

```
> restart:
with(plots):
with(plottools):
with(Statistics):
```

```
> # Let us take note of the initial time, just for efficiency stats

mTime:= time():
```

```
> # let us set precision to a number of digits.
```

```
Digits:=40;
digits:=Digits:
```

*Digits := 40*

(1)

```
> # let us fix the value of c and alpha. In homogeous units we have
```

```
cv := 299792458:
MEv:= 597237*10^19:
Gv:=667430*10^(-16):
alv:=2*MEv*Gv/cv^2:
```

```
Surface:= 3189000; # in meter
```

*Surface := 3189000*

(2)

```
> # The Lagrangian for material points in a Schwarzschild
gravitational field (in polar coordinates)
```

```
-(1-al/r)*c^2+dr^2/(1-al/r)+r^2*dte^2:
sqrt(-%):
L:=%;
```

$$L := \sqrt{\left(1 - \frac{al}{r}\right) c^2 - \frac{dr^2}{1 - \frac{al}{r}} - r^2 dte^2}$$

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```
> diff(L, dr):
simplify(%):
radsimp(%) assuming r=3*alv, al=alv:
pr:=%;
```

```
diff(L, dte):
simplify(%):
radsimp(%) assuming r=3*alv, al=alv:
pte:=%;
```

$$pr := - \frac{r^2 dr}{\sqrt{(r^3 dte^2 al - r^4 dte^2 + al^2 c^2 - 2 al c^2 r + c^2 r^2 - dr^2 r^2) r (r - al)}}$$



$$pte := \frac{r^3 (-r + al) dte}{\sqrt{(r^3 dte^2 al - r^4 dte^2 + al^2 c^2 - 2 al c^2 r + c^2 r^2 - dr^2 r^2) r (r - al)}} \quad (4)$$

> # Total energy first integral

pr\*dr+pte\*dte-L:

%^2:

simplify(%):

radsimp(%) assuming r>alv, al=alv:

factor(%):

subs([dr^2=dr2, dte^2=dte2], %):

cons1:= %-c^4\*ep2;

$$cons1 := -c^4 ep2 - \frac{(-r + al)^3 c^4}{(r^3 dte2 al - r^4 dte2 + al^2 c^2 - 2 al c^2 r + c^2 r^2 - dr2 r^2) r} \quad (5)$$

> # Angular momentum first integral

pte^2:

simplify(%):

radsimp(%) assuming r>alv, al=alv:

factor(%):

subs([dr^2=dr2, dte^2=dte2], %):

cons2:=%-k2;

$$cons2 := - \frac{dte2 (-r + al) r^5}{r^3 dte2 al - r^4 dte2 + al^2 c^2 - 2 al c^2 r + c^2 r^2 - dr2 r^2} - k2 \quad (6)$$

> # Weierstrass equations

[cons1, cons2]:

solve(%, [dr2, dte2]):

op(%):

factor(%):

#%;

subs(%, [dr2, dr2/dte2]):

#simplify(%):

wPhi, wPsi := op(%):

wPhi;

wPsi;

$$\frac{(-r + al)^2 (c^2 ep2 r^3 + al r^2 - r^3 + k2 al - k2 r)}{ep2 r^5} \quad (7)$$

$$\frac{(c^2 ep2 r^3 + al r^2 - r^3 + k2 al - k2 r) r}{k2}$$

> # now we need to slow down and put Weierstrass in a smart form  
# so that later we can analytically integrate Weierstrass equations.

# essentially we want to factorize it in first order polynomials

>

# Remember that: -c < ep < 0



```
> numer(wPsi)/r:
collect(%, r):
P:=%;
```

$$P := (c^2 ep2 - 1) r^3 + al r^2 - k2 r + k2 al \quad (8)$$

```
> P + (c^2*ep2-1)*(rp-r)*(r-rm)*(r-r0):
collect(%, r):
[subs(r=0, %), subs(r=0, diff(%, r)), subs(r=0, diff(%, r, r))]:
simplify(%):
solve(%, [ep2, k2, r0]):
op(%):
Soll := %;
```

$$Soll := \left[ ep2 = -\frac{al^2 rm + al^2 rp - al rm^2 - 2 al rm rp - al rp^2 + rm^2 rp + rm rp^2}{c^2 (al rm^2 + al rm rp + al rp^2 - rm^2 rp - rm rp^2)}, k2 = -\frac{al rm^2 rp^2}{al rm^2 + al rm rp + al rp^2 - rm^2 rp - rm rp^2}, r0 = -\frac{al rm rp}{al rm + al rp - rp rm} \right] \quad (9)$$

```
> wPhi:
subs(Soll, %):
simplify(%):
factor(%):
map(collect, %, r):
mPhi := %;
```

$$mPhi := \frac{c^2 (al^2 - 2 al r + r^2) (r - rm) (r - rp) al ((al rm + al rp - rp rm) r + al rm rp)}{r^5 (rm + rp) (al - rp) (al - rm)} \quad (10)$$

```
> wPsi:
subs(Soll, %):
simplify(%):
factor(%):
map(collect, %, r):
mPsi := %;
```

$$mPsi := \frac{(r - rp) r (r - rm) ((al rm + al rp - rp rm) r + al rm rp)}{rm^2 rp^2} \quad (11)$$

```
> (1-al/r)*c^2/mPhi - 1/(1-al/r) - r^2/mPsi:
sqrt(%)/c:
simplify(%) assuming c=cv, al=alv, rm=10*alv, rp=30*alv, r=20*alv:
radsimp(%) assuming c=cv, al=alv, rm=10*alv, rp=30*alv, r=20*alv:
factor(%):
dtau := %;
```

$$dtau := \frac{r^3 |^2 \sqrt{-al rm^2 - al rm rp - al rp^2 + rm^2 rp + rm rp^2}}{\sqrt{-al r rm - al r rp - al rm rp + r rm rp} \sqrt{al} \sqrt{rp - r} \sqrt{r - rm} c} \quad (12)$$

## Utilities

```
> # Utility procedures
```



```

> Real:= proc(x)
local X:
  if type(x, list) then
    map(Real, x):
  else
    X := Im(x):
    if X > 10^(-Digits+2) or X < -10^(-Digits+2) then
      printf("Imaginary part neglected: %a\n", X);
    end if:
    Re(x):
  end if:
end:

```

```

> BranchWith := proc(S, t)
local TSv:
  TSv:= halfT(S):
  t-tS(S, 0):
  subs(r=minS(S), %):
  evalf(%):
  %/TSv:
  evalf(%):
  floor(%):
end:

```

```

> # When solving equations we parameterize t and theta in terms of
# r, which is not monotonic
# thus orbits are broken in branches which need to be glue
# together to get an orbit.
# That for satellites going along bounded orbits (and similarly
# later for light rays) can be done by the following procedure,
# best understood by checking later usage.

```

```

> Branch := proc(fr, branch, F, f0)
  if(floor(branch) mod 2 = 0) then
    fr:
    f0 + % + 2*F*floor(branch/2):
  else
    fr:
    f0 + 2*F*(floor(branch/2)+1) - %:
  end if:
  evalf(%):
end:

```

```

> isOutgoing:=proc(b)
  if b mod 2 = 0 then
    return true:
  end if:
  return false:
end:

```

```

> myRange:= proc(x1, x2)
  if x1<x2 then
    return x1..x2;
  else

```



```

    return x2..x1;
end if:
end:

```

```
> # Utility ciclic permutations
```

```

Cycle:= proc(list)
  local x, n, N, r:
  x:= list[1]:
  N:= nops(list):
  r:= list:
  for n from 1 to N-1 do
    r[n] := list[n+1]:
  end do:
  r[N]:= x:
  r;
end:

```

```

> CycleUntil:=proc(list, n)
  local r:
  if not(n in list) then
    printf("code %d is not in the list %a\n", n, list);
    return list:
  end if:
  r:= list;
  do
    r:= Cycle(r):
  until r[1]=n:
  r:
end:

```

```

> CreateList := proc(n, v := none)
  local k:
  [seq(v, 1..n)]:
end:

```

```

> NextEvent:=proc(Sat)
  global NextSignalAvailable:
  3*NextSignalAvailable[Sat+1] +Sat +1:
end:

```

```

> Q:= proc(x)
  round(x*10^(Digits-5))/10^(Digits-5):
end:

```

```

> #
# Now let us formalize satellietes
#
> # Global variables

```



```

MaxSatellites := 3;

MaxGenerations:= 5;

MaxSignals := 2^MaxGenerations-1;
MaxSatellites := 3
MaxGenerations := 5
MaxSignals := 31

```

(13)

```

> MaxT:= 1/2; # Indietro
MaxT :=  $\frac{1}{2}$ 

```

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```

> Ingoing := -1;
Outgoing:= 1;

Clockwise      := -1;
Counterclockwise := 1;

Ingoing := -1
Outgoing := 1
Clockwise := -1
Counterclockwise := 1

```

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```

> #isScattering := true;

```

```

> Infalling:= 1;
SameScattering:= 2;
OtherScattering := 3;

Infalling := 1
SameScattering := 2
OtherScattering := 3

```

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```

> Side:= 1;
Front:= 2;
Behind:= 3;

Side := 1
Front := 2
Behind := 3

```

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```

>
# Verbose Flags

```

```

> ProducePlots:= true;
DebugOn := true;
DebugTimeOn := true;

```



```

DebugFlowControlOn := true;
DebugSolutionsOn := true;
        ProducePlots := true
        DebugOn := true
        DebugTimeOn := true
        DebugFlowControlOn := true
        DebugSolutionsOn := true

```

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```
> BestKnownSolution:= none:
```

```

> ClearTypeToDo:= proc()
  global TypeToDo:
  TypeToDo := [
    true, true, true, true,
    true, true, true, true,
    true, true, true, true
  ]:
end:

```

```
ClearTypeToDo():
```

```

> TypeToDo ;
[true, true, true, true, true, true, true, true, true, true, true, true]

```

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## SearchSignal procedures

Searching for a signal is a complicated issue with many attempts and tries.  
Never look for two signals at the same time.

Let us describe a search by a structure on which we can do many operations  
and then eventually save the result out of the structure and then

```

> TargetSat      := 1:      # -1, 0, 1, 2
TargetP         := 2:      # [r, te, t]
TargetBranch    := 3:      # none is no sat
TargetCol       := 4:      # black, red, blue, green

SourceSat       := 5:      # 0, 1, 2
SourceBranch    := 6:      # ..., -1, 0, 1, ...
SourceCol       := 7:      # black, red, blue, green
PreviousSourceBranch:= 8:  # true | false(D)

Gen             := 9:      # 0(D), 1, 2, ...

IndrGuessMax    :=10:
IndrGuessMin    :=11:
SourceCrossing:=12:

IndrmGuess      :=13:
IndK2Guess      :=14:
Indsv           :=15:

```



```

Indkv      :=16:
Indscos    :=17:

Intervalr1 :=18:
Intervalr2 :=19:
Intervalrm :=20:
IntervalK2  :=21:

RisP       :=22:
Risr       :=23:
RisBranch  :=24:
Risrm      :=25:
RisK2      :=26:

RayType     :=27:      # Infalling | SameScattering |
OtherScattering
RayBranch   :=28:      # Ingoing | Outgoing (at Target)
RayClock    :=29:      # Clockwise | Counterclockwise
SolveType   :=30:      # Side | Front | Behind

> SearchSignalClear := [

    none,      #x TargetSat      := 1:      # -1, 0, 1, 2
    none,      #x TargetP       := 2:      # [r, te, t]
    none,      #x TargetBranch  := 3:      # none is no sat
    none,      #x TargetCol     := 4:      # black, red, blue,
green
    none,      #x SourceSat      := 5:      # 0, 1, 2
    none,      #x SourceBranch  := 6:      # ..., -1, 0, 1, ..
.
    none,      #x SourceCol     := 7:      # black, red, blue,
green
    false,     # PreviousSourceBranch:= 8: # true | false(D)

    0,         #x Gen           := 9:      # 0(D), 1, 2, ...

    none,      # IndrGuessMax   :=10:
    none,      # IndrGuessMin   :=11:
    none,      # SourceCrossing:=12:

    none,      # IndrmGuess     :=13:
    none,      # IndK2Guess     :=14:
    none,      # Indsv          :=15:
    none,      # Indkv          :=16:
    none,      # Indscos        :=17:

    none,      # Intervalr1     :=18:
    none,      # Intervalr2     :=19:
    none,      # Intervalrm     :=20:
    none,      # IntervalK2     :=21:

    none,      # RisP           :=22:
    none,      # Risr           :=23:
    none,      # RisBranch      :=24:
    none,      # Risrm          :=25:

```



```

    none,          # Risk2          :=26:
    none,          # RayType         :=27:      # Infalling |
SameScattering | OtherScattering
    none,          # RayBranch       :=28:      # Ingoing | Outgoing
(at Target)
    none,          # RayClock        :=29:      # Clockwise |
Counterclockwise
    none          # SolveType       :=30:      # Side | Front |
Behind
]:

```

```

SearchSignal := SearchSignalClear:

```

```

> ClearSearchSignal := proc()
  global SearchSignal;
  SearchSignal := SearchSignalClear:
end:

```

```

> SaveSearchSignal := proc()
  return SearchSignal:
end:

```

```

> RestoreSearchSignal := proc(saved)
  global SearchSignal;
  SearchSignal := saved:
end:

```

```

> Get := proc(field)
  global SearchSignalFields:
  SearchSignal[field];
end:

```

```

> Set := proc(field, val)
  global SearchSignal;
  #print("Set %d = %a", field, val);
  SearchSignal[field] := val;
end:

```

```

> NewSearch:= proc()
  Set(RisP, none):
  Set(Risr, none):
  Set(RisBranch, none):
  Set(Risrm, none):
  Set(Risk2, none):

```

```

  #Set(RayType, none):      #check they are not set in
LinearGuess
  #Set(RayBranch, none):
  #Set(RayClock, none):
  #Set(SolveType, none):
end:

```



```

> SetFreeTarget := proc(P)
    Set(TargetSat, -1):
    Set(TargetP, P):
    Set(TargetCol, black):
end:

> SetTargetPoint := proc(Sat, Pv)
    Set(TargetSat, Sat):
    Set(TargetP, Pv):
    Set(TargetBranch, BranchWith(Sat, Pv[3])):
    if Sat = 1 then
        Set(TargetCol, red):
    elif Sat = 2 then
        Set(TargetCol, blue):
    elif Sat = 3 then
        Set(TargetCol, green):
    else
        Set(TargetCol, black):
    end if:
end:

> SetSourceSat := proc(Sat)
    Set(SourceSat, Sat):
    Set(SourceBranch, BranchWith(Sat, Get(TargetP)[3])):
    if Sat = 1 then
        Set(SourceCol, red):
    elif Sat = 2 then
        Set(SourceCol, blue):
    elif Sat = 3 then
        Set(SourceCol, green):
    else
        Set(SourceCol, black):
    end if:
end:

> SetGeneration := proc(G)
    Set(Gen, G):
end:

> GetGeneration := proc()
    Get(Gen):
end:

```

```

# if Pr<rSat.min      |   P  <--  S
#
#      Scattering      Infalling
# sv<0      Nope      Nope
# 0<sv<1      Out Other      Nope
# sv>1      In Same      In (single branch)
#
#
# if Pr>rSat.max      |   S  -->  P
#
#      Scattering      Infalling

```



# sv<0	Out Same	Out (single branch)
# 0<sv<1	Out Other	Nope
# sv>1	Nope	Nope
#		

```

> LinearGuess:= proc()
  local targetR, targetTe, targetT:
  local sourceSat, sourceBranch:
  local rGuessMax, rGuessMin, d2, sv, rmGuess, kv, scos:
  local mmin, mmax, tm, tM, rguess:

  targetR, targetTe, targetT := op(Get(TargetP)):
  sourceSat := Get(SourceSat):
  sourceBranch := Get(SourceBranch):
  mmin := minS(sourceSat):
  mmax := maxS(sourceSat):

  tS(sourceSat, sourceBranch)-targetT:
  evalf(%):
  fsolve(%, r=mmin..mmax, fulldigits):
  rGuessMax:=Q(%):
  Set(IndrGuessMax, rGuessMax):

  BranchWith(sourceSat, targetT- MaxT):
  tS(sourceSat, %)-(targetT - MaxT):
  evalf(%):
  fsolve(%, r=mmin..mmax, fulldigits):
  rGuessMin:= %:
  Set(IndrGuessMin, rGuessMin):
  # notice: it is guaranteed that rGuessMin < rGuessMax (it
depends on the satallite branch)
  # what is guaranteed is that t at rGuessMin is less than t at
rGuessMax
  # Therefore be careful when writing Ranges.

  tS(sourceSat, sourceBranch):
  subs(r=mmin, %):
  evalf(%):
  tm:=%:

  tS(sourceSat, sourceBranch):
  subs(r=mmax, %):
  Real(%):
  simplify(%):
  evalf(%):
  tM:=%:

  #printf("mmin= %a, mmax=%a, rGuessMax= %a, rGuessMin=%a\n", mmin,
mmax, rGuessMax, rGuessMin);
  # tm< targetT - MaxT (< targetT < tM ) ?
  if targetT-MaxT < tm and tm < targetT then
    Set(PreviousSourceBranch,false):
    Set(SourceCrossing, true):
    Set(Intervalr1, myRange(mmin, rGuessMax)):
    Set(Intervalr2, myRange(mmin, rGuessMin)):

```



```

    printf("two intervals r = %a or r = %a\n", myRange(mmin,
rGuessMax), myRange(mmin, rGuessMin)):
    elif targetT-MaxT < tM and tM < targetT then
        Set(PreviousSourceBranch,false):
        Set(SourceCrossing, true):
        Set(Intervalr1, myRange(rGuessMax, mmax)):
        Set(Intervalr2, myRange(rGuessMin, mmax)):
        printf("two intervals r = %a or r = %a\n", myRange
(rGuessMax, mmax), myRange(rGuessMin, mmax)):
    else
        Set(PreviousSourceBranch,false):
        Set(SourceCrossing, false):
        Set(Intervalr1, myRange(rGuessMax, rGuessMin)):
        Set(Intervalr2, none):
        printf("one interval r = %a\n", myRange(rGuessMax, rGuessMin))
:
end if:

(s*targetR*cos(targetTe)+(1-s)*xS(sourceSat, sourceBranch))^2
+(s*targetR*sin(targetTe) + (1-s)*yS(sourceSat, sourceBranch))
^2:
subs(r=rGuessMax, %):
d2:= evalf(%):
diff(%, s):
solve(%, s):
sv:=%;
Set(Indsv, sv):

subs(s=sv, d2):
sqrt(%):
evalf(%):
rmGuess:=%; # many times
the linear approximation is a good starting point to locate rm.
Set(IndrmGuess, rmGuess):

# printf("[xS, yS]=%a\n", [xS(sourceSat, sourceBranch), yS
(sourceSat, sourceBranch)]);
# printf("Target=%a\n", [targetR*cos(targetTe), targetR*sin
(targetTe)]);
# printf("Target=%a\n", [targetR*cos(targetTe), targetR*sin
(targetTe)]);
xS(sourceSat, sourceBranch)*targetR*sin(targetTe)-yS(sourceSat,
sourceBranch)*targetR*cos(targetTe):
subs(r=rGuessMax, %):
evalf(%):
kv:=%;
Set(Indkv, kv):

targetR*cos(targetTe)*xS(sourceSat, sourceBranch)+targetR*sin
(targetTe)*yS(sourceSat, sourceBranch):
subs(r=rGuessMax, %):
scos:= %; # >0 target
and source are on the same side of BH
Set(Indscos, scos):

if kv > 0 then

```



```

    Set(RayClock, Counterclockwise):
else
    Set(RayClock, Clockwise):
end if:
Set(Intervalrm, rcrit..min(targetR, mmax)):
Set(IntervalK2, 0..K2crit):

# forse è più importante stare larghi così da sapere che in sides
# non ci possono essere infalling
# e lasciare la possibilità tutta in front e behind

    if rmGuess > 4*alv then
        Set(SolveType, Side):
        if targetR > mmax then
S ---> P
            if sv<0 then
                Set(RayBranch, Outgoing):
                Set(RayType, SameScattering):
            elif sv>0 and sv<1 then
                Set(RayBranch, Outgoing):
                Set(RayType, OtherScattering):
            else
                # sv>1
                printf("It cannot be s>1 and | S ---> P\n");
            end if:
        elif targetR < mmin then
P <--- S
            if sv<0 then
                printf("It cannot be s<0 and | P <--- S\n");
            elif sv>0 and sv<1 then
                Set(RayBranch, Outgoing):
                Set(RayType, OtherScattering):
            else
                # sv>1
                Set(RayBranch, Ingoing):
                Set(RayType, SameScattering):
            end if:
        else
            printf("The user is in the satellite ring. Too messy to be
considered. At least now.\n");
        end if:
    else
        if scos > 0 then
            # same side of BH
            Set(SolveType, Front):
            if rmGuess > rcrit then
                if targetR > mmax then
--> P
                    Set(RayBranch, Outgoing):
                    Set(RayType, SameScattering):
                elif targetR < mmin then
-- S
                    Set(RayBranch, Ingoing):
                    Set(RayType, SameScattering):
                end if:
            else
                if targetR > mmax then
--> P
                    Set(RayBranch, Outgoing):
                    Set(RayType, Infalling):

```



```

        elif targetR < mmin then                                # | P <-
-- S
    Set(RayBranch, Ingoing):
    Set(RayType, Infalling):
    end if:
end if:
else                                # opposite sides of BH
    Set(SolveType, Behind):
    if targetR > mmax then          # | S --
-> P
        Set(RayBranch, Outgoing):
        Set(RayType, OtherScattering ):
    elif targetR < mmin then        # | P <--
- S
        Set(RayBranch, Outgoing):
        Set(RayType, OtherScattering ):
    end if:
end if:
end if:
end:

> SetPreviousSourceBranch := proc()
    local Crossing, Int2, sourceBranch, Sat:
    Crossing := Get(SourceCrossing):
    Set(SourceCrossing, false):
    Sat := Get(SourceSat):
    if Crossing then
        Int2 := Get(Intervalr2):
        Set(Intervalr1, Int2):
        Set(Intervalr2, none):
        sourceBranch := Get(SourceBranch):
        Set(SourceBranch, sourceBranch-1):
        if (sourceBranch mod 2) = 0 then
            # even then Outgoing, I crossed rm
            Set(rGuessMax, minS(Sat))
        else
            # odd then Ingoing, I crossed rp
            Set(rGuessMax, maxS(Sat))
        end if:
    end if:
end:

> OtherClock := proc()
    local rayClock:
    rayClock := Get(RayClock):
    Set(RayClock, -rayClock):
end:

> CreateHint:= proc()
    local h1, h2, h3, h4, h5, h6, h7, h8:
    h1:= Get(Risr):
    h6:= Get(Intervalr1):
    printf("r=%g in [%a]\n",h1, h6):
    h2:= Get(RayType):
    if h2 = Infalling then
        h7:= Get(IntervalK2):

```



```

    h5:= Get(RisK2);
    printf("Infalling ray (K2=%g) in [%a].\n", h5, h7);
  elif h2 = SameScattering then
    h7:= Get(Intervalrm);
    h5:= Get(Risrm);
    printf("Scattering ray (rm=%g) in [%a]: target and source on
the same branch.\n", h5, h7);
  elif h2 = OtherScattering then
    h7:= Get(Intervalrm);
    h5:= Get(Risrm);
    printf("Scattering ray (rm=%g) in [%a]: target and source on
the different branches.\n", h5, h7);
  else
    printf("Type of ray unknown.\n");
  end if:
  h3:= Get(RayClock):
  if h3 = Clockwise then
    printf("Clockwise ray.\n");
  else
    printf("Counterclockwise ray.\n");
  end if:
  h4:= Get(RayBranch):
  if h4 = Outgoing then
    printf("Ray outgoing at target.\n");
  else
    printf("Ray outgoing at target.\n");
  end if:
  h8 := Get(SolveType):
  if h8 = Side then
    printf("Solve Side.\n");
  elif h8 = Front then
    printf("Solve Front.\n");
  elif h8 = Behind then
    printf("Solve Behind.\n");
  else
    printf("Solve type unknown.\n");
  end if:
  [h1, h2, h3, h4, h5, h6, h7, h8]:
end:

```

```

> UseHint:= proc(h)
  printf("hint used Hint := %a\n", h);
  if type(h, list) and nops(h)=8 then
    Set(Risr, h[1]):
    Set(IndrGuessMax, h[1]):
    Set(RayType, h[2]):
    if h[2] = Infalling then
      Set(RisK2, h[5]):
      Set(IndK2Guess, h[5]):
      Set(IntervalK2, h[7]):
    elif h[2] = SameScattering then
      Set(Risrm, h[5]):
      Set(IndrmGuess, h[5]):
      Set(Intervalrm, h[7]):
    elif h[2] = OtherScattering then
      Set(Risrm, h[5]):

```



```

        Set(IndrmGuess, h[5]):
        Set(Intervalrm, h[7]):
    else
        printf("Type of ray unknown.\n");
    end if:
    Set(RayClock, h[3]):
    Set(RayBranch, h[4]):
    Set(Intervalr1, h[6]):
    Set(SolveType, h[8]):
else
    printf("No hint used.\n");
    return:
end if:
end:

```

```

> ComputeSat:= proc(tein, tin, tauin, rmv, rpv)
    local tr, ter, taur, Te, T, Tau;

    mPhi:
    1/%:
    simplify(%) assuming c=cv, al=alv, rm=rmv, rp=rpv, r=(rmv+rpv)/2:
    sqrt(%):
    radsimp(%) assuming c=cv, al=alv, rm=rmv, rp=rpv, r=(rmv+rpv)/2:
    #%;
    subs(r=R, %):
    subs([c=cv, al=alv, rm=rmv, rp=rpv], %):
    int(%, R=rmv..r) assuming r>rmv, r<rpv:
    simplify(%) assuming r>rmv, r<rpv:
    radsimp(%) assuming r>rmv, r<rpv:
    tr:= %:

    mPsi:
    1/%:
    simplify(%) assuming c=cv, al=alv, rm=rmv, rp=rpv, r=(rmv+rpv)/2:
    sqrt(%):
    radsimp(%) assuming c=cv, al=alv, rm=rmv, rp=rpv, r=(rmv+rpv)/2:
    #%;
    subs(r=R, %):
    subs([c=cv, al=alv, rm=rmv, rp=rpv], %):
    int(%, R=rmv..r) assuming r>rmv, r<rpv :
    simplify(%) assuming r>rmv, r<rpv:
    radsimp(%) assuming r>rmv, r<rpv:
    ter := %:

    dtau :
    subs(r=R, %):
    subs([c=cv, al=alv, rm=rmv, rp=rpv], %):
    int(%, R=rmv..r) assuming r>rmv, r<rpv :
    simplify(%) assuming r>rmv, r<rpv:
    radsimp(%) assuming r>rmv, r<rpv:
    taur := %:

```



```

ter:
subs(r=rpv, %):
simplify(%):
Te:=%:

tr:
subs(r=rpv, %):
simplify(%):
T:=%:

taur:
subs(r=rpv, %):
simplify(%):
Tau:=%:
[tr, ter, taur, Te, T, Tau]:
end:

> ComputeSat0:= proc()
  global te0in, t0in, tau0in, rm0v, rp0v;
  global t0r, te0r, tau0r, Te0, T0, Tau0;
  ComputeSat(te0in, t0in, tau0in, rm0v, rp0v):
  t0r, te0r, tau0r, Te0, T0, Tau0 := op(%):
end:

> ComputeSat1:= proc()
  global telin, tlin, tau1in, rmlv, rp1v;
  global t1r, telr, tau1r, Te1, T1, Tau1;
  ComputeSat(telin, tlin, tau1in, rmlv, rp1v):
  t1r, telr, tau1r, Te1, T1, Tau1 := op(%):
end:

> ComputeSat2:= proc()
  global te2in, t2in, tau2in, rm2v, rp2v;
  global t2r, te2r, tau2r, Te2, T2, Tau2;
  ComputeSat(te2in, t2in, tau2in, rm2v, rp2v):
  t2r, te2r, tau2r, Te2, T2, Tau2 := op(%):
end:

```

```

> # First satellite

```

```

> rp0v := Surface+20900000;
rm0v := Surface+20100000;

t0in := -1500;
te0in := Pi/6;
tau0in := -1500;

```

```

rp0v := 24089000
rm0v := 23289000

```



```
> Sat0 := [te0in, t0in, tau0in, rm0v, rp0v];
```

$$Sat0 := \left[ \frac{\pi}{6}, -1500, -1500, 23289000, 24089000 \right] \quad (21)$$

```
>
# for each satellite we can analytically integrate Weierstrass
equations
# This is painfully detailed and sensitive.
# Small changes end up with Maple find worse representations of the
integrals.
```

```
> te0in:
evalf(%);
```

```
t0in:
evalf(%);
```

```
tau0in:
evalf(%);
```

```
rp0v:
evalf(%);
```

```
rm0v:
evalf(%);
```

0.5235987755982988730771072305465838140329

−1500.

−1500.

$2.4089000 \times 10^7$

$2.3289000 \times 10^7$

(22)

```
> ComputeSat0():
```

```
t0r:
evalf(%):
%:
```

```
te0r:
evalf(%):
%:
```

```
tau0r:
evalf(%):
%:
```

```
Te0:
evalf(%):
%;
```



```
# 3.14159265535486
# 3.1415926553548523984
# 3.141592655354852398254265390134094198563
#
3.14159265535485239825426539013409419856260322061411445326054936278
37502485449557
#
3.14159265535485239825426539013409419856260322061411445326054936278
375024854495562322064808510137802951136526164386486406
```

```
T0:
evalf(%) :
%;
```

```
# 18142.3587447772
# 18142.358744777252186
# 18142.35874477725218605913669834122217866
#
18142.3587447772521860591366983412221786668982957166911430921766664
28555043660612
#
18142.3587447772521860591366983412221786668982957166911430921766664
285550436606122065933274111800343050009373669231962657
```

```
Tau0:
evalf(%) :
%;
```

```
# 18142.3587396824
# 18142.358739682192163
# 18142.35873968219216302445852030230007633
#
18142.3587396821921630244585203023000763226421341640380742732892052
57465944332626
#
18142.3587396821921630244585203023000763226421341640380742732892052
574659443326254156449373758670831322675204918861768195
```

```
3.141592655354852398254265390134094198563
18142.35874477725218605913669834122217866
18142.35873968219216302445852030230007633
```

(23)

```
> # (half-)periods
```

```
> evalf(2*T0);
%/60/60;
```



```
# 36284.71748955450437211827339668244435732
#
36284.7174895545043721182733966824443573337965914333822861843533328
57110087321224
```

```
36284.71748955450437211827339668244435732
10.07908819154291788114396483241179009926
```

(24)

```
> evalf(2*(Te0-Pi)); # Precession (in radiant)
```

```
# 3.530118319583244013709182628732*10^(-9)
#
3.53011831958324401370918262873086764247801726457120954095186768451
74933*10^(-9)
```

```
3.530118319583244013709182628732 × 10-9
```

(25)

```
> evalf(2*Tau0);
```

```
# 36284.71747936438432604891704060460015266
#
36284.7174793643843260489170406046001526452842683280761485465784105
14931888665252
```

```
evalf(Tau0/T0); # Slowing time factor
```

```
# 0.9999999997191622051624669169824116350436
#
0.99999999971916220516246691698241163504326993418205012998397463349
053450494301666
```

```
36284.71747936438432604891704060460015266
0.9999999997191622051624669169824116350436
```

(26)

```
>
# Second satellite
```

```
> rplv := Surface+21900000;
rmlv := Surface+21100000;
```

```
tlin := -1600;
telin := -Pi/6;
taulin := -1600;
```

```
rplv := 25089000
rmlv := 24289000
```

(27)

```
> Sat1 := [telin, tlin, tau1n, rmlv, rplv];
```

```
Sat1 :=  $\left[-\frac{\pi}{6}, -1600, -1600, 24289000, 25089000\right]$ 
```

(28)

```
> ComputeSat1();
```



```
t1r:
evalf(%):
%:
```

```
telr:
evalf(%):
%:
```

```
taulr:
evalf(%):
%:
```

```
Tel:
evalf(%):
%;
```

```
T1:
evalf(%):
%;
```

```
Taul:
evalf(%):
%;
```

3.141592655283322340782345120016275877553  
19303.18211302455233332661930535341676357  
19303.18210782306326017425688729853980400

(29)

```
> evalf(2*T1);
%/60/60;
# 38606.36422604910466665323861070683352712
```

38606.36422604910466665323861070683352712  
10.72399006279141796295923294741856486864

(30)

```
> evalf(2*(Tel-Pi)); # Precession (in radiant)
# 3.387058204639403473473545986712*10^(-9)
```

$3.387058204639403473473545986712 \times 10^{-9}$

(31)

```
> evalf(2*Taul);
# 38606.36421564612652034851377459707960800
evalf(Taul/T1); # Slowing time factor
# 0.9999999997305372221690469175902316869746
```



$$\begin{aligned} & 38606.36421564612652034851377459707960800 \\ & 0.9999999997305372221690469175902316869746 \end{aligned} \quad (32)$$

```
> # Terzo satellite
```

```
> rp2v := Surface+22900000;  
rm2v := Surface+22100000;
```

```
t2in := -1400:  
te2in := 0:  
tau2in := -1400:
```

$$\begin{aligned} rp2v &:= 26089000 \\ rm2v &:= 25289000 \end{aligned} \quad (33)$$

```
> Sat2 := [te2in, t2in, tau2in, rm2v, rp2v];
```

$$Sat2 := [0, -1400, -1400, 25289000, 26089000] \quad (34)$$

```
> OrbitalParameters := [Sat0, Sat1, Sat2];
```

$$OrbitalParameters := \left[ \left[ \frac{\pi}{6}, -1500, -1500, 23289000, 24089000 \right], \left[ -\frac{\pi}{6}, -1600, -1600, 24289000, 25089000 \right], [0, -1400, -1400, 25289000, 26089000] \right] \quad (35)$$

```
> ComputeSat2():
```

```
t2r:  
evalf(%):  
%:
```

```
te2r:  
evalf(%):  
%:
```

```
tau2r:  
evalf(%):  
%:
```

```
Te2:  
evalf(%):  
%;
```

```
T2:  
evalf(%):  
%;
```

```
Tau2:  
evalf(%):  
%;
```

$$\begin{aligned} & 3.141592655217365427848202944116826336967 \\ & 20487.75898732513113742426435899419444116 \end{aligned}$$



20487.75898201934745186745874168545755717

(36)

```
> evalf(2*T2);  
%/60/60;  
# 40975.51797465026227484852871798838888234
```

