156620929,
361.5258025667092103169732633904738767505,
401.8817390456693557768710363035942033610,
389.5900151636768193215427971322770902995,
328.4693989408461215949802910517759824053,
```

```
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39,
336.61215841896814562480780852436779244 74,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47,
302.31384315568848376796265994291495826 87,
328.46938514156800214590184162368586747 42,
343.81340625835421703979201086489793557 49,
375.73285290259096995720321354942573682 57,
328.11709295063993725853989951371541810 04,
292.99969139254502518039123300665093977 38,
358.64341561447603893795491132280271468 75, none,
360.06173467160513542876082998491529336 62,
336.59441032923378877125889532758231191 84, none,
324.65521224447868382413500029721146060 63,
331.93806792484347669800425013761897135 57, none, none, none]

1 --> 2  target = [34.49522661180257458109116786770173620946,
3.89713131601424241951869124928835260208 7,
373.78081885327030896824587483845189398 40]
one interval r = 21.06068473219091639771843962183254967460 ..
26.26979834298526009718440450997903927624
Time Approximations 0.033.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [-.1e-37, -.3e-37, .532e-34]Solution in 0.793s

Time Plot 0 s.
Exiting SolveHard() after 1.509r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
```

```
358.9736282454463824384881274363054177873,
398.3314710415374756920777858672284170219,
371.4838739522632354877372474948321997139,
336.6121584189681456248078085243677924474,
361.5088834769288039786980517638161672734,
324.6714499348439172499544771246216041447,
302.3138431556884837679626599429149582687,
328.4693851415680021459018416236858674742,
343.8134062583542170397920108648979355749,
375.7328529025909699572032135494257368257,
328.1170929506399372585398995137154181004,
292.9996913925450251803912330066509397738,
358.6434156144760389379549113228027146875,
299.8986620591996120988448654379693860079,
360.0617346716051354287608299849152933662,
336.5944103292337887712588953275823119184, none,
324.6552122444786838241350002972114606063,
331.9380679248434766980042501376189713557, none, none, none]

0 --> 2  target = [33.8136249542771019204861990247761341446 1,
3.7256489936499387436937114831736222029 78,
325.8920997374894294913104098096601269387]
two intervals r = 18.552270489938343459745819795570481567 40 ..
19000000000053373378762371264157325261/1000000000000000000000000000000000
00000 or r = 12.491969358333498622255801000424205439 15 ..
19000000000053373378762371264157325261/1000000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=4e-38
Equations at solution: [-.87e-37, .4e-37, -.2489e-34]Solution in 4.983s

Time Plot 0 s.
Exiting SolveHard() after 6.453r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349368579283630159537618620960893,
441.6429597321200479750508265642403736448,
436.9174816554759519225805573751992503187,
422.9849339755614622825261200365156620929,
361.5258025667092103169732633904738767505,
401.8817390456693557768710363035942033610,
389.5900151636768193215427971322770902995,
```

```
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39,
336.61215841896814562480780852436779244 74,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47,
302.31384315568848376796265994291495826 87,
328.46938514156800214590184162368586747 42,
343.81340625835421703979201086489793557 49,
375.73285290259096995720321354942573682 57,
328.11709295063993725853989951371541810 04,
292.99969139254502518039123300665093977 38,
358.64341561447603893795491132280271468 75,
299.89866205919961209884486543796938600 79,
360.06173467160513542876082998491529336 62,
336.59441032923378871258895327582311918 4, none,
324.65521224447868382413500029721146060 63,
331.93806792484347669800425013761897135 57, none, none,
289.54595773606357723673386203967562015 24]

1 --> 2  target = [33.81362495427710192048619902477613414461,
3.72564899364993874369371148317362220297 8,
325.89209973748942949131040980966012693 87]
one interval r = 20.37468935117987178084044049809819104062 ..
25.37892165318196481051759372378792498830
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., -.356e-34]Solution in 0.591s

Time Plot 0 s.
Exiting SolveHard() after 1.174r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.64295973212004797505082656424037364 48,
436.91748165547595192258055737519925031 87,
422.98493397556146228252612003651566209 29,
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
```

```
389.5900151636768193215427971322770902995,
328.4693989408461215949802910517759824053,
401.5075715838129149548652182609728511727,
358.9736282454463824384881274363054177873,
398.3314710415374756920777858672284170219,
371.4838739522632354877372474948321997139,
336.6121584189681456248078085243677924474,
361.5088834769288039786980517638161672734,
324.6714499348439172499544771246216041447,
302.3138431556884837679626599429149582687,
328.4693851415680021459018416236858674742,
343.8134062583542170397920108648979355749,
375.7328529025909699572032135494257368257,
328.1170929506399372585398995137154181004,
292.9996913925450251803912330066509397738,
358.6434156144760389379549113228027146875,
299.8986620591996120988448654379693860079,
360.0617346716051354287608299849152933662,
336.5944103292337887712588953275823119184,
256.1075318723175604934874921676340895119,
324.6552122444786838241350002972114606063,
331.9380679248434766980042501376189713557, none, none,
289.5459577360635772367338620396756201524]

1 --> 0  target = [17.93041369694352511264703027513930719175,
4.686508701852568921640967845234639806580,
353.3054109573244905104867622722315775952]
one interval r = 20.73150479102468747914571012644687507852 ..
25.90675353535472257589163176752367802188
Time Approximations 0.037.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.5e-38
Equations at solution: [.1e-37, .25e-37, .178e-34]Solution in 5.274s

Time Plot 0 s.
Exiting SolveHard() after 6.001r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
441.6429597321200479750508265642403736448,
436.9174816554759519225805573751992503187,
422.9849339755614622825261200365156620929,
```

```
361.52580256670921031697326339047387675 05,
401.88173904566935577687103630359420336 10,
389.59001516367681932154279713227709029 95,
328.46939894084612159498029105177598240 53,
401.50757158381291495486521826097285117 27,
358.97362824544638243848812743630541778 73,
398.33147104153747569207778586722841702 19,
371.48387395226323548773724749483219971 39,
336.61215841896814562480780852436779244 74,
361.50888347692880397869805176381616727 34,
324.67144993484391724995447712462160414 47,
302.31384315568848376796265994291495826 87,
328.46938514156800214590184162368586747 42,
343.81340625835421703979201086489793557 49,
375.73285290259096995720321354942573682 57,
328.11709295063993725853989951371541810 04,
292.99969139254502518039123300665093977 38,
358.64341561447603893795491132280271468 75,
299.89866205919961209884486543796938600 79,
360.06173467160513542876082998491529336 62,
336.59441032923378877125889532758231191 84,
256.10753187231756049348749216763408951 19,
324.65521224447868382413500029721146060 63,
331.93806792484347669800425013761897135 57,
304.79958326223923711461318190879372065 26, none,
289.54595773606357723673386203967562015 24]

2 --> 0  target = [17.93041369694352511264703027513930719175,
4.68650870185256892164096784523463980658 0,
353.30541095732449051048676227223157759 52]
one interval r = 31.37435487015804015540287754437267885013 ..
34.20127520041193882199129317707872034414
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [-.5e-37, .8e-37, -.39e-35]Solution in 0.409s

Time Plot 0 s.
Exiting SolveHard() after 0.695r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493685792836301595376186209608 93,
```

```
441.642959732120047975050826564240373644 8,
436.917481655475951922580557375199250318 7,
422.984933975561462282526120036515662092 9,
361.525802566709210316973263390473876750 5,
401.881739045669355776871036303594203361 0,
389.590015163676819321542797132277090299 5,
328.469398940846121594980291051775982405 3,
401.507571583812914954865218260972851172 7,
358.973628245446382438488127436305417787 3,
398.331471041537475692077785867228417021 9,
371.483873952263235487737247494832199713 9,
336.612158418968145624807808524367792447 4,
361.508883476928803978698051763816167273 4,
324.671449934843917249954477124621604144 7,
302.313843155688483767962659942914958268 7,
328.469385141568002145901841623685867474 2,
343.813406258354217039792010864897935574 9,
375.732852902590969957203213549425736825 7,
328.117092950639937258539899513715418100 4,
292.999691392545025180391233006650939773 8,
358.643415614476038937954911322802714687 5,
299.898662059199612098844865437969386007 9,
360.061734671605135428760829984915293366 2,
336.594410329233788771258895327582311918 4,
256.107531872317560493487492167634089511 9,
324.655212244478683824135000297211460606 3,
331.938067924843476698004250137618971355 7,
304.799583262239237114613181908793720652 6,
323.461691776506777017829817880301586934 9,
289.545957736063577236733862039675620152 4]

Cascade time 166.962
counts: 28, 28

 Iteration 86

Start Generation 1
1 --> 0  target = [11.999999999883187097178102827227148091 00,
6.217012502996920409168582329134007862951,
485.549080897276217646277306479275978929 8]
one interval r = 23.408503016535478751739747405293971703 71 ..
27.675780464293456322748379367880816990 20
Time Approximations 0.043.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.64e-37
Equations at solution: [-.9e-37, .264e-36, .7e-36]Solution in 1.072s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.298r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999883187097178102827227148091 00,
6.2170125029969204091685823291340078629 51,
485.54908089727621764627730647927597892 98]
one interval r = 32.628147792168002190257735270289719179 64 ..
36.102483889424312164756211342824954749 66
Time Approximations 0.024.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .71e-35]Solution in 4.467s

Time Plot 0 s.
Exiting SolveHard() after 4.893r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844808393545475189386820979410 84,
6.5834347216371783424657604076912257776 30,
467.78730595802748599636734218174775965 78]
one interval r = 32.419789556666737738536848284902706409 85 ..
35.851524173724730824871251360894336526 42
Time Approximations 0.021.
```

```
hint used Hint := [34.9450788879772991463756999080445721474 1, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, -.118e-34]Solution in 0.674s

Time Plot 0 s.
Exiting SolveHard() after 1.072r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17, none, none,
401.88173904213125864720282490683345139 83, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684480839354547518938682097941084,
6.58343472163717834246576040769122577763 0,
467.78730595802748599636734218174775965 78]
two intervals r = 12.92327160818117852153576308770237547963 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000 or r = 18.39424858022885449304623838177435114694 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=0
Equations at solution: [0., 0., .11449e-34]Solution in 45.419s

Time Plot 0 s.
Exiting SolveHard() after 46.853r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23, none,
401.88173904213125864720282490683345139 83, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962826169510337481891791834107669,
4.12565179687164029739004773459755718 5824,
440.67123064929028948202390544141766088 24]
two intervals r = 14.35659705107787715187988402316225633039 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000 or r = 17.70352613799059109510304283480281511569 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000
Time Approximations 0.054.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., .26706e-34]Solution in 1.35s

Time Plot 0 s.
Exiting SolveHard() after 6.113r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23, none,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962826169510337481891791834107669,
4.12565179687164029739004773459755718 5824,
440.67123064929028948202390544141766088 24]
one interval r = 22.39761154364273868214595905107989616069 ..
```

```
27.23722351598250563287524572467175903560
Time Approximations 0.039.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 5.062 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064385006719055127849479899906147, rm =
14.37818770421913044651333504719387739331}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, -.161e-34]Solution in
13.166s

Time Plot 0 s.
Exiting SolveHard() after 14.103r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888800335230765073559057575874034,
4.00486908185131939910674926305942221037 2,
404.86224501279890378726758639852018544 29]
two intervals r = 16.08011007746263710674068520852953714148 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000 or r = 16.41579812686601414983294933892961512263 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
```

```
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.34e-37, 0., .13734e-34]Solution in 5.598s

Time Plot 0 s.
Exiting SolveHard() after 6.744r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87, none, none,
358.97362823875403957888168064343688969 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888003352307650735590575758740 34,
4.0048690818513193991067492630594222103 72,
404.86224501279890378726758639852018544 29]
one interval r = 21.641943994111365625219308125629248283 11 ..
26.763306600423992713600398413853225608 29
Time Approximations 0.059.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., .648e-34]Solution in 5.08s

Time Plot 0 s.
Exiting SolveHard() after 6.175r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58, none,
358.97362823875403957888168064343688969 19, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941768178194973467685601332290377,
5.5896371830012696357841903601088758229 13,
443.83065884561761738881487098578465093 07]
one interval r = 22.46725374477634999723128440373027642388 ..
27.27388428355668254344950066122281828710
Time Approximations 0.041.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., -.133e-34]Solution in 1.032s

Time Plot 0 s.
Exiting SolveHard() after 2.047r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58, none,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

2 --> 0  target = [14.19258941768178194973467685601332290377,
5.5896371830012696357841903601088758229 13,
443.83065884561761738881487098578465093 07]
```

```
one interval r = 32.15575279506190922646775408333615543644 ..
35.50872228739577701926670519933811489181
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., -.35e-35]Solution in 0.472s

Time Plot 0 s.
Exiting SolveHard() after 0.831r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136500536340997020033660447411306,
5.18778357853469446138888935920431896483 2,
408.65773862589404404573805165034261229 73]
one interval r = 21.71840114648887081023339444039970223697 ..
26.81849303511453168243537691291802270961
Time Approximations 0.06.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
```

```
Accepted {r=26.4632, rm=15.9013} with Delta=1.32e-37
Equations at solution: [-.1e-37, -.132e-36, .190e-34]Solution in 1.014s

Time Plot 0 s.
Exiting SolveHard() after 6.222r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81, none, none,
361.50888347062034258152690478881533697 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136500536340997020033660447411306,
5.18778357853469446138888935920431896483 2,
408.65773862589404404573805165034261229 73]
one interval r = 31.80828598755696553323374004421854638711 ..
35.00011460046341826866315251714567648988
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .443e-34]Solution in 0.42s

Time Plot 0 s.
Exiting SolveHard() after 0.75r=34.4952 in [32.91337941 .. 35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
```

```
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25, none,
361.50888347062034258152690478881533697 25, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110540371398202658832874984837924,
6.1962625653689113645053776454382698077 46,
385.44474379274251395832899438273584790 25]
one interval r = 31.608360975416725765114237141124799158 37 ..
34.663727956118370194435114588912201538 50
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, -.7e-36]Solution in 4.356s

Time Plot 0 s.
Exiting SolveHard() after 4.628r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25, none,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110540371398202658832874984837924,
6.196262565368911364505377645438269807746,
385.44474379274251395832899438273584790 25]
two intervals r = 16.875634087414661269411584317558229864 16 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000 or r = 15.556404937953113396782760135634696489 68 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 8.266 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175110347837507682179865252372956, rm
= 2.33653277404195615534987994323246984595 1}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.179e-37, 0., -.48056e-34]Solution in 33.676s

Time Plot 0 s.
Exiting SolveHard() after 34.865r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.198988747227500052833632938636180985 83,
```

```
4.883810779867955829905615431083633487052,
376.61967855777010258264730292046343482 56]
one interval r = 21.11001304878324115938312687206740419169 ..
26.31784243475495527272150751545516442432
Time Approximations 0.036.

hint used Hint := [25.872049912068339523744720204705511942 63, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=1.27e-37
Equations at solution: [.4e-37, .127e-36, .243e-34]Solution in 0.876s

Time Plot 0 s.
Exiting SolveHard() after 1.594r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89, none,
328.46938513543186224156211489560234313 85, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874722750005283363293863618098583,
4.883810779867955829905615431083633487052,
376.61967855777010258264730292046343482 56]
one interval r = 31.53899497716110473706965834930027990623 ..
34.53618386095551672169212458763153749116
Time Approximations 0.018.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
```

```
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=9.30e-36
Equations at solution: [-.714e-35, .930e-35, .458e-34]Solution in
0.509s

Time Plot 0 s.
Exiting SolveHard() after 0.801r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89, none,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017542417955502930471434686904176,
6.0258135493288644582225491475108673118 19,
351.42702948428198925425161500027113629 28]
one interval r = 31.362302061192586786748129170050679498 38 ..
34.174466406179392450663491802490450473 24
Time Approximations 0.016.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .224e-34]Solution in 0.53s
```

```
Time Plot 0 s.
Exiting SolveHard() after 4.447r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89, none,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59, none, none,
292.99969138383561940197799066283216656 86, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175424179555029304714346869041 76,
6.0258135493288644582225491475108673118 19,
351.42702948428198925425161500027113629 28]
two intervals r = 17.981355144292865006621877932931839094 05 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000 or r = 13.846080154171770351943675946078817215 89 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.834 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713589777791751295556264619909 04, rm
= 2.7345009930904225517383864838220153687 29}});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [-.19e-37, .1e-37, -.11037e-34]Solution in
20.61s
```

```
Time Plot 0 s.
Exiting SolveHard() after 21.7r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89,
302.31384314925573900123657678387544288 75,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59, none, none,
292.99969138383561940197799066283216656 86, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418751216256053711128952111598 11,
6.3779438739179852257601507950571658197 99,
423.28832784030141133465269526563187228 27]
one interval r = 31.946618176025909405092108482079193035 36 ..
35.212123086545478641425301623736584128 82
Time Approximations 0.02.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=8e-38
Equations at solution: [-.7e-37, .8e-37, -.29e-35]Solution in 0.655s

Time Plot 0 s.
Exiting SolveHard() after 1.012r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89,
302.31384314925573900123657678387544288 75,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59, none, none,
292.99969138383561940197799066283216656 86, none, none,
360.06173466894394931754485659718880182 16, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941875121625605371112895211159811,
6.37794387391798522576015079505716581979 9,
423.28832784030141133465269525631872282 7]
two intervals r = 15.22886702417368615409753066062725531640 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000 or r = 17.12965777070041785673512460573935695560 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000
Time Approximations 0.063.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 8.037 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.51537054112856771125959128263723702667, rm
= 2.06406829868160196674525004619466186318 1}});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [.63e-37, .2e-37, .29177e-34]Solution in 33.863s

Time Plot 0 s.
Exiting SolveHard() after 39.133r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89,
302.31384314925573900123657678387544288 75,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59,
375.73285290267061578112438985118545528 09, none,
292.99969138383561940197799066283216656 86, none, none,
360.06173466894394931754485659718880182 16, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234344432527858819514089417147206,
4.0035598155518505418573356977973532680 75,
404.47973593999584209849498835506011911 50]
two intervals r = 16.09683966356555952846322026732716229924 ..
189999999999151416948757371153289064 19/10000000000000000000000000000000
00000 or r = 16.39988649107092619967633806408092440288 ..
189999999999151416948757371153289064 19/10000000000000000000000000000000
00000
Time Approximations 0.054.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.33e-37, .1e-37, .769e-36]Solution in 5.711s

Time Plot 0 s.
Exiting SolveHard() after 6.808r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
```

```
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89,
302.31384314925573900123657678387544288 75,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59,
375.73285290267061578112438985118545528 09, none,
292.99969138383561940197799066283216656 86,
358.64341560737168900207869478080072133 88, none,
360.06173466894394931754485659718880182 16, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234344432527858819514089417147206,
4.0035598155518505418573356977973532680 75,
404.47973593999584209849498835506011911 50]
one interval r = 21.634296299886843140751442943882126283 95 ..
26.757681698924149548955241218796136838 65
Time Approximations 0.053.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, -.206e-34]Solution in 1.14s

Time Plot 0 s.
Exiting SolveHard() after 2.208r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
```

```
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89,
302.31384314925573900123657678387544288 75,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59,
375.73285290267061578112438985118545528 09,
328.11709294406218144128606794019120301 89,
292.99969138383561940197799066283216656 86,
358.64341560737168900207869478080072133 88, none,
360.06173466894394931754485659718880182 16, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954458496562877548725598795097917,
6.19617723022222679350919106964848946604 4,
385.42734025645193778270990768219736212 11]
one interval r = 31.60822049097003750056663615747564342311 ..
34.66347615049437840993528618516822354922
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .394e-34]Solution in 0.575s

Time Plot 0 s.
Exiting SolveHard() after 4.538r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
```

```
398.3314710415857420077573000813427779581,
371.4838739425313038795136981524259834625,
336.6121584121698857476917557471017262190,
361.5088834706203425815269047888153369725,
324.6714499259026972830331088145746488689,
302.3138431492557390012365767838754428875,
328.4693851354318622415621148956023431385,
343.8134062483659481306842645900847414059,
375.7328529026706157811243898511854552809,
328.1170929440621814412860679401912030189,
292.9996913838356194019779906628321665686,
358.6434156073716890020786947808007213388, none,
360.0617346689439493175448565971888018216, none, none,
324.6552122356514741043387677079701139681, none, none, none, none]

0 --> 1  target = [26.46318954458496562877548725598795097917,
6.196177230222226793509191069648489466044,
385.4273402564519377827099076821973621211]
two intervals r = 16.87629600280264346396793770907258963696 ..
189999999999151416948757371153289064 19/10000000000000000000000000000000
00000 or r = 15.555590006436711914908665167231139896 40 ..
189999999999151416948757371153289064 19/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 8.041 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852522156976862294304313930874668, rm
= 2.33669042810768043430476988563057100473 6}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.359e-37, 0., -.12978e-34]Solution in 33.864s

Time Plot 0 s.
Exiting SolveHard() after 35.064r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
```

```
389.590015156904654095085181263459557948 7,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89,
302.31384314925573900123657678387544288 75,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59,
375.73285290267061578112438985118545528 09,
328.11709294406218144128606794019120301 89,
292.99969138383561940197799066283216656 86,
358.64341560737168900207869478080072133 88, none,
360.06173466894394931754485659718880182 16,
336.59441032256020602721922812280376569 05, none,
324.65521223565147410433876770797011396 81, none, none, none, none]

0 --> 2  target = [34.49522661165990119715947727639343875614,
3.897131315966131544779980549967184453289,
373.78081884321210091955482088229546388 33]
two intervals r = 17.29769086215287282816792732789052558264 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000 or r = 14.99436407420167351529909820372865542886 ..
189999999999151416948757371153289064 19/100000000000000000000000000000000
00000
Time Approximations 0.092.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [-.72e-37, .1e-37, .25626e-34]Solution in 5.257s

Time Plot 0 s.
Exiting SolveHard() after 6.977r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
```

```
389.590015156904654095085181263459557948 7,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89,
302.31384314925573900123657678387544288 75,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59,
375.73285290267061578112438985118545528 09,
328.11709294406218144128606794019120301 89,
292.99969138383561940197799066283216656 86,
358.64341560737168900207869478080072133 88, none,
360.06173466894394931754485659718880182 16,
336.59441032256020602721922812280376569 05, none,
324.65521223565147410433876770797011396 81,
331.93806791253467180949223015875810050 42, none, none, none]

1 --> 2  target = [34.49522661165990119715947727639343875614,
3.897131315966131544779980549967184453289,
373.78018184321210091955482088229546388 33]
one interval r = 21.06068473202292564498395591449246628346 ..
26.26979834281946113336841907875609328513
Time Approximations 0.035.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., .773e-34]Solution in 0.763s

Time Plot 0 s.
Exiting SolveHard() after 1.488r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.590015156904654095085181263459557948 7,
```

```
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89,
302.31384314925573900123657678387544288 75,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59,
375.73285290267061578112438985118545528 09,
328.11709294406218144128606794019120301 89,
292.99969138383561940197799066283216656 86,
358.64341560737168900207869478080072133 88,
299.89866204726016370209288807314749141 29,
360.06173466894394931754485659718880182 16,
336.59441032256020602721922812280376569 05, none,
324.65521223565147410433876770797011396 81,
331.93806791253467180949223015875810050 42, none, none, none]

0 --> 2  target = [33.81362495412317038310206309480082701981,
3.72564899360327038576746970933616023477 8,
325.89209972823358046931209991563817113 21]
two intervals r = 18.552270489939854201221508066198984017 47 ..
189999999999151416948757371153289064 19/1000000000000000000000000000000000
00000 or r = 12.491969357839851039161699887457975154 67 ..
189999999999151416948757371153289064 19/1000000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [.69e-37, -.3e-37, -.19674e-34]Solution in
1.125s

Time Plot 0 s.
Exiting SolveHard() after 6.504r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
```

```
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89,
302.31384314925573900123657678387544288 75,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59,
375.73285290267061578112438985118545528 09,
328.11709294406218144128606794019120301 89,
292.99969138383561940197799066283216656 86,
358.64341560737168900207869478080072133 88,
299.89866204726016370209288807314749141 29,
360.06173466894394931754485659718880182 16,
336.59441032256020602721922812280376569 05, none,
324.65521223565147410433876770797011396 81,
331.93806791253467180949223015875810050 42, none, none,
289.54595772443463064945077862744796276 48]

1 --> 2  target = [33.81362495412317038310206309480082701981,
3.72564899360327038576746970933616023477 8,
325.89209972823358046931209991563817113 21]
one interval r = 20.37468935108232582524396032386154498485 ..
25.37892165300186001294735902609980760079
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, .261e-34]Solution in 4.243s

Time Plot 0 s.
Exiting SolveHard() after 4.801r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
```

```
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89,
302.31384314925573900123657678387544288 75,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59,
375.73285290267061578112438985118545528 09,
328.11709294406218144128606794019120301 89,
292.99969138383561940197799066283216656 86,
358.64341560737168900207869478080072133 88,
299.89866204726016370209288807314749141 29,
360.06173466894394931754485659718880182 16,
336.59441032256020602721922812280376569 05,
256.10753186116650556603920025762535436 63,
324.65521223565147410433876770797011396 81,
331.93806791253467180949223015875810050 42, none, none,
289.54595772443463064945077862744796276 48]

1 --> 0  target = [17.93041369695690594758917945721485600405,
4.68650870200846282006782531643126868663 5,
353.30541095040119672753556868980066406 06]
one interval r = 20.73150479092697442451267343292399244520 ..
25.90675353523266870772432210212900272886
Time Approximations 0.039.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=7.1e-38
Equations at solution: [.3e-37, .71e-37, .78e-35]Solution in 0.672s

Time Plot 0 s.
Exiting SolveHard() after 1.411r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
```

```
441.642959731185492565535216922661915024 7,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89,
302.31384314925573900123657678387544288 75,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59,
375.73285290267061578112438985118545528 09,
328.11709294406218144128606794019120301 89,
292.99969138383561940197799066283216656 86,
358.64341560737168900207869478080072133 88,
299.89866204726016370209288807314749141 29,
360.06173466894394931754485659718880182 16,
336.59441032256020602721922812280376569 05,
256.10753186116650556603920025762535436 63,
324.65521223565147410433876770797011396 81,
331.93806791253467180949223015875810050 42,
304.79958325604393721919861359380463667 96, none,
289.54595772443463064945077862744796276 48]

2 --> 0  target = [17.93041369695690594758917945721485600405,
4.68650870200846282006782531643126868663 5,
353.30541095040119672753556868980066406 06]
one interval r = 31.37435486998401369561069403372413770163 ..
34.20127520030394725466472568216661165048
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.7e-36]Solution in 0.355s

Time Plot 0 s.
Exiting SolveHard() after 0.616r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493513960850141011495715221441 72,
441.64295973118549256553521692266191502 47,
436.91748165164473169752583444886286228 17,
422.98493397459573417919219781173081648 23,
361.52580256028193297011101209881459134 06,
401.88173904213125864720282490683345139 83,
389.59001515690465409508518126345955794 87,
328.46939893470636754559192216778076605 58,
401.50757157980948914752841283174236632 75,
358.97362823875403957888168064343688969 19,
398.33147104158574200775730008134277795 81,
371.48387394253130387951369815242598346 25,
336.61215841216988574769175574710172621 90,
361.50888347062034258152690478881533697 25,
324.67144992590269728303310881457464886 89,
302.31384314925573900123657678387544288 75,
328.46938513543186224156211489560234313 85,
343.81340624836594813068426459008474140 59,
375.73285290267061578112438985118545528 09,
328.11709294406218144128606794019120301 89,
292.99969138383561940197799066283216656 86,
358.64341560737168900207869478080072133 88,
299.89866204726016370209288807314749141 29,
360.06173466894394931754485659718880182 16,
336.59441032256020602721922812280376569 05,
256.10753186116650556603920025762535436 63,
324.65521223565147410433876770797011396 81,
331.93806791253467180949223015875810050 42,
304.79958325604393721919861359380463667 96,
323.46169176626601627824285270014566417 93,
289.54595772443463064945077862744796276 48]

Cascade time 276.757
counts: 28, 28

 Iteration 87

Start Generation 1
1 --> 0  target = [12.00000000005986150144087835133393693200,
6.21701250287082013369231798948069649208 7,
485.54908089219924512276593779473723206 43]
one interval r = 23.40850301639997481699734005530617197519 ..
27.67578046412159720988384659891629178853
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
```

```
Accepted {r=27.5236, rm=6.49211} with Delta=2.5e-38
Equations at solution: [.1e-37, -.25e-37, -.116e-36]Solution in 4.802s

Time Plot 0 s.
Exiting SolveHard() after 5.977r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000005986150144087835133393693200,
6.217012502870820133692317989480696492087,
485.549080892199245122765937794737232 0643]
one interval r = 32.628147792141128936682078959225693732 49 ..
36.10248388934878511684526420944897161248
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, -.93e-35]Solution in 0.605s

Time Plot 0 s.
Exiting SolveHard() after 1.037r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684463371032547780366792904760550,
6.583434721547223807677359932753108199921,
467.787305953030077984936353564696698 5619]
```

```
one interval r = 32.41978955664621821272974778331101239323 ..
35.85152417365193396724450906972044232177
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.121e-34]Solution in 0.636s

Time Plot 0 s.
Exiting SolveHard() after 0.999r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39, none, none,
401.88173903619183596452557191585627099 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684463371032547780366792904760550,
6.58343472154722380767735993275310819992 1,
467.78730595303007798493635356469669856 19]
```

"Imaginary part neglected: ", $1.10311211488726617719536011599434034301 4 \times 10^{-17}$

```
two intervals r = 12.92327160836992453226905380903906943300 ..
23749999999874649999240741619454327 03/125000000000000000000000000000000
000 or r = 18.39424858011794740583332322979870700035 ..
23749999999874649999240741619454327 03/125000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.2e-38
```

```
Equations at solution: [.1e-37, .22e-37, -.967e-35]Solution in 43.473s

Time Plot 0 s.
Exiting SolveHard() after 48.561r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54, none,
401.88173903619183596452557191585627099 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962817833712064104094194789090868,
4.12565179666377806385866737454019983 1447,
440.67123064335187328751071785823309295 53]
```

"Imaginary part neglected: ", $1.10311211488726617719536011599434034301 4 \times 10^{-17}$

```
two intervals r = 14.35659705131599911930617971925638028963 ..
23749999999874649999240741619454327 03/12500000000000000000000000000000000
000 or r = 17.70352613780603591052512378328770282785 ..
23749999999874649999240741619454327 03/12500000000000000000000000000000000
000
Time Approximations 0.052.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [-.14e-37, -.1e-37, -.828e-35]Solution in 1.345s

Time Plot 0 s.
Exiting SolveHard() after 6.353r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
```

```
422.98493396965485924203583391481360563 54, none,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962817833712064104094194789090868,
4.12565179666377806385866737454019983144 7,
440.67123064335187328751071785823309295 53]
one interval r = 22.39761154352130074832218306441186907306 ..
27.23722351579290800892413982649060325456
Time Approximations 0.041.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 5.14 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064357854275774206670947519519732, rm =
14.37818770035240492244023515370858865421}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, .24182e-34]Solution in 12.602s

Time Plot 0 s.
Exiting SolveHard() after 13.542r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888792224603366092919878109835967,
4.00486908164225533452330277438353570619 7,
404.86224500667927424690225906392804724 04]
```

"Imaginary part neglected: ", $1.10311211488726617719536011599434034301 4 \times 10^{-17}$

```
two intervals r = 16.08011007767543226380304097060467540079 ..
237499999998746499992407416194543270 3/125000000000000000000000000000000
000 or r = 16.41579812660543118716307195702131401762 ..
237499999998746499992407416194543270 3/125000000000000000000000000000000
000
Time Approximations 0.06.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.67e-37, -.1e-37, .850e-35]Solution in 5.769s

Time Plot 0 s.
Exiting SolveHard() after 6.93r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50, none, none,
358.97362823133274759152355018160956544 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888792224603366092919878109835967,
4.00486908164225533452330277438353570619 7,
404.86224500667927424690225906392804724 04]
one interval r = 21.64194399402093559129405355529797938570 ..
26.76330660022666616957152962780215939753
Time Approximations 0.058.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
```

```
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.1856e-35]Solution in
4.854s

Time Plot 0 s.
Exiting SolveHard() after 5.944r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33, none,
358.97362823133274759152355018160956544 64, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941787709781175061743672674760093,
5.58963718289330770801663131551316386612 1,
443.83065884052847179490798491141087414 38]
one interval r = 22.46725374467094752174307261176152227459 ..
27.27388428337743972046094469318330925675
Time Approximations 0.037.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., .3230e-35]Solution in 1.029s

Time Plot 0 s.
Exiting SolveHard() after 2.011r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
```

```
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33, none,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941787709781175061743672674760093,
5.5896371828933077080166313155131638661 21,
443.83065884052847179490798491141087414 38]
one interval r = 32.15575279504799566129838497725506062354 ..
35.50872228732427373645313261850491785474
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=1.1e-37
Equations at solution: [.14e-36, -.11e-36, .121e-34]Solution in 0.465s

Time Plot 0 s.
Exiting SolveHard() after 0.824r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136527453429323417722564170472000,
5.1877835784143760118662694114910551714 45,
408.65773861862485167607447033430388689 94]
```

```
one interval r = 21.71840114637110420440588160349620239240 ..
26.81849303490108126896787980448468186457
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.6e-38
Equations at solution: [0., .26e-37, .130e-36]Solution in 1.051s

Time Plot 0 s.
Exiting SolveHard() after 5.711r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56, none, none,
361.50888346348265963497235840862079506 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136527453429323417722564170472000,
5.18778357841437601186626941149105517 1445,
408.65773861862485167607447033430388689 94]
one interval r = 31.80828598753487953843225087145869871664 ..
35.00011460036506288087258762990309863284
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
```

```
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.84e-35]Solution in 0.412s

Time Plot 0 s.
Exiting SolveHard() after 0.742r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82, none,
361.50888346348265963497235840862079506 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110518410145638638151401909779512,
6.19626255626204577053122123829582460634 6,
385.44474378533376958353279453231140115 26]
one interval r = 31.60836097540382472422898725852729723818 ..
34.66372795602182874016530199909099553511
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.278e-34]Solution in 4.666s

Time Plot 0 s.
Exiting SolveHard() after 4.927r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82, none,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110518410145638638151401909779512,
6.19626256526204577053122123829582460634 6,
385.44474378533376958353279453231140115 26]

"Imaginary part neglected: ", 1.10311211488726617719536011599434034301 4 × 10^-17
two intervals r = 16.87563408764878749951344946162161026684 ..
23749999999874649999240741619454327 03/125000000000000000000000000000000
000 or r = 15.55640493759806211328810441560245693019 ..
23749999999874649999240741619454327 03/125000000000000000000000000000000
000
Time Approximations 0.063.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 7.777 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175120398663017799410604979070380, rm
= 2.33653277419236923462750290534857203023 9}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.718e-37, 0., -.3108e-34]Solution in 33.698s

Time Plot 0 s.
Exiting SolveHard() after 34.904r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
```

```
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874745181693657852219038593494774,
4.8838107797534188197448962374266045173 22,
376.61967855014304341578051827833974383 22]
one interval r = 21.11001304869841258990966548549849735420 ..
26.31784243453059644241676405731004246747
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.4637e-35]Solution in 0.868s

Time Plot 0 s.
Exiting SolveHard() after 1.578r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
```

```
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26, none,
328.46938512806722636126486121304236439 54, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874745181693657852219038593494774,
4.8838107797534188197448962374266045173 22,
376.61967855014304341578051827833974383 22]
one interval r = 31.53899497715059120441053896809875413895 ..
34.53618386085746688290502692372982534699
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.125e-35
Equations at solution: [-.864e-35, .1125e-34, -.100e-34]Solution in
4.455s

Time Plot 0 s.
Exiting SolveHard() after 4.743r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26, none,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017519903563419116853816328651378,
6.0258135492183035757737661732082064115 15,
351.42702947670794884112433995847415051 83]
```

```
one interval r = 31.36230206119444630097639799344718969427 ..
34.17446640608738894209160897797664728046
Time Approximations 0.015.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .176e-34]Solution in 0.556s

Time Plot 0 s.
Exiting SolveHard() after 0.818r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26, none,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82, none, none,
292.99969137540887116275263969976001154 57, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017519903563419116853816328651378,
6.02581354921830357577376617320820641151 5,
351.42702947670794884112433995847415051 83]
```

"Imaginary part neglected: ", $1.10311211488726617719536011599434034301 4 \times 10^{-17}$

```
two intervals r = 17.98135514446029751038671660609780046911 ..
23749999999874649999240741619454327 03/125000000000000000000000000000000000
000 or r = 13.84608015375777748602558974270141363383 ..
23749999999874649999240741619454327 03/125000000000000000000000000000000000
000
```

```
Time Approximations 0.053.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 6.949 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713622047395476287917053695826 51, rm
= 2.7345009933487979605765329342181065063 12}});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [-.73e-37, .3e-37, .36e-36]Solution in 21.002s

Time Plot 0 s.
Exiting SolveHard() after 22.088r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82, none, none,
292.99969137540887116275263969976001154 57, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418570059596355861938356104143 36,
6.3779438738249926684792348732616492113 86,
423.28832783522279247957996397863257096 86]
one interval r = 31.946618176018898718728690871775365340 53 ..
35.212123086476768729313515937170576169 71
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
```

```
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, .59e-35]Solution in 4.646s

Time Plot 0 s.
Exiting SolveHard() after 5.005r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82, none, none,
292.99969137540887116275263969976001154 57, none, none,
360.06173466289512869935076904679272707 36, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941857005959635586193835610414336,
6.37794387382499266847923487326164921138 6,
423.28832783522279247957996397863257096 86]
```

"Imaginary part neglected: ", $1.10311211488726617719536011599434034301 4 \times 10^{-17}$

```
two intervals r = 15.22886702435731698581979526174725032786 ..
23749999999874649999240741619454327 03/12500000000000000000000000000000
000 or r = 17.12965777051270317977250343587325919498 ..
23749999999874649999240741619454327 03/12500000000000000000000000000000
000
Time Approximations 0.065.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
```

```
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 7.644 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541231802131006887762411814479 70, rm
= 2.06406829875840727123633989350951776 0547}});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [-.47e-37, -.2e-37, .99e-36]Solution in 33.479s

Time Plot 0 s.
Exiting SolveHard() after 34.819r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82,
375.73285289778012317605239643958330755 55, none,
292.99969137540887116275263969976001154 57, none, none,
360.06173466289512869935076904679272707 36, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234335769795125543539057605152851,
4.00355981534146264572567918167109944838 7,
404.47973593349136455766861845495261434 79]
```

"Imaginary part neglected: ", $1.10311211488726617719536011599434034301 4 \times 10^{-17}$

```
two intervals r = 16.09683966379466595506025967288007027299 ..
2374999999987464999924074161945432703/125000000000000000000000000000000
000 or r = 16.39988649079360110801949141007421209560 ..
```

```
2374999999987464999924074161945432703/125000000000000000000000000000000
000
Time Approximations 0.06.

hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [-.17e-37, 0., -.719e-35]Solution in 5.703s

Time Plot 0 s.
Exiting SolveHard() after 6.814r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82,
375.73285289778012317605239643958330755 55, none,
292.99969137540887116275263969976001154 57,
358.64341559961617054717201249473509418 66, none,
360.06173466289512869935076904679272707 36, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234335769795125543539057605152851,
4.0035598153414626457256791816710994483 87,
404.47973593349136455766861845495261434 79]
one interval r = 21.63429629978913365438257205494360270821 ..
26.75768169872111826673263905246519456217
Time Approximations 0.055.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
```

```
16.7792152583743340383255391208949113668O, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, .74883e-34]Solution in
5.023s

Time Plot 0 s.
Exiting SolveHard() after 6.099r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82,
375.73285289778012317605239643958330755 55,
328.11709293634097540807326411992949704 27,
292.99969137540887116275263969976001154 57,
358.64341559961617054717201249473509418 66, none,
360.06173466289512869935076904679272707 36, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954436667183145882378874173392447,
6.19617723011576019395493112870942595725 5,
385.42734024912485040238773670108490393 50]
one interval r = 31.60822049095780353691408372085436729907 ..
34.66347615039902155911461976119786360725
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=1.4e-37
Equations at solution: [-.10e-36, .14e-36, .275e-34]Solution in 0.575s

Time Plot 0 s.
Exiting SolveHard() after 0.881r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82,
375.73285289778012317605239643958330755 55,
328.11709293634097540807326411992949704 27,
292.99969137540887116275263969976001154 57,
358.64341559961617054717201249473509418 66, none,
360.06173466289512869935076904679272707 36, none, none,
324.65521222747513408815321521879534120 05, none, none, none, none]

0 --> 1  target = [26.46318954436667183145882378874173392447,
6.19617723011576019395493112870942595725 5,
385.42734024912485040238773670108490393 50]
```

"Imaginary part neglected: ", $1.10311211488726617719536011599434034301 4 \times 10^{-17}$

```
two intervals r = 16.87629600303363053158511588167589260630 ..
23749999999874649999240741619454327 03/125000000000000000000000000000000
000 or r = 15.55559000608545058338628417343642386885 ..
23749999999874649999240741619454327 03/125000000000000000000000000000000
000
Time Approximations 0.066.
```

```
hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 8.632 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852532050939973883363634402066013, rm
= 2.33669042825739177681385307506951545112 4}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.718e-37, 0., -.413e-35]Solution in 34.646s

Time Plot 0 s.
Exiting SolveHard() after 35.843r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82,
375.73285289778012317605239643958330755 55,
328.11709293634097540807326411992949704 27,
292.99969137540887116275263969976001154 57,
358.64341559961617054717201249473509418 66, none,
360.06173466289512869935076904679272707 36,
336.59441031550174514805053892664619628 07, none,
324.65521222747513408815321521879534120 05, none, none, none, none]

0 --> 2  target = [34.49522661154533190809176049955847246463,
3.89713131574710846637645938692385450465 6,
373.78081883440111454040637270466632398 10]
```

"Imaginary part neglected: ", $1.10311211488726617719536011599434034301 4 \times 10^{-17}$

```
two intervals r = 17.29769086241181174623838140685319839902 ..
237499999998746499992407416194543270 3/125000000000000000000000000000000000
000 or r = 14.99436407375606494483117824628564401235 ..
237499999998746499992407416194543270 3/125000000000000000000000000000000000
000
Time Approximations 0.104.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., .1942e-34]Solution in 1.175s

Time Plot 0 s.
Exiting SolveHard() after 6.713r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82,
375.73285289778012317605239643958330755 55,
328.11709293634097540807326411992949704 27,
292.99969137540887116275263969976001154 57,
358.64341559961617054717201249473509418 66, none,
360.06173466289512869935076904679272707 36,
336.59441031550174514805053892664619628 07, none,
324.65521222747513408815321521879534120 05,
331.93806790262370745103951610021066693 28, none, none, none]
```

```
1 --> 2  target = [34.49522661154533190809176049955847246463,
3.89713131574710846637645938692385450465 6,
373.78081883440111454040637270466623981 0]
one interval r = 21.06068473192154337288925932346108694962 ..
26.26979834257461879011194044608410002623
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [.2e-37, .5e-37, .23169e-34]Solution in 0.766s

Time Plot 0 s.
Exiting SolveHard() after 1.513r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82,
375.73285289778012317605239643958330755 55,
328.11709293634097540807326411992949704 27,
292.99969137540887116275263969976001154 57,
358.64341559961617054717201249473509418 66,
299.89866203744056025233240704577969028 61,
360.06173466289512869935076904679272707 36,
336.59441031550174514805053892664619628 07, none,
324.65521222747513408815321521879534120 05,
331.93806790262370745103951610021066693 28, none, none, none]
```

```
0 --> 2  target = [33.81362495402456288697143842380555708898,
3.72564899338428580358843844437849259029 2,
325.89209971974173957011600470672493762 45]
```

"Imaginary part neglected: ", $1.10311211488726617719536011599434034301 4 \times 10^{-17}$

```
two intervals r = 18.552270490064053988243764069749521676 20 ..
237499999998746499992407416194543270 3/125000000000000000000000000000000000
000 or r = 12.49196935737584803836559926763183298338 ..
237499999998746499992407416194543270 3/125000000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1e-38
Equations at solution: [.36e-37, -.1e-37, -.5e-37]Solution in 1.123s

Time Plot 0 s.
Exiting SolveHard() after 6.234r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82,
375.73285289778012317605239643958330755 55,
328.11709293634097540807326411992949704 27,
292.99969137540887116275263969976001154 57,
358.64341559961617054717201249473509418 66,
299.89866203744056025233240704577969028 61,
```

```
360.06173466289512869935076904679272707 36,
336.59441031550174514805053892664619628 07, none,
324.65521222747513408815321521879534120 05,
331.93806790262370745103951610021066693 28, none, none,
289.54595771468271927135577440926594076 21]

1 --> 2  target = [33.81362495402456288697143842380555708898,
3.72564899338428580358843844437849259029 2,
325.89209971974173957011600470672493762 45]
one interval r = 20.37468935106138423062363306787369069637 ..
25.37892165275941936165630505496540319775
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=8e-38
Equations at solution: [.6e-37, .8e-37, -.11218e-34]Solution in 4.448s

Time Plot 0 s.
Exiting SolveHard() after 5.014r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82,
375.73285289778012317605239643958330755 55,
328.11709293634097540807326411992949704 27,
292.99969137540887116275263969976001154 57,
358.64341559961617054717201249473509418 66,
```

```
299.89866203744056025233240704577969028 61,
360.06173466289512869935076904679272707 36,
336.59441031550174514805053892664619628 07,
256.10753185167762715169243834693533217 37,
324.65521222747513408815321521879534120 05,
331.93806790262370745103951610021066693 28, none, none,
289.54595771468271927135577440926594076 21]

1 --> 0  target = [17.93041369712160706041805295896645025140,
4.6865087019009826792728872334850491005 76,
353.30541094308176340252736853802719780 23]
one interval r = 20.73150479087938247032994260066865919376 ..
25.906753535012247517309590647092477543 05
Time Approximations 0.034.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.26216e-34]Solution in
0.711s

Time Plot 0 s.
Exiting SolveHard() after 1.473r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82,
375.73285289778012317605239643958330755 55,
```

```
328.11709293634097540807326411992949704 27,
292.99969137540887116275263969976001154 57,
358.64341559961617054717201249473509418 66,
299.89866203744056025233240704577969028 61,
360.06173466289512869935076904679272707 36,
336.59441031550174514805053892664619628 07,
256.10753185167762715169243834693533217 37,
324.65521222747513408815321521879534120 05,
331.93806790262370745103951610021066693 28,
304.79958324909193890947987901133435552 82, none,
289.54595771468271927135577440926594076 21]

2 --> 0  target = [17.93041369712160706041805295896645025140,
4.68650870190098267927288723348504910057 6,
353.30541094308176340252736853802719780 23]
one interval r = 31.37435486998661963674572574630051766689 ..
34.20127520021515638802714701339247163771
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=9e-38
Equations at solution: [.6e-37, -.9e-37, -.103e-34]Solution in 0.356s

Time Plot 0 s.
Exiting SolveHard() after 0.619r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493025894563918160212766609059 20,
441.64295972629328511431900451176393957 99,
436.91748164588015110001487838265414811 39,
422.98493396965485924203583391481360563 54,
361.52580255306486076414729768559044611 41,
401.88173903619183596452557191585627099 47,
389.59001514985575784707543857505285878 50,
328.46939892733941491012930759550259175 33,
401.50757157349363820641376571921151949 14,
358.97362823133274759152355018160956544 64,
398.33147103662469451404825957733735061 56,
371.48387393396231438073584500909750955 82,
336.61215840502807053542687191440863131 86,
361.50888346348265963497235840862079506 88,
324.67144991765018016329237606481250564 26,
302.31384314209366149314670575056656564 05,
```

```
328.46938512806722636126486121304236439 54,
343.81340623919801769080757733423886834 82,
375.73285289778012317605239643958330755 55,
328.11709293634097540807326411992949704 27,
292.99969137540887116275263969976001154 57,
358.64341559961617054717201249473509418 66,
299.89866203744056025233240704577969028 61,
360.06173466289512869935076904679272707 36,
336.59441031550174514805053892664619628 07,
256.10753185167762715169243834693533217 37,
324.65521222747513408815321521879534120 05,
331.93806790262370745103951610021066693 28,
304.79958324909193890947987901133435552 82,
323.46169175720920398708739423141631176 30,
289.54595771468271927135577440926594076 21]

Cascade time 278.779
counts: 28, 28

 Iteration 88

Start Generation 1
1 --> 0  target = [11.99999999993593392166612898341831716200,
6.21701250287623405077702783312555822343 0,
485.54908089563061082953921335694878914 05]
one interval r = 23.40850301657135407827010920922089038370 ..
27.67578046429026821743817790816675671907
Time Approximations 0.041.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=0
Equations at solution: [0., 0., .12e-35]Solution in 1.044s

Time Plot 0 s.
Exiting SolveHard() after 2.19r=27.5236 in [25.56992694 .. 27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.99999999993593392166612898341831716200,
```

```
6.217012502876234050777027833125558223430,
485.549080895630610829539213356948789140 5]
one interval r = 32.628147792117092891080980105865267187 20 ..
36.102483889435365816309565594011297852 71
Time Approximations 0.023.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., .68e-35]Solution in 5.17s

Time Plot 0 s.
Exiting SolveHard() after 5.578r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434933607174159839376075973821 3174,
441.642959729366701570101758189767473905 8,
436.917481648301629720938260741438748984 2, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.523596844800742813226520545503233758 22,
6.583434721518268048903751081718836346437,
467.787305956424576000798423927947806990 8]
one interval r = 32.419789556612984835757909463733105521 76 ..
35.851524173731696165434102234463623532 52
Time Approximations 0.021.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.233e-34]Solution in 0.673s

Time Plot 0 s.
Exiting SolveHard() after 1.068r=34.9451 in [33.70078237 ..
```

```
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42, none, none,
401.88173903867218884379058358898042193 87, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844800742813226520545503233758 22,
6.583434721518268048903751081718836346437,
467.78730595642457600079842392794780699 08]
two intervals r = 12.923271608270576981371384826381365572 45 ..
190000000000545721857780001738754001 67/100000000000000000000000000000000
00000 or r = 18.394248580320715856791945744283006374 14 ..
190000000000545721857780001738754001 67/100000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.2e-38
Equations at solution: [-.2e-37, -.32e-37, .9011e-35]Solution in
46.271s

Time Plot 0 s.
Exiting SolveHard() after 47.671r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93, none,
401.88173903867218884379058358898042193 87, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628239807328466822406923515451 27,
4.125651796608458273564409387729130534769,
```

```
440.67123064618446658307787422538564107 32]
two intervals r = 14.35659705128682275247250828335764270175 ..
190000000000545721857780001738754001 67/100000000000000000000000000000000000
00000 or r = 17.70352613800772491094292481375261211434 ..
190000000000545721857780001738754001 67/100000000000000000000000000000000000
00000
Time Approximations 0.051.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., -.22906e-34]Solution in 5.285s

Time Plot 0 s.
Exiting SolveHard() after 6.402r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93, none,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962823980732846682240692351545127,
4.12565179660845827356440938772913053476 9,
440.67123064618446658307787422538564107 32]
one interval r = 22.39761154367932612788146191931514851862 ..
27.23722351595189941387844444491006414457
Time Approximations 0.042.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
```

```
in partial time = 1.298 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064370916196504071886018584698807, rm =
14.37818769936219967805526209009345924019}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.2e-38
Equations at solution: [0., -.52e-37, -.516e-34]Solution in 9.064s

Time Plot 0 s.
Exiting SolveHard() after 10.001r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978319309160709714 5, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888797007918173061087304277923035,
4.00486908158755135820892148099138352772 4,
404.86224500957360131371025024547999407 88]
two intervals r = 16.08011007769445778465244900336960253475 ..
19000000000054572185778000173875400167/10000000000000000000000000000000
00000 or r = 16.41579812681424367516533490252521562102 ..
19000000000054572185778000173875400167/10000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.18e-37, -.1e-37, -.692e-36]Solution in 1.497s

Time Plot 0 s.
Exiting SolveHard() after 6.714r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45, none, none,
358.97362823350740695504213590326279865 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888797007918173061087304277923035,
4.0048690815875513582089214809913835277 24,
404.86224500957360131371025024547999407 88]
one interval r = 21.641943994178527377511600429667598676 17 ..
26.763306600384087304282191926867574336 98
Time Approximations 0.047.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [0., .26e-37, -.218e-34]Solution in 1.048s

Time Plot 0 s.
Exiting SolveHard() after 6.053r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60, none,
358.97362823350740695504213590326279865 29, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941779962434624324953483376650936,
5.589637182893619970272577188837164157539,
443.83065884419743086162667791380543936 91]
```

```
one interval r = 22.4672537448475049176993151053627787249 7 ..
27.27388428354630546491550572207041979923
Time Approximations 0.043.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., -.27e-37, -.22e-35]Solution in 1.029s

Time Plot 0 s.
Exiting SolveHard() after 2.025r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60, none,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941779962434624324953483376650936,
5.589637182893619970272577188837164157539,
443.83065884419743086162667791380543936 91]
one interval r = 32.15575279500599321086373274647426331511 ..
35.50872228739901225246692914535250018364
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=4e-38
```

```
Equations at solution: [-.5e-37, .4e-37, -.189e-34]Solution in 0.488s

Time Plot 0 s.
Exiting SolveHard() after 4.748r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136532143293434259644826603240743,
5.18778357839717741941424876849257671353 2,
408.65773862078159921652735704898384329 44]
one interval r = 21.71840114651404413054080485460612866138 ..
26.81849303504807100187992607522106899361
Time Approximations 0.06.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.32e-37
Equations at solution: [-.1e-37, -.132e-36, .168e-34]Solution in 1.04s

Time Plot 0 s.
Exiting SolveHard() after 2.205r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
```

```
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22, none, none,
361.50888346543933209813537624576505322 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136532143293434259644826603240743,
5.18778357839717741941424876849257671353 2,
408.65773862078159921652735704898384329 44]
one interval r = 31.80828598746204206032936325536378946577 ..
35.00011460040378856637132500790882509818
Time Approximations 0.02.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, -.323e-34]Solution in 4.228s

Time Plot 0 s.
Exiting SolveHard() after 4.504r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62, none,
361.50888346543933209813537624576505322 69, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 1  target = [26.46347110533029930139787475365480017137,
6.196262565231027003724460906136686979191,
385.44474378757845550087263928051965178 59]
one interval r = 31.60836097532256395167792297509039635664 ..
34.66372795605197775616305244632357683514
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .146e-34]Solution in 0.596s

Time Plot 0 s.
Exiting SolveHard() after 0.905r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62, none,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110533029930139787475365480017137,
6.196262565231027003724460906136686979191,
385.44474378757845550087263928051965178 59]
two intervals r = 16.87563408771794873497712727523836364273 ..
19000000000054572185778000173875400167/10000000000000000000000000000000
00000 or r = 15.55640493777271431260427821848582293498 ..
19000000000054572185778000173875400167/10000000000000000000000000000000
00000
Time Approximations 0.062.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.359e-37, 0., -.34014e-34]Solution in 5.014s

Time Plot 0 s.
Exiting SolveHard() after 6.188r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874753262459929467357824549600332,
4.88381077973910337610787598259768800 6135,
376.61967855231559099531946096617021000 66]
one interval r = 21.11001304884003685526198959971765905790 ..
26.31784243467495294588566094512034188065
Time Approximations 0.035.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, -.244e-34]Solution in 0.878s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.567r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03, none,
328.46938513005700371222236590827838231 23, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874753262459929467357824549600332,
4.8838107797391033761078759825976880061 35,
376.61967855231559099531946096617021000 66]
one interval r = 31.53899497706543407989230943030752873246 ..
34.53618386088274520411299938253371569423
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.35e-36
Equations at solution: [.257e-35, -.335e-35, .30e-35]Solution in 0.504s

Time Plot 0 s.
Exiting SolveHard() after 0.794r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
```

```
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03, none,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017534426267165659427520670743420,
6.0258135491889430693473296060069768623 61,
351.42702947900677843056883029782124249 63]
one interval r = 31.362302061101203817817844117463443916 74 ..
34.174466406103326161074567871866560207 69
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.35e-35]Solution in 0.542s

Time Plot 0 s.
Exiting SolveHard() after 0.774r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
```

```
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03, none,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11, none, none,
292.99969137683635139607148919610724837 35, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175344262671656594275206707434 20,
6.0258135491889430693473296060069768623 61,
351.42702947900677843056883029782124249 63]
two intervals r = 17.981355144560732486231947048073380319 23 ..
19000000000054572185778000173875400167/10000000000000000000000000000000
00000 or r = 13.846080153907235702684919060132482533 15 ..
19000000000054572185778000173875400167/10000000000000000000000000000000
00000
Time Approximations 0.046.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [0., 0., .7557e-35]Solution in 1.175s

Time Plot 0 s.
Exiting SolveHard() after 6.143r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03,
302.31384314454360370765820760078748762 32,
328.46938513005700371222236590827838231 23,
```

```
343.8134062401847705157340296220588910611, none, none,
292.9996913768363513960714891961072483735, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941874109717813290588125387376403,
6.37794387380005709838252283905849131080 7,
423.2883278390715003950899832726704226410]
one interval r = 31.9466181759686248651413039936662964078 2 ..
35.21212308654602010096267746862203812680
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=6e-38
Equations at solution: [.5e-37, -.6e-37, .158e-34]Solution in 0.606s

Time Plot 0 s.
Exiting SolveHard() after 0.972r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349336071741598393760759738213174,
441.6429597293667015701017581897674739058,
436.9174816483016297209382607414387489842,
422.9849339732533082872146643981383966693,
361.5258025550066616353557354908727579009,
401.8817390386721888437905835889804219387,
389.5900151520219379597839193091607097145,
328.4693989293285935347873805648095091960,
401.5075715759609002260662426962185509977,
358.9736282335074069550421359032627986529,
398.3314710401327287168202638222866670622,
371.4838739350280993166440045787102375262,
336.6121584074297341454452509547147456476,
361.5088834654393320981353762457650532269,
324.6714499190371564139774904927638584803,
302.3138431445436037076582076007874876232,
328.4693851300570037122223659082783823123,
343.8134062401847705157340296220588910611, none, none,
292.9996913768363513960714891961072483735, none, none,
360.0617346657920592891223349169364231756, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941874109717813290588125387376403,
6.37794387380005709838252283905849131080 7,
```

```
423.2883278390715003950899832726704226410]
two intervals r = 15.22886702430429539816142515771725118234 ..
190000000000545721857780001738754001 67/1000000000000000000000000000000000
00000 or r = 17.12965777075465260347522529944564313788 ..
190000000000545721857780001738754001 67/1000000000000000000000000000000000
00000
Time Approximations 0.076.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.31e-37, .1e-37, .11433e-34]Solution in 1.335s

Time Plot 0 s.
Exiting SolveHard() after 7.374r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03,
302.31384314454360370765820760087487623 2,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11,
375.73285290186345780047819378632788565 77, none,
292.99969137683635139607148919610724837 35, none, none,
360.06173466579205928912233491693642317 56, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234340517669633087658277655486566,
4.0035598152867168413041074682292454381 14,
404.47973593637232357767297297886757777 80]
two intervals r = 16.09683966381479447366037463080121966492 ..
190000000000545721857780001738754001 67/1000000000000000000000000000000000
```

```
00000 or r = 16.3998864910018360310354587926563152292 1 ..
190000000000545721857780001738754001 67/100000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.7615110884558244986626642365610012490 6, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.18e-37, 0., -.18845e-34]Solution in 1.498s

Time Plot 0 s.
Exiting SolveHard() after 6.552r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03,
302.31384314454360370765820760078748762 32,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11,
375.73285290186345780047819378632788565 77, none,
292.99969137683635139607148919610724837 35,
358.64341560177885894099319215297366353 58, none,
360.06173466579205928912233491693642317 56, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234340517669633087658277655486566,
4.0035598152867168413041074682292454381 14,
404.47973593637232357767297297886757777 80]
one interval r = 21.63429629994642589913190989674878559702 ..
26.75768169887832507890606438090345069619
Time Approximations 0.056.
```

```
hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, .101e-34]Solution in 5.094s

Time Plot 0 s.
Exiting SolveHard() after 6.176r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382 13174,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03,
302.31384314454360370765820760078748762 32,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11,
375.73285290186345780047819378632788565 77,
328.11709293831772432111554934346026888 58,
292.99969137683635139607148919610724837 35,
358.64341560177885894099319215297366353 58, none,
360.06173466579205928912233491693642317 56, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954451311589665234101173482613884,
6.19617723008481713744371178083708031044 5,
385.42734025138483642188968823014135542 26]
one interval r = 31.608220490876659573293338103225072788 30 ..
34.663476150429384437975392710832856278 36
Time Approximations 0.019.

hint used Hint := [33.813392675100157147469018161182119369 13, 3, 1, 1,
11.783209121413521870719438174471899656 27, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, -.307e-34]Solution in 0.6s

Time Plot 0 s.
Exiting SolveHard() after 0.924r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03,
302.31384314454360370765820760078748762 32,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11,
375.73285290186345780047819378632788565 77,
328.11709293831772432111554934346026888 58,
292.99969137683635139607148919610724837 35,
358.64341560177885894099319215297366353 58, none,
360.06173466579205928912233491693642317 56, none, none,
324.65521222887638154023170088191562520 23, none, none, none, none]

0 --> 1  target = [26.46318954451311589665234101173482613884,
6.19617723008481713744371178083708031044 5,
385.42734025138483642188968823014135542 26]
two intervals r = 16.87629600310222829905679596647019183103 ..
19000000000054572185778000173875400167/10000000000000000000000000000000
00000 or r = 15.55559000626081039712100258633944478827 ..
19000000000054572185778000173875400167/10000000000000000000000000000000
00000
Time Approximations 0.066.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.359e-37, 0., .11800e-34]Solution in 5.117s

Time Plot 0 s.
Exiting SolveHard() after 6.326r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03,
302.31384314454360370765820760078748762 32,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11,
375.73285290186345780047819378632788565 77,
328.11709293831772432111554934346026888 58,
292.99969137683635139607148919610724837 35,
358.64341560177885894099319215297366353 58, none,
360.06173466579205928912233491693642317 56,
336.59441031791900943450293048305521637 38, none,
324.65521222887638154023170088191562520 23, none, none, none, none]

0 --> 2  target = [34.49522661155893220831305938300014047493,
3.8971313156876145621607245284567116525 89,
373.78081883584982048616805190529937613 49]
two intervals r = 17.29769086252021891084464902763017507154 ..
19000000000054572185778000173875400167/10000000000000000000000000000000
00000 or r = 14.99436407388427676710830953356150467243 ..
19000000000054572185778000173875400167/10000000000000000000000000000000
00000
Time Approximations 0.092.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.36e-37, 0., -.12900e-34]Solution in 5.258s

Time Plot 0 s.
Exiting SolveHard() after 6.976r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03,
302.31384314454360370765820760078748762 32,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11,
375.73285290186345780047819378632788565 77,
328.11709293831772432111554934346026885 8,
292.99969137683635139607148919610724837 35,
358.64341560177885894099319215297366353 58, none,
360.06173466579205928912233491693642317 56,
336.59441031791900943450293048305521637 38, none,
324.65521222887638154023170088191562520 23,
331.93806790350102741944041394875909308 94, none, none, none]

1 --> 2  target = [34.49522661155893220831305938300014047493,
3.89713131568761456216072452845671165 2589,
373.78081883584982048616805190529937613 49]
one interval r = 21.06068473205046551833017261660803557750 ..
26.26979834270645298501685027782703623195
Time Approximations 0.035.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
```

```
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., -.86e-35]Solution in 0.81s

Time Plot 0 s.
Exiting SolveHard() after 1.538r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03,
302.31384314454360370765820760078748762 32,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11,
375.73285290186345780047819378632788565 77,
328.11709293831772432111554934346026885 8,
292.99969137683635139607148919610724837 35,
358.64341560177885894099319215297366353 58,
299.89866203809287585270444264937381058 74,
360.06173466579205928912233491693642317 56,
336.59441031791900943450293048305521637 38, none,
324.65521222887638154023170088191562520 23,
331.93806790350102741944041394875909308 94, none, none, none]

0 --> 2  target = [33.81362495402161021357662290399069885698,
3.72564899332607534550257557221836123589 0,
325.89209972151915883225773879514136382 67]
two intervals r = 18.55227049019499422742139458751849531124 ..
19000000000054572185778000173875400167/10000000000000000000000000000000
00000 or r = 12.49196935745857173097245436636737395189 ..
19000000000054572185778000173875400167/10000000000000000000000000000000
00000
Time Approximations 0.038.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=2e-38
Equations at solution: [-.34e-37, .2e-37, .27457e-34]Solution in 1.229s

Time Plot 0 s.
Exiting SolveHard() after 7.306r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03,
302.31384314454360370765820760078748762 32,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11,
375.73285290186345780047819378632788565 77,
328.11709293831772432111554934346026888 58,
292.99969137683635139607148919610724837 35,
358.64341560177885894099319215297366353 58,
299.89866203809287585270444264937381058 74,
360.06173466579205928912233491693642317 56,
336.59441031791900943450293048305521637 38, none,
324.65521222887638154023170088191562520 23,
331.93806790350102741944041394875909308 94, none, none,
289.54595771580919839204426852228019160 16]

1 --> 2  target = [33.81362495402161021357662290399069885698,
3.72564899332607534550257557221836123589 0,
325.89209972151915883225773879514136382 67]
one interval r = 20.37468935119185732377894345576254758584 ..
25.37892165289353578009829605145564619830
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
```

```
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, -.359e-34]Solution in 0.55s

Time Plot 0 s.
Exiting SolveHard() after 1.112r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382286667062 2,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03,
302.31384314454360370765820760078748762 32,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11,
375.73285290186345780047819378632788565 77,
328.11709293831772432111554934346026888 58,
292.99969137683635139607148919610724837 35,
358.64341560177885894099319215297366353 58,
299.89866203809287585270444264937381058 74,
360.06173466579205928912233491693642317 56,
336.59441031791900943450293048305521637 38,
256.10753185261594024370457907726391833 78,
324.65521222887638154023170088191562520 23,
331.93806790350102741944041394875909308 94, none, none,
289.54595771580919839204426852228019160 16]

1 --> 0  target = [17.930413697217158143953602082933176325 41,
4.6865087018909432120854924740309668467 31,
353.30541094549729334472567432426493627 53]
one interval r = 20.731504791022220101531034529144128352 30 ..
25.906753535159695444519149074614456349 19
Time Approximations 3.896.
```

```
hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.006244454766213453020624512246743570 53, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., -.218e-34]Solution in 0.67s

Time Plot 0 s.
Exiting SolveHard() after 5.256r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03,
302.31384314454360370765820760078748762 32,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11,
375.73285290186345780047819378632788565 77,
328.11709293831772432111554934346026888 58,
292.99969137683635139607148919610724837 35,
358.64341560177885894099319215297366353 58,
299.89866203809287585270444264937381058 74,
360.06173466579205928912233491693642317 56,
336.59441031791900943450293048305521637 38,
256.10753185261594024370457907726391833 78,
324.65521222887638154023170088191562520 23,
331.93806790350102741944041394875909308 94,
304.79958325133495736824660620080231714 60, none,
289.54595771580919839204426852228019160 16]

2 --> 0  target = [17.930413697217158143953602082933176325 41,
4.6865087018909432120854924740309668467 31,
353.30541094549729334472567432426493627 53]
one interval r = 31.374354869894775710222547073645088706 62 ..
```

```
34.20127520023360311445551066070247572969
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [-.6e-37, .8e-37, -.21e-35]Solution in 0.345s

Time Plot 0 s.
Exiting SolveHard() after 0.641r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493360717415983937607597382131 74,
441.64295972936670157010175818976747390 58,
436.91748164830162972093826074143874898 42,
422.98493397325330828721466439813839666 93,
361.52580255500666163535573549087275790 09,
401.88173903867218884379058358898042193 87,
389.59001515202193795978391930916070971 45,
328.46939892932859353478738056480950919 60,
401.50757157596090022606624269621855099 77,
358.97362823350740695504213590326279865 29,
398.33147104013272871682026382228666706 22,
371.48387393502809931664400457871023752 62,
336.61215840742973414544525095471474564 76,
361.50888346543933209813537624576505322 69,
324.67144991903715641397749049276385848 03,
302.31384314454360370765820760078748762 32,
328.46938513005700371222236590827838231 23,
343.81340624018477051573402962205889106 11,
375.73285290186345780047819378632788565 77,
328.11709293831772432111554934346026888 58,
292.99969137683635139607148919610724837 35,
358.64341560177885894099319215297366353 58,
299.89866203809287585270444264937381058 74,
360.06173466579205928912233491693642317 56,
336.59441031791900943450293048305521637 38,
256.10753185261594024370457907726391833 78,
324.65521222887638154023170088191562520 23,
331.93806790350102741944041394875909308 94,
304.79958325133495736824660620080231714 60,
323.46169175835225945638190508906959321 56,
289.54595771580919839204426852228019160 16]
```

```
Cascade time 166.745
counts: 28, 28

 Iteration 89

Start Generation 1
1 --> 0  target = [11.999999999986334901024231357509763094 00,
6.217012502968362132163451647872821342787,
485.54908090142286632063263716347396856 21]
one interval r = 23.408503016606928460920698920108434230 53 ..
27.675780464418572308997224586733399013 13
Time Approximations 0.046.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.5e-38
Equations at solution: [.2e-37, -.55e-37, .14e-35]Solution in 1.051s

Time Plot 0 s.
Exiting SolveHard() after 2.219r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999986334901024231357509763094 00,
6.217012502968362132163451647872821342787,
485.54908090142286632063263716347396856 21]
```

"Imaginary part neglected: ", $3.183223432221338457742730483908255797113 \times 10^{-17}$

```
one interval r = 32.628147792208299378733482447140477140 16 ..
36.102483889466333613818219584186514988 19
Time Approximations 0.024.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
```

```
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.72e-35]Solution in 4.447s

Time Plot 0 s.
Exiting SolveHard() after 4.86r=35.4632 in [33.94922194 .. 36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684493956059856633873382236605662,
6.58343472157304880662266527028209602 9091,
467.78730596238646856314751591318937454 19]
```

"Imaginary part neglected: ", $3.18322343222133845774273048390825579 7113 \times 10^{-17}$

```
one interval r = 32.41978955670748770212200643756517483041 ..
35.85152417377103779790378532692197080586
Time Approximations 0.023.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=5e-38
Equations at solution: [-.6e-37, .5e-37, -.65e-35]Solution in 0.657s

Time Plot 0 s.
Exiting SolveHard() after 1.037r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30, none, none,
```

```
401.881739047694376354145380450653322912 7, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684493956059856633873382236605662,
6.583434721573048806622665270282096029091,
467.78730596238646856314751591318937454 19]
two intervals r = 12.92327160826322381559788215711449089923 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000 or r = 18.39424858037945690877238890592842064223 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=7.7e-38
Equations at solution: [-.6e-37, -.77e-37, .1227e-34]Solution in
46.334s

Time Plot 0 s.
Exiting SolveHard() after 47.74r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39, none,
401.881739047694376354145380450653322912 7, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962832033718246764358903060761895,
4.125651796949248981558694109323208733953,
440.67123065439214051658050797417972287 50]
two intervals r = 14.35659705110792019080331560980354924826 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000 or r = 17.70352613819386241172251661129156782056 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
```

```
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.28e-37, -.2e-37, .337e-35]Solution in 5.17s

Time Plot 0 s.
Exiting SolveHard() after 6.256r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39, none,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962832033718246764358903060761895,
4.1256517969492489815586941093232087339 53,
440.67123065439214051658050797417972287 50]
one interval r = 22.397611543695634764410346890750671674 98 ..
27.237223516127157999402785213481960453 27
Time Approximations 0.036.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.279 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070644002088138053361627590537522 47, rm =
14.378187704528709088389300444451753800 87}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.489e-34]Solution in 8.739s

Time Plot 0 s.
Exiting SolveHard() after 9.616r=26.4635 in [24.64256576 ..
```

```
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888073991303868969799978786714 79,
4.0048690819318328243584477451820483692 66,
404.86224501862446795362292255803722556 66]
two intervals r = 16.0801100774568384817054202464762668897 0 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000 or r = 16.4157981271519638287866334606284469718 1 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000
Time Approximations 0.053.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [0., 0., .29e-36]Solution in 1.51s

Time Plot 0 s.
Exiting SolveHard() after 6.618r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85, none, none,
358.97362824598872462440219694381562916 56, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888807399130386896979997878671479,
4.004869081931832824358447745182048369266,
404.86224501862446795362292255803722556 66]
one interval r = 21.641943994139292663918081179223627665 50 ..
26.763306600583873965841523360255654 69490
Time Approximations 0.058.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., .64e-35]Solution in 5.432s

Time Plot 0 s.
Exiting SolveHard() after 6.527r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01, none,
358.97362824598872462440219694381562916 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941771679012610805295763796150884,
5.589637182990331512767511733099226661442,
443.83065885064397336945037260123993661 62]
one interval r = 22.467253744830767489408198580830643327 50 ..
27.273884283699838658061613126015223498 09
Time Approximations 0.039.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
```

```
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, -.54e-35]Solution in 1.041s

Time Plot 0 s.
Exiting SolveHard() after 2.07r=27.0204 in [24.71083344 .. 27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01, none,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941771679012610805295763796150884,
5.5896371829903315127675117330992266614 42,
443.83065885064397336945037260123993661 62]
```

"Imaginary part neglected: ", $3.1832234322213384577427304839082557971 13 \times 10^{-17}$

```
one interval r = 32.15575279510645891623694508853403306163 ..
35.50872228745315311174639487869673513399
Time Approximations 0.022.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., .142e-34]Solution in 4.969s

Time Plot 0 s.
Exiting SolveHard() after 5.332r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136497821969079296676607594812887,
5.18778357853682628047460321175321780928 8,
408.65773863217508658929296534306956481 43]
one interval r = 21.71840114653067339673318144673691505114 ..
26.81849303528049154413935199719780259213
Time Approximations 0.058.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .243e-34]Solution in 1.047s

Time Plot 0 s.
Exiting SolveHard() after 2.162r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45, none, none,
```

```
361.50888347717509578069973987991681214 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136497821969079296676607594812887,
5.18778357853682628047460321175321780928 8,
408.65773863217508658929296534306956481 43]
```

"Imaginary part neglected: ", $3.18322343222133845774273048390825579711 3 \times 10^{-17}$

```
one interval r = 31.80828598760634730190355154021879019718 ..
35.00011460054054964813457529534636831732
Time Approximations 0.021.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .148e-34]Solution in 0.423s

Time Plot 0 s.
Exiting SolveHard() after 0.738r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28, none,
361.50888347717509578069973987991681214 10, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110558445977787668386409784899777,
6.19626256532224704396547876835684117836 8,
385.44474379976540154495570301709218769 42]
```

"Imaginary part neglected: ", $3.18322343222133845774273048390825579711 3 \times 10^{-17}$

```
one interval r = 31.60836097546576355892162077228245975874 ..
34.66372795620669383409709489238569385801
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, .137e-34]Solution in 0.593s

Time Plot 0 s.
Exiting SolveHard() after 4.996r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28, none,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110558445977787668386409784899777,
6.19626256532224704396547876835684117836 8,
385.44474379976540154495570301709218769 42]
two intervals r = 16.87563408737047322141899372603010585795 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000 or r = 15.55640493832717691033432130491439051917 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000
Time Approximations 0.062.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
```

```
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.359e-37, 0., -.1479e-34]Solution in 1.198s

Time Plot 0 s.
Exiting SolveHard() after 2.394r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874719106855208355896252909530457,
4.8838107798748787423173333018444290007 09,
376.61967856479509940786532675632973364 55]
one interval r = 21.11001304879583565933481627808866716172 ..
26.31784243493665088931067833615047046367
Time Approximations 0.034.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.471e-34]Solution in 0.872s

Time Plot 0 s.
Exiting SolveHard() after 5.501r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
```

```
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55, none,
328.46938514271518750491222576615373078 13, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874719106855208355896252909530457,
4.8838107798748787423173333018444290007 09,
376.61967856479509940786532675632973364 55]
```

"Imaginary part neglected: ", $3.18322343222133845774273048390825579711 3 \times 10^{-17}$

```
one interval r = 31.53899497720728696475952904479423040126 ..
34.53618386104387349961008018664099491160
Time Approximations 0.019.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=6.71e-36
Equations at solution: [.514e-35, -.671e-35, .454e-34]Solution in
0.491s

Time Plot 0 s.
Exiting SolveHard() after 0.802r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
```

```
441.642959735245599026017019711850225799 7,
436.917481656501774965811264002521968813 0,
422.984933979917352316883298711529465683 9,
361.525802566915167700891917848267591846 6,
401.881739047694376354145380450653322912 7,
389.590015163420933451478226079214182818 5,
328.469398941991922577430715000575773930 1,
401.507571585777674891964012253079814906 1,
358.973628245988724624402196943815629165 6,
398.331471046786104388767358582956844184 5,
371.483873949809648882125113823252011252 8,
336.612158420199844865437734932212624445 8,
361.508883477175095780699739879916812141 0,
324.671449934028056847213461705895513755 5, none,
328.469385142715187504912225766153730781 3,
343.813406256456211225826267131795071694 2, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.872050175618976180638026104998727212 45,
6.025813549288266109166906163271680460479,
351.427029491986340145839676355428192482 5]
```

"Imaginary part neglected: ", $3.183223432221338457742730483908255797113 \times 10^{-17}$

```
one interval r = 31.362302061234437636350596931975285869 85 ..
34.174466406276931932491173559432452695 15
Time Approximations 0.018.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.1e-37, .3e-37, .246e-34]Solution in 0.546s

Time Plot 0 s.
Exiting SolveHard() after 0.781r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434939648231854214849436163489820 6,
441.642959735245599026017019711850225799 7,
436.917481656501774965811264002521968813 0,
422.984933979917352316883298711529465683 9,
361.525802566915167700891917848267591846 6,
401.881739047694376354145380450653322912 7,
389.590015163420933451478226079214182818 5,
```

```
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55, none,
328.46938514271518750491222576615373078 13,
343.81340625645621122582626713179507169 42, none, none,
292.99969139262290331210091306346129864 36, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017561897618063802610499872721245,
6.0258135492882661091669061632716804604 79,
351.42702949198634014583967635542819248 25]
two intervals r = 17.98135514426478635181520633539988940328 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000 or r = 13.84608015463424920808282550344034252523 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [-.17e-37, .1e-37, -.969e-35]Solution in 1.178s

Time Plot 0 s.
Exiting SolveHard() after 6.211r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
```

```
361.508883477175095780699739879916812141 0,
324.671449934028056847213461705895513755 5,
302.313843157846958469218923740531378219 2,
328.469385142715187504912225766153730781 3,
343.813406256456211225826267131795071694 2, none, none,
292.999691392622903312100913063461298643 6, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418904480366824855729329726112 36,
6.377943873861711207642616140768057150376,
423.288327845841336118846515800954174655 4]
```

"Imaginary part neglected: ", $3.18322343222133845774273048390825579711 3 \times 10^{-17}$

```
one interval r = 31.946618176071740959147473925873871244 92 ..
35.212123086611316504896337056702912448 38
Time Approximations 0.021.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.150e-34]Solution in 0.598s

Time Plot 0 s.
Exiting SolveHard() after 0.957r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434939648231854214849436163489820 6,
441.642959735245599026017019711850225799 7,
436.917481656501774965811264002521968813 0,
422.984933979917352316883298711529465683 9,
361.525802566915167700891917848267591846 6,
401.881739047694376354145380450653322912 7,
389.590015163420933451478226079214182818 5,
328.469398941991922577430715000575773930 1,
401.507571585777674891964012253079814906 1,
358.973628245988724624402196943815629165 6,
398.331471046786104388767358582956844184 5,
371.483873949809648882125113823252011252 8,
336.612158420199844865437734932212624445 8,
361.508883477175095780699739879916812141 0,
324.671449934028056847213461705895513755 5,
302.313843157846958469218923740531378219 2,
328.469385142715187504912225766153730781 3,
```

```
343.8134062564562112258262671317950716942, none, none,
292.9996913926229033121009130634612986436, none, none,
360.0617346756533858978677261003829262324, none, none, none, none,
none, none, none]

0 --> 1  target = [27.0203794189044803668248557293297261123 6,
6.3779438738617112076426161407680571503 76,
423.2883278458413361188465158009541746554]
two intervals r = 15.22886702417881970098416040061322112275 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000 or r = 17.12965777094467898522529319235059166305 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.31e-37, -.1e-37, -.1725e-34]Solution in
1.349s

Time Plot 0 s.
Exiting SolveHard() after 6.641r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349396482318542148494361634898206,
441.6429597352455990260170197118502257997,
436.9174816565017749658112640025219688130,
422.9849339799173523168832987115294656839,
361.5258025669151677008919178482675918466,
401.8817390476943763541453804506533229127,
389.5900151634209334514782260792141828185,
328.4693989419919225774307150005757739301,
401.5075715857776748919640122530798149061,
358.9736282459887246244021969438156291656,
398.3314710467861043887673585829568441845,
371.4838739498096488821251138232520112528,
336.6121584201998448654377349322126244458,
361.5088834771750957806997398799168121410,
324.6714499340280568472134617058955137555,
302.3138431578469584692189237405313782192,
328.4693851427151875049122257661537307813,
343.8134062564562112258262671317950716942,
375.7328529092694539100328204904621095992, none,
292.9996913926229033121009130634612986436, none, none,
```

```
360.06173467565338589786772610038292623 24, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234352097977469246915006985794189,
4.0035598156337871620970635770628196787 64,
404.47973594623552994425150837618167180 55]
two intervals r = 16.09683966354177315254696178828673771231 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000 or r = 16.39988649137477380101354754430629729971 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000
Time Approximations 0.052.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.35e-37, -.1e-37, .475e-35]Solution in 1.508s

Time Plot 0 s.
Exiting SolveHard() after 6.43r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55,
302.31384315784695846921892374053137821 92,
328.46938514271518750491222576615373078 13,
343.81340625645621122582626713179507169 42,
375.73285290926945391003282049046210959 92, none,
292.99969139262290331210091306346129864 36,
358.64341561496482467239902662666852621 67, none,
360.06173467565338589786772610038292623 24, none, none, none, none,
none, none, none]
```

```
1 --> 2  target = [34.93953234352097977469246915006985794189,
4.0035598156337871620970635770628196787 64,
404.47973594623552994425150837618167180 55]
one interval r = 21.63429629992257296298917697824385856587 ..
26.75768169909017150008059311210475372206
Time Approximations 0.058.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.630e-34]Solution in 5.77s

Time Plot 0 s.
Exiting SolveHard() after 6.845r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55,
302.31384315784695846921892374053137821 92,
328.46938514271518750491222576615373078 13,
343.81340625645621122582626713179507169 42,
375.73285290926945391003282049046210959 92,
328.11709295172896613836973170417149680 56,
292.99969139262290331210091306346129864 36,
358.64341561496482467239902662666852621 67, none,
360.06173467565338589786772610038292623 24, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954476440755748647195513331454200,
6.19617723017516792210346193492142674723 8,
385.42734026339410155375507484854064741 49]
```

"Imaginary part neglected: ", $3.183223432221338457742730483908255797113 \times 10^{-17}$

```
one interval r = 31.608220491018418133776082280431664129 91 ..
34.663476150581534182664780378052726012 40
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .160e-34]Solution in 0.601s

Time Plot 0 s.
Exiting SolveHard() after 0.903r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55,
302.31384315784695846921892374053137821 92,
328.46938514271518750491222576615373078 13,
343.81340625645621122582626713179507169 42,
375.73285290926945391003282049046210959 92,
328.11709295172896613836973170417149680 56,
292.99969139262290331210091306346129864 36,
358.64341561496482467239902662666852621 67, none,
360.06173467565338589786772610038292623 24, none, none,
324.65521224370153072467974139069293524 88, none, none, none, none]

0 --> 1  target = [26.46318954476440755748647195513331454200,
6.19617723017516792210346193492142674723 8,
385.42734026339410155375507484854064741 49]
two intervals r = 16.876296002761540008522517967607105070 95 ..
```

```
9500000000006951072847740434755264409/500000000000000000000000000000000
000 or r = 15.555590006807028052651921082988261198 80 ..
9500000000006951072847740434755264409/500000000000000000000000000000000
000
Time Approximations 0.063.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.180e-37, 0., .717e-35]Solution in 5.173s

Time Plot 0 s.
Exiting SolveHard() after 6.367r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55,
302.31384315784695846921892374053137821 92,
328.46938514271518750491222576615373078 13,
343.81340625645621122582626713179507169 42,
375.73285290926945391003282049046210959 92,
328.11709295172896613836973170417149680 56,
292.99969139262290331210091306346129864 36,
358.64341561496482467239902662666852621 67, none,
360.06173467565338589786772610038292623 24,
336.59441033050778271474788419074270576 60, none,
324.65521224370153072467974139069293524 88, none, none, none, none]

0 --> 2  target = [34.49522661175626701550816318552916538107,
3.897131316053266997986172252379634973358,
373.78081885079379714204684234609959618 46]
two intervals r = 17.297690862100036743975161562335543387 56 ..
```

```
9500000000006951072847740434755264409/500000000000000000000000000000000
000 or r = 14.994364074624313542303997079174408371 66 ..
9500000000006951072847740434755264409/500000000000000000000000000000000
000
Time Approximations 0.085.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.18e-37, 0., -.3070e-34]Solution in 5.488s

Time Plot 0 s.
Exiting SolveHard() after 7.189r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55,
302.31384315784695846921892374053137821 92,
328.46938514271518750491222576615373078 13,
343.81340625645621122582626713179507169 42,
375.73285290926945391003282049046210959 92,
328.11709295172896613836973170417149680 56,
292.99969139262290331210091306346129864 36,
358.64341561496482467239902662666852621 67, none,
360.06173467565338589786772610038292623 24,
336.59441033050778271474788419074270576 60, none,
324.65521224370153072467974139069293524 88,
331.93806792136550268500639647353880080 40, none, none, none]

1 --> 2  target = [34.49522661175626701550816318552916538107,
3.897131316053266997986172252379634973358,
373.78081885079379714204684234609959618 46]
```

```
one interval r = 21.06068473204095532932053132864278949267 ..
26.26979834301084987843707615870771819462
Time Approximations 0.033.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, -.251e-34]Solution in 0.82s

Time Plot 0 s.
Exiting SolveHard() after 1.533r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55,
302.31384315784695846921892374053137821 92,
328.46938514271518750491222576615373078 13,
343.81340625645621122582626713179507169 42,
375.73285290926945391003282049046210959 92,
328.11709295172896613836973170417149680 56,
292.99969139262290331210091306346129864 36,
358.64341561496482467239902662666852621 67,
299.89866205614212867572667326617580670 19,
360.06173467565338589786772610038292623 24,
336.59441033050778271474788419074270576 60, none,
324.65521224370153072467974139069293524 88,
331.93806792136550268500639647353880080 40, none, none, none]

0 --> 2  target = [33.81362495422965382023700562149655738493,
3.72564899369456830345198076976383578 5597,
325.89209973668624518719225271783301878 49]
```

```
two intervals r = 18.552270489936787626192710285178723637 85 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000 or r = 12.491969358351145186173961092940634615 19 ..
95000000000069510728477404347552644 09/50000000000000000000000000000000000
000
Time Approximations 0.042.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=0
Equations at solution: [0., 0., -.902e-35]Solution in 5.167s

Time Plot 0 s.
Exiting SolveHard() after 6.619r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55,
302.31384315784695846921892374053137821 92,
328.46938514271518750491222576615373078 13,
343.81340625645621122582626713179507169 42,
375.73285290926945391003282049046210959 92,
328.11709295172896613836973170417149680 56,
292.99969139262290331210091306346129864 36,
358.64341561496482467239902662666852621 67,
299.89866205614212867572667326617580670 19,
360.06173467565338589786772610038292623 24,
336.59441033050778271474788419074270576 60, none,
324.65521224370153072467974139069293524 88,
331.93806792136550268500639647353880080 40, none, none,
289.54595773408743020054377350004121660 77]
```

```
1 --> 2  target = [33.81362495422965382023700562149655738493,
3.72564899369456830345198076976383578559 7,
325.89209973668624518719225271783301878 49]
one interval r = 20.37468935103348524876465915266709206736 ..
25.37892165320978800110506012194316423545
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.2e-37, -.2e-37, -.203e-34]Solution in 0.591s

Time Plot 0 s.
Exiting SolveHard() after 1.162r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55,
302.31384315784695846921892374053137821 92,
328.46938514271518750491222576615373078 13,
343.81340625645621122582626713179507169 42,
375.73285290926945391003282049046210959 92,
328.11709295172896613836973170417149680 56,
292.99969139262290331210091306346129864 36,
358.64341561496482467239902662666852621 67,
299.89866205614212867572667326617580670 19,
360.06173467565338589786772610038292623 24,
336.59441033050778271474788419074270576 60,
256.10753187081400980531559283337465213 90,
324.65521224370153072467974139069293524 88,
331.93806792136550268500639647353880080 40, none, none,
```

```
289.54595773408743020054377350004121660 77]