40975.51797465026227484852871798838888232

11.38208832629173952079125797721899691176

(37)

```
> evalf(2*(Te2-Pi)); # Precession (in radiant)  
# 3.255144378771119121674646905539*10^(-9)
```

$3.255144378771119121674646905539 \times 10^{-9}$

(38)

```
> evalf(2*Tau2);  
# 40975.51796403869490373491748337091511433  
evalf(Tau2/T2); # Slowing time factor  
# 0.9999999997410266447960824347153030507972
```

40975.51796403869490373491748337091511433

0.9999999997410266447960824347153030507972

(39)

## Satellites

### Satellite 0

```
> # Satellite 0  
> xS0 := proc(branchv)  
  r*cos(Branch(te0r,branchv, Te0, te0in)):  
end:  
  
> yS0 := proc(branchv)  
  r*sin(Branch(te0r,branchv, Te0, te0in)):  
end:  
  
> tS0 := proc(branchv)  
  Branch(t0r, branchv, T0, t0in):  
end:  
  
> minS0:= proc()  
  rm0v;
```



```
end:
```

```
> maxS0:= proc()  
    rp0v;  
end:
```

```
> halfT0:= proc()  
    T0;  
end:
```

```
> halfTe0:= proc()  
    Te0;  
end:
```

```
> halfTau0:= proc()  
    Tau0;  
end:
```

## ▼ Satellite 1

```
> xS1 := proc(branchv)  
    r*cos(Branch(telr,branchv, Tel, telin)):  
end:
```

```
> yS1 := proc(branchv)  
    r*sin(Branch(telr,branchv, Tel, telin)):  
end:
```

```
> tS1 := proc(branchv)  
    Branch(tlr, branchv, T1, tlin):  
end:
```

```
> minS1:= proc()  
    rmlv;  
end:
```

```
> maxS1:= proc()  
    rplv;  
end:
```

```
> halfT1:= proc()  
    T1;  
end:
```

```
> halfTel:= proc()  
    Tel;  
end:
```

```
> halfTau1:= proc()  
    Tau1;  
end:
```



## Satellite 2

```
> xS2 := proc(branchv)
    r*cos(Branch(te2r,branchv, Te2, te2in)):
end:

> yS2 := proc(branchv)
    r*sin(Branch(te2r,branchv, Te2, te2in)):
end:

> tS2 := proc(branchv)
    Branch(t2r, branchv, T2, t2in):
end:

> minS2:= proc()
    rm2v;
end:

> maxS2:= proc()
    rp2v;
end:

> halfT2:= proc()
    T2;
end:

> halfTe2:= proc()
    Te2;
end:

> halfTau2:= proc()
    Tau2;
end:
```

```
> rSat:= proc(S, rv, branchv)
    rv:
end:

> teSat:= proc(S, rv, branchv)
    if S=0 then
        Branch(te0r,branchv, Te0, te0in):
    elif S=1 then
        Branch(te1r,branchv, Te1, telin):
    elif S=2 then
        Branch(te2r,branchv, Te2, te2in):
    end if:
    subs(r=rv, %):
    evalf(%):
end:
```



end:

```
> tSat:= proc(S, rv, branchv)
  if S=0 then
    Branch(t0r, branchv, T0, t0in):
  elif S=1 then
    Branch(t1r, branchv, T1, t1in):
  elif S=2 then
    Branch(t2r, branchv, T2, t2in):
  end if:
  subs(r=rv, %):
  evalf(%):
end:
```

```
> tauSat:= proc(S, rv, branchv)
  if S=0 then
    Branch(tau0r, branchv, Tau0, tau0in):
  elif S=1 then
    Branch(tau1r, branchv, Tau1, tau1in):
  elif S=2 then
    Branch(tau2r, branchv, Tau2, tau2in):
  end if:
  subs(r=rv, %):
  evalf(%):
end:
```

```
> xS := proc(S, b)
  if S = 0 then
    xS0(b):
  elif S = 1 then
    xS1(b):
  elif S = 2 then
    xS2(b):
  end if:
end:
```

```
> yS := proc(S, b)
  if S = 0 then
    yS0(b):
  elif S = 1 then
    yS1(b):
  elif S = 2 then
    yS2(b):
  end if:
end:
```

```
> tS := proc(S, b)
  if S = 0 then
    tS0(b):
  elif S = 1 then
    tS1(b):
  elif S = 2 then
    tS2(b):
  end if:
end:
```



```
> maxS := proc(S)
  if S = 0 then
    maxS0():
  elif S = 1 then
    maxS1():
  elif S = 2 then
    maxS2():
  end if:
end:
```

```
> minS := proc(S)
  if S = 0 then
    minS0():
  elif S = 1 then
    minS1():
  elif S = 2 then
    minS2():
  end if:
end:
```

```
> halfT:= proc(S)
  if S = 0 then
    halfT0():
  elif S = 1 then
    halfT1():
  elif S = 2 then
    halfT2():
  end if:
end:
```

```
> halfTe:= proc(S)
  if S = 0 then
    halfTe0():
  elif S = 1 then
    halfTe1():
  elif S = 2 then
    halfTe2():
  end if:
end:
```

```
> halfTau:= proc(S)
  if S = 0 then
    halfTau0():
  elif S = 1 then
    halfTau1():
  elif S = 2 then
    halfTau2():
  end if:
end:
```

```
> tS(0, -1):
```



```

subs(r=minS(0)+3000., %):
evalf(%):
%;

# -2195.79945653386
# -2195.7994565338638080
# -2195.799456533863808051453426653406049608
#
-2195.7994565338638080514534266534060496078310514216505707216055497
581263612107142
#
-2195.7994565338638080514534266534060496078310514216505707216055497
5812636121071426711939722345969745313355541543905829501

```

-2195.799456533863808051453426653406049608

(40)

```
> # Plot satellites worldlines
```

```
> [xS(0, -1), yS(0, -1), tS(0, -1), r=minS(0)..maxS(0)]:
evalf(%):
c0_1n := spacecurve(%, axes=box, color = red):
```

```
> [xS(0, 0), yS(0, 0), tS(0, 0), r=minS(0)..maxS(0)]:
evalf(%):
c0_0 := spacecurve(%, axes=box, color = red, thickness = 4):
```

```
> [xS(0, 1), yS(0, 1), tS(0, 1), r=minS(0)..maxS(0)]:
evalf(%):
c0_1 := spacecurve(%, axes=box, color = red):
```

```
> [xS(1, -1), yS(1, -1), tS(1, -1), r=rmlv..maxS(1)]:
evalf(%):
c1_1n := spacecurve(%, axes=box, color = blue):
```

```
> [xS(1, 0), yS(1, 0), tS(1, 0), r=minS(1)..maxS(1)]:
evalf(%):
c1_0 := spacecurve(%, axes=box, color = blue, thickness = 4):
```

```
> [xS(1, 1), yS(1, 1), tS(1, 1), r=minS(1)..maxS(1)]:
evalf(%):
c1_1 := spacecurve(%, axes=box, color = blue):
```

```
> [xS(2, -1), yS(2, -1), tS(2, -1), r=minS(2)..maxS(2)]:
evalf(%):
c2_1n := spacecurve(%, axes=box, color = green):
```

```
> [xS(2, 0), yS(2, 0), tS(2, 0), r=minS(2)..maxS(2)]:
evalf(%):
```



```
c2_0 := spacecurve(% , axes=box, color = green, thickness = 4):
```

```
> [xS(2, 1), yS(2, 1), tS(2, 1), r=minS(2)..maxS(2)]:  
evalf(%):  
c2_1 := spacecurve(% , axes=box, color = green):
```

```
> BH := spacecurve([0,0,s, s=-5000..5000],  
axes=box, color = black, linestyle = dash):
```

```
> [R*cos(te), R*sin(te), t]:  
subs(R=Surface, %):  
plot3d(% , te=0..2*Pi, t=-50000..50000):  
S := %:
```

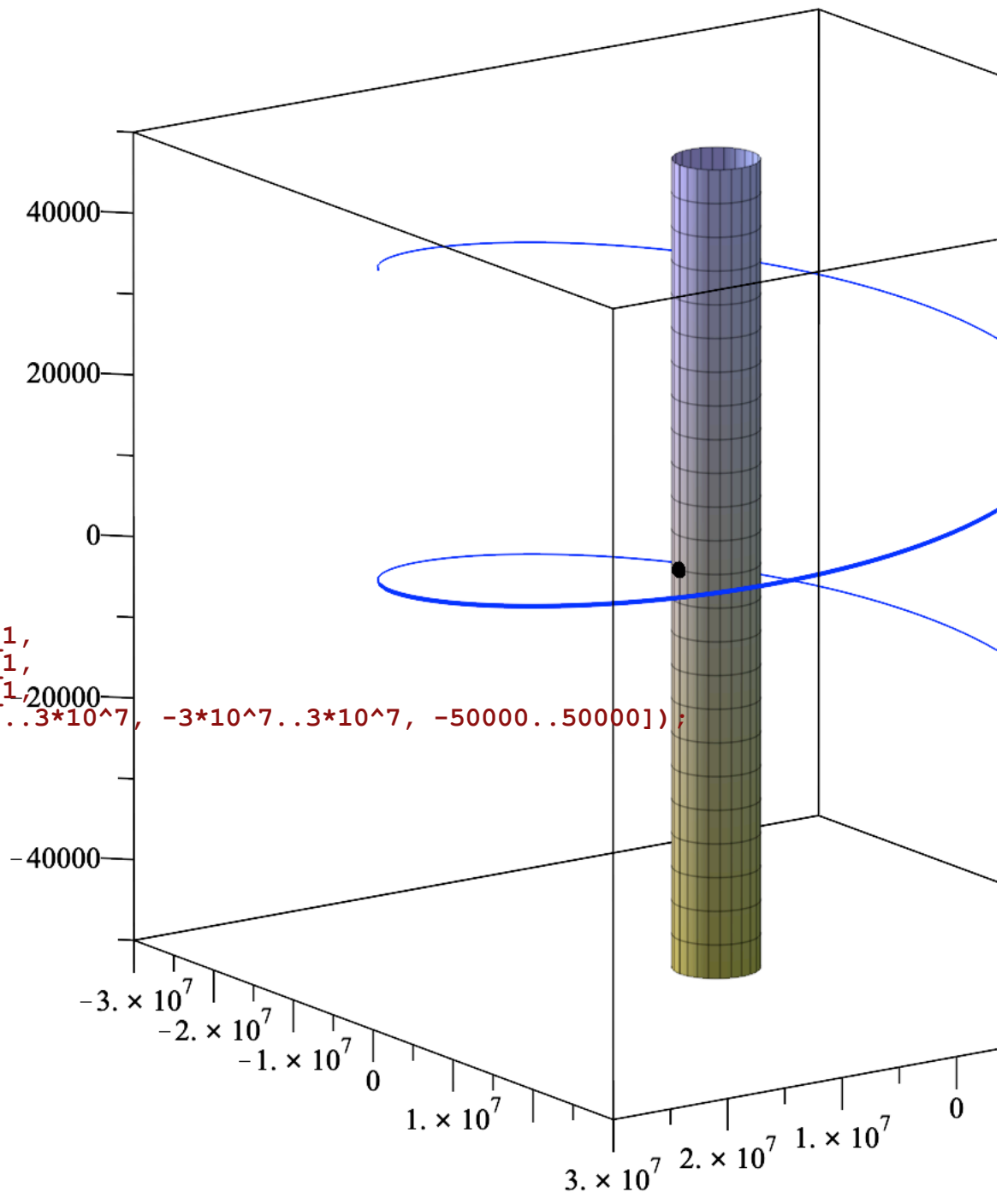
```
> ux := Surface+1000:  
uy := 0:  
ut := 0:
```

```
user := pointplot3d([ux, uy, ut], color=black, symbol =  
solidcircle, symbolsize = 10):
```

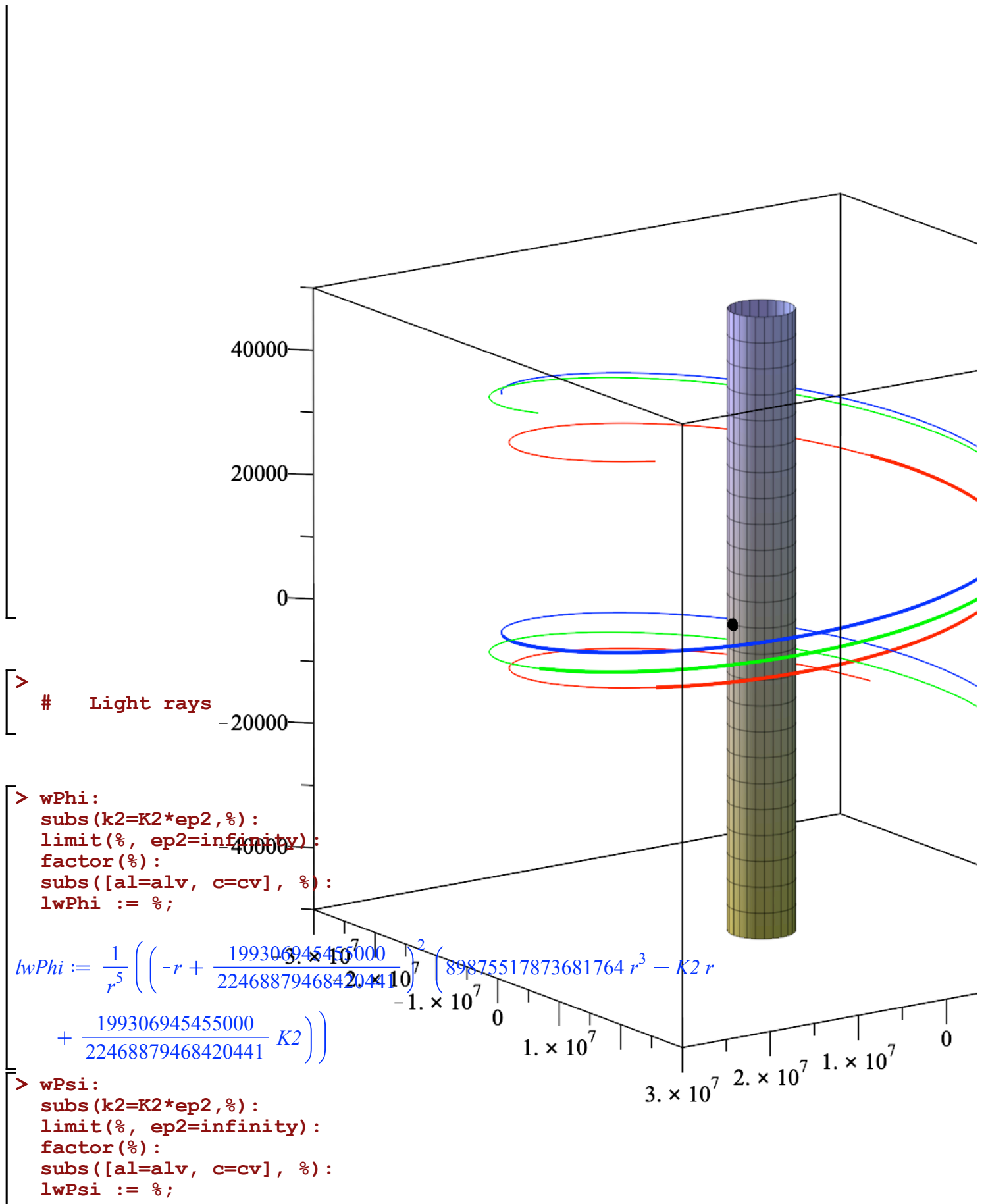
```
> display(BH, S, user,  
c1_1n,c1_0,c1_1,  
view=[ -3*10^7..3*10^7, -3*10^7..3*10^7, -50000..50000]  
);
```



```
> display(BH, S,
user,
c0_1n,c0_0,c0_1,
c1_1n,c1_0,c1_1,
c2_1n,c2_0,c2_1,
view=[ -3*10^7..3*10^7, -3*10^7..3*10^7, -50000..50000]);
```









$$lwPsi := \frac{\left( 89875517873681764 r^3 - K2 r + \frac{199306945455000}{22468879468420441} K2 \right) r}{K2} \quad (42)$$

```
> # Again we need to put Weierstrass in a form suitable for later
integration
# even though in this case we have both scattering and infalling
rays.
```

```
wPsi:
subs(k2=K2*ep2,%):
limit(%, ep2=infinity):
factor(%):
numer(%) / r:
lp:=%;

subs(r=al, %):
#subs([al=alv, c=cv], %):
evalf(%);
```

$$lp := \frac{c^2 r^3 + K2 al - K2 r}{al^3 c^2} \quad (43)$$

```
> lp:
diff(%, r):
solve(%, r):
[%][1]:
subs([al=alv, c=cv], %):
rcrit:=%;
```

$$rcrit := \frac{\sqrt{3} \sqrt{K2}}{899377374} \quad (44)$$

```
> lp:
subs(r=rcrit, %):
subs(K2=K^2, %):
radsimp(%):
%/K^2:
simplify(%):
solve(%, K):
[%][1]:
%^2:
#%;
subs([al=alv, c=cv], %):
K2crit:=%;
evalf(%);
```

$$K2crit := \frac{1072527979678263319239675000000}{22468879468420441}$$

$$4.773393266832375251471851872452971366257 \times 10^{13} \quad (45)$$

```
> rcrit:
subs(K2=K2crit, %):
simplify(%):
radsimp(%) assuming al>0, c>0:
```



```
rcrit:= %;
```

```
%-a1*3/2:
```

```
subs([a1=alv, c=cv], %):
```

```
radsimp(%):
```

```
%;
```

$$rcrit := \frac{298960418182500}{22468879468420441} 0$$

(46)

```
> subs(K2=K2crit, lp):
```

```
subs([a1=alv, c=cv], %):
```

```
subs(r=rcrit, %):
```

```
radsimp(%):
```

```
%;
```

0

(47)

```
> #
```

```
# One real solution
```

```
# 0 < K2 < K2crit
```

```
#
```

```
# To be honest we should work better for the integration of  
infalling branches
```

```
# (to be improved, being currently numerical)
```

```
#
```

```
> lwPhi:
```

```
subs([a1=alv, c=cv], %):
```

```
sqrt(%):
```

```
radsimp(% assuming r>0:
```

```
simplify(% assuming r>0:
```

```
1/%:
```

```
subs(r=R, %):
```

```
Int(%, R=r0..r) assuming K2>0:
```

```
inltr:=t0+ branch*%;
```

```
#
```

```
# branch = 1 : Outgoing
```

```
# branch = -1 : Ingoing
```

$$inltr := \left( \int_{r0}^r (3368000302171748692416989 R^5)^{1/2} \right) / \left( (22468879468420441 R - 199306945455000) \right.$$

(48)

$$\left. \left( (-22468879468420441 R + 199306945455000) K2 \right. \right.$$

$$\left. \left. + 2019402178265622557315950186537924 R^3 \right)^{1/2} \right) dR \right) branch + t0$$



```

> lwPsi:
subs([al=alv, c=cv], %):
sqrt(%):
radsimp(%) assuming r>0, K2>0:
simplify(%) assuming r>0, K2>0:
1/%:
subs(r=R, %):
Int(%, R=r0..r):
#%;
inlter:=te0+ clock*branch*%;

```

$$\begin{aligned}
inlter := & \left( \int_{r0}^r (149896229 \sqrt{K2}) / \right. \\
& \left( \sqrt{R} ( (-22468879468420441 R + 199306945455000) K2 \right. \\
& \left. \left. + 2019402178265622557315950186537924 R^3 \right)^{1/2} \right) dR \Big) branch \, clock + te0
\end{aligned} \tag{49}$$

```

> # Test

```

```

inltr:
subs([t0=13, r0=5*10^6, te0=Pi/4], %):
subs([branch=Ingoing], %):
subs([K2=1/3*K2crit], %):
subs([r=4*10^6], %):
#subs([al=alv, c=cv], %):
evalf(%):
%;

# 13.0033356409586
# 13.003335640958583963
# 13.00333564095858396346014332501596545067
#
13.0033356409585839634601433250159654506745716515247157709883911852
27116102166949
#
13.0033356409585839634601433250159654506745716515247157709883911852
271161021669488704448725304601998169620433156250366028
13.00333564095858396346014332501596545067

```

(50)

```

> # Test

```

```

inltr:
subs([t0=13, r0=5*10^6, te0=Pi/4], %):
subs([branch=Ingoing, clock=Clockwise], %):
subs([branch=Ingoing], %):
subs([K2=1/3*K2crit], %):
subs([r=4*10^6], %):
#subs([al=alv, c=cv], %):
evalf(%):
%;

```



```

# 0.785398162732171
# 0.78539816273217165752
# 0.7853981627321716575185171785762213111637
#
0.78539816273217165751851717857622131116379327021445572240516128527
790383719947160
#
0.78539816273217165751851717857622131116379327021445572240516128527
7903837199471595095447571793144393708942829965589722622

```

$$0.7853981627321716575185171785762213111637 \quad (51)$$

```

>
# Three real solutions
# K2crit < K2

```

```

> lp;

```

$$c^2 r^3 + K2 al - K2 r \quad (52)$$

```

> lp-cv^2*(r+rh+rm)*(r-rh)*(r-rm):
expand(%):
collect(%, r):
#%;
[subs(r=0, %), subs(r=0, diff(%, r))]:
solve(%, [K2, rh]):
op(%):
allvalues(%):
[%][2]:
subs([al=alv, c=cv], %):
simplify(%):
lSol:=%;

```

$$\begin{aligned}
lSol := & \left[ K2 = \frac{2019402178265622557315950186537924 \, rm^3}{-199306945455000 + 22468879468420441 \, rm}, rh \right. \\
& = \frac{1}{-398613890910000 + 44937758936840882 \, rm} \left( (199306945455000 \right. \\
& - 22468879468420441 \, rm \\
& + (504850544566405639328987546634481 \, rm^2 + 8956407469294884458788091310000 \, rm \\
& \left. - 119169775519807035471075000000)^{1/2} \right) \, rm \left. \right]
\end{aligned} \quad (53)$$

```

> subs(lSol, rh):
#subs([al=alv, c=cv], %):
radsimp(%):
rhv:=%;

```

$$\begin{aligned}
rhv := & - \left( (-199306945455000 + 22468879468420441 \, rm \right. \\
& - (504850544566405639328987546634481 \, rm^2 + 8956407469294884458788091310000 \, rm \\
& \left. - 119169775519807035471075000000)^{1/2} \right) \, rm \left. \right) / (2 (-199306945455000
\end{aligned} \quad (54)$$



+ 22468879468420441  $rm$ ))

> # I could also try to use the numerical integration with  $K2(rm)$   
# which seems pretty stable.

```
subs(lSol, K2):
#subs([al=alv, c=cv], %):
radsimp(%):
K2v:=%;
```

$$K2v := \frac{2019402178265622557315950186537924 \, rm^3}{-199306945455000 + 22468879468420441 \, rm} \quad (55)$$

> # verify  $r0$  is greater than  $al$  and less than  $3/2*al$  (result must be true)

```
[alv, rhv, 3/2*alv]:
subs([rm=2*rcrit], %):
subs([al=alv, c=cv], %):
evalf(%):
evalb( %[1]< %[2] and %[2] < %[3] ):
%;
```

$true$  (56)

> lwPhi;  
cv^2;

$$\frac{1}{r^5} \left( \left( -r + \frac{199306945455000}{22468879468420441} \right)^2 \left( 89875517873681764 \, r^3 - K2 \, r + \frac{199306945455000}{22468879468420441} K2 \right) \right) \quad (57)$$

> subs([al=alv, c=cv], lp)=cv^2\*(r+rh+rm)\*(r-rh)\*(r-rm):  
%;

$$89875517873681764 \, r^3 - K2 \, r + \frac{199306945455000}{22468879468420441} K2 = 89875517873681764 \, (r + rh + rm) \, (r - rh) \, (r - rm) \quad (58)$$

```
> lwPhi:
subs([subs([al=alv, c=cv], -lp)=-cv^2*(r+rh+rm)*(r-rh)*(r-rm)], %):
subs([subs([al=alv, c=cv], lp)=cv^2*(r+rh+rm)*(r-rh)*(r-rm)], %):
#subs(rh=rhv, %):
#subs([al=alv, c=cv], %):
simplify(%):
lPhi:=%;
```

$$lPhi := \frac{4 \, (22468879468420441 \, r - 199306945455000)^2 \, (r + rh + rm) \, (r - rh) \, (r - rm)}{22468879468420441 \, r^5} \quad (59)$$

> # It is convenient to integrate from  $rm$  'cos it removes divergences

```
#
#      ltr = t0 + b*int      = tm + b*int      (t0,
```







```
# for r=r0 lter=te0
# it is regular in r=rm
```

```
> lPsi:
simplify(%):
sqrt(%):
radsimp(%) assuming r>rm, rm>rh, rh<3/2*alv, rh>alv:
simplify(%) assuming r>rm, rm>rh, rh<3/2*alv, rh>alv:
1/%:
#%;
subs(r=R, %):
int(%, R=rm..r) assuming r>rm, rm>3/2*alv, rh<3/2*alv, rh>alv:
#subs(rh=rhv, %):
simplify(%) assuming r>rm, rm>3/2*alv:
radsimp(%) assuming r>rm, rm>3/2*alv:
simplify(%) assuming r>rm, rm>3/2*alv:
te0+ clock*branch*(% -subs(r=r0, %)):
radsimp(%) assuming r>rm, r0>rm, rm>3/2*alv:
simplify(%) assuming r>rm, r0>rm, rm>3/2*alv:
lter:=%:
```

```
[> # test solving
```

```
> targetR:=3000.:
targetTe:=978.:
targetT:=17.:
```

```
> # Let us build an example with a known solution.
```

```
[r, lter, ltr]:
subs([t0=targetT, r0=targetR, te0=targetTe], %):
subs([branch=Ingoing, clock=Clockwise], %):
subs([rm=80., rh= subs(rm=80., rhv)], %):
subs(r=7000., %):
#evalf(%, Digits+20):
evalf(%):
sourceR, sourceTe, sourceT:= op(%):
%;
```

```
7000., 978.0152418532877030366037032464994130979,
16.99998665537716076852575689408595928621
```

(61)

```
> # Now forget the solution and remember only target and a line
through the source parameterized by R
```

```
[r*cos(lter), r*sin(lter), ltr, rh=rhv]:
subs([t0=targetT, r0=targetR, te0=targetTe], %):
subs([branch=Ingoing, clock=Clockwise], %):
```

```
subs(r=rm, %):
simplify(%): # importante questo: migliora ltr
xm, ym, tm, rrm:= op(%):
```



```

subs(rm=80, %%):
subs(rh= evalf(subs(rm=80., rhv), Digits+20), %):
#evalf(%, Digits+20):
evalf(%):
%;

```

```

[−66.93668249421465362995469700806238946839,
 43.81187666225560733387153104558708478183,
 17.00001000350636079243929685927086178896,
 0.008870355470337533261253400710622787479284
 = 0.00887035547033753326125340071062278747]

```

(62)

```

> [r*cos(lter), r*sin(lter), ltr, rh=rhv]:
subs([t0=sourceT, r0=R, te0=sourceTe], %):
subs([branch=Ingoing, clock=Clockwise], %):
subs(r=rm, %):
simplify(%):
xm1, ym1, tm1, rrm1:= op(%): # (rm, rh, R)

```

```

> [xm-xm1, ym-ym1, tm-tm1, rrm-rrm1]:
subs(rm=80., %):
subs(R=7000., %):
subs(rh= evalf(subs(rm=80., rhv)), %):
#evalf(%, Digits+20):
evalf(%):
%;

```

$$[-8.77 \times 10^{-36}, -1.338 \times 10^{-35}, 6.96808911367645 \times 10^{-32}, 0. = 0.]$$

(63)

```

> evalf(subs(rm=80, rhv), 60);
R;
rh=rhv;
0.0088703554703375332612534007106227874792843908459855221144
R

```

$$rh = - \left( (-199306945455000 + 22468879468420441 \, rm - (504850544566405639328987546634481 \, rm^2 + 8956407469294884458788091310000 \, rm - 119169775519807035471075000000)^{1/2} ) \, rm \right) / (2 (-199306945455000 + 22468879468420441 \, rm))$$

(64)

```

> # Let us regard rh as an unknown and attach the equation rh=rhv(rm)

```

```

{xm-xm1, tm-tm1, rh=rhv}:
evalf(%):
fsolve(%, {rm=78, rh=0.008, R=7003}, {rm=70..90, rh=0.007..0.009,
R=6800..7200}, fulldigits):
sol:=%:
if type(sol, set) then
{xm-xm1, ym-ym1, tm-tm1}:
subs(sol, %):
#evalf(%, Digits+20):

```















```
# 13.0000100069483
# 13.000010006948296459
# 13.00001000694829645934878047262925045635
#
13.0000100069482964593487804726292504563469259080836097560060021808
48670229387471
#
13.0000100069482964593487804726292504563469259080836097560060021808
486702293874707942541871684323815155008409344186458339
```

13.00001000694829645934878047238083337551

(70)

```
> ltr:
subs(rh=rhv, %):
subs([t0=13, r0=5000], %):
subs(branch = Ingoing, %):
subs([rm=4000.], %):
evalf(%, Digits+20):
#Digits:=Digits-10:
#fsolve(%%-13, r=4995, fulldigits):
#fsolve(%%-13, r=4995, r=4900..5100):
#fsolve(%%-13, r=4995):
fsolve(%%-13, r=4995, r=4900..5100, fulldigits):
#Digits:=Digits+10:
evalf(%%):
evalf(%):
%;
```

0.00002668512761565531977733421446904376348052 EllipticPi $\left(\sqrt{r-4000}.$

(71)

$\sqrt{-\frac{0.5000016631897861154836962166410016806343}{-1. r + 0.008870355361338870508842177985926087791260}},$

$2.181080190355556137424704784938620323597 \times 10^{-17},$

$0.002105982651313650602634789894614995186713 \Big)$

+ 13.00001000694829645934878047238083337551

− 5.917637874533978315658143033909332208825

$\times 10^{-11} \text{ EllipticPi}\left(\sqrt{r-4000}.$

$\sqrt{-\frac{0.5000016631897861154836962166410016806343}{-1. r + 0.008870355361338870508842177985926087791260}},$

$1.999993347262985066571114515541467057995,$

$0.002105982651313650602634789894614995186713 \Big)$



$$\begin{aligned}
& - 0.00004002769142387666760690810604905868503905 \operatorname{EllipticF} \left( \sqrt{r - 4000.} \right. \\
& \left. \sqrt{-\frac{0.5000016631897861154836962166410016806343}{-1. r + 0.008870355361338870508842177985926087791260}}, \right. \\
& \left. 0.002105982651313650602634789894614995186713 \right) \\
& + 0.00001334259339621387660176484053482141727520 \operatorname{EllipticE} \left( \sqrt{r - 4000.} \right. \\
& \left. \sqrt{-\frac{0.5000016631897861154836962166410016806343}{-1. r + 0.008870355361338870508842177985926087791260}}, \right. \\
& \left. 0.002105982651313650602634789894614995186713 \right) \\
& - 4.219284467614066249451817817879501313856 \\
& \times 10^{-7} \sqrt{r} \sqrt{r - 4000.} (0.0001249998614008511570430253152799165266138 r \\
& + 0.5000005543965953718278987388803338935447) \\
& ^{1/2} \sqrt{-\frac{0.5000016631897861154836962166410016806343}{-1. r + 0.008870355361338870508842177985926087791260}}
\end{aligned}$$

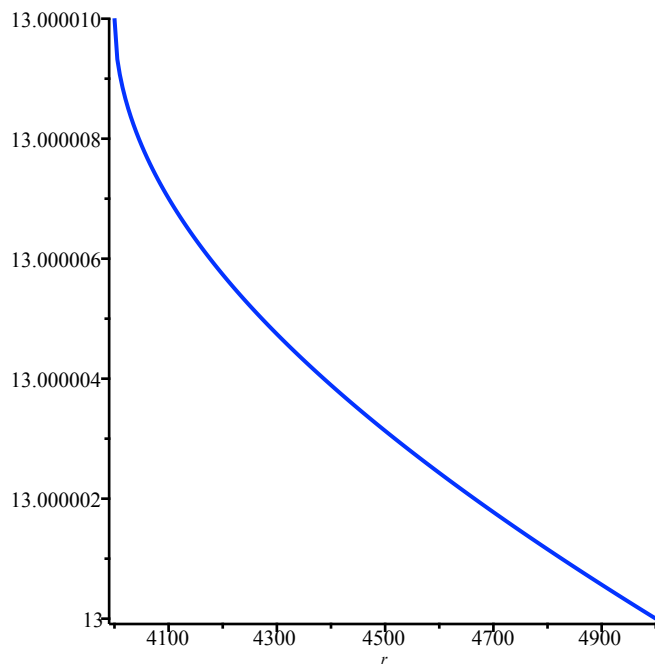
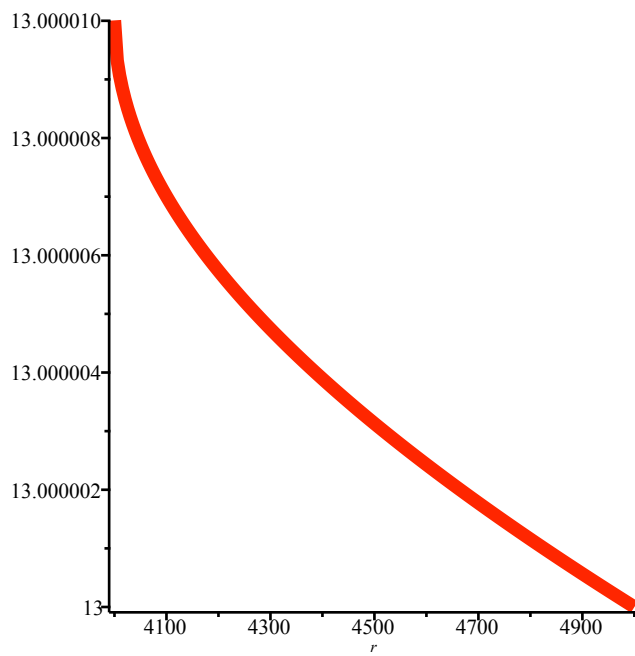
```

> ltr:
subs(rh=rhv, %):
subs([t0=13, r0=5000], %):
subs(branch = Ingoing, %):
subs([rm=4000.], %):
evalf(%):
P1:=plot(%, r=4000..5000, color=red, thickness=5):
evalf(%);

ltr:
subs(rh=rhv, %):
subs([t0=13, r0=5000], %):
subs(branch = Ingoing, %):
subs([rm=4000.], %):
evalf(%, Digits+20):
P2:=plot(%, r=4000..5000, color = blue):
evalf(%);

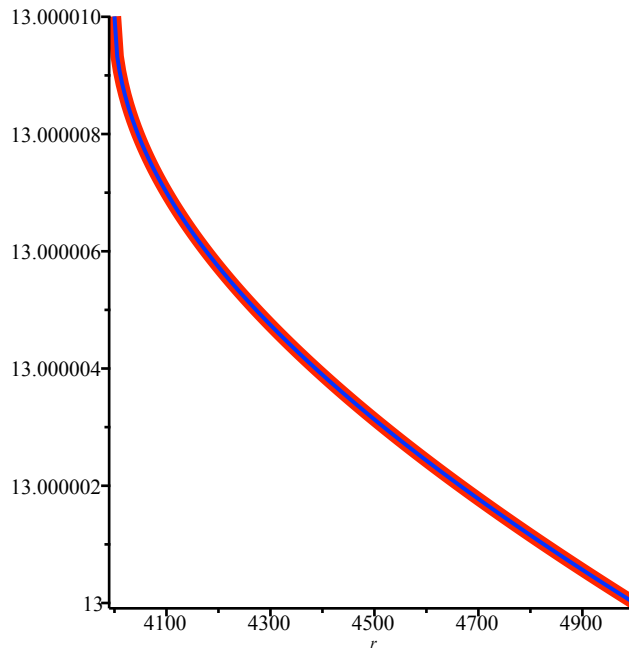
```





```
> display(P1, P2);
```





```
> # Test
lter:
subs(rh=rhv, %):
subs([te0=Pi/4, r0=5000], %):
subs([branch = Ingoing, clock=Clockwise], %):
subs([rm=4000], %):
subs(r=5000, %):
evalf(%):
%;

# 0.785398163397448
# 0.78539816339744830962
# 0.7853981633974483096156608458198757210492
#
0.78539816339744830961566084581987572104929234984377645524373614807
695410157155225
#
0.78539816339744830961566084581987572104929234984377645524373614807
6954101571552249657008706335529266995537021628320576662
#

lter:
subs(rh=rhv, %):
subs([te0=Pi/4, r0=5000], %):
subs([branch = Ingoing, clock=Clockwise], %):
subs([rm=4000], %):
subs(r=4000, %):
evalf(%):
%;

# 0.141896019726870
# 0.14189601972687309574
# 0.1418960197268730957269044953340687714983
#
0.14189601972687309572690449533406877149804469972737078694229940889
431121598109676
```



```
#
0.14189601972687309572690449533406877149804469972737078694229940889
4311215981096748541328487539441416277889566945152031824
#
```

```
0.7853981633974483096156608458198757210497
0.1418960197268730957269044953340687714980
```