1 --> 0  target = [17.93041369692225675881455346218830860969,
4.68650870202085573242841746517912442710 1,
353.30541095824976248973765035453691664 92]
one interval r = 20.73150479091688917204902269600159456693 ..
25.90675353543011279335486215192371234950
Time Approximations 0.036.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=7.2e-38
Equations at solution: [-.3e-37, -.72e-37, -.78e-35]Solution in 4.803s

Time Plot 0 s.
Exiting SolveHard() after 5.514r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55,
302.31384315784695846921892374053137821 92,
328.46938514271518750491222576615373078 13,
343.81340625645621122582626713179507169 42,
375.73285290926945391003282049046210959 92,
328.11709295172896613836973170417149680 56,
292.99969139262290331210091306346129864 36,
358.64341561496482467239902662666852621 67,
299.89866205614212867572667326617580670 19,
360.06173467565338589786772610038292623 24,
336.59441033050778271474788419074270576 60,
256.10753187081400980531559283337465213 90,
```

```
324.65521224370153072467974139069293524 88,
331.93806792136550268500639647353880080 40,
304.79958326411007534551427731926419354 20, none,
289.54595773408743020054377350004121660 77]

2 --> 0  target = [17.93041369692225675881455346218830860969,
4.68650870202085573242841746517912442710 1,
353.30541095824976248973765035453691664 92]
```

"Imaginary part neglected: ", $3.18322343222133845774273048390825579711 3 \times 10^{-17}$

```
one interval r = 31.37435487002753124933661806687636883696 ..
34.20127520040362208954996390074291235907
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.172e-34]Solution in 0.393s

Time Plot 0 s.
Exiting SolveHard() after 0.673r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493964823185421484943616348982 06,
441.64295973524559902601701971185022579 97,
436.91748165650177496581126400252196881 30,
422.98493397991735231688329871152946568 39,
361.52580256691516770089191784826759184 66,
401.88173904769437635414538045065332291 27,
389.59001516342093345147822607921418281 85,
328.46939894199192257743071500057577393 01,
401.50757158577767489196401225307981490 61,
358.97362824598872462440219694381562916 56,
398.33147104678610438876735858295684418 45,
371.48387394980964888212511382325201125 28,
336.61215842019984486543773493221262444 58,
361.50888347717509578069973987991681214 10,
324.67144993402805684721346170589551375 55,
302.31384315784695846921892374053137821 92,
328.46938514271518750491222576615373078 13,
343.81340625645621122582626713179507169 42,
375.73285290926945391003282049046210959 92,
328.11709295172896613836973170417149680 56,
```

```
292.99969139262290331210091306346129864 36,
358.6434156149648246723990266266685262167,
299.8986620561421286757266732661758067019,
360.0617346756533858978677261003829262324,
336.5944103305077827147478841907427057660,
256.1075318708140098053155928333746521390,
324.6552122437015307246797413906929352488,
331.9380679213655026850063964735388008040,
304.7995832641100753455142773192641935420,
323.4616917751683988168892650256788443610,
289.5459577340874302005437735000412166077]

Cascade time 166.757
counts: 28, 28

 Iteration 90

Start Generation 1
1 --> 0  target = [11.999999999960216530054796316801255490 00,
6.217012503088092219494687749962305309865,
485.5490809071597133847784384735221732698]
one interval r = 23.40850301674978174986237967585031695580 ..
27.67578046435260147625880621697626979294
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.57e-37
Equations at solution: [-.5e-37, .157e-36, .18e-35]Solution in 1.039s

Time Plot 0 s.
Exiting SolveHard() after 2.182r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499995865275821,
441.64295974141948220535041301222195283 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999960216530054796316801255490 00,
6.217012503088092219494687749962305309865,
485.5490809071597133847784384735221732698]
one interval r = 32.62814779222479587339335027078385434704 ..
```

```
36.10248388955422359446515889635245444858
Time Approximations 0.024.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [.6e-37, -.4e-37, .27e-35]Solution in 4.32s

Time Plot 0 s.
Exiting SolveHard() after 4.729r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684489205096783824818078765042635,
6.58343472159868388808545350677078582934 8,
467.78730596881164852861500770295367980 89]
one interval r = 32.41978955672447331446096794421214476604 ..
35.85152417386594251670466034177338129671
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, -.353e-34]Solution in 0.676s

Time Plot 0 s.
Exiting SolveHard() after 1.066r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52, none, none,
401.88173905328967833869196812524960224 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844892050967838248180787650426 35,
6.5834347215986838880854535067707858293 48,
467.78730596881164852861500770295367980 89]
two intervals r = 12.923271608200274778540800608195013423 21 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000 or r = 18.394248580609863791902009459272876278 60 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=6.5e-38
Equations at solution: [.5e-37, .65e-37, -.322e-35]Solution in 47.97s

Time Plot 0 s.
Exiting SolveHard() after 49.399r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54, none,
401.88173905328967833869196812524960224 62, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628373068050348413952496851823 57,
4.1256517968848743604652133290567930661 78,
440.67123065822260767385888947902844207 80]
two intervals r = 14.356597051188927186154891252002926293 01 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000 or r = 17.703526138389575803894304325240404482 56 ..
```

```
475000000003843557464147939468306064 3/250000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.29e-37, -.2e-37, .993e-35]Solution in 5.431s

Time Plot 0 s.
Exiting SolveHard() after 6.533r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54, none,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962837306805034841395249685182357,
4.12565179688487436046521332905679306617 8,
440.67123065822260767385888947902844207 80]
one interval r = 22.39761154383052317219110572515373595072 ..
27.23722351607043555468570482074060040886
Time Approximations 0.038.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.282 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064396900248253246884929665315957, rm =
```

```
14.37818770387945774453101493345932987067}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.32e-37
Equations at solution: [-.1e-37, -.132e-36, -.664e-34]Solution in
8.852s

Time Plot 0 s.
Exiting SolveHard() after 9.767r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888814206540189176212044487300356,
4.0048690818733511867652991330653818289 77,
404.86224502406319735371288166124197987 4]
two intervals r = 16.08011007747540886565247925131432146299 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000 or r = 16.41579812743957961936718981374848829471 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000
Time Approximations 0.052.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.17e-37, .1e-37, .1030e-34]Solution in 1.484s

Time Plot 0 s.
Exiting SolveHard() after 6.596r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
```

```
441.642959741419482205350413012221952839 7,
436.917481660525429544413079613061155005 2,
422.984933987963953711787611973788750395 4,
361.525802569386692436169991750241149400 9,
401.881739053289678338691968125249602246 2,
389.590015165117357034364733116126050442 4, none, none,
358.973628248899186974091893465443132816 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888142065401891762120444873003 56,
4.004869081873351186765299133065381828977,
404.862245024063197353712881661264197987 4]
one interval r = 21.641943994330138990363058617356004601 92 ..
26.763306600573212979285682228141872146 28
Time Approximations 0.046.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.242e-34]Solution in 1.037s

Time Plot 0 s.
Exiting SolveHard() after 5.931r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434945305192670418124999586527582 1,
441.642959741419482205350413012221952839 7,
436.917481660525429544413079613061155005 2,
422.984933987963953711787611973788750395 4,
361.525802569386692436169991750241149400 9,
401.881739053289678338691968125249602246 2,
389.590015165117357034364733116126050442 4,
328.469398945937350547110648016426731704 0, none,
358.973628248899186974091893465443132816 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589417566560973952780731698301532 86,
5.589637183138732469093480501811631243212,
443.830658858898039181166126854089606238 2]
one interval r = 22.467253745061076283803813621150755186 85 ..
27.273884283692001289394712890844505827 69
Time Approximations 0.043.
```

```
hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [0., -.81e-37, .59e-35]Solution in 1.01s

Time Plot 0.001 s.
Exiting SolveHard() after 1.984r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40, none,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941756656097395278073169830153286,
5.589637183138732469093480501811631243212,
443.83065885898039181166126854089606238 2]
one interval r = 32.15575279513194512623749451208441769143 ..
35.50872228757019075052006326617389434466
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, .235e-34]Solution in 0.486s

Time Plot 0 s.
Exiting SolveHard() after 4.798r=34.9395 in [33.37332721 ..
```

```
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136516781285502528938301114306926,
5.18778357860895742648627101917057181873 7,
408.65773863374092080860594905905171422 22]
one interval r = 21.71840114664117627483510412835194362734 ..
26.81849303521111030888206397977430171260
Time Approximations 0.061.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [-.1e-37, -.79e-37, -.158e-34]Solution in 1.029s

Time Plot 0 s.
Exiting SolveHard() after 2.183r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
```

```
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21, none, none,
361.50888347970289636907552699794315931 28, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136516781285502528938301114306926,
5.1877835786089574264862710191705718187 37,
408.65773863374092080860594905905171422 22]
one interval r = 31.80828598755155529695667445644910779760 ..
35.00011460055336502416580067232913955154
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .124e-34]Solution in 4.28s

Time Plot 0 s.
Exiting SolveHard() after 4.548r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95, none,
361.50888347970289636907552699794315931 28, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110554379148817146628890006855337,
6.1962625653282880967070853143094551329 99,
385.44474380237977361234715550474970397 32]
one interval r = 31.60836097541348592259964845089284059472 ..
```

```
34.663727956229063683218928879462317959 62
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=9e-38
Equations at solution: [-.6e-37, .9e-37, .33e-35]Solution in 0.586s

Time Plot 0 s.
Exiting SolveHard() after 0.895r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95, none,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110554379148817146628890006855337,
6.196262565328288096707085314309455132999,
385.44474380237977361234715550474970397 32]
two intervals r = 16.87563408751080724753816756722807375526 ..
47500000000384355746414793946830606 43/2500000000000000000000000000000000
000 or r = 15.55640493850285920875569526239445741785 ..
47500000000384355746414793946830606 43/2500000000000000000000000000000000
000
Time Approximations 0.066.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
```

```
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.358e-37, 0., -.2450e-34]Solution in 5.319s

Time Plot 0 s.
Exiting SolveHard() after 6.501r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874732215210951011813849685537156,
4.883810779964301809569690097473446777394,
376.61967856764800417130396771135221292 52]
one interval r = 21.11001304895140333214332430549768357432 ..
26.31784243490549851829485875760236619854
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [-.2e-37, -.75e-37, -.129e-34]Solution in 0.876s

Time Plot 0 s.
Exiting SolveHard() after 1.59r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703787339 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68, none,
328.46938514666256165172710201889683750 63, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874732215210951011813849685537156,
4.8838107799643018095696900974734467773 94,
376.61967856764800417130396771135221292 52]
one interval r = 31.53899497715420750760445294429220966335 ..
34.53618386106744721972920708854060891413
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=5.48e-36
Equations at solution: [-.422e-35, .548e-35, .156e-34]Solution in
0.502s

Time Plot 0 s.
Exiting SolveHard() after 0.783r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
```

```
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68, none,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017562855252749863476972747593215,
6.0258135493014640311609894584982650462 94,
351.42702949611630315659969958663383756 20]
one interval r = 31.362302061181929016033840937450495880 10 ..
34.174466406312026825215231069757820608 58
Time Approximations 0.017.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.73e-35]Solution in 4.255s

Time Plot 0 s.
Exiting SolveHard() after 4.486r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
```

```
324.67144993597392225388711846864309459 68, none,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37, none, none,
292.99969139594437624894079882720953847 23, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175628552527498634769727475932 15,
6.0258135493014640311609894584982650462 94,
351.42702949611630315659969958663383756 20]
two intervals r = 17.981355144361740826281749453659075537 36 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000 or r = 13.846080154894058919788368716928225338 86 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [.54e-37, -.1e-37, .1635e-34]Solution in 1.182s

Time Plot 0 s.
Exiting SolveHard() after 2.278r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37, none, none,
292.99969139594437624894079882720953847 23, none, none, none, none,
none, none, none, none, none, none]
```

```
2 --> 1  target = [27.02037941893584153832249327684369676698,
6.37794387390205853964699097962389485295 5,
423.28832785554533027753907190498799330 25]
one interval r = 31.946618176100311679431571432007005503 28 ..
35.21212308674534236320424168799980480718
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, .4e-36]Solution in 0.586s

Time Plot 0 s.
Exiting SolveHard() after 0.916r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37, none, none,
292.99969139594437624894079882720953847 23, none, none,
360.06173468427405057422096785264949164 72, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941893584153832249327684369676698,
6.37794387390205853964699097962389485295 5,
423.28832785554533027753907190498799330 25]
two intervals r = 15.22886702397904220125394039786811661250 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000 or r = 17.12965777136342215427116492220088915795 ..
```

```
475000000003843557464147939468306064 3/250000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [.63e-37, .2e-37, -.2657e-34]Solution in 1.266s

Time Plot 0 s.
Exiting SolveHard() after 6.904r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652 75821,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37,
375.73285292084823767052595529042298504 44, none,
292.99969139594437624894079882720953847 23, none, none,
360.06173468427405057422096785264949164 72, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343587630240599970566402961239 49,
4.0035598155749966412445265462405796423 38,
404.47973595158237752497282600106865696 87]
two intervals r = 16.096839663564621511688432691690176784 02 ..
475000000003843557464147939468306064 3/250000000000000000000000000000000
000 or r = 16.399886491658994879529771412416025416 32 ..
475000000003843557464147939468306064 3/250000000000000000000000000000000
000
Time Approximations 0.052.
```

```
hint used Hint := [17.2111140106151193856920846960623014381 1, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., -.1740e-34]Solution in 1.521s

Time Plot 0 s.
Exiting SolveHard() after 7.226r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37,
375.73285292084823767052595529042298504 44, none,
292.99969139594437624894079882720953847 23,
358.64341561779477884394863930179324101 35, none,
360.06173468427405057422096785264949164 72, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234358763024059997056640296123949,
4.0035598155749966412445265462405796423 38,
404.47973595158237752497282600106865696 87]
one interval r = 21.63429630011176013468614599741138507745 ..
26.75768169907846091160181972875150140087
Time Approximations 0.056.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
```

```
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=1.01e-37
Equations at solution: [.2e-37, .101e-36, .329e-34]Solution in 5.054s

Time Plot 0 s.
Exiting SolveHard() after 6.136r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37,
375.73285292084823767052595529042298504 44,
328.11709295558959644366367670847178711 06,
292.99969139594437624894079882720953847 23,
358.64341561779477884394863930179324101 35, none,
360.06173468427405057422096785264949164 72, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954472468648616477733060072533823,
6.19617723018149243812666037476537860916 4,
385.42734026606635961571601273811601052 43]
one interval r = 31.60822049096660252454538106189813890076 ..
34.66347615060473719507772841900306120118
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=8e-38
Equations at solution: [-.5e-37, .8e-37, -.22e-35]Solution in 0.597s

Time Plot 0 s.
Exiting SolveHard() after 0.924r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37,
375.73285292084823767052595529042298504 44,
328.11709295558959644366367670847178711 06,
292.99969139594437624894079882720953847 23,
358.64341561779477884394863930179324101 35, none,
360.06173468427405057422096785264949164 72, none, none,
324.65521224570138933209472372582234050 41, none, none, none, none]

0 --> 1  target = [26.46318954472468648616477733060072533823,
6.19617723018149243812666037476537860916 4,
385.42734026606635961571601273811601052 43]
two intervals r = 16.87629600289967087339683574982126460675 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000 or r = 15.55559000698542526064705509510494649030 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000
Time Approximations 0.064.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.1432e-36, 0., -.2001e-34]Solution in 5.27s

Time Plot 0 s.
Exiting SolveHard() after 6.472r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37,
375.73285292084823767052595529042298504 44,
328.11709295558959644366367670847178711 06,
292.99969139594437624894079882720953847 23,
358.64341561779477884394863930179324101 35, none,
360.06173468427405057422096785264949164 72,
336.59441033473114462116114286267712540 51, none,
324.65521224570138933209472372582234050 41, none, none, none, none]

0 --> 2  target = [34.495226611724754503611995492293532 03,
3.8971313159729193318647631557340233403 06,
373.78081884987756164537179186720347156 48]
two intervals r = 17.297690862360386838229469440695450943 52 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000 or r = 14.994364074627813406106932382806520881 00 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000
Time Approximations 0.093.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.36e-37, 0., -.262e-35]Solution in 5.329s

Time Plot 0 s.
Exiting SolveHard() after 7.051r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37,
375.73285292084823767052595529042298504 44,
328.11709295558959644366367670847178711 06,
292.99969139594437624894079882720953847 23,
358.64341561779477884394863930179324101 35, none,
360.06173468427405057422096785264949164 72,
336.59441033473114462116114286267712540 51, none,
324.65521224570138933209472372582234050 41,
331.93806791864052905321547237807086041 31, none, none, none]

1 --> 2  target = [34.495226611724754503611995492293532 03,
3.897131315972919331864763155734023340306,
373.78081884987756164537179186720347156 48]
one interval r = 21.06068473213299695896829793836373941510 ..
26.26979834291738392749157416197191776759
Time Approximations 0.047.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
```

```
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [-.2e-37, -.5e-37, -.188e-34]Solution in 0.795s

Time Plot 0 s.
Exiting SolveHard() after 1.511r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37,
375.73285292084823767052595529042298504 44,
328.11709295558959644366367670847178711 06,
292.99969139594437624894079882720953847 23,
358.64341561779477884394863930179324101 35,
299.89866205424170119729866859291031467 52,
360.06173468427405057422096785264949164 72,
336.59441033473114462116114286267712540 51, none,
324.65521224570138933209472372582234050 41,
331.93806791864052905321547237807086041 31, none, none, none]

0 --> 2  target = [33.81362495422335495875862435950297610411,
3.72564899362351964797737083481073788079 8,
325.89209973839474286104973520870102956 78]
two intervals r = 18.552270490089792616214739124686547826 39 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000 or r = 12.491969358476190566893813335563177101 26 ..
475000000003843557464147939468306064 3/2500000000000000000000000000000000
000
Time Approximations 0.038.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1.0e-37
Equations at solution: [.243e-36, -.10e-36, .1467e-34]Solution in
1.206s

Time Plot 0 s.
Exiting SolveHard() after 6.789r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37,
375.73285292084823767052595529042298504 44,
328.11709295558959644366367670847178711 06,
292.99969139594437624894079882720953847 23,
358.64341561779477884394863930179324101 35,
299.89866205424170119729866859291031467 52,
360.06173468427405057422096785264949164 72,
336.59441033473114462116114286267712540 51, none,
324.65521224570138933209472372582234050 41,
331.93806791864052905321547237807086041 31, none, none,
289.54595773362633911691004061789437584 92]

1 --> 2  target = [33.81362495422335495875862435950297610411,
3.72564899362351964797737083481073788 0798,
325.89209973839474286104973520870102956 78]
one interval r = 20.37468935119285636971991795616215478585 ..
25.37892165319131815027967112964008944807
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
```

```
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.2e-37, .2e-37, .281e-34]Solution in 0.542s

Time Plot 0 s.
Exiting SolveHard() after 1.097r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37,
375.73285292084823767052595529042298504 44,
328.11709295558959644366367670847178711 06,
292.99969139594437624894079882720953847 23,
358.64341561779477884394863930179324101 35,
299.89866205424170119729866859291031467 52,
360.06173468427405057422096785264949164 72,
336.59441033473114462116114286267712540 51,
256.10753187130894652829173257637921268 80,
324.65521224570138933209472372582234050 41,
331.93806791864052905321547237807086041 31, none, none,
289.54595773362633911691004061789437584 92]

1 --> 0  target = [17.93041369701732974348151023501089499767,
4.68650870212344917088171181455072352820 9,
353.30541096241663461543949450657405455 76]
one interval r = 20.73150479110195246473028682890203104605 ..
25.90675353543889797821215068016258419065
Time Approximations 0.031.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=1.42e-37
Equations at solution: [.6e-37, .142e-36, .158e-34]Solution in 0.658s

Time Plot 0 s.
Exiting SolveHard() after 5.501r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37,
375.73285292084823767052595529042298504 44,
328.11709295558959644366367670847178711 06,
292.99969139594437624894079882720953847 23,
358.64341561779477884394863930179324101 35,
299.89866205424170119729866859291031467 52,
360.06173468427405057422096785264949164 72,
336.59441033473114462116114286267712540 51,
256.10753187130894652829173257637921268 80,
324.65521224570138933209472372582234050 41,
331.93806791864052905321547237807086041 31,
304.79958326941213630378846591716280476 40, none,
289.54595773362633911691004061789437584 92]

2 --> 0  target = [17.93041369701732974348151023501089499767,
4.68650870212344917088171181455072352820 9,
353.30541096241663461543949450657405455 76]
one interval r = 31.37435486997593731431244406210136438597 ..
34.20127520043977799334873263485212980047
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
```

```
17.863534069716629640044658426895081929 62, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., .125e-34]Solution in 0.343s

Time Plot 0 s.
Exiting SolveHard() after 0.635r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494530519267041812499958652758 21,
441.64295974141948220535041301222195283 97,
436.91748166052542954441307961306115500 52,
422.98493398796395371178761197378875039 54,
361.52580256938669243616999175024114940 09,
401.88173905328967833869196812524960224 62,
389.59001516511735703436473311612605044 24,
328.46939894593735054711064801642673170 40,
401.50757159128307618619247228613806380 25,
358.97362824889918697409189346544313281 60,
398.33147105615235443517770625544074120 21,
371.48387394918917091528970594307088795 95,
336.61215842436418251751792921703783739 38,
361.50888347970289636907552699794315931 28,
324.67144993597392225388711846864309459 68,
302.31384316349215827039757528346476525 85,
328.46938514666256165172710201889683750 63,
343.81340625665190234169438088828188700 37,
375.73285292084823767052595529042298504 44,
328.11709295558959644366367670847178711 06,
292.99969139594437624894079882720953847 23,
358.64341561779477884394863930179324101 35,
299.89866205424170119729866859291031467 52,
360.06173468427405057422096785264949164 72,
336.59441033473114462116114286267712540 51,
256.10753187130894652829173257637921268 80,
324.65521224570138933209472372582234050 41,
331.93806791864052905321547237807086041 31,
304.79958326941213630378846591716280476 40,
323.46169177636989627024704023791408747 84,
289.54595773362633911691004061789437584 92]

Cascade time 167.476
counts: 28, 28

 Iteration 91
```

```
Start Generation 1
1 --> 0  target = [11.999999999948074845154949298826011523 00,
6.217012502920087864815596070147664115576,
485.549080895447999482939418855610822819 0]
one interval r = 23.40850301652460950529872907600174893963 ..
27.67578046428485970381315238208343481166
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.2e-38
Equations at solution: [.2e-37, -.52e-37, -.10e-35]Solution in 1.032s

Time Plot 0 s.
Exiting SolveHard() after 2.195r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.642959729335174425707604711149684662 6, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999948074845154949298826011523 00,
6.217012502920087864815596070147664115576,
485.549080895447999482939418855610822819 0]
one interval r = 32.62814779212041652115179996289515470375 ..
36.10248388935098619693060575070902070535
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., .176e-34]Solution in 0.587s

Time Plot 0 s.
```

```
Exiting SolveHard() after 1.005r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684479400011718438834254866757261,
6.58343472160962913520393128665126711 2219,
467.78730595592435321844640474576592269 50]
one interval r = 32.41978955661820597012669966862991297537 ..
35.85152417364895955260145984154703582623
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, -.127e-34]Solution in 0.657s

Time Plot 0 s.
Exiting SolveHard() after 5.021r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72, none, none,
401.88173903854747405780272936039477208 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684479400011718438834254866757261,
6.58343472160962913520393128665126711 2219,
467.78730595592435321844640474576592269 50]
two intervals r = 12.92327160830165375330400060322391925037 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000 or r = 18.39424858032030452454600575078248853612 ..
```

```
190000000000645358498214410799714611 69/1000000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=6.7e-38
Equations at solution: [.6e-37, .67e-37, -.153e-35]Solution in 46.369s

Time Plot 0 s.
Exiting SolveHard() after 47.787r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99, none,
401.88173903854747405780272936039477208 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962817594447235331878470215105303,
4.12565179667972831228623256655138164937 9,
440.67123064633576615970201026135776259 01]
two intervals r = 14.35659705128532593080814512926160741927 ..
190000000000645358498214410799714611 69/1000000000000000000000000000000000
00000 or r = 17.70352613802205118986872336018961961225 ..
190000000000645358498214410799714611 69/1000000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=5e-38
Equations at solution: [-.98e-37, -.5e-37, -.3122e-34]Solution in
6.033s
```

```
Time Plot 0 s.
Exiting SolveHard() after 7.125r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99, none,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962817594447235331878470215105303,
4.1256517966797283122862325665513816493 79,
440.67123064633576615970201026135776259 01]
one interval r = 22.397611543618721721360662019045171082 60 ..
27.237223515953284072798606100418590313 69
Time Approximations 0.038.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.281 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643753593497457756167247815803 10, rm =
14.378187701418442824207014649444552060 81}});
Accepted {r=26.4635, rm=16.5329} with Delta=7.9e-38
Equations at solution: [0., .79e-37, .469e-34]Solution in 8.909s

Time Plot 0 s.
Exiting SolveHard() after 9.814r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
```

```
361.525802555297304400727012052456455383 6,
401.881739038547474057802729360394772085 1,
389.590015152703709320863677872990100373 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887906719093924182961791713064 79,
4.004869081655596328146877858925338612017,
404.862245008873958862652755902567465308 2]
two intervals r = 16.080110077732995530188863303846826639 61 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000 or r = 16.415798126794351435838956232517772491 75 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.34e-37, 0., -.2639e-34]Solution in 1.48s

Time Plot 0 s.
Exiting SolveHard() after 6.694r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434933638343593264026070410017241 1,
441.642959729335174425707604711149684662 6,
436.917481649008154020626455145712996427 2,
422.984933972138618041523877681752985939 9,
361.525802555297304400727012052456455383 6,
401.881739038547474057802729360394772085 1,
389.590015152703709320863677872990100373 3, none, none,
358.973628233460373210298532447579273014 6, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887906719093924182961791713064 79,
4.004869081655596328146877858925338612017,
404.862245008873958862652755902567465308 2]
one interval r = 21.641943994082063437042529399335023103 11 ..
26.763306600373145730747412544922240829 48
Time Approximations 0.058.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
```

```
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=5.2e-38
Equations at solution: [-.2e-37, -.52e-37, -.435e-34]Solution in 4.918s

Time Plot 0 s.
Exiting SolveHard() after 6.002r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95, none,
358.97362823346037321029853244757927301 46, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941787827568615765880587697959407,
5.58963718292334155121993837762421613 7911,
443.83065884280629765317950241937195981 28]
one interval r = 22.46725374475442455851870249686996522130 ..
27.27388428352984806655623632195976086992
Time Approximations 0.039.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=1.34e-37
Equations at solution: [.1e-37, -.134e-36, .61e-35]Solution in 1.044s

Time Plot 0 s.
Exiting SolveHard() after 2.052r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
```

```
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.642959729335174425707604711149684 6626,
436.917481649008154020626455145712996427 2,
422.984933972138618041523877681752985939 9,
361.525802555297304400727012052456455383 6,
401.881739038547474057802729360394772085 1,
389.590015152703709320863677872990100373 3,
328.469398928814245402286549165132457469 5, none,
358.973628233460373210298532447579273014 6,
398.331471038535002618733713869023774213 4, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589417878275686157658805876979594 07,
5.589637182923341551219938377624216137911,
443.830658842806297653179502419371959812 8]
one interval r = 32.155752795009127370979558159960951382 44 ..
35.508722287311827093693987692792617193 71
Time Approximations 0.021.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, -.114e-34]Solution in 4.294s

Time Plot 0 s.
Exiting SolveHard() after 4.66r=34.9395 in [33.37332721 .. 35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.642959729335174425707604711149684 6626,
436.917481649008154020626455145712996427 2,
422.984933972138618041523877681752985939 9,
361.525802555297304400727012052456455383 6,
401.881739038547474057802729360394772085 1,
389.590015152703709320863677872990100373 3,
328.469398928814245402286549165132457469 5,
401.507571575711758219844583773039684400 5,
358.973628233460373210298532447579273014 6,
398.331471038535002618733713869023774213 4, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136530651397157573395102450465693,
5.18778357845025954718197434831699679958 3,
408.65773862128771241428926130054460077 77]
one interval r = 21.71840114644382569330129250379768983633 ..
26.81849303505485563201756719501786522113
Time Approximations 0.061.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=5.3e-38
Equations at solution: [.1e-37, .53e-37, -.56e-35]Solution in 1.054s

Time Plot 0 s.
Exiting SolveHard() after 2.194r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34, none, none,
361.50888346574144388363101235608547422 94, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136530651397157573395102450465693,
5.18778357845025954718197434831699679958 3,
408.65773862128771241428926130054460077 77]
one interval r = 31.80828598749366876095161235038441157851 ..
35.00011460035654638008862599145457332399
Time Approximations 0.02.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .100e-34]Solution in 0.421s

Time Plot 0 s.
Exiting SolveHard() after 0.73r=34.4952 in [32.91337941 .. 35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10, none,
361.50888346574144388363101235608547422 94, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110532556766451930382336081882955,
6.19626256532457058607001597885033686970 5,
385.44474378741150877026024331123403972 00]
one interval r = 31.60836097535349147591844107705827448612 ..
34.66372795600334441187579191874780089663
Time Approximations 0.016.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.61e-35]Solution in 0.546s

Time Plot 0 s.
Exiting SolveHard() after 4.867r=33.8136 in [32.62689490 ..
34.66372796]
```

```
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10, none,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110532556766451930382336081882955,
6.19626256532457058607001597885033686970 5,
385.44474378741150877026024331123403972 00]
two intervals r = 16.875634087732784060656505419652276985 12 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000 or r = 15.556404937773515739540514416750481749 22 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., .1956e-34]Solution in 1.183s

Time Plot 0 s.
Exiting SolveHard() after 2.357r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
```

```
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874755036096417009017986070903246,
4.8838107797850324067784462044494519042 05,
376.61967855205922188648410231996471489 18]
one interval r = 21.11001304873851671072075341672164634754 ..
26.31784243466769754446366497168362834628
Time Approximations 0.034.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, -.232e-34]Solution in 0.852s

Time Plot 0 s.
Exiting SolveHard() after 5.57r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04, none,
```

```
328.46938512954288872872378479737107196 36, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874755036096417009017986070903246,
4.8838107797850324067784462044494519042 05,
376.61967855205922188648410231996471489 18]
one interval r = 31.53899497709757782997488517143309024004 ..
34.53618386083600822188577696006446142827
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.08e-36
Equations at solution: [.236e-35, -.308e-35, -.232e-34]Solution in
0.491s

Time Plot 0 s.
Exiting SolveHard() after 0.802r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04, none,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017531925885506299698741705588476,
6.0258135492784846107493930772707431203 24,
351.42702947801181404850701122763299958 53]
one interval r = 31.36230206113361633677554242629355266931 ..
34.17446640605515640585432974812654907575
```

```
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [.1e-37, -.3e-37, .308e-34]Solution in 0.547s

Time Plot 0 s.
Exiting SolveHard() after 0.781r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04, none,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04, none, none,
292.99969137617544983134185705797012605 98, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017531925885506299698741705588476,
6.02581354927848461074939307727074312 0324,
351.42702947801181404850701122763299958 53]
two intervals r = 17.98135514459660156939479253871380889770 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000 or r = 13.84608015386171773436709652696787410166 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [.54e-37, -.2e-37, -.3576e-34]Solution in 1.167s

Time Plot 0 s.
Exiting SolveHard() after 6.245r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04, none, none,
292.99969137617544983134185705797012605 98, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941871309165426841932628065901790,
6.37794387388415072208233534756284782376 5,
423.28832783696261367440496562545210192 70]
one interval r = 31.94661817597138660106582235923652211507 ..
35.21212308645565703930972644523723184760
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
```

```
Equations at solution: [.4e-37, -.5e-37, .20e-35]Solution in 0.597s

Time Plot 0 s.
Exiting SolveHard() after 0.954r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04, none, none,
292.99969137617544983134185705797012605 98, none, none,
360.06173466412639786943224959718267426 78, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941871309165426841932628065901790,
6.37794387388415072208233534756284782376 5,
423.28832783696261367440496562545210192 70]
two intervals r = 15.22886702441363750474959085222068340966 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000 or r = 17.12965777068856908342331100144797235105 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000
Time Approximations 0.063.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [-.46e-37, -.1e-37, -.792e-35]Solution in 1.311s

Time Plot 0 s.
```

```
Exiting SolveHard() after 6.674r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04,
375.73285289899311760743277359963135203 18, none,
292.99969137617544983134185705797012605 98, none, none,
360.06173466412639786943224959718267426 78, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343340109649858414325195655128 56,
4.0035598153543224457564485702037813349 60,
404.47973593554546495133738827678847020 46]
two intervals r = 16.096839663858828168588447366027869381 56 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000 or r = 16.399886490976557168303187566969076214 07 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., .1974e-34]Solution in 1.491s

Time Plot 0 s.
Exiting SolveHard() after 6.729r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04,
375.73285289899311760743277359963135203 18, none,
292.99969137617544983134185705797012605 98,
358.64341560162185450403105930420011022 47, none,
360.06173466412639786943224959718267426 78, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234334010964985841432519565512856,
4.0035598153543224457564485702037813349 60,
404.47973593554546495133738827678847020 46]
one interval r = 21.63429629984724002566091072991384461021 ..
26.75768169886547121246549754273676413543
Time Approximations 0.056.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., -.519e-34]Solution in 5.856s

Time Plot 0 s.
Exiting SolveHard() after 6.936r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04,
375.73285289899311760743277359963135203 18,
328.11709293768604390036711647243243348 75,
292.99969137617544983134185705797012605 98,
358.64341560162185450403105930420011022 47, none,
360.06173466412639786943224959718267426 78, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954450857330367622865423676357298,
6.19617723017841870505395934273173229949 9,
385.42734025122968756224106254594925816 57]
one interval r = 31.60822049090768617072783370842511504691 ..
34.66347615038092807657193613058700615382
Time Approximations 0.03.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, .115e-34]Solution in 0.613s

Time Plot 0 s.
Exiting SolveHard() after 0.939r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
```

```
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04,
375.73285289899311760743277359963135203 18,
328.11709293768604390036711647243243348 75,
292.99969137617544983134185705797012605 98,
358.64341560162185450403105930420011022 47, none,
360.06173466412639786943224959718267426 78, none, none,
324.65521222901590965725598516093972577 03, none, none, none, none]

0 --> 1  target = [26.46318954450857330367622865423676357298,
6.19617723017841870505395934273173229949 9,
385.42734025122968756224106254594925816 57]
two intervals r = 16.87629600311661449483935382820825173531 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000 or r = 15.55559000626216298462347592959334197111 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000
Time Approximations 0.065.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.358e-37, 0., .3200e-34]Solution in 5.045s

Time Plot 0 s.
Exiting SolveHard() after 6.245r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
```

```
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04,
375.73285289899311760743277359963135203 18,
328.11709293768604390036711647243243348 75,
292.99969137617544983134185705797012605 98,
358.64341560162185450403105930420011022 47, none,
360.06173466412639786943224959718267426 78,
336.59441031711674731414208955326328600 57, none,
324.65521222901590965725598516093972577 03, none, none, none, none]

0 --> 2  target = [34.49522661152937300250867589106852670241,
3.8971313157608623042873716863720159358 08,
373.78081883671348648867420075114949471 31]
two intervals r = 17.29769086249935983795072149759681094248 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000 or r = 14.99436407393517864940607049627087489737 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000
Time Approximations 0.086.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., -.1080e-34]Solution in 5.242s

Time Plot 0 s.
Exiting SolveHard() after 6.917r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
```

```
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04,
375.73285289899311760743277359963135203 18,
328.11709293768604390036711647243243348 75,
292.99969137617544983134185705797012605 98,
358.64341560162185450403105930420011022 47, none,
360.06173466412639786943224959718267426 78,
336.59441031711674731414208955326328600 57, none,
324.65521222901590965725598516093972577 03,
331.93806790480957712096077063759598909 40, none, none, none]

1 --> 2  target = [34.49522661152937300250867589106852670241,
3.8971313157608623042873716863720159358 08,
373.78081883671348648867420075114949471 31]
one interval r = 21.06068473196689363221112794116593450005 ..
26.26979834271788175037383418335836897853
Time Approximations 0.034.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., .26e-35]Solution in 0.799s

Time Plot 0 s.
Exiting SolveHard() after 1.507r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
```

```
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04,
375.73285289899311760743277359963135203 18,
328.11709293768604390036711647243243348 75,
292.99969137617544983134185705797012605 98,
358.64341560162185450403105930420011022 47,
299.89866203900122646370916495605470000 31,
360.06173466412639786943224959718267426 78,
336.59441031711674731414208955326328600 57, none,
324.65521222901590965725598516093972577 03,
331.93806790480957712096077063759598909 40, none, none, none]

0 --> 2  target = [33.81362495399054213997256329834137223519,
3.7256489933944515737802877283704293977 45,
325.89209972107647622690842425436386358 27]
two intervals r = 18.552270490212475333099050806191137810 74 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000 or r = 12.491969357440561879422200824435068185 58 ..
19000000000064535849821441079971461169/10000000000000000000000000000000
00000
Time Approximations 0.037.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [-.120e-36, .5e-37, .4265e-34]Solution in 1.214s

Time Plot 0 s.
Exiting SolveHard() after 6.575r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
```

```
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04,
375.73285289899311760743277359963135203 18,
328.11709293768604390036711647243243348 75,
292.99969137617544983134185705797012605 98,
358.64341560162185450403105930420011022 47,
299.89866203900122646370916495605470000 31,
360.06173466412639786943224959718267426 78,
336.59441031711674731414208955326328600 57, none,
324.65521222901590965725598516093972577 03,
331.93806790480957712096077063759598909 40, none, none,
289.54595771594190864521606166350604031 69]

1 --> 2  target = [33.81362495399054213997256329834137223519,
3.72564899339445157378028772837042939774 5,
325.89209972107647622690842425436386358 27]
one interval r = 20.37468935106702931870043335268413646141 ..
25.37892165287287626060377767906617382225
Time Approximations 0.028.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.2e-37, -.2e-37, -.379e-34]Solution in 0.547s

Time Plot 0 s.
Exiting SolveHard() after 1.112r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
```

```
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04,
375.73285289899311760743277359963135203 18,
328.11709293768604390036711647243243348 75,
292.99969137617544983134185705797012605 98,
358.64341560162185450403105930420011022 47,
299.89866203900122646370916495605470000 31,
360.06173466412639786943224959718267426 78,
336.59441031711674731414208955326328600 57,
256.10753185231512117879769868039820149 74,
324.65521222901590965725598516093972577 03,
331.93806790480957712096077063759598909 40, none, none,
289.54595771594190864521606166350604031 69]

1 --> 0  target = [17.93041369725525971177827891072360336182,
4.68650870193075410101879618643838904685 3,
353.30541094444318364605817893468497800 48]
one interval r = 20.73150479089824242122248841116713435430 ..
25.90675353513404473754160638977818186561
Time Approximations 0.034.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, .51e-35]Solution in 4.643s

Time Plot 0 s.
Exiting SolveHard() after 5.362r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04,
375.73285289899311760743277359963135203 18,
328.11709293768604390036711647243243348 75,
292.99969137617544983134185705797012605 98,
358.64341560162185450403105930420011022 47,
299.89866203900122646370916495605470000 31,
360.06173466412639786943224959718267426 78,
336.59441031711674731414208955326328600 57,
256.10753185231512117879769868039820149 74,
324.65521222901590965725598516093972577 03,
331.93806790480957712096077063759598909 40,
304.79958325001622998851709754686830176 15, none,
289.54595771594190864521606166350604031 69]

2 --> 0  target = [17.93041369725525971177827891072360336182,
4.68650870193075410101879618643838904685 3,
353.30541094444318364605817893468497800 48]
one interval r = 31.37435486992639244984505462288556107412 ..
34.20127520018389899895577494320528917367
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, -.311e-34]Solution in 0.388s

Time Plot 0 s.
Exiting SolveHard() after 0.662r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343493363834359326402607041001724 11,
441.64295972933517442570760471114968466 26,
436.91748164900815402062645514571299642 72,
422.98493397213861804152387768175298593 99,
361.52580255529730440072701205245645538 36,
401.88173903854747405780272936039477208 51,
389.59001515270370932086367787299010037 33,
328.46939892881424540228654916513245746 95,
401.50757157571175821984458377303968440 05,
358.97362823346037321029853244757927301 46,
398.33147103853500261873371386902377421 34,
371.48387393643306199378064906114817882 10,
336.61215840661542432983954119172419949 48,
361.50888346574144388363101235608547422 94,
324.67144991916568813459518186634559904 04,
302.31384314294161361098236088205089594 15,
328.46938512954288872872378479737107196 36,
343.81340624095125233664045711975177512 04,
375.73285289899311760743277359963135203 18,
328.11709293768604390036711647243243348 75,
292.99969137617544983134185705797012605 98,
358.64341560162185450403105930420011022 47,
299.89866203900122646370916495605470000 31,
360.06173466412639786943224959718267426 78,
336.59441031711674731414208955326328600 57,
256.10753185231512117879769868039820149 74,
324.65521222901590965725598516093972577 03,
331.93806790480957712096077063759598909 40,
304.79958325001622998851709754686830176 15,
323.46169175842262323455374497474937535 35,
289.54595771594190864521606166350604031 69]

Cascade time 166.573
counts: 28, 28

 Iteration 92

Start Generation 1
1 --> 0  target = [12.00000000001936856423393654480353084100,
6.21701250292896418857472124613436634369 7,
485.54908089987980038356832567888239894 27]
one interval r = 23.40850301666764857295495339915482263148 ..
27.67578046441939486914414652529474554086
Time Approximations 0.044.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937859 4, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
```

```
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=7.7e-38
Equations at solution: [.2e-37, -.77e-37, -.19e-35]Solution in 1.064s

Time Plot 0 s.
Exiting SolveHard() after 2.2r=27.5236 in [25.56992694 .. 27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000001936856423393654480353084100,
6.217012502928964188574721246134366343697,
485.549080899879800383568325678882398942 7]
one interval r = 32.628147792222515782709975376258551755 03 ..
36.102483889454632325831450959888264838 34
Time Approximations 0.028.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=5e-38
Equations at solution: [.9e-37, -.5e-37, -.111e-34]Solution in 4.461s

Time Plot 0 s.
Exiting SolveHard() after 4.877r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684492664156539077680187329340432,
6.583434721678478726735374019181505412802,
467.787305959863895689134198863922355073 5]
```

```
one interval r = 32.41978955671298813678618263895356179801 ..
35.85152417374652235318507650759986542608
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.11e-35]Solution in 0.657s

Time Plot 0 s.
Exiting SolveHard() after 1.047r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32, none, none,
401.88173904317520407388921335923740175 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684492664156539077680187329340432,
6.58343472167847872673537401918150541280 2,
467.78730595986389568913419886392235507 35]
two intervals r = 12.92327160835708001644561955824521088874 ..
19000000000015445512232971188585003469/10000000000000000000000000000000
00000 or r = 18.39424858032940861509587232452280004820 ..
19000000000015445512232971188585003469/10000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=8.8e-38
Equations at solution: [.7e-37, .88e-37, .1183e-34]Solution in 48.518s

Time Plot 0 s.
```

```
Exiting SolveHard() after 49.931r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60, none,
401.88173904317520407388921335923740175 81, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962830592761463906279525999114004,
4.1256517968712707322679613967039548085 13,
440.67123065245012613317728861699026967 36]
two intervals r = 14.35659705117656295509104965209074905754 ..
19000000000015445512232971188585003469/10000000000000000000000000000000
00000 or r = 17.70352613813594827264462193816733660128 ..
19000000000015445512232971188585003469/10000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.29e-37, -.2e-37, .1221e-34]Solution in 5.232s

Time Plot 0 s.
Exiting SolveHard() after 6.32r=15.9119 in [14.35659706 .. 18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60, none,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962830592761463906279525999114004,
```

```
4.125651796871270732267961396703954808513,
440.671230652450126133177288616990269673 6]
one interval r = 22.39761154378680868349606502815207243089 ..
27.23722351611413251084917316056803390100
Time Approximations 0.038.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.28 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064399538446754303517582811120478, rm =
14.37818770487443909396401936685372802511}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .62e-35]Solution in 9.172s

Time Plot 0 s.
Exiting SolveHard() after 10.084r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888802291303841032082886368589374,
4.00486908184437010643080152550833549250 8,
404.86224501395069881909641761683361294 29]
two intervals r = 16.08011007764067043684740169570543536338 ..
19000000000015445512232971188585003469/10000000000000000000000000000000
00000 or r = 16.41579812695378890967843125030823262444 ..
19000000000015445512232971188585003469/10000000000000000000000000000000
00000
Time Approximations 0.052.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.35e-37, .1e-37, .1702e-34]Solution in 1.498s

Time Plot 0 s.
Exiting SolveHard() after 6.852r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60, none, none,
358.97362824075393912811481662698828812 83, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888022913038410320828863685893 74,
4.0048690818443701064308015255083354925 08,
404.86224501395069881909641761683361294 29]
one interval r = 21.641943994211542337833384574408956882 14 ..
26.763306600528065158472536240585694745 21
Time Approximations 0.05.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., -.400e-34]Solution in 1.056s