(72)

## Light rays

```
> # Procedures for rays
>
#   Infalling rays are made of one branch only.

> InFallingRayThrough := proc(r0v, te0v, t0v, branchv, clockv)
    [r*cos(inlter), r*sin(inlter), inltr]:
    subs([r0=r0v, te0=te0v, t0=t0v], %):
    subs([branch=branchv, clock=clockv], %):
    evalf(%, Digits+20):
    #evalf(%):
end:

> ScatteringBranchThrough := proc(r0v, te0v, t0v, branchv, clockv)
    [r*cos(lter), r*sin(lter), ltr]:
    subs([r0=r0v, te0=te0v, t0=t0v], %):
    subs([branch=branchv, clock=clockv], %):
    evalf(%, Digits+20):
    #evalf(%):
end:

> ScatteringOtherBranchThrough := proc(r0v, te0v, t0v, branchv,
    clockv) local Tm, Tem, Rm:
    [r, lter, ltr]:
    subs([r0=r0v, te0=te0v, t0=t0v], %):
    subs([branch=branchv, clock=clockv], %):
    subs(r=rm, %):
    evalf(%, Digits+20):
    Rm, Tem, Tm := op(%):
    ScatteringBranchThrough(Rm, Tem, Tm, -branchv, clockv)
end:

> ScatteringRayThroughEm := proc(rmv, tem, tm, clockv := Clockwise)
:
local crm0, crm1, eq1, eq2, dt:

    ScatteringBranchThrough(rmv, tem, tm, Ingoing, clockv):
    subs(rm=rmv, %):
    eq1:=%:
    subs(r= maxS(2), %):
```



```

dt:= evalf(%[3]):

ScatteringOtherBranchThrough(rmv, tem, tm, Ingoing, clockv):
subs(rm=rmv, %):
eq2:=%:
subs(r= maxS(2), %)[3]- dt:
dt:= evalf(%):
printf("dt= %g\n", dt);

[op(eq1), r=rmv..5*10^6]:
crm0:= spacecurve(%, axes=box, color = black, linestyle = solid)
:
[op(eq2), r=rmv..5*10^6]:
crm1:= spacecurve(%, axes=box, color = black, linestyle = solid)
:
[crm0, crm1], dt:
end:

> ScatteringRayThrough := proc(rmv, clockv := Clockwise):
  ScatteringRayThroughEm(rmv, 0, 0, clockv):
end:

> rayMin:= proc()
  local sourceR, targetR, t:
  global SearchSignal:

  sourceR := SearchSignal[Risr]:
  targetR := SearchSignal[TargetP][1]:
  t := SearchSignal[RayType]:

  if t = Infalling or t = SameScattering then
    return min(sourceR, targetR);
  elif t = OtherScattering then
    return SearchSignal[Risrm];
  else
    printf("Ray type unknown %d. \n", t);
  end if:
end:

```

## ▼ Procedure for plotting

```

> # Try to get a segment from Sat 0 (r=13 on branch 1) to Sat 2

> AddPlotPointCartesian:= proc(P, PSat, G)
  local col:
  global Plots:
  if PSat = -1 then
    col := black:
  elif PSat = 0 then
    col := red:
  elif PSat = 1 then

```



```

    col := blue:
  elif PSat = 2 then
    col := green:
  end if:
  P:
  #pointplot3d(% , color=col, symbol = solidcircle, symbolsize =
6+3*G):
  pointplot3d(% , color=col, symbol = solidcircle, symbolsize = 6):
  Plots := [op(Plots), %]:
end:

```

```

> AddPlotPointPolar:= proc(P, PSat, G)  # Polar
  local R, Te, T, col:
  global Plots:
  R, Te, T := op(P):
  [R*cos(Te), R*sin(Te), T]:
  AddPlotPointCartesian(% , PSat, G):
end:

```

```

> AddPlotRayScatteringSame := proc(Through, rlv, S1, G1, rmv,
branchv, clockv)
  local eq:
  global Plots:
  ScatteringBranchThrough(Through[1], Through[2], Through[3],
branchv, clockv):
  subs(rh=rhv, %):
  subs(rm=rmv, %):
  eq:= %:
  subs(r=rlv, eq):
  AddPlotPointCartesian(% , S1, G1+1):
  if rlv > Through[1] then
    eq:=[op(eq), r=Through[1]..rlv]:
  else
    eq:=[op(eq), r=rlv..Through[1]]:
  end if:
  spacecurve(eq, axes=box, color = black, linestyle = solid,
thickness = 1):
  Plots := [op(Plots), %]:
end:

```

```

> AddPlotRayScatteringOther := proc(Through, rlv, S1, G1, rmv,
branchv, clockv)
  local eq:
  global Plots:
  ScatteringBranchThrough(Through[1], Through[2], Through[3],
branchv, clockv):
  subs(rh=rhv, %):
  subs(rm=rmv, %):
  eq:= %:
  [op(eq), r=Through[1]..rmv]:
  spacecurve(% , axes=box, color = black, linestyle = solid,
thickness = 1):
  Plots := [op(Plots), %]:

  ScatteringOtherBranchThrough(Through[1], Through[2], Through[3],

```



```

branchv, clockv):
    subs(rh=rhv, %):
    subs(rm=rmv, %):
    eq:= %:
    subs(r=r1v, eq):
    AddPlotPointCartesian(%, S1, G1+1):
    [op(eq), r=rmv..r1v]:
    spacecurve(%, axes=box, color = black, linestyle = solid,
thickness = 1):
    Plots := [op(Plots), %];
end:

```

```

> AddPlotRayInfalling := proc(Through, r1v, S1, G1, K2v, branchv,
clockv)
    local eq:
    global Plots:
    #printf("Step 1\n");
    InFallingRayThrough(Through[1], Through[2], Through[3], branchv,
clockv):
    subs(K2=K2v, %):
    eq:= %:
    subs(r=r1v, eq):
    AddPlotPointCartesian(%, S1, G1):
    if r1v > Through[1] then
        eq:=[op(eq), r=Through[1]..r1v]:
    else
        eq:=[op(eq), r=r1v..Through[1]]:
    end if:
    spacecurve(eq, axes=box, color = black, linestyle = solid,
thickness = 1):
    Plots := [op(Plots), %];
    #printf("Step out\n");
end:

```

```

> PlotLinear := proc()
    local sourceSat, sourceBranch, rGuessMax, rGuessMin, Previous,
targetP, St:
    global OtherPlots:
    St := time():
    sourceSat := Get(SourceSat):
    sourceBranch := Get(SourceBranch):
    rGuessMax := Get(IndrGuessMax):
    rGuessMin := Get(IndrGuessMin):
    Previous:= Get(PreviousSourceBranch):
    targetP := Get(TargetP):

    [xS(sourceSat, sourceBranch), yS(sourceSat, sourceBranch), tS
(sourceSat, sourceBranch)]:
    subs([r=rGuessMax], %):
    pointplot3d(%, color=black, symbol = solidcircle, symbolsize =
9):
    OtherPlots := [op(OtherPlots), %]:
    if Previous then
        [xS(sourceSat, sourceBranch-1), yS(sourceSat,
sourceBranch-1), tS(sourceSat, sourceBranch-1)]:
        subs([r=rGuessMin], %):
    end if:
end:

```



```

        pointplot3d(% , color=black, symbol = solidcircle, symbolsize
= 9):
        OtherPlots := [op(OtherPlots), %]:
    else
        [xS(sourceSat, sourceBranch), yS(sourceSat, sourceBranch),
tS(sourceSat, sourceBranch)]:
        subs([r=rGuessMin],%):
        pointplot3d(% , color=black, symbol = solidcircle, symbolsize
= 9):
        OtherPlots := [op(OtherPlots), %]:
    end if:
    [s*targetP[1]*cos(targetP[2]) + (1-s)*xS(sourceSat,
sourceBranch),
    s*targetP[1]*sin(targetP[2]) + (1-s)*yS(sourceSat,
sourceBranch),
    targetP[3],
    s=0..1
]:
    subs(r=rGuessMax, %):
    spacecurve(% , axes=box, color = black, linestyle = solid,
thickness = 1):
    OtherPlots := [op(OtherPlots), %]:
    printf("Time Approximations %g.\n\n", time()-St);
end:

```

```

> AddPlotRay:= proc()
    local sourceSat, targetP, risr,risrm, risK2, Gen, rayBranchv,
rayClockv, rayTypev, St:

    St:=time():
    if Get(Risr) = none then
        printf("No solution found\n");
    else
        sourceSat:= Get(SourceSat):
        targetP:= Get(TargetP):
        risr:= Get(Risr):
        risrm:= Get(Risrm):
        risK2:= Get(RisK2):
        Gen:= GetGeneration():
        rayBranchv:= Get(RayBranch):
        rayClockv:= Get(RayClock):
        rayTypev:= Get(RayType):

        if ProducePlots then
            if rayTypev = Infalling then
                AddPlotRayInfalling(targetP, risr, sourceSat,
GetGeneration(), risK2, rayBranchv, rayClockv):
            elif rayTypev = SameScattering then
                AddPlotRayScatteringSame(targetP, risr, sourceSat,
GetGeneration(), risrm, rayBranchv, rayClockv):
            elif rayTypev = OtherScattering then
                AddPlotRayScatteringOther(targetP, risr, sourceSat,
GetGeneration(), risrm, rayBranchv, rayClockv):
            else
                printf("Ray type unknown\n"):
            end if:
        end if:
    end if:
end:

```



```

        end if:
    end if:
    printf("Time Plot %g s.\n", time()-St);
end:

```

```
> # Test
```

```

InFallingRayThrough(5000, Pi/4, 13, Ingoing, Clockwise):
subs([K2=2/3*K2crit], %):
subs([r=3/2*alv], %):
evalf(%):
#Real(%):
%;

```

```
[0.006753650832658289110492708704834141286334,
```

(73)

```

-0.01146409219959293458726220347622928362856,

```

```

13.00001667863762159014472123266724105055]

```

```
> # this is NOT very regular and relatively fast and robust
```

```
> ScatteringBranchThrough(5000, Pi/4, 13, Ingoing, Clockwise):
```

```

subs(rh=rhv, %):
subs(rm=4000., %):
subs(r=5000., %):
evalf(%, Digits+20):
%[1];

```

```

ScatteringBranchThrough(5000, Pi/4, 13, Ingoing, Clockwise)[1] =
ScatteringOtherBranchThrough(5000, Pi/4, 13, Ingoing, Clockwise)[1]
:

```

```

subs(rh=rhv, %):
subs(rm=4000., %):
subs(r=4000., %):
evalf(%, Digits+20):
%;

```

```
ScatteringOtherBranchThrough(5000, Pi/4, 13, Ingoing, Clockwise):
```

```

subs(rh=rhv, %):
subs(rm=4000., %):
subs(r=5000., %):
evalf(%, Digits+20):
%[1];

```

```
3535.53390593273762200422181052424519642568785391720592383860
```

```
3959.79856005754575396418084386035267550845102316102114177839
```

```
= 3959.79856005754575396418084386035267550845102316102114177839
```

```
4384.05706731970397754626260654312539796270280057122159162479
```

(74)

```
> ScatteringBranchThrough(5000, Pi/4, 13, Ingoing, Clockwise):
```

```

subs(rh=rhv, %):
subs(rm=4000., %):
subs(r=5000., %):
evalf(%, Digits+20):

```



```
ScatteringBranchThrough(5000, Pi/4, 13, Ingoing, Clockwise)[2] =
ScatteringOtherBranchThrough(5000, Pi/4, 13, Ingoing, Clockwise)[2]
:
subs(rh=rhv, %):
subs(rm=4000., %):
subs(r=4000., %):
evalf(%, Digits+20):
%;
```

3535.53390593273762200422181052424519642408573652990753709360

= 565.681327043934881053447799446184147363128886193245649097492

(75)

```
ScatteringBranchThrough(5000, Pi/4, 13, Ingoing, Clockwise)[3] =
ScatteringOtherBranchThrough(5000, Pi/4, 13, Ingoing, Clockwise)[3]
:
subs(rh=rhv, %):
subs(rm=4000., %):
subs(r=4000., %):
evalf(%, Digits+20):
%;
```

12.9999999999999999999999997515831103884990219913970367879

$$= 13.00001000694829645934878047213241629467$$

(76)

```
> #13.000010006948296459349 08955520080230862
#13.000010006948296459349 11518252446975528
#13.000010006948296459348780472629250456346925908083609756006002180
848 627745658915
#13.000010006948296459348780472629250456346925908083609756006002180
```



848 585261930359

```
> #
# Some plots to test
#

> PM := pointplot3d([5*10^6*cos(Pi/4), 5*10^6*sin(Pi/4), 13], color=
blue, symbol = solidcircle, symbolsize = 10):

> ScatteringBranchThrough(5*10^6, Pi/4, 13, Ingoing, Clockwise):
subs(rh=rhv, %):
subs(rm=4*10^6, %):
[op(%), r=4*10^6..5*10^7]:
cr1:= spacecurve(% , axes=box, color = black, linestyle = solid):

> ScatteringOtherBranchThrough(5*10^6, Pi/4, 13, Ingoing, Clockwise):
subs(rh=rhv, %):
subs(rm=4*10^6, %):
[op(%), r=4*10^6..5*10^7]:
cr2:= spacecurve(% , axes=box, color = black, linestyle = solid):

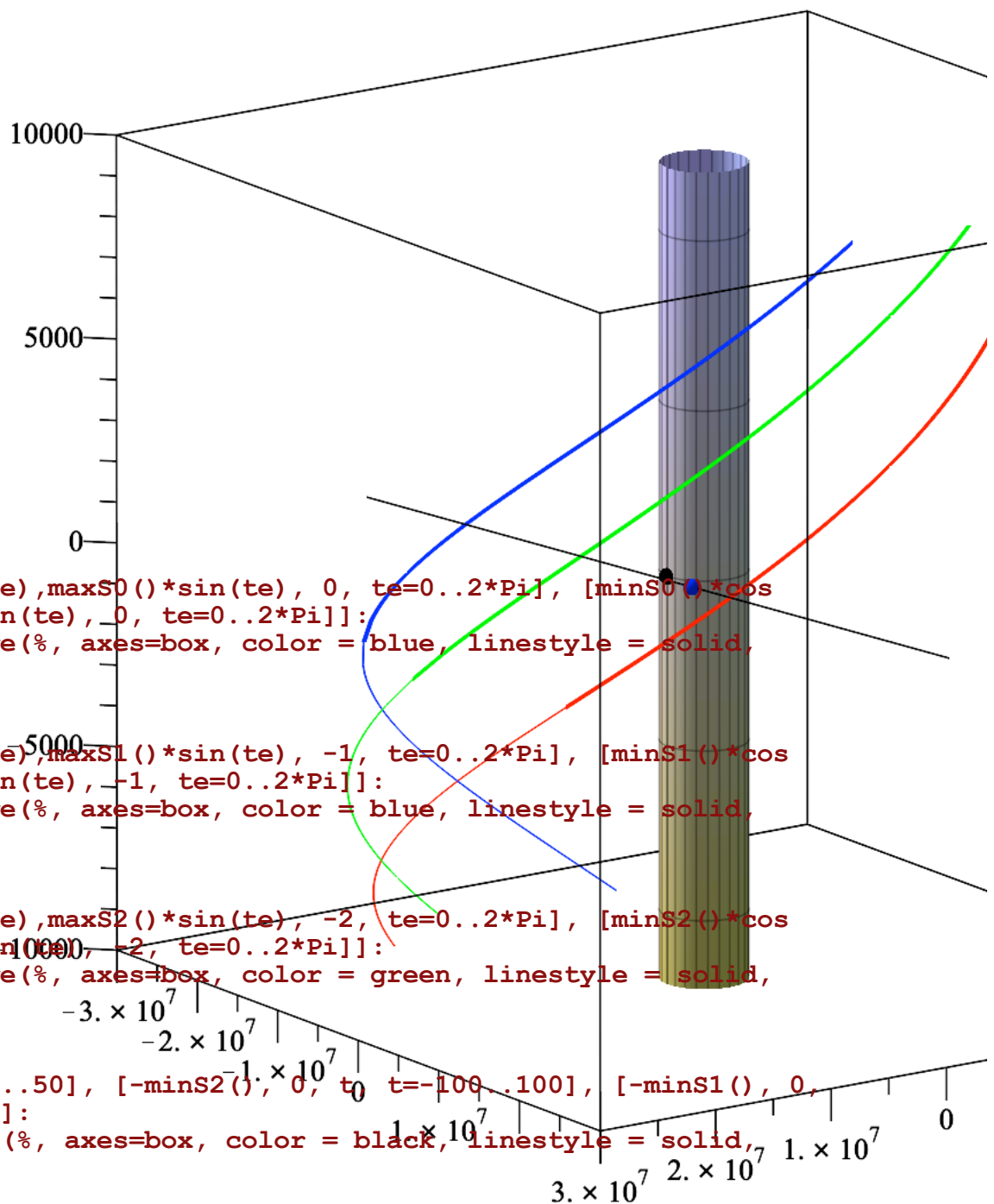
> display(BH, S,
user,
c0_1n,c0_0,c0_1,c1_1n, c1_0,c1_1, c2_1n,c2_0,c2_1,

PM, cr1, cr2,

view=[ -3*10^7..3*10^7, -3*10^7..3*10^7, -10000..10000]

);
```





```

> [[maxS0()*cos(te),maxS0()*sin(te), 0, te=0..2*Pi], [minS0()*cos
(te),minS0()*sin(te), 0, te=0..2*Pi]]:
cs0:= spacecurve(%, axes=box, color = blue, linestyle = solid,
thickness=1):

[[maxS1()*cos(te),maxS1()*sin(te), -1, te=0..2*Pi], [minS1()*cos
(te),minS1()*sin(te), -1, te=0..2*Pi]]:
cs1:= spacecurve(%, axes=box, color = blue, linestyle = solid,
thickness=1):

[[maxS2()*cos(te),maxS2()*sin(te), -2, te=0..2*Pi], [minS2()*cos
(te),minS2()*sin(te), -2, te=0..2*Pi]]:
cs2:= spacecurve(%, axes=box, color = green, linestyle = solid,
thickness=1):

[[x,0, 1, x=-50..50], [-minS2(), 0, t, t=-100..100], [-minS1(), 0,
t, t=-100..100]]:
cs:= spacecurve(%, axes=box, color = black, linestyle = solid,
thickness=1):

> P1 := pointplot3d([minS2(), 0, 0], color=black, symbol =
solidcircle, symbolsize = 10):
P2 := pointplot3d([-minS2(), 0, 0], color=black, symbol =

```



```
solidcircle, symbolsize = 10):
```

```
> rmv:= 4*10^3;
ScatteringBranchThrough(minS2(), 0, 0, Ingoing, Clockwise):
subs(rh=rhv, %):
subs(rm=rmv, %):
[op(%), r=rmv..50*10^6]:
r1:= spacecurve(%, axes=box, color = black, linestyle = solid):

ScatteringOtherBranchThrough(minS2(), 0, 0, Ingoing, Clockwise):
subs(rh=rhv, %):
subs(rm=rmv, %):
[op(%), r=rmv..50*10^6]:
r2:= spacecurve(%, axes=box, color = black, linestyle = solid):

ScatteringBranchThrough(minS2(), 0, 0, Ingoing, Clockwise):
subs(rh=rhv, %):
subs(rm=rmv, %):
subs(r=maxS2(), %):
%[3]:
evalf(%):
tt0:=%:

ScatteringOtherBranchThrough(minS2(), 0, 0, Ingoing, Clockwise):
subs(rh=rhv, %):
subs(rm=rmv, %):
subs(r=maxS2(), %):
%[3]-tt0:
evalf(%):
MaxT22:= %; # That is the worst case scenario, with some
overestimation.
#It should be better than this.
```

$rmv := 4000$

$MaxT22 := 0.1740470721371413957087506466848811112107$

(77)

```
> display(BH, S,
#user,
c0_1n,c0_0,c0_1,
c1_1n,c1_0,c1_1,
c2_1n,c2_0,c2_1,
P1, P2,
cs0,cs1,cs2, cs,
r1, r2,
#op(CC),
view=[ -3*10^7..3*10^7, -3*10^7..3*10^7, -5000..5000]);
```



```
> MaxT:= 1/2;
```

$$MaxT := \frac{1}{2}$$

```
> # Then for any exchange, we should not need to go back in time
# further than 90 and closer to BH than 3.5
#
# I'm not saying that being stricter on limits gets us faster
# though.
# I would be much more first test than decide about it.
```

## Procedures for solving signals

```
> Equations:= proc()
  local rayTypev, rayBranchv, rayClockv, sourceSat, sourceBranch,
  targetR, targetTe, targetT:
  local eqs, xx1, yy1, tt1, xx2, yy2, tt2, rs, tes, ts:

  rayTypev := Get(RayType):
  rayBranchv := Get(RayBranch):
  rayClockv := Get(RayClock):
  sourceSat := Get(SourceSat):
  sourceBranch := Get(SourceBranch):
```



```

targetR, targetTe, targetT := op(Get(TargetP)):

rs:= rSat(sourceSat, s, sourceBranch):
tes:= teSat(sourceSat, s, sourceBranch):
ts:= tSat(sourceSat, s, sourceBranch):

if rayTypev = Infalling then
    InFallingRayThrough(targetR, targetTe, targetT, rayBranchv,
rayClockv):
    xx1, yy1, tt1 := op(%): # (r, K2)

    InFallingRayThrough(rs, tes, ts, rayBranchv, rayClockv):
    subs(s=r, %): # (r, rm)
    xx2, yy2, tt2 := op(%): # (r, s, K2)
    [xx1-xx2, yy1-yy2, tt1-tt2]: # (r, K2)
    eqs:=%:
elif rayTypev = SameScattering then
    ScatteringBranchThrough(targetR, targetTe, targetT,
rayBranchv, rayClockv):
    xx1, yy1, tt1 := op(%): # (rh, rm, r)

    ScatteringBranchThrough(rs, tes, ts, rayBranchv, rayClockv):
    xx2, yy2, tt2 := op(%):
    [xx1-xx2, yy1-yy2, tt1-tt2]: # (s, rm)
    subs(rh=rhv, %):
    subs(r=rm, %):
    subs(s=r, %): # (r, rm)
    eqs:=%:
elif rayTypev = OtherScattering then
    ScatteringBranchThrough(targetR, targetTe, targetT,
rayBranchv, rayClockv):
    xx1, yy1, tt1 := op(%):

    ScatteringBranchThrough(rs, tes, ts, -rayBranchv, rayClockv):
    xx2, yy2, tt2 := op(%):
    [xx1-xx2, yy1-yy2, tt1-tt2]: # (s, rm)
    subs(rh=rhv, %):
    subs(r=rm, %):
    subs(s=r, %): # (r, rm)
    eqs:=%:
else
end if:
eqs:
end:

```

```

> SolveOne:= proc()
    local sourceSat, sourceBranch:
    local targetR, targetTe, targetT:
    local sv, rGuessMax, rGuessMin, rGuess, rmGuess, kv, scos,
Previous, Crossing:
    local rayBranchv, rayClockv, rayTypev, SolveTypev:
    local eqs, xx, yy, tt, avoidSols, pstring, ris, interval, Delta,
St:
    local SignalSave:

```



```

St:= time():
sourceSat := Get(SourceSat):
sourceBranch := Get(SourceBranch):
sv := Get(Indsv):
rGuessMax := Get(IndrGuessMax):
rGuessMin := Get(IndrGuessMin):
Previous := Get(PreviousSourceBranch):
Crossing := Get(SourceCrossing):
rmGuess := Get(IndrmGuess):
kv := Get(Indkv):
scos := Get(Indscos):
rayTypeev := Get(RayType):
SolveTypeev := Get(SolveType):
targetR, targetTe, targetT := op(Get(TargetP)):
rayBranchv := Get(RayBranch):
rayClockv := Get(RayClock):

if DebugFlowControlOn then
    pstring:= "I search for ":
    if rayTypeev = Infalling then
        pstring := cat(pstring,"an infalling ray "):
    elif rayTypeev = SameScattering then
        pstring := cat(pstring,"an scattering ray on same branch "):
    elif rayTypeev = OtherScattering then
        pstring := cat(pstring,"an scattering ray on opposite
branches "):
    else
        pstring := cat(pstring,"an unknown ray "):
    end if:
    if sv<0 then
        pstring := cat(pstring,"with sv<0 (%g) "):
    elif sv>0 and sv<1 then
        pstring := cat(pstring,"with 0<sv<1 (%g) "):
    else # sv>1
        pstring := cat(pstring,"with sv>1 (%g) "):
    end if:

    if targetR < minS(sourceSat) then
        pstring := cat(pstring,"|    P <--- S \n"):
    elif targetR > minS(sourceSat) then
        pstring := cat(pstring,"|    S ---> P \n"):
    else
        pstring := cat(pstring,"mixing source and target.\n"):
    end if:
    printf(pstring, sv);
    printf("rGuessMin=%g    rGuessMax=%g    rmGuess=%g    k=%g    scos=
%g\n", rGuessMin, rGuessMax, rmGuess, kv, scos);
    pstring := "branch ":
    if rayBranchv = Outgoing then
        pstring := cat(pstring," outgoing at target, "):
    else
        pstring := cat(pstring," ingoing at target, "):
    end if:
    if rayClockv = Clockwise then
        pstring := cat(pstring,"Clockwise\n"):
    else
        pstring := cat(pstring,"Counterclockwise\n"):

```



```

    end if:
    printf(pstring) ;
end if:

if rayTypev = Infalling then
    interval:={r= Get(Intervalr1), K2 = Get(IntervalK2)}:
elif rayTypev = SameScattering then
    interval:={r= Get(Intervalr1), rm = Get(Intervalrm)}:
elif rayTypev = OtherScattering then
    interval:={r= Get(Intervalr1), rm = Get(Intervalrm)}:
else
    interval:={}:
end if:
Equations():
subs(rh=rhv, %):
#evalf(%, Digits+20):
evalf(%):
eqs:= %:
avoidSols := {}:
ris := "searching":
do
    if rayTypev = Infalling then
        printf("(infalling) fsolve({eqs1, eqs3}, {r=%a, K2=0}, %a,
avoid=%a);\n", rGuessMax, interval, avoidSols);
        fsolve({eqs[1], eqs[3]}, {r=rGuessMax, K2=0}, interval,
avoid=avoidSols, fulldigits):
        ris:= %:
    else
        # il lato alto di Intervalr1
        #rGuess:= op(2, Get(Intervalr1)):
        #printf("Step1 (%d)\n", Digits);
        rGuess:= Get(IndrGuessMax):
        #interval:= {r=myRange(Get(IndrGuessMin), rGuess), rm=
rmGuess-500000..rmGuess+500000}:

        printf("(Scattering) fsolve(eqs, {r=%a, rm=%a}, %a, avoid=%a)
; \n", evalf(rGuess), evalf(rmGuess), evalf(interval), avoidSols)
;

        fsolve({eqs[1], eqs[3]}, {r=rGuess, rm=rmGuess}, interval,
avoid=avoidSols, fulldigits):
        #fsolve({eqs[1], eqs[3]}, {r=rGuess, rm=rmGuess}, interval,
avoid=avoidSols):
        #fsolve({eqs[1], eqs[3]}, {r=rGuess, rm=rmGuess}, avoid=
avoidSols):
        ris:= %:
    end if:
    if type(ris, 'set') then
        eqs[2]:
        subs(ris, %):
        Delta := abs(evalf(%)):
        if Delta > 10^(-Digits+20) then
            avoidSols:= {op(avoidSols), ris}:                # forse posso
restringere l'intervallo?
            if DebugSolutionsOn then
                printf("Rejected {r=%g, rm=%g} for Delta=%g\n", subs
(ris, r), subs(ris, rm), Delta);

```



```

        if DebugTimeOn then
            printf("in partial time = %g s \n", time()-St);
        end if:
    end if:
    ris:="rejected":
else
    if DebugSolutionsOn then
        if rayTypeev = Infalling then
            print(ris);
            printf("Accepted {r=%g, K2=%g} with Delta=%g\n",
subs(ris, r), subs(ris, K2), Delta);
        else
            printf("Accepted {r=%g, rm=%g} with Delta=%g\n",
subs(ris, r), subs(ris, rm), Delta);
        end if:
        subs(ris, eqs):
        printf("Equations at solution: %a\n", %):
    end if:
end if:
else
    ris:= {}:
end if:
until type(ris, set):
if nops(ris)> 0 then
    [r, teSat(sourceSat, r, sourceBranch), tSat(sourceSat, r,
sourceBranch)]:
    subs(r=subs(ris, r), %):
    Set(RisP, %):
    Set(Risr, subs(ris, r)):
    Set(RisBranch, sourceBranch):

    if rayTypeev = Infalling then
        Set(RisK2, subs(ris, K2)):
        ris:= [sourceSat, subs(ris, r), subs(ris, K2), sourceBranch,
rayTypeev];
    else
        Set(Risrm, subs(ris, rm)):
        ris:= [sourceSat, subs(ris, r), subs(ris, rm), sourceBranch,
rayTypeev];
    end if:
else
    # if no solution found
    # Se crossing vando a cercare sull'altro Ramo
    # Per -clock e infalling/scattering invece lascio fare al
chiamante
    #
    if Crossing then
        if DebugFlowControlOn and DebugTimeOn then
            printf("Turn to the other branch [nested SolveOne()]\n");
        end if:
        SignalSave := SaveSearchSignal():
        SetPreviousSourceBranch():
        ris := SolveOne():
        RestoreSearchSignal(SignalSave):
    else
        # ris:= {}:
    end if:

```



```

end if:
if DebugFlowControlOn and DebugTimeOn then
    printf("Solution in %gs\n\n", time()-St);
end if:
ris;
end:

```

```

> CompareSolution:= proc()
    local Newrm, Bestrm:
    global SearchSignal, BestKnownSolution:

    Newrm := SearchSignal[Risrm]:
    if not(Newrm = none) and BestKnownSolution = none then
        BestKnownSolution := SearchSignal:
    elif not(Newrm = none) then
        Bestrm := BestKnownSolution[Risrm]:
        if Bestrm < Newrm then
            BestKnownSolution := SearchSignal:
        end if:
    end if:
end:

```

```

> ValidateSolution:= proc() # -> Bool
    local sourceR, targetR, t:
    global SearchSignal, BestKnownSolution:

    sourceR := SearchSignal[Risr]:
    targetR := SearchSignal[TargetP][1]:
    if sourceR = none then
        return false:
    end if:
    if rayMin() > 4*alv then
        BestKnownSolution := SearchSignal:
        return true:
    end if:
    CompareSolution():
    return false:
end:

```

```

> AcceptBestSolution := proc()
    global SearchSignal, BestKnownSolution:
    SearchSignal := BestKnownSolution:
end:

```

```

> #(Infalling0 | SameScattering1 | OtherScattering2) + (Ingoing0 +
    Outgoing1) + (Orario0 | Antitorario1)

```

```

Sol2Code:=proc()
    local r:
    global SearchSignal:

    if Get(RayClock) = Clockwise then
        r:= 0*1:
    else

```







```

Clockwise
  Set(RayType, SameScattering);
  Set(RayClock, Clockwise);
  Set(RayBranch, Ingoing);
elif n = 6 then                                # SameScattering Ingoing
Counter-clockwise
  Set(RayType, SameScattering);
  Set(RayClock, Counter-clockwise);
  Set(RayBranch, Ingoing);
elif n = 7 then                                # SameScattering Outgoing
Clockwise
  Set(RayType, SameScattering);
  Set(RayClock, Clockwise);
  Set(RayBranch, Outgoing);
elif n = 8 then                                # SameScattering Outgoing
Counter-clockwise
  Set(RayType, SameScattering);
  Set(RayClock, Counter-clockwise);
  Set(RayBranch, Outgoing);

elif n = 9 then                                # OtherScattering Ingoing
Clockwise
  Set(RayType, OtherScattering);
  Set(RayClock, Clockwise);
  Set(RayBranch, Ingoing);
elif n = 10 then                               # OtherScattering Ingoing
Counter-clockwise
  Set(RayType, OtherScattering);
  Set(RayClock, Counter-clockwise);
  Set(RayBranch, Ingoing);
elif n = 11 then                               # OtherScattering Outgoing
Clockwise
  Set(RayType, OtherScattering);
  Set(RayClock, Clockwise);
  Set(RayBranch, Outgoing);
elif n = 12 then                               # OtherScattering Outgoing
Counter-clockwise
  Set(RayType, OtherScattering);
  Set(RayClock, Counter-clockwise);
  Set(RayBranch, Outgoing);
else
  printf("Type Code unknown %d\n", n);
end if:
end:

```

```

> SolveHard:= proc(TargetSat, TargetPoint, SourceSat, Gen, hint:=
none)
  global Plots, OtherPlots, TypeToDo, BestKnownSolution:
  local sol, OriginalSS, loop, n, targetR, sourceMin, sourceMax,
recCodes, St:
  St:= time():
  printf("%d --> %d  target = %a\n", SourceSat, TargetSat,
TargetPoint);
  sol:=[]:

  Plots:=[];

```



```

OtherPlots:=[];
ClearSearchSignal():
ClearTypeToDo();

targetR:= TargetPoint[1]:
sourceMin:= minS(SourceSat):
sourceMax:= maxS(SourceSat):

SetTargetPoint(TargetSat, TargetPoint):
SetSourceSat(SourceSat):
SetGeneration(Gen):
LinearGuess():
PlotLinear():

# Plot the ray through Target at rmGuess

[TargetPoint[1]*cos(TargetPoint[2]), TargetPoint[1]*sin
(TargetPoint[2]), TargetPoint[3]]:
pointplot3d(%, color=orange, symbol = solidcircle, symbolsize =
9):
OtherPlots:=[op(OtherPlots), %]:

OriginalSS := SaveSearchSignal():
if type(hint, list) then
    # hint ha anche informazioni su guess e forse intervals
    UseHint(hint):
    sol := SolveOne():
    DoneCode():
    if ValidateSolution() then
        AddPlotRay():
        printf("Exiting SolveHard() after %g", time()-St);
        return sol:
    end if:
    if type(BestKnownSolution, list) then
        AcceptBestSolution():
        return sol:
    end if:
end if:
RestoreSearchSignal(OriginalSS):
OriginalSS := SaveSearchSignal():
loop:= true:
if targetR > sourceMax then
    recCodes:= [12, 11, 7, 3, 4, 8]:
    n:= Sol2Code():
    recCodes:= CycleUntil(recCodes, n);
elif targetR < sourceMax then
    recCodes:= [12, 11, 5, 1, 2, 6]:
    n:= Sol2Code():
    recCodes:= CycleUntil(recCodes, n);
else
    printf("Tagent is Source ring should not happen\n"):
    printf("Exiting SolveHard() after %g", time()-St);
    return []:
end if:
BestKnownSolution := none:
do
    n:= recCodes[1]:

```



```

printf("Try code %d\n", n);
Code2Sol(n);
sol := SolveOne();
DoneCode():
if nops(sol) > 0 then
    if ValidateSolution() then
        AddPlotRay():
        printf("Exiting SolveHard() after %g", time()-St);
        return sol:
    end if:
end if:
recCodes := [op(2..nops(recCodes), recCodes)]:
until nops(recCodes) = 0:

printf("Warning: Going to unrecommended types\n");
for n from 1 to 12 do
    if TypeToDo[n] then
        printf("Try unrecommended code %d\n", n);
        Code2Sol(n);
        sol := SolveOne():
        DoneCode():
        if ValidateSolution() then
            AddPlotRay():
            printf("Exiting SolveHard() after %g", time()-St);
            return sol:
        end if:
    end if:
end do:
printf("Could not find a validate solution\n");
AcceptBestSolution():

if SearchSignal = none then
    printf("No solution\n");
    sol:=[]:
else # [sourceSat, subs(ris, r), subs(ris, rm),
sourceBranch, rayTypev];
    if Get(RayType) = Infalling then
        sol:=[Get(SourceSat), Get(Risr), Get(RisK2), Get
(SourceBranch), Get(RayType)]:
    else
        sol:=[Get(SourceSat), Get(Risr), Get(Risrm), Get
(SourceBranch), Get(RayType)]:
    end if:
end if:
printf("Exiting SolveHard() after %g\n", time()-St);
# Possiamo ricostruire la BestSolution if any se no
sol:
end:

```

>

> # perché i - su clock e branch?

```

DrawGuess := proc(Through, branchv, clockv, rmv)
global Plots:
local eq:

```



```

# deal with infalling as well

    ScatteringBranchThrough(Through[1], Through[2], Through[3],
branchv, clockv ):
    subs(rh=rhv, %):
    subs(rm=rmv, %):
    subs(r=Through[1], %):
    printf("P=%a= %a\n", %, [Through[1]*cos(Through[2]), Through[1]
*sin(Through[2]), Through[3]]);
    ScatteringBranchThrough(Through[1], Through[2], Through[3],
branchv, clockv):
    subs(rh=rhv, %):
    subs(rm=rmv, %):
    [op(%), r=rmv..Through[1]]:
    spacecurve(%, axes=box, color = orange, linestyle = solid,
thickness = 3):
    Plots := [op(Plots), %];

    ScatteringOtherBranchThrough(Through[1], Through[2], Through
[3], branchv, clockv):
    subs(rh=rhv, %):
    subs(rm=rmv, %):
    [op(%), r=rmv..3*10^7]:
    spacecurve(%, axes=box, color = orange, linestyle = solid,
thickness = 3):
    Plots := [op(Plots), %];
end:

> AcceptSolution:= proc()
    local sourceSat, gen, k;
    global NextSignalAvailable, ListPlots, Plots, OtherPlots,
ListP, ListHints, ListTau:

    sourceSat := SearchSignal[SourceSat];
    gen := SearchSignal[Gen];

    ListPlots[gen+1] := [op(ListPlots[gen+1]), op(Plots)]:
    Plots:= []:
    OtherPlots:= []:

    k := 3*NextSignalAvailable[sourceSat+1] +sourceSat + 1:
    ListHints[k]:= CreateHint():
    ListP[k]:= Get(RisP):
    ListTau[k]:= tauSat(sourceSat, Get(Risr), Get(RisBranch));
    NextSignalAvailable[sourceSat+1]:= NextSignalAvailable
[sourceSat+1] +1;
    k:
end:

> SetBasePoint:= proc(Sat, rv, branchv)
    local n:
    global Plots, OtherPlots, NextSignalAvailable, ListHints, ListP,
ListTau, ListPlots, MaxSignals, MaxGenerations:

    Plots:=[]:

```



```

OtherPlots:=[]:

NextSignalAvailable:=CreateList(3, 0);
#ListHints:=CreateList(MaxSignals, none);
ListP:=CreateList(MaxSignals, none);
ListTau:=CreateList(MaxSignals, none);
ListPlots:= CreateList(MaxGenerations, []):

n := 3*NextSignalAvailable[Sat+1]:
[r, teSat(Sat, r, branchv), tSat(Sat, r, branchv)]:
subs(r=rv, %):
evalf(%):
ListP[n+1]:= %:
tauSat(Sat, rv, branchv):
evalf(%):
ListTau[n+1]:= %:
NextSignalAvailable[Sat+1]:= NextSignalAvailable[Sat+1]+1:
AddPlotPointPolar(ListP[n+1], Sat, 0): # AddPlotPointPolar:=
proc(P, PSat, G)
ListPlots[1] := Plots:
Plots:=[]:
OtherPlots:=[]:
n+1:
end:

```

```

> Cascade:= proc(Sat, rv, branchv)
local Gen, NewSignals, OldSignals, n, St, event:
global ListHints, ListTau, ListP, MaxGenerations:

St:= time():

Gen := 0:
NewSignals:= []:

n:= SetBasePoint(Sat, rv, branchv):
NewSignals:= [op(NewSignals), n]:

do
Gen := Gen+1:
printf("Start Generation %d\n", Gen):
OldSignals := NewSignals:
NewSignals:= []:

for n in OldSignals do
event:= NextEvent(n mod 3):
SolveHard((n-1) mod 3, ListP[n], n mod 3, Gen, ListHints
[event]):
if nops(%) > 0 then
AcceptSolution():
NewSignals:= [op(NewSignals), %]:

printf("\nTau %a\n\n", ListTau);
else
printf("\nNo solution found\n\n");
end if:

```







```
Start Generation 1
1 --> 0 target = [23889000.,
2.632556434601106999967951373508400147983,
10510.45814019397412810699008640310343639]
one interval r = 24850248.88301387793152980010731189259064 ..
621256961929631488519759813441433737669/25000000000000000000000000000000
0000
Time Approximations 0.041.
```

```
Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.533028) | P <--- S
rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.03965e+07
k=5.43204e+14 scos=2.39474e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
24850278.47718525954079039253765734950676, rm=
20396532.00022429663988381376027621048948}, {r =
24850248.88301387793152980010731189259064 ..
24850278.47718525954079039253765734950676, rm =
.1330553304194287328500223794129351168576e-1 .. 23889000.}, avoid={})
);
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=5e-32
Equations at solution: [-.7e-31, -.5e-31, .31e-34]
Solution in 0.529s
```

```
Time Plot 1.722 s.
Exiting SolveHard() after 3.469r=2.48503e+07 in
[24850248.88301387793152980010731189259064 ..
621256961929631488519759813441433737669/25000000000000000000000000000000
0000]
Scattering ray (rm=2.03964e+07) in
[298960418182500/22468879468420441 .. 23889000.]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none]
```

```
2 --> 0 target = [23889000.,
2.632556434601106999967951373508400147983,
10510.45814019397412810699008640310343639]
one interval r = 25795878.07759835662287683941521694813384 ..
161224421923802901739910205661944662513/62500000000000000000000000000000
000
Time Approximations 0.028.
```

```
Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.63282) | P <--- S
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07
k=4.31733e+14 scos=4.39724e+14
```











```
Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030, none,  
10510.33729983843119480224801106566318860, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none]
```

```
0 --> 2 target = [25795903.79985511561316781642113695093491,  
1.856318223578679476954424130626621939862,  
10510.39514388006538100371792982892667986]  
one interval r = 23888966.51012875395781144697926248654402 ..  
597224906318887365922168983423648563467/25000000000000000000000000000000  
0000  
Time Approximations 0.048.
```

```
Try code 11  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.36718) | S ---> P  
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.28604e+07  
k=-4.31732e+14 scos=4.39724e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=  
23888996.25275549463688675933694594253868, rm=  
22860406.58721842027484008163007284959497}, {r =  
23888966.51012875395781144697926248654402 ..  
23888996.25275549463688675933694594253868, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}  
);  
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=2.0e-31  
Equations at solution: [.21e-30, .20e-30, -.72806541321e-34]  
Solution in 0.974s
```

```
Time Plot 1.618 s.  
Exiting SolveHard() after 3.551r=2.38890e+07 in  
[23888966.51012875395781144697926248654402 ..  
597224906318887365922168983423648563467/25000000000000000000000000000000  
0000]  
Scattering ray (rm=2.28605e+07) in  
[298960418182500/22468879468420441 .. 24089000]: target and source  
on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030, none,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none]
```

```
1 --> 2 target = [25795903.79985511561316781642113695093491,  
1.856318223578679476954424130626621939862,  
10510.39514388006538100371792982892667986]
```



```
one interval r = 24850245.15428919258543501803067780064530 ..
1242513737429890818728656221501545064373/50000000000000000000000000000000
00000
Time Approximations 0.052.
```

```
Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.239828) | S ---> P
rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07
k=2.37383e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
24850274.74859781637457312443003090128746, rm=
24743529.26572090240829457163611182636715}, {r =
24850245.15428919258543501803067780064530 ..
24850274.74859781637457312443003090128746, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}
);
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.4949e-31, 0., -.2e-35]
Solution in 0.885s
```

```
Time Plot 0.875 s.
Exiting SolveHard() after 2.498r=2.48503e+07 in
[24850245.15428919258543501803067780064530 ..
1242513737429890818728656221501545064373/50000000000000000000000000000000
00000]
Scattering ray (rm=2.47435e+07) in
[298960418182500/22468879468420441 .. 25089000]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
Start Generation 3
0 --> 2 target = [25795900.39533807920941978281062023785801,
1.856309428678043266814392150358620934802,
10510.33730295667902500448016714561852920]
one interval r = 23888963.06934924828335479031706673264809 ..
597224820303770076400571671810948623397/25000000000000000000000000000000
0000
Time Approximations 0.046.
```

```
Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.36718) | S ---> P
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.28604e+07
```











```
000000]
Scattering ray (rm=2.03964e+07) in
[298960418182500/22468879468420441 ..
23888989.43146328644175807501347600997649]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,
10510.30529788717935851166146004846522174, none,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]
```

```
2 --> 0 target = [23888989.43146328644175807501347600997649,
2.632526185857223384039587002846869614072,
10510.28046964806592275046837426848454175]
one interval r = 25795867.61967723367271871049090258979880 ..
1289794852506034629127592665641586312049/50000000000000000000000000000000
00000
Time Approximations 0.032.
```

```
Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07
k=4.31731e+14 scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
25795897.05012069258255185331283172624098, rm=
22860409.41793325550947758079808518242822}, {r =
25795867.61967723367271871049090258979880 ..
25795897.05012069258255185331283172624098, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.43146328644175807501347600997649}, avoid={});
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=8e-32
Equations at solution: [-.8e-31, -.8e-31, .34e-34]
Solution in 0.32s
```

```
Time Plot 1.615 s.
Exiting SolveHard() after 3.029r=2.57959e+07 in
[25795867.61967723367271871049090258979880 ..
1289794852506034629127592665641586312049/50000000000000000000000000000000
00000]
Scattering ray (rm=2.28604e+07) in
[298960418182500/22468879468420441 ..
23888989.43146328644175807501347600997649]: target and source on the
different branches.
Counterclockwise ray.
```



Ray outgoing at target.  
Solve Side.

Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none]

1 --> 0 target = [23888992.50558472677941562199764206716244,  
2.632534984402235609731825352106670503722,  
10510.33214920907894172487061349142712895]  
one interval r = 24850241.42564446195929857082855634229261 ..  
1242513551004516068014443586172817582043/50000000000000000000000000000000  
00000  
Time Approximations 0.547.

Try code 12  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.533028) | P <--- S  
rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.03965e+07  
k=5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=  
24850271.02009032136028887172345635164086, rm=  
20396533.77303752801474467359952408294903}, {r =  
24850241.42564446195929857082855634229261 ..  
24850271.02009032136028887172345635164086, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888992.50558472677941562199764206716244}, avoid={});  
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=6e-32  
Equations at solution: [.9e-31, .6e-31, -.48e-34]  
Solution in 0.431s

Time Plot 1.77 s.  
Exiting SolveHard() after 3.466r=2.48503e+07 in  
[24850241.42564446195929857082855634229261 ..  
1242513551004516068014443586172817582043/50000000000000000000000000000000  
00000]  
Scattering ray (rm=2.03964e+07) in  
[298960418182500/22468879468420441 ..  
23888992.50558472677941562199764206716244]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,



[illegible]

```
Time Plot 1.603 s.  
Exiting SolveHard() after 3.005r=2.57959e+07 in  
[25795870.66161245800881604810046494170563 ..  
2579590009198804353979547140796504033263/10000000000000000000000000000000000000000]  
Scattering ray (rm=2.28604e+07) in  
[298960418182500/22468879468420441 ..  
23888992.50558472677941562199764206716244]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,



```
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917, none,  
10510.24330971172246516729985032379126358, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none,  
none, none]
```

```
2 --> 1 target = [24850272.85449189132526402658550193546657,  
1.476967822333829106402975511591159392512,  
10510.36314214321830785127830686208023579]  
one interval r = 25795872.48589893272346549352544011957533 ..  
644897547905845320097763426011396162913/25000000000000000000000000000000  
0000  
Time Approximations 0.031.
```

```
Try code 11  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.760172) | P <--- S  
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07  
k=-2.37383e+14 scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=  
25795901.91623381280391053704045584651652, rm=  
24743526.91905590224386272111992570359323}, {r =  
25795872.48589893272346549352544011957533 ..  
25795901.91623381280391053704045584651652, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850272.85449189132526402658550193546657}, avoid={}));  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [-.22269e-30, 0., .51e-34]  
Solution in 0.358s
```

```
Time Plot 1.382 s.  
Exiting SolveHard() after 2.311r=2.57959e+07 in  
[25795872.48589893272346549352544011957533 ..  
644897547905845320097763426011396162913/25000000000000000000000000000000  
0000]  
Scattering ray (rm=2.47435e+07) in  
[298960418182500/22468879468420441 ..  
24850272.85449189132526402658550193546657]: target and source on the  
different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,
```



```

10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
10510.26914997049867919619036826661055917, none,
10510.24330971172246516729985032379126358,
10510.33113809489794039050928383230867111, none, none, none, none,
none, none, none, none, none, none, none, none, none, none,
none]

0 --> 1 target = [24850272.85449189132526402658550193546657,
1.476967822333829106402975511591159392512,
10510.36314214321830785127830686208023579]
one interval r = 23888964.60644587167575168019887114912430 ..
1194449717458466618404658405105548818961/50000000000000000000000000000000
00000
Time Approximations 0.043.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S ---> P
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.03965e+07
k=-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
23888994.34916933236809316810211097637922, rm=
20396533.33694427575538318401158701311191}, {r =
23888964.60644587167575168019887114912430 ..
23888994.34916933236809316810211097637922, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={})
);
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1.2e-31
Equations at solution: [-.21e-30, -.12e-30, .65962938835e-34]
Solution in 0.449s

Time Plot 1.757 s.
Exiting SolveHard() after 3.666r=2.38890e+07 in
[23888964.60644587167575168019887114912430 ..
1194449717458466618404658405105548818961/50000000000000000000000000000000
00000]
Scattering ray (rm=2.03966e+07) in
[298960418182500/22468879468420441 .. 24089000]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,
10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
10510.26914997049867919619036826661055917,

```



```
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none,  
none]
```

Start Generation 4

```
1 --> 0 target = [23888989.06496329485872911197089488868350,  
2.632525136888364227325281476146718182654,  
10510.27430837455143119298455217139668111]  
one interval r = 24850238.00203928112673210949390165384854 ..  
99401070386444582936187413117559617381/400000000000000000000000000000  
00
```

Time Approximations 0.038.

Try code 12

```
I search for an scattering ray on opposite branches with 0<sv<1  
(0.533028) | P <--- S  
rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.03965e+07  
k=5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=  
24850267.59661114573404685327938990434525, rm=  
20396534.58688838957072525672242744114298}, {r =  
24850238.00203928112673210949390165384854 ..  
24850267.59661114573404685327938990434525, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.06496329485872911197089488868350}, avoid={});  
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=8e-32  
Equations at solution: [-.14e-30, -.8e-31, .45e-34]  
Solution in 0.458s
```

Time Plot 1.79 s.

```
Exiting SolveHard() after 3.501r=2.48503e+07 in  
[24850238.00203928112673210949390165384854 ..  
99401070386444582936187413117559617381/400000000000000000000000000000  
00]
```

```
Scattering ray (rm=2.03964e+07) in  
[298960418182500/22468879468420441 ..  
23888989.06496329485872911197089488868350]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,
```



```
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111, none,  
10510.18546892759931749020200814673581561, none, none, none, none,  
none, none, none, none, none, none, none, none, none]
```

```
2 --> 0 target = [23888989.06496329485872911197089488868350,  
2.632525136888364227325281476146718182654,  
10510.27430837455143119298455217139668111]  
one interval r = 25795867.25701511448801980095492062593890 ..  
644897417186666629737809369304875630137/25000000000000000000000000000000  
0000  
Time Approximations 0.032.
```

```
Try code 12  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.632821) | P <--- S  
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07  
k=4.31731e+14 scos=4.39725e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=  
25795896.68746666518951237477219502520548, rm=  
22860409.57002084574800412379704066967983}, {r =  
25795867.25701511448801980095492062593890 ..  
25795896.68746666518951237477219502520548, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.06496329485872911197089488868350}, avoid={});  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=0  
Equations at solution: [0., 0., .11e-34]  
Solution in 0.317s
```

```
Time Plot 1.564 s.  
Exiting SolveHard() after 2.971r=2.57959e+07 in  
[25795867.25701511448801980095492062593890 ..  
644897417186666629737809369304875630137/25000000000000000000000000000000  
0000]  
Scattering ray (rm=2.28604e+07) in  
[298960418182500/22468879468420441 ..  
23888989.06496329485872911197089488868350]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,  
10510.27430450924162391988961859334736825,
```







```

0 --> 1 target = [24850269.43101309861241733616879334665430,
1.476958531724740943417498052240302069225,
10510.30530118318637486325253716065365327]
one interval r = 23888961.16565299817948948119449178210506 ..
2388899090855127150824991682392263419777/100000000000000000000000000000000
000000
Time Approximations 0.045.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S ---> P
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.03965e+07
k=-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
23888990.90855127150824991682392263419777, rm=
20396534.15080526694809002090915887830537}, {r =
23888961.16565299817948948119449178210506 ..
23888990.90855127150824991682392263419777, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}
);
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1e-32
Equations at solution: [-.1e-31, -.1e-31, .11157188429e-34]
Solution in 0.472s

Time Plot 1.81 s.
Exiting SolveHard() after 3.76r=2.38890e+07 in
[23888961.16565299817948948119449178210506 ..
2388899090855127150824991682392263419777/100000000000000000000000000000000
000000]
Scattering ray (rm=2.03966e+07) in
[298960418182500/22468879468420441 .. 24089000]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,
10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
10510.26914997049867919619036826661055917,
10510.27430450924162391988961859334736825,
```



```

10510.24330971172246516729985032379126358,
10510.33113809489794039050928383230867111,
10510.21646359961425315844565703156723271,
10510.18546892759931749020200814673581561,
10510.21130922484649961353409212373837542, none, none,
10510.27329709823633588538635678513651667, none, none, none, none,
none, none, none, none, none, none]

2 --> 1 target = [24850262.70323078644925875505002433594091,
1.476940274031262782949409178233029376476,
10510.19163349172132567315655665283194565]
one interval r = 25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/20000000000000000000000000000000
0000
Time Approximations 0.548.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.76017) | P <--- S
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07
k=-2.37384e+14 scos=5.95461e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
25795891.82119371218082845151233599574945, rm=
24743514.34236388601195499498420225667633}, {r =
25795862.39063358446748128489577933480278 ..
25795891.82119371218082845151233599574945, rm =
.1330553304194287328500223794129351168576e-1 ..
24850262.70323078644925875505002433594091}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.2475e-31, 0., .7e-35]
Solution in 0.339s

Time Plot 0.898 s.
Exiting SolveHard() after 2.367r=2.57959e+07 in
[25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/20000000000000000000000000000000
0000]
Scattering ray (rm=2.47435e+07) in
[298960418182500/22468879468420441 ..
24850262.70323078644925875505002433594091]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,
10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,

```







```

10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
10510.26914997049867919619036826661055917,
10510.27430450924162391988961859334736825,
10510.24330971172246516729985032379126358,
10510.33113809489794039050928383230867111,
10510.21646359961425315844565703156723271,
10510.18546892759931749020200814673581561,
10510.21130922484649961353409212373837542,
10510.10279600720328866736745742636090095, none,
10510.27329709823633588538635678513651667, none, none,
10510.15962933478753614217473212128017612, none, none, none, none,
none, none, none]

0 --> 2 target = [25795893.34215400624221088590056918107762,
1.856291208228086723992130094043171908925,
10510.21747360711085261323132545308200478]
one interval r = 23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/40000000000000000000000000000000
Time Approximations 0.047.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.367179) | S --> P
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.28604e+07
k=-4.31730e+14 scos=4.39725e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
23888985.68416735818079539166696781319750, rm=
22860410.97294351506648977721019738349107}, {r =
23888955.94100364913487175812205942139954 ..
23888985.68416735818079539166696781319750, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}
);
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=4e-32
Equations at solution: [-.5e-31, -.4e-31, .17869651403e-34]
Solution in 0.474s

Time Plot 1.568 s.
Exiting SolveHard() after 3.565r=2.38890e+07 in
[23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/40000000000000000000000000000000
]
Scattering ray (rm=2.28605e+07) in
[298960418182500/22468879468420441 .. 24089000]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,

```



```

10510.30529788717935851166146004846522174,  

10510.21747048889388355633134516173064786,  

10510.27430496999043214669943582338314253,  

10510.19163019574467350866023355115823455,  

10510.26914997049867919619036826661055917,  

10510.27430450924162391988961859334736825,  

10510.24330971172246516729985032379126358,  

10510.33113809489794039050928383230867111,  

10510.21646359961425315844565703156723271,  

10510.18546892759931749020200814673581561,  

10510.21130922484649961353409212373837542,  

10510.10279600720328866736745742636090095, none,  

10510.27329709823633588538635678513651667,  

10510.15447580454509166514822644094075171, none,  

10510.15962933478753614217473212128017612, none, none, none, none,  

none, none, none]

1 --> 2 target = [25795893.34215400624221088590056918107762,  

1.856291208228086723992130094043171908925,  

10510.21747360711085261323132545308200478]  

one interval r = 24850234.63796912173141920432208117660707 ..  

2485026423266479739181704602953513976831/1000000000000000000000000  

000000  

Time Approximations 0.036.

Try code 12  

I search for an scattering ray on opposite branches with 0<sv<1  

(0.23983) | S ---> P  

rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07  

k=2.37384e+14 scos=5.95462e+14  

branch outgoing at target, Counterclockwise  

(Scattering) fsolve(eqs, {r=  

24850264.23266479739181704602953513976831, rm=  

24743516.23722280464959361155393002428550}, {r =  

24850234.63796912173141920432208117660707 ..  

24850264.23266479739181704602953513976831, rm =  

.1330553304194287328500223794129351168576e-1 .. 25089000.}), avoid={}  

);  

Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0  

Equations at solution: [.4949e-31, 0., -.7e-35]  

Solution in 0.337s

Time Plot 1.399 s.  

Exiting SolveHard() after 3.025r=2.48503e+07 in  

[24850234.63796912173141920432208117660707 ..  

2485026423266479739181704602953513976831/1000000000000000000000000  

000000]  

Scattering ray (rm=2.47435e+07) in  

[298960418182500/22468879468420441 .. 25089000]: target and source  

on the different branches.  

Counterclockwise ray.  

Ray outgoing at target.  

Solve Side.

Tau [10510.45813678936207842014764182324596788,  

10510.36930058682532618475501145707984185,  

10510.39514076180265431188536776817562044,
```



```

10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,
10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
10510.26914997049867919619036826661055917,
10510.27430450924162391988961859334736825,
10510.24330971172246516729985032379126358,
10510.33113809489794039050928383230867111,
10510.21646359961425315844565703156723271,
10510.18546892759931749020200814673581561,
10510.21130922484649961353409212373837542,
10510.10279600720328866736745742636090095,
10510.18546846172437709400611835600963181,
10510.27329709823633588538635678513651667,
10510.15447580454509166514822644094075171, none,
10510.15962933478753614217473212128017612, none, none, none, none,
none, none, none]

2 --> 1 target = [24850265.76205359199870736703593497152527,
1.476948574973150678424900072270774720410,
10510.24331300771292254440409885192330801]
one interval r = 25795865.43257821616570179673528437159660 ..
644897371576761841392917646201273823237/25000000000000000000000000000000
0000
Time Approximations 0.032.

Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.76017) | P <--- S
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07
k=-2.37384e+14 scos=5.95462e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
25795894.86307047365571670584805095292948, rm=
24743518.13202615342765793575105213867182}, {r =
25795865.43257821616570179673528437159660 ..
25795894.86307047365571670584805095292948, rm =
.1330553304194287328500223794129351168576e-1 ..
24850265.76205359199870736703593497152527}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.7424e-31, 0., -.38e-34]
Solution in 0.356s

Time Plot 1.39 s.
Exiting SolveHard() after 2.316r=2.57959e+07 in
[25795865.43257821616570179673528437159660 ..
644897371576761841392917646201273823237/25000000000000000000000000000000
0000]
Scattering ray (rm=2.47435e+07) in
[298960418182500/22468879468420441 ..
24850265.76205359199870736703593497152527]: target and source on the
different branches.
Clockwise ray.

```







```
0000]
Scattering ray (rm=2.03966e+07) in
[298960418182500/22468879468420441 .. 24089000]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,
10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
10510.26914997049867919619036826661055917,
10510.27430450924162391988961859334736825,
10510.24330971172246516729985032379126358,
10510.33113809489794039050928383230867111,
10510.21646359961425315844565703156723271,
10510.18546892759931749020200814673581561,
10510.21130922484649961353409212373837542,
10510.10279600720328866736745742636090095,
10510.18546846172437709400611835600963181,
10510.27329709823633588538635678513651667,
10510.15447580454509166514822644094075171, none,
10510.15962933478753614217473212128017612,
10510.15447547815947286212833271267435954, none,
10510.21130888350685915386493858978930221, none, none, none, none]
```

```
0 --> 2 target = [25795896.38402523646204355452114396262515,
1.856299066265858109439961038732420613264,
10510.26915308872895791524093204162206305]
one interval r = 23888959.01529623342321627788824672003212 ..
59722471895759389095528762418719523933/250000000000000000000000000000
00
Time Approximations 0.047.
```

```
Try code 11
I search for an scattering ray on opposite branches with 0<sv<1
(0.367179) | S --> P
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.28604e+07
k=-4.31731e+14 scos=4.39725e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
23888988.75830375563821150496748780957320, rm=
22860409.69727600069935150120919868735041}, {r =
23888959.01529623342321627788824672003212 ..
23888988.75830375563821150496748780957320, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={})
);
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.4e-31
Equations at solution: [-.15e-30, -.14e-30, .49600392670e-34]
```



```
Time Plot 1.038 s.  
Exiting SolveHard() after 3.554r=2.38890e+07 in  
[23888959.01529623342321627788824672003212 ..  
59722471895759389095528762418719523933/250000000000000000000000000000  
00]  
Scattering ray (rm=2.28605e+07) in  
[298960418182500/22468879468420441 .. 24089000]: target and source  
on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,  
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111,  
10510.21646359961425315844565703156723271,  
10510.18546892759931749020200814673581561,  
10510.21130922484649961353409212373837542,  
10510.10279600720328866736745742636090095,  
10510.18546846172437709400611835600963181,  
10510.27329709823633588538635678513651667,  
10510.15447580454509166514822644094075171, none,  
10510.15962933478753614217473212128017612,  
10510.15447547815947286212833271267435954, none,  
10510.21130888350685915386493858978930221,  
10510.20615520675546656272235083800212186, none, none, none]
```

[illegible]

```

Try code 12
I search for an scattering ray on opposite branches with  $0 < \text{sv} < 1$ 
(0.239829) | S ---> P
rGuessMin=2.48502e+07    rGuessMax=2.48503e+07    rmGuess=2.47435e+07
k=2.37384e+14    scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
24850267.29147974977450284930859388709426, rm=

```















```

10510.27329709823633588538635678513651667,
10510.15447580454509166514822644094075171,
10510.23714797607061291086956144565493086,
10510.15962933478753614217473212128017612,
10510.15447547815947286212833271267435954,
10510.29913613974234190309901279325515298,
10510.21130888350685915386493858978930221,
10510.20615520675546656272235083800212186, none, none,
10510.26814323592336180387677929719916736]

1 --> 0 target = [23888989.06493588745880515057802801897934,
2.632525136809920840883118370193127663532,
10510.27430791380262283822338748373982610]
one interval r = 24850238.00201200929744389905914961384924 ..
1242513379829193745424075414664369797603/50000000000000000000000000000000
00000
Time Approximations 0.04.

Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.533028) | P <--- S
rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.03965e+07
k=5.43203e+14 scos=2.39475e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
24850267.59658387490848150829328739595206, rm=
20396534.58689487248581145152492543949363}, {r =
24850238.00201200929744389905914961384924 ..
24850267.59658387490848150829328739595206, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06493588745880515057802801897934}, avoid={}));
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=8e-32
Equations at solution: [-.12e-30, -.8e-31, .63e-34]
Solution in 0.456s

Time Plot 1.891 s.
Exiting SolveHard() after 3.561r=2.48503e+07 in
[24850238.00201200929744389905914961384924 ..
1242513379829193745424075414664369797603/50000000000000000000000000000000
00000]
Scattering ray (rm=2.03964e+07) in
[298960418182500/22468879468420441 ..
23888989.06493588745880515057802801897934]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,
10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,

```



```
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,  
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111,  
10510.21646359961425315844565703156723271,  
10510.18546892759931749020200814673581561,  
10510.21130922484649961353409212373837542,  
10510.10279600720328866736745742636090095,  
10510.18546846172437709400611835600963181,  
10510.27329709823633588538635678513651667,  
10510.15447580454509166514822644094075171,  
10510.23714797607061291086956144565493086,  
10510.15962933478753614217473212128017612,  
10510.15447547815947286212833271267435954,  
10510.29913613974234190309901279325515298,  
10510.21130888350685915386493858978930221,  
10510.20615520675546656272235083800212186,  
10510.18546846685091064707627195110501083, none,  
10510.26814323592336180387677929719916736]
```

[illegible]

```
Try code 12
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57959e+07    rGuessMax=2.57959e+07    rmGuess=2.28604e+07
k=4.31731e+14    scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
25795896.68743954540111976106317781487286, rm=
22860409.57003221906527138862981536625147}, {r =
25795867.25698799409451243993168295190626 ..
25795896.68743954540111976106317781487286, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06493588745880515057802801897934}, avoid={}));
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=4e-32
Equations at solution: [-.5e-31, -.4e-31, .15e-34]
Solution in 0.318s
```

```
Time Plot 1.593 s.  
Exiting SolveHard() after 3.054r=2.57959e+07 in  
[25795867.25698799409451243993168295190626 ..  
1289794834371977270055988053158890743643/5000000000000000000000000000000000000000]  
Scattering ray (rm=2.28604e+07) in  
[298960418182500/22468879468420441 ..  
23888989.06493588745880515057802801897934]: target and source on the  
different branches.  
Counterclockwise ray.
```



Ray outgoing at target.  
Solve Side.

Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,  
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111,  
10510.21646359961425315844565703156723271,  
10510.18546892759931749020200814673581561,  
10510.21130922484649961353409212373837542,  
10510.10279600720328866736745742636090095,  
10510.18546846172437709400611835600963181,  
10510.27329709823633588538635678513651667,  
10510.15447580454509166514822644094075171,  
10510.23714797607061291086956144565493086,  
10510.15962933478753614217473212128017612,  
10510.15447547815947286212833271267435954,  
10510.29913613974234190309901279325515298,  
10510.21130888350685915386493858978930221,  
10510.20615520675546656272235083800212186,  
10510.18546846685091064707627195110501083,  
10510.21130876409839922259159893924009430,  
10510.26814323592336180387677929719916736]

Cascade time 97.223

[10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,  
10510.27430450924162391988961859334736825,

(81)



10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111,  
10510.21646359961425315844565703156723271,  
10510.18546892759931749020200814673581561,  
10510.21130922484649961353409212373837542,  
10510.10279600720328866736745742636090095,  
10510.18546846172437709400611835600963181,  
10510.27329709823633588538635678513651667,  
10510.15447580454509166514822644094075171,  
10510.23714797607061291086956144565493086,  
10510.15962933478753614217473212128017612,  
10510.15447547815947286212833271267435954,  
10510.29913613974234190309901279325515298,  
10510.21130888350685915386493858978930221,  
10510.20615520675546656272235083800212186,  
10510.18546846685091064707627195110501083,  
10510.21130876409839922259159893924009430,  
10510.26814323592336180387677929719916736]

```
> display(BH, S,  
  c0_1n,c0_0,c0_1,  
  c1_1n,c1_0,c1_1,  
  c2_1n,c2_0,c2_1,  
  op(ListPlots[1]), op(ListPlots[2]), op(ListPlots[3]), op(ListPlots  
  [4]),  
  #op(Plots),  
  #op(OtherPlots),  
  view=[ -3*10^7..3*10^7, -3*10^7..3*10^7, 10510.1..10510.5]);
```











$$2.286036617838899487248578273534549876757 \times 10^7,$$

$$2.579586761967723367271871049090258979880 \times 10^7$$

$$\frac{1289794852506034629127592665641586312049}{5000000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$[..2.388898943146328644175807501347600997649 \times 10^7, 1],$$

$$\left[ 2.388898906496329485872911197089488868350 \times 10^7, 3, -1, 1, \right.$$

$$2.286045280984339454018821965990898965218 \times 10^7,$$

$$2.388896306934924828335479031706673264809 \times 10^7$$

$$\dots \frac{597224820303770076400571671810948623397}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 24089000, 1],$$

$$[2.485026270323078644925875505002433594091 \times 10^7, 3, 1, 1,$$

$$2.039643789679703248883954261208042438711 \times 10^7,$$

$$2.485023836672641289161845524403523557614 \times 10^7$$

$$\therefore \frac{2485026796128485539651696969285982263711}{1000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$..2.388898943146328644175807501347600997649 \times 10^7, 1],$$

$$[2.579589638402523646204355452114396262515 \times 10^7, 3, 1, 1,$$

$$2.286036490258708217946452135489353704915 \times 10^7,$$

$$2.579587066161245800881604810046494170563 \times 10^7$$

$$\therefore \frac{2579590009198804353979547140796504033263}{100000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$..2.388899250558472677941562199764206716244 \times 10^7, 1 \Big],$$

$$\left[ 2.388898906493588745880515057802801897934 \times 10^7, 3, -1, 1, \right.$$







$$\begin{aligned} & \dots \frac{99401070386444582936187413117559617381}{40000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & \dots 2.388898906496329485872911197089488868350 \times 10^7, 1 \Big], \\ & \left[ 2.579589297949951637755150436992208534955 \times 10^7, 3, 1, 1, \right. \\ & 2.286036633048982198263378361971969356546 \times 10^7, \\ & 2.579586725701511448801980095492062593890 \times 10^7 \\ & \dots \frac{644897417186666629737809369304875630137}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & \dots 2.388898906496329485872911197089488868350 \times 10^7, 1 \Big], \\ & \left[ 2.388897886275416794827744619407470166172 \times 10^7, 3, -1, 1, \right. \\ & 2.039663360486737240849086583158368073964 \times 10^7, \\ & 2.388895440382902066202638611670198664112 \times 10^7 \\ & \dots \frac{298612301838385292588525592212900916387}{12500000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 2.4089000, 1 \Big], \\ & \left[ 2.485026233852743516297758053367205548154 \times 10^7, 3, 1, 1, \right. \\ & 2.474350581299238129247202614818714422035 \times 10^7, \\ & 2.485023463796912173141920432208117660707 \times 10^7 \\ & \dots \frac{2485026423266479739181704602953513976831}{10000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 2.5089000, 1 \Big], \\ & \left[ 2.579589662812623062600675830146784169746 \times 10^7, 3, -1, 1, \right. \\ & 2.474353075497470617154487139175541092326 \times 10^7, \\ & 2.579586908129891002682354003571513818579 \times 10^7 \\ & \dots \frac{2579589851170975665166085604166659164517}{10000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & \dots 2.485026943101309861241733616879334665430 \times 10^7, 1 \Big], \end{aligned}$$



$$2.286045576745222490936139362415353022295 \times 10^7,$$

$$2.388895594100364913487175812205942139954 \times 10^7$$

$$\dots \frac{9555594273666943272318156666787125279}{4000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 24089000, 1],$$

$$\left[ 2.485026539735153163244735697452997613798 \times 10^7, 3, 1, 1, \right.$$

$$2.474350960268659785926677493114233979369 \times 10^7,$$

$$2.485023769689665459305925255508686501973 \times 10^7$$

$$\dots \frac{1242513364573987488725142465429694354713}{5000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 25089000, 1],$$

$$\left[ 2.579588993759639200267360170740297062535 \times 10^7, 3, -1, 1, \right.$$

$$2.474352241979937237431350459260520845964 \times 10^7,$$

$$2.579586239063358446748128489577933480278 \times 10^7$$

$$\frac{515917836423874243616569030246719914989}{20000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$[..2.485026270323078644925875505002433594091 \times 10^7, 1],$$

$$\left[ 2.388898193692575516559285951162594064654 \times 10^7, 3, -1, 1, \right.$$

$$2.039663287781872303409295730403849988110 \times 10^7,$$

$$2.388895747813172092635353311983299926098 \times 10^7$$

$$\dots \frac{597224680530433446694670341991304966889}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 24089000, 1],$$

$$\left[ 2.485026906631200499486567135274096895840 \times 10^7, 3, 1, 1, \right.$$

$$2.474351414830449237551739504443492551457 \times 10^7,$$

$$2.485024136598119958086914129896830053765 \times 10^7$$



$$\begin{aligned} & \dots \frac{2485027096042925490424856544845137371367}{10000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} ..25089000, 1 \Big], \\ & [2.579589297947942498571214559240875822627 \times 10^7, 3, -1, 1, \\ & 2.474352620943107487634918939497374544011 \times 10^7, \\ & 2.579586543257821616570179673528437159660 \times 10^7 \\ & \dots \frac{644897371576761841392917646201273823237}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & ..2.485026576205359199870736703593497152527 \times 10^7, 1 \Big], \\ & [2.388898501109652402578398653593278001387 \times 10^7, 3, -1, 1, \\ & 2.286045449191911041368999760273437756943 \times 10^7, \\ & 2.388895901529623342321627788824672003212 \times 10^7 \\ & \dots \frac{59722471895759389095528762418719523933}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} ..24089000, 1 \Big], \\ & [2.485026233852773859450921827957797170818 \times 10^7, 3, 1, 1, \\ & 2.039643798350304935080318674410988058300 \times 10^7, \\ & 2.485023800201200929744389905914961384924 \times 10^7 \\ & \dots \frac{1242513379829193745424075414664369797603}{50000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & ..2.388898906493588745880515057802801897934 \times 10^7, 1 \Big], \\ & [2.579589297947239655457473800077316699564 \times 10^7, 3, 1, 1, \\ & 2.286036633050119628977322107538139299992 \times 10^7, \\ & 2.579586725698799409451243993168295190626 \times 10^7 \\ & \dots \frac{1289794834371977270055988053158890743643}{50000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & ..2.388898906493588745880515057802801897934 \times 10^7, 1 \Big], \end{aligned}$$







$$\left[ \begin{aligned} & \left[ 13.41829467, 2, 1, 1, 12.66403723, 12.09244395 \dots 18.12235563, \frac{3}{2} \dots 20, 1 \right], \\ & \left[ 28.55371953, 3, -1, 1, 18.23928734, 27.32888176 \dots 29.33437772, \frac{3}{2} \dots 30, 1 \right], \\ & \left[ 34.04329235, 3, 1, 1, 13.19952918, 33.07960633 \dots 35.04036788, \frac{3}{2} \dots 28.57854263, 1 \right], \\ & \left[ 15.37532903, 2, -1, 1, 15.25553472, 13.57904887 \dots 18.95882153, \frac{3}{2} \dots 20, 1 \right], \\ & \left[ 27.91054939, 3, -1, 1, 18.28194106, 26.47344578 \dots 28.84320324, \frac{3}{2} \dots 30, 1 \right], \\ & \left[ 35.32189384, 3, 1, 1, 12.83511624, 34.40089320 \dots 36.32321658, \frac{3}{2} \dots 29.60365284, 1 \right], \\ & \left[ 16.19548605, 3, 1, 1, 15.83834737, 14.63245592 \dots 19.40154367, \frac{3}{2} \dots 20, 1 \right], \\ & \left[ 27.06673807, 3, -1, 1, 18.29599023, 25.41214835 \dots 28.13785667, \frac{3}{2} \dots 30, 1 \right], \\ & \left[ 34.60825269, 3, 1, 1, 13.05351763, 33.65602173 \dots 35.61673250, \frac{3}{2} \dots 29.10532394, 1 \right], \\ & \left[ 17.00004437, 2, -1, 1, 16.34341807, 15.64116418 \dots 19.72383047, \frac{3}{2} \dots 20, 1 \right], \\ & \left[ 28.27511071, 3, -1, 1, 16.13212079, 26.23773143 \dots 28.69508492, \frac{3}{2} \dots 16.13405333, 1 \right], \\ & \left[ 34.61952959, 3, 1, 1, 16.12727049, 33.19571921 \dots 35.15878287, \frac{3}{2} \dots 16.13405333, 1 \right], \\ & \left[ 18.47234754, 2, -1, 1, 16.88870166, 17.60249431 \dots 20, \frac{3}{2} \dots 20, 1 \right] \end{aligned} \right]$$

**> Tau10:=ListTau;**

*T10* := [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,  
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,

(83)



10510.33113809489794039050928383230867111,  
 10510.21646359961425315844565703156723271,  
 10510.18546892759931749020200814673581561,  
 10510.21130922484649961353409212373837542,  
 10510.10279600720328866736745742636090095,  
 10510.18546846172437709400611835600963181,  
 10510.27329709823633588538635678513651667,  
 10510.15447580454509166514822644094075171,  
 10510.23714797607061291086956144565493086,  
 10510.15962933478753614217473212128017612,  
 10510.15447547815947286212833271267435954,  
 10510.29913613974234190309901279325515298,  
 10510.21130888350685915386493858978930221,  
 10510.20615520675546656272235083800212186,  
 10510.18546846685091064707627195110501083,  
 10510.21130876409839922259159893924009430,  
 10510.26814323592336180387677929719916736]

> # 334.332 10 digits, 30 signals without hints  
 # 402.226 40 digits, 30 signals with hints  
 # 478.446 40 digits, 30 signals with hints

#+1470.84s Digits 80, 30 signals with hints.  
 #+2680.41s Digits 120, 30 signals with hints.

> ListP;

[[[2.3889000  $\times 10^7$ , 2.632556434601106999967951373508400147983,  
 10510.45814019397412810699008640310343639],  
 [2.485027321919080441727298772802245902434  $\times 10^7$ ,  
 1.476968812052973768854931710912166405062,  
 10510.36930388284943964962986012895802509],  
 [2.579590379985511561316781642113695093491  $\times 10^7$ ,  
 1.856318223578679476954424130626621939862,  
 10510.39514388006538100371792982892667986],  
 [2.388898943146328644175807501347600997649  $\times 10^7$ ,  
 2.632526185857223384039587002846869614072,  
 10510.28046964806592275046837426848454175],  
 [2.485027285449189132526402658550193546657  $\times 10^7$ ,  
 1.476967822333829106402975511591159392512,

(84)



10510.36314214321830785127830686208023579],  
[2.579590039533807920941978281062023785801  $\times 10^7$ ,  
1.856309428678043266814392150358620934802,  
10510.33730295667902500448016714561852920],  
[2.388899250558472677941562199764206716244  $\times 10^7$ ,  
2.632534984402235609731825352106670503722,  
10510.33214920907894172487061349142712895],  
[2.485026943101309861241733616879334665430  $\times 10^7$ ,  
1.476958531724740943417498052240302069225,  
10510.30530118318637486325253716065365327],  
[2.579589334215400624221088590056918107762  $\times 10^7$ ,  
1.856291208228086723992130094043171908925,  
10510.21747360711085261323132545308200478],  
[2.388898906496329485872911197089488868350  $\times 10^7$ ,  
2.632525136888364227325281476146718182654,  
10510.27430837455143119298455217139668111],  
[2.485026270323078644925875505002433594091  $\times 10^7$ ,  
1.476940274031262782949409178233029376476,  
10510.19163349172132567315655665283194565],  
[2.579589638402523646204355452114396262515  $\times 10^7$ ,  
1.856299066265858109439961038732420613264,  
10510.26915308872895791524093204162206305],  
[2.388898906493588745880515057802801897934  $\times 10^7$ ,  
2.632525136809920840883118370193127663532,  
10510.27430791380262283822338748373982610],  
[2.485026576205359199870736703593497152527  $\times 10^7$ ,  
1.476948574973150678424900072270774720410,  
10510.24331300771292254440409885192330801],  
[2.579590003265730627104171495990540839944  $\times 10^7$ ,  
1.856308491764749423379286180252072848042,  
10510.33114121314418368243534285689405493],  
[2.388898562428976346864819081258538077918  $\times 10^7$ ,



2.632515289280424206131912197926326469939,  
10510.21646700415918947844933270059772262 ],  
[ 2.485026233855500957466342744981057733590  $\times 10^7$ ,  
1.476939284387011891994733060386469058011,  
10510.18547222357432378741608618331333162 ],  
[ 2.579589297949951637755150436992208534955  $\times 10^7$ ,  
1.856290271387187972181639013079578187667,  
10510.21131234306188188303663680330418308 ],  
[ 2.388897886275416794827744619407470166172  $\times 10^7$ ,  
2.632495937139278365787357031758438379818,  
10510.10279941171665913868204367584729103 ],  
[ 2.485026233852743516297758053367205548154  $\times 10^7$ ,  
1.476939284312181442761458511933570344463,  
10510.18547175769938326677043841340214688 ],  
[ 2.579589662812623062600675830146784169746  $\times 10^7$ ,  
1.856299696850400409360845210716945284409,  
10510.27330021646768266593203519288941732 ],  
[ 2.388898193694517027845377679197924121306  $\times 10^7$ ,  
2.632504735732313595979470005326364074828,  
10510.15447920907281378055866420134300440 ],  
[ 2.485026539735153163244735697452997613798  $\times 10^7$ ,  
1.476947585254050921513325282894279988209,  
10510.23715127205942429607669743097876350 ],  
[ 2.579588993759639200267360170740297062535  $\times 10^7$ ,  
1.856282413285235542277892438065723972356,  
10510.15963245298960864198697440313036003 ],  
[ 2.388898193692575516559285951162594064654  $\times 10^7$ ,  
2.632504735676745772370268426291614807656,  
10510.15447888268719488690045381920543437 ],  
[ 2.485026906631200499486567135274096895840  $\times 10^7$ ,  
1.476957542004040463083358424683822000041,  
10510.29913943574771225996952465456145640 ],



```
> Digits:= 40;  
ListTau:=CreateList(31, 0);  
ListP:=CreateList(31, []);  
#ListHints:=CreateList(31, none);  
Digits := 40  
ListTau := [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]  
ListP := [[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ]], (85)  
[ ],[ ],[ ],[ ],[ ],[ ],[ ],[ ]]  
> Cascade(BaseSat, BaseR, BaseBranch) ;
```

---

```
Start Generation 1  
1 --> 0 target = [23889000.,  
2.632556434601106999967951373508400147983,  
10510.45814019397412810699008640310343639]  
one interval r = 24850248.88301387793152980010731189259064 ..  
621256961929631488519759813441433737669/250000000000000000000000000000  
0000  
Time Approximations 0.042.  
  
hint used Hint := [24850273.21919080441727298772802245902434, 3, 1,  
1, 20396435.39659592643000634328381896732011,  
24850248.88301387793152980010731189259064 ..  
621256961929631488519759813441433737669/250000000000000000000000000000  
0000, 298960418182500/22468879468420441 .. 23889000., 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.533028) | P <--- S
```







Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=9e-32  
Equations at solution: [-.10e-30, -.9e-31, .33e-34]  
Solution in 0.2s

Time Plot 1.549 s.  
Exiting SolveHard() after 2.323r=2.57959e+07 in  
[25795878.07759835662287683941521694813384 ..  
161224421923802901739910205661944662513/625000000000000000000000000000  
000]  
Scattering ray (rm=2.28604e+07) in  
[298960418182500/22468879468420441 .. 23889000.]: target and source  
on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none,  
none, none]

Start Generation 2  
2 --> 1 target = [24850273.21919080441727298772802245902434,  
1.476968812052973768854931710912166405062,  
10510.36930388284943964962986012895802509]  
one interval r = 25795872.84858705064741864171344489129222 ..  
2579590227891383795486680697763551821623/100000000000000000000000000000  
000000  
Time Approximations 0.031.

hint used Hint := [25795900.39533807920941978281062023785801, 3, -1,  
1, 24743535.44822274617462265333263319921312,  
25795872.84858705064741864171344489129222 ..  
2579590227891383795486680697763551821623/100000000000000000000000000000  
000000, 298960418182500/22468879468420441 ..  
24850273.21919080441727298772802245902434, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.760172) | P <--- S  
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07  
k=-2.37383e+14 scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=  
25795900.39533807920941978281062023785801, rm=  
24743535.44822274617462265333263319921312}, {r =  
25795872.84858705064741864171344489129222 ..  
25795902.27891383795486680697763551821623, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850273.21919080441727298772802245902434}, avoid={});  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [-.9897e-31, 0., .34e-34]  
Solution in 0.189s

Time Plot 0.894 s.  
Exiting SolveHard() after 2.184r=2.57959e+07 in  
[25795872.84858705064741864171344489129222 ..







```

Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030, none,
10510.33729983843119480224801106566318860, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2 target = [25795903.79985511561316781642113695093491,
1.856318223578679476954424130626621939862,
10510.39514388006538100371792982892667986]
one interval r = 23888966.51012875395781144697926248654402 ..
597224906318887365922168983423648563467/25000000000000000000000000000000
0000
Time Approximations 0.563.

hint used Hint := [23888992.50558472677941562199764206716244, 3, -1,
1, 22860451.38218918783573792125602312359251,
23888966.51012875395781144697926248654402 ..
597224906318887365922168983423648563467/25000000000000000000000000000000
0000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.36718) | S ---> P
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07
k=-4.31732e+14 scos=4.39724e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
23888992.50558472677941562199764206716244, rm=
22860451.38218918783573792125602312359251}, {r =
23888966.51012875395781144697926248654402 ..
23888996.25275549463688675933694594253868, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={})
);
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=2.0e-31
Equations at solution: [.21e-30, .20e-30, -.72806541321e-34]
Solution in 0.245s

Time Plot 1.574 s.
Exiting SolveHard() after 3.312r=2.38890e+07 in
[23888966.51012875395781144697926248654402 ..
597224906318887365922168983423648563467/25000000000000000000000000000000
0000]
Scattering ray (rm=2.28605e+07) in
[298960418182500/22468879468420441 .. 24089000]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030, none,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

```



```
1 --> 2 target = [25795903.79985511561316781642113695093491,
1.856318223578679476954424130626621939862,
10510.39514388006538100371792982892667986]
one interval r = 24850245.15428919258543501803067780064530 ..
1242513737429890818728656221501545064373/50000000000000000000000000000000
00000
Time Approximations 0.04.
```

```
hint used Hint := [24850272.85449189132526402658550193546657, 3, 1,
1, 24743518.84163067386800243860950511726701,
24850245.15428919258543501803067780064530 ..
1242513737429890818728656221501545064373/50000000000000000000000000000000
00000, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.239828) | S ---> P
rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07
k=2.37383e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
24850272.85449189132526402658550193546657, rm=
24743518.84163067386800243860950511726701}, {r =
24850245.15428919258543501803067780064530 ..
24850274.74859781637457312443003090128746, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={})
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.4949e-31, 0., -.2e-35]
Solution in 0.23s
```

```
Time Plot 1.391 s.
Exiting SolveHard() after 2.317r=2.48503e+07 in
[24850245.15428919258543501803067780064530 ..
1242513737429890818728656221501545064373/50000000000000000000000000000000
00000]
Scattering ray (rm=2.47435e+07) in
[298960418182500/22468879468420441 .. 25089000]: target and source
on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
Start Generation 3
0 --> 2 target = [25795900.39533807920941978281062023785801,
1.856309428678043266814392150358620934802,
10510.33730295667902500448016714561852920]
one interval r = 23888963.06934924828335479031706673264809 ..
597224820303770076400571671810948623397/25000000000000000000000000000000
```



Time Approximations 0.046.

Time Plot 1.603 s.

[illegible]

```
hint used Hint := [24850269.43101309861241733616879334665430, 3, 1,
1, 24743514.60014700975744022125022662330031,
```



























```
1, 20396631.19196530471597662319541189563767,
23888964.60644587167575168019887114912430 ..
1194449717458466618404658405105548818961/50000000000000000000000000000000
00000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with  $0 < \text{sv} < 1$ 
(0.466972) | S --> P
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07
k=-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
23888989.06493588745880515057802801897934, rm=
20396631.19196530471597662319541189563767}, {r =
23888964.60644587167575168019887114912430 ..
23888994.34916933236809316810211097637922, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}
);
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1.2e-31
Equations at solution: [-.21e-30, -.12e-30, .65962938835e-34]
Solution in 0.254s
```

```
Time Plot 1.746 s.
Exiting SolveHard() after 3.549r=2.38890e+07 in
[23888964.60644587167575168019887114912430 ..
1194449717458466618404658405105548818961/50000000000000000000000000000000
00000]
Scattering ray (rm=2.03966e+07) in
[298960418182500/22468879468420441 .. 24089000]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,
10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
10510.26914997049867919619036826661055917,
10510.27430450924162391988961859334736825,
10510.24330971172246516729985032379126358,
10510.33113809489794039050928383230867111, none, none, none, none,
none, none, none, none, none, none, none, none, none, none,
none]
```

```
Start Generation 4
1 --> 0 target = [23888989.06496329485872911197089488868350,
2.632525136888364227325281476146718182654,
10510.27430837455143119298455217139668111]
one interval r = 24850238.00203928112673210949390165384854 ..
99401070386444582936187413117559617381/40000000000000000000000000000000
00
```



Time Approximations 0.04.

```
hint used Hint := [24850262.33855500957466342744981057733590, 3, 1,
1, 20396437.98349656584284330028372043025521,
24850238.00203928112673210949390165384854 ..
99401070386444582936187413117559617381/400000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888989.06496329485872911197089488868350, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.533028) | P <--- S
rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07
k=5.43203e+14 scos=2.39475e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
24850262.33855500957466342744981057733590, rm=
20396437.98349656584284330028372043025521}, {r =
24850238.00203928112673210949390165384854 ..
24850267.59661114573404685327938990434525, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06496329485872911197089488868350}, avoid={});
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=8e-32
Equations at solution: [-.14e-30, -.8e-31, .45e-34]
Solution in 0.232s
```

Time Plot 1.815 s.

```
Exiting SolveHard() after 3.317r=2.48503e+07 in
[24850238.00203928112673210949390165384854 ..
99401070386444582936187413117559617381/400000000000000000000000000000
00]
Scattering ray (rm=2.03964e+07) in
[298960418182500/22468879468420441 ..
23888989.06496329485872911197089488868350]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,
10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
10510.26914997049867919619036826661055917,
10510.27430450924162391988961859334736825,
10510.24330971172246516729985032379126358,
10510.33113809489794039050928383230867111, none,
10510.18546892759931749020200814673581561, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
2 --> 0 target = [23888989.06496329485872911197089488868350,
2.632525136888364227325281476146718182654,
```



```
10510.27430837455143119298455217139668111]
one interval r = 25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/25000000000000000000000000000000
0000
Time Approximations 0.033.
```

```
hint used Hint := [25795892.97949951637755150436992208534955, 3, 1,
1, 22860366.33048982198263378361971969356546,
25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/25000000000000000000000000000000
0000, 298960418182500/22468879468420441 ..
23888989.06496329485872911197089488868350, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07
k=4.31731e+14 scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
25795892.97949951637755150436992208534955, rm=
22860366.33048982198263378361971969356546}, {r =
25795867.25701511448801980095492062593890 ..
25795896.68746666518951237477219502520548, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06496329485872911197089488868350}, avoid={}));
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=0
Equations at solution: [0., 0., .11e-34]
Solution in 0.706s
```

```
Time Plot 1.023 s.
Exiting SolveHard() after 2.321r=2.57959e+07 in
[25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/25000000000000000000000000000000
0000]
Scattering ray (rm=2.28604e+07) in
[298960418182500/22468879468420441 ..
23888989.06496329485872911197089488868350]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,
10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
10510.26914997049867919619036826661055917,
10510.27430450924162391988961859334736825,
10510.24330971172246516729985032379126358,
10510.33113809489794039050928383230867111, none,
10510.18546892759931749020200814673581561,
```



```

10510.21130922484649961353409212373837542, none, none, none, none, none, none, none]
2 --> 1 target = [24850269.43101309861241733616879334665430,
1.476958531724740943417498052240302069225,
10510.30530118318637486325253716065365327]
one interval r = 25795869.08129891002682354003571513818579 ..
2579589851170975665166085604166659164517/1000000000000000000000000000000000
000000
Time Approximations 0.031.

hint used Hint := [25795896.62812623062600675830146784169746, 3, -1,
1, 24743530.75497470617154487139175541092326,
25795869.08129891002682354003571513818579 ..
2579589851170975665166085604166659164517/1000000000000000000000000000000000
000000, 298960418182500/22468879468420441 ..
24850269.43101309861241733616879334665430, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.760171) | P <--- S
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07
k=-2.37384e+14 scos=5.95462e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
25795896.62812623062600675830146784169746, rm=
24743530.75497470617154487139175541092326}, {r =
25795869.08129891002682354003571513818579 ..
25795898.51170975665166085604166659164517, rm =
.1330553304194287328500223794129351168576e-1 ..
24850269.43101309861241733616879334665430}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.24743e-30, 0., .99e-34]
Solution in 0.2s

Time Plot 1.421 s.
Exiting SolveHard() after 2.705r=2.57959e+07 in
[25795869.08129891002682354003571513818579 ..
2579589851170975665166085604166659164517/1000000000000000000000000000000000
000000]
Scattering ray (rm=2.47435e+07) in
[298960418182500/22468879468420441 ..
24850269.43101309861241733616879334665430]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,
10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
```



```

10510.26914997049867919619036826661055917,  

10510.27430450924162391988961859334736825,  

10510.24330971172246516729985032379126358,  

10510.33113809489794039050928383230867111, none,  

10510.18546892759931749020200814673581561,  

10510.21130922484649961353409212373837542, none, none,  

10510.27329709823633588538635678513651667, none, none, none, none,  

none, none, none, none, none, none]  
  

0 --> 1 target = [24850269.43101309861241733616879334665430,  

1.476958531724740943417498052240302069225,  

10510.30530118318637486325253716065365327]  

one interval r = 23888961.16565299817948948119449178210506 ..  

2388899090855127150824991682392263419777/1000000000000000000000000  

000000  

Time Approximations 0.046.  
  

hint used Hint := [23888985.62428976346864819081258538077918, 3, -1,  

1, 20396632.00572718336484608918971225516342,  

23888961.16565299817948948119449178210506 ..  

2388899090855127150824991682392263419777/1000000000000000000000000  

000000, 298960418182500/22468879468420441 .. 24089000, 1]  

I search for an scattering ray on opposite branches with 0<sv<1  

(0.466972) | S ---> P  

rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07  

k=-5.43203e+14 scos=2.39475e+14  

branch outgoing at target, Clockwise  

(Scattering) fsolve(eqs, {r=  

23888985.62428976346864819081258538077918, rm=  

20396632.00572718336484608918971225516342}, {r =  

23888961.16565299817948948119449178210506 ..  

23888990.90855127150824991682392263419777, rm =  

.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}  

);  

Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1e-32  

Equations at solution: [-.1e-31, -.1e-31, .11157188429e-34]  

Solution in 0.252s  
  

Time Plot 1.785 s.  

Exiting SolveHard() after 3.5r=2.38890e+07 in  

[23888961.16565299817948948119449178210506 ..  

2388899090855127150824991682392263419777/1000000000000000000000000  

000000]  

Scattering ray (rm=2.03966e+07) in  

[298960418182500/22468879468420441 .. 24089000]: target and source  

on the different branches.  

Clockwise ray.  

Ray outgoing at target.  

Solve Side.  
  

Tau [10510.45813678936207842014764182324596788,  

10510.36930058682532618475501145707984185,  

10510.39514076180265431188536776817562044,  

10510.28046624350321269977944246351856030,  

10510.36313884719584037870310490843560311,  

10510.33729983843119480224801106566318860,  

10510.33214580450188010420862572621511170,
```



```

10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
10510.26914997049867919619036826661055917,
10510.27430450924162391988961859334736825,
10510.24330971172246516729985032379126358,
10510.33113809489794039050928383230867111,
10510.21646359961425315844565703156723271,
10510.18546892759931749020200814673581561,
10510.21130922484649961353409212373837542, none, none,
10510.27329709823633588538635678513651667, none, none, none, none,
none, none, none, none, none, none]

2 --> 1 target = [24850262.70323078644925875505002433594091,
1.476940274031262782949409178233029376476,
10510.19163349172132567315655665283194565]
one interval r = 25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/20000000000000000000000000000000
0000
Time Approximations 0.032.

hint used Hint := [25795889.93759639200267360170740297062535, 3, -1,
1, 24743522.41979937237431350459260520845964,
25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/20000000000000000000000000000000
0000, 298960418182500/22468879468420441 ..
24850262.70323078644925875505002433594091, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.76017) | P <--- S
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07
k=-2.37384e+14 scos=5.95461e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
25795889.93759639200267360170740297062535, rm=
24743522.41979937237431350459260520845964}, {r =
25795862.39063358446748128489577933480278 ..
25795891.82119371218082845151233599574945, rm =
.1330553304194287328500223794129351168576e-1 ..
24850262.70323078644925875505002433594091}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.2475e-31, 0., .7e-35]
Solution in 0.723s

Time Plot 0.866 s.
Exiting SolveHard() after 2.19r=2.57959e+07 in
[25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/20000000000000000000000000000000
0000]
Scattering ray (rm=2.47435e+07) in
[298960418182500/22468879468420441 ..
24850262.70323078644925875505002433594091]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

```



```
Tau [10510.15813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,  
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111,  
10510.21646359961425315844565703156723271,  
10510.18546892759931749020200814673581561,  
10510.21130922484649961353409212373837542, none, none,  
10510.27329709823633588538635678513651667, none, none,  
10510.15962933478753614217473212128017612, none, none, none, none,  
none, none, none]  
  
0 --> 1 target = [24850262.70323078644925875505002433594091,  
1.476940274031262782949409178233029376476,  
10510.19163349172132567315655665283194565]  
one interval r = 23888954.40382902066202638611670198664112 ..  
298612301838385292588525592212900916387/12500000000000000000000000  
0000  
Time Approximations 0.047.  
  
hint used Hint := [23888978.86275416794827744619407470166172, 3, -1,  
1, 20396633.60486737240849086583158368073964,  
23888954.40382902066202638611670198664112 ..  
298612301838385292588525592212900916387/12500000000000000000000000  
0000, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.466972) | S ---> P  
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07  
k=-5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=  
23888978.86275416794827744619407470166172, rm=  
20396633.60486737240849086583158368073964}, {r =  
23888954.40382902066202638611670198664112 ..  
23888984.14707082340708204737703207331096, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}  
);  
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=5e-32  
Equations at solution: [.9e-31, .5e-31, -.48097927114e-34]  
Solution in 0.263s  
  
Time Plot 1.792 s.  
Exiting SolveHard() after 3.587r=2.38890e+07 in  
[23888954.40382902066202638611670198664112 ..  
298612301838385292588525592212900916387/12500000000000000000000000  
0000]  
Scatteringray (rm=2.03966e+07) in
```



[298960418182500/22468879468420441 .. 24089000]: target and source  
on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,  
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111,  
10510.21646359961425315844565703156723271,  
10510.18546892759931749020200814673581561,  
10510.21130922484649961353409212373837542,  
10510.10279600720328866736745742636090095, none,  
10510.27329709823633588538635678513651667, none, none,  
10510.15962933478753614217473212128017612, none, none, none, none,  
none, none, none]

0 --> 2 target = [25795893.34215400624221088590056918107762,  
1.856291208228086723992130094043171908925,  
10510.21747360711085261323132545308200478]  
one interval r = 23888955.94100364913487175812205942139954 ..  
9555594273666943272318156666787125279/40000000000000000000000000000000  
Time Approximations 0.05.

hint used Hint := [23888981.93694517027845377679197924121306, 3, -1,  
1, 22860455.76745222490936139362415353022295,  
23888955.94100364913487175812205942139954 ..  
9555594273666943272318156666787125279/40000000000000000000000000000000  
, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.367179) | S ---> P  
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07  
k=-4.31730e+14 scos=4.39725e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=  
23888981.93694517027845377679197924121306, rm=  
22860455.76745222490936139362415353022295}, {r =  
23888955.94100364913487175812205942139954 ..  
23888985.68416735818079539166696781319750, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}  
);  
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=4e-32  
Equations at solution: [-.5e-31, -.4e-31, .17869651403e-34]  
Solution in 0.76s















1.476948574973150678424900072270774720410,  
10510.24331300771292254440409885192330801]  
one interval r = 23888957.47813172092635353311983299926098 ..  
597224680530433446694670341991304966889/25000000000000000000000000000000  
0000  
Time Approximations 0.046.

hint used Hint := [23888981.93692575516559285951162594064654, 3, -1,  
1, 20396632.87781872303409295730403849988110,  
23888957.47813172092635353311983299926098 ..  
597224680530433446694670341991304966889/25000000000000000000000000000000  
0000, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.466972) | S ---> P  
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07  
k=-5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=  
23888981.93692575516559285951162594064654, rm=  
20396632.87781872303409295730403849988110}, {r =  
23888957.47813172092635353311983299926098 ..  
23888987.22121733786778681367965219867556, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}  
);  
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=9e-32  
Equations at solution: [.17e-30, .9e-31, -.49839784331e-34]  
Solution in 0.802s

Time Plot 1.672 s.  
Exiting SolveHard() after 3.98r=2.38890e+07 in  
[23888957.47813172092635353311983299926098 ..  
597224680530433446694670341991304966889/25000000000000000000000000000000  
0000]  
Scattering ray (rm=2.03966e+07) in  
[298960418182500/22468879468420441 .. 24089000]: target and source  
on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,  
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111,  
10510.21646359961425315844565703156723271,  
10510.18546892759931749020200814673581561,



```

10510.21130922484649961353409212373837542,
10510.10279600720328866736745742636090095,
10510.18546846172437709400611835600963181,
10510.27329709823633588538635678513651667,
10510.15447580454509166514822644094075171, none,
10510.15962933478753614217473212128017612,
10510.15447547815947286212833271267435954, none,
10510.21130888350685915386493858978930221, none, none, none, none]

0 --> 2 target = [25795896.38402523646204355452114396262515,
1.856299066265858109439961038732420613264,
10510.26915308872895791524093204162206305]
one interval r = 23888959.01529623342321627788824672003212 ..
59722471895759389095528762418719523933/2500000000000000000000000000000000
00
Time Approximations 0.539.

hint used Hint := [23888985.01109652402578398653593278001387, 3, -1,
1, 22860454.49191911041368999760273437756943,
23888959.01529623342321627788824672003212 ..
59722471895759389095528762418719523933/2500000000000000000000000000000000
00, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.367179) | S --> P
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07
k=-4.31731e+14 scos=4.39725e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
23888985.01109652402578398653593278001387, rm=
22860454.49191911041368999760273437756943}, {r =
23888959.01529623342321627788824672003212 ..
23888988.75830375563821150496748780957320, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}
);
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.4e-31
Equations at solution: [-.15e-30, -.14e-30, .49600392670e-34]
Solution in 0.231s

Time Plot 1.579 s.
Exiting SolveHard() after 3.252r=2.38890e+07 in
[23888959.01529623342321627788824672003212 ..
59722471895759389095528762418719523933/2500000000000000000000000000000000
00]
Scattering ray (rm=2.28605e+07) in
[298960418182500/22468879468420441 .. 24089000]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,

```



```

10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
10510.26914997049867919619036826661055917,
10510.27430450924162391988961859334736825,
10510.24330971172246516729985032379126358,
10510.33113809489794039050928383230867111,
10510.21646359961425315844565703156723271,
10510.18546892759931749020200814673581561,
10510.21130922484649961353409212373837542,
10510.10279600720328866736745742636090095,
10510.18546846172437709400611835600963181,
10510.27329709823633588538635678513651667,
10510.15447580454509166514822644094075171, none,
10510.15962933478753614217473212128017612,
10510.15447547815947286212833271267435954, none,
10510.21130888350685915386493858978930221,
10510.20615520675546656272235083800212186, none, none, none]

1 --> 2 target = [25795896.38402523646204355452114396262515,
1.856299066265858109439961038732420613264,
10510.26915308872895791524093204162206305]
one interval r = 24850237.69689665459305925255508686501973 ..
1242513364573987488725142465429694354713/50000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [24850265.39735153163244735697452997613798, 3, 1,
1, 24743509.60268659785926677493114233979369,
24850237.69689665459305925255508686501973 ..
1242513364573987488725142465429694354713/50000000000000000000000000000000
00000, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.239829) | S ---> P
rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07
k=2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
24850265.39735153163244735697452997613798, rm=
24743509.60268659785926677493114233979369}, {r =
24850237.69689665459305925255508686501973 ..
24850267.29147974977450284930859388709426, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={})
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.2475e-31, 0., .18e-34]
Solution in 0.224s

Time Plot 1.419 s.
Exiting SolveHard() after 2.357r=2.48503e+07 in
[24850237.69689665459305925255508686501973 ..
1242513364573987488725142465429694354713/50000000000000000000000000000000
00000]
Scattering ray (rm=2.47435e+07) in
[298960418182500/22468879468420441 .. 25089000]: target and source
on the different branches.

```



Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

```
Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,  
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111,  
10510.21646359961425315844565703156723271,  
10510.18546892759931749020200814673581561,  
10510.21130922484649961353409212373837542,  
10510.10279600720328866736745742636090095,  
10510.18546846172437709400611835600963181,  
10510.27329709823633588538635678513651667,  
10510.15447580454509166514822644094075171,  
10510.23714797607061291086956144565493086,  
10510.15962933478753614217473212128017612,  
10510.15447547815947286212833271267435954, none,  
10510.21130888350685915386493858978930221,  
10510.20615520675546656272235083800212186, none, none, none]
```

[illegible]

```
hint used Hint := [23888988.69843567795653101185231580305160, 3, -1,  
1, 22860452.96192886545581288218987601576608,  
23888962.70280479206299436081506076835447 ..  
2388899244562496946280763877837409340903/10000000000000000000000000  
000000, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.36718) | S ---> P  
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07  
k=-4.31731e+14 scos=4.39724e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=  
23888988.69843567795653101185231580305160, rm=  
22860452.96192886545581288218987601576608}, {r =  
23888962.70280479206299436081506076835447 ..  
23888992.44562496946280763877837409340903, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}  
);
```











```

10510.26814323592336180387677929719916736]

1 --> 0  target = [23888989.06493588745880515057802801897934,
2.632525136809920840883118370193127663532,
10510.27430791380262283822338748373982610]
one interval r = 24850238.00201200929744389905914961384924 ..
1242513379829193745424075414664369797603/50000000000000000000000000000000
00000
Time Approximations 0.041.

hint used Hint := [24850262.33852773859450921827957797170818, 3, 1,
1, 20396437.98350304935080318674410988058300,
24850238.00201200929744389905914961384924 ..
1242513379829193745424075414664369797603/50000000000000000000000000000000
00000, 298960418182500/22468879468420441 ..
23888989.06493588745880515057802801897934, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.533028) | P <--- S
rGuessMin=2.48502e+07  rGuessMax=2.48503e+07  rmGuess=2.03964e+07
k=5.43203e+14  scos=2.39475e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
24850262.33852773859450921827957797170818, rm=
20396437.98350304935080318674410988058300}, {r =
24850238.00201200929744389905914961384924 ..
24850267.59658387490848150829328739595206, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06493588745880515057802801897934}, avoid={});
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=8e-32
Equations at solution: [-.12e-30, -.8e-31, .63e-34]
Solution in 0.24s

Time Plot 1.787 s.
Exiting SolveHard() after 2.755r=2.48503e+07 in
[24850238.00201200929744389905914961384924 ..
1242513379829193745424075414664369797603/50000000000000000000000000000000
00000]
Scattering ray (rm=2.03964e+07) in
[298960418182500/22468879468420441 ..
23888989.06493588745880515057802801897934]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678936207842014764182324596788,
10510.36930058682532618475501145707984185,
10510.39514076180265431188536776817562044,
10510.28046624350321269977944246351856030,
10510.36313884719584037870310490843560311,
10510.33729983843119480224801106566318860,
10510.33214580450188010420862572621511170,
10510.30529788717935851166146004846522174,
10510.21747048889388355633134516173064786,
10510.27430496999043214669943582338314253,
10510.19163019574467350866023355115823455,
10510.26914997049867919619036826661055917,

```



```
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111,  
10510.21646359961425315844565703156723271,  
10510.18546892759931749020200814673581561,  
10510.21130922484649961353409212373837542,  
10510.10279600720328866736745742636090095,  
10510.18546846172437709400611835600963181,  
10510.27329709823633588538635678513651667,  
10510.15447580454509166514822644094075171,  
10510.23714797607061291086956144565493086,  
10510.15962933478753614217473212128017612,  
10510.15447547815947286212833271267435954,  
10510.29913613974234190309901279325515298,  
10510.21130888350685915386493858978930221,  
10510.20615520675546656272235083800212186,  
10510.18546846685091064707627195110501083, none,  
10510.26814323592336180387677929719916736]
```

```
2 --> 0 target = [23888989.06493588745880515057802801897934,  
2.632525136809920840883118370193127663532,  
10510.27430791380262283822338748373982610]  
one interval r = 25795867.25698799409451243993168295190626 ..  
1289794834371977270055988053158890743643/50000000000000000000000000000000  
00000
```

Time Approximations 0.032.

```
hint used Hint := [25795892.97947239655457473800077316699564, 3, 1,  
1, 22860366.33050119628977322107538139299992,  
25795867.25698799409451243993168295190626 ..  
1289794834371977270055988053158890743643/50000000000000000000000000000000  
00000, 298960418182500/22468879468420441 ..  
23888989.06493588745880515057802801897934, 1]
```

I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.632821) | P <--- S

rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07  
k=4.31731e+14 scos=4.39725e+14

branch outgoing at target, Counterclockwise

(Scattering) fsolve(eqs, {r=

25795892.97947239655457473800077316699564, rm=

22860366.33050119628977322107538139299992}, {r =

25795867.25698799409451243993168295190626 ..

25795896.68743954540111976106317781487286, rm =

.1330553304194287328500223794129351168576e-1 ..

23888989.06493588745880515057802801897934}, avoid={});

Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=4e-32

Equations at solution: [-.5e-31, -.4e-31, .15e-34]

Solution in 0.205s

Time Plot 1.525 s.

Exiting SolveHard() after 2.806r=2.57959e+07 in

[25795867.25698799409451243993168295190626 ..

1289794834371977270055988053158890743643/50000000000000000000000000000000  
00000]

Scattering ray (rm=2.28604e+07) in

[298960418182500/22468879468420441 ..

23888989.06493588745880515057802801897934]: target and source on the



different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,  
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111,  
10510.21646359961425315844565703156723271,  
10510.18546892759931749020200814673581561,  
10510.21130922484649961353409212373837542,  
10510.10279600720328866736745742636090095,  
10510.18546846172437709400611835600963181,  
10510.27329709823633588538635678513651667,  
10510.15447580454509166514822644094075171,  
10510.23714797607061291086956144565493086,  
10510.15962933478753614217473212128017612,  
10510.15447547815947286212833271267435954,  
10510.29913613974234190309901279325515298,  
10510.21130888350685915386493858978930221,  
10510.20615520675546656272235083800212186,  
10510.18546846685091064707627195110501083,  
10510.21130876409839922259159893924009430,  
10510.26814323592336180387677929719916736]

Cascade time 88.928

[10510.45813678936207842014764182324596788,  
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,

(86)



10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111,  
10510.21646359961425315844565703156723271,  
10510.18546892759931749020200814673581561,  
10510.21130922484649961353409212373837542,  
10510.10279600720328866736745742636090095,  
10510.18546846172437709400611835600963181,  
10510.27329709823633588538635678513651667,  
10510.15447580454509166514822644094075171,  
10510.23714797607061291086956144565493086,  
10510.15962933478753614217473212128017612,  
10510.15447547815947286212833271267435954,  
10510.29913613974234190309901279325515298,  
10510.21130888350685915386493858978930221,  
10510.20615520675546656272235083800212186,  
10510.18546846685091064707627195110501083,  
10510.21130876409839922259159893924009430,  
10510.268143235923361803876779297199167361

```
> HintsOriginal:=ListHints;
```

$$\begin{aligned} HintsOriginal := & \left[ none, \left[ 2.485027321919080441727298772802245902434 \times 10^7, 3, 1, 1, \right. \right. \\ & 2.039643539659592643000634328381896732011 \times 10^7, \\ & 2.485024888301387793152980010731189259064 \times 10^7 \\ & .. \frac{621256961929631488519759813441433737669}{250000000000000000000000000000}, \frac{298960418182500}{22468879468420441} .. 2.3889000 \\ & \times 10^7, 1 \Big], \left[ 2.579590379985511561316781642113695093491 \times 10^7, 3, 1, 1, \right. \\ & 2.286036179220184767639985483105161990097 \times 10^7, \\ & 2.579587807759835662287683941521694813384 \times 10^7 \\ & .. \frac{161224421923802901739910205661944662513}{625000000000000000000000000000}, \frac{298960418182500}{22468879468420441} .. 2.3889000 \\ & \times 10^7, 1 \Big], \left[ 2.388898943146328644175807501347600997649 \times 10^7, 3, -1, 1, \right. \end{aligned}$$







$$2.286036617838899487248578273534549876757 \times 10^7,$$

$$2.579586761967723367271871049090258979880 \times 10^7$$

$$\frac{1289794852506034629127592665641586312049}{5000000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$[..2.388898943146328644175807501347600997649 \times 10^7, 1],$$

$$\left[ 2.388898906496329485872911197089488868350 \times 10^7, 3, -1, 1, \right.$$

$$2.286045280984339454018821965990898965218 \times 10^7,$$

$$2.388896306934924828335479031706673264809 \times 10^7$$

$$\dots \frac{597224820303770076400571671810948623397}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 24089000, 1],$$

$$[2.485026270323078644925875505002433594091 \times 10^7, 3, 1, 1,$$

$$2.039643789679703248883954261208042438711 \times 10^7,$$

$$2.485023836672641289161845524403523557614 \times 10^7$$

$$\therefore \frac{2485026796128485539651696969285982263711}{1000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$..2.388898943146328644175807501347600997649 \times 10^7, 1],$$

$$[2.579589638402523646204355452114396262515 \times 10^7, 3, 1, 1,$$

$$2.286036490258708217946452135489353704915 \times 10^7,$$

$$2.579587066161245800881604810046494170563 \times 10^7$$

$$\therefore \frac{2579590009198804353979547140796504033263}{100000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$..2.388899250558472677941562199764206716244 \times 10^7, 1 \Big],$$

$$\left[ 2.388898906493588745880515057802801897934 \times 10^7, 3, -1, 1, \right.$$



$$2.039663119196530471597662319541189563767 \times 10^7,$$

$$2.388896460644587167575168019887114912430 \times 10^7$$

$$\dots \frac{1194449717458466618404658405105548818961}{50000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 24089000, 1 \Big],$$

$$\Big[ 2.485026576205359199870736703593497152527 \times 10^7, 3, 1, 1,$$

$$2.039643716957127706091945345483725838621 \times 10^7,$$

$$2.485024142564446195929857082855634229261 \times 10^7$$

$$\dots \frac{1242513551004516068014443586172817582043}{50000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$\dots 2.388899250558472677941562199764206716244 \times 10^7, 1 \Big],$$

$$\Big[ 2.579590003265730627104171495990540839944 \times 10^7, 3, -1, 1,$$

$$2.474353499638995296918328323923452726440 \times 10^7,$$

$$2.579587248589893272346549352544011957533 \times 10^7$$

$$\dots \frac{644897547905845320097763426011396162913}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$\dots 2.485027285449189132526402658550193546657 \times 10^7, 1 \Big],$$

$$\Big[ 2.388898562428976346864819081258538077918 \times 10^7, 3, -1, 1,$$

$$2.039663200572718336484608918971225516342 \times 10^7,$$

$$2.388896116565299817948948119449178210506 \times 10^7$$

$$\dots \frac{2388899090855127150824991682392263419777}{100000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 24089000, 1 \Big],$$

$$\Big[ 2.485026233855500957466342744981057733590 \times 10^7, 3, 1, 1,$$

$$2.039643798349656584284330028372043025521 \times 10^7,$$

$$2.485023800203928112673210949390165384854 \times 10^7$$



$$\begin{aligned} & \dots \frac{99401070386444582936187413117559617381}{40000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & \dots 2.388898906496329485872911197089488868350 \times 10^7, 1 \Big], \\ & \left[ 2.579589297949951637755150436992208534955 \times 10^7, 3, 1, 1, \right. \\ & 2.286036633048982198263378361971969356546 \times 10^7, \\ & 2.579586725701511448801980095492062593890 \times 10^7 \\ & \dots \frac{644897417186666629737809369304875630137}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & \dots 2.388898906496329485872911197089488868350 \times 10^7, 1 \Big], \\ & \left[ 2.388897886275416794827744619407470166172 \times 10^7, 3, -1, 1, \right. \\ & 2.039663360486737240849086583158368073964 \times 10^7, \\ & 2.388895440382902066202638611670198664112 \times 10^7 \\ & \dots \frac{298612301838385292588525592212900916387}{12500000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 2.4089000, 1 \Big], \\ & \left[ 2.485026233852743516297758053367205548154 \times 10^7, 3, 1, 1, \right. \\ & 2.474350581299238129247202614818714422035 \times 10^7, \\ & 2.485023463796912173141920432208117660707 \times 10^7 \\ & \dots \frac{2485026423266479739181704602953513976831}{10000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 2.5089000, 1 \Big], \\ & \left[ 2.579589662812623062600675830146784169746 \times 10^7, 3, -1, 1, \right. \\ & 2.474353075497470617154487139175541092326 \times 10^7, \\ & 2.579586908129891002682354003571513818579 \times 10^7 \\ & \dots \frac{2579589851170975665166085604166659164517}{10000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & \dots 2.485026943101309861241733616879334665430 \times 10^7, 1 \Big], \end{aligned}$$



$$2.286045576745222490936139362415353022295 \times 10^7,$$

$$2.388895594100364913487175812205942139954 \times 10^7$$

$$\dots \frac{9555594273666943272318156666787125279}{4000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 24089000, 1],$$

$$\left[ 2.485026539735153163244735697452997613798 \times 10^7, 3, 1, 1, \right.$$

$$2.474350960268659785926677493114233979369 \times 10^7,$$

$$2.485023769689665459305925255508686501973 \times 10^7$$

$$\dots \frac{1242513364573987488725142465429694354713}{5000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 25089000, 1],$$

$$\left[ 2.579588993759639200267360170740297062535 \times 10^7, 3, -1, 1, \right.$$

$$2.474352241979937237431350459260520845964 \times 10^7,$$

$$2.579586239063358446748128489577933480278 \times 10^7$$

$$\frac{515917836423874243616569030246719914989}{20000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$[..2.485026270323078644925875505002433594091 \times 10^7, 1],$$

$$\left[ 2.388898193692575516559285951162594064654 \times 10^7, 3, -1, 1, \right.$$

$$2.039663287781872303409295730403849988110 \times 10^7,$$

$$2.388895747813172092635353311983299926098 \times 10^7$$

$$\dots \frac{597224680530433446694670341991304966889}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 24089000, 1],$$

$$\left[ 2.485026906631200499486567135274096895840 \times 10^7, 3, 1, 1, \right.$$

$$2.474351414830449237551739504443492551457 \times 10^7,$$

$$2.485024136598119958086914129896830053765 \times 10^7$$



$$\begin{aligned} & \dots \frac{2485027096042925490424856544845137371367}{10000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} ..25089000, 1], \\ [ & 2.579589297947942498571214559240875822627 \times 10^7, 3, -1, 1, \\ & 2.474352620943107487634918939497374544011 \times 10^7, \\ & 2.579586543257821616570179673528437159660 \times 10^7 \\ & \dots \frac{644897371576761841392917646201273823237}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & ..2.485026576205359199870736703593497152527 \times 10^7, 1], \\ [ & 2.388898501109652402578398653593278001387 \times 10^7, 3, -1, 1, \\ & 2.286045449191911041368999760273437756943 \times 10^7, \\ & 2.388895901529623342321627788824672003212 \times 10^7 \\ & \dots \frac{59722471895759389095528762418719523933}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} ..24089000, 1], \\ [ & 2.485026233852773859450921827957797170818 \times 10^7, 3, 1, 1, \\ & 2.039643798350304935080318674410988058300 \times 10^7, \\ & 2.485023800201200929744389905914961384924 \times 10^7 \\ & \dots \frac{1242513379829193745424075414664369797603}{50000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & ..2.388898906493588745880515057802801897934 \times 10^7, 1], \\ [ & 2.579589297947239655457473800077316699564 \times 10^7, 3, 1, 1, \\ & 2.286036633050119628977322107538139299992 \times 10^7, \\ & 2.579586725698799409451243993168295190626 \times 10^7 \\ & \dots \frac{1289794834371977270055988053158890743643}{50000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & ..2.388898906493588745880515057802801897934 \times 10^7, 1], \end{aligned}$$



```
> ListP;
```

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10510.21747360711085261323132545308200478],  
[2.388898906496329485872911197089488868350  $\times 10^7$ ,  
2.632525136888364227325281476146718182654,  
10510.27430837455143119298455217139668111],  
[2.485026270323078644925875505002433594091  $\times 10^7$ ,  
1.476940274031262782949409178233029376476,  
10510.19163349172132567315655665283194565],  
[2.579589638402523646204355452114396262515  $\times 10^7$ ,  
1.856299066265858109439961038732420613264,  
10510.26915308872895791524093204162206305],  
[2.388898906493588745880515057802801897934  $\times 10^7$ ,  
2.632525136809920840883118370193127663532,  
10510.27430791380262283822338748373982610],  
[2.485026576205359199870736703593497152527  $\times 10^7$ ,  
1.476948574973150678424900072270774720410,  
10510.24331300771292254440409885192330801],  
[2.579590003265730627104171495990540839944  $\times 10^7$ ,  
1.856308491764749423379286180252072848042,  
10510.33114121314418368243534285689405493],  
[2.388898562428976346864819081258538077918  $\times 10^7$ ,  
2.632515289280424206131912197926326469939,  
10510.21646700415918947844933270059772262],  
[2.485026233855500957466342744981057733590  $\times 10^7$ ,  
1.476939284387011891994733060386469058011,  
10510.18547222357432378741608618331333162],  
[2.579589297949951637755150436992208534955  $\times 10^7$ ,  
1.856290271387187972181639013079578187667,  
10510.21131234306188188303663680330418308],  
[2.388897886275416794827744619407470166172  $\times 10^7$ ,  
2.632495937139278365787357031758438379818,  
10510.10279941171665913868204367584729103],  
[2.485026233852743516297758053367205548154  $\times 10^7$ ,



1.476939284312181442761458511933570344463,  
10510.18547175769938326677043841340214688],  
[2.579589662812623062600675830146784169746  $\times 10^7$ ,  
1.856299696850400409360845210716945284409,  
10510.27330021646768266593203519288941732],  
[2.388898193694517027845377679197924121306  $\times 10^7$ ,  
2.632504735732313595979470005326364074828,  
10510.15447920907281378055866420134300440],  
[2.485026539735153163244735697452997613798  $\times 10^7$ ,  
1.476947585254050921513325282894279988209,  
10510.23715127205942429607669743097876350],  
[2.579588993759639200267360170740297062535  $\times 10^7$ ,  
1.856282413285235542277892438065723972356,  
10510.15963245298960864198697440313036003],  
[2.388898193692575516559285951162594064654  $\times 10^7$ ,  
2.632504735676745772370268426291614807656,  
10510.15447888268719488690045381920543437],  
[2.485026906631200499486567135274096895840  $\times 10^7$ ,  
1.476957542004040463083358424683822000041,  
10510.29913943574771225996952465456145640],  
[2.579589297947942498571214559240875822627  $\times 10^7$ ,  
1.856290271335286132301953878872779740164,  
10510.21131200172224133545801523733254263],  
[2.388898501109652402578398653593278001387  $\times 10^7$ ,  
2.632513534255812389126480133535401951085,  
10510.20615861129754021002998679026322401],  
[2.485026233852773859450921827957797170818  $\times 10^7$ ,  
1.476939284313004884366752566883988035157,  
10510.18547176282591682121004935559081749],  
[2.579589297947239655457473800077316699564  $\times 10^7$ ,  
1.856290271317129674530763579789604514168,  
10510.21131188231378137343192100793454908],



```
[2.388898869843567795653101185231580305160 × 107,  
2.632524087841062612764952030294829834920,  
10510.26814664048264971782564984723048132]]
```

```
> Tau31Original:=ListTau;
```

```
Tau31Original := [10510.45813678936207842014764182324596788,
```

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```
10510.36930058682532618475501145707984185,  
10510.39514076180265431188536776817562044,  
10510.28046624350321269977944246351856030,  
10510.36313884719584037870310490843560311,  
10510.33729983843119480224801106566318860,  
10510.33214580450188010420862572621511170,  
10510.30529788717935851166146004846522174,  
10510.21747048889388355633134516173064786,  
10510.27430496999043214669943582338314253,  
10510.19163019574467350866023355115823455,  
10510.26914997049867919619036826661055917,  
10510.27430450924162391988961859334736825,  
10510.24330971172246516729985032379126358,  
10510.33113809489794039050928383230867111,  
10510.21646359961425315844565703156723271,  
10510.18546892759931749020200814673581561,  
10510.21130922484649961353409212373837542,  
10510.10279600720328866736745742636090095,  
10510.18546846172437709400611835600963181,  
10510.27329709823633588538635678513651667,  
10510.15447580454509166514822644094075171,  
10510.23714797607061291086956144565493086,  
10510.15962933478753614217473212128017612,  
10510.15447547815947286212833271267435954,  
10510.29913613974234190309901279325515298,  
10510.21130888350685915386493858978930221,  
10510.20615520675546656272235083800212186,  
10510.18546846685091064707627195110501083,  
10510.21130876409839922259159893924009430,  
10510.26814323592336180387677929719916736]
```

```
> PurifyTau := proc (Tau)  
  local T, n, Tr;
```



```

T:= Tau[1..3]:
Tr:=CreateList(31, 0):

for n from 0 to nops(Tau)-1 do
  Tr[n+1] := -Tau[n+1] +T[1+ (n mod 3)]:
end do:
Tr[4..MaxSignals];
end:

```

```

> ObsSignals:=PurifyTau(Tau31Original);
ObsSignals := [0.17767054585886572036819935972740758,
0.00616173962948580605190654864423874, 0.05784092337145950963735670251243184,
0.12599098486019831593901609703085618, 0.06400269964596767309355140861462011,
0.17767027290877075555402260644497258, 0.18383181937164627344820599986282535,
0.17767039108065267609477790592160730, 0.12599079130397511569499950156506127,
0.18383228012045450025802322989859963, 0.12599087510286101745516113328857827,
0.06400266690471392137608393586694933, 0.24167318974782526170198479167873517,
0.18383165922600869455300331034402624, 0.18383153695615469835127564443724502,
0.35534078215878975278018439688506693, 0.18383212510094909074889310107021004,
0.12184366356631842649901098303910377, 0.30366098481698675499941538230521617,
0.13215261075471327388545001142491099, 0.23551142701511816971063564689544432,
0.30366131120260555801930911057160834, 0.07016444708298428165599866382468887,
0.18383187829579515802042917838631823, 0.25198158260661185742529098524384602,
0.18383211997441553767873950597483102, 0.18383199770425508929376882893552614,
0.18999355343871661627086252604680052]

```

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```

> [xS(0, 1),yS(0, 1), tS(0, 1)]:
subs(r=BaseR, %):
evalf(%):
P0:=%;

P0 := [−347872.5347758159415648681665589043635417,
−2.388646699910953027770045775437464303397 × 107,
22774.25934936053024401128331027934092093]

```

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```

> Rumor:= proc(x, si, n:= 1)
  if type(x, list) then
    return map(Rumor, x, si, n):
  else
    x+Sample(Normal(0,si), n)[1]:
    Q(%):
  end if:

```







[illegible]

$$\left[ \begin{array}{l} \textcolor{brown}{> \text{MaxT}:=1/2;} \\ \textcolor{blue}{MaxT} := \frac{1}{2} \end{array} \right. \quad (97)$$

```
> [xS(0, -1), yS(0, -1), tS(0, -1), r=minS(0)..maxS(0)-0.00001]:  
evalf(%):  
c0_1n := spacecurve(% , axes=box, color = red):  
  
> [xS(0, 0), yS(0, 0), tS(0, 0), r=minS(0)..maxS(0)-0.00001]:  
evalf(%):  
c0_0 := spacecurve(% , axes=box, color = red, thickness = 4):  
  
> [xS(0, 1), yS(0, 1), tS(0, 1), r=minS(0)..maxS(0)-0.00001]:  
evalf(%):  
c0_1 := spacecurve(% , axes=box, color = red):
```

```
> [xS(0, 0), yS(0, 0), tS(0, 0), r=minS(0)..maxS(0)-0.00001]:  
evalf(%):  
c0_0 := spacecurve(% , axes=box, color = red, thickness = 4):  
  
> [xS(0, 1), yS(0, 1), tS(0, 1), r=minS(0)..maxS(0)-0.00001]:  
evalf(%):  
c0_1 := spacecurve(% , axes=box, color = red):
```

```
> [xS(0, 1), yS(0, 1), tS(0, 1), r=minS(0)..maxS(0)-0.00001]:  
evalf(%):  
c0_1 := spacecurve(%, axes=box, color = red):
```

```
> [xS(1, -1), yS(1, -1), tS(1, -1), r=rm1v..maxS(1)]:  
evalf(%):  
c1 ln := spacecurve(% , axes=box, color = blue):
```

```
> [xS(1, 0), yS(1, 0), tS(1, 0), r=minS(1)..maxS(1)]:  
evalf(%):  
c1 0 := spacecurve(%, axes=box, color = blue, thickness = 4):
```

```
> [xS(1, 1), yS(1, 1), tS(1, 1), r=minS(1)..maxS(1)]:  
evalf(%):  
c1 1 := spacecurve(%, axes=box, color = blue):
```

```
> [xS(2, -1), yS(2, -1), tS(2, -1), r=minS(2).. $\max$ S(2)]:  
evalf(%):
```



```
c2_1n := spacecurve(% , axes=box, color = green):
```

```
> [xS(2, 0), yS(2, 0), tS(2, 0), r=minS(2)..maxS(2)]:
```

```
evalf(%):
```

```
c2_0 := spacecurve(% , axes=box, color = green, thickness = 4):
```

```
> [xS(2, 1), yS(2, 1), tS(2, 1), r=minS(2)..maxS(2)]:
```

```
evalf(%):
```

```
c2_1 := spacecurve(% , axes=box, color = green):
```

```
> [xS(2, 1), yS(2, 1), tS(2, 1)]:
```

```
subs( r=minS(2)+3000, %):
```

```
evalf(%):
```

```
%;
```

```
[2.509633250971791423506592344643737497830 × 107,  
-3.139961554171601162464902239629135445563 × 106,  
38788.71922578421490894396141216712926245]
```

(98)

```
> display(BH, S,
```

```
user,
```

```
c0_1n, c0_0, c0_1,
```

```
c1_1n, c1_0, c1_1,
```

```
c2_1n, c2_0, c2_1,
```

```
view=[ -3*107..3*107, -3*107..3*107, 100..10510]);
```











$$2.286036617838899487248578273534549876757 \times 10^7,$$

$$2.579586761967723367271871049090258979880 \times 10^7$$

$$\frac{1289794852506034629127592665641586312049}{5000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$[..2.388898943146328644175807501347600997649 \times 10^7, 1],$$

$$\left[ 2.388898906496329485872911197089488868350 \times 10^7, 3, -1, 1, \right.$$

$$2.286045280984339454018821965990898965218 \times 10^7,$$

$$2.388896306934924828335479031706673264809 \times 10^7$$

$$\dots \frac{597224820303770076400571671810948623397}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 24089000, 1],$$

$$[2.485026270323078644925875505002433594091 \times 10^7, 3, 1, 1,$$

$$2.039643789679703248883954261208042438711 \times 10^7,$$

$$2.485023836672641289161845524403523557614 \times 10^7$$

$$\therefore \frac{2485026796128485539651696969285982263711}{1000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$..2.388898943146328644175807501347600997649 \times 10^7, 1],$$

$$[2.579589638402523646204355452114396262515 \times 10^7, 3, 1, 1,$$

$$2.286036490258708217946452135489353704915 \times 10^7,$$

$$2.579587066161245800881604810046494170563 \times 10^7$$

$$\therefore \frac{2579590009198804353979547140796504033263}{100000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$..2.388899250558472677941562199764206716244 \times 10^7, 1 \Big],$$

$$\left[ 2.388898906493588745880515057802801897934 \times 10^7, 3, -1, 1, \right.$$







$$\begin{aligned} & \dots \frac{99401070386444582936187413117559617381}{40000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & \dots 2.388898906496329485872911197089488868350 \times 10^7, 1 \Big], \\ & \left[ 2.579589297949951637755150436992208534955 \times 10^7, 3, 1, 1, \right. \\ & 2.286036633048982198263378361971969356546 \times 10^7, \\ & 2.579586725701511448801980095492062593890 \times 10^7 \\ & \dots \frac{644897417186666629737809369304875630137}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & \dots 2.388898906496329485872911197089488868350 \times 10^7, 1 \Big], \\ & \left[ 2.388897886275416794827744619407470166172 \times 10^7, 3, -1, 1, \right. \\ & 2.039663360486737240849086583158368073964 \times 10^7, \\ & 2.388895440382902066202638611670198664112 \times 10^7 \\ & \dots \frac{298612301838385292588525592212900916387}{12500000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 2.4089000, 1 \Big], \\ & \left[ 2.485026233852743516297758053367205548154 \times 10^7, 3, 1, 1, \right. \\ & 2.474350581299238129247202614818714422035 \times 10^7, \\ & 2.485023463796912173141920432208117660707 \times 10^7 \\ & \dots \frac{2485026423266479739181704602953513976831}{10000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 2.5089000, 1 \Big], \\ & \left[ 2.579589662812623062600675830146784169746 \times 10^7, 3, -1, 1, \right. \\ & 2.474353075497470617154487139175541092326 \times 10^7, \\ & 2.579586908129891002682354003571513818579 \times 10^7 \\ & \dots \frac{2579589851170975665166085604166659164517}{10000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \\ & \dots 2.485026943101309861241733616879334665430 \times 10^7, 1 \Big], \end{aligned}$$



$$2.286045576745222490936139362415353022295 \times 10^7,$$

$$2.388895594100364913487175812205942139954 \times 10^7$$

$$\dots \frac{9555594273666943272318156666787125279}{4000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 24089000, 1],$$

$$\left[ 2.485026539735153163244735697452997613798 \times 10^7, 3, 1, 1, \right.$$

$$2.474350960268659785926677493114233979369 \times 10^7,$$

$$2.485023769689665459305925255508686501973 \times 10^7$$

$$\dots \frac{1242513364573987488725142465429694354713}{5000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 25089000, 1],$$

$$\left[ 2.579588993759639200267360170740297062535 \times 10^7, 3, -1, 1, \right.$$

$$2.474352241979937237431350459260520845964 \times 10^7,$$

$$2.579586239063358446748128489577933480278 \times 10^7$$

$$\frac{515917836423874243616569030246719914989}{20000000000000000000000000000000}, \frac{298960418182500}{22468879468420441}$$

$$[..2.485026270323078644925875505002433594091 \times 10^7, 1],$$

$$\left[ 2.388898193692575516559285951162594064654 \times 10^7, 3, -1, 1, \right.$$

$$2.039663287781872303409295730403849988110 \times 10^7,$$

$$2.388895747813172092635353311983299926098 \times 10^7$$

$$\dots \frac{597224680530433446694670341991304966889}{25000000000000000000000000000000}, \frac{298960418182500}{22468879468420441} \dots 24089000, 1],$$

$$\left[ 2.485026906631200499486567135274096895840 \times 10^7, 3, 1, 1, \right.$$

$$2.474351414830449237551739504443492551457 \times 10^7,$$

$$2.485024136598119958086914129896830053765 \times 10^7$$



[illegible]











```
Tau [10510.45813678928226660477102574814051205,  
10510.36930058675694383261265624655327347,  
10510.39514076173560537623842221650667075, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none]  
  
Start Generation 2  
2 --> 1 target = [24850273.21919079356362490230678462799050,  
1.476968812147896056910374220682530277520,  
10510.36930388274200031451993406344570704]  
Imaginary part neglected:  
-.1026155300751533131360866346426467533500e-14  
one interval r = 25795872.84858704378856951185306734762016 ..  
515918045578276621923499741794085540997/20000000000000000000000000000000  
00000  
Time Approximations 0.034.  
  
hint used Hint := [25795900.39533807920941978281062023785801, 3,  
-1, 1, 24743535.44822274617462265333263319921312,  
25795872.84858705064741864171344489129222 ..  
2579590227891383795486680697763551821623/10000000000000000000000000000000  
0000000, 298960418182500/22468879468420441 ..  
24850273.21919080441727298772802245902434, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.760172) | P <--- S  
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07  
k=-2.37383e+14 scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=  
25795900.39533807920941978281062023785801, rm=  
24743535.44822274617462265333263319921312}, {r =  
25795872.84858705064741864171344489129222 ..  
25795902.27891383795486680697763551821623, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850273.21919080441727298772802245902434}, avoid={}));  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [-.32167e-30, 0., .109e-33]  
Solution in 0.855s  
  
Time Plot 0.867 s.  
Exiting SolveHard() after 2.328r=2.57959e+07 in  
[25795872.84858705064741864171344489129222 ..  
2579590227891383795486680697763551821623/10000000000000000000000000000000  
0000000]  
Scattering ray (rm=2.47435e+07) in  
[298960418182500/22468879468420441 ..  
24850273.21919080441727298772802245902434]: target and source on  
the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.  
  
Tau [10510.45813678928226660477102574814051205,  
10510.36930058675694383261265624655327347,  
10510.39514076173560537623842221650667075, none, none,  
10510.33729983839454577741391701189418867, none, none, none, none,
```



```

none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 1 target = [24850273.21919079356362490230678462799050,
1.476968812147896056910374220682530277520,
10510.36930388274200031451993406344570704]
one interval r = 23888964.97298890373227748037881108056209 ..
1194449735784687082112662898054501817003/500000000000000000000000000000
000000
Time Approximations 0.049.

hint used Hint := [23888989.43146328644175807501347600997649, 3,
-1, 1, 20396631.10527514558221865082982151479995,
23888964.97298890387184335817087833215779 ..
597224867892343544545306555277700499657/250000000000000000000000000000
00000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S ---> P
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07
k=-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
23888989.43146328644175807501347600997649, rm=
20396631.10527514558221865082982151479995}, {r =
23888964.97298890387184335817087833215779 ..
23888994.71569374178181226221110801998628, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid=
{});
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1e-32
Equations at solution: [-.2e-31, -.1e-31, .3e-35]
Solution in 0.354s

Time Plot 1.917 s.
Exiting SolveHard() after 3.847r=2.38890e+07 in
[23888964.97298890387184335817087833215779 ..
597224867892343544545306555277700499657/250000000000000000000000000000
00000]
Scattering ray (rm=2.03966e+07) in
[298960418182500/22468879468420441 .. 24089000]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678928226660477102574814051205,
10510.36930058675694383261265624655327347,
10510.39514076173560537623842221650667075,
10510.28046624342051857199311024576509888, none,
10510.33729983839454577741391701189418867, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]

0 --> 2 target = [25795903.79985510788987103616920668140469,
1.856318223483248027101135056499671459414,
10510.39514387994325233570484161180106706]
one interval r = 23888966.51012875294442503553851009715646 ..
2388899625275549362355161779093185737573/1000000000000000000000000000000