Time Plot 0 s.
Exiting SolveHard() after 6.522r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75, none,
358.97362824075393912811481662698828812 83, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941795732286746199372354408540041,
5.5896371829079893922573168810763019414 77,
443.83065884540077142294931766337063768 42]
one interval r = 22.46725374484597806074666233613937048142 ..
27.27388428364944814233875557195107901710
Time Approximations 0.043.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., .182e-34]Solution in 5.096s

Time Plot 0 s.
Exiting SolveHard() after 6.077r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75, none,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941795732286746199372354408540041,
```

```
5.589637182907989392257316881076301941477,
443.830658845400771422949317663370637684 2]
one interval r = 32.155752795087421051657593092558294889 68 ..
35.508722287391005843571864083385427202 85
Time Approximations 0.02.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .90e-35]Solution in 0.495s

Time Plot 0 s.
Exiting SolveHard() after 0.872r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434937790704468343566657799667598 6,
441.642959733078879002038349190697097367 2,
436.917481654654254704348831200312356103 2,
422.984933974550884405863646743191806846 0,
361.525802562555358266019403766244594866 2,
401.881739043175204073889213359237401758 1,
389.590015160624578089473462227732133906 0,
328.469398935100834465668962721775492575,
401.507571580575748725740126307375927968 6,
358.973628240753939128114816626988288128 3,
398.331471039875525198238003307132489785 5, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365068719030891807843177980870 24,
5.187783578494755655153316034551989452614,
408.657738629667188904408583392182719933 4]
one interval r = 21.718401146642214222197875153047646340 86 ..
26.818493035256690956995655561786320080 73
Time Approximations 0.06.

hint used Hint := [26.463189544581466506563323041523995778 98, 3, -1, 1,
15.901285154143411587657101096863708215 21, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.31e-37
Equations at solution: [.2e-37, .131e-36, .20e-35]Solution in 1.049s

Time Plot 0 s.
Exiting SolveHard() after 2.165r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55, none, none,
361.50888347291841032725175409490880069 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136506871903089180784317798087024,
5.1877835784947556551533160345519894526 14,
408.65773862966718890440858339218271993 34]
one interval r = 31.80828598762271063126713428188348078173 ..
35.00011460052053937411751428391463023182
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.51e-35]Solution in 0.434s

Time Plot 0 s.
Exiting SolveHard() after 5.016r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
371.48387394677664678848353693288181440 58, none,
361.50888347291841032725175409490880069 73, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110552366333413348918050516312968,
6.19626256541463058121947036027061724754 5,
385.44474379497956591212303222437885772 88]
one interval r = 31.60836097546808130706350065698814141442 ..
34.66372795615591031031502443993953586660
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=0
Equations at solution: [0., 0., -.28e-35]Solution in 0.567s

Time Plot 0 s.
Exiting SolveHard() after 0.877r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
371.48387394677664678848353693288181440 58, none,
```

```
361.5088834729184103272517540949088006973,
324.6714499272501354978020051662286954194, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110552366333413348918050516312968,
6.1962625654146305812194703602706172475 45,
385.44474379497956591212303222437885772 88]
two intervals r = 16.875634087536883101367311315818678676 51 ..
19000000000015445512232971188585003469/10000000000000000000000000000000000
00000 or r = 15.556404938095377586339664382390188026 55 ..
19000000000015445512232971188585003469/10000000000000000000000000000000000
00000
Time Approximations 0.062.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., .4020e-34]Solution in 5.192s

Time Plot 0 s.
Exiting SolveHard() after 6.392r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349377907044683435666577996675986,
441.6429597330788790020383491906970973672,
436.9174816546542547043488312003123561032,
422.9849339745508844058636467431918068460,
361.5258025625553582660194037662445948662,
401.8817390431752040738892133592374017581,
389.5900151606245780894734622277321339060,
328.4693989351008344656668962721775492575,
401.5075715805757487257401263073759279686,
358.9736282407539391281148166269882881283,
398.3314710398755251982380033071324897855,
371.4838739467766467884835369328818144058,
336.6121584124848967651219262593955593893,
361.5088834729184103272517540949088006973,
324.6714499272501354978020051662286954194, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874735524889842230712199902672272,
4.8838107798136689829540895358972833101 88,
376.6196785597523859563322036537895129436]
one interval r = 21.110013048897490959454672598392398193 18 ..
```

```
26.31784243487066422914854699796339355548
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.222e-34]Solution in 0.865s

Time Plot 0 s.
Exiting SolveHard() after 1.582r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
371.48387394677664678848353693288181440 58,
336.61215841248489676512192625939555938 93,
361.50888347291841032725175409490880069 73,
324.67144992725013549780200516622869541 94, none,
328.46938513582681463109151539647588188 57, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874735524889842230712199902672272,
4.88381077981366898295408953589728331018 8,
376.61967855975238595633220365378951294 36]
one interval r = 31.53899497721027094616222430353271785915 ..
34.53618386099036514770181463017718168585
Time Approximations 0.018.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=7.82e-36
Equations at solution: [-.600e-35, .782e-35, .307e-34]Solution in
0.493s

Time Plot 0 s.
Exiting SolveHard() after 0.782r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
371.48387394677664678848353693288181440 58,
336.61215841248489676512192625939555938 93,
361.50888347291841032725175409490880069 73,
324.67144992725013549780200516622869541 94, none,
328.46938513582681463109151539647588188 57,
343.81340625110863231069023129112382506 35, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017550876260505584387803539852490,
6.0258135493658791092435220475583421950 42,
351.42702948458918557904038767151234492 30]
one interval r = 31.362302061230392183197712920178750181 11 ..
34.174466406193072852408746799420111210 84
Time Approximations 0.017.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [-.3e-37, .6e-37, .178e-34]Solution in 4.794s

Time Plot 0 s.
Exiting SolveHard() after 5.028r=33.3686 in [32.23723258 ..
34.17446642]
```

```
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
371.48387394677664678848353693288181440 58,
336.61215841248489676512192625939555938 93,
361.50888347291841032725175409490880069 73,
324.67144992725013549780200516622869541 94, none,
328.46938513582681463109151539647588188 57,
343.81340625110863231069023129112382506 35, none, none,
292.99969138335642767857982365192048954 24, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017550876260505584387803539852490,
6.0258135493658791092435220475583421950 42,
351.42702948458918557904038767151234492 30]
two intervals r = 17.981355144454947571849774382875954832 86 ..
19000000000015445512232971188585003469/10000000000000000000000000000000000
00000 or r = 13.846080154227291253405105447984807195 24 ..
19000000000015445512232971188585003469/10000000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=1e-38
Equations at solution: [.19e-37, -.1e-37, .1038e-34]Solution in 1.157s

Time Plot 0 s.
Exiting SolveHard() after 2.218r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
371.48387394677664678848353693288181440 58,
336.61215841248489676512192625939555938 93,
361.50888347291841032725175409490880069 73,
324.67144992725013549780200516622869541 94,
302.31384314772322658290962950247090459 80,
328.46938513582681463109151539647588188 57,
343.81340625110863231069023129112382506 35, none, none,
292.99969138335642767857982365192048954 24, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941881740462638786339938993835496,
6.37794387394317949675047821933460598729 1,
423.28832783843921804579898806219721450 35]
one interval r = 31.94661817603760090177863516616230348997 ..
35.21212308651922261816680513456457241472
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=8e-38
Equations at solution: [.7e-37, -.8e-37, -.13e-35]Solution in 4.721s

Time Plot 0 s.
Exiting SolveHard() after 5.054r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
```

```
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
371.48387394677664678848353693288181440 58,
336.61215841248489676512192625939555938 93,
361.50888347291841032725175409490880069 73,
324.67144992725013549780200516622869541 94,
302.31384314772322658290962950247090459 80,
328.46938513582681463109151539647588188 57,
343.81340625110863231069023129112382506 35, none, none,
292.99969138335642767857982365192048954 24, none, none,
360.06173466648407143300364112225389265 55, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941881740462638786339938993835496,
6.3779438739431794967504782193346059872 91,
423.28832783843921804579898806219721450 35]
two intervals r = 15.22886702451110161743525456125002066608 ..
19000000000015445512232971188585003469/10000000000000000000000000000000
00000 or r = 17.12965777067795362455624687433895084722 ..
19000000000015445512232971188585003469/10000000000000000000000000000000
00000
Time Approximations 0.066.

hint used Hint := [16.53336698818830587377417370329609603012, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [-.15e-37, 0., -.2833e-34]Solution in 1.338s

Time Plot 0 s.
Exiting SolveHard() after 2.676r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
```

```
371.48387394677664678848353693288181440 58,
336.61215841248489676512192625939555938 93,
361.50888347291841032725175409490880069 73,
324.67144992725013549780200516622869541 94,
302.31384314772322658290962950247090459 80,
328.46938513582681463109151539647588188 57,
343.81340625110863231069023129112382506 35,
375.73285289862961311875745639689293923 86, none,
292.99969138335642767857982365192048954 24, none, none,
360.06173466648407143300364112225389265 55, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234345981491040150396562152595130,
4.0035598155439306588479274431078990585 25,
404.47973594086376592420650822581220626 04]
two intervals r = 16.09683966375570766084845119551122449304 ..
19000000000015445512232971188585003469/10000000000000000000000000000000
00000 or r = 16.39988649114693582882521424164478474871 ..
19000000000015445512232971188585003469/10000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [-.17e-37, 0., -.1172e-34]Solution in 1.535s

Time Plot 0 s.
Exiting SolveHard() after 6.723r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
371.48387394677664678848353693288181440 58,
336.61215841248489676512192625939555938 93,
361.50888347291841032725175409490880069 73,
```

```
324.67144992725013549780200516622869541 94,
302.31384314772322658290962950247090459 80,
328.46938513582681463109151539647588188 57,
343.81340625110863231069023129112382506 35,
375.73285289862961311875745639689293923 86, none,
292.99969138335642767857982365192048954 24,
358.64341560912622991876383547065203771 48, none,
360.06173466648407143300364112225389265 55, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234345981491040150396562152595130,
4.0035598155439306588479274431078990585 25,
404.47973594086376592420650822581220626 04]
one interval r = 21.634296299981346645906427043061826264 57 ..
26.757681699024006982995840118397072616 26
Time Approximations 0.049.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., -.681e-34]Solution in 1.055s

Time Plot 0 s.
Exiting SolveHard() after 6.598r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
371.48387394677664678848353693288181440 58,
336.61215841248489676512192625939555938 93,
361.50888347291841032725175409490880069 73,
324.67144992725013549780200516622869541 94,
302.31384314772322658290962950247090459 80,
328.46938513582681463109151539647588188 57,
343.81340625110863231069023129112382506 35,
```

```
375.7328528986296131187574563968929392386,
328.1170929441949744537228395485974717517,
292.9996913833564276785798236519204895424,
358.6434156091262299187638354706520377148, none,
360.0617346664840714330036411222538926555, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954470532516934829153532038772812,
6.196177230268070897280745657874501066610,
385.42734025871434008016345023500486553 23]
one interval r = 31.608220491021597247166832808382414990 87 ..
34.66347615053228727579778286637707248997
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, .84e-35]Solution in 0.561s

Time Plot 0 s.
Exiting SolveHard() after 0.869r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
371.48387394677664678848353693288181440 58,
336.61215841248489676512192625939555938 93,
361.50888347291841032725175409490880069 73,
324.67144992725013549780200516622869541 94,
302.31384314772322658290962950247090459 80,
328.46938513582681463109151539647588188 57,
343.81340625110863231069023129112382506 35,
375.7328528986296131187574563968929392386,
328.1170929441949744537228395485974717517,
292.9996913833564276785798236519204895424,
```

```
358.6434156091262299187638354706520377148, none,
360.0617346664840714330036411222538926555, none, none,
324.6552122370225478233684056879388065221, none, none, none, none]

0 --> 1  target = [26.46318954470532516934829153532038772812,
6.1961772302680708972807456578745010666 10,
385.4273402587143400801634502350048655323]
two intervals r = 16.87629600292389418096323340337280393493 ..
19000000000015445512232971188585003469/10000000000000000000000000000000000
00000 or r = 15.55559000658017171389581369976405591192 ..
19000000000015445512232971188585003469/10000000000000000000000000000000000
00000
Time Approximations 0.066.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., .548e-35]Solution in 5.947s

Time Plot 0 s.
Exiting SolveHard() after 7.16r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349377907044683435666577996675986,
441.6429597330788790020383491906970973672,
436.9174816546542547043488312003123561032,
422.9849339745508844058636467431918068460,
361.5258025625553582660194037662445948662,
401.8817390431752040738892133592374017581,
389.5900151606245780894734622277321339060,
328.4693989351008344656668962721775492575,
401.5075715805757487257401263073759279686,
358.9736282407539391281148166269882881283,
398.3314710398755251982380033071324897855,
371.4838739467766467884835369328818144058,
336.6121584124848967651219262593955593893,
361.5088834729184103272517540949088006973,
324.6714499272501354978020051662286954194,
302.3138431477232265829096295024709045980,
328.4693851358268146310915153964758818857,
343.8134062511086323106902312911238250635,
375.7328528986296131187574563968929392386,
328.1170929441949744537228395485974717517,
292.9996913833564276785798236519204895424,
358.6434156091262299187638354706520377148, none,
```

```
360.06173466648407143300364112225389265 55,
336.59441032290110612368344826631329877 19, none,
324.65521223702254782336840568793880652 21, none, none, none, none]

0 --> 2  target = [34.49522661173030971844821046737607387995,
3.8971313159707896707920588330391517232 24,
373.78081884763845523775807481065001847 24]
two intervals r = 17.297690862195481781814419917336343217 49 ..
19000000000015445512232971188585003469/10000000000000000000000000000000000
00000 or r = 14.994364074457876243713511388712687146 14 ..
19000000000015445512232971188585003469/10000000000000000000000000000000000
00000
Time Approximations 0.089.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.36e-37, 0., .2875e-34]Solution in 5.205s

Time Plot 0 s.
Exiting SolveHard() after 6.932r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
371.48387394677664678848353693288181440 58,
336.61215841248489676512192625939555938 93,
361.50888347291841032725175409490880069 73,
324.67144992725013549780200516622869541 94,
302.31384314772322658290962950247090459 80,
328.46938513582681463109151539647588188 57,
343.81340625110863231069023129112382506 35,
375.73285289862961311875745639689293923 86,
328.11709294419497445372283954859747175 17,
292.99969138335642767857982365192048954 24,
358.64341560912622991876383547065203771 48, none,
```

```
360.06173466648407143300364112225389265 55,
336.5944103229011061236834482663132987719, none,
324.6552122370225478233684056879388065221,
331.9380679173770903112767767698329173128, none, none, none]

1 --> 2  target = [34.49522661173030971844821046737607387995,
3.8971313159707896707920588330391517232 24,
373.7808188476384552377580748106500184724]
one interval r = 21.06068473217894130198494445519350123863 ..
26.26979834297675247903562748277726123395
Time Approximations 0.037.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [-.1e-37, -.3e-37, .299e-34]Solution in 0.822s

Time Plot 0 s.
Exiting SolveHard() after 1.578r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349377907044683435666577996675986,
441.6429597330788790020383491906970973672,
436.9174816546542547043488312003123561032,
422.9849339745508844058636467431918068460,
361.5258025625553582660194037662445948662,
401.8817390431752040738892133592374017581,
389.5900151606245780894734622277321339060,
328.4693989351008344656668962721775492575,
401.5075715805757487257401263073759279686,
358.9736282407539391281148166269882881283,
398.3314710398755251982380033071324897855,
371.4838739467766467884835369328818144058,
336.6121584124848967651219262593955593893,
361.5088834729184103272517540949088006973,
324.6714499272501354978020051662286954194,
302.3138431477232265829096295024709045980,
328.4693851358268146310915153964758818857,
343.8134062511086323106902312911238250635,
375.7328528986296131187574563968929392386,
328.1170929441949744537228395485974717517,
292.9996913833564276785798236519204895424,
358.6434156091262299187638354706520377148,
299.8986620506404531410951232936940614328,
```

```
360.06173466648407143300364112225389265 55,
336.5944103229011061236834482663132987719, none,
324.6552122370225478233684056879388065221,
331.9380679173770903112767767698329173128, none, none, none]

0 --> 2  target = [33.81362495415606208120349547563926180520,
3.7256489935977970725939409714999752423 81,
325.8920997296975340242447127957769570927]
two intervals r = 18.552270490057995258634100275391025537 82 ..
19000000000015445512232971188585003469/1000000000000000000000000000000000
00000 or r = 12.49196935796212948136942155301957796054 ..
19000000000015445512232971188585003469/1000000000000000000000000000000000
00000
Time Approximations 0.039.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1e-38
Equations at solution: [.34e-37, -.1e-37, .956e-35]Solution in 1.222s

Time Plot 0 s.
Exiting SolveHard() after 6.617r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493779070446834356665779966759 86,
441.64295973307887900203834919069709736 72,
436.91748165465425470434883120031235610 32,
422.98493397455088440586364674319180684 60,
361.52580256255535826601940376624459486 62,
401.88173904317520407388921335923740175 81,
389.59001516062457808947346222773213390 60,
328.46939893510083446566689627217754925 75,
401.50757158057574872574012630737592796 86,
358.97362824075393912811481662698828812 83,
398.33147103987552519823800330713248978 55,
371.48387394677664678848353693288181440 58,
336.61215841248489676512192625939555938 93,
361.50888347291841032725175409490880069 73,
324.67144992725013549780200516622869541 94,
302.31384314772322658290962950247090459 80,
328.46938513582681463109151539647588188 57,
343.81340625110863231069023129112382506 35,
375.73285289862961311875745639689293923 86,
328.11709294419497445372283954859747175 17,
292.99969138335642767857982365192048954 24,
```

```
358.6434156091262299187638354706520377148,
299.8986620506404531410951232936940614328,
360.0617346664840714330036411222538926555,
336.5944103229011061236834482663132987719, none,
324.6552122370225478233684056879388065221,
331.9380679173770903112767767698329173128, none, none,
289.5459577266230072837370892662745097767]

1 --> 2  target = [33.8136249541560620812034954756392618052 0,
3.7256489935977970725939409714999752423 81,
325.8920997296975340242447127957769570927]
one interval r = 20.374689351182145756394845830148630727 71 ..
25.37892165310736399184962653590451239308
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=6e-38
Equations at solution: [.5e-37, .6e-37, -.470e-34]Solution in 0.55s

Time Plot 0 s.
Exiting SolveHard() after 1.116r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349377907044683435666577996675986,
441.6429597330788790020383491906970973672,
436.9174816546542547043488312003123561032,
422.9849339745508844058636467431918068460,
361.5258025625553582660194037662445948662,
401.8817390431752040738892133592374017581,
389.5900151606245780894734622277321339060,
328.4693989351008344656668962721775492575,
401.5075715805757487257401263073759279686,
358.9736282407539391281148166269882881283,
398.3314710398755251982380033071324897855,
371.4838739467766467884835369328818144058,
336.6121584124848967651219262593955593893,
361.5088834729184103272517540949088006973,
324.6714499272501354978020051662286954194,
302.3138431477232265829096295024709045980,
328.4693851358268146310915153964758818857,
343.8134062511086323106902312911238250635,
375.7328528986296131187574563968929392386,
328.1170929441949744537228395485974717517,
```

```
292.99969138335642767857982365192048954 24,
358.6434156091262299187638354706520377148,
299.8986620506404531410951232936940614328,
360.0617346664840714330036411222538926555,
336.5944103229011061236834482663132987719,
256.1075318617988871985525651676960695703,
324.6552122370225478233684056879388065221,
331.9380679173770903112767767698329173128, none, none,
289.5459577266230072837370892662745097767]

1 --> 0  target = [17.9304136971227524683956797902192272 6822,
4.6865087019421074737697426108106996552 37,
353.3054109506364786654886011241653572562]
one interval r = 20.7315047910119778725535891434899804263 8 ..
25.9067535353161229330689239255996670300 9
Time Approximations 0.029.

hint used Hint := [25.4020715378188039710536173439704738 2754, 3, -1, 1,
17.0062444547662134530206245122467435705 3, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.332e-34]Solution in 0.669s

Time Plot 0 s.
Exiting SolveHard() after 5.339r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349377907044683435666577996675986,
441.6429597330788790020383491906970973672,
436.9174816546542547043488312003123561032,
422.9849339745508844058636467431918068460,
361.5258025625553582660194037662445948662,
401.8817390431752040738892133592374017581,
389.5900151606245780894734622277321339060,
328.4693989351008344656668962721775492575,
401.5075715805757487257401263073759279686,
358.9736282407539391281148166269882881283,
398.3314710398755251982380033071324897855,
371.4838739467766467884835369328818144058,
336.6121584124848967651219262593955593893,
361.5088834729184103272517540949088006973,
324.6714499272501354978020051662286954194,
302.3138431477232265829096295024709045980,
328.4693851358268146310915153964758818857,
343.8134062511086323106902312911238250635,
```

```
375.7328528986296131187574563968929392386,
328.1170929441949744537228395485974717517,
292.9996913833564276785798236519204895424,
358.6434156091262299187638354706520377148,
299.8986620506404531410951232936940614328,
360.0617346664840714330036411222538926555,
336.5944103229011061236834482663132987719,
256.1075318617988871985525651676960695703,
324.6552122370225478233684056879388065221,
331.9380679173770903112767767698329173128,
304.7995832546749554147889184378965418048, none,
289.5459577266230072837370892662745097767]

2 --> 0  target = [17.93041369712275246839567979021922726822,
4.686508701942107473769742610810699655237,
353.3054109506364786654886011241653572562]
one interval r = 31.37435487002125773768497721571548204986 ..
34.20127520031637625649905471915922682762
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=6e-38
Equations at solution: [-.4e-37, .6e-37, -.450e-34]Solution in 0.337s

Time Plot 0 s.
Exiting SolveHard() after 0.608r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349377907044683435666577996675986,
441.6429597330788790020383491906970973672,
436.9174816546542547043488312003123561032,
422.9849339745508844058636467431918068460,
361.5258025625553582660194037662445948662,
401.8817390431752040738892133592374017581,
389.5900151606245780894734622277321339060,
328.4693989351008344656668962721775492575,
401.5075715805757487257401263073759279686,
358.9736282407539391281148166269882881283,
398.3314710398755251982380033071324897855,
371.4838739467766467884835369328818144058,
336.6121584124848967651219262593955593893,
361.5088834729184103272517540949088006973,
324.6714499272501354978020051662286954194,
```

```
302.31384314772322658290962950247090459 80,
328.46938513582681463109151539647588188 57,
343.81340625110863231069023129112382506 35,
375.73285289862961311875745639689293923 86,
328.11709294419497445372283954859747175 17,
292.99969138335642767857982365192048954 24,
358.64341560912622991876383547065203771 48,
299.89866205064045314109512329369406143 28,
360.06173466648407143300364112225389265 55,
336.59441032290110612368344826631329877 19,
256.10753186179888719855256516769606957 03,
324.65521223702254782336840568793880652 21,
331.93806791737709031127677676983291731 28,
304.79958325467495541478891843789654180 48,
323.46169176748624663569225516542886263 19,
289.54595772662300728373708926627450977 67]

Cascade time 170.172
counts: 28, 28

 Iteration 93

Start Generation 1
1 --> 0  target = [12.00000000014197097501586951234425534900,
6.2170125029054690811885088666617435874 10,
485.54908089876004035791263133244781590 27]
one interval r = 23.40850301659603820303562695253303835440 ..
27.67578046432010932047971536233447208981
Time Approximations 0.05.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.32e-37
Equations at solution: [-.4e-37, .132e-36, -.19e-37]Solution in 5.817s

Time Plot 0 s.
Exiting SolveHard() after 7r=27.5236 in [25.56992694 .. 27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768 39894,
441.64295973149949458425980874499063336 65, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0  target = [12.00000000014197097501586951234425534900,
6.217012502905469081188508866661743587410,
485.549080898760040357912631332447815902 7]
one interval r = 32.628147792255944939527537278470558589 06 ..
36.102483889441776229272486964408700442 26
Time Approximations 0.022.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., .10620e-34]Solution in 0.567s

Time Plot 0 s.
Exiting SolveHard() after 0.983r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684482748838732658862690275088784,
6.583434721704908270330981044389141898085,
467.787305958601844888113145682367766887 6]
one interval r = 32.419789556748424042771353528125469733 91 ..
35.851524173734252786446747798824777014 17
Time Approximations 0.023.

hint used Hint := [34.945078887977299146375699908044572147 41, 3, 1, 1,
10.936518216516446421969092460790970158 68, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., -.8717e-35]Solution in 0.658s

Time Plot 0 s.
```

```
Exiting SolveHard() after 1.042r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82, none, none,
401.88173904405370496322023971022269916 02, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.523596844827488387326588626902750887 84,
6.583434721704908270330981044389141898085,
467.787305958601844888113145682367766887 6]
two intervals r = 12.923271608509218049601370602829778334 42 ..
475000000002935724066684205330837821 7/250000000000000000000000000000000
000 or r = 18.394248580388104850868161486498719036 51 ..
475000000002935724066684205330837821 7/250000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.6e-38
Equations at solution: [-.4e-37, -.56e-37, -.206e-35]Solution in
43.009s

Time Plot 0 s.
Exiting SolveHard() after 48.543r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89, none,
401.88173904405370496322023971022269916 02, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628333156763044752703480943773 64,
```

```
4.125651797085081065141566446650858547615,
440.671230653636878948145422481103541728 1]
two intervals r = 14.35659705121873152447503346801942546117 ..
475000000002935724066684205330837821 7/250000000000000000000000000000000
000 or r = 17.70352613824647858967143119344337033056 ..
475000000002935724066684205330837821 7/250000000000000000000000000000000
000
Time Approximations 0.046.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., -.1624e-34]Solution in 1.345s

Time Plot 0 s.
Exiting SolveHard() after 6.34r=15.9119 in [14.35659706 .. 18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89, none,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962833315676304475270348094377364,
4.125651797085081065141566446650858547615,
440.671230653636878948145422481103541728 1]
one interval r = 22.39761154377413724823616739760315271046 ..
27.23722351604569584128562175044608401781
Time Approximations 0.037.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
```

```
in partial time = 1.238 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070644008298102136514206714224890 30, rm =
14.378187707960609877650934882295556376 52}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .47841e-34]Solution in 8.944s

Time Plot 0 s.
Exiting SolveHard() after 13.823r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768 39894,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888052908522390781825087618842 00,
4.0048690820575015116907499136262456266 73,
404.86224501489546347301258414705895120 51]
two intervals r = 16.080110077713761025874213504585596817 33 ..
475000000002935724066684205330837821 7/250000000000000000000000000000000
000 or r = 16.415798127051616045003027792204495001 60 ..
475000000002935724066684205330837821 7/250000000000000000000000000000000
000
Time Approximations 0.058.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.69e-37, -.1e-37, .1623e-34]Solution in 5.392s

Time Plot 0 s.
Exiting SolveHard() after 6.533r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05, none, none,
358.97362824465553607817522265496425744 95, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888805290852239078182508761884200,
4.0048690820575015116907499136262456266 73,
404.86224501489546347301258414705895120 51]
one interval r = 21.641943994197178830879402587467509076 89 ..
26.763306600466093476465723784081046541 16
Time Approximations 0.056.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, .63595e-34]Solution in 1.129s

Time Plot 0 s.
Exiting SolveHard() after 2.217r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55, none,
358.97362824465553607817522265496425744 95, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.192589418154143443846698116694475853 61,
5.5896371828891014024153986516849343621 74,
443.83065884357975982899495977921907218 27]
```

```
one interval r = 22.46725374476647056824864110714528030253 ..
27.27388428354561954377458181069859819183
Time Approximations 0.041.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [0., .27e-37, .1630e-35]Solution in 1.015s

Time Plot 0 s.
Exiting SolveHard() after 5.967r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55, none,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941815414344384669811669447585361,
5.589637182889101402415398651684934362174,
443.83065884357975982899495977921907218 27]
one interval r = 32.15575279512188588629859637183664271117 ..
35.50872228737440228859287664656923504176
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
```

```
Equations at solution: [-.3e-37, .2e-37, -.12574e-34]Solution in 0.482s

Time Plot 0 s.
Exiting SolveHard() after 0.868r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136501023459188694861754211676342,
5.18778357854437322619095608221038632569 9,
408.65773863351115864221681533341553429 20]
one interval r = 21.71840114668633582648146413481241125986 ..
26.81849303523546416398614076089145526548
Time Approximations 0.054.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.58e-37
Equations at solution: [-.1e-37, -.158e-36, -.640e-36]Solution in
1.005s

Time Plot 0 s.
Exiting SolveHard() after 6.047r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
```

```
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18, none, none,
361.50888347632809021496159989008198944 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136501023459188694861754211676342,
5.18778357854437322619095608221038632569 9,
408.65773863351115864221681533341553429 20]
one interval r = 31.80828598771676267866345930797004191854 ..
35.00011460059190553489044878369927703649
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.5e-37, -.5e-37, -.17621e-34]Solution in 0.398s

Time Plot 0 s.
Exiting SolveHard() after 0.724r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768 39894,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81, none,
361.50888347632809021496159989008198944 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 1  target = [26.46347110551550071476568287419565059435,
6.19626256546437711543309847802514917058 9,
385.44474379894662730005581819505107535 54]
one interval r = 31.60836097556206736848659178308110818523 ..
34.66372795623305377467664959504443478359
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .2507e-35]Solution in 0.569s

Time Plot 0 s.
Exiting SolveHard() after 0.836r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768 39894,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81, none,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110551550071476568287419565059435,
6.19626256546437711543309847802514917058 9,
385.44474379894662730005581819505107535 54]
two intervals r = 16.87563408750231168961098967182090158062 ..
475000000002935724066684205330837821 7/2500000000000000000000000000000000
000 or r = 15.55640493832917220798074717816230989512 ..
475000000002935724066684205330837821 7/2500000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.179e-37, 0., -.2071e-34]Solution in 1.154s

Time Plot 0 s.
Exiting SolveHard() after 6.882r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874733886530227336217017977396250,
4.8838107798613782234734982739082358054 64,
376.61967856352456257508764486416663215 35]
one interval r = 21.11001304893430925985462549579669294932 ..
26.31784243486336848608586071899762391772
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=5.1e-38
Equations at solution: [.1e-37, .51e-37, -.22786e-34]Solution in 4.82s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.55r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98, none,
328.46938513913222408677338384866763053 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874733886530227336217017977396250,
4.88381077986137822347349827390823580546 4,
376.61967856352456257508764486416663215 35]
one interval r = 31.53899497730213663688414855506192481658 ..
34.53618386106614606616381448962051380430
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=4.41e-36
Equations at solution: [.338e-35, -.441e-35, .9143e-35]Solution in
0.528s

Time Plot 0 s.
Exiting SolveHard() after 0.804r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
```

```
441.642959731499494584259087449906333665,
436.917481655770203461415427400195954578 2,
422.984933973146576027142598655218653908 9,
361.525802566068347735817186316881336459 0,
401.881739044053704963220239710222699160 2,
389.590015164589271845421249555801517260 5,
328.469398938409432633982668901085892205 5,
401.507571581840196522081607910336100425 6,
358.973628244655536078175222654964257449 5,
398.331471037367260795377581040070669911 8,
371.483873953322281822670749409098445528 1,
336.612158416570759122197674384896855301 5,
361.508883476328090214961599890081989440 4,
324.671449933177621975967729074150418929 8, none,
328.469385139132240867733838486676305360,
343.813406257887148902543204629943446044 1, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.872050175512168313220574655502879015 64,
6.025813549414713604897370075628159808604,
351.427029488346697822291623429486908476 3]
one interval r = 31.362302061320071102669453522464566592 79 ..
34.174466406272585284131691953761815770 84
Time Approximations 0.016.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, .2822e-35]Solution in 0.537s

Time Plot 0 s.
Exiting SolveHard() after 0.788r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434937084023799486586767587683989 4,
441.642959731499494584259087449906333665,
436.917481655770203461415427400195954578 2,
422.984933973146576027142598655218653908 9,
361.525802566068347735817186316881336459 0,
401.881739044053704963220239710222699160 2,
389.590015164589271845421249555801517260 5,
328.469398938409432633982668901085892205 5,
401.507571581840196522081607910336100425 6,
358.973628244655536078175222654964257449 5,
```

```
398.331471037367260795377581040070669911 8,
371.483873953322281822670749409098445528 1,
336.612158416570759122197674384896855301 5,
361.508883476328090214961599890081989440 4,
324.671449933177621975967729074150418929 8, none,
328.469385139132224086773383848667630536 0,
343.813406257887148902543204629943446044 1, none, none,
292.999691389143792176032527672942354227 1, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175512168313220574655502879015 64,
6.025813549414713604897370075628159808604,
351.427029488346697822291623429486908476 3]
two intervals r = 17.981355144469570137641158561262911041 08 ..
475000000002935724066684205330837821 7/250000000000000000000000000000000
000 or r = 13.846080154451550708488480306877762237 27 ..
475000000002935724066684205330837821 7/250000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [-.71e-37, .2e-37, .1155e-34]Solution in 5.095s

Time Plot 0 s.
Exiting SolveHard() after 6.178r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434937084023799486586767587683989 4,
441.642959731499494584259087449906333665,
436.917481655770203461415427400195954578 2,
422.984933973146576027142598655218653908 9,
361.525802566068347735817186316881336459 0,
401.881739044053704963220239710222699160 2,
389.590015164589271845421249555801517260 5,
328.469398938409432633982668901085892205 5,
401.507571581840196522081607910336100425 6,
358.973628244655536078175222654964257449 5,
398.331471037367260795377581040070669911 8,
371.483873953322281822670749409098445528 1,
336.612158416570759122197674384896855301 5,
361.508883476328090214961599890081989440 4,
324.671449933177621975967729074150418929 8,
302.313843151568189651783093358977447838 9,
```

```
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41, none, none,
292.99969138914379217603252767294235422 71, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941870887810186009082180138659129,
6.37794387396385009171018303677943452474 4,
423.28832783621758318013704089248606671 67]
one interval r = 31.94661817607275038466527209130398246894 ..
35.21212308650014964951659963170662105076
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.7670e-35]Solution in 0.624s

Time Plot 0 s.
Exiting SolveHard() after 0.989r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768 39894,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98,
302.31384315156818965178309335897744783 89,
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41, none, none,
292.99969138914379217603252767294235422 71, none, none,
360.06173466655426293702947420444343314 62, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941870887810186009082180138659129,
```

```
6.37794387396385009171018303677943452474,
423.28832783621758318013704089248606671 67]
two intervals r = 15.22886702472903139359289898705844548494 ..
475000000002935724066684205330837821 7/2500000000000000000000000000000000
000 or r = 17.12965777066567092522733972238912819856 ..
475000000002935724066684205330837821 7/2500000000000000000000000000000000
000
Time Approximations 0.083.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [0., 0., .311e-35]Solution in 1.164s

Time Plot 0 s.
Exiting SolveHard() after 6.445r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768 39894,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98,
302.31384315156818965178309335897744783 89,
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41,
375.73285289627661735742968980811630157 08, none,
292.99969138914379217603252767294235422 71, none, none,
360.06173466655426293702947420444343314 62, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234349559893210496706625858331231,
4.00355981575841439810733964251199848120 1,
404.47973594220309070041032290343682421 97]
two intervals r = 16.09683966381172189708044896733555585603 ..
```

```
475000000002935724066684205330837821 7/250000000000000000000000000000000
000 or r = 16.39988649126110969663190170116093517464 ..
475000000002935724066684205330837821 7/250000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [-.18e-37, 0., -.1046e-34]Solution in 5.533s

Time Plot 0 s.
Exiting SolveHard() after 6.617r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768 39894,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98,
302.31384315156818965178309335897744783 89,
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41,
375.73285289627661735742968980811630157 08, none,
292.99969138914379217603252767294235422 71,
358.64341561336950577514109117860815514 59, none,
360.06173466655426293702947420444343314 62, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234349559893210496706625858331231,
4.0035598157584143981073396425119984812 01,
404.47973594220309070041032290343682421 97]
one interval r = 21.63429629997489597498545285947227507781 ..
26.75768169896794117356790993312625910732
Time Approximations 0.052.
```

```
hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.2469e-35]Solution in 1.113s

Time Plot 0 s.
Exiting SolveHard() after 2.157r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768 39894,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98,
302.31384315156818965178309335897744783 89,
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41,
375.73285289627661735742968980811630157 08,
328.11709294786715302832794429723775505 75,
292.99969138914379217603252767294235422 71,
358.64341561336950577514109117860815514 59, none,
360.06173466655426293702947420444343314 62, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954469545150799792064438331728326,
6.19617723031729639356830602899477531417 3,
385.42734026257513403410158860246064052 87]
one interval r = 31.608220491114724396805788124342920119 54 ..
34.663476150607896096087042698677724871 56
Time Approximations 0.018.

hint used Hint := [33.813392675100157147469018161182119369 13, 3, 1, 1,
11.783209121413521870719438174471899656 27, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=0
Equations at solution: [0., 0., -.4362e-35]Solution in 4.961s

Time Plot 0 s.
Exiting SolveHard() after 5.248r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98,
302.31384315156818965178309335897744783 89,
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41,
375.73285289627661735742968980811630157 08,
328.11709294786715302832794429723775505 75,
292.99969138914379217603252767294235422 71,
358.64341561336950577514109117860815514 59, none,
360.06173466655426293702947420444343314 62, none, none,
324.65521224285091248506511883390577809 10, none, none, none, none]

0 --> 1  target = [26.46318954469545150799792064438331728326,
6.19617723031729639356830602899477531417 3,
385.42734026257513403410158860246064052 87]
two intervals r = 16.87629600289338648985084072809772534276 ..
475000000002935724066684205330837821 7/2500000000000000000000000000000000
000 or r = 15.55559000680899699115358075670356882510 ..
475000000002935724066684205330837821 7/2500000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [.1076e-36, -.1e-37, -.1515e-34]Solution in
1.242s

Time Plot 0 s.
Exiting SolveHard() after 2.463r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98,
302.31384315156818965178309335897744783 89,
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41,
375.73285289627661735742968980811630157 08,
328.11709294786715302832794429723775505 75,
292.99969138914379217603252767294235422 71,
358.64341561336950577514109117860815514 59, none,
360.06173466655426293702947420444343314 62,
336.59441032687857917965340103132016685 27, none,
324.65521224285091248506511883390577809 10, none, none, none, none]

0 --> 2  target = [34.49522661184936554914882624262048030167,
3.8971313162044212312566839947007916703 66,
373.78081885437964752069086312500755603 73]
two intervals r = 17.297690862080707339269714381626693727 25 ..
475000000002935724066684205330837821 7/2500000000000000000000000000000000
000 or r = 14.994364074832346304924812060171102041 34 ..
475000000002935724066684205330837821 7/2500000000000000000000000000000000
000
Time Approximations 0.084.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [.72e-37, -.1e-37, .2984e-34]Solution in 1.255s

Time Plot 0 s.
Exiting SolveHard() after 6.932r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98,
302.31384315156818965178309335897744783 89,
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41,
375.73285289627661735742968980811630157 08,
328.11709294786715302832794429723775505 75,
292.99969138914379217603252767294235422 71,
358.64341561336950577514109117860815514 59, none,
360.06173466655426293702947420444343314 62,
336.59441032687857917965340103132016685 27, none,
324.65521224285091248506511883390577809 10,
331.93806792640680602937684238929605690 53, none, none, none]

1 --> 2  target = [34.49522661184936554914882624262048030167,
3.897131316204421231256683994700791670366,
373.78081885437964752069086312500755603 73]
one interval r = 21.06068473226617312239163528825136621546 ..
26.26979834302127876989763877592018689570
Time Approximations 0.038.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [-.2e-37, -.5e-37, .52164e-34]Solution in 4.909s

Time Plot 0 s.
Exiting SolveHard() after 5.682r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98,
302.31384315156818965178309335897744783 89,
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41,
375.73285289627661735742968980811630157 08,
328.11709294786715302832794429723775505 75,
292.99969138914379217603252767294235422 71,
358.64341561336950577514109117860815514 59,
299.89866205928017747903593084383448010 81,
360.06173466655426293702947420444343314 62,
336.59441032687857917965340103132016685 27, none,
324.65521224285091248506511883390577809 10,
331.93806792640680602937684238929605690 53, none, none, none]

0 --> 2  target = [33.81362495427213243221963261604554024803,
3.72564899383026578844778704561171266260 3,
325.89209973581114273642521511636447085 37]
two intervals r = 18.552270490060846623522890962779189414 51 ..
475000000002935724066684205330837821 7/2500000000000000000000000000000000
000 or r = 12.491969358287186980774301617042276006 40 ..
475000000002935724066684205330837821 7/2500000000000000000000000000000000
000
Time Approximations 0.042.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
```

```
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1e-38
Equations at solution: [-.17e-37, .1e-37, -.2801e-34]Solution in 5.348s

Time Plot 0 s.
Exiting SolveHard() after 6.813r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768 39894,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98,
302.31384315156818965178309335897744783 89,
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41,
375.73285289627661735742968980811630157 08,
328.11709294786715302832794429723775505 75,
292.99969138914379217603252767294235422 71,
358.64341561336950577514109117860815514 59,
299.89866205928017747903593084383448010 81,
360.06173466655426293702947420444343314 62,
336.59441032687857917965340103132016685 27, none,
324.65521224285091248506511883390577809 10,
331.93806792640680602937684238929605690 53, none, none,
289.54595773511763713614484243495585947 22]

1 --> 2  target = [33.81362495427213243221963261604554024803,
3.72564899383026578844778704561171266260 3,
325.89209973581114273642521511636447085 37]
one interval r = 20.37468935122649467707074480100597118248 ..
25.37892165316821299560243703268912325388
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
```

```
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, -.44384e-34]Solution in
0.585s

Time Plot 0 s.
Exiting SolveHard() after 1.112r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768398 94,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98,
302.31384315156818965178309335897744783 89,
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41,
375.73285289627661735742968980811630157 08,
328.11709294786715302832794429723775505 75,
292.99969138914379217603252767294235422 71,
358.64341561336950577514109117860815514 59,
299.89866205928017747903593084383448010 81,
360.06173466655426293702947420444343314 62,
336.59441032687857917965340103132016685 27,
256.10753186985076465263878308819096075 85,
324.65521224285091248506511883390577809 10,
331.93806792640680602937684238929605690 53, none, none,
289.54595773511763713614484243495585947 22]

1 --> 0  target = [17.93041369712947481170602360396606573850,
4.68650870199140166161825862342352878969 4,
353.30541095459958744315175877775972686 08]
one interval r = 20.73150479104454133003685755097842370684 ..
25.90675353532252615811965221775284995197
```

```
Time Approximations 0.034.

hint used Hint := [25.402071537818803971053617343970473827 54, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., .13592e-34]Solution in 0.675s

Time Plot 0 s.
Exiting SolveHard() after 1.376r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768 39894,
441.64295973149949458425980874499063336 65,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98,
302.31384315156818965178309335897744783 89,
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41,
375.73285289627661735742968980811630157 08,
328.11709294786715302832794429723775505 75,
292.99969138914379217603252767294235422 71,
358.64341561336950577514109117860815514 59,
299.89866205928017747903593084383448010 81,
360.06173466655426293702947420444343314 62,
336.59441032687857917965340103132016685 27,
256.10753186985076465263878308819096075 85,
324.65521224285091248506511883390577809 10,
331.93806792640680602937684238929605690 53,
304.79958325815575636467612933224261094 15, none,
289.54595773511763713614484243495585947 22]

2 --> 0  target = [17.93041369712947481170602360396606573850,
4.68650870199140166161825862342352878969 4,
```

```
353.30541095459958744315175877775972686 08]
one interval r = 31.37435487011244782892939857036243028495 ..
34.20127520039854266197652268053565275120
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.14757e-34]Solution in 0.362s

Time Plot 0 s.
Exiting SolveHard() after 0.619r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493708402379948658676758768 39894,
441.64295973149949458425980874499063336 5,
436.91748165577020346141542740019595457 82,
422.98493397314657602714259865521865390 89,
361.52580256606834773581718631688133645 90,
401.88173904405370496322023971022269916 02,
389.59001516458927184542124955580151726 05,
328.46939893840943263398266890108589220 55,
401.50757158184019652208160791033610042 56,
358.97362824465553607817522265496425744 95,
398.33147103736726079537758104007066991 18,
371.48387395332228182267074940909844552 81,
336.61215841657075912219767438489685530 15,
361.50888347632809021496159989008198944 04,
324.67144993317762197596772907415041892 98,
302.31384315156818965178309335897744783 89,
328.46938513913222408677338384866763053 60,
343.81340625788714890254320462994344604 41,
375.73285289627661735742968980811630157 08,
328.11709294786715302832794429723775505 75,
292.99969138914379217603252767294235422 71,
358.64341561336950577514109117860815514 59,
299.89866205928017747903593084383448010 81,
360.06173466655426293702947420444343314 62,
336.59441032687857917965340103132016685 27,
256.10753186985076465263878308819096075 85,
324.65521224285091248506511883390577809 10,
331.93806792640680602937684238929605690 53,
304.79958325815575636467612933224261094 15,
323.46169177456907057243772687405815460 23,
```