```



Time Approximations 0.05.

```

Time Plot 1.744 s.
Exiting SolveHard() after 3.727r=2.38890e+07 in
[23888966.51012875395781144697926248654402 ..
597224906318887365922168983423648563467/250000000000000000000000000000
00000]
Scattering ray (rm=2.28605e+07) in
[298960418182500/22468879468420441 .. 24089000]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

```

```
hint used Hint := [24850272.85449189132526402658550193546657, 3, 1,  
1, 24743518.84163067386800243860950511726701,  
24850245.15428919258543501803067780064530 ..  
1242513737429890818728656221501545064373/5000000000000000000000000000]
```







```

branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
23888989.06496329485872911197089488868350, rm=
22860452.80984339454018821965990898965218}, {r =
23888963.06934924828335479031706673264809 ..
23888992.81215080305602286687243794493588, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid=
{});
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1e-32
Equations at solution: [-.2e-31, -.1e-31, .1e-35]
Solution in 0.399s

Time Plot 1.77 s.
Exiting SolveHard() after 3.933r=2.38890e+07 in
[23888963.06934924828335479031706673264809 ..
597224820303770076400571671810948623397/2500000000000000000000000000000000000000]
00000]
Scattering ray (rm=2.28605e+07) in
[298960418182500/22468879468420441 .. 24089000]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678928226660477102574814051205,
10510.36930058675694383261265624655327347,
10510.39514076173560537623842221650667075,
10510.28046624342051857199311024576509888,
10510.36313884712847965775569317684443514,
10510.33729983839454577741391701189418867,
10510.33214580438980764823267660892487223, none, none,
10510.27430496990875955382588600356772761, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none]

1 --> 2 target = [25795900.39533807327544378112689742037988,
1.856309428582616439361810046725514222772,
10510.33730295658729624728775968678009451]
one interval r = 24850241.73069461639109989150678709642668 ..
2485027132512924832475540643882267471443/10000000000000000000000000000000000000000
0000000
Time Approximations 0.041.

hint used Hint := [24850269.43101309861241733616879334665430, 3, 1,
1, 24743514.60014700975744022125022662330031,
24850241.73069462631526237165506015616915 ..
1242513566256462912427735010225393921567/5000000000000000000000000000000000000000
000000, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.239829) | S --> P
rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07
k=2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
24850269.43101309861241733616879334665430, rm=
24743514.60014700975744022125022662330031}, {r =
24850241.73069462631526237165506015616915 ..

```



[illegible]



















```

branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
25795900.03265730627104171495990540839944, rm=
24743534.99638995296918328323923452726440}, {r =
25795872.48589893272346549352544011957533 ..
25795901.91623381280391053704045584651652, rm =
.1330553304194287328500223794129351168576e-1 ..
24850272.85449189132526402658550193546657}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.29692e-30, 0., -.47e-34]
Solution in 0.351s

Time Plot 1.56 s.
Exiting SolveHard() after 2.505r=2.57959e+07 in
[25795872.48589893272346549352544011957533 ..
644897547905845320097763426011396162913/250000000000000000000000000000
00000]
Scattering ray (rm=2.47435e+07) in
[298960418182500/22468879468420441 ..
24850272.85449189132526402658550193546657]: target and source on
the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678928226660477102574814051205,
10510.36930058675694383261265624655327347,
10510.39514076173560537623842221650667075,
10510.28046624342051857199311024576509888,
10510.36313884712847965775569317684443514,
10510.33729983839454577741391701189418867,
10510.33214580438980764823267660892487223,
10510.30529788714239771809335500698244005,
10510.21747048882395230960657176617286961,
10510.27430496990875955382588600356772761,
10510.19163019567340884644099825129471966,
10510.26914997039936966730963356197247364, none,
10510.2433097116218220253584095370937230,
10510.33113809486231299723047499935891749, none, none, none, none,
none, none, none, none, none, none, none, none, none, none,
none]

0 --> 1 target = [24850272.85449188053207901056340126504694,
1.476967822428751558555360627314858900107,
10510.36314214311189014736359718401163674]
one interval r = 23888964.60644587159695944975655547987032 ..
2388899434916933228930472349101298390609/100000000000000000000000000000
0000000
Time Approximations 0.047.

hint used Hint := [23888989.06493588745880515057802801897934, 3,
-1, 1, 20396631.19196530471597662319541189563767,
23888964.60644587167575168019887114912430 ..
1194449717458466618404658405105548818961/500000000000000000000000000000
000000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S --> P

```











Time Approximations 0.032.

```
hint used Hint := [25795892.97949951637755150436992208534955, 3, 1,
1, 22860366.33048982198263378361971969356546,
25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/250000000000000000000000000000
00000, 298960418182500/22468879468420441 ..
23888989.06496329485872911197089488868350, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07
k=4.31731e+14 scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
25795892.97949951637755150436992208534955, rm=
22860366.33048982198263378361971969356546}, {r =
25795867.25701511448801980095492062593890 ..
25795896.68746666518951237477219502520548, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06496329485872911197089488868350}, avoid={});
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=4.5e-31
Equations at solution: [-.46e-30, -.45e-30, .172e-33]
Solution in 0.971s
```

Time Plot 1.039 s.

```
Exiting SolveHard() after 2.608r=2.57959e+07 in
[25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/250000000000000000000000000000
00000]
Scattering ray (rm=2.28604e+07) in
[298960418182500/22468879468420441 ..
23888989.06496329485872911197089488868350]: target and source on
the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678928226660477102574814051205,
10510.36930058675694383261265624655327347,
10510.39514076173560537623842221650667075,
10510.28046624342051857199311024576509888,
10510.36313884712847965775569317684443514,
10510.33729983839454577741391701189418867,
10510.33214580438980764823267660892487223,
10510.30529788714239771809335500698244005,
10510.21747048882395230960657176617286961,
10510.27430496990875955382588600356772761,
10510.19163019567340884644099825129471966,
10510.26914997039936966730963356197247364,
10510.27430450915995142240204558056617525,
10510.24330971162182220253584095370937230,
10510.33113809486231299723047499935891749, none,
10510.18546892752907436199946414207076900,
10510.21130922477758990004537996233678879, none, none, none, none,
none, none, none, none, none, none, none, none]
```

2 --> 1 target = [24850269.43101308961849505293763786076077,



1.476958531819668278500821832305837912777,  
10510.30530118311035708672525493140330468]  
Imaginary part neglected:  
-.1026155300751533131360866346426467533500e-14  
one interval r = 25795869.08129890501747366643281812774670 ..  
515917970234195032848540396832604510299/200000000000000000000000000000  
00000

Time Approximations 0.033.

hint used Hint := [25795896.62812623062600675830146784169746, 3,  
-1, 1, 24743530.75497470617154487139175541092326,  
25795869.08129891002682354003571513818579 ..  
2579589851170975665166085604166659164517/100000000000000000000000000000  
0000000, 298960418182500/22468879468420441 ..  
24850269.43101309861241733616879334665430, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.760171) | P <--- S  
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07  
k=-2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=  
25795896.62812623062600675830146784169746, rm=  
24743530.75497470617154487139175541092326}, {r =  
25795869.08129891002682354003571513818579 ..  
25795898.51170975665166085604166659164517, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850269.43101309861241733616879334665430}, avoid={});  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [-.61859e-30, 0., .153e-33]  
Solution in 0.336s

Time Plot 1.549 s.  
Exiting SolveHard() after 3.118r=2.57959e+07 in  
[25795869.08129891002682354003571513818579 ..  
2579589851170975665166085604166659164517/100000000000000000000000000000  
0000000]  
Scattering ray (rm=2.47435e+07) in  
[298960418182500/22468879468420441 ..  
24850269.43101309861241733616879334665430]: target and source on  
the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678928226660477102574814051205,  
10510.36930058675694383261265624655327347,  
10510.39514076173560537623842221650667075,  
10510.28046624342051857199311024576509888,  
10510.36313884712847965775569317684443514,  
10510.33729983839454577741391701189418867,  
10510.33214580438980764823267660892487223,  
10510.30529788714239771809335500698244005,  
10510.21747048882395230960657176617286961,  
10510.27430496990875955382588600356772761,  
10510.19163019567340884644099825129471966,  
10510.26914997039936966730963356197247364,  
10510.27430450915995142240204558056617525,



[illegible]



```

10510.27430496990875955382588600356772761,
10510.19163019567340884644099825129471966,
10510.26914997039936966730963356197247364,
10510.27430450915995142240204558056617525,
10510.24330971162182220253584095370937230,
10510.33113809486231299723047499935891749,
10510.21646359956298056179601266091691668,
10510.18546892752907436199946414207076900,
10510.21130922477758990004537996233678879, none, none,
10510.27329709823110843605343208653981099, none, none, none, none,
none, none, none, none, none, none]

2 --> 1 target = [24850262.70323077542487007924319295537749,
1.476940274126184608013264725247634791396,
10510.19163349161100402796898051515748909]
Imaginary part neglected:
-.1026155300751533131360866346426467533500e-14
one interval r = 25795862.39063357743891837328184779886274 ..
1289794591059685257621331052809837646519/500000000000000000000000000000
000000
Time Approximations 0.034.

hint used Hint := [25795889.93759639200267360170740297062535, 3,
-1, 1, 24743522.41979937237431350459260520845964,
25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/200000000000000000000000000000
00000, 298960418182500/22468879468420441 ..
24850262.70323078644925875505002433594091, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.76017) | P <--- S
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07
k=-2.37384e+14 scos=5.95461e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
25795889.93759639200267360170740297062535, rm=
24743522.41979937237431350459260520845964}, {r =
25795862.39063358446748128489577933480278 ..
25795891.82119371218082845151233599574945, rm =
.1330553304194287328500223794129351168576e-1 ..
24850262.70323078644925875505002433594091}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.17320e-30, 0., -.69e-34]
Solution in 0.341s

Time Plot 1.528 s.
Exiting SolveHard() after 2.457r=2.57959e+07 in
[25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/200000000000000000000000000000
00000]
Scattering ray (rm=2.47435e+07) in
[298960418182500/22468879468420441 ..
24850262.70323078644925875505002433594091]: target and source on
the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

```



```

Tau [10510.45813678928226660477102574814051205,
10510.36930058675694383261265624655327347,
10510.39514076173560537623842221650667075,
10510.28046624342051857199311024576509888,
10510.36313884712847965775569317684443514,
10510.33729983839454577741391701189418867,
10510.33214580438980764823267660892487223,
10510.30529788714239771809335500698244005,
10510.21747048882395230960657176617286961,
10510.27430496990875955382588600356772761,
10510.19163019567340884644099825129471966,
10510.26914997039936966730963356197247364,
10510.27430450915995142240204558056617525,
10510.24330971162182220253584095370937230,
10510.33113809486231299723047499935891749,
10510.21646359956298056179601266091691668,
10510.18546892752907436199946414207076900,
10510.21130922477758990004537996233678879, none, none,
10510.27329709823110843605343208653981099, none, none,
10510.15962933474800479716466300877790128, none, none, none, none,
none, none, none]

0 --> 1 target = [24850262.70323077542487007924319295537749,
1.476940274126184608013264725247634791396,
10510.19163349161100402796898051515748909]
one interval r = 23888954.40382902035099521959594167651252 ..
47777968294141646192132929835672982311/200000000000000000000000000000
000
Time Approximations 0.048.

hint used Hint := [23888978.86275416794827744619407470166172, 3,
-1, 1, 20396633.60486737240849086583158368073964,
23888954.40382902066202638611670198664112 ..
298612301838385292588525592212900916387/125000000000000000000000000000
00000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S ---> P
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07
k=-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
23888978.86275416794827744619407470166172, rm=
20396633.60486737240849086583158368073964}, {r =
23888954.40382902066202638611670198664112 ..
23888984.14707082340708204737703207331096, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid=
{});
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1.0e-31
Equations at solution: [-.18e-30, -.10e-30, .55e-34]
Solution in 0.355s

Time Plot 1.892 s.
Exiting SolveHard() after 3.888r=2.38890e+07 in
[23888954.40382902066202638611670198664112 ..
298612301838385292588525592212900916387/125000000000000000000000000000
00000]
Scattering ray (rm=2.03966e+07) in

```







Time Plot 1.747 s.  
Exiting SolveHard() after 3.791r=2.38890e+07 in  
[23888955.94100364913487175812205942139954 ..  
9555594273666943272318156666787125279/400000000000000000000000000000  
0]  
Scattering ray (rm=2.28605e+07) in  
[298960418182500/22468879468420441 .. 24089000]: target and source  
on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678928226660477102574814051205,  
10510.36930058675694383261265624655327347,  
10510.39514076173560537623842221650667075,  
10510.28046624342051857199311024576509888,  
10510.36313884712847965775569317684443514,  
10510.33729983839454577741391701189418867,  
10510.33214580438980764823267660892487223,  
10510.30529788714239771809335500698244005,  
10510.21747048882395230960657176617286961,  
10510.27430496990875955382588600356772761,  
10510.19163019567340884644099825129471966,  
10510.26914997039936966730963356197247364,  
10510.27430450915995142240204558056617525,  
10510.24330971162182220253584095370937230,  
10510.33113809486231299723047499935891749,  
10510.21646359956298056179601266091691668,  
10510.18546892752907436199946414207076900,  
10510.21130922477758990004537996233678879,  
10510.10279600711771223183720764661380097, none,  
10510.27329709823110843605343208653981099,  
10510.15447580443013689942757281320268247, none,  
10510.15962933474800479716466300877790128, none, none, none, none,  
none, none, none]

1 --> 2 target = [25795893.34215399834919737492890586442363,  
1.856291208132654835857558501540490716346,  
10510.21747360698584163413966705612461421]  
one interval r = 24850234.63796910983717898554805847838168 ..  
155314151454154909362578131238694401423/6250000000000000000000000000  
0000  
Time Approximations 0.038.

hint used Hint := [24850262.33852743516297758053367205548154, 3, 1,  
1, 24743505.81299238129247202614818714422035,  
24850234.63796912173141920432208117660707 ..  
2485026423266479739181704602953513976831/1000000000000000000000000000  
0000000, 298960418182500/22468879468420441 .. 25089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.23983) | S ---> P  
rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07  
k=2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=  
24850262.33852743516297758053367205548154, rm=  
24743505.81299238129247202614818714422035}, {r =







```
25795865.43257821616570179673528437159660 ..
644897371576761841392917646201273823237/2500000000000000000000000000000000000000
00000, 298960418182500/22468879468420441 ..
24850265.76205359199870736703593497152527, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.76017) | P <--- S
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07
k=-2.37384e+14 scos=5.95462e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
25795892.97947942498571214559240875822627, rm=
24743526.20943107487634918939497374544011}, {r =
25795865.43257821616570179673528437159660 ..
25795894.86307047365571670584805095292948, rm =
.1330553304194287328500223794129351168576e-1 ..
24850265.76205359199870736703593497152527}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.37115e-30, 0., .82e-34]
Solution in 0.341s
```

```
Time Plot 1.514 s.
Exiting SolveHard() after 2.454r=2.57959e+07 in
[25795865.43257821616570179673528437159660 ..
644897371576761841392917646201273823237/2500000000000000000000000000000000000000
00000]
Scattering ray (rm=2.47435e+07) in
[298960418182500/22468879468420441 ..
24850265.76205359199870736703593497152527]: target and source on
the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678928226660477102574814051205,
10510.36930058675694383261265624655327347,
10510.39514076173560537623842221650667075,
10510.28046624342051857199311024576509888,
10510.36313884712847965775569317684443514,
10510.33729983839454577741391701189418867,
10510.33214580438980764823267660892487223,
10510.30529788714239771809335500698244005,
10510.21747048882395230960657176617286961,
10510.27430496990875955382588600356772761,
10510.19163019567340884644099825129471966,
10510.26914997039936966730963356197247364,
10510.27430450915995142240204558056617525,
10510.24330971162182220253584095370937230,
10510.33113809486231299723047499935891749,
10510.21646359956298056179601266091691668,
10510.18546892752907436199946414207076900,
10510.21130922477758990004537996233678879,
10510.10279600711771223183720764661380097,
10510.18546846165413405188156484516996124,
10510.27329709823110843605343208653981099,
10510.15447580443013689942757281320268247, none,
10510.15962933474800479716466300877790128, none, none,
10510.21130888343794949010388960006649069, none, none, none, none]
```



```
0 --> 1 target = [24850265.76205357923551187889741644691503,
1.476948575068067784652549033081002489236,
10510.24331300757322259666390079080882505]
one interval r = 23888957.47813171886767684821728313834524 ..
2388898722121733580921450079396643594553/1000000000000000000000000
0000000
Time Approximations 0.046.
```

```
hint used Hint := [23888981.93692575516559285951162594064654, 3,
-1, 1, 20396632.87781872303409295730403849988110,
23888957.47813172092635353311983299926098 ..
597224680530433446694670341991304966889/2500000000000000000000000
00000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S ---> P
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07
k=-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
23888981.93692575516559285951162594064654, rm=
20396632.87781872303409295730403849988110}, {r =
23888957.47813172092635353311983299926098 ..
23888987.22121733786778681367965219867556, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid=
{});
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1.3e-31
Equations at solution: [-.23e-30, -.13e-30, .54e-34]
Solution in 0.341s
```

```
Time Plot 1.855 s.
Exiting SolveHard() after 3.833r=2.38890e+07 in
[23888957.47813172092635353311983299926098 ..
597224680530433446694670341991304966889/2500000000000000000000000
00000]
Scattering ray (rm=2.03966e+07) in
[298960418182500/22468879468420441 .. 24089000]: target and source
on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678928226660477102574814051205,
10510.36930058675694383261265624655327347,
10510.39514076173560537623842221650667075,
10510.28046624342051857199311024576509888,
10510.36313884712847965775569317684443514,
10510.33729983839454577741391701189418867,
10510.33214580438980764823267660892487223,
10510.30529788714239771809335500698244005,
10510.21747048882395230960657176617286961,
10510.27430496990875955382588600356772761,
10510.19163019567340884644099825129471966,
10510.26914997039936966730963356197247364,
10510.27430450915995142240204558056617525,
10510.24330971162182220253584095370937230,
10510.33113809486231299723047499935891749,
10510.21646359956298056179601266091691668,
```



```
10510.18546892752907436199946414207076900,  
10510.21130922477758990004537996233678879,  
10510.10279600711771223183720764661380097,  
10510.18546846165413405188156484516996124,  
10510.27329709823110843605343208653981099,  
10510.15447580443013689942757281320268247, none,  
10510.15962933474800479716466300877790128,  
10510.15447547804451814969854273566634718, none,  
10510.21130888343794949010388960006649069, none, none, none, none]
```

```
0 --> 2 target = [25795896.38402522683983536439942161568499,  
1.856299066170421754244817176559072489400,  
10510.26915308857456865398574618581602030]  
one interval r = 23888959.01529623049071406502520181905549 ..  
2388898875830375270585805955385198922989/1000000000000000000000000  
0000000  
Time Approximations 0.046.
```

```
hint used Hint := [23888985.01109652402578398653593278001387, 3,  
-1, 1, 22860454.49191911041368999760273437756943,  
23888959.01529623342321627788824672003212 ..  
59722471895759389095528762418719523933/2500000000000000000000000000  
000, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.367179) | S --> P  
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07  
k=-4.31731e+14 scos=4.39725e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=  
23888985.01109652402578398653593278001387, rm=  
22860454.49191911041368999760273437756943}, {r =  
23888959.01529623342321627788824672003212 ..  
23888988.75830375563821150496748780957320, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid=  
{});  
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.1e-31  
Equations at solution: [-.12e-30, -.11e-30, .45e-34]  
Solution in 0.428s
```

```
Time Plot 1.732 s.  
Exiting SolveHard() after 3.802r=2.38890e+07 in  
[23888959.01529623342321627788824672003212 ..  
59722471895759389095528762418719523933/2500000000000000000000000000  
000]  
Scattering ray (rm=2.28605e+07) in  
[298960418182500/22468879468420441 .. 24089000]: target and source  
on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678928226660477102574814051205,  
10510.36930058675694383261265624655327347,  
10510.39514076173560537623842221650667075,  
10510.28046624342051857199311024576509888,  
10510.36313884712847965775569317684443514,  
10510.33729983839454577741391701189418867,
```



```

10510.33214580438980764823267660892487223,
10510.30529788714239771809335500698244005,
10510.21747048882395230960657176617286961,
10510.27430496990875955382588600356772761,
10510.19163019567340884644099825129471966,
10510.26914997039936966730963356197247364,
10510.27430450915995142240204558056617525,
10510.24330971162182220253584095370937230,
10510.33113809486231299723047499935891749,
10510.21646359956298056179601266091691668,
10510.18546892752907436199946414207076900,
10510.21130922477758990004537996233678879,
10510.10279600711771223183720764661380097,
10510.18546846165413405188156484516996124,
10510.27329709823110843605343208653981099,
10510.15447580443013689942757281320268247, none,
10510.15962933474800479716466300877790128,
10510.15447547804451814969854273566634718, none,
10510.21130888343794949010388960006649069,
10510.20615520661113356087935323833302835, none, none, none]

1 --> 2 target = [25795896.38402522683983536439942161568499,
1.856299066170421754244817176559072489400,
10510.26915308857456865398574618581602030]
one interval r = 24850237.69689664095995603695435735222709 ..
1242513364573986807094965730166701418729/500000000000000000000000000000
000000
Time Approximations 0.039.

hint used Hint := [24850265.39735153163244735697452997613798, 3, 1,
1, 24743509.60268659785926677493114233979369,
24850237.69689665459305925255508686501973 ..
1242513364573987488725142465429694354713/500000000000000000000000000000
000000, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.239829) | S --> P
rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07
k=2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=
24850265.39735153163244735697452997613798, rm=
24743509.60268659785926677493114233979369}, {r =
24850237.69689665459305925255508686501973 ..
24850267.29147974977450284930859388709426, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}), avoid=
{});
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.17320e-30, 0., -.3138510182e-35]
Solution in 0.993s

Time Plot 0.883 s.
Exiting SolveHard() after 2.616r=2.48503e+07 in
[24850237.69689665459305925255508686501973 ..
1242513364573987488725142465429694354713/500000000000000000000000000000
000000]
Scattering ray (rm=2.47435e+07) in
[298960418182500/22468879468420441 .. 25089000]: target and source

```



on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

```
Tau [10510.45813678928226660477102574814051205,
10510.36930058675694383261265624655327347,
10510.39514076173560537623842221650667075,
10510.28046624342051857199311024576509888,
10510.36313884712847965775569317684443514,
10510.33729983839454577741391701189418867,
10510.33214580438980764823267660892487223,
10510.30529788714239771809335500698244005,
10510.21747048882395230960657176617286961,
10510.27430496990875955382588600356772761,
10510.19163019567340884644099825129471966,
10510.26914997039936966730963356197247364,
10510.27430450915995142240204558056617525,
10510.24330971162182220253584095370937230,
10510.33113809486231299723047499935891749,
10510.21646359956298056179601266091691668,
10510.18546892752907436199946414207076900,
10510.21130922477758990004537996233678879,
10510.10279600711771223183720764661380097,
10510.18546846165413405188156484516996124,
10510.27329709823110843605343208653981099,
10510.15447580443013689942757281320268247,
10510.23714797597099157038295782028366001,
10510.15962933474800479716466300877790128,
10510.15447547804451814969854273566634718, none,
10510.21130888343794949010388960006649069,
10510.20615520661113356087935323833302835, none, none, none]
```

```
0 --> 2 target = [25795900.03265730039719736133907340057404,
1.856308491669322751268699689331613423512,
10510.33114121305347655679848373184609635]
one interval r = 23888962.70280479291877943774403586151201 ..
597224811140624257963725487321767425023/25000000000000000000000000000000
00000
```

Time Approximations 0.049.

```
hint used Hint := [23888988.69843567795653101185231580305160, 3,
-1, 1, 22860452.96192886545581288218987601576608,
23888962.70280479206299436081506076835447 ..
2388899244562496946280763877837409340903/10000000000000000000000000000000
0000000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.36718) | S ---> P
rGuessMin=2.38890e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07
k=-4.31731e+14 scos=4.39724e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=
23888988.69843567795653101185231580305160, rm=
22860452.96192886545581288218987601576608}, {r =
23888962.70280479206299436081506076835447 ..
23888992.44562496946280763877837409340903, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid=
```











10510.20615520661113356087935323833302835, none, none,  
10510.26814323584271084088165998894118362]

1 --> 0 target = [23888989.06493588729429015355245697550403,  
2.632525136926025776583219923131090342452,  
10510.27430791369476398713489140903800894]  
one interval r = 24850238.00201199841849165063781367157715 ..  
2485026759658386402992757702582840849809/1000000000000000000000000  
0000000  
Time Approximations 0.042.

hint used Hint := [24850262.33852773859450921827957797170818, 3, 1,  
1, 20396437.98350304935080318674410988058300,  
24850238.00201200929744389905914961384924 ..  
1242513379829193745424075414664369797603/5000000000000000000000000  
000000, 298960418182500/22468879468420441 ..  
23888989.06493588745880515057802801897934, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.533028) | P <--- S  
rGuessMin=2.48502e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07  
k=5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=  
24850262.33852773859450921827957797170818, rm=  
20396437.98350304935080318674410988058300}, {r =  
24850238.00201200929744389905914961384924 ..  
24850267.59658387490848150829328739595206, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.06493588745880515057802801897934}, avoid={});  
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=1.0e-31  
Equations at solution: [-.18e-30, -.10e-30, .53585653726e-34]  
Solution in 0.966s

Time Plot 1.867 s.  
Exiting SolveHard() after 3.586r=2.48503e+07 in  
[24850238.00201200929744389905914961384924 ..  
1242513379829193745424075414664369797603/5000000000000000000000000  
000000]  
Scattering ray (rm=2.03964e+07) in  
[298960418182500/22468879468420441 ..  
23888989.06493588745880515057802801897934]: target and source on  
the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678928226660477102574814051205,  
10510.36930058675694383261265624655327347,  
10510.39514076173560537623842221650667075,  
10510.28046624342051857199311024576509888,  
10510.36313884712847965775569317684443514,  
10510.33729983839454577741391701189418867,  
10510.33214580438980764823267660892487223,  
10510.30529788714239771809335500698244005,  
10510.21747048882395230960657176617286961,  
10510.27430496990875955382588600356772761,  
10510.19163019567340884644099825129471966,



```
10510.26914997039936966730963356197247364,  
10510.27430450915995142240204558056617525,  
10510.24330971162182220253584095370937230,  
10510.33113809486231299723047499935891749,  
10510.21646359956298056179601266091691668,  
10510.18546892752907436199946414207076900,  
10510.21130922477758990004537996233678879,  
10510.10279600711771223183720764661380097,  
10510.18546846165413405188156484516996124,  
10510.27329709823110843605343208653981099,  
10510.15447580443013689942757281320268247,  
10510.23714797597099157038295782028366001,  
10510.15962933474800479716466300877790128,  
10510.15447547804451814969854273566634718,  
10510.29913613970640274144652400254147134,  
10510.21130888343794949010388960006649069,  
10510.20615520661113356087935323833302835,  
10510.18546846678066761425962119565146542, none,  
10510.26814323584271084088165998894118362]
```

```
2 --> 0 target = [23888989.06493588729429015355245697550403,  
2.632525136926025776583219923131090342452,  
10510.27430791369476398713489140903800894]  
Imaginary part neglected:  
-.1026155300751533131360866346426467533500e-14  
one interval r = 25795867.25698798721094006208844569514947 ..  
2579589668743953851770523662922381313433/1000000000000000000000000  
0000000  
Time Approximations 0.032.
```

```
hint used Hint := [25795892.97947239655457473800077316699564, 3, 1,  
1, 22860366.33050119628977322107538139299992,  
25795867.25698799409451243993168295190626 ..  
1289794834371977270055988053158890743643/500000000000000000000000  
000000, 298960418182500/22468879468420441 ..  
23888989.06493588745880515057802801897934, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.632821) | P <--- S  
rGuessMin=2.57959e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07  
k=4.31731e+14 scos=4.39725e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=  
25795892.97947239655457473800077316699564, rm=  
22860366.33050119628977322107538139299992}, {r =  
25795867.25698799409451243993168295190626 ..  
25795896.68743954540111976106317781487286, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.06493588745880515057802801897934}, avoid={});  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=3.7e-31  
Equations at solution: [-.38e-30, -.37e-30, .147e-33]  
Solution in 0.289s
```

```
Time Plot 1.666 s.  
Exiting SolveHard() after 2.532r=2.57959e+07 in  
[25795867.25698799409451243993168295190626 ..  
1289794834371977270055988053158890743643/500000000000000000000000  
000000]
```



Scattering ray (rm=2.28604e+07) in  
[298960418182500/22468879468420441 ..  
23888989.06493588745880515057802801897934]: target and source on  
the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678928226660477102574814051205,  
10510.36930058675694383261265624655327347,  
10510.39514076173560537623842221650667075,  
10510.28046624342051857199311024576509888,  
10510.36313884712847965775569317684443514,  
10510.33729983839454577741391701189418867,  
10510.33214580438980764823267660892487223,  
10510.30529788714239771809335500698244005,  
10510.21747048882395230960657176617286961,  
10510.27430496990875955382588600356772761,  
10510.19163019567340884644099825129471966,  
10510.26914997039936966730963356197247364,  
10510.27430450915995142240204558056617525,  
10510.24330971162182220253584095370937230,  
10510.33113809486231299723047499935891749,  
10510.21646359956298056179601266091691668,  
10510.18546892752907436199946414207076900,  
10510.21130922477758990004537996233678879,  
10510.10279600711771223183720764661380097,  
10510.18546846165413405188156484516996124,  
10510.27329709823110843605343208653981099,  
10510.15447580443013689942757281320268247,  
10510.23714797597099157038295782028366001,  
10510.15962933474800479716466300877790128,  
10510.15447547804451814969854273566634718,  
10510.29913613970640274144652400254147134,  
10510.21130888343794949010388960006649069,  
10510.20615520661113356087935323833302835,  
10510.18546846678066761425962119565146542,  
10510.21130876402948960448870901709072836,  
10510.26814323584271084088165998894118362]

Cascade time 99.246

[10510.45813678928226660477102574814051205,  
10510.36930058675694383261265624655327347,  
10510.39514076173560537623842221650667075,  
10510.28046624342051857199311024576509888,  
10510.36313884712847965775569317684443514,  
10510.33729983839454577741391701189418867,  
10510.33214580438980764823267660892487223,  
10510.30529788714239771809335500698244005,  
10510.21747048882395230960657176617286961,  
10510.27430496990875955382588600356772761,

(101)



[illegible]