```
289.54595773511763713614484243495585947 22]

Cascade time 167.643
counts: 28, 28

 Iteration 94

Start Generation 1
1 --> 0  target = [11.999999999843502478843839862088803210 00,
6.217012503185402070363495237721556342199,
485.549080903863974693780396858141738830 1]
one interval r = 23.408503016801509589753642196312486731 54 ..
27.675780464387496883744139569293600376 13
Time Approximations 0.045.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=8.0e-38
Equations at solution: [.3e-37, -.80e-37, -.1e-36]Solution in 1.056s

Time Plot 0 s.
Exiting SolveHard() after 2.187r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.642959739040622526794498202911615274 6, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999843502478843839862088803210 00,
6.217012503185402070363495237721556342199,
485.549080903863974693780396858141738830 1]
one interval r = 32.628147792156878980355585840700713227 54 ..
36.102483889438345237759656281983845290 73
Time Approximations 0.024.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
```

```
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., -.14e-35]Solution in 4.456s

Time Plot 0 s.
Exiting SolveHard() after 4.869r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684492509829442308209171967573574,
6.58343472156411181057505523446737939 3899,
467.78730596630838886065592411550417126 59]
one interval r = 32.41978955666999860375294856626432222586 ..
35.85152417376378753618903444551198061034
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [-.2e-37, .2e-37, .104e-34]Solution in 0.693s

Time Plot 0 s.
Exiting SolveHard() after 1.088r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33, none, none,
401.88173905116727578801953885370984810 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
0 --> 1  target = [27.523596844925098294423082091719675735 74,
6.5834347215641118105750552344673793938 99,
467.78730596630838886065592411550417126 59]
two intervals r = 12.923271608047379506875538750381480434 90 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000 or r = 18.394248580498837787457938929108930583 90 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=2.1e-38
Equations at solution: [.2e-37, .21e-37, -.48e-35]Solution in 47.9s

Time Plot 0 s.
Exiting SolveHard() after 49.358r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80, none,
401.88173905116727578801953885370984810 74, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628250608793761146831914600702 80,
4.1256517968736811967916752107149885846 63,
440.67123065401958170387504563977822619 92]
two intervals r = 14.356597051136249553044663260956827930 55 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000 or r = 17.703526138214502966163050745655580893 41 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., -.113e-34]Solution in 1.382s

Time Plot 0 s.
Exiting SolveHard() after 2.462r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80, none,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962825060879376114683191460070280,
4.12565179687368119679167521071498858466 3,
440.67123065401958170387504563977822619 92]
one interval r = 22.39761154390218161764198719468776580043 ..
27.23722351607394846438819004196752356011
Time Approximations 0.04.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.261 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064394337436452832519508970104741, rm =
14.37818770318094854381111305814301292154}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [-.1e-37, -.26e-37, .194e-34]Solution in 9.107s

Time Plot 0 s.
Exiting SolveHard() after 14.085r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888805575351006799154156255441223,
4.0048690818686135062921935336854238942 10,
404.86224502195503028515866021056788361 41]
two intervals r = 16.08011007733638665274671892177702887140 ..
19000000000058164468683560518939718807/1000000000000000000000000000000000
00000 or r = 16.41579812731046059094488967613307741377 ..
19000000000058164468683560518939718807/1000000000000000000000000000000000
00000
Time Approximations 0.059.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [.17e-37, 0., -.225e-34]Solution in 5.848s

Time Plot 0 s.
Exiting SolveHard() after 7.021r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80, none, none,
358.97362824514417341572993607063154288 36, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888805575351006799154156255441223,
4.0048690818686135062921935336854238942 10,
```

```
404.86224502195503028515866021056788361 41]
one interval r = 21.641943994483674748240657840172960632 21 ..
26.763306600594971755142593667953108075 21
Time Approximations 0.055.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.4e-38
Equations at solution: [.1e-37, .24e-37, .173e-34]Solution in 1.133s

Time Plot 0 s.
Exiting SolveHard() after 2.198r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21, none,
358.97362824514417341572993607063154288 36, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941730193213849014461376741111327,
5.589637183267974940747028278573262227626,
443.83065885874401071272692252349033829 17]
one interval r = 22.467253745218978717968627427406785194 87 ..
27.273884283743356551221101725427890067 97
Time Approximations 0.036.

hint used Hint := [27.020379418720783127960221698280162527 17, 2, -1,
-1, 13.575921446493761927382499512296927627 48, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
```

```
Equations at solution: [0., 0., -.17e-35]Solution in 1.02s

Time Plot 0 s.
Exiting SolveHard() after 6.029r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21, none,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941730193213849014461376741111327,
5.589637183267974940747028278573262227626,
443.83065885874401071272692252349033829 17]
one interval r = 32.155752795107675632911108545927094665 52 ..
35.508722287505525886325188024739033135 25
Time Approximations 0.021.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, .113e-34]Solution in 0.465s

Time Plot 0 s.
Exiting SolveHard() after 0.855r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
```

```
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136517090898185219435276689144653,
5.18778357867349070915648633972371030323 6,
408.65773862841077875888832038203767971 46]
one interval r = 21.71840114672545554236627165649221452196 ..
26.81849303518689684151075660128773935076
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [-.1e-37, -.79e-37, -.152e-34]Solution in 1.029s

Time Plot 0 s.
Exiting SolveHard() after 6.673r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89, none, none,
361.50888347626727374695273779965702888 37, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136517090898185219435276689144653,
5.18778357867349070915648633972371030323 6,
408.65773862841077875888832038203767971 46]
```

```
one interval r = 31.80828598748445448380209184114453986658 ..
35.00011460041968501086600013442498289005
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, -.239e-34]Solution in 0.413s

Time Plot 0 s.
Exiting SolveHard() after 0.742r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13, none,
361.50888347626727374695273779965702888 37, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110554079046072388953077421813978,
6.196262565287354765570933038411427392523,
385.44474379879738411146746558057667215 41]
one interval r = 31.60836097536865123580743923950612168412 ..
34.66372795612481424739437631397829448482
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=6e-38
Equations at solution: [-.3e-37, .6e-37, -.318e-34]Solution in 0.59s

Time Plot 0 s.
Exiting SolveHard() after 0.859r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13, none,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110554079046072388953077421813978,
6.19626256528735476557093303841142739252 3,
385.44474379879738411146746558057667215 41]
two intervals r = 16.875634087439065023273429864723774380 95 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000 or r = 15.556404938293612053990869867119643877 16 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.154e-34]Solution in 1.172s

Time Plot 0.001 s.
Exiting SolveHard() after 6.358r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874724921038684139792013955658634,
4.8838107800455405386943943891792153999 19,
376.61967856411466963443418830763234044 91]
one interval r = 21.11001304910940901842666232646559446228 ..
26.31784243490256802477063923831524223162
Time Approximations 0.038.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.6e-38
Equations at solution: [0., .26e-37, -.130e-34]Solution in 5.433s

Time Plot 0 s.
Exiting SolveHard() after 6.182r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
```

```
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61, none,
328.46938514517308344087737148225167705 04, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.198988747249210386841397920139556586 34,
4.8838107800455405386943943891792153999 19,
376.61967856411466963443418830763234044 91]
one interval r = 31.538994977112142942639128156770557695 17 ..
34.536183860965534127926448986022837702 92
Time Approximations 0.016.

hint used Hint := [34.089767995907079678444172504001644261 52, 3, 1, 1,
17.198984348938819986702586587288643071 01, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=5.78e-36
Equations at solution: [-.445e-35, .578e-35, -.120e-34]Solution in
0.532s

Time Plot 0 s.
Exiting SolveHard() after 0.829r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61, none,
328.46938514517308344087737148225167705 04,
```

```
343.81340625154945154723895011159718619 60, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017566184460301869713305256211886,
6.02581354926994596072640874455821536274 6,
351.42702949452285390299024723255833091 68]
one interval r = 31.36230206115896000326832900685403411869 ..
34.17446640624264734751955074462118957090
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .54e-35]Solution in 0.53s

Time Plot 0 s.
Exiting SolveHard() after 0.781r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61, none,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60, none, none,
292.99969139470628644867069824632856714 93, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017566184460301869713305256211886,
6.02581354926994596072640874455821536274 6,
351.42702949452285390299024723255833091 68]
two intervals r = 17.98135514423630562969374663790904959006 ..
19000000000058164468683560518939718807/1000000000000000000000000000000000
```

```
00000 or r = 13.846080154761516466809336077524119092 55 ..
190000000000581644686835605189397188 07/100000000000000000000000000000000
00000
Time Approximations 0.049.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [-.54e-37, .2e-37, .98e-35]Solution in 5.308s

Time Plot 0 s.
Exiting SolveHard() after 6.383r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60, none, none,
292.99969139470628644867069824632856714 93, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379419010849641622795607368841277 34,
6.3779438738869248806439312956708801337 72,
423.28832785727943640356835747095786700 21]
one interval r = 31.946618176097156454435144558224660865 69 ..
35.212123086711535330601799965565623150 23
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., .16e-35]Solution in 0.628s

Time Plot 0 s.
Exiting SolveHard() after 0.997r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60, none, none,
292.99969139470628644867069824632856714 93, none, none,
360.06173468614416177075435907073268085 08, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379419010849641622795607368841277 34,
6.3779438738869248806439312956708801337 72,
423.28832785727943640356835747095786700 21]
two intervals r = 15.22886702364252671849816231719404366449 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000 or r = 17.12965777138155550142007396238193438207 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000
Time Approximations 0.066.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
```

```
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.15e-37, .1e-37, -.117e-34]Solution in 5.217s

Time Plot 0 s.
Exiting SolveHard() after 6.589r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60,
375.73285292490578588573951358924324103 11, none,
292.99969139470628644867069824632856714 93, none, none,
360.06173468614416177075435907073268085 08, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343505185232111131039380693588 01,
4.0035598155711510177500404896183709867 60,
404.47973594973634865090330179671299254 92]
two intervals r = 16.096839663414379033323481300558817483 87 ..
190000000000581644686835605189397188 07/100000000000000000000000000000000
00000 or r = 16.399886491540585601180065180192407205 38 ..
190000000000581644686835605189397188 07/100000000000000000000000000000000
00000
Time Approximations 0.056.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [.17e-37, 0., .184e-34]Solution in 1.574s

Time Plot 0 s.
Exiting SolveHard() after 2.706r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60,
375.73285292490578588573951358924324103 11, none,
292.99969139470628644867069824632856714 93,
358.64341561426568233088853455156551500 65, none,
360.06173468614416177075435907073268085 08, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234350518523211113103938069358801,
4.0035598155711510177500404896183709867 60,
404.47973594973634865090330179671299254 92]
one interval r = 21.634296300270929350427823067250625886 33 ..
26.757681699104042989623701840615297155 86
Time Approximations 0.065.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=4.9e-38
Equations at solution: [-.2e-37, -.49e-37, -.16e-35]Solution in 1.067s

Time Plot 0 s.
```

```
Exiting SolveHard() after 6.324r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60,
375.73285292490578588573951358924324103 11,
328.11709295434148257091112609536978539 43,
292.99969139470628644867069824632856714 93,
358.64341561426568233088853455156551500 65, none,
360.06173468614416177075435907073268085 08, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954472149163815398410782346535792,
6.19617723014050055618722733232017586743 1,
385.42734026247210760799032065034147389 78]
one interval r = 31.608220490921676811094256069140034058 62 ..
34.663476150500319296161035225009766634 51
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .141e-34]Solution in 0.58s

Time Plot 0 s.
Exiting SolveHard() after 0.892r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
```

```
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60,
375.73285292490578585739513589243241031 1,
328.11709295434148257091112609536978539 43,
292.99969139470628644867069824632856714 93,
358.64341561426568233088853455156551500 65, none,
360.06173468614416177075435907073268085 08, none, none,
324.65521224259399150828381834438238298 54, none, none, none, none]

0 --> 1  target = [26.46318954472149163815398410782346535792,
6.19617723014050055618722733232017586743 1,
385.42734026247210760799032065034147389 78]
two intervals r = 16.87629600282838126850277202099790839943 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000 or r = 15.55559000677560601867188741340573475103 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.359e-37, 0., .150e-34]Solution in 1.156s

Time Plot 0 s.
Exiting SolveHard() after 6.544r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60,
375.73285292490578588573951358924324103 11,
328.11709295434148257091112609536978539 43,
292.99969139470628644867069824632856714 93,
358.64341561426568233088853455156551500 65, none,
360.06173468614416177075435907073268085 08,
336.59441033316747835554398156045595808 16, none,
324.65521224259399150828381834438238298 54, none, none, none, none]

0 --> 2  target = [34.49522661157930126538548595377862642498,
3.8971313159520221704216917032567457300 59,
373.78081884328799404607407440884064676 93]
two intervals r = 17.297690862392413978342496891877554773 67 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000 or r = 14.994364074258988991715722273810267585 91 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000
Time Approximations 0.082.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [0., 0., -.37e-35]Solution in 1.222s

Time Plot 0 s.
Exiting SolveHard() after 7.031r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60,
375.73285292490578588573951358924324103 11,
328.11709295434148257091112609536978539 43,
292.99969139470628644867069824632856714 93,
358.64341561426568233088853455156551500 65, none,
360.06173468614416177075435907073268085 08,
336.59441033316747835554398156045595808 16, none,
324.65521224259399150828381834438238298 54,
331.93806791093997634091193751355566553 55, none, none, none]

1 --> 2  target = [34.49522661157930126538548595377862642498,
3.8971313159520221704216917032567457300 59,
373.78081884328799404607407440884064676 93]
one interval r = 21.06068473224171860109419615586633554868 ..
26.26979834286227751840145345209800898024
Time Approximations 0.037.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [-.3e-37, -.7e-37, -.280e-34]Solution in 0.779s

Time Plot 0 s.
Exiting SolveHard() after 1.532r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60,
375.73285292490578588573951358924324103 11,
328.11709295434148257091112609536978539 43,
292.99969139470628644867069824632856714 93,
358.64341561426568233088853455156551500 65,
299.89866204863352978790202893262196576 87,
360.06173468614416177075435907073268085 08,
336.59441033316747835554398156045595808 16, none,
324.65521224259399150828381834438238298 54,
331.93806791093997634091193751355566553 55, none, none, none]

0 --> 2  target = [33.81362495413749203216078092918019033868,
3.72564899361408049363407457909013723883 6,
325.89209973528563767507338463079892564 39]
two intervals r = 18.552270490004661639173123467609564996 03 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000 or r = 12.491969358254598481354685147744482893 71 ..
19000000000058164468683560518939718807/10000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [.86e-37, -.3e-37, .64e-35]Solution in 1.237s

Time Plot 0 s.
Exiting SolveHard() after 7.216r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
```

```
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60,
375.73285292490578585739513589243241031 1,
328.11709295434148257091112609536978539 43,
292.99969139470628644867069824632856714 93,
358.64341561426568233088853455156551500 65,
299.89866204863352978790202893262196576 87,
360.06173468614416177075435907073268085 08,
336.59441033316747835554398156045595808 16, none,
324.65521224259399150828381834438238298 54,
331.93806791093997634091193751355566553 55, none, none,
289.54595772899506303911539780217820605 24]

1 --> 2  target = [33.81362495413749203216078092918019033868,
3.7256489936140804936340745790901372388 36,
325.89209973528563767507338463079892564 39]
one interval r = 20.37468935141430779073507155048022035411 ..
25.37892165320107922104824270535204380579
Time Approximations 0.029.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, -.202e-34]Solution in 4.538s

Time Plot 0 s.
Exiting SolveHard() after 5.113r=24.3395 in [22.07732228 ..
25.37892164]
```

```
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60,
375.73285292490578588573951358924324103 11,
328.11709295434148257091112609536978539 43,
292.99969139470628644867069824632856714 93,
358.64341561426568233088853455156551500 65,
299.89866204863352978790202893262196576 87,
360.06173468614416177075435907073268085 08,
336.59441033316747835554398156045595808 16,
256.10753186887787742743274908929331431 62,
324.65521224259399150828381834438238298 54,
331.93806791093997634091193751355565653 55, none, none,
289.54595772899506303911539780217820605 24]

1 --> 0  target = [17.93041369688116400848495433166740614552,
4.6865087022223566875522971939873602114 99,
353.30541096117448900023252888347852718 46]
one interval r = 20.73150479132158516985631690136634971862 ..
25.90675353547828375939082468965945021595
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=4.6e-38
Equations at solution: [.2e-37, .46e-37, -.43e-35]Solution in 0.696s

Time Plot 0 s.
```

```
Exiting SolveHard() after 1.446r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60,
375.73285292490578588573951358924324103 11,
328.11709295434148257091112609536978539 43,
292.99969139470628644867069824632856714 93,
358.64341561426568233088853455156551500 65,
299.89866204863352978790202893262196576 87,
360.06173468614416177075435907073268085 08,
336.59441033316747835554398156045595808 16,
256.10753186887787742743274908929331431 62,
324.65521224259399150828381834438238298 54,
331.93806791093997634091193751355566553 55,
304.79958327024831270727765145996769465 71, none,
289.54595772899506303911539780217820605 24]

2 --> 0  target = [17.93041369688116400848495433166740614552,
4.68650870222235668755229719398736021149 9,
353.30541096117448900023252888347852718 46]
one interval r = 31.37435486995493019997854196899927375535 ..
34.20127520037507338591815796635243277228
Time Approximations 0.016.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
```

```
Equations at solution: [-.2e-37, .3e-37, -.94e-35]Solution in 0.347s

Time Plot 0 s.
Exiting SolveHard() after 0.607r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494173598647688419922529462416 20,
441.64295973904062252679449820291161527 46,
436.91748165635087226088770361969693442 33,
422.98493398738809673826770855521743442 80,
361.52580256596260071740203531390037143 99,
401.88173905116727578801953885370984810 74,
389.59001515964409272017225879658899495 80,
328.46939894444786936675407287796037503 21,
401.50757158941711643862624593839574518 89,
358.97362824514417341572993607063154288 36,
398.33147105789809616719877971309659641 89,
371.48387394268650170706626097616147612 13,
336.61215842281264672371826530311384093 45,
361.50888347626727374695273779965702888 37,
324.67144993287758576309178672441377704 61,
302.31384316384570147427224570419186685 34,
328.46938514517308344087737148225167705 04,
343.81340625154945154723895011159718619 60,
375.73285292490578588573951358924324103 11,
328.11709295434148257091112609536978539 43,
292.99969139470628644867069824632856714 93,
358.64341561426568233088853455156551500 65,
299.89866204863352978790202893262196576 87,
360.06173468614416177075435907073268085 08,
336.59441033316747835554398156045595808 16,
256.10753186887787742743274908929331431 62,
324.65521224259399150828381834438238298 54,
331.93806791093997634091193751355566553 55,
304.79958327024831270727765145996769465 71,
323.46169177320829502865222420687469347 22,
289.54595772899506303911539780217820605 24]

Cascade time 166.015
counts: 28, 28

 Iteration 95

Start Generation 1
1 --> 0  target = [11.99999999989385729552564359659043549600,
6.21701250302037956989170447873446785 1848,
485.54908089274929221260062534402242406 29]
one interval r = 23.40850301637509764517975718331724357811 ..
27.67578046425968387157457702948019266414
Time Approximations 0.04.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
```

```
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=8.0e-38
Equations at solution: [-.2e-37, .80e-37, -.967e-36]Solution in 1.029s

Time Plot 0 s.
Exiting SolveHard() after 2.202r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999893857295525643596590435496 00,
6.217012503020379569891704478734467851848,
485.549080892749292212600625344022424062 9]
one interval r = 32.628147792098620632016299386873869335 82 ..
36.102483889329355685808055646974741136 25
Time Approximations 3.892.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, -.16e-35]Solution in 0.615s

Time Plot 0 s.
Exiting SolveHard() after 4.888r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
```

```
441.642959727226402239572463734699358119 7,
436.91748164427541584788943617592482843 59, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684477467526031805707006710173839,
6.58343472157182022697069254609501237617 3,
467.78730595417095423633970900558444227 68]
one interval r = 32.41978955660857064411703802331535029600 ..
35.85152417363973098289968534561251558050
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., -.38e-35]Solution in 0.635s

Time Plot 0 s.
Exiting SolveHard() after 1.041r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.642959727226402239572463734699358119 7,
436.91748164427541584788943617592482843 59, none, none,
401.88173903610291922429861265423165936 01, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684477467526031805707006710173839,
6.58343472157182022697069254609501237617 3,
467.78730595417095423633970900558444227 68]
two intervals r = 12.92327160815474788782594549974249268295 ..
18999999999795734908093059834757192803/10000000000000000000000000000000
00000 or r = 18.39424858006590834683063976168557408800 ..
18999999999795734908093059834757192803/10000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
```

```
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=4.2e-38
Equations at solution: [-.4e-37, -.42e-37, .987e-35]Solution in 48.691s

Time Plot 0 s.
Exiting SolveHard() after 50.124r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74, none,
401.88173903610291922429861265423165936 01, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962812731528812237563819389297317,
4.12565179660363038376626742933641836844 5,
440.67123064192707844258758905872453645 91]
two intervals r = 14.35659705122531421004083389173608087211 ..
18999999999795734908093059834757192803/10000000000000000000000000000000
00000 or r = 17.70352613770047096595035500822596716976 ..
18999999999795734908093059834757192803/10000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=2e-38
Equations at solution: [-.27e-37, -.2e-37, .3607e-34]Solution in 1.384s

Time Plot 0 s.
Exiting SolveHard() after 2.479r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74, none,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962812731528812237563819389297317,
4.12565179660363038376626742933641836844 5,
440.67123064192707844258758905872453645 91]
one interval r = 22.39761154339808063606534935315657896207 ..
27.23722351589857088935144683196163257185
Time Approximations 0.038.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.268 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064363865824330119117982349279269, rm =
14.37818769982637750399185971046140629011}});
Accepted {r=26.4635, rm=16.5329} with Delta=0
Equations at solution: [0., 0., .30829e-34]Solution in 9.003s

Time Plot 0 s.
Exiting SolveHard() after 14.026r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.94507888789058432630931551201207417696,
4.00486908158713061990071204100933642674 4,
```

```
404.86224500680282837394623966736764409 16]
two intervals r = 16.08011007750936335508038029962993133126 ..
189999999997957349080930598347571928 03/1000000000000000000000000000000000
00000 or r = 16.41579812656530096180202148522397694972 ..
189999999997957349080930598347571928 03/1000000000000000000000000000000000
00000
Time Approximations 0.053.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.34e-37, -.1e-37, .781e-35]Solution in 1.708s

Time Plot 0 s.
Exiting SolveHard() after 7.177r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24, none, none,
358.97362822932133178805123456258607972 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888789058432630931551201207417696,
4.0048690815871306199007120410093364267 44,
404.86224500680282837394623966736764409 16]
one interval r = 21.64194399388727534325959150568536320052 ..
26.76330660033573826374859183794511969030
Time Approximations 0.055.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
```

```
Accepted {r=25.8721, rm=16.7767} with Delta=0
Equations at solution: [0., 0., -.14994e-34]Solution in 5.364s

Time Plot 0 s.
Exiting SolveHard() after 6.455r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86, none,
358.97362822932133178805123456258607972 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941755452449901118744398324341089,
5.58963718305817827542879735381733494180 4,
443.83065884357255830005790113364190175 76]
one interval r = 22.46725374465024499350453702419541362097 ..
27.27388428353581368335993978622770371576
Time Approximations 0.039.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=0
Equations at solution: [0., 0., .7548e-35]Solution in 1.023s

Time Plot 0 s.
Exiting SolveHard() after 2.045r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
```

```
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86, none,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589417554524499011187443983243410 89,
5.589637183058178275428797353817334941804,
443.83065884357255830005790113364190175 76]
one interval r = 32.155752795027462640091413139149939116 96 ..
35.508722287337834419643158711820329038 26
Time Approximations 0.021.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., -.209e-34]Solution in 0.462s

Time Plot 0 s.
Exiting SolveHard() after 0.825r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931365277511362776772028310395899 89,
5.187783578494362469689825382451411606168,
408.65773861495313159196873799561137836 63]
one interval r = 21.718401146165133043993237405810978372 45 ..
26.818493034957366640314253789848065125 21
```

```
Time Approximations 0.062.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.11e-37
Equations at solution: [-.1e-37, -.211e-36, -.39860e-34]Solution in
1.038s

Time Plot 0 s.
Exiting SolveHard() after 6.559r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42, none, none,
361.50888346077953899769043993081787353 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136527751136277677202831039589989,
5.18778357849436246968982538245141160 6168,
408.65773861495313159196873799561137836 63]
one interval r = 31.80828598744582082949015654856090118524 ..
35.00011460027869501907484052243083716860
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
```

```
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, -.97e-35]Solution in 0.434s

Time Plot 0 s.
Exiting SolveHard() after 0.771r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01, none,
361.50888346077953899769043993081787353 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110523831875213760808792445049593,
6.19626256527271008886558805623225800176 7,
385.44474378270371783968592026781741577 73]
one interval r = 31.60836097532547971902040293042145933761 ..
34.66372795594862724001272142323365391488
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .152e-34]Solution in 5.38s

Time Plot 0 s.
Exiting SolveHard() after 5.656r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01, none,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110523831875213760808792445049593,
6.1962625652727100888655880562322580017 67,
385.44474378270371783968592026781741577 73]
two intervals r = 16.875634087594335592119155412232633770 14 ..
189999999997957349080930598347571928 03/100000000000000000000000000000000
00000 or r = 15.556404937439222703038792836046943139 05 ..
189999999997957349080930598347571928 03/100000000000000000000000000000000
00000
Time Approximations 0.06.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 8.437 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175112996195918028891807727763058, rm
= 2.33653277412281366169723606633387432392}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.180e-37, 0., .524e-35]Solution in 32.24s

Time Plot 0 s.
Exiting SolveHard() after 33.42r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
```

```
401.88173903610291922429861265423165936 01,
389.5900151466178181542187057269941213724,
328.46939892600281738368139245881242335 86,
401.5075715733903336023142379289147557875,
358.9736282293213317880512345625860797233,
398.3314710408249976369272219545597172442,
371.4838739277702480616505007043588966501,
336.6121584041853180840074043163154261623,
361.5088834607795389976904399308178735351,
324.6714499139492278363711620588830833127, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874738490972710207728940821702608,
4.883810779844540019605092672055290140853,
376.6196785477523613862317354567282819063]
one interval r = 21.11001304848735318687452775469148876774 ..
26.31784243458183958608473714681122188023
Time Approximations 0.032.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.4e-38
Equations at solution: [-.1e-37, -.24e-37, .6184e-35]Solution in 0.828s

Time Plot 0 s.
Exiting SolveHard() after 5.712r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.1634349309726956971256048638574717220,
441.6429597272264022395724637346993581197,
436.9174816442754158478894361759248284359,
422.9849339727698746531595655579984195874,
361.5258025503153327782000387604502721357,
401.88173903610291922429861265423165936 01,
389.5900151466178181542187057269941213724,
328.46939892600281738368139245881242335 86,
401.5075715733903336023142379289147557875,
358.9736282293213317880512345625860797233,
398.3314710408249976369272219545597172442,
371.4838739277702480616505007043588966501,
336.6121584041853180840074043163154261623,
361.5088834607795389976904399308178735351,
324.6714499139492278363711620588830833127, none,
328.4693851267323290289487795857743366329, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874738490972710207728940821702608,
4.883810779844540019605092672055290140853,
376.61967854775236138623173545672828190 63]
one interval r = 31.53899497707463790715688199635941148835 ..
34.53618386078702314247842628201796144076
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=7.54e-36
Equations at solution: [.579e-35, -.754e-35, .203e-34]Solution in
0.479s

Time Plot 0 s.
Exiting SolveHard() after 0.774r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27, none,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017525175864928837185519397940831,
6.025813549237345219487826843789672083268,
351.42702947553544743402223178576110736 90]
one interval r = 31.36230206112787435194897719707697970238 ..
34.17446640603242729219045291514192300421
Time Approximations 0.017.
```

```
hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=0
Equations at solution: [0., 0., .100e-34]Solution in 0.542s

Time Plot 0 s.
Exiting SolveHard() after 0.774r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27, none,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73, none, none,
292.99969137304425504412843857958755870 72, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017525175864928837185519397940831,
6.02581354923734521948782684378967208326 8,
351.42702947553544743402223178576110736 90]
two intervals r = 17.98135514435138258283407231608291994284 ..
189999999997957349080930598347571928 03/10000000000000000000000000000000
00000 or r = 13.84608015367719891313166262227810127332 ..
189999999997957349080930598347571928 03/10000000000000000000000000000000
00000
Time Approximations 0.042.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
```

```
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 8.014 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713519160046035669116687215054 40, rm
= 2.7345009932087780937691100532501922028 18}});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [0., 0., .1331e-34]Solution in 18.414s

Time Plot 0 s.
Exiting SolveHard() after 23.867r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73, none, none,
292.99969137304425504412843857958755870 72, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941874529763772556399515902911196,
6.37794387386793304639158240433151752566 0,
423.28832783973118357333423820681349634 21]
one interval r = 31.946618176008524504710742854212166668 89 ..
35.212123086510132604809178592916889033 01
Time Approximations 0.019.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=0
Equations at solution: [0., 0., .53e-35]Solution in 0.616s

Time Plot 0 s.
Exiting SolveHard() after 5.241r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73, none, none,
292.99969137304425504412843857958755870 72, none, none,
360.06173466590926998405650309217506264 66, none, none, none, none,
none, none, none]

0 --> 1  target = [27.02037941874529763772556399515902911196,
6.37794387386793304639158240433151752566 0,
423.28832783973118357333423820681349634 21]
two intervals r = 15.22886702397472746015562300108365255159 ..
189999999997957349080930598347571928 03/10000000000000000000000000000000
00000 or r = 17.12965777062012284157784801984279781308 ..
189999999997957349080930598347571928 03/10000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
```

```
in partial time = 8.211 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370540953468007170995285276756936 11, rm
= 2.0640682986766549931174704734421283215 62}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.32e-37, .1e-37, .1938e-34]Solution in 35.786s

Time Plot 0 s.
Exiting SolveHard() after 37.124r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73,
375.73285290445947525124172405899102572 9, none,
292.99969137304425504412843857958755870 72, none, none,
360.06173466590926998405650309217506264 66, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343325790031038770865945537606 45,
4.0035598152862860779457409632933779214 37,
404.47973593360020306437901419232030161 39]
two intervals r = 16.096839663629354042187694536322258454 69 ..
189999999997957349080930598347571928 03/10000000000000000000000000000000
00000 or r = 16.399886490753059110811303241513278871 02 ..
189999999997957349080930598347571928 03/10000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [0., 0., .2849e-34]Solution in 6.059s

Time Plot 0 s.
Exiting SolveHard() after 7.144r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73,
375.73285290445947525124127240589910257 29, none,
292.99969137304425504412843857958755870 72,
358.64341559759152392590825583680669078 82, none,
360.06173466590926998405650309217506264 66, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234332579003103877086594553760645,
4.0035598152862860779457409632933779214 37,
404.47973593360020306437901419232030161 39]
one interval r = 21.63429629965471225556957277384023100578 ..
26.75768169882978643237396559846359173329
Time Approximations 0.052.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.0e-38
Equations at solution: [-.2e-37, -.50e-37, .64622e-34]Solution in
1.122s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.187r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73,
375.73285290445947525124127240589910257 29,
328.11709293499066026602582692412476866 92,
292.99969137304425504412843857958755870 72,
358.64341559759152392590825583680669078 82, none,
360.06173466590926998405650309217506264 66, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954442164795146126496425633481165,
6.19617723012665912940681060282143580904 5,
385.42734024654253660480362556478725772 88]
one interval r = 31.608220490879844909989038698015769154 31 ..
34.663476150326509363334666543393173967 72
Time Approximations 0.019.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, .193e-34]Solution in 4.862s

Time Plot 0 s.
Exiting SolveHard() after 5.154r=33.8134 in [32.62668594 ..
34.66347615]
```

```
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73,
375.73285290445947525124127240589910257 29,
328.11709293499066026602582692412476866 92,
292.99969137304425504412843857958755870 72,
358.64341559759152392590825583680669078 82, none,
360.06173466590926998405650309217506264 66, none, none,
324.65521222381871172643433108429283017 34, none, none, none, none]

0 --> 1  target = [26.46318954442164795146126496425633481165,
6.19617723012665912940681060282143580904 5,
385.42734024654253660480362556478725772 88]
two intervals r = 16.87629600297735515923469811857488149099 ..
189999999997957349080930598347571928 03/10000000000000000000000000000000
00000 or r = 15.55559000592884399094108908725819410358 ..
189999999997957349080930598347571928 03/10000000000000000000000000000000
00000
Time Approximations 0.061.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 8.878 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852524557912187182368238043134683, rm
= 2.33669042818738590097777476596502657068 5}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
```

```
Equations at solution: [-.179e-37, 0., -.2610e-34]Solution in 32.033s

Time Plot 0 s.
Exiting SolveHard() after 33.203r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73,
375.73285290445947525124127240589910257 29,
328.11709293499066026602582692412476866 92,
292.99969137304425504412843857958755870 72,
358.64341559759152392590825583680669078 82, none,
360.06173466590926998405650309217506264 66,
336.59441031470764815180860887226755525 38, none,
324.65521222381871172643433108429283017 34, none, none, none, none]

0 --> 2  target = [34.49522661142090696656665192159190276665,
3.8971313156699376750758727439034950673 23,
373.78018188282824609037204509058314823646]
two intervals r = 17.29769086247172929239669686313688584028 ..
189999999997957349080930598347571928 03/10000000000000000000000000000000
00000 or r = 14.99436407342350612234725082663509266844 ..
189999999997957349080930598347571928 03/10000000000000000000000000000000
00000
Time Approximations 0.084.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
```

```
Equations at solution: [-.36e-37, 0., .1937e-34]Solution in 1.205s

Time Plot 0 s.
Exiting SolveHard() after 8.113r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73,
375.73285290445947525124127240589910257 29,
328.11709293499066026602582692412476866 92,
292.99969137304425504412843857958755870 72,
358.64341559759152392590825583680669078 82, none,
360.06173466590926998405650309217506264 66,
336.59441031470764815180860887226755525 38, none,
324.65521222381871172643433108429283017 34,
331.93806789512666382392712321808179193 64, none, none, none]

1 --> 2  target = [34.49522661142090696656665192159190276665,
3.8971313156699376750758727439034950673 23,
373.78081882828246090372045090583148236 46]
one interval r = 21.06068473164382096308431143859467875820 ..
26.26979834256013044359881451604369096111
Time Approximations 0.037.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=0
Equations at solution: [0., 0., .8417e-35]Solution in 4.818s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.585r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73,
375.73285290445947525124127240589910257 29,
328.11709293499066026602582692412476866 92,
292.99969137304425504412843857958755870 72,
358.64341559759152392590825583680669078 82,
299.89866203036741176200244925043098571 44,
360.06173466590926998405650309217506264 66,
336.59441031470764815180860887226755525 38, none,
324.65521222381871172643433108429283017 34,
331.93806789512666382392712321808179193 64, none, none, none]

0 --> 2  target = [33.81362495393396819618617231995888125090,
3.72564899331527724625567356690000676328 5,
325.89209971616734120555166661169406012 40]
two intervals r = 18.552270489999591599116278318618751808 15 ..
189999999997957349080930598347571928 03/10000000000000000000000000000000
00000 or r = 12.491969357183092929469936639523100156 43 ..
189999999997957349080930598347571928 03/10000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=18.8546, rm=16.5667} with Delta=7e-38
Equations at solution: [.173e-36, -.7e-37, -.2783e-34]Solution in
1.176s

Time Plot 0 s.
Exiting SolveHard() after 2.615r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73,
375.73285290445947525124127240589910257 29,
328.11709293499066026602582692412476866 92,
292.99969137304425504412843857958755870 72,
358.64341559759152392590825583680669078 82,
299.89866203036741176200244925043098571 44,
360.06173466590926998405650309217506264 66,
336.59441031470764815180860887226755525 38, none,
324.65521222381871172643433108429283017 34,
331.93806789512666382392712321808179193 64, none, none,
289.54595770944467527251642858158311835 21]

1 --> 2  target = [33.81362495393396819618617231995888125090,
3.72564899331527724625567356690000676328 5,
325.89209971616734120555166661169406012 40]
one interval r = 20.37468935080436702343774297938990320087 ..
25.37892165274218594389658865029496858168
Time Approximations 0.026.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
```

```
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=0
Equations at solution: [0., 0., .34859e-34]Solution in 0.565s

Time Plot 0 s.
Exiting SolveHard() after 5.309r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73,
375.73285290445947525124127240589910257 29,
328.11709293499066026602582692412476866 92,
292.99969137304425504412843857958755870 72,
358.64341559759152392590825583680669078 82,
299.89866203036741176200244925043098571 44,
360.06173466590926998405650309217506264 66,
336.59441031470764815180860887226755525 38,
256.10753184695768057359315412318921490 17,
324.65521222381871172643433108429283017 34,
331.93806789512666382392712321808179193 64, none, none,
289.54595770944467527251642858158311835 21]

1 --> 0  target = [17.93041369701275310669247498738104147825,
4.68650870200051726235821638786300112738 9,
353.30541094190361654817002987215594411 68]
one interval r = 20.73150479066822583488125142488554276791 ..
25.90675353506637081892785521210767049331
Time Approximations 0.033.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=6.8e-38
Equations at solution: [-.3e-37, -.68e-37, -.26183e-34]Solution in
0.671s

Time Plot 0 s.
Exiting SolveHard() after 1.415r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73,
375.73285290445947525124127240589910257 29,
328.11709293499066026602582692412476866 92,
292.99969137304425504412843857958755870 72,
358.64341559759152392590825583680669078 82,
299.89866203036741176200244925043098571 44,
360.06173466590926998405650309217506264 66,
336.59441031470764815180860887226755525 38,
256.10753184695768057359315412318921490 17,
324.65521222381871172643433108429283017 34,
331.93806789512666382392712321808179193 64,
304.79958324898296070604544455799291117 58, none,
289.54595770944467527251642858158311835 21]

2 --> 0  target = [17.93041369701275310669247498738104147825,
4.68650870200051726235821638786300112738 9,
353.30541094190361654817002987215594411 68]
one interval r = 31.37435486991999361983382098814260124084 ..
34.20127520016026868361015190277160041449
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
```

```
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .467e-34]Solution in 0.358s

Time Plot 0 s.
Exiting SolveHard() after 0.631r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343493097269569712560486385747172 20,
441.64295972722640223957246373469935811 97,
436.91748164427541584788943617592482843 59,
422.98493397276987465315956555799841958 74,
361.52580255031533277820003876045027213 57,
401.88173903610291922429861265423165936 01,
389.59001514661781815421870572699412137 24,
328.46939892600281738368139245881242335 86,
401.50757157339033360231423792891475578 75,
358.97362822932133178805123456258607972 33,
398.33147104082499763692722195455971724 42,
371.48387392777024806165050070435889665 01,
336.61215840418531808400740431631542616 23,
361.50888346077953899769043993081787353 51,
324.67144991394922783637116205888308331 27,
302.31384314264752071370870459597264887 54,
328.46938512673232902894877958577433663 29,
343.81340623387938662373783325495706787 73,
375.73285290445947525124127240589910257 29,
328.11709293499066026602582692412476866 92,
292.99969137304425504412843857958755870 72,
358.64341559759152392590825583680669078 82,
299.89866203036741176200244925043098571 44,
360.06173466590926998405650309217506264 66,
336.59441031470764815180860887226755525 38,
256.10753184695768057359315412318921490 17,
324.65521222381871172643433108429283017 34,
331.93806789512666382392712321808179193 64,
304.79958324898296070604544455799291117 58,
323.46169175285287122035687859698762590 98,
289.54595770944467527251642858158311835 21]

Cascade time 282.581
counts: 28, 28

 Iteration 96

Start Generation 1
```

```
1 --> 0  target = [11.999999999904984205426029998506778725 00,
6.217012502913368682966756400487179127331,
485.549080907439650616725773593845633120 3]
one interval r = 23.408503016731081119311522627349912796 59 ..
27.675780464285915407070194610927610830 16
Time Approximations 0.045.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.09e-37
Equations at solution: [.6e-37, -.209e-36, -.21e-35]Solution in 1.039s

Time Plot 0 s.
Exiting SolveHard() after 2.198r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434945005327323548241115163596 6472,
441.642959739087430367777871892178674597 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999904984205426029998506778725 00,
6.217012502913368682966756400487179127331,
485.549080907439650616725773593845633120 3]
one interval r = 32.628147792326583321154615783788393873 71 ..
36.102483889665400479051039448315395598 83
Time Approximations 0.023.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=1e-38
Equations at solution: [-.2e-37, .1e-37, .13601e-34]Solution in 0.611s

Time Plot 0 s.
Exiting SolveHard() after 5.281r=35.4632 in [33.94922194 ..
```

```
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684480184574712086548692306198503,
6.58343472183860573359100323822612920998 4,
467.78730596620563365623522287082988750 02]
one interval r = 32.41978955679317526414072670897000463240 ..
35.85152417393478160000529287361515655567
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, .15325e-34]Solution in 0.633s

Time Plot 0 s.
Exiting SolveHard() after 1.005r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21, none, none,
401.88173904739924185842268739923191164 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684480184574712086548692306198503,
6.58343472183860573359100323822612920998 4,
467.78730596620563365623522287082988750 02]
two intervals r = 12.92327160826545830078044022658124571924 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000000
000 or r = 18.39424858041673115037506993938328251036 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000000
```

```
000
Time Approximations 0.045.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=1.0e-38
Equations at solution: [.1e-37, .10e-37, .825e-35]Solution in 47.874s

Time Plot 0 s.
Exiting SolveHard() after 49.309r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83, none,
401.88173904739924185842268739923191164 27, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229628504453780163296715003054761 44,
4.125651796878695122577996888177232485862,
440.671230660120987378999073888368894875 4]
two intervals r = 14.356597050980911331068719010655535595 22 ..
37999999999992736221996002553513632 29/2000000000000000000000000000000000
000 or r = 17.703526138321714328981484944749658557 35 ..
37999999999992736221996002553513632 29/2000000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [-.57e-37, -.3e-37, .399e-35]Solution in 5.694s

Time Plot 0 s.
```

```
Exiting SolveHard() after 6.771r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83, none,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962850445378016329671500305476144,
4.12565179687869512257799688817723248586 2,
440.67123066012098737899907388836889487 54]
one interval r = 22.39761154385422478488114954139736568300 ..
27.23722351603165462451017618331286614214
Time Approximations 0.037.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.291 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064401336439043165404363960192258, rm =
14.37818770702703798821084612561759115468}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.05e-37
Equations at solution: [.1e-37, .105e-36, .993e-34]Solution in 8.978s

Time Plot 0 s.
Exiting SolveHard() after 9.878r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
```

```
401.881739047399241858422687399231911642 7,
389.590015168531644170291544877434629023 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888162116008239468310565226179 99,
4.004869081841990066700339088184802872771,
404.862245018410456192810033813929031672 6]
two intervals r = 16.080110077583041406519383822447933867 14 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000000
000 or r = 16.415798127105087530598863605820170946 89 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.18e-37, .1e-37, .4112e-34]Solution in 1.736s

Time Plot 0 s.
Exiting SolveHard() after 7.17r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434945005327323548241115163596647 2,
441.642959739087430367777871892178674597 3,
436.917481662024266912430573826029026352 1,
422.984933977234982258210109066118680098 3,
361.525802567332143935678638899231049073 4,
401.881739047399241858422687399231911642 7,
389.590015168531644170291544877434629023 0, none, none,
358.973628246085823705432825783838472043 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888162116008239468310565226179 99,
4.004869081841990066700339088184802872771,
404.862245018410456192810033813929031672 6]
one interval r = 21.641943994203934392363047116065538024 33 ..
26.763306600435427926034119488860574410 44
Time Approximations 0.05.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
```

```
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [-.1e-37, -.49e-37, .51e-35]Solution in 1.063s

Time Plot 0 s.
Exiting SolveHard() after 6.761r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66, none,
358.97362824608582370543282578383847204 38, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941802181957518076357784775552540,
5.5896371828134510382348521735806961500 96,
443.83065884812355885224836292268668473 44]
one interval r = 22.46725374480446844307042543625329658275 ..
27.27388428350682032990478295919377614891
Time Approximations 0.043.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [-.1e-37, .81e-37, .51e-35]Solution in 1.013s

Time Plot 0 s.
Exiting SolveHard() after 2.01r=27.0204 in [24.71083344 .. 27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66, none,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941802181957518076357784775552540,
5.5896371828134510382348521735806961500 96,
443.83065884812355885224836292268668473 44]
one interval r = 32.15575279511666187874705230624144107182 ..
35.50872228751937860950035724843683589312
Time Approximations 0.018.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.5020e-35]Solution in 0.461s

Time Plot 0 s.
Exiting SolveHard() after 5.146r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
```

```
none, none, none, none]

1 --> 0  target = [15.91193136484501818495253666819043615327,
5.18778357845698917922122938042299560333 4,
408.65773863787436690657126069174646565 47]
one interval r = 21.71840114671140469493881283926663302012 ..
26.81849303521516933830790693144644719857
Time Approximations 0.057.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=7.9e-38
Equations at solution: [-.1e-37, -.79e-37, -.42e-35]Solution in 0.98s

Time Plot 0 s.
Exiting SolveHard() after 2.116r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359 66472,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98, none, none,
361.50888347776212948362870096238512437 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136484501818495253666819043615327,
5.18778357845698917922122938042299560333 4,
408.65773863787436690657126069174646565 47]
one interval r = 31.80828598768858917403578496988207728425 ..
35.00011460071550792674761653921374957069
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [-.5e-37, .5e-37, -.33980e-34]Solution in 0.411s

Time Plot 0 s.
Exiting SolveHard() after 0.691r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54, none,
361.50888347776212948362870096238512437 16, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110545480632139578948621829925697,
6.19626256556721996771911534548312106089 3,
385.44474380004995428814755400150273233 69]
one interval r = 31.60836097549436316398880628977489720043 ..
34.66372795629647699636385871265693380360
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=1.1e-37
Equations at solution: [.8e-37, -.11e-36, .8163e-35]Solution in 0.572s

Time Plot 0 s.
Exiting SolveHard() after 5.313r=33.8136 in [32.62689490 ..
34.66372796]
```

```
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54, none,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110545480632139578948621829925697,
6.1962625655672199677191153454831210608 93,
385.44474380004995428814755400150273233 69]
two intervals r = 16.875634087449680452704820119966884122 42 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000
000 or r = 15.556404938312012322246704865092262116 27 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000
000
Time Approximations 0.062.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 8.236 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175114655214071839808420877679038, rm
= 2.33653277399263440816531888684775311153 8}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.179e-37, 0., -.53e-36]Solution in 36.012s

Time Plot 0 s.
Exiting SolveHard() after 37.188r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874725383433342255511534705647398,
4.8838107797430482787405671025655913314 52,
376.61967856524216590071262507960477251 46]
one interval r = 21.11001304889939113446924560695553616507 ..
26.31784243481530488913035851543517409145
Time Approximations 0.036.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=7.5e-38
Equations at solution: [.2e-37, .75e-37, .86e-35]Solution in 0.857s

Time Plot 0.001 s.
Exiting SolveHard() after 1.557r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
```

```
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48, none,
328.46938513763248829609567823695198079 94, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874725383433342255511534705647398,
4.8838107797430482787405671025655913314 52,
376.61967856524216590071262507960477251 46]
one interval r = 31.53899497723567307930648647826931207248 ..
34.53618386113348197197449608395717033479
Time Approximations 0.018.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.87e-36
Equations at solution: [.296e-35, -.387e-35, .48181e-34]Solution in
0.505s

Time Plot 0 s.
Exiting SolveHard() after 0.797r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48, none,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017540690702839788572762982002499,
```

```
6.025813549502529764773919119042064283120,
351.427029486598592330802900388890783685 9]
one interval r = 31.362302061221714137010717771399475115 18 ..
34.174466406276146030973509979188200366 96
Time Approximations 0.017.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [-.3e-37, .6e-37, .37922e-34]Solution in 0.557s

Time Plot 0 s.
Exiting SolveHard() after 0.8r=33.3686 in [32.23723258 .. 34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48, none,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37, none, none,
292.99969138334206011563476019376748823 86, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175406907028397885727629820024 99,
6.025813549502529764773919119042064283120,
351.427029486598592330802900388890783685 9]
two intervals r = 17.981355144457436063538743056243723012 05 ..
37999999999992736221996002553513632 29/2000000000000000000000000000000000
000 or r = 13.846080154349847310563002547609743754 00 ..
37999999999992736221996002553513632 29/2000000000000000000000000000000000
000
Time Approximations 0.047.
```

```
hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 7.237 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713688765329173382376908425652 27, rm
= 2.7345009930815228364398802177573394949 19}});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [.19e-37, 0., -.210e-35]Solution in 21.768s

Time Plot 0 s.
Exiting SolveHard() after 27.411r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359 66472,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37, none, none,
292.99969138334206011563476019376748823 86, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418648732717060672048381160585 50,
6.3779438740713393259408865701785295355 77,
423.28832783819104525790061357378315104 28]
one interval r = 31.946618176029864423605250534633601625 60 ..
35.212123086597110462860216099260562392 43
Time Approximations 0.018.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [.3e-37, -.3e-37, .2426e-35]Solution in 0.599s

Time Plot 0 s.
Exiting SolveHard() after 0.918r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37, none, none,
292.99969138334206011563476019376748823 86, none, none,
360.06173466445084304665889499470208250 68, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418648732717060672048381160585 50,
6.3779438740713393259408865701785295355 77,
423.28832783819104525790061357378315104 28]
two intervals r = 15.228867024693029122963791651177774965 01 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000
000 or r = 17.129657770627310594378837455468377605 86 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000
000
Time Approximations 0.064.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
```

```
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 8.143 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541464694470263619986079540050 26, rm
= 2.06406829870609332552222924011039046975 1}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.48e-37, .1e-37, .2031e-34]Solution in 35.054s

Time Plot 0 s.
Exiting SolveHard() after 36.408r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37,
375.73285289391327966277932363660309184 80, none,
292.99969138334206011563476019376748823 86, none, none,
360.06173466445084304665889499470208250 68, none, none, none, none,
none, none, none]

0 --> 2  target = [34.93953234358982751074940272683546661967,
4.0035598155394240447436496022003048451 10,
404.47973594469969354935332928890780068 01]
two intervals r = 16.09683966372515016483333855054649754003 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000
000 or r = 16.39988649127295809974066569676662866591 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
```

```
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.34e-37, -.1e-37, -.2086e-34]Solution in
6.297s

Time Plot 0 s.
Exiting SolveHard() after 7.404r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37,
375.73285289391327966277932363660309184 80, none,
292.99969138334206011563476019376748823 86,
358.64341561392087837493553850947369147 97, none,
360.06173466445084304665889499470208250 68, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234358982751074940272683546661967,
4.0035598155394240447436496022003048451 10,
404.47973594469969354935332928890780068 01]
one interval r = 21.63429629996118913683783514952135287808 ..
26.75768169892246807710657858439570003593
Time Approximations 0.053.

hint used Hint := [25.86532280054226848442580721202864937576, 3, -1, 1,
16.77921525837433403832553912089491136680, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.1e-38
Equations at solution: [.2e-37, .51e-37, -.106e-34]Solution in 1.106s

Time Plot 0 s.
Exiting SolveHard() after 2.115r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37,
375.73285289391327966277932363660309184 80,
328.11709294542467822992971711039167984 80,
292.99969138334206011563476019376748823 86,
358.64341561392087837493553850947369147 97, none,
360.06173466445084304665889499470208250 68, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954463759496148530830672893314344,
6.19617723042099767956568966429214649112 2,
385.42734026385357788177649036213500031 67]
one interval r = 31.60822049104842673917759818682712793928 ..
34.66347615067384326911673208132740002184
Time Approximations 0.019.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=1.1e-37
```

```
Equations at solution: [.8e-37, -.11e-36, .9141e-35]Solution in 4.628s

Time Plot 0 s.
Exiting SolveHard() after 4.921r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37,
375.73285289391327966277932363660309184 80,
328.11709294542467822992971711039167984 80,
292.99969138334206011563476019376748823 86,
358.64341561392087837493553850947369147 97, none,
360.06173466445084304665889499470208250 68, none, none,
324.65521223997548963180722489924737427 36, none, none, none, none]

0 --> 1  target = [26.46318954463759496148530830672893314344,
6.19617723042099767956568966429214649112 2,
385.42734026385357788177649036213500031 67]
two intervals r = 16.87629600283407317793447931059365550205 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000
000 or r = 15.55559000680006656741936422396359169156 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000
000
Time Approximations 0.061.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
```

```
in partial time = 8.078 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852526288131125128485710123193759, rm
= 2.33669042805749273899506867495167400415 9}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [0., 0., -.318e-35]Solution in 31.05s

Time Plot 0 s.
Exiting SolveHard() after 32.221r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359 66472,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37,
375.73285289391327966277932363660309184 80,
328.11709294542467822992971711039167984 80,
292.99969138334206011563476019376748823 86,
358.64341561392087837493553850947369147 97, none,
360.06173466445084304665889499470208250 68,
336.59441032383085849542930096986251428 40, none,
324.65521223997548963180722489924737427 36, none, none, none, none]

0 --> 2  target = [34.49522661192156003257556839933776219199,
3.8971313159847352134689963180236444605 36,
373.78081885654557287987414145208906353 35]
two intervals r = 17.29769086197762364074626430280477413546 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000
000 or r = 14.99436407488835840246847762023728821963 ..
37999999999992736221996002553513632 29/20000000000000000000000000000000
000
Time Approximations 0.084.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
```

```
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [.18e-37, 0., -.1968e-34]Solution in 1.249s

Time Plot 0 s.
Exiting SolveHard() after 7.39r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37,
375.73285289391327966277932363660309184 80,
328.11709294542467822992971711039167984 80,
292.99969138334206011563476019376748823 86,
358.64341561392087837493553850947369147 97, none,
360.06173466445084304665889499470208250 68,
336.59441032383085849542930096986251428 40, none,
324.65521223997548963180722489924737427 36,
331.93806792670161478807710130108874418 32, none, none, none]

1 --> 2  target = [34.49522661192156003257556839933776219199,
3.8971313159847352134689963180236444605 36,
373.78018885654557287987414145208906353 35]
one interval r = 21.06068473223830649590066910227509339563 ..
26.26979834298168984918141515529314298281
Time Approximations 4.419.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
```

```
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=5e-38
Equations at solution: [-.2e-37, -.5e-37, .393e-34]Solution in 0.732s

Time Plot 0 s.
Exiting SolveHard() after 5.878r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37,
375.73285289391327966277932363660309184 80,
328.11709294542467822992971711039167984 80,
292.99969138334206011563476019376748823 86,
358.64341561392087837493553850947369147 97,
299.89866205652448007150965320771468852 49,
360.06173466445084304665889499470208250 68,
336.59441032383085849542930096986251428 40, none,
324.65521223997548963180722489924737427 36,
331.93806792670161478807710130108874418 32, none, none, none]

0 --> 2  target = [33.81362495424432193391577470701573627669,
3.7256489935921608098236239520417276282 36,
325.89209973279778684330941314627851677 48]
two intervals r = 18.552270490028108819532524493667420525 05 ..
37999999999992736221996002553513632 29/2000000000000000000000000000000000
000 or r = 12.491969358180713917759914905952829508 26 ..
37999999999992736221996002553513632 29/2000000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
```

```
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=5e-38
Equations at solution: [-.120e-36, .5e-37, .57e-36]Solution in 1.132s

Time Plot 0 s.
Exiting SolveHard() after 2.574r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37,
375.73285289391327966277932363660309184 80,
328.11709294542467822992971711039167984 80,
292.99969138334206011563476019376748823 86,
358.64341561392087837493553850947369147 97,
299.89866205652448007150965320771468852 49,
360.06173466445084304665889499470208250 68,
336.59441032383085849542930096986251428 40, none,
324.65521223997548963180722489924737427 36,
331.93806792670161478807710130108874418 32, none, none,
289.54595773091065677633350445219886022 17]

1 --> 2  target = [33.81362495424432193391577470701573627669,
3.72564899359216080982362395204172762823 6,
325.89209973279778684330941314627851677 48]
one interval r = 20.37468935112498798082977062833009592057 ..
25.37892165303843005756765622794772688343
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
```

```
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=9e-38
Equations at solution: [.7e-37, .9e-37, .377e-34]Solution in 0.607s

Time Plot 0 s.
Exiting SolveHard() after 5.455r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37,
375.73285289391327966277932363660309184 80,
328.11709294542467822992971711039167984 80,
292.99969138334206011563476019376748823 86,
358.64341561392087837493553850947369147 97,
299.89866205652448007150965320771468852 49,
360.06173466445084304665889499470208250 68,
336.59441032383085849542930096986251428 40,
256.10753186236421471829458291716929579 02,
324.65521223997548963180722489924737427 36,
331.93806792670161478807710130108874418 32, none, none,
289.54595773091065677633350445219886022 17]

1 --> 0  target = [17.93041369715899047289525164787487494893,
4.68650870183052103036639870654241914227 3,
353.30541095145919358533639656343873318 08]
one interval r = 20.73150479093145461567486018423513260864 ..
25.90675353519155406881716300807730848403
Time Approximations 0.033.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.358e-34]Solution in 0.663s

Time Plot 0 s.
Exiting SolveHard() after 1.379r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37,
375.73285289391327966277932363660309184 80,
328.11709294542467822992971711039167984 80,
292.99969138334206011563476019376748823 86,
358.64341561392087837493553850947369147 97,
299.89866205652448007150965320771468852 49,
360.06173466445084304665889499470208250 68,
336.59441032383085849542930096986251428 40,
256.10753186236421471829458291716929579 02,
324.65521223997548963180722489924737427 36,
331.93806792670161478807710130108874418 32,
304.79958325169174686910069835337499807 79, none,
289.54595773091065677633350445219886022 17]

2 --> 0  target = [17.93041369715899047289525164787487494893,
4.68650870183052103036639870654241914227 3,
353.30541095145919358533639656343873318 08]
one interval r = 31.37435487000543869050086743180462685971 ..
34.20127520038323898937190720053683581700
Time Approximations 0.017.
```

```
hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=1.1e-37
Equations at solution: [.8e-37, -.11e-36, -.1896e-35]Solution in 0.353s

Time Plot 0 s.
Exiting SolveHard() after 0.617r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494500532732354824111516359664 72,
441.64295973908743036777787189217867459 73,
436.91748166202426691243057382602902635 21,
422.98493397723498225821010906611868009 83,
361.52580256733214393567863889923104907 34,
401.88173904739924185842268739923191164 27,
389.59001516853164417029154487743462902 30,
328.46939893690510289211269965603411660 66,
401.50757158418952605406668518695561703 96,
358.97362824608582370543282578383847204 38,
398.33147103947567053841333951080601053 98,
371.48387395534654992844336109125030106 54,
336.61215841334440556225547044765504262 34,
361.50888347776212948362870096238512437 16,
324.67144993013886965108096108337417441 48,
302.31384314561777294636280263836595392 56,
328.46938513763248829609567823695198079 94,
343.81340625755306924003422448139619968 37,
375.73285289391327966277932363660309184 80,
328.11709294542467822992971711039167984 80,
292.99969138334206011563476019376748823 86,
358.64341561392087837493553850947369147 97,
299.89866205652448007150965320771468852 49,
360.06173466445084304665889499470208250 68,
336.59441032383085849542930096986251428 40,
256.10753186236421471829458291716929579 02,
324.65521223997548963180722489924737427 36,
331.93806792670161478807710130108874418 32,
304.79958325169174686910069835337499807 79,
323.46169176995884819940941422595268214 92,
289.54595773091065677633350445219886022 17]

Cascade time 278.746
counts: 28, 28
```

```
 Iteration 97

Start Generation 1
1 --> 0  target = [12.00000000006687436857647698861852864700,
6.217012502929908747684585807292750450200,
485.549080884343019075673302271718838046 7]
one interval r = 23.40850301626174064403463406146056482893 ..
27.67578046423763012661874368527509627273
Time Approximations 0.046.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=5.2e-38
Equations at solution: [.2e-37, -.52e-37, .566e-36]Solution in 1.045s

Time Plot 0 s.
Exiting SolveHard() after 2.199r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000006687436857647698861852864700,
6.217012502929908747684585807292750450200,
485.549080884343019075673302271718838046 7]
one interval r = 32.62814779199476514486914982996631885476 ..
36.10248388917422840288268468035748636211
Time Approximations 0.023.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=6e-38
Equations at solution: [-.9e-37, .6e-37, -.113e-34]Solution in 0.584s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.01r=35.4632 in [33.94922194 .. 36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684473327945587256002828247852062,
6.58343472155715621566550569536987656 3523,
467.78730594542714504495687028701202694 71]
one interval r = 32.41978955650777345296862698287774208200 ..
35.85152417348072613449970606552581141959
Time Approximations 0.019.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=7e-38
Equations at solution: [-.8e-37, .7e-37, -.193e-34]Solution in 0.661s

Time Plot 0 s.
Exiting SolveHard() after 5.496r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42, none, none,
401.88173902884056943782573110264723733 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684473327945587256002828247852062,
6.58343472155715621566550569536987656 3523,
467.78730594542714504495687028701202694 71]
two intervals r = 12.92327160837539913475555601173218795118 ..
9499999999877207623701792981813553099/50000000000000000000000000000000
000 or r = 18.39424857987315858209088107792413651824 ..
```

```
9499999999877207623701792981813553099/500000000000000000000000000000000
000
Time Approximations 0.044.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=8.6e-38
Equations at solution: [.7e-37, .86e-37, .1669e-34]Solution in 48.128s

Time Plot 0 s.
Exiting SolveHard() after 49.615r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333556955215705736721694674 6, none,
401.88173902884056943782573110264723733 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.463229627999983389906749360966659257 67,
4.125651796680640733559940241862727593589,
440.67123063523809233660818614504862614 91]
two intervals r = 14.356597051352518147477107707741335918 34 ..
9499999999877207623701792981813553099/500000000000000000000000000000000
000 or r = 17.703526137494365062066406567047914800 90 ..
9499999999877207623701792981813553099/500000000000000000000000000000000
000
Time Approximations 0.048.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=1e-38
Equations at solution: [.29e-37, .1e-37, .2624e-34]Solution in 1.32s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.439r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46, none,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962799998338990674936096665925767,
4.1256517966806407335599402418627275935 89,
440.67123063523809233660818614504862614 91]
one interval r = 22.397611543350644689766865045557948740 56 ..
27.237223515857867347924635716803730226 16
Time Approximations 0.04.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.241 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070643588359027256612742232973504 05, rm =
14.378187700655369675572608465941603127 71}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., .26e-37, .10998e-34]Solution in 9.346s

Time Plot 0 s.
Exiting SolveHard() after 14.229r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
```

```
361.525802545412088488324562044231680519 6,
401.881739028840569437825731102647237335 3,
389.590015141709043840319493228390408187 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887754955337606505578229795562 23,
4.004869081660336198624371075347427416702,
404.862244999238510702279535743014296577 6]
two intervals r = 16.080110077666490615866973340260376029 79 ..
949999999987720762370179298181355309 9/500000000000000000000000000000000000
000 or r = 16.415798126246691180595045447708897632 23 ..
949999999987720762370179298181355309 9/500000000000000000000000000000000000
000
Time Approximations 0.059.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [-.18e-37, -.1e-37, .2525e-34]Solution in 6.434s

Time Plot 0 s.
Exiting SolveHard() after 7.603r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434922852666634346733700831425988 3,
441.642959718879562388685895845758506456 7,
436.917481637865906857437419529803155464 2,
422.984933963333556955215705736721694674 6,
361.525802545412088488324562044231680519 6,
401.881739028840569437825731102647237335 3,
389.590015141709043840319493228390408187 8, none, none,
358.973628223676867681680857257475597417 0, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078887754955337606505578229795562 23,
4.004869081660336198624371075347427416702,
404.862244999238510702279535743014296577 6]
one interval r = 21.641943993853941686487159957318541795 81 ..
26.763306600261681062420491915605284228 86
Time Approximations 0.053.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
```

```
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=5.1e-38
Equations at solution: [-.2e-37, -.51e-37, -.21601e-34]Solution in
5.28s

Time Plot 0 s.
Exiting SolveHard() after 6.343r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64, none,
358.97362822367686768168085725747559741 70, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941784020118917549154546396833911,
5.58963718297116167160989851872906406631 5,
443.83065883383031987323606108773062205 71]
one interval r = 22.46725374453297550116521279579481664106 ..
27.27388428346212966526254486248041452853
Time Approximations 0.036.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.3e-38
Equations at solution: [.1e-37, -.53e-37, -.2967e-35]Solution in 1.005s

Time Plot 0 s.
Exiting SolveHard() after 1.954r=27.0204 in [24.71083344 ..
27.27388429]
```

```
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64, none,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941784020118917549154546396833911,
5.589637182971161671609898518729064066315,
443.83065883383031987323606108773062205 71]
one interval r = 32.15575279492634725195437623660570850754 ..
35.50872228716624070659170222391215121709
Time Approximations 0.02.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=4e-38
Equations at solution: [-.5e-37, .4e-37, -.3e-36]Solution in 0.471s

Time Plot 0 s.
Exiting SolveHard() after 0.823r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
```

```
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136532011473483154412478936420574,
5.18778357847210391722939650624200715857 1,
408.65773861000993123075711362528673709 08]
one interval r = 21.71840114618095268906627891401898879637 ..
26.81849303492322801450692538287689459663
Time Approximations 0.058.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.6e-38
Equations at solution: [.1e-37, .26e-37, -.652e-36]Solution in 1.019s

Time Plot 0 s.
Exiting SolveHard() after 6.13r=26.4632 in [23.93303356 .. 26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333556955215705736721694674 6,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34, none, none,
361.50888345583204662297973355434767137 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136532011473483154412478936420574,
5.18778357847210391722939650624200715857 1,
408.65773861000993123075711362528673709 08]
one interval r = 31.80828598740629096634387041384471296531 ..
35.00011460018054275774696451010955553637
Time Approximations 0.018.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
```

```
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=5e-38
Equations at solution: [.4e-37, -.5e-37, -.6e-36]Solution in 0.4s

Time Plot 0 s.
Exiting SolveHard() after 0.718r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93, none,
361.50888345583204662297973355434767137 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110519198315453102993222769870225,
6.19626256527222011952523755242936624 6379,
385.44474377748931151367935711346033157 48]
one interval r = 31.60836097529115412484634870201591415906 ..
34.66372795584997581982214007926040944255
Time Approximations 0.019.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=5e-38
Equations at solution: [-.4e-37, .5e-37, -.56e-35]Solution in 0.58s

Time Plot 0 s.
```

```
Exiting SolveHard() after 0.844r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93, none,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110519198315453102993222769870225,
6.19626256527222011952523755242936624637 9,
385.44474377748931151367935711346033157 48]
two intervals r = 16.87563408764315512541221090742740016517 ..
94999999998772076237017929818135530 99/50000000000000000000000000000000
000 or r = 15.55640493718616322183272636766621856369 ..
94999999998772076237017929818135530 99/50000000000000000000000000000000
000
Time Approximations 0.054.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 8.172 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175114227246808665507609487387241, rm
= 2.33653277413697237560144027909350985 5662}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [-.179e-37, 0., -.385e-35]Solution in 32.129s

Time Plot 0 s.
Exiting SolveHard() after 38.21r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874744717728978353645964310215 88,
4.88381077981796564354432668524664068286 4,
376.61967854204870943702390818834054002 30]
one interval r = 21.11001304851954062627936404480774273152 ..
26.31784243452467213260437481901684161958
Time Approximations 4.105.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=2.5e-38
Equations at solution: [0., .25e-37, .19964e-34]Solution in 0.853s

Time Plot 0 s.
Exiting SolveHard() after 5.672r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
```

```
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03, none,
328.46938512104712558885267102404171523 98, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874744717772897835364596431021588,
4.8838107798179656435443266852466406828 64,
376.61967854204870943702390818834054002 30]
one interval r = 31.53899497703953882150191252490658266383 ..
34.53618386068273454476184528263900118825
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=2.78e-36
Equations at solution: [-.214e-35, .278e-35, .166e-34]Solution in
0.486s

Time Plot 0 s.
Exiting SolveHard() after 0.797r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333556955215705736721694674 6,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03, none,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017518220458455559276293673457395,
6.025813549231617838480180755072956633744,
351.42702946951319491992557337470709979 77]
one interval r = 31.362302061099935987821739159346070937 50 ..
34.174466405928110326593570584542630054 83
Time Approximations 0.017.

hint used Hint := [33.368553550020083888860664127893394910 74, 3, 1, 1,
12.142802994561336759524256526823424721 44, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=1.2e-37
Equations at solution: [.6e-37, -.12e-36, -.65e-35]Solution in 0.537s

Time Plot 0 s.
Exiting SolveHard() after 0.789r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03, none,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95, none, none,
292.99969136850343864742143044961333767 14, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017518220458455559276293673457395,
6.025813549231617838480180755072956633744,
351.42702946951319491992557337470709979 77]
two intervals r = 17.981355144403349590796054864081370793 89 ..
9499999999877207623701792981813553099/50000000000000000000000000000000
000 or r = 13.846080153333067415374951007956516161 06 ..
```

```
9499999999877207623701792981813553099/500000000000000000000000000000000
000
Time Approximations 4.406.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 3.205 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.913570713507302015630228277402784626 70, rm
= 2.7345009932446907923153208779263181228 73}});
Accepted {r=18.6878, rm=15.3648} with Delta=4e-38
Equations at solution: [-.106e-36, .4e-37, .238e-34]Solution in 16.097s

Time Plot 0 s.
Exiting SolveHard() after 21.538r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95, none, none,
292.99969136850343864742143044961333767 14, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.020379418644374763378723152106339017 88,
6.3779438738437966298623002915852984167 42,
423.28832782927992952058166590873291262 78]
one interval r = 31.946618175910647184307649429927648387 83 ..
35.212123086330304114952767797181563818 79
Time Approximations 0.02.
```

```
hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=8e-38
Equations at solution: [-.7e-37, .8e-37, .230e-34]Solution in 0.627s

Time Plot 0 s.
Exiting SolveHard() after 0.984r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95, none, none,
292.99969136850343864742143044962133767 14, none, none,
360.06173465712229469278983123493431959 58, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418644374763378723152106339017 88,
6.37794387384379662986230029158529841674 2,
423.28832782927992952058166590873291262 78]
two intervals r = 15.22886702428340354968740101828235730675 ..
9499999999877207623701792981813553099/50000000000000000000000000000000
000 or r = 17.12965777024216667483256251814859700910 ..
9499999999877207623701792981813553099/50000000000000000000000000000000
000
Time Approximations 0.063.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.69073378973079182825919789070949638294, 15.22886699 .. 19, 3/2 ..
19, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 8.38 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541157283483032380079220173596 62, rm
= 2.0640682987155493397197443745917877301 08}});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [.48e-37, .2e-37, -.279e-35]Solution in 30.973s

Time Plot 0 s.
Exiting SolveHard() after 36.562r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95,
375.73285289336087185361758422993570048 84, none,
292.99969136850343864742143044962133767 14, none, none,
360.06173465712229469278983123493431959 58, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343191599892485697248566894382 74,
4.0035598153598089410533002619978395718 68,
404.47973592613137261189921845200830667 00]
two intervals r = 16.096839663781996513103589642935320573 32 ..
949999999987720762370179298181355309 9/50000000000000000000000000000000000
000 or r = 16.399886490437522729606563529017099931 17 ..
949999999987720762370179298181355309 9/50000000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
```

```
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [.35e-37, .1e-37, -.816e-35]Solution in 1.693s

Time Plot 0 s.
Exiting SolveHard() after 7.021r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95,
375.73285289336087185361758422993570048 84, none,
292.99969136850343864742143044962133767 14,
358.64341559202933209870310432731086375 91, none,
360.06173465712229469278983123493431959 58, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343191599892485697248566894382 74,
4.0035598153598089410533002619978395718 68,
404.47973592613137261189921845200830667 00]
one interval r = 21.634296299623695234914374020792758336 37 ..
26.757681698756916218996483834924220710 02
Time Approximations 0.059.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
```

```
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., .3801e-35]Solution in 5.349s

Time Plot 0 s.
Exiting SolveHard() after 6.429r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95,
375.73285289336087185361758422993570048 84,
328.11709292939515831252671634698863890 09,
292.99969136850343864742143044961333767 14,
358.64341559202933209870310432731086375 91, none,
360.06173465712229469278983123493431959 58, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954437457049442823817845970272978,
6.19617723012594545601084188270997991432 9,
385.42734024128261318224863200606458374 38]
one interval r = 31.60822049084515774212065791038914186624 ..
34.66347615022720211804168893572012996728
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
```

```
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=1.4e-37
Equations at solution: [-.10e-36, .14e-36, .103e-34]Solution in 0.59s

Time Plot 0 s.
Exiting SolveHard() after 0.918r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95,
375.73285289336087185361758422993570048 84,
328.11709292939515831252671634698863890 09,
292.99969136850343864742143044962133767 14,
358.64341559202933209870310432731086375 91, none,
360.06173465712229469278983123493431959 58, none, none,
324.65521221994492601769412989765583236 81, none, none, none, none]

0 --> 1  target = [26.46318954437457049442823817845970272978,
6.19617723012594545601084188270997991 4329,
385.42734024128261318224863200606458374 38]
two intervals r = 16.87629600302789903765614381344007760977 ..
94999999998772076237017929818135530 99/50000000000000000000000000000000
000 or r = 15.55559000567362499262832829424612784620 ..
94999999998772076237017929818135530 99/50000000000000000000000000000000
000
Time Approximations 0.064.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 12.365 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852525873010947204619212367714485, rm
= 2.33669042820196035114623098263128097923 9}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [-.179e-37, 0., -.1681e-34]Solution in 35.624s

Time Plot 0 s.
Exiting SolveHard() after 36.832r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95,
375.73285289336087185361758422993570048 84,
328.11709292939515831252671634698863890 09,
292.99969136850343864742143044962133767 14,
358.64341559202933209870310432731086375 91, none,
360.06173465712229469278983123493431959 58,
336.59441030913578154255638340706972850 16, none,
324.65521221994492601769412989765583236 81, none, none, none, none]

0 --> 2  target = [34.49522661135477451028194998619263340242,
3.8971313157580676189416831447754672412 16,
373.78018182519157725922740669443846786 23]
two intervals r = 17.29769086244253317043325726260290687554 ..
9499999999877207623701792981813553099/5000000000000000000000000000000
000 or r = 14.99436407325720490021994507187114055404 ..
9499999999877207623701792981813553099/5000000000000000000000000000000
000
Time Approximations 0.092.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
```

```
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=1e-38
Equations at solution: [-.18e-37, .1e-37, -.712e-35]Solution in 5.656s

Time Plot 0 s.
Exiting SolveHard() after 7.358r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95,
375.73285289336087185361758422993570048 84,
328.11709292939515831252671634698863890 09,
292.99969136850343864742143044962133767 14,
358.64341559202933209870310432731086375 91, none,
360.06173465712229469278983123493431959 58,
336.59441030913578154255638340706972850 16, none,
324.65521221994492601769412989765583236 81,
331.93806789337302809304294071897973744 86, none, none, none]

1 --> 2  target = [34.49522661135477451028194998619263340242,
3.8971313157580676189416831447754672412 16,
373.78081882519157725922740669443846786 23]
one interval r = 21.06068473172406418169899403211900670352 ..
26.26979834254657621428344367529379800446
Time Approximations 0.032.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
```

```
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [.1e-37, .3e-37, -.19786e-34]Solution in 0.785s

Time Plot 0 s.
Exiting SolveHard() after 1.47r=25.3005 in [23.14060343 .. 26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95,
375.73285289336087185361758422993570048 84,
328.11709292939515831252671634698863890 09,
292.99969136850343864742143044962133767 14,
358.64341559202933209870310432731086375 91,
299.89866202881006148910101265253577940 79,
360.06173465712229469278983123493431959 58,
336.59441030913578154255638340706972850 16, none,
324.65521221994492601769412989765583236 81,
331.93806789337302809304294071897973744 86, none, none, none]

0 --> 2  target = [33.81362495386233723498585996300402926172,
3.72564899339930094701916409819512450479 7,
325.89209971210565496627222311808814368 56]
two intervals r = 18.55227048998928888380063460887442852629 ..
9499999999877207623701792981813553099/50000000000000000000000000000000
000 or r = 12.49196935692400013610039724082566978594 ..
9499999999877207623701792981813553099/50000000000000000000000000000000
000
Time Approximations 0.037.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
```

```
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=1e-38
Equations at solution: [.17e-37, -.1e-37, -.84e-35]Solution in 1.11s

Time Plot 0 s.
Exiting SolveHard() after 7.23r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95,
375.73285289336087185361758422993570048 84,
328.11709292939515831252671634698863890 09,
292.99969136850343864742143044962133767 14,
358.64341559202933209870310432731086375 91,
299.89866202881006148910101265253577940 79,
360.06173465712229469278983123493431959 58,
336.59441030913578154255638340706972850 16, none,
324.65521221994492601769412989765583236 81,
331.93806789337302809304294071897973744 86, none, none,
289.54595770678981621676481575100432284 66]

1 --> 2  target = [33.81362495386233723498585996300402926172,
3.72564899339930094701916409819512450479 7,
325.89209971210565496627222311808814368 56]
one interval r = 20.37468935091263622078019150951101729100 ..
25.37892165270787460109265037886334583697
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=4e-38
Equations at solution: [.3e-37, .4e-37, .37231e-34]Solution in 0.536s

Time Plot 0 s.
Exiting SolveHard() after 1.084r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95,
375.73285289336087185361758422993570048 84,
328.11709292939515831252671634698863890 09,
292.99969136850343864742143044962133767 14,
358.64341559202933209870310432731086375 91,
299.89866202881006148910101265253577940 79,
360.06173465712229469278983123493431959 58,
336.59441030913578154255638340706972850 16,
256.10753184451856873855802093968588119 22,
324.65521221994492601769412989765583236 81,
331.93806789337302809304294071897973744 86, none, none,
289.54595770678981621676481575100432284 66]

1 --> 0  target = [17.93041369705420629207593383073034695906,
4.68650870197681423831451350993585815 3637,
353.30541093633011769239177404705273832 98]
one interval r = 20.73150479072819260110877432323617861219 ..
25.90675353500550418591391471289715583370
Time Approximations 0.034.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
```

```
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, -.6606e-35]Solution in 5.004s

Time Plot 0 s.
Exiting SolveHard() after 5.708r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95,
375.73285289336087185361758422993570048 84,
328.11709292939515831252671634698863890 09,
292.99969136850343864742143044962133767 14,
358.64341559202933209870310432731086375 91,
299.89866202881006148910101265253577940 79,
360.06173465712229469278983123493431959 58,
336.59441030913578154255638340706972850 16,
256.10753184451856873855802093968588119 22,
324.65521221994492601769412989765583236 81,
331.93806789337302809304294071897973744 86,
304.79958324347325292306294627680436416 73, none,
289.54595770678981621676481575100432284 66]

2 --> 0  target = [17.93041369705420629207593383073034695906,
4.68650870197681423831451350993585815 3637,
353.30541093633011769239177404705273832 98]
one interval r = 31.37435486989423551155810225997695869318 ..
34.20127520006199756347424608384921861229
Time Approximations 0.016.
```

```
hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=2.0e-37
Equations at solution: [.12e-36, -.20e-36, .113e-34]Solution in 0.384s

Time Plot 0 s.
Exiting SolveHard() after 0.663r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492285266663434673370083142598 83,
441.64295971887956238868589584575850645 67,
436.91748163786590685743741952980315546 42,
422.98493396333355695521570573672169467 46,
361.52580254541208848832456204423168051 96,
401.88173902884056943782573110264723733 53,
389.59001514170904384031949322839040818 78,
328.46939892031907551312128554154179360 64,
401.50757156622142868351793308520761449 44,
358.97362822367686768168085725747559741 70,
398.33147103082265880163895078313545094 34,
371.48387392488535732927469456399002735 93,
336.61215839865982923968166401427140111 62,
361.50888345583204662297973355434767137 60,
324.67144991011788879095383504798060360 03,
302.31384313639286807157567717481997163 13,
328.46938512104712558885267102404171523 98,
343.81340623044823972760914413254012314 95,
375.73285289336087185361758422993570048 84,
328.11709292939515831252671634698863890 09,
292.99969136850343864742143044962133767 14,
358.64341559202933209870310432731086375 91,
299.89866202881006148910101265253577940 79,
360.06173465712229469278983123493431959 58,
336.59441030913578154255638340706972850 16,
256.10753184451856873855802093968588119 22,
324.65521221994492601769412989765583236 81,
331.93806789337302809304294071897973744 86,
304.79958324347325292306294627680436416 73,
323.46169174957417663901064800315584393 42,
289.54595770678981621676481575100432284 66]

Cascade time 278.732
counts: 28, 28
```

```
 Iteration 98

Start Generation 1
1 --> 0  target = [11.999999999859079701747044276183239281 00,
6.217012503052800228535287701965370225156,
485.549080905617642325082677733688570068 7]
one interval r = 23.408503016701527828426320347014862635 40 ..
27.675780464373662041881272561834556794 57
Time Approximations 0.045.

hint used Hint := [27.523596844799965841529785954236081127 68, 2, -1,
-1, 6.492111403073555466582512703889939378594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=2.8e-38
Equations at solution: [0., .28e-37, -.3e-36]Solution in 1.062s

Time Plot 0 s.
Exiting SolveHard() after 2.256r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [11.999999999859079701747044276183239281 00,
6.217012503052800228535287701965370225156,
485.549080905617642325082677733688570068 7]
one interval r = 32.628147792214531173643924305138808573 01 ..
36.102483889451622267375017436514540542 61
Time Approximations 0.022.

hint used Hint := [35.463229628253790402151219742545983048 78, 3, 1, 1,
9.620026104439904675370322880257400169732, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=4e-38
Equations at solution: [-.6e-37, .4e-37, .48e-35]Solution in 0.619s
```

```
Time Plot 0 s.
Exiting SolveHard() after 5.39r=35.4632 in [33.94922194 .. 36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
2 --> 1  target = [27.52359684490091727573009090064053815106,
6.58343472168746649823961205342096148958 2,
467.78730596629372730574905694580679813 85]
one interval r = 32.41978955670959205948230163287914227105 ..
35.85152417375540917754221934533660480018
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=0
Equations at solution: [0., 0., .207e-34]Solution in 0.63s

Time Plot 0 s.
Exiting SolveHard() after 1.003r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53, none, none,
401.88173904817254385152850054791262226 95, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684490091727573009090064053815106,
6.58343472168746649823961205342096148958 2,
467.78730596629372730574905694580679813 85]
two intervals r = 12.92327160814994477109109826078220711695 ..
11875000000051668133647329655423494 07/6250000000000000000000000000000000
00 or r = 18.39424858051286696291143175152954992315 ..
```

```
11875000000051668133647329655423494 07/625000000000000000000000000000000
00
Time Approximations 0.045.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.139007733779466776968183543403560819 58, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=6.5e-38
Equations at solution: [-.6e-37, -.65e-37, -.1627e-34]Solution in
48.367s

Time Plot 0 s.
Exiting SolveHard() after 49.799r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81, none,
401.88173904817254385152850054791262226 95, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962827303889942535764593456953231,
4.12565179674660517588664309918201289 2034,
440.67123065579732025456538809351511262 97]
two intervals r = 14.35659705113655605335992894690531083048 ..
11875000000051668133647329655423494 07/625000000000000000000000000000000
00 or r = 17.70352613828070817431056307858261431466 ..
11875000000051668133647329655423494 07/625000000000000000000000000000000
00
Time Approximations 0.05.

hint used Hint := [15.911931365105695951377706985834321313 26, 2, -1, 1,
15.844784736186364983412155432818143035 59, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=0
Equations at solution: [0., 0., .733e-35]Solution in 5.64s
```

```
Time Plot 0 s.
Exiting SolveHard() after 6.748r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81, none,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962827303889942535764593456953231,
4.12565179674660517588664309918201289 2034,
440.67123065579732025456538809351511262 97]
one interval r = 22.39761154374809558654325148218262756733 ..
27.23722351607275705397445467884887449290
Time Approximations 0.039.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.327 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064394241975507563038124707588158, rm =
14.37818770326192491220652798065318634066}});
Accepted {r=26.4635, rm=16.5329} with Delta=5.3e-38
Equations at solution: [-.1e-37, -.53e-37, .213e-34]Solution in 9.959s

Time Plot 0 s.
Exiting SolveHard() after 10.89r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
```

```
361.525802563686935477834382921524448041 8,
401.881739048172543851528500547912622269 5,
389.590015161700778190688955234315671814 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888015707738674759975223704822 08,
4.004869081725289365001530497909180494653,
404.862245018942492572572877466498465062 2]
two intervals r = 16.080110077543232166790230653645214483 35 ..
118750000000516681336473296554234940 7/625000000000000000000000000000000
00 or r = 16.415798127199113480217273416464555789 97 ..
118750000000516681336473296554234940 7/625000000000000000000000000000000
00
Time Approximations 0.057.

hint used Hint := [17.198988747356282559236517292459382700 93, 2, -1, 1,
16.754921831705671242433031556784761016 88, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=0
Equations at solution: [-.17e-37, 0., -.5811e-34]Solution in 1.486s

Time Plot 0 s.
Exiting SolveHard() after 6.865r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434943707262870697463069781856124 2,
441.642959738982336494659341682924180521 2,
436.917481658042694401467286616715367375 3,
422.984933982993787483439410653330606768 1,
361.525802563686935477834382921524448041 8,
401.881739048172543851528500547912622269 5,
389.590015161700778190688955234315671814 3, none, none,
358.973628243077149305877845791134285924 8, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.945078888015707738674759975223704822 08,
4.004869081725289365001530497909180494653,
404.862245018942492572572877466498465062 2]
one interval r = 21.641943994185409931011899798868469144 16 ..
26.763306600525132885191064497110071788 05
Time Approximations 0.054.

hint used Hint := [25.872050175403105209417695955142983100 90, 3, -1, 1,
```

```
16.776718390804723703243908834102746068 98, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [.1e-37, .49e-37, -.30e-35]Solution in 5.7s

Time Plot 0 s.
Exiting SolveHard() after 6.771r=25.8721 in [23.84730094 ..
26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68, none,
358.97362824307714930587784579113428592 48, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941765502851974140317351346691199,
5.58963718304917451612694228335716211 2686,
443.83065885376462500939820080979213505 16]
one interval r = 22.46725374491968398961589546821305925373 ..
27.27388428366416535968644074924746831027
Time Approximations 0.04.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=5.4e-38
Equations at solution: [0., -.54e-37, -.86e-35]Solution in 1.021s

Time Plot 0 s.
Exiting SolveHard() after 2.013r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
```

```
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68, none,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941765502851974140317351346691199,
5.5896371830491745161269422833571621126 86,
443.83065885376462500939820080979213505 16]
one interval r = 32.15575279509880401424655247705387636889 ..
35.50872228743009862912086765463490286175
Time Approximations 0.022.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=0
Equations at solution: [0., 0., -.59e-35]Solution in 4.58s

Time Plot 0 s.
Exiting SolveHard() after 4.951r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
```

```
398.33147104910956979717512627614002648 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136516513716540199517437992553183,
5.18778357854116745021443795352369247813 2,
408.65773863025330484678010511539959582 09]
one interval r = 21.71840114652955987307226258324567907453 ..
26.81849303518853688172210521487031063710
Time Approximations 0.055.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=5.3e-38
Equations at solution: [.1e-37, .53e-37, .42e-35]Solution in 1.022s

Time Plot 0 s.
Exiting SolveHard() after 2.117r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60, none, none,
361.50888347412682265714178534645684298 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136516513716540199517437992553183,
5.18778357854116745021443795352369247813 2,
408.65773863025330484678010511539959582 09]
one interval r = 31.80828598755055331488705209684638336781 ..
35.00011460044930503566100908127748310189
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
```

```
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=1.0e-37
Equations at solution: [.9e-37, -.10e-36, -.125e-34]Solution in 0.419s

Time Plot 0 s.
Exiting SolveHard() after 0.734r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52, none,
361.50888347412682265714178534645684298 56, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110547322545696596010423938283110,
6.19626256540082758945155762364348839 8813,
385.44474379650620727876622448535603154 78]
one interval r = 31.60836097540258087483905518741001355557 ..
34.66372795609908543289996448565672321759
Time Approximations 0.017.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, .88e-35]Solution in 0.614s

Time Plot 0 s.
```

```
Exiting SolveHard() after 5.348r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52, none,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110547322545696596010423938283110,
6.19626256540082758945155762364348839881 3,
385.44474379650620727876622448535603154 78]
two intervals r = 16.87563408759159457848119959027592208593 ..
11875000000051668133647329655423494 07/6250000000000000000000000000000000
00 or r = 15.55640493820250892333145603374243445283 ..
11875000000051668133647329655423494 07/6250000000000000000000000000000000
00
Time Approximations 0.062.

hint used Hint := [17.93041369709740979309933253273271318826, 3, 1, 1,
15.70097251556007075830079142443830067567, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.180e-37, 0., -.4075e-34]Solution in 1.18s

Time Plot 0 s.
Exiting SolveHard() after 2.396r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
```

```
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874739721418289855590619300418333,
4.8838107798746015392876614605066635886 56,
376.61967856165020583135090502817864396 62]
one interval r = 21.11001304879406572233458649328671081844 ..
26.31784243482766733574666369918741030501
Time Approximations 0.032.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.195e-34]Solution in 0.835s

Time Plot 0 s.
Exiting SolveHard() after 6.092r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
```

```
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88, none,
328.46938513844874427170427656193021434 87, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874739721418289855590619300418333,
4.8838107798746015392876614605066635886 56,
376.61967856165020583135090502817864396 62]
one interval r = 31.53899497714682933185919037581344680219 ..
34.53618386093911768136731148426547256766
Time Approximations 0.016.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=4.44e-36
Equations at solution: [-.341e-35, .444e-35, -.515e-34]Solution in
0.463s

Time Plot 0 s.
Exiting SolveHard() after 0.759r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88, none,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017549008659175557420324167957581,
6.0258135493598061055772785135326649974 63,
351.42702948764706156676938352373756863 44]
```

```
one interval r = 31.36230206117152850168706535170586674011 ..
34.17446640615871198293912118443183539216
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=6e-38
Equations at solution: [.3e-37, -.6e-37, .68e-35]Solution in 0.543s

Time Plot 0 s.
Exiting SolveHard() after 0.762r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88, none,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51, none, none,
292.99969138497758932078091587900175635 68, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017549008659175557420324167957581,
6.02581354935980610557727851353266499746 3,
351.42702948764706156676938352373756863 44]
two intervals r = 17.98135514447049261568092964155386531489 ..
11875000000051668133647329655423494 07/6250000000000000000000000000000000
00 or r = 13.84608015442182923278472273774482771884 ..
11875000000051668133647329655423494 07/6250000000000000000000000000000000
00
Time Approximations 0.045.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
```

```
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=3e-38
Equations at solution: [-.109e-36, .3e-37, -.35e-35]Solution in 1.175s

Time Plot 0 s.
Exiting SolveHard() after 6.97r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88,
302.31384315251172901859815487181461921 83,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51, none, none,
292.99969138497758932078091587900175635 68, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941886934860424730903931608807588,
6.37794387396876296695142207039203985027 0,
423.28832784829430841445338632651120731 41]
one interval r = 31.946618176056499688009162481193472965 51 ..
35.212123086581561580430748440411281770 51
Time Approximations 0.019.

hint used Hint := [34.327159798933221351566072928006859218 91, 3, 1, 1,
11.395769599984320296717245861504673207 76, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
```

```
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .100e-34]Solution in 0.62s

Time Plot 0 s.
Exiting SolveHard() after 0.983r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88,
302.31384315251172901859815487181461921 83,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51, none, none,
292.99969138497758932078091587900175635 68, none, none,
360.06173467459702927348842625163952966 24, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418869348604247309039316088075 88,
6.3779438739687629669514220703920398502 70,
423.28832784829430841445338632651120731 41]
two intervals r = 15.22886702416184060645715130803256136788 ..
11875000000051668133647329655423494 07/62500000000000000000000000000000000
00 or r = 17.12965777107208002867925842028105786814 ..
11875000000051668133647329655423494 07/62500000000000000000000000000000000
00
Time Approximations 0.062.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=16.5334, rm=15.6907} with Delta=2e-38
Equations at solution: [-.47e-37, -.2e-37, -.2152e-34]Solution in
```

```
1.308s

Time Plot 0 s.
Exiting SolveHard() after 7.006r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88,
302.31384315251172901859815487181461921 83,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51,
375.73285291067470669118201649193976568 07, none,
292.99969138497758932078091587900175635 68, none, none,
360.06173467459702927348842625163952966 24, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343450290963329487241676177775 31,
4.0035598154243005264748387075570127618 29,
404.47973594569468771934337721553513342 98]
two intervals r = 16.096839663665669552327104015658236049 31 ..
11875000000051668133647329655423494 07/62500000000000000000000000000000000
00 or r = 16.399886491386067747792658126793356303 20 ..
11875000000051668133647329655423494 07/62500000000000000000000000000000000
00
Time Approximations 0.058.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [-.17e-37, 0., .4482e-34]Solution in 5.537s

Time Plot 0 s.
```

```
Exiting SolveHard() after 6.598r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88,
302.31384315251172901859815487181461921 83,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51,
375.73285291067470669118201649193976568 07, none,
292.99969138497758932078091587900175635 68,
358.64341561130944809646229660547904767 03, none,
360.06173467459702927348842625163952966 24, none, none, none, none,
none, none, none]

1 --> 2  target = [34.939532343450290963329487241676177775 31,
4.0035598154243005264748387075570127618 29,
404.47973594569468771934337721553513342 98]
one interval r = 21.634296299951712139630607999700597582 61 ..
26.757681699018888019168830581848659104 12
Time Approximations 0.055.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=5.1e-38
Equations at solution: [.1e-37, .51e-37, .122e-34]Solution in 1.129s

Time Plot 0 s.
Exiting SolveHard() after 2.198r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
```

```
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88,
302.31384315251172901859815487181461921 83,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51,
375.73285291067470669118201649193976568 07,
328.11709294666580645884386342126173291 92,
292.99969138497758932078091587900175635 68,
358.64341561130944809646229660547904767 03, none,
360.06173467459702927348842625163952966 24, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954465616857554093467212201745576,
6.19617723025465537470130411243560383125 6,
385.42734026032001633692153741089860447 50]
one interval r = 31.60822049095673325103733976561975602533 ..
34.66347615047660639759873659558027458069
Time Approximations 0.015.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [.2e-37, -.3e-37, -.178e-34]Solution in 0.551s

Time Plot 0 s.
Exiting SolveHard() after 5.5r=33.8134 in [32.62668594 .. 34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
```

```
441.642959738982336494659341682924180521 2,
436.917481658042694401467286616715367375 3,
422.984933982993787483439410653330606768 1,
361.525802563686935477834382921524448041 8,
401.881739048172543851528500547912622269 5,
389.590015161700778190688955234315671814 3,
328.469398937720165963306648063986594576 8,
401.507571585415719234722887737848977218 8,
358.973628243077149305877845791134285924 8,
398.331471049109569797175126276140026486 0,
371.483873944503633545343321829543603645 2,
336.612158415786278584580812580931677728 2,
361.508883474126822657141785346456842985 6,
324.671449927499380519034841562646773338 8,
302.313843152511729018598154871814619218 3,
328.469385138448744271704276561930214348 7,
343.813406249641817892914391642163055285 1,
375.732852910674706691182016491939765680 7,
328.117092946665806458843863421261732919 2,
292.999691384977589320780915879001756356 8,
358.643415611309448096462296605479047670 3, none,
360.061734674597029273488426251639529662 4, none, none,
324.655212237345507164652015175085303492 1, none, none, none, none]

0 --> 1  target = [26.463189544656168575540934672122017455 76,
6.196177230254655374701304112435603831256,
385.427340260320016336921537410898604475 0]
two intervals r = 16.876296002975600462996929502025009681 29 ..
118750000000516681336473296554234940 7/625000000000000000000000000000000
00 or r = 15.555590006691008842955907489133163373 83 ..
118750000000516681336473296554234940 7/625000000000000000000000000000000
00
Time Approximations 0.064.

hint used Hint := [17.930920830435909701694400819009216331 81, 3, 1, 1,
15.700860627013397511253157405159644880 12, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [-.896e-37, .1e-37, -.333e-35]Solution in 1.204s

Time Plot 0 s.
Exiting SolveHard() after 2.345r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434943707262870697463069781856124 2,
441.642959738982336494659341682924180521 2,
```

```
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88,
302.31384315251172901859815487181461921 83,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51,
375.73285291067470669118201649193976568 07,
328.11709294666580645884386342126173291 92,
292.99969138497758932078091587900175635 68,
358.64341561130944809646229660547904767 03, none,
360.06173467459702927348842625163952966 24,
336.59441032628307046329494200420008535 23, none,
324.65521223734550716465201517508530349 21, none, none, none, none]

0 --> 2  target = [34.49522661161652331708141148328340095149,
3.8971313158257188526023193105195394902 24,
373.78081884519547240933369716682714704 31]
two intervals r = 17.297690862386834134147545828046125564 32 ..
11875000000051668133647329655423494 07/6250000000000000000000000000000000
00 or r = 14.994364074371586051177843968476137361 43 ..
11875000000051668133647329655423494 07/6250000000000000000000000000000000
00
Time Approximations 0.098.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [0., 0., -.2179e-34]Solution in 1.279s

Time Plot 0 s.
Exiting SolveHard() after 7.48r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
```

```
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88,
302.31384315251172901859815487181461921 83,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51,
375.73285291067470669118201649193976568 07,
328.11709294666580645884386342126173291 92,
292.99969138497758932078091587900175635 68,
358.64341561130944809646229660547904767 03, none,
360.06173467459702927348842625163952966 24,
336.59441032628307046329494200420008535 23, none,
324.65521223734550716465201517508530349 21,
331.93806791312921440578460716090052681 72, none, none, none]

1 --> 2  target = [34.49522661161652331708141148328340095149,
3.8971313158257188526023193105195394902 24,
373.78081884519547240933369716682714704 31]
one interval r = 21.06068473199914274439301406860976146936 ..
26.26979834286026708430542842946407122639
Time Approximations 0.031.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=3e-38
Equations at solution: [-.1e-37, -.3e-37, -.415e-34]Solution in 0.756s

Time Plot 0 s.
Exiting SolveHard() after 5.718r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
```

```
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88,
302.31384315251172901859815487181461921 83,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51,
375.73285291067470669118201649193976568 07,
328.11709294666580645884386342126173291 92,
292.99969138497758932078091587900175635 68,
358.64341561130944809646229660547904767 03,
299.89866204641406142466370729630750681 70,
360.06173467459702927348842625163952966 24,
336.59441032628307046329494200420008535 23, none,
324.65521223734550716465201517508530349 21,
331.93806791312921440578460716090052681 72, none, none, none]

0 --> 2  target = [33.81362495408046462541744673748118189000,
3.72564899346126668085965352038580269063 3,
325.89209972983185612229316883843801091 80]
two intervals r = 18.552270490142016457479544696608416310 45 ..
11875000000051668133647329655423494 07/62500000000000000000000000000000000
00 or r = 12.491969358000964092400185197907100033 22 ..
11875000000051668133647329655423494 07/62500000000000000000000000000000000
00
Time Approximations 0.043.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=6e-38
Equations at solution: [-.157e-36, .6e-37, -.69e-35]Solution in 5.604s

Time Plot 0 s.
Exiting SolveHard() after 7.101r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
```

```
441.642959738982336494659341682924180521 2,
436.917481658042694401467286616715367375 3,
422.984933982993787483439410653330606768 1,
361.525802563686935477834382921524448041 8,
401.881739048172543851528500547912622269 5,
389.590015161700778190688955234315671814 3,
328.469398937720165963306648063986594576 8,
401.507571585415719234722887737848977218 8,
358.973628243077149305877845791134285924 8,
398.331471049109569797175126276140026486 0,
371.483873944503633545343321829543603645 2,
336.612158415786278584580812580931677728 2,
361.508883474126822657141785346456842985 6,
324.671449927499380519034841562646773338 8,
302.313843152511729018598154871814619218 3,
328.469385138448744271704276561930214348 7,
343.813406249641817892914391642163055285 1,
375.732852910674706691182016491939765680 7,
328.117092946665806458843863421261732919 2,
292.999691384977589320780915879001756356 8,
358.643415611309448096462296605479047670 3,
299.898662046414061424663707296307506817 0,
360.061734674597029273488426251639529662 4,
336.594410326283070463294942004200085352 3, none,
324.655212237345507164652015175085303492 1,
331.938067913129214405784607160900526817 2, none, none,
289.545957724612473966936367380489687645 4]

1 --> 2  target = [33.813624954080464625417446737481181890 00,
3.725648993461266680859653520385802690633,
325.892099729831856122293168838438010918 0]
one interval r = 20.374689351030988800077897769858657778 82 ..
25.378921653032680748529278101029037113 67
Time Approximations 0.026.

hint used Hint := [24.339499030253814217461077660681155334 23, 3, -1, 1,
17.272158782512944212267074944284355815 74, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [-.1e-37, -.2e-37, -.186e-34]Solution in 0.606s

Time Plot 0 s.
Exiting SolveHard() after 1.142r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88,
302.31384315251172901859815487181461921 83,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51,
375.73285291067470669118201649193976568 07,
328.11709294666580645884386342126173291 92,
292.99969138497758932078091587900175635 68,
358.64341561130944809646229660547904767 03,
299.89866204641406142466370729630750681 70,
360.06173467459702927348842625163952966 24,
336.59441032628307046329494200420008535 23,
256.10753185993313687636140969994541126 58,
324.65521223734550716465201517508530349 21,
331.93806791312921440578460716090052681 72, none, none,
289.54595772461247396693636738048968764 54]

1 --> 0  target = [17.93041369714098011339328408166378095249,
4.6865087020120867392233057445541244626 97,
353.30541095355527312516005797339199252 12]
one interval r = 20.73150479091058955441651786458938934354 ..
25.90675353529460244006329769492951805522
Time Approximations 0.036.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=0
Equations at solution: [0., 0., .190e-34]Solution in 0.673s

Time Plot 0 s.
Exiting SolveHard() after 1.374r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88,
302.31384315251172901859815487181461921 83,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51,
375.73285291067470669118201649193976568 07,
328.11709294666580645884386342126173291 92,
292.99969138497758932078091587900175635 68,
358.64341561130944809646229660547904767 03,
299.89866204641406142466370729630750681 70,
360.06173467459702927348842625163952966 24,
336.59441032628307046329494200420008535 23,
256.10753185993313687636140969994541126 58,
324.65521223734550716465201517508530349 21,
331.93806791312921440578460716090052681 72,
304.79958325834146734378071393684491849 68, none,
289.54595772461247396693636738048968764 54]

2 --> 0  target = [17.93041369714098011339328408166378095249,
4.68650870201208673922330574455412446269 7,
353.30541095355527312516005797339199252 12]
one interval r = 31.37435486996182425839774773085854920375 ..
34.20127520028003585181276552609951193088
Time Approximations 0.018.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=0
Equations at solution: [0., 0., .204e-34]Solution in 0.363s

Time Plot 0 s.
Exiting SolveHard() after 0.626r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
```

```
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494370726287069746306978185612 42,
441.64295973898233649465934168292418052 12,
436.91748165804269440146728661671536737 53,
422.98493398299378748343941065333060676 81,
361.52580256368693547783438292152444804 18,
401.88173904817254385152850054791262226 95,
389.59001516170077819068895523431567181 43,
328.46939893772016596330664806398659457 68,
401.50757158541571923472288773784897721 88,
358.97362824307714930587784579113428592 48,
398.33147104910956979717512627614002648 60,
371.48387394450363354534332182954360364 52,
336.61215841578627858458081258093167772 82,
361.50888347412682265714178534645684298 56,
324.67144992749938051903484156264677333 88,
302.31384315251172901859815487181461921 83,
328.46938513844874427170427656193021434 87,
343.81340624964181789291439164216305528 51,
375.73285291067470669118201649193976568 07,
328.11709294666580645884386342126173291 92,
292.99969138497758932078091587900175635 68,
358.64341561130944809646229660547904767 03,
299.89866204641406142466370729630750681 70,
360.06173467459702927348842625163952966 24,
336.59441032628307046329494200420008535 23,
256.10753185993313687636140969994541126 58,
324.65521223734550716465201517508530349 21,
331.93806791312921440578460716090052681 72,
304.79958325834146734378071393684491849 68,
323.46169176675809535879940591634439491 72,
289.54595772461247396693636738048968764 54]

Cascade time 170
counts: 28, 28

 Iteration 99

Start Generation 1
1 --> 0  target = [12.00000000004306312483639792830987726900,
6.21701250293571504254363638764751612 2411,
485.54908088961360926812448097653040541 22]
one interval r = 23.40850301628831326833916306527376522959 ..
27.67578046410653900810047925338291446537
Time Approximations 0.046.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
```

```
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=0
Equations at solution: [0., 0., -.12e-35]Solution in 1.09s

Time Plot 0 s.
Exiting SolveHard() after 2.306r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000004306312483639792830987726900,
6.21701250293571504254363638764751612241 1,
485.54908088961360926812448097653040541 22]
one interval r = 32.62814779192691485178941417994975831544 ..
36.10248388925645426916985928074333328352
Time Approximations 0.025.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.62002610443990467537032288025740016973 2, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .57e-35]Solution in 4.889s

Time Plot 0 s.
Exiting SolveHard() after 5.317r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
```

```
2 --> 1  target = [27.52359684461045048722898371841382396832,
6.58343472168372432448865061377600975290 5,
467.78730594994723379914621563111367739 27]
one interval r = 32.41978955641894602586510040626693666998 ..
35.85152417354420896803714098685056597651
Time Approximations 0.021.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=7e-38
Equations at solution: [.8e-37, -.7e-37, -.127e-34]Solution in 0.658s

Time Plot 0 s.
Exiting SolveHard() after 1.029r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32, none, none,
401.88173903388556699432875444119787710 22, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684461045048722898371841382396832,
6.58343472168372432448865061377600975290 5,
467.78730594994723379914621563111367739 27]
two intervals r = 12.92327160836050040292263570563334320035 ..
189999999997443965666334687677879133 43/100000000000000000000000000000000
00000 or r = 18.39424857993082896087590088280359103407 ..
189999999997443965666334687677879133 43/100000000000000000000000000000000
00000
Time Approximations 0.044.

hint used Hint := [14.19258941788061374381621459708876997324, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=5.4e-38
```

```
Equations at solution: [-.5e-37, -.54e-37, -.73e-35]Solution in 48.968s

Time Plot 0 s.
Exiting SolveHard() after 50.42r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02, none,
401.88173903388556699432875444119787710 22, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962808778875608895361040962408502,
4.12565179687725220556758550652158018803 6,
440.67123064237631278112230828541385265 49]
two intervals r = 14.35659705117143407093335641653777471280 ..
189999999997443965666334687677879133 43/100000000000000000000000000000000
00000 or r = 17.70352613767026052235494855227449987511 ..
189999999997443965666334687677879133 43/100000000000000000000000000000000
00000
Time Approximations 0.05.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=3e-38
Equations at solution: [-.43e-37, -.3e-37, .267e-34]Solution in 1.418s

Time Plot 0 s.
Exiting SolveHard() after 2.518r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02, none,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.463229628087788756088953610409624085 02,
4.1256517968772522055675855065215801880 36,
440.671230642376312781122308285413852 6549]
one interval r = 22.39761154342338349042813766786944784021 ..
27.23722351578664312457005557441827267502
Time Approximations 0.037.

hint used Hint := [26.46347110539976775688849031731477848509, 3, -1, 1,
16.53292398122155720971187607751147172520, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.242 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.41507064365643084898375996874206388879, rm =
14.37818770480156280395252641002629257453}});
Accepted {r=26.4635, rm=16.5329} with Delta=2.6e-38
Equations at solution: [0., -.26e-37, .462e-34]Solution in 9.276s

Time Plot 0 s.
Exiting SolveHard() after 14.438r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078887797643263534041834378051715 85,
4.0048690818514493912952253268376025184 57,
404.86224500457388487596888061349010446 84]
two intervals r = 16.08011007756570485864165341495828656616 ..
18999999999744396566633468767787913343/10000000000000000000000000000000
00000 or r = 16.41579812643613966611631616850297534820 ..
18999999999744396566633468767787913343/10000000000000000000000000000000
00000
```

```
Time Approximations 0.078.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.52e-37, .1e-37, -.26e-35]Solution in 1.733s

Time Plot 0 s.
Exiting SolveHard() after 7.423r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93, none, none,
358.97362823161155478677705152469278238 43, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888779764326353404183437805171585,
4.0048690818514493912952253268376025184 57,
404.86224500457388487596888061349010446 84]
one interval r = 21.641943993882698216288416305646692783 3 ..
26.763306600197808200423986384023373075 93
Time Approximations 0.056.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=4.9e-38
Equations at solution: [.1e-37, .49e-37, .342e-34]Solution in 5.72s

Time Plot 0 s.
Exiting SolveHard() after 6.842r=25.8721 in [23.84730094 ..
26.76330661]
```

```
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39, none,
358.97362823161155478677705152469278238 43, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941791943928359582832396393227861,
5.5896371829300028082763200516789549887 46,
443.83065883600621151450804845098772478 23]
one interval r = 22.46725374449626828540751425713585742572 ..
27.27388428333090058362637516772645367086
Time Approximations 0.038.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=8.1e-38
Equations at solution: [-.1e-37, .81e-37, -.118e-34]Solution in 1.043s

Time Plot 0 s.
Exiting SolveHard() after 2.069r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39, none,
358.97362823161155478677705152469278238 43,
398.33147103095386581565206574707567454 2, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.19258941791943928359582832396393227861,
5.589637182930002808276320051678954988746,
443.83065883600621151450804845098772478 23]
one interval r = 32.15575279479674483970536036215759289683 ..
35.50872228718420541175034135907383197409
Time Approximations 0.022.

hint used Hint := [34.93953234341508060366636866791499227215, 3, 1, 1,
13.44293633391471214310583527459503641144, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [-.3e-37, .2e-37, .98e-35]Solution in 0.487s

Time Plot 0 s.
Exiting SolveHard() after 0.863r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.91193136502654545109422818242665729137,
5.187783578509172543880089159005018359116,
408.65773861965111181099171987354049091 55]
one interval r = 21.71840114629785642579261114118160120880 ..
26.81849303491838943195222351259539417145
Time Approximations 0.062.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
```

```
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=1.58e-37
Equations at solution: [.1e-37, .158e-36, .471e-34]Solution in 1.06s

Time Plot 0 s.
Exiting SolveHard() after 6.321r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42, none, none,
361.50888346343316762679839310148233594 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136502654545109422818242665729137,
5.18778357850917254388008915900501835911 6,
408.65773861965111181099171987354049091 55]
one interval r = 31.80828598732556065595495868305927108233 ..
35.00011460028777859167265209325930042583
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=1.2e-37
Equations at solution: [-.12e-36, .12e-36, .181e-34]Solution in 0.432s

Time Plot 0 s.
Exiting SolveHard() after 0.78r=34.4952 in [32.91337941 .. 35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96, none,
361.50888346433316762679839310148233594 51, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110518569895490062578978674815374,
6.19626256541414241167751187547552601864 9,
385.44474378548586378870382095710622882 23]
one interval r = 31.608360975177392758966903084517607636 74 ..
34.663727955920044689145169941921188367 57
Time Approximations 0.021.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, -.252e-34]Solution in 5.247s

Time Plot 0 s.
Exiting SolveHard() after 5.532r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
```

```
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96, none,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110518569895490062578978674815374,
6.1962625654141424116775118754755260186 49,
385.44474378548586378870382095710622882 23]
two intervals r = 16.875634087443456009045308919194010543 79 ..
189999999997443965666334687677879133 43/10000000000000000000000000000000
00000 or r = 15.556404937538263528803762849446121778 12 ..
189999999997443965666334687677879133 43/10000000000000000000000000000000
00000
Time Approximations 0.063.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4683, rm=2.33653} for Delta=36.149
in partial time = 8.566 s
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46834175102937956764528730642222330171, rm
= 2.33653277403167674677459569312788113283 8}});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [0., 0., -.91e-35]Solution in 32.677s

Time Plot 0 s.
Exiting SolveHard() after 33.889r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
```

```
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874724692331112253363979821046726,
4.8838107798323889343167919490258760694 51,
376.61967855035137753774352168927302283 47]
one interval r = 21.11001304859112424544917989648333044474 ..
26.31784243453161144431214054335324503061
Time Approximations 0.032.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=0
Equations at solution: [0., 0., -.420e-34]Solution in 0.888s

Time Plot 0 s.
Exiting SolveHard() after 5.699r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58, none,
328.46938512704697194505303132986568668 79, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874724692331112253363979821046726,
4.8838107798323889343167919490258760694 51,
376.61967855035137753774352168927302283 47]
one interval r = 31.53899497692139082514163489413390459237 ..
34.53618386075191287962600313573713519987
Time Approximations 0.017.
```

```
hint used Hint := [34.0897679959070796784441725040016442615 2, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=3.11e-36
Equations at solution: [-.239e-35, .311e-35, -.84e-35]Solution in
0.496s

Time Plot 0 s.
Exiting SolveHard() after 0.815r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58, none,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017517423557431522010318019935228,
6.0258135493653267459576879334926489637 52,
351.42702947580957714660311237712874156 25]
one interval r = 31.362302060949904238293466254750669085 47 ..
34.174466405952752832788057068342193200 45
Time Approximations 0.016.

hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=8e-38
Equations at solution: [-.5e-37, .8e-37, -.158e-34]Solution in 0.533s

Time Plot 0 s.
Exiting SolveHard() after 0.762r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084678040822433000 2,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58, none,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96, none, none,
292.99969137529729657965411643240224067 14, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.872050175174235574315220103180199352 28,
6.0258135493653267459576879334926489637 52,
351.42702947580957714660311237712874156 25]
two intervals r = 17.981355144294714529971708638009669136 68 ..
189999999997443965666334687677879133 43/1000000000000000000000000000000
00000 or r = 13.846080153673032114701152135765691539 01 ..
189999999997443965666334687677879133 43/1000000000000000000000000000000
00000
Time Approximations 0.043.

hint used Hint := [18.687780860957178516374898837283989935 43, 3, 1, 1,
15.364767191755816971778688874745711183 96, 17.98135512 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.9136, rm=2.7345} for Delta=34.0544
in partial time = 7.409 s
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
```

```
= 3/2 .. 19}, avoid={{r = 18.91357071346675747239414183529864525869, rm
= 2.73450099309545170272459374201220966955 9}});
Accepted {r=18.6878, rm=15.3648} with Delta=2e-38
Equations at solution: [-.55e-37, .2e-37, -.64e-35]Solution in 18.447s

Time Plot 0 s.
Exiting SolveHard() after 24.126r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58,
302.31384313995442195952250222877849978 95,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96, none, none,
292.99969137529729657965411643240224067 14, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941849910785691460157917736968065,
6.37794387394922588237669711003607818478 4,
423.28832782954208450026327374721778065 29]
one interval r = 31.94661817575086320750237333724648127393 ..
35.21212308630962921630033475115398632664
Time Approximations 0.017.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
Accepted {r=34.3272, rm=11.3958} with Delta=8e-38
Equations at solution: [-.7e-37, .8e-37, .101e-34]Solution in 0.594s

Time Plot 0 s.
Exiting SolveHard() after 5.792r=34.3272 in [33.10127385 ..
35.21212310]
```

```
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58,
302.31384313995442195952250222877849978 95,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96, none, none,
292.99969137529729657965411643240224067 14, none, none,
360.06173465821445122606569785148517954 63, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418499107856914601579177369680 65,
6.3779438739492258823766971100360781847 84,
423.28832782954208450026327374721778065 29]
two intervals r = 15.22886702443182665080840332903486224381 ..
189999999997443965666334687677879133 43/100000000000000000000000000000000
00000 or r = 17.12965777021412818993471926780449485063 ..
189999999997443965666334687677879133 43/100000000000000000000000000000000
00000
Time Approximations 0.064.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=17.5154, rm=2.06407} for Delta=34.8889
in partial time = 8.139 s
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={{r = 17.515370541230776384337420518196415861 40, rm
= 2.0640682986946984054241328788368753760 00}});
Accepted {r=16.5334, rm=15.6907} with Delta=1e-38
Equations at solution: [.15e-37, .1e-37, .301e-34]Solution in 36.319s

Time Plot 0 s.
```

```
Exiting SolveHard() after 37.732r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58,
302.31384313995442195952250222877849978 95,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96,
375.73285289066070559066383429203011673 91, none,
292.99969137529729657965411643240224067 14, none, none,
360.06173465821445122606569785148517954 63, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343234359331944590540542622488 26,
4.0035598155509942698559668128861091003 26,
404.47973593148645273461208376257507355 57]
two intervals r = 16.096839663680340468810652090269135089 25 ..
189999999997443965666334687677879133 43/100000000000000000000000000000000
00000 or r = 16.399886490628540193294724705791928855 27 ..
189999999997443965666334687677879133 43/100000000000000000000000000000000
00000
Time Approximations 0.057.

hint used Hint := [17.211114010615119385692084696062301438 11, 2, -1, 1,
16.761511088455824498662664236561001249 06, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.2111, rm=16.7615} with Delta=1e-38
Equations at solution: [-.33e-37, -.1e-37, -.332e-34]Solution in 6.387s

Time Plot 0 s.
Exiting SolveHard() after 7.507r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
```

```
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58,
302.31384313995442195952250222877849978 95,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96,
375.73285289066070559066383429203011673 91, none,
292.99969137529729657965411643240224067 14,
358.64341559998348639671610773400569989 18, none,
360.06173465821445122606569785148517954 63, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234323435933194459054054262248826,
4.0035598155509942698559668128861091003 26,
404.47973593148645273461208376257507355 57]
one interval r = 21.634296299652747244003111841782628529 62 ..
26.757681698693626606780042990597670995 47
Time Approximations 0.056.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=0
Equations at solution: [0., 0., .406e-34]Solution in 1.112s

Time Plot 0 s.
Exiting SolveHard() after 2.166r=25.8653 in [23.83864811 ..
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58,
302.31384313995442195952250222877849978 95,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96,
375.73285289066070559066383429203011673 91,
328.11709293541531932998025127017671164 60,
292.99969137529729657965411643240224067 14,
358.64341559998348639671610773400569989 18, none,
360.06173465821445122606569785148517954 63, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954436739595392744131008922884658,
6.19617723026759317510185770714830031752 5,
385.42734024922313751456210986204811419 50]
one interval r = 31.60822049073093079289683494124488075456 ..
34.66347615029645000133668564720401208390
Time Approximations 0.018.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .269e-34]Solution in 5.237s

Time Plot 0 s.
Exiting SolveHard() after 5.519r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
```

```
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58,
302.31384313995442195952250222877849978 95,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96,
375.73285289066070559066383429203011673 91,
328.11709293541531932998025127017671164 60,
292.99969137529729657965411643240224067 14,
358.64341559998348639671610773400569989 18, none,
360.06173465821445122606569785148517954 63, none, none,
324.65521222825125191453185179491545548 41, none, none, none, none]

0 --> 1  target = [26.46318954436739595392744131008922884658,
6.19617723026759317510185770714830031752 5,
385.42734024922313751456210986204811419 50]
two intervals r = 16.87629600283034962619193305138567182562 ..
189999999997443965666334687677879133 43/10000000000000000000000000000000
00000 or r = 15.55559000602314695021073888640556281200 ..
189999999997443965666334687677879133 43/10000000000000000000000000000000
00000
Time Approximations 0.055.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Rejected {r=18.4687, rm=2.33669} for Delta=36.1487
in partial time = 8.517 s
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={{r = 18.46866852514691807917755755930033792151, rm
= 2.33669042809714709821424100664514092824 1}});
Accepted {r=17.9309, rm=15.7009} with Delta=0
Equations at solution: [.180e-37, 0., -.411e-34]Solution in 31.662s

Time Plot 0 s.
Exiting SolveHard() after 32.774r=17.9309 in [16.87629601 .. 19]
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386581565206574707567454 2,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58,
302.31384313995442195952250222877849978 95,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96,
375.73285289066070559066383429203011673 91,
328.11709293541531932998025127017671164 60,
292.99969137529729657965411643240224067 14,
358.64341559998348639671610773400569989 18, none,
360.06173465821445122606569785148517954 63,
336.59441031437209346172213287810518284 23, none,
324.65521222825125191453185179491545548 41, none, none, none, none]

0 --> 2  target = [34.49522661148260115950477622425804647935,
3.8971313159746327724610874314347128323 84,
373.78081883768196794556484896407242281 51]
two intervals r = 17.297690862102442402816290862840333325 64 ..
189999999997443965666334687677879133 43/10000000000000000000000000000000000
00000 or r = 14.994364073861066588268799814743288862 45 ..
189999999997443965666334687677879133 43/10000000000000000000000000000000000
00000
Time Approximations 0.081.

hint used Hint := [18.059890259455304895973508660914791147 76, 2, -1, 1,
17.068404768465135028650316079952068631 20, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [-.36e-37, 0., .249e-34]Solution in 1.201s

Time Plot 0 s.
Exiting SolveHard() after 6.976r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58,
302.31384313995442195952250222877849978 95,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96,
375.73285289066070559066383429203011673 91,
328.11709293541531932998025127017671164 60,
292.99969137529729657965411643240224067 14,
358.64341559998348639671610773400569989 18, none,
360.06173465821445122606569785148517954 63,
336.59441031437209346172213287810518284 23, none,
324.65521222825125191453185179491545548 41,
331.93806790773684625140446011680696114 88, none, none, none]

1 --> 2  target = [34.49522661148260115950477622425804647935,
3.8971313159746327724610874314347128323 84,
373.78081883768196794556484896407242281 51]
one interval r = 21.06068473186571184136307859591288269071 ..
26.26979834262742295517372377352381955321
Time Approximations 0.039.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=7e-38
Equations at solution: [.3e-37, .7e-37, .221e-34]Solution in 4.921s

Time Plot 0 s.
Exiting SolveHard() after 5.652r=25.3005 in [23.14060343 ..
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
```

```
441.642959723149628414840823335519711099 3,
436.917481644677028515601893652240438733 2,
422.984933965407846790846878040822433000 2,
361.525802553067678634142387591323973826 7,
401.881739033885566994328754441197877102 2,
389.590015150891243281975554560925664399 3,
328.469398926320960708213178890246906273 9,
401.507571571285685695164730560080266137 8,
358.973628231611554786777051524692782384 3,
398.331471030953865881565206574707567454 2,
371.483873936925523008809853323459835359 6,
336.612158403953308044169703639731669182 2,
361.508883463433167626798393101482335945 1,
324.671449918476481695600982894580663905 8,
302.313843139954421959522502228778499789 5,
328.469385127046971945053031329865686687 9,
343.813406241777246454311576738467974669 6,
375.732852890660705590663834292030116739 1,
328.117092935415319329980251270176711646 0,
292.999691375297296579654116432402240671 4,
358.643415599983486396716107734005699891 8,
299.898662041384261679717818564706900273 9,
360.061734658214451226065697851485179546 3,
336.594410314372093461722132878105182842 3, none,
324.655212228251251914531851794915455484 1,
331.938067907736846251404460116806961148 8, none, none, none]

0 --> 2  target = [33.813624953904366713066047785474218137 40,
3.725648993603431703662226661646978973176,
325.892099720833831489180741244338877190 2]
two intervals r = 18.552270489865183887266069520102823880 91 ..
189999999997443965666334687677879133 43/100000000000000000000000000000000
00000 or r = 12.491969357420855029806999511877487346 93 ..
189999999997443965666334687677879133 43/100000000000000000000000000000000
00000
Time Approximations 0.045.

hint used Hint := [18.854629705150053956301497983584903567 95, 2, -1, 1,
16.566705844729602590090281135302151805 39, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=3e-38
Equations at solution: [.70e-37, -.3e-37, -.302e-34]Solution in 1.171s

Time Plot 0 s.
Exiting SolveHard() after 2.62r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58,
302.31384313995442195952250222877849978 95,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96,
375.73285289066070559066383429203011673 91,
328.11709293541531932998025127017671164 60,
292.99969137529729657965411643240224067 14,
358.64341559998348639671610773400569989 18,
299.89866204138426167971781856470690027 39,
360.06173465821445122606569785148517954 63,
336.59441031437209346172213287810518284 23, none,
324.65521222825125191453185179491545548 41,
331.93806790773684625140446011680696114 88, none, none,
289.54595771798041457324262975489693191 46]

1 --> 2  target = [33.81362495390436671306604778547421813740,
3.72564899360343170366222666164697897 3176,
325.89209972083383148918074124433887719 02]
one interval r = 20.37468935094083592593673722882305655022 ..
25.37892165276589362452587434647771157421
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=2e-38
Equations at solution: [.1e-37, .2e-37, -.263e-34]Solution in 0.58s

Time Plot 0 s.
Exiting SolveHard() after 5.089r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58,
302.31384313995442195952250222877849978 95,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96,
375.73285289066070559066383429203011673 91,
328.11709293541531932998025127017671164 60,
292.99969137529729657965411643240224067 14,
358.64341559998348639671610773400569989 18,
299.89866204138426167971781856470690027 39,
360.06173465821445122606569785148517954 63,
336.59441031437209346172213287810518284 23,
256.10753185362838623632805644647435251 86,
324.65521222825125191453185179491545548 41,
331.93806790773684625140446011680696114 88, none, none,
289.54595771798041457324262975489693191 46]

1 --> 0  target = [17.93041369696463424854601088284472583132,
4.68650870196398000831340972720355641729 8,
353.30541094186594760979735872882430602 59]
one interval r = 20.73150479073936632066174357348674921967 ..
25.90675353498218413377367971903377005711
Time Approximations 0.032.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=2.3e-38
Equations at solution: [-.1e-37, -.23e-37, -.29e-35]Solution in 0.66s

Time Plot 0 s.
Exiting SolveHard() after 1.368r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58,
302.31384313995442195952250222877849978 95,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96,
375.73285289066070559066383429203011673 91,
328.11709293541531932998025127017671164 60,
292.99969137529729657965411643240224067 14,
358.64341559998348639671610773400569989 18,
299.89866204138426167971781856470690027 39,
360.06173465821445122606569785148517954 63,
336.59441031437209346172213287810518284 23,
256.10753185362838623632805644647435251 86,
324.65521222825125191453185179491545548 41,
331.93806790773684625140446011680696114 88,
304.79958324657428022856815037617550236 71, none,
289.54595771798041457324262975489693191 46]

2 --> 0  target = [17.93041369696463424854601088284472583132,
4.68650870196398000831340972720355641729 8,
353.30541094186594760979735872882430602 59]
one interval r = 31.37435486974048632111346732140674283989 ..
34.20127520007696933585254770780438211282
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [.6e-37, -.8e-37, .388e-34]Solution in 0.357s

Time Plot 0 s.
Exiting SolveHard() after 0.629r=33.7963 in [32.25770943 ..
34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
```

```
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343492783583924865650513578202997 05,
441.64295972314962841484082333551971109 93,
436.91748164467702851560189365224043873 32,
422.98493396540784679084687804082243300 02,
361.52580255306767863414238759132397382 67,
401.88173903388556699432875444119787710 22,
389.59001515089124328197555456092566439 93,
328.46939892632096070821317889024690627 39,
401.50757157128568569516473056008026613 78,
358.97362823161155478677705152469278238 43,
398.33147103095386588156520657470756745 42,
371.48387393692552300880985332345983535 96,
336.61215840395330804416970363973166918 22,
361.50888346343316762679839310148233594 51,
324.67144991847648169560098289458066390 58,
302.31384313995442195952250222877849978 95,
328.46938512704697194505303132986568668 79,
343.81340624177724645431157673846797466 96,
375.73285289066070559066383429203011673 91,
328.11709293541531932998025127017671164 60,
292.99969137529729657965411643240224067 14,
358.64341559998348639671610773400569989 18,
299.89866204138426167971781856470690027 39,
360.06173465821445122606569785148517954 63,
336.59441031437209346172213287810518284 23,
256.10753185362838623632805644647435251 86,
324.65521222825125191453185179491545548 41,
331.93806790773684625140446011680696114 88,
304.79958324657428022856815037617550236 71,
323.46169175871152884167063038285401469 75,
289.54595771798041457324262975489693191 46]

Cascade time 285.039
counts: 28, 28

 Iteration 100

Start Generation 1
1 --> 0  target = [12.00000000009745708705064127549949285800,
6.21701250272860367035801702252157145346 0,
485.54908090564274276396627408194754704 21]
one interval r = 23.40850301674022875078902437788767618007 ..
27.67578046442331439360798038228945450700
Time Approximations 0.043.

hint used Hint := [27.52359684479996584152978595423608112768, 2, -1,
-1, 6.49211140307355546658251270388993937 8594, 25.56992694 ..
27.67578046, 3/2 .. 12., 1]
I search for an scattering ray on same branch with sv>1 (1.50031) |   P
 <--- S
rGuessMin=23.4085   rGuessMax=27.5236   rmGuess=6.49211  k=-143.44
scos=299.535
```

```
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.6758, rm=6.49211}, {r = 25.56992694 ..
27.67578046, rm = 3/2 .. 12.}, avoid={});
Accepted {r=27.5236, rm=6.49211} with Delta=1.32e-37
Equations at solution: [-.4e-37, .132e-36, -.10e-35]Solution in 1.04s

Time Plot 0 s.
Exiting SolveHard() after 2.241r=27.5236 in [25.56992694 ..
27.67578046]
Scattering ray (rm=6.49211) in [3/2 .. 12.]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [12.00000000009745708705064127549949285800,
6.2170125027286036703580170225215714534 60,
485.54908090564274276396627408194754704 21]
one interval r = 32.628147792278785461161508248006974977 02 ..
36.10248388953458282079039326428835403508
Time Approximations 0.021.

hint used Hint := [35.46322962825379040215121974254598304878, 3, 1, 1,
9.6200261044399046753703228802574001697 32, 33.94922194 .. 36.10248389,
3/2 .. 12., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.828638) |   P  <--- S
rGuessMin=32.6281   rGuessMax=35.4632   rmGuess=9.62003  k=403.284
scos=-158.271
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=36.1025, rm=9.62003}, {r = 33.94922194 ..
36.10248389, rm = 3/2 .. 12.}, avoid={});
Accepted {r=35.4632, rm=9.62003} with Delta=0
Equations at solution: [0., 0., -.6e-36]Solution in 0.6s

Time Plot 0 s.
Exiting SolveHard() after 5.349r=35.4632 in [33.94922194 ..
36.10248389]
Scattering ray (rm=9.62003) in [3/2 .. 12.]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 2
```

```
2 --> 1  target = [27.52359684494833115857561045778366636045,
6.58343472138436956408063014451959150084 4,
467.78730596633716999737834734684350327 74]
one interval r = 32.41978955677321251477323546257629376874 ..
35.85152417383690046883193078592324932526
Time Approximations 0.022.

hint used Hint := [34.94507888797729914637569990804457214741, 3, 1, 1,
10.93651821651644642196909246079097015868, 33.70078237 .. 35.85152418,
3/2 .. 27.52359685, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.576367) |   P  <--- S
rGuessMin=32.4198   rGuessMax=34.9451   rmGuess=10.9365  k=689.037
scos=-706.35
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.8515, rm=10.9365}, {r = 33.70078237 ..
35.85152418, rm = 3/2 .. 27.52359685}, avoid={});
Accepted {r=34.9451, rm=10.9365} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .204e-34]Solution in 0.65s

Time Plot 0 s.
Exiting SolveHard() after 1.019r=34.9451 in [33.70078237 ..
35.85152418]
Scattering ray (rm=10.9365) in [3/2 .. 27.52359685]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47, none, none,
401.88173905293347191692442090859043238 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 1  target = [27.52359684494833115857561045778366636045,
6.58343472138436956408063014451959150084 4,
467.78730596633716999737834734684350327 74]
```

"Imaginary part neglected: ", $1.1031121149100328835762296022560208589 94 \times 10^{-17}$

```
two intervals r = 12.92327160839834409014299437984536954743 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000
000 or r = 18.39424858058648163986950179833197255740 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [14.192589417880613743816214597088769973 24, 2, 1, 1,
14.13900773377946677696818354340356081958, 12.92327158 .. 18.68550893,
3/2 .. 19, 1]
I search for an scattering ray on same branch with sv<0 (-0.315768) |
S  ---> P
rGuessMin=18.3942   rGuessMax=14.1926   rmGuess=14.139  k=217.686
scos=281.304
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=18.6855, rm=14.139}, {r = 12.92327158 ..
18.68550893, rm = 3/2 .. 19}, avoid={});
Accepted {r=14.1926, rm=14.139} with Delta=3.3e-38
Equations at solution: [.3e-37, .33e-37, .1571e-34]Solution in 49.036s

Time Plot 0 s.
Exiting SolveHard() after 50.467r=14.1926 in [12.92327158 ..
18.68550893]
Scattering ray (rm=14.139) in [3/2 .. 19]: target and source on the
same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94, none,
401.88173905293347191692442090859043238 33, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]

0 --> 2  target = [35.46322962841386402868659521626829139938,
4.1256517968753483963086902751755341552 07,
440.67123066013431530293354386262545702 73]
```

"Imaginary part neglected: ", $1.10311211491003288357622960225602085899 4 \times 10^{-17}$

```
two intervals r = 14.35659705115924685494651928853015755570 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000
000 or r = 17.70352613847010936819168260449124749036 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000
000
Time Approximations 0.054.

hint used Hint := [15.91193136510569595137770698583432131326, 2, -1, 1,
15.84478473618636498341215543281814303559, 14.35659706 .. 18.96093397,
3/2 .. 19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.100126) |
 S  ---> P
rGuessMin=17.7035   rGuessMax=15.9119   rmGuess=15.8448  k=-503.657
scos=74.4631
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=18.9609, rm=15.8448}, {r = 14.35659706 ..
18.96093397, rm = 3/2 .. 19}, avoid={});
Accepted {r=15.9119, rm=15.8448} with Delta=5e-38
Equations at solution: [.98e-37, .5e-37, -.764e-35]Solution in 1.381s

Time Plot 0 s.
Exiting SolveHard() after 2.493r=15.9119 in [14.35659706 ..
18.96093397]
Scattering ray (rm=15.8448) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94, none,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

1 --> 2  target = [35.46322962841386402868659521626829139938,
4.1256517968753483963086902751755341552 07,
440.67123066013431530293354386262545702 73]
one interval r = 22.397611543884861265009498447308286200 65 ..
27.237223516167376500797958681051202069 84
Time Approximations 0.041.

hint used Hint := [26.463471105399767756888490317314778485 09, 3, -1, 1,
16.532923981221557209711876077511471725 20, 24.64256576 .. 27.23722351,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.422652) |   S  ---> P
rGuessMin=22.3976   rGuessMax=26.4635   rmGuess=16.5329  k=-698.357
scos=-667.307
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={});
Rejected {r=26.4151, rm=14.3782} for Delta=5.35408
in partial time = 1.254 s
(Scattering) fsolve(eqs, {r=27.2372, rm=16.5329}, {r = 24.64256576 ..
27.23722351, rm = 3/2 .. 28}, avoid={{r =
26.415070644008519720903313124642094008 80, rm =
14.378187701411384265201169911762182875 48}});
Accepted {r=26.4635, rm=16.5329} with Delta=1.85e-37
Equations at solution: [-.1e-37, -.185e-36, -.174e-34]Solution in
9.303s

Time Plot 0 s.
Exiting SolveHard() after 14.617r=26.4635 in [24.64256576 ..
27.23722351]
Scattering ray (rm=16.5329) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]

Start Generation 3
0 --> 2  target = [34.945078888161108500053711425299092884 66,
```

```
4.004869081856547194409842681311083140488,
404.86224502378304541651104033018695564 78]
```

"Imaginary part neglected: ", $1.10311211491003288357622960225602085899 4 \times 10^{-17}$

```
two intervals r = 16.08011007755356438290465066483149704812 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000 or r = 16.41579812744455621259308115980899721776 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000
Time Approximations 0.062.

hint used Hint := [17.19898874735628255923651729245938270093, 2, -1, 1,
16.75492183170567124243303155678476101688, 16.08011004 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0257633) |
  S  ---> P
rGuessMin=16.4158   rGuessMax=17.199   rmGuess=16.7549  k=-511.6
scos=232.423
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7549}, {r = 16.08011004 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.199, rm=16.7549} with Delta=1e-38
Equations at solution: [.35e-37, .1e-37, .41e-36]Solution in 6.132s

Time Plot 0 s.
Exiting SolveHard() after 7.287r=17.199 in [16.08011004 .. 19]
Scattering ray (rm=16.7549) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11, none, none,
358.97362825276669573792411084973577570 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2  target = [34.94507888816110850005371142529909288466,
4.004869081856547194409842681311083140488,
404.86224502378304541651104033018695564 78]
one interval r = 21.64194399432704901495330550411046097852 ..
26.76330660063645034350727872407268870969
Time Approximations 0.053.

hint used Hint := [25.87205017540310520941769595514298310090, 3, -1, 1,
16.77671839080472370324390883410274606898, 23.84730094 .. 26.76330661,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420199) |   S  ---> P
rGuessMin=21.6419   rGuessMax=25.8721   rmGuess=16.7767  k=-706.355
```

```
scos=-612.983
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7633, rm=16.7767}, {r = 23.84730094 ..
26.76330661, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8721, rm=16.7767} with Delta=2.3e-38
Equations at solution: [.1e-37, .23e-37, -.121e-34]Solution in 1.115s

Time Plot 0 s.
Exiting SolveHard() after 2.16r=25.8721 in [23.84730094 .. 26.76330661]
Scattering ray (rm=16.7767) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26, none,
358.97362825276669573792411084973577570 60, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 0  target = [14.19258941790152050256641749628767384886,
5.5896371827467035300878890039791440299 71,
443.83065885379789719010446730859423855 60]
one interval r = 22.46725374496156562721658873611112239431 ..
27.27388428370861045855766833850926181936
Time Approximations 0.036.

hint used Hint := [27.02037941872078312796022169828016252717, 2, -1,
-1, 13.57592144649376192738249951229692762748, 24.71083344 ..
27.27388429, 3/2 .. 14.19258939, 1]
I search for an scattering ray on same branch with sv>1 (1.09677) |   P
 <--- S
rGuessMin=22.4673   rGuessMax=27.0204   rmGuess=13.5759  k=-299.351
scos=245.408
branch  ingoing at target, Clockwise
(Scattering) fsolve(eqs, {r=27.2739, rm=13.5759}, {r = 24.71083344 ..
27.27388429, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=27.0204, rm=13.5759} with Delta=2.7e-38
Equations at solution: [.1e-37, -.27e-37, .168e-34]Solution in 0.979s

Time Plot 0 s.
Exiting SolveHard() after 6.966r=27.0204 in [24.71083344 ..
27.27388429]
Scattering ray (rm=13.5759) in [3/2 .. 14.19258939]: target and source
on the same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
```

```
441.642959738848635079965679957703182644 7,
436.917481662314399545971295291830363054 7,
422.984933982920437563403716980644181059 4,
361.525802573733493929065373069319444558 8,
401.881739052933471916924420908590432383 3,
389.590015170591340534810205658971584641 1,
328.469398948264660857484029586406404452 6, none,
358.973628252766695737924110849735775706 0,
398.331471049001975924724840576893400556 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

2 --> 0  target = [14.192589417901520502566417496287673848 86,
5.589637182746703530087889003979144029971,
443.830658853797897190104467308594238556 0]
one interval r = 32.155752795161268569293217144551887404 57 ..
35.508722287509205367083093465640201755 30
Time Approximations 0.021.

hint used Hint := [34.939532343415080603666368667914992272 15, 3, 1, 1,
13.442936333914712143105835274595036411 44, 33.37332721 .. 35.50872230,
3/2 .. 14.19258939, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.894037) |   P  <--- S
rGuessMin=32.1558   rGuessMax=34.9395   rmGuess=13.4429  k=500.498
scos=58.9797
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.5087, rm=13.4429}, {r = 33.37332721 ..
35.50872230, rm = 3/2 .. 14.19258939}, avoid={});
Accepted {r=34.9395, rm=13.4429} with Delta=2e-38
Equations at solution: [.3e-37, -.2e-37, .78e-35]Solution in 0.458s

Time Plot 0 s.
Exiting SolveHard() after 0.842r=34.9395 in [33.37332721 ..
35.50872230]
Scattering ray (rm=13.4429) in [3/2 .. 14.19258939]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.163434943684490158785589424844992182 7,
441.642959738848635079965679957703182644 7,
436.917481662314399545971295291830363054 7,
422.984933982920437563403716980644181059 4,
361.525802573733493929065373069319444558 8,
401.881739052933471916924420908590432383 3,
389.590015170591340534810205658971584641 1,
328.469398948264660857484029586406404452 6,
401.507571591221969274001182632467242279 9,
358.973628252766695737924110849735775706 0,
398.331471049001975924724840576893400556 3, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

1 --> 0  target = [15.911931364965882623086493937600984717 20,
5.187783578347000057729995030974433171368,
```

```
408.65773863972805027437577888340453874 59]
one interval r = 21.71840114676610628425458369498324663054 ..
26.81849303536540319261594664948785598229
Time Approximations 0.062.

hint used Hint := [26.46318954458146650656332304152399577898, 3, -1, 1,
15.90128515414341158765710109686370821521, 23.93303356 .. 26.81849303,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.8876) |   P  <--- S
rGuessMin=21.7184   rGuessMax=26.4632   rmGuess=15.9013  k=-384.251
scos=185.616
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.8185, rm=15.9013}, {r = 23.93303356 ..
26.81849303, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=26.4632, rm=15.9013} with Delta=2.7e-38
Equations at solution: [-.1e-37, -.27e-37, -.272e-34]Solution in 5.437s

Time Plot 0 s.
Exiting SolveHard() after 6.533r=26.4632 in [23.93303356 ..
26.81849303]
Scattering ray (rm=15.9013) in [3/2 .. 15.91193137]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63, none, none,
361.50888348392922428777190542878270079 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 0  target = [15.91193136496588262308649393760098471720,
5.18778357834700005772999503097443317 1368,
408.65773863972805027437577888340453874 59]
one interval r = 31.80828598769799128705229578120106656978 ..
35.00011460066221668744452529908520379526
Time Approximations 0.019.

hint used Hint := [34.49522661165008082072914924801977531221, 3, 1, 1,
15.76385589023872534302387520454213741716, 32.91337941 .. 35.00011460,
3/2 .. 15.91193137, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.96562) |   P  <--- S
rGuessMin=31.8083   rGuessMax=34.4952   rmGuess=15.7639  k=512.772
scos=217.311
branch  outgoing at target, Counterclockwise
```

```
(Scattering) fsolve(eqs, {r=35.0001, rm=15.7639}, {r = 32.91337941 ..
35.00011460, rm = 3/2 .. 15.91193137}, avoid={});
Accepted {r=34.4952, rm=15.7639} with Delta=1.0e-37
Equations at solution: [-.10e-36, .10e-36, .195e-34]Solution in 0.439s

Time Plot 0 s.
Exiting SolveHard() after 0.756r=34.4952 in [32.91337941 ..
35.00011460]
Scattering ray (rm=15.7639) in [3/2 .. 15.91193137]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69, none,
361.50888348392922428777190542878270079 44, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none]

2 --> 1  target = [26.46347110568212429299630999666838029371,
6.19626256514984348230668972087769664398 9,
385.44474380702434940529987400090947684 59]
one interval r = 31.60836097554765665902340914243388162390 ..
34.66372795632488834266050013578047549307
Time Approximations 0.018.

hint used Hint := [33.81362495408772233776384102941444868670, 3, 1, 1,
11.78302906874267690462414906858869172132, 32.62689490 .. 34.66372796,
3/2 .. 26.46347110, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581737) |   P  <--- S
rGuessMin=31.6084   rGuessMax=33.8136   rmGuess=11.783  k=708.892
scos=-582.197
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6637, rm=11.783}, {r = 32.62689490 ..
34.66372796, rm = 3/2 .. 26.46347110}, avoid={});
Accepted {r=33.8136, rm=11.783} with Delta=9e-38
Equations at solution: [.6e-37, -.9e-37, -.118e-34]Solution in 0.578s

Time Plot 0 s.
Exiting SolveHard() after 0.862r=33.8136 in [32.62689490 ..
34.66372796]
Scattering ray (rm=11.783) in [3/2 .. 26.46347110]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
```

```
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69, none,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

0 --> 1  target = [26.46347110568212429299630999666838029371,
6.19626256514984348230668972087769664398 9,
385.44474380702434940529987400090947684 59]
```

"Imaginary part neglected: ", $1.10311211491003288357622960225602085899 4 \times 10^{-17}$

```
two intervals r = 16.875634087396731433052916320974340988 59 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000 or r = 15.556404938733662745252835745490047518 85 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000
Time Approximations 0.069.

hint used Hint := [17.930413697097409793099332532732713188 26, 3, 1, 1,
15.700972515560070758300791424438300675 67, 16.87563409 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.198546) |   S  ---> P
rGuessMin=15.5564   rGuessMax=17.9304   rmGuess=15.701  k=421.37
scos=147.947
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.701}, {r = 16.87563409 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9304, rm=15.701} with Delta=0
Equations at solution: [.895e-37, 0., -.258e-35]Solution in 5.467s

Time Plot 0 s.
Exiting SolveHard() after 6.668r=17.9304 in [16.87563409 .. 19]
Scattering ray (rm=15.701) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
```

```
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]

Start Generation 4
1 --> 0  target = [17.19898874723201322581899261710351740473,
4.8838107796809334357385055545017032335 09,
376.61967857193528551565853639426899417 42]
one interval r = 21.11001304902194695664863442630878852525 ..
26.31784243503885852751140366944287554313
Time Approximations 0.037.

hint used Hint := [25.87204991206833952374472020470551194263, 3, -1, 1,
16.76106391218669896442664507234311360129, 23.20517308 .. 26.31784245,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.775838) |   P  <--- S
rGuessMin=21.11   rGuessMax=25.872   rmGuess=16.7611  k=-432.176
scos=134.564
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.3178, rm=16.7611}, {r = 23.20517308 ..
26.31784245, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=25.872, rm=16.7611} with Delta=9.8e-38
Equations at solution: [.2e-37, .98e-37, .327e-34]Solution in 0.883s

Time Plot 0 s.
Exiting SolveHard() after 1.597r=25.872 in [23.20517308 .. 26.31784245]
Scattering ray (rm=16.7611) in [3/2 .. 17.19898872]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03, none,
328.46938514898590264051473121464379856 42, none, none, none, none,
```

```
none, none, none, none, none, none, none, none, none, none]

2 --> 0  target = [17.19898874723201322581899261710351740473,
4.883810779680933435738505554501703233509,
376.61967857193528551565853639426899417 42]
one interval r = 31.53899497728540309132780591808370918195 ..
34.53618386116049529421027525502000542937
Time Approximations 0.017.

hint used Hint := [34.08976799590707967844417250400164426152, 3, 1, 1,
17.19898434893881998670258658728864307101, 32.52213872 .. 34.53618387,
3/2 .. 17.19898872, 1]
I search for an scattering ray on opposite branches with sv>1 (1.04453)
|   P  <--- S
rGuessMin=31.539   rGuessMax=34.0898   rmGuess=17.199  k=492.219
scos=332.478
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.5362, rm=17.199}, {r = 32.52213872 ..
34.53618387, rm = 3/2 .. 17.19898872}, avoid={});
Accepted {r=34.0898, rm=17.199} with Delta=1.025e-35
Equations at solution: [-.787e-35, .1025e-34, -.625e-34]Solution in
0.492s

Time Plot 0 s.
Exiting SolveHard() after 0.777r=34.0898 in [32.52213872 ..
34.53618387]
Scattering ray (rm=17.199) in [3/2 .. 17.19898872]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03, none,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1  target = [25.87205017573166103248114265616953208245,
6.025813549114122203401542672368254377071,
351.42702949868839982374607793548873685 18]
one interval r = 31.36230206130120519880059785260435994447 ..
34.17446640638722074670372621844693078698
Time Approximations 0.014.
```

```
hint used Hint := [33.36855355002008388886066412789339491074, 3, 1, 1,
12.14280299456133675952425652682342472144, 32.23723258 .. 34.17446642,
3/2 .. 25.87205019, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.586276) |   P  <--- S
rGuessMin=31.3623   rGuessMax=33.3686   rmGuess=12.1428  k=710.716
scos=-525.954
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.1745, rm=12.1428}, {r = 32.23723258 ..
34.17446642, rm = 3/2 .. 25.87205019}, avoid={});
Accepted {r=33.3686, rm=12.1428} with Delta=3e-38
Equations at solution: [-.2e-37, .3e-37, .463e-34]Solution in 0.546s

Time Plot 0 s.
Exiting SolveHard() after 5.746r=33.3686 in [32.23723258 ..
34.17446642]
Scattering ray (rm=12.1428) in [3/2 .. 25.87205019]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03, none,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73, none, none,
292.99969140052062028276934799385184690 15, none, none, none, none,
none, none, none, none, none, none]

0 --> 1  target = [25.87205017573166103248114265616953208245,
6.02581354911412220340154267236825437707 1,
351.42702949868839982374607793548873685 18]
```

"Imaginary part neglected: ", $1.10311211491003288357622960225602085899 4 \times 10^{-17}$

```
two intervals r = 17.98135514434852823511527517103777315874 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000 or r = 13.84608015503778689879969052306112695810 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000
Time Approximations 0.05.

hint used Hint := [18.68778086095717851637489883728398993543, 3, 1, 1,
15.36476719175581697177868887474571118396, 17.98135512 .. 19, 3/2 ..
```

```
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.281836) |   S  ---> P
rGuessMin=13.8461   rGuessMax=18.6878   rmGuess=15.3648  k=454.38
scos=99.8164
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.3648}, {r = 17.98135512 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.6878, rm=15.3648} with Delta=0
Equations at solution: [0., 0., .870e-35]Solution in 1.17s

Time Plot 0 s.
Exiting SolveHard() after 2.254r=18.6878 in [17.98135512 .. 19]
Scattering ray (rm=15.3648) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73, none, none,
292.99969140052062028276934799385184690 15, none, none, none, none,
none, none, none, none, none, none]

2 --> 1  target = [27.02037941891218198214824948180385429900,
6.37794387366622138979080123116983146985 5,
423.28832784836624211923416606971331474 01]
one interval r = 31.94661817611854634468847456591233699503 ..
35.21212308665939251141447742201913939269
Time Approximations 0.02.

hint used Hint := [34.32715979893322135156607292800685921891, 3, 1, 1,
11.39576959998432029671724586150467320776, 33.10127385 .. 35.21212310,
3/2 .. 27.02037943, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.578366) |   P  <--- S
rGuessMin=31.9466   rGuessMax=34.3272   rmGuess=11.3958  k=702.811
scos=-641.33
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=35.2121, rm=11.3958}, {r = 33.10127385 ..
35.21212310, rm = 3/2 .. 27.02037943}, avoid={});
```

```
Accepted {r=34.3272, rm=11.3958} with Delta=2e-38
Equations at solution: [.2e-37, -.2e-37, -.105e-34]Solution in 4.798s

Time Plot 0 s.
Exiting SolveHard() after 5.103r=34.3272 in [33.10127385 ..
35.21212310]
Scattering ray (rm=11.3958) in [3/2 .. 27.02037943]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73, none, none,
292.99969140052062028276934799385184690 15, none, none,
360.06173467959780448972657057044543569 34, none, none, none, none,
none, none, none]

0 --> 1  target = [27.020379418912181982148249481803854299 00,
6.3779438736662213897908012311698314698 55,
423.28832784836624211923416606971331474 01]
```

"Imaginary part neglected: ", $1.10311211491003288357622960225602085899 4 \times 10^{-17}$

```
two intervals r = 15.22886702439499236556193872732407536722 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000 or r = 17.12965777112582799728017389857487856432 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000
Time Approximations 0.066.

hint used Hint := [16.533366988188305873774173703296096030 12, 3, 1, 1,
15.690733789730791828259197890709496382 94, 15.22886699 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.0394878) |   S  ---> P
rGuessMin=17.1297   rGuessMax=16.5334   rmGuess=15.6907  k=353.537
scos=210.559
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.6907}, {r = 15.22886699 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=16.5334, rm=15.6907} with Delta=0
Equations at solution: [-.15e-37, 0., .1460e-34]Solution in 1.298s

Time Plot 0 s.
Exiting SolveHard() after 2.685r=16.5334 in [15.22886699 .. 19]
Scattering ray (rm=15.6907) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73,
375.73285291075698207789331549186123363 14, none,
292.99969140052062028276934799385184690 15, none, none,
360.06173467959780448972657057044543569 34, none, none, none, none,
none, none, none]

0 --> 2  target = [34.939532343611155513754536892758385584 88,
4.0035598155592258909804080111486646627 80,
404.47973595160390842013869273724728575 55]
```

"Imaginary part neglected: ", $1.10311211491003288357622960225602085899 4 \times 10^{-17}$

```
two intervals r = 16.09683966362951093451420367238146777456 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000 or r = 16.39988649167644062930707565861474273433 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000
Time Approximations 0.057.

hint used Hint := [17.21111401061511938569208469606230143811, 2, -1, 1,
16.76151108845582449866266423656100124906, 16.09683967 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with 0<sv<1 (0.0248847) |
  S  ---> P
rGuessMin=16.3999   rGuessMax=17.2111   rmGuess=16.7615  k=-511.46
scos=233.924
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.7615}, {r = 16.09683967 .. 19, rm
= 3/2 .. 19}, avoid={});
```

```
Accepted {r=17.2111, rm=16.7615} with Delta=0
Equations at solution: [-.18e-37, 0., .305e-35]Solution in 1.53s

Time Plot 0 s.
Exiting SolveHard() after 6.989r=17.2111 in [16.09683967 .. 19]
Scattering ray (rm=16.7615) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73,
375.73285291075698207789331549186123363 14, none,
292.99969140052062028276934799385184690 15,
358.64341562192467270455192194488573866 37, none,
360.06173467959780448972657057044543569 34, none, none, none, none,
none, none, none]

1 --> 2  target = [34.93953234361115551375453689275838558488,
4.0035598155592258909804080111486646627 80,
404.47973595160390842013869273724728575 55]
one interval r = 21.634296300114604703399335239376234985 40 ..
26.757681699146050701078005996300374066 11
Time Approximations 0.056.

hint used Hint := [25.865322800542268484425807212028649375 76, 3, -1, 1,
16.779215258374334038325539120894911366 80, 23.83864811 .. 26.75768170,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.420165) |   S  ---> P
rGuessMin=21.6343   rGuessMax=25.8653   rmGuess=16.7792  k=-706.416
scos=-612.385
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.7577, rm=16.7792}, {r = 23.83864811 ..
26.75768170, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.8653, rm=16.7792} with Delta=1.73e-37
Equations at solution: [.5e-37, .173e-36, .419e-34]Solution in 5.599s

Time Plot 0 s.
Exiting SolveHard() after 6.693r=25.8653 in [23.83864811 ..
```

```
26.75768170]
Scattering ray (rm=16.7792) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73,
375.73285291075698207789331549186123363 14,
328.11709295819483689371259454040143996 65,
292.99969140052062028276934799385184690 15,
358.64341562192467270455192194488573866 37, none,
360.06173467959780448972657057044543569 34, none, none, none, none,
none, none, none]

2 --> 1  target = [26.46318954486101657208351902604758053314,
6.19617723000244104110357654351439735421 0,
385.42734027058701557088841135707654297 68]
one interval r = 31.60822049109977266371161321713358998330 ..
34.66347615069877363988392833447993267361
Time Approximations 0.017.

hint used Hint := [33.81339267510015714746901816118211936913, 3, 1, 1,
11.78320912141352187071943817447189965627, 32.62668594 .. 34.66347615,
3/2 .. 26.46318954, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.581739) |   P  <--- S
rGuessMin=31.6082   rGuessMax=33.8134   rmGuess=11.7832  k=708.893
scos=-582.169
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.6635, rm=11.7832}, {r = 32.62668594 ..
34.66347615, rm = 3/2 .. 26.46318954}, avoid={});
Accepted {r=33.8134, rm=11.7832} with Delta=6e-38
Equations at solution: [.4e-37, -.6e-37, -.208e-34]Solution in 0.589s

Time Plot 0 s.
Exiting SolveHard() after 0.892r=33.8134 in [32.62668594 ..
34.66347615]
Scattering ray (rm=11.7832) in [3/2 .. 26.46318954]: target and source
on the different branches.
```

```
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73,
375.73285291075698207789331549186123363 14,
328.11709295819483689371259454040143996 65,
292.99969140052062028276934799385184690 15,
358.64341562192467270455192194488573866 37, none,
360.06173467959780448972657057044543569 34, none, none,
324.65521225203995464462676748969333766 94, none, none, none, none]

0 --> 1  target = [26.46318954486101657208351902604758053314,
6.19617723000244104110357654351439735421 0,
385.42734027058701557088841135707654297 68]
```

"Imaginary part neglected: ", $1.10311211491003288357622960225602085899 4 \times 10^{-17}$

```
two intervals r = 16.87629600279032982891507663220007583645 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000 or r = 15.55559000721044293422076862765712681119 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000
Time Approximations 0.067.

hint used Hint := [17.93092083043590970169440081900921633181, 3, 1, 1,
15.70086062701339751125315740515964488012, 16.87629601 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.1986) |   S  ---> P
rGuessMin=15.5556   rGuessMax=17.9309   rmGuess=15.7009  k=421.393
scos=147.92
branch  outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=19, rm=15.7009}, {r = 16.87629601 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=17.9309, rm=15.7009} with Delta=1e-38
Equations at solution: [-.179e-37, .1e-37, .3506e-34]Solution in 6.213s

Time Plot 0 s.
Exiting SolveHard() after 7.401r=17.9309 in [16.87629601 .. 19]
```

```
Scattering ray (rm=15.7009) in [3/2 .. 19]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73,
375.73285291075698207789331549186123363 14,
328.11709295819483689371259454040143996 65,
292.99969140052062028276934799385184690 15,
358.64341562192467270455192194488573866 37, none,
360.06173467959780448972657057044543569 34,
336.59441033684045631825607000379294905 38, none,
324.65521225203995464462676748969333766 94, none, none, none, none]

0 --> 2  target = [34.49522661190754521848490914460289710040,
3.8971313159938803097086206389208465683 41,
373.78081886033534801001066294891706880 89]
```

"Imaginary part neglected: ", $1.10311211491003288357622960225602085899 4 \times 10^{-17}$

```
two intervals r = 17.29769086206195429293859156870567775013 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000 or r = 14.99436407515655229173453256664057237563 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000
Time Approximations 0.087.

hint used Hint := [18.05989025945530489597350866091479114776, 2, -1, 1,
17.06840476846513502865031607995206863120, 17.29769086 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.0522555) |
 S  ---> P
rGuessMin=14.9944   rGuessMax=18.0599   rmGuess=17.0684  k=-489.404
scos=341.35
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=17.0684}, {r = 17.29769086 .. 19, rm
= 3/2 .. 19}, avoid={});
Accepted {r=18.0599, rm=17.0684} with Delta=0
Equations at solution: [0., 0., -.1877e-34]Solution in 1.269s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.977r=18.0599 in [17.29769086 .. 19]
Scattering ray (rm=17.0684) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73,
375.73285291075698207789331549186123363 14,
328.11709295819483689371259454040143996 65,
292.99969140052062028276934799385184690 15,
358.64341562192467270455192194488573866 37, none,
360.06173467959780448972657057044543569 34,
336.59441033684045631825607000379294905 38, none,
324.65521225203995464462676748969333766 94,
331.93806793201955598008783583468090585 913, none, none, none]

1 --> 2  target = [34.49522661190754521848490914460289710040,
3.897131315993880309708620638920846568341,
373.78018186033534801001066294891706880 89]
one interval r = 21.06068473230791001542736439152750514481 ..
26.26979834315602758128582330069984899436
Time Approximations 0.036.

hint used Hint := [25.30046634035072136692300883420836359084, 3, -1, 1,
16.97472952764178754508574613836907474818, 23.14060343 .. 26.26979834,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.416878) |   S  ---> P
rGuessMin=21.0607   rGuessMax=25.3005   rmGuess=16.9747  k=-709.872
scos=-563.248
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=26.2698, rm=16.9747}, {r = 23.14060343 ..
26.26979834, rm = 3/2 .. 28}, avoid={});
Accepted {r=25.3005, rm=16.9747} with Delta=1.4e-37
Equations at solution: [-.6e-37, -.14e-36, .209e-34]Solution in 0.806s

Time Plot 0 s.
Exiting SolveHard() after 5.707r=25.3005 in [23.14060343 ..
```

```
26.26979834]
Scattering ray (rm=16.9747) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73,
375.73285291075698207789331549186123363 14,
328.11709295819483689371259454040143996 65,
292.99969140052062028276934799385184690 15,
358.64341562192467270455192194488573866 37,
299.89866206642524957917551088341996147 27,
360.06173467959780448972657057044543569 34,
336.59441033684045631825607000379294905 38, none,
324.65521225203995464462676748969333766 94,
331.93806793201955598008783584680905859 13, none, none, none]

0 --> 2  target = [33.81362495436291251529868461598462663264,
3.72564899363245935499815688400127737526 2,
325.89209974508524590756732975006040641 50]

"Imaginary part neglected: ", 1.10311211491003288357622960225602085899 4 × 10^-17
two intervals r = 18.55227049002040975982385449715382633346 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000 or r = 12.49196935882990321193328416031114805111 ..
95000000000966527663706348732605123 57/50000000000000000000000000000000000
000
Time Approximations 0.049.

hint used Hint := [18.85462970515005395630149798358490356795, 2, -1, 1,
16.56670584472960259009028113530215180539, 18.55227050 .. 19, 3/2 ..
19, 1]
I search for an scattering ray on same branch with sv<0 (-0.206409) |
S  ---> P
rGuessMin=12.492   rGuessMax=18.8546   rmGuess=16.5667  k=-425.512
scos=460.944
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=19, rm=16.5667}, {r = 18.55227050 .. 19, rm
```

```
= 3/2 .. 19}, avoid={});
Accepted {r=18.8546, rm=16.5667} with Delta=2e-38
Equations at solution: [-.69e-37, .2e-37, .1203e-34]Solution in 5.478s

Time Plot 0 s.
Exiting SolveHard() after 6.983r=18.8546 in [18.55227050 .. 19]
Scattering ray (rm=16.5667) in [3/2 .. 19]: target and source on the
same branch.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73,
375.73285291075698207789331549186123363 14,
328.11709295819483689371259454040143996 65,
292.99969140052062028276934799385184690 15,
358.64341562192467270455192194488573866 37,
299.89866206642524957917551088341996147 27,
360.06173467959780448972657057044543569 34,
336.59441033684045631825607000379294905 38, none,
324.65521225203995464462676748969333766 94,
331.93806793201955598008783584680905859 13, none, none,
289.54595774374353966175475885750593639 22]

1 --> 2  target = [33.81362495436291251529868461598462663264,
3.72564899363245935499815688400127737526 2,
325.89209974508524590756732975006040641 50]
one interval r = 20.37468935125218744215230939279965479491 ..
25.37892165337466303970094824707218570494
Time Approximations 0.027.

hint used Hint := [24.33949903025381421746107766068115533423, 3, -1, 1,
17.27215878251294421226707494428435581574, 22.07732228 .. 25.37892164,
3/2 .. 28, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.409254) |   S  ---> P
rGuessMin=20.3747   rGuessMax=24.3395   rmGuess=17.2722  k=-710.181
scos=-481.737
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.3789, rm=17.2722}, {r = 22.07732228 ..
```

```
25.37892164, rm = 3/2 .. 28}, avoid={});
Accepted {r=24.3395, rm=17.2722} with Delta=6e-38
Equations at solution: [-.4e-37, -.6e-37, .222e-34]Solution in 0.588s

Time Plot 0 s.
Exiting SolveHard() after 1.156r=24.3395 in [22.07732228 ..
25.37892164]
Scattering ray (rm=17.2722) in [3/2 .. 28]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73,
375.73285291075698207789331549186123363 14,
328.11709295819483689371259454040143996 65,
292.99969140052062028276934799385184690 15,
358.64341562192467270455192194488573866 37,
299.89866206642524957917551088341996147 27,
360.06173467959780448972657057044543569 34,
336.59441033684045631825607000379294905 38,
256.10753188000484204427455060434001061 59,
324.65521225203995464462676748969333766 94,
331.93806793201955598008783583468090585913, none, none,
289.54595774374353966175475885750593639 22]

1 --> 0  target = [17.93041369700260401958592793901482104046,
4.68650870182447104880800931511863333 6896,
353.30541096499798093482795445438517024 24]
one interval r = 20.73150479113055366786029441029008987983 ..
25.90675353554231047754309117349992796739
Time Approximations 0.037.

hint used Hint := [25.40207153781880397105361734397047382754, 3, -1, 1,
17.00624445476621345302062451224674357053, 22.67806074 .. 25.90675353,
3/2 .. 17.93041370, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.721805) |   P  <--- S
rGuessMin=20.7315   rGuessMax=25.4021   rmGuess=17.0062  k=-453.132
scos=102.222
```

```
branch  outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25.9068, rm=17.0062}, {r = 22.67806074 ..
25.90675353, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=25.4021, rm=17.0062} with Delta=7.1e-38
Equations at solution: [-.3e-37, -.71e-37, .106e-34]Solution in 0.684s

Time Plot 0 s.
Exiting SolveHard() after 1.404r=25.4021 in [22.67806074 ..
25.90675353]
Scattering ray (rm=17.0062) in [3/2 .. 17.93041370]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73,
375.73285291075698207789331549186123363 14,
328.11709295819483689371259454040143996 65,
292.99969140052062028276934799385184690 15,
358.64341562192467270455192194488573866 37,
299.89866206642524957917551088341996147 27,
360.06173467959780448972657057044543569 34,
336.59441033684045631825607000379294905 38,
256.10753188000484204427455060434001061 59,
324.65521225203995464462676748969333766 94,
331.93806793201955598008783584680905859 13,
304.79958326995262183784140174307017312 10, none,
289.54595774374353966175475885750593639 22]

2 --> 0  target = [17.93041369700260401958592793901482104046,
4.68650870182447104880800931511863333689 6,
353.30541096499798093482795445438517024 24]
one interval r = 31.37435487009521750281004897098636010597 ..
34.20127520051459531829668991349695316218
Time Approximations 0.017.

hint used Hint := [33.79632952269190120147214620388869542508, 2, 1, -1,
17.86353406971662964004465842689508192962, 32.25770943 .. 34.20127520,
3/2 .. 17.93041370, 1]
I search for an scattering ray on same branch with sv>1 (1.11221) |   P
```

```
 <--- S
rGuessMin=31.3744   rGuessMax=33.7963   rmGuess=17.8635  k=465.49
scos=399.232
branch  ingoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=34.2013, rm=17.8635}, {r = 32.25770943 ..
34.20127520, rm = 3/2 .. 17.93041370}, avoid={});
Accepted {r=33.7963, rm=17.8635} with Delta=8e-38
Equations at solution: [.6e-37, -.8e-37, -.432e-34]Solution in 4.586s

Time Plot 0 s.
Exiting SolveHard() after 4.84r=33.7963 in [32.25770943 .. 34.20127520]
Scattering ray (rm=17.8635) in [3/2 .. 17.93041370]: target and source
on the same branch.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [462.16343494368449015878558942484499218 27,
441.64295973884863507996567995770318264 47,
436.91748166231439954597129529183036305 47,
422.98493398292043756340371698064418105 94,
361.52580257373349392906537306931944455 88,
401.88173905293347191692442090859043238 33,
389.59001517059134053481020565897158464 11,
328.46939894826466085748402958640640445 26,
401.50757159122196927400118263246724227 99,
358.97362825276669573792411084973577570 60,
398.33147104900197592472484057689340055 63,
371.48387395933230624016876319650484180 69,
336.61215842659987874357148932867937202 14,
361.50888348392922428777190542878270079 44,
324.67144994242808305214129342756808611 03,
302.31384316365598769098270068677621859 62,
328.46938514898590264051473121464379856 42,
343.81340626583631619856420472276206965 73,
375.73285291075698207789331549186123363 14,
328.11709295819483689371259454040143996 65,
292.99969140052062028276934799385184690 15,
358.64341562192467270455192194488573866 37,
299.89866206642524957917551088341996147 27,
360.06173467959780448972657057044543569 34,
336.59441033684045631825607000379294905 38,
256.10753188000484204427455060434001061 59,
324.65521225203995464462676748969333766 94,
331.93806793201955598008783584680905859 13,
304.79958326995262183784140174307017312 10,
323.46169178430247584877546502677374019 22,
289.54595774374353966175475885750593639 22]

Cascade time 171.529
counts: 28, 28
```

```
> H;
```

$$[2.09343615952533803165785378796139906032 0 \times 10^{-15}, \quad 2.29131587463036265005293784239979902147 4 \times 10^{-16}, \tag{89}$$

$1.0398542036793371049171171702380666219 94 \times 10^{-15}$,
$1.3883366602285208330868166452583221290 56 \times 10^{-17}$,
$3.3778430498574027967981251389365545346 80 \times 10^{-16}$,
$2.9508241518390208036763665400933781032 76 \times 10^{-16}$,
$1.4876108630342605510254937512057531502 20 \times 10^{-16}$,
$8.9009869478638230002865622880506079702 86 \times 10^{-16}$,
$8.1884048248709687634982577947394561558 38 \times 10^{-16}$,
$6.7063366593663153402486233899074902545 92 \times 10^{-17}$,
$7.7286158605550290428008623228214491698 57 \times 10^{-16}$,
$1.4488026141735844906784669554993047351 38 \times 10^{-16}$,
$4.7697461528980132499424366181036273398 98 \times 10^{-16}$,
$1.9515994295231985813631197498087426313 54 \times 10^{-15}$,
$1.6444781661794576856992801886658986117 96 \times 10^{-16}$,
$3.9787768285359399692099940326328493577 89 \times 10^{-15}$,
$1.0459262051612830517245212672931838743 79 \times 10^{-16}$,
$1.1892905124725137202441388572348054613 51 \times 10^{-15}$,
$1.0581640680252944994466432385080729709 91 \times 10^{-15}$,
$3.0280578678477578332819884292754208821 71 \times 10^{-16}$,
$4.4241146221285815643906838264610678241 17 \times 10^{-17}$,
$1.5916799816973407216675861377805312091 60 \times 10^{-17}$,
$1.1103795649201627620527095274949164821 13 \times 10^{-15}$,
$1.1462362797830620390651985470169521163 67 \times 10^{-18}$,
$1.1024096975800656259957135131082207276 76 \times 10^{-15}$,
$2.9729068975323267511192771605006563105 71 \times 10^{-15}$,
$2.0877052230365475951793775174445870963 64 \times 10^{-16}$,
$1.0515458310820598781279781407721586002 86 \times 10^{-16}$,
$1.1925169425547398652467966570159644262 47 \times 10^{-16}$,
$4.5149842783395141643965972674664781939 73 \times 10^{-15}$,
$1.0925702496529758114954165508616261883 67 \times 10^{-16}$,
$7.2439541240804785081746571867479537659 73 \times 10^{-17}$,
$1.1485327050940802898967871436883249131 66 \times 10^{-17}$,
$1.7124834796412103879891702761801978048 95 \times 10^{-17}$,
$1.2542842020045100928754496949811329079 59 \times 10^{-15}$,
$8.6662022981864656950717872949542264651 92 \times 10^{-16}$,
$4.6937371519514082053337001233924690427 83 \times 10^{-17}$,
$4.5498267961717891861495447288396169105 56 \times 10^{-16}$,

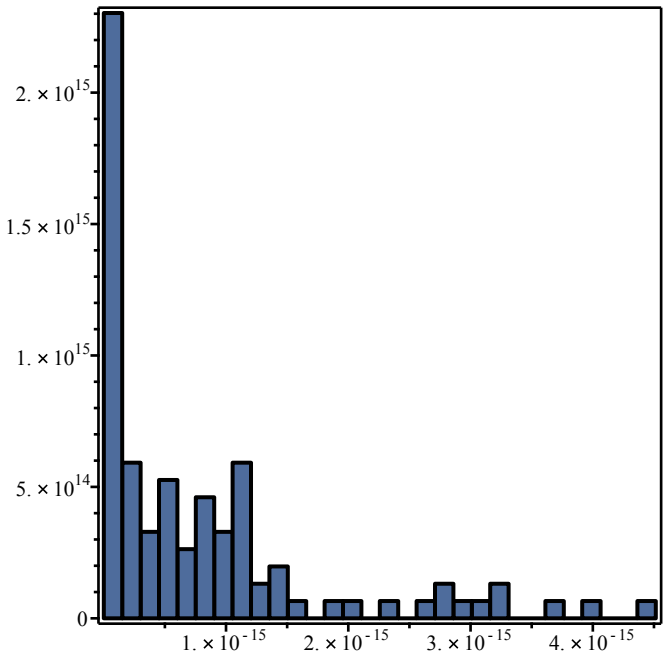
$6.6215651281364197693352887964972958121 09 \times 10^{-16}$,
$2.4079966992040826218863318685629548753 27 \times 10^{-15}$,
$1.2315018788332638577171480701207629233 69 \times 10^{-15}$,
$5.5345926977387800839044334545667820074 38 \times 10^{-16}$,
$8.3276469845524586958238099538700848973 71 \times 10^{-16}$,
$2.6982296619094964375240049882705769980 09 \times 10^{-16}$,
$6.7567655432540339977273038535041034090 21 \times 10^{-16}$,
$8.0817533677658183926628521927737127702 28 \times 10^{-16}$,
$5.4999902487364063662563444686534153705 85 \times 10^{-16}$,
$1.1121042822412235342487683215385783324 78 \times 10^{-15}$,
$1.4361717580024142982346121113172031229 56 \times 10^{-16}$,
$5.0552079056490770241585844173784572193 41 \times 10^{-16}$,
$1.0583893023072744746352448193665304605 26 \times 10^{-15}$,
$7.1992766077578040541784090691266647217 33 \times 10^{-16}$,
$1.2861770229473692281800560118610149830 45 \times 10^{-17}$,
$3.2304854836804271629488224366119891032 02 \times 10^{-15}$,
$9.2334372484944704692689520767019303808 33 \times 10^{-16}$,
$9.5443133663091548235571713419780480449 79 \times 10^{-16}$,
$2.1232702450427988427034270349175504348 87 \times 10^{-16}$,
$3.3764336654455998098631621313039854646 78 \times 10^{-16}$,
$1.0662717035041420794443324595900613138 77 \times 10^{-15}$,
$4.2529727389090578579746398260502377513 24 \times 10^{-16}$,
$1.4810223526740978267490721718062712010 06 \times 10^{-15}$,
$3.0323325815426010272041885962774513150 54 \times 10^{-16}$,
$2.6144547102122318776431843935222029565 09 \times 10^{-18}$,
$6.7015365231550688779652662479872993947 68 \times 10^{-16}$,
$1.8709501295326593607879968465413281683 03 \times 10^{-16}$,
$2.4284788480921412058700109499376492017 03 \times 10^{-18}$,
$2.1577398843249635529188468710090131229 68 \times 10^{-17}$,
$8.7153280025849422350966772345746641909 16 \times 10^{-17}$,
$3.0977270146462730849958236074754974244 41 \times 10^{-15}$,
$1.0250046753862980982541369238661950140 71 \times 10^{-15}$,
$2.6080960891459294907649244919522154716 87 \times 10^{-18}$,
$2.7243703511407609138344184284041473878 28 \times 10^{-15}$,
$1.3576357884684754498848492282441621607 26 \times 10^{-15}$,
$8.7905843866890846130770169431909575936 05 \times 10^{-16}$,

$2.7254316390090695885795147162971861871 91 \times 10^{-16}$,
$2.5059672706455013994195367118895856280 61 \times 10^{-17}$,
$1.3623241719444697246100901264834710365 66 \times 10^{-16}$,
$3.5682746203429165273792810815308388375 66 \times 10^{-17}$,
$6.0106929168338874977522504782729515566 78 \times 10^{-19}$,
$1.1450831761617680562375979653692536655 54 \times 10^{-16}$,
$1.4622760944934620392687838144304239755 10 \times 10^{-15}$,
$4.7529160273169244668220265281712309876 59 \times 10^{-19}$,
$4.7186428119551579725604259364728192799 51 \times 10^{-16}$,
$6.0641431519178415827299261600784332617 52 \times 10^{-17}$,
$5.3038280233252329000988900551063937780 97 \times 10^{-17}$,
$3.2274065302972213057746225143188171717 36 \times 10^{-15}$,
$1.9080826317010445999201578254805627099 81 \times 10^{-17}$,
$4.5830519464002547381718091734163595407 48 \times 10^{-16}$,
$1.0998811783706200258396767481843794772 25 \times 10^{-15}$,
$9.9783853555387710174807706767326578986 91 \times 10^{-16}$,
$6.1639034729827783596965015443769077357 65 \times 10^{-19}$,
$1.0645311153500816403055478230634146748 53 \times 10^{-15}$,
$3.6851711360565283859752668973767506877 54 \times 10^{-18}$,
$2.5612140683572733403715001202945931045 89 \times 10^{-15}$,
$4.1024676866811120752316500786767617557 39 \times 10^{-17}$,
$2.7986213536577969873148154741194357617 61 \times 10^{-15}$,
$1.8410167574878787675951374741395172654 24 \times 10^{-16}$,
$5.7518602591442249095302224291203445780 58 \times 10^{-16}$,
$1.6021244376124963033744697354546953939 61 \times 10^{-15}$,
$7.8562933062908044380048772098093901841 69 \times 10^{-17}$,
$3.7608831197695357214287959803656004882 54 \times 10^{-15}$]

```
> nops(H);
```

$$101 \tag{90}$$

```
> Histogram(H);
```


2. × 10^15
1.5 × 10^15
1. × 10^15
5. × 10^14
0
1. × 10^-15
2. × 10^-15
3. × 10^-15
4. × 10^-15