[2.485027285449188053207901056340126504694  $\times 10^7$ ,  
1.476967822428751558555360627314858900107,  
10510.36314214311189014736359718401163674],  
[2.579590039533807327544378112689742037988  $\times 10^7$ ,  
1.856309428582616439361810046725514222772,  
10510.33730295658729624728775968678009451],  
[2.388899250558472480658669404448922600131  $\times 10^7$ ,  
2.632534984518335369781116140963680479043,  
10510.33214920894068291528529915291954195],  
[2.485026943101308961849505293763786076077  $\times 10^7$ ,  
1.476958531819668278500821832305837912777,  
10510.30530118311035708672525493140330468],  
[2.579589334215399834919737492890586442363  $\times 10^7$ ,  
1.856291208132654835857558501540490716346,  
10510.21747360698584163413966705612461421],  
[2.388898906496329469420844096828926528171  $\times 10^7$ ,  
2.632525137004469163009143380898398519884,  
10510.27430837444357224651007926317166688],  
[2.485026270323077542487007924319295537749  $\times 10^7$ ,  
1.476940274126184608013264725247634791396,  
10510.19163349161100402796898051515748909],  
[2.579589638402522683983536439942161568499  $\times 10^7$ ,  
1.856299066170421754244817176559072489400,  
10510.26915308857456865398574618581602030],  
[2.388898906493588729429015355245697550403  $\times 10^7$ ,  
2.632525136926025776583219923131090342452,  
10510.27430791369476398713489140903800894],  
[2.485026576205357923551187889741644691503  $\times 10^7$ ,  
1.476948575068067784652549033081002489236,  
10510.24331300757322259666390079080882505],  
[2.579590003265730039719736133907340057404  $\times 10^7$ ,  
1.856308491669322751268699689331613423512,



10510.33114121305347655679848373184609635],  
[2.388898562428976511246495378157716928937  $\times 10^7$ ,  
2.632515289396534317475991135074738068410,  
10510.21646700408173052820720741456533446],  
[2.485026233855499861073278342030887771175  $\times 10^7$ ,  
1.476939284481933881140047454461195650955,  
10510.18547222346502367624547422352559497],  
[2.579589297949950854466363750577138770615  $\times 10^7$ ,  
1.856290271291756239374055951524030417251,  
10510.21131234293789243718130272808802766],  
[2.388897886275416755153191303530735058434  $\times 10^7$ ,  
2.632495937255382636833400315190115110600,  
10510.10279941160489634954978669764144984],  
[2.485026233852742419905203095220230381181  $\times 10^7$ ,  
1.476939284407103431920598985378486296794,  
10510.18547175759008324167781697043383453],  
[2.579589662812622654149501997771584016474  $\times 10^7$ ,  
1.856299696754978359658614888784306091091,  
10510.27330021640737548424888947406751137],  
[2.388898193694516813413454323487770203483  $\times 10^7$ ,  
2.632504735848412865304601027823412663829,  
10510.15447920893167266122784492042992534],  
[2.485026539735151892971422373749110151438  $\times 10^7$ ,  
1.476947585348968191836752112429258503277,  
10510.23715127192074597261417802117575159],  
[2.579588993759638589899408624130030433227  $\times 10^7$ ,  
1.856282413189808276546276535638494981249,  
10510.15963245289499756461784955137954747],  
[2.388898193692575302127679590107424125108  $\times 10^7$ ,  
2.632504735792845041704472318081047906997,  
10510.15447888254605382086049820382144238],  
[2.485026906631199606140762882772173793919  $\times 10^7$ ,



1.476957542098967962263938732300222751859,  
 10510.29913943567271611535813158500123072 ],  
 [ 2.579589297947941715282720559675859140903  $\times 10^7$ ,  
 1.856290271239854399501932044106913726285,  
 10510.21131200159825193933034434660211384 ],  
 [ 2.388898501109652013391500318899032870084  $\times 10^7$ ,  
 2.632513534371906656749288751555122916844,  
 10510.20615861112702085456866511163339400 ],  
 [ 2.485026233852772763058421962248294531644  $\times 10^7$ ,  
 1.476939284407926873527388108689712304531,  
 10510.18547176271661680542533067049506287 ],  
 [ 2.579589297947238872169248542041152805698  $\times 10^7$ ,  
 1.856290271221697941737684210562315759514,  
 10510.21131188218979202296240919653647032 ],  
 [ 2.388898869843567785278139321847313400094  $\times 10^7$ ,  
 2.632524087957167722383291470646720269362,  
 10510.26814664037581240122989116028220920 ]]  
 [ 10510.45813678928226660477102574814051205,  
 10510.36930058675694383261265624655327347,  
 10510.39514076173560537623842221650667075,  
 10510.28046624342051857199311024576509888,  
 10510.36313884712847965775569317684443514,  
 10510.33729983839454577741391701189418867,  
 10510.33214580438980764823267660892487223,  
 10510.30529788714239771809335500698244005,  
 10510.21747048882395230960657176617286961,  
 10510.27430496990875955382588600356772761,  
 10510.19163019567340884644099825129471966,  
 10510.26914997039936966730963356197247364,  
 10510.27430450915995142240204558056617525,  
 10510.24330971162182220253584095370937230,  
 10510.33113809486231299723047499935891749,  
 10510.21646359956298056179601266091691668,  
 10510.18546892752907436199946414207076900,  
 10510.21130922477758990004537996233678879,

(102)



```
10510.10279600711771223183720764661380097,  
10510.18546846165413405188156484516996124,  
10510.27329709823110843605343208653981099,  
10510.15447580443013689942757281320268247,  
10510.23714797597099157038295782028366001,  
10510.15962933474800479716466300877790128,  
10510.15447547804451814969854273566634718,  
10510.29913613970640274144652400254147134,  
10510.21130888343794949010388960006649069,  
10510.20615520661113356087935323833302835,  
10510.18546846678066761425962119565146542,  
10510.21130876402948960448870901709072836,  
10510.26814323584271084088165998894118362]
```

```
> NewSignals:= PurifyTau(ListTau);
```

```
NewSignals := [0.17767054586174803277791550237541317,  
0.00616173962846417485696306970883833, 0.05784092334105959882450520461248208,  
0.12599098489245895653834913921563982, 0.06400269961454611451930123957083342,  
0.17767027291165306663185045033380114, 0.18383181937350705094513974457278444,  
0.17767039108353498617165799525855381, 0.12599079133623570892878865453419711,  
0.18383228012231518236898016757433680, 0.12599087513512163007681529284390117,  
0.06400266687329237900794721714775326, 0.24167318971928604297501308722359537,  
0.18383165922786947061319210448250447, 0.18383153695801547619304225416988196,  
0.35534078216455437293381810152671108, 0.18383212510280978073109140138331223,  
0.12184366350449694018499012996685976, 0.30366098485212970534345293493782958,  
0.13215261078595226222969842626961346, 0.23551142698760057907375920772876947,  
0.30366131123774845507248301247416487, 0.07016444705054109116613224401180213,  
0.18383187829765588613453261644018006, 0.25198158267113304389167250980748370,  
0.18383211997627621835303505090180805, 0.18383199770611577174971319941594239,  
0.18999355343955576388936575919932843]
```

(103)

```
> ObsSignals;
```

```
[0.17767054585886572036819935972740758, 0.00616173962948580605190654864423874,  
0.05784092337145950963735670251243184, 0.12599098486019831593901609703085618,  
0.06400269964596767309355140861462011, 0.17767027290877075555402260644497258,  
0.18383181937164627344820599986282535, 0.17767039108065267609477790592160730,  
0.12599079130397511569499950156506127, 0.18383228012045450025802322989859963,  
0.12599087510286101745516113328857827, 0.06400266690471392137608393586694933,  
0.24167318974782526170198479167873517, 0.18383165922600869455300331034402624,  
0.18383153695615469835127564443724502, 0.35534078215878975278018439688506693,
```

(104)



```
0.18383212510094909074889310107021004, 0.12184366356631842649901098303910377,
0.30366098481698675499941538230521617, 0.13215261075471327388545001142491099,
0.23551142701511816971063564689544432, 0.30366131120260555801930911057160834,
0.07016444708298428165599866382468887, 0.18383187829579515802042917838631823,
0.25198158260661185742529098524384602, 0.18383211997441553767873950597483102,
0.18383199770425508929376882893552614, 0.18999355343871661627086252604680052]
```

```
> FitFunction:= proc(s1, s2)
  local n, k, P:
  P:= 0.0:
  n := nops(s1):
  printf("counts: %d, %d\n ", n, nops(s2));
  for k from 1 to n do
    P:= P+ (s1[n]-s2[n])^2:
  end do:
  P:
end:
```

```
> FitFunction(NewSignals, ObsSignals);
H:= [%];
```

```
# 0.0003 -> Phi ~ 12.28161979412227036196898376335373904316
```

```
# 10^(-15) -> Phi ~ 7*10^(-26)
```

```
counts: 28, 28
```

$$H := [1.971672431791013634866669684351828446301 \times 10^{-23}] \quad (105)$$

```
> ProducePlots:= false:
```

```
> Stat:=proc()
  global te0in, t0in, tau0in, rm0v, rp0v:
  global telin, tlin, tau1in, rmlv, rp1v:
  global te2in, t2in, tau2in, rm2v, rp2v:
  global H, BaseR, MaxT, ListTau, ListP, BaseSat, BaseBranch,
  ObsSignals, ListHints, HintsOriginal:
  local NewSignals:

  ListHints:= HintsOriginal:
  ListTau:=CreateList(31, 0):
  ListP:=CreateList(31, []):
  #
  Rumor(BaseROriginal, 10^(-10)):
  BaseR:=%:

  Rumor(Sat0, 10^(-10)):
  te0in, t0in, tau0in, rm0v, rp0v := op(%):
  ComputeSat0():
```



$\lceil \cdot \rceil \geq H;$ 

```
> evalf(BaseROriginal);  
Sat0;  
Sat1;  
Sat2;
```

```
> # HereHere
```

## Iteration 1

```
Start Generation 1
1 --> 0  target = [23888999.99999999989630389048167149031081,
2.632556434661917927608210461285439357231,
10510.45814019400141376533994813705334636]
one interval r = 24835349.43352073347400087267515731448309 ..
2485027847718526709072192528787929838919/1000000000000000000000000000000
000
Time Approximations 0.037.

hint used Hint := [24850273.21919080441727298772802245902434, 3, 1, 1,
20396435.39659592643000634328381896732011,
24850248.88301387793152980010731189259064 ..
```























```
(Scattering) fsolve(eqs, {r=23888989.06496329485872911197089488868350,
rm=22860452.80984339454018821965990898965218}, {r =
23888963.06934924828335479031706673264809 ..
23888992.81215080305602286687243794493588, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={});
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=3.0e-31
Equations at solution: [.31e-30, .30e-30, -.108663074002e-33]
Solution in 0.399s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.288r=2.38890e+07 in
[23888963.06934924828335479031706673264809 ..
597224820303770076400571671810948623397/2500000000000000000000000000000000
0]
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678944242001671184086773476800,
10510.36930058692371237634766420582120505,
10510.39514076165434656325405307424286284,
10510.28046624356371105066247575283169983,
10510.36313884729203957386907803459205855,
10510.33729983825909943170848255074907818,
10510.33214580458397924448755880606587243, none, none,
10510.27430497004874370166927236627703638, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]
```

```
1 --> 2 target = [25795900.39533807191863401603414042575375,
1.856309428750374862928727220645591502993,
10510.33730295668340182251060678320667179]
one interval r = 24835342.15555717971550636579390619649459 ..
2485027132512926444263114808544162110609/1000000000000000000000000000000000
000
Time Approximations 0.037.
```

```
hint used Hint := [24850269.43101309861241733616879334665430, 3, 1, 1,
24743514.60014700975744022125022662330031,
24850241.73069462631526237165506015616915 ..
1242513566256462912427735010225393921567/5000000000000000000000000000000000
00, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.239829) | S --> P
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=
2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850269.43101309861241733616879334665430,
rm=24743514.60014700975744022125022662330031}, {r =
24850241.73069462631526237165506015616915 ..
24850271.32512925824855470020450787843134, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={});
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.27218e-30, 0., .72535720609e-34]
Solution in 0.335s
```







Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888989.43146328644175807501347600997649]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678944242001671184086773476800,  
10510.36930058692371237634766420582120505,  
10510.39514076165434656325405307424286284,  
10510.28046624356371105066247575283169983,  
10510.36313884729203957386907803459205855,  
10510.33729983825909943170848255074907818,  
10510.33214580458397924448755880606587243,  
10510.30529788725177007207568075770432126, none,  
10510.27430497004874370166927236627703638,  
10510.19163019582321647064300607748399615, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none]

2 --> 0 target = [23888989.43146328515769294080022690083289,  
2.632526185918030933328191890954929669607,  
10510.28046964807336516313155971723215809]  
one interval r = 25781103.42398365932058875462776018587017 ..  
2579589705012068547218935852552136268821/1000000000000000000000000000000  
000  
Time Approximations 0.034.

hint used Hint := [25795893.34215400624221088590056918107762, 3, 1, 1,  
22860366.17838899487248578273534549876757,  
25795867.61967723367271871049090258979880 ..  
1289794852506034629127592665641586312049/500000000000000000000000000000  
00, 298960418182500/22468879468420441 ..  
23888989.43146328644175807501347600997649, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.632821) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31731e+14 scos=4.39725e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795893.34215400624221088590056918107762,  
rm=22860366.17838899487248578273534549876757}, {r =  
25795867.61967723367271871049090258979880 ..  
25795897.05012069258255185331283172624098, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.43146328644175807501347600997649}, avoid={});  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1.1e-31  
Equations at solution: [-.11e-30, -.11e-30, .17891556820e-34]  
Solution in 0.244s

Time Plot 0 s.  
Exiting SolveHard() after 0.823r=2.57959e+07 in  
[25795867.61967723367271871049090258979880 ..  
1289794852506034629127592665641586312049/500000000000000000000000000000  
00]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23888989.43146328644175807501347600997649]: target and source on the  
different branches.



Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

[illegible]

```
1 --> 0   target = [23888992.50558472678026208585856820454121,  
2.632534984463046836597014199022746177667,  
10510.33214920910798492693569733562367818]  
one interval r = 24835341.84514836974964306457890500130474 ..  
24850271020090329014316783803828674627/1000000000000000000000000000  
Time Approximations 0.039.
```

```
hint used Hint := [24850265.76205359199870736703593497152527, 3, 1, 1,  
20396437.16957127706091945345483725838621,  
24850241.42564446195929857082855634229261 ..  
1242513551004516068014443586172817582043/5000000000000000000000000000  
00, 298960418182500/22468879468420441 ..  
23888992.50558472677941562199764206716244, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.533028) | P <--- S  
rGuessMin=2.48353e+07    rGuessMax=2.48503e+07    rmGuess=2.03964e+07    k=  
5.43203e+14    scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850265.76205359199870736703593497152527,  
rm=20396437.16957127706091945345483725838621}, {r =  
24850241.42564446195929857082855634229261 ..  
24850271.02009032136028887172345635164086, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888992.50558472677941562199764206716244}}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=1.5e-31  
Equations at solution: [.27e-30, .15e-30, -.73099523739e-34]  
Solution in 0.375s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 1.759r=2.48503e+07 in  
[24850241.42564446195929857082855634229261 ..  
1242513551004516068014443586172817582043/50000000000000000000000000000000  
00]  
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888992.50558472677941562199764206716244]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```



```
Tau [10510.45813678944242001671184086773476800,
10510.36930058692371237634766420582120505,
10510.39514076165434656325405307424286284,
10510.28046624356371105066247575283169983,
10510.36313884729203957386907803459205855,
10510.33729983825909943170848255074907818,
10510.33214580458397924448755880606587243,
10510.30529788725177007207568075770432126,
10510.21747048872573259267125122487784566,
10510.27430497004874370166927236627703638,
10510.19163019582321647064300607748399615, none, none,
10510.24330971182260890021436749299395174, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0   target = [23888992.50558472678026208585856820454121,  
2.632534984463046836597014199022746177667,  
10510.33214920910798492693569733562367818]  
one interval r = 25781106.49760329040975345631928243198275 ..  
257959000919880377008737363016314629139/1000000000000000000000000000  
0
```

Time Approximations 0.03.

```
hint used Hint := [25795896.38402523646204355452114396262515, 3, 1, 1,  
22860364.90258708217946452135489353704915,  
25795870.66161245800881604810046494170563 ..  
2579590009198804353979547140796504033263/100000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
23888992.50558472677941562199764206716244, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.63282) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31731e+14 scos=4.39724e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795896.38402523646204355452114396262515,  
rm=22860364.90258708217946452135489353704915}, {r =  
25795870.66161245800881604810046494170563 ..  
25795900.09198804353979547140796504033263, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888992.50558472677941562199764206716244}, avoid={{}});  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1.3e-31  
Equations at solution: [.13e-30, .13e-30, -.38927202828e-34]  
Solution in 0.268s
```

Time Plot 0 s.

```
Exiting SolveHard() after 1.558r=2.57959e+07 in  
[25795870.66161245800881604810046494170563 ..  
2579590009198804353979547140796504033263/10000000000000000000000000000000  
000]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23888992.50558472677941562199764206716244]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

Tau [10510.45813678944242001671184086773476800,



```
10510.36930058692371237634766420582120505,  
10510.39514076165434656325405307424286284,  
10510.28046624356371105066247575283169983,  
10510.36313884729203957386907803459205855,  
10510.33729983825909943170848255074907818,  
10510.33214580458397924448755880606587243,  
10510.30529788725177007207568075770432126,  
10510.21747048872573259267125122487784566,  
10510.27430497004874370166927236627703638,  
10510.19163019582321647064300607748399615,  
10510.26914997035212898869267993991496724, none,  
10510.24330971182260890021436749299395174, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none,  
none]
```

```
2 --> 1 target = [24850272.85449189815856429155061883810535,  
1.476967822248808109343816436707125402098,  
10510.36314214323348482200853320972547968]  
one interval r = 25781108.34089176649661379904921281689356 ..  
2579590191623380614883023462795987217113/1000000000000000000000000000000  
000  
Time Approximations 0.031.
```

```
hint used Hint := [25795900.03265730627104171495990540839944, 3, -1, 1,  
24743534.99638995296918328323923452726440,  
25795872.48589893272346549352544011957533 ..  
644897547905845320097763426011396162913/2500000000000000000000000000000  
0, 298960418182500/22468879468420441 ..  
24850272.85449189132526402658550193546657, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.760172) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=  
-2.37383e+14 scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=25795900.03265730627104171495990540839944,  
rm=24743534.99638995296918328323923452726440}, {r =  
25795872.48589893272346549352544011957533 ..  
25795901.91623381280391053704045584651652, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850272.85449189132526402658550193546657}, avoid={});  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.14846e-30, 0., -.12883400053e-34]  
Solution in 0.316s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 0.869r=2.57959e+07 in  
[25795872.48589893272346549352544011957533 ..  
644897547905845320097763426011396162913/2500000000000000000000000000000  
0]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 24850272.85449189132526402658550193546657]: target and source on the  
different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678944242001671184086773476800,
```











```

10510.36313884729203957386907803459205855,
10510.33729983825909943170848255074907818,
10510.33214580458397924448755880606587243,
10510.30529788725177007207568075770432126,
10510.21747048872573259267125122487784566,
10510.27430497004874370166927236627703638,
10510.19163019582321647064300607748399615,
10510.26914997035212898869267993991496724,
10510.27430450929993527620908220893075554,
10510.24330971182260890021436749299395174,
10510.33113809472365802239474533022147286, none,
10510.18546892767567365813448374780641658, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 0 target = [23888989.06496329344458267186137128068427,
2.632525136949171404307486367688541244880,
10510.27430837455668680973393359371314481]
one interval r = 25781103.05754416335379565326955804244083 ..
2579589668746665795043259573770552511089/1000000000000000000000000000000000000000
000
Time Approximations 0.029.

hint used Hint := [25795892.97949951637755150436992208534955, 3, 1, 1,
22860366.33048982198263378361971969356546,
25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/2500000000000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
23888989.06496329485872911197089488868350, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=
4.31731e+14 scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795892.97949951637755150436992208534955,
rm=22860366.33048982198263378361971969356546}, {r =
25795867.25701511448801980095492062593890 ..
25795896.68746666518951237477219502520548, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06496329485872911197089488868350}, avoid={});
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=5e-32
Equations at solution: [-.6e-31, -.5e-31, .29041090254e-34]
Solution in 0.252s

Time Plot 0 s.
Exiting SolveHard() after 1.507r=2.57959e+07 in
[25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/2500000000000000000000000000000000000000
0]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23888989.06496329485872911197089488868350]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678944242001671184086773476800,
10510.36930058692371237634766420582120505,

```



```

10510.39514076165434656325405307424286284,
10510.28046624356371105066247575283169983,
10510.36313884729203957386907803459205855,
10510.33729983825909943170848255074907818,
10510.33214580458397924448755880606587243,
10510.30529788725177007207568075770432126,
10510.21747048872573259267125122487784566,
10510.27430497004874370166927236627703638,
10510.19163019582321647064300607748399615,
10510.26914997035212898869267993991496724,
10510.27430450929993527620908220893075554,
10510.24330971182260890021436749299395174,
10510.33113809472365802239474533022147286, none,
10510.18546892767567365813448374780641658,
10510.21130922467616185732613607588081288, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1 target = [24850269.43101310403780663977737130222823,
1.476958531639716125513212990586405231697,
10510.30530118317776419922465667874040101]
one interval r = 25781104.90082927573358479115742779385423 ..
161224365698185928727621431134541313521/625000000000000000000000000000
Time Approximations 0.032.

hint used Hint := [25795896.62812623062600675830146784169746, 3, -1, 1,
24743530.75497470617154487139175541092326,
25795869.08129891002682354003571513818579 ..
2579589851170975665166085604166659164517/1000000000000000000000000000000
000, 298960418182500/22468879468420441 ..
24850269.43101309861241733616879334665430, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.760171) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=
-2.37384e+14 scos=5.95462e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25795896.62812623062600675830146784169746,
rm=24743530.75497470617154487139175541092326}, {r =
25795869.08129891002682354003571513818579 ..
25795898.51170975665166085604166659164517, rm =
.1330553304194287328500223794129351168576e-1 ..
24850269.43101309861241733616879334665430}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.17321e-30, 0., -.34020736843e-34]
Solution in 0.303s

Time Plot 0 s.
Exiting SolveHard() after 0.842r=2.57959e+07 in
[25795869.08129891002682354003571513818579 ..
2579589851170975665166085604166659164517/1000000000000000000000000000000
000]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 24850269.43101309861241733616879334665430]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

```



```
Tau [10510.45813678944242001671184086773476800,  
10510.36930058692371237634766420582120505,  
10510.39514076165434656325405307424286284,  
10510.28046624356371105066247575283169983,  
10510.36313884729203957386907803459205855,  
10510.33729983825909943170848255074907818,  
10510.33214580458397924448755880606587243,  
10510.30529788725177007207568075770432126,  
10510.21747048872573259267125122487784566,  
10510.27430497004874370166927236627703638,  
10510.19163019582321647064300607748399615,  
10510.26914997035212898869267993991496724,  
10510.27430450929993527620908220893075554,  
10510.24330971182260890021436749299395174,  
10510.33113809472365802239474533022147286, none,  
10510.18546892767567365813448374780641658,  
10510.21130922467616185732613607588081288, none, none,  
10510.27329709803826586967642853979680895, none, none, none, none,  
none, none, none, none, none, none]  
  
0 --> 1 target = [24850269.43101310403780663977737130222823,  
1.476958531639716125513212990586405231697,  
10510.30530118317776419922465667874040101]  
one interval r = 23873935.37540036203113986695197705679266 ..  
298612386356890865866032887305222255193/1250000000000000000000000000  
0  
Time Approximations 0.045.  
  
hint used Hint := [23888985.62428976346864819081258538077918, 3, -1, 1,  
20396632.00572718336484608918971225516342,  
23888961.16565299817948948119449178210506 ..  
2388899090855127150824991682392263419777/1000000000000000000000000000  
000, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.466972) | S ---> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=  
-5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888985.62428976346864819081258538077918,  
rm=20396632.00572718336484608918971225516342}, {r =  
23888961.16565299817948948119449178210506 ..  
23888990.90855127150824991682392263419777, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={});  
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1.0e-31  
Equations at solution: [.17e-30, .10e-30, -.81906088896e-34]  
Solution in 0.363s  
  
Time Plot 0 s.  
Exiting SolveHard() after 1.924r=2.38890e+07 in  
[23888961.16565299817948948119449178210506 ..  
2388899090855127150824991682392263419777/1000000000000000000000000000  
000]  
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.
```



```

Tau [10510.45813678944242001671184086773476800,
10510.36930058692371237634766420582120505,
10510.39514076165434656325405307424286284,
10510.28046624356371105066247575283169983,
10510.36313884729203957386907803459205855,
10510.33729983825909943170848255074907818,
10510.33214580458397924448755880606587243,
10510.30529788725177007207568075770432126,
10510.21747048872573259267125122487784566,
10510.27430497004874370166927236627703638,
10510.19163019582321647064300607748399615,
10510.26914997035212898869267993991496724,
10510.27430450929993527620908220893075554,
10510.24330971182260890021436749299395174,
10510.33113809472365802239474533022147286,
10510.21646359964877690034093489239714209,
10510.18546892767567365813448374780641658,
10510.21130922467616185732613607588081288, none, none,
10510.27329709803826586967642853979680895, none, none, none, none,
none, none, none, none, none, none]

2 --> 1 target = [24850262.70323079223760247772378139786078,
1.476940273946238949900071846678906343805,
10510.19163349171884641069886588584790644]
one interval r = 25781098.14047725057568411323624479522167 ..
2579589182119370448644361834160550882303/1000000000000000000000000000000
000
Time Approximations 0.03.

hint used Hint := [25795889.93759639200267360170740297062535, 3, -1, 1,
24743522.41979937237431350459260520845964,
25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/2000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
24850262.70323078644925875505002433594091, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.76017) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=
-2.37384e+14 scos=5.95461e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25795889.93759639200267360170740297062535,
rm=24743522.41979937237431350459260520845964}, {r =
25795862.39063358446748128489577933480278 ..
25795891.82119371218082845151233599574945, rm =
.1330553304194287328500223794129351168576e-1 ..
24850262.70323078644925875505002433594091}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.29690e-30, 0., -.52047379278e-34]
Solution in 0.324s

Time Plot 0 s.
Exiting SolveHard() after 1.594r=2.57959e+07 in
[25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/2000000000000000000000000000000
0]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 24850262.70323078644925875505002433594091]: target and source on the

```



different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678944242001671184086773476800,  
10510.36930058692371237634766420582120505,  
10510.39514076165434656325405307424286284,  
10510.28046624356371105066247575283169983,  
10510.36313884729203957386907803459205855,  
10510.33729983825909943170848255074907818,  
10510.33214580458397924448755880606587243,  
10510.30529788725177007207568075770432126,  
10510.21747048872573259267125122487784566,  
10510.27430497004874370166927236627703638,  
10510.19163019582321647064300607748399615,  
10510.26914997035212898869267993991496724,  
10510.27430450929993527620908220893075554,  
10510.24330971182260890021436749299395174,  
10510.33113809472365802239474533022147286,  
10510.21646359964877690034093489239714209,  
10510.18546892767567365813448374780641658,  
10510.21130922467616185732613607588081288, none, none,  
10510.27329709803826586967642853979680895, none, none,  
10510.15962933459559753629439868067034152, none, none, none, none,  
none, none, none]

0 --> 1 target = [24850262.70323079223760247772378139786078,  
1.476940273946238949900071846678906343805,  
10510.19163349171884641069886588584790644]  
one interval r = 23873928.44821858556238367177871592728866 ..  
1194449207353541076640665368025952497573/500000000000000000000000000000000000  
00

Time Approximations 0.047.

hint used Hint := [23888978.86275416794827744619407470166172, 3, -1, 1,  
20396633.60486737240849086583158368073964,  
23888954.40382902066202638611670198664112 ..  
298612301838385292588525592212900916387/1250000000000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]

I search for an scattering ray on opposite branches with  $0 < sv < 1$

(0.466972) | S --> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=  
-5.43203e+14 scos=2.39475e+14

branch outgoing at target, Clockwise

(Scattering) fsolve(eqs, {r=23888978.86275416794827744619407470166172,  
rm=20396633.60486737240849086583158368073964}, {r =

23888954.40382902066202638611670198664112 ..

23888984.14707082340708204737703207331096, rm =

.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));

Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1e-32

Equations at solution: [.2e-31, .1e-31, -.12054891271e-34]

Solution in 1.032s

Time Plot 0 s.

Exiting SolveHard() after 1.941r=2.38890e+07 in

[23888954.40382902066202638611670198664112 ..



```

298612301838385292588525592212900916387/1250000000000000000000000000000000
0]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678944242001671184086773476800,
10510.36930058692371237634766420582120505,
10510.39514076165434656325405307424286284,
10510.28046624356371105066247575283169983,
10510.36313884729203957386907803459205855,
10510.33729983825909943170848255074907818,
10510.33214580458397924448755880606587243,
10510.30529788725177007207568075770432126,
10510.21747048872573259267125122487784566,
10510.27430497004874370166927236627703638,
10510.19163019582321647064300607748399615,
10510.26914997035212898869267993991496724,
10510.27430450929993527620908220893075554,
10510.24330971182260890021436749299395174,
10510.33113809472365802239474533022147286,
10510.21646359964877690034093489239714209,
10510.18546892767567365813448374780641658,
10510.21130922467616185732613607588081288,
10510.10279600724394380471276705093947948, none,
10510.27329709803826586967642853979680895, none, none,
10510.15962933459559753629439868067034152, none, none, none, none,
none, none, none]

0 --> 2 target = [25795893.34215399918355581382028868765829,
1.856291208300418919856463892003291992875,
10510.21747360711917383814221550950469857]
one interval r = 23873930.02298357355838231922658359045726 ..
1194449284208367847449849979133350007483/500000000000000000000000000000000
00
Time Approximations 0.048.

hint used Hint := [23888981.93694517027845377679197924121306, 3, -1, 1,
22860455.76745222490936139362415353022295,
23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/400000000000000000000000000000000,
298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.367179) | S --> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31730e+14 scos=4.39725e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888981.93694517027845377679197924121306,
rm=22860455.76745222490936139362415353022295}, {r =
23888955.94100364913487175812205942139954 ..
23888985.68416735818079539166696781319750, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=6e-32
Equations at solution: [-.7e-31, -.6e-31, .19071747348e-34]
Solution in 1.045s

```







```
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.32166e-30, 0., -.68273671114e-34]
Solution in 0.332s
```

Time Plot 0 s.

Exiting SolveHard() after 1.021r=2.48503e+07 in

[24850234.63796912173141920432208117660707 ..

`2485026423266479739181704602953513976831/1000000000000000000000000000000  
000]`

Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 . . 25089000]: target and source on the different branches.

Counterclockwise ray.

Ray outgoing at target.

Solve Side.

```
Tau [10510.45813678944242001671184086773476800,
10510.36930058692371237634766420582120505,
10510.39514076165434656325405307424286284,
10510.28046624356371105066247575283169983,
10510.36313884729203957386907803459205855,
10510.33729983825909943170848255074907818,
10510.33214580458397924448755880606587243,
10510.30529788725177007207568075770432126,
10510.21747048872573259267125122487784566,
10510.27430497004874370166927236627703638,
10510.19163019582321647064300607748399615,
10510.26914997035212898869267993991496724,
10510.27430450929993527620908220893075554,
10510.24330971182260890021436749299395174,
10510.33113809472365802239474533022147286,
10510.21646359964877690034093489239714209,
10510.18546892767567365813448374780641658,
10510.21130922467616185732613607588081288,
10510.10279600724394380471276705093947948,
10510.18546846180073306841260750697802765,
10510.27329709803826586967642853979680895,
10510.15447580460734762105091195474193578, none,
10510.15962933459559753629439868067034152, none, none, none, none,
none, none, none]
```

```
2 --> 1 target = [24850265.76205359906553910092732758786831,
1.476948574888130314961502941605595781280,
10510.24331300773204405288392296352078265]
one interval r = 25781101.21410470538842893929847981970965 ..
2579589486307046723277793277834358384863/1000000000000000000000000000000
000
```

Time Approximations 0.03.

```
hint used Hint := [25795892.97947942498571214559240875822627, 3, -1, 1,
24743526.20943107487634918939497374544011,
25795865.43257821616570179673528437159660 ..
644897371576761841392917646201273823237/25000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
24850265.76205359199870736703593497152527, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.76017) | P <--- S
```

```

rGuessMin=2.57811e+07    rGuessMax=2.57959e+07    rmGuess=2.47435e+07    k=

```











1.856299066338193589775327311327804340868,  
10510.26915308875887989631979082506670442]  
one interval r = 23873933.17245664589367586817750358836586 ..  
477779775166075113826613703085311914583/2000000000000000000000000000000000  
0

Time Approximations 0.047.

hint used Hint := [23888985.01109652402578398653593278001387, 3, -1, 1,  
22860454.49191911041368999760273437756943,  
23888959.01529623342321627788824672003212 ..  
59722471895759389095528762418719523933/2500000000000000000000000000000000,  
298960418182500/22468879468420441 .. 24089000, 1]

I search for an scattering ray on opposite branches with  $0 < sv < 1$

(0.367179) | S --> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=  
-4.31731e+14 scos=4.39725e+14

branch outgoing at target, Clockwise

(Scattering) fsolve(eqs, {r=23888985.01109652402578398653593278001387,  
rm=22860454.49191911041368999760273437756943}, {r =

23888959.01529623342321627788824672003212 ..

23888988.75830375563821150496748780957320, rm =

.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));

Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1e-32

Equations at solution: [.1e-31, .1e-31, -.16506226939e-34]

Solution in 1.015s

Time Plot 0 s.

Exiting SolveHard() after 1.909r=2.38890e+07 in

[23888959.01529623342321627788824672003212 ..

59722471895759389095528762418719523933/2500000000000000000000000000000000]

Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 ..

. 24089000]: target and source on the different branches.

Clockwise ray.

Ray outgoing at target.

Solve Side.

Tau [10510.45813678944242001671184086773476800,

10510.36930058692371237634766420582120505,

10510.39514076165434656325405307424286284,

10510.28046624356371105066247575283169983,

10510.36313884729203957386907803459205855,

10510.33729983825909943170848255074907818,

10510.33214580458397924448755880606587243,

10510.30529788725177007207568075770432126,

10510.21747048872573259267125122487784566,

10510.27430497004874370166927236627703638,

10510.19163019582321647064300607748399615,

10510.26914997035212898869267993991496724,

10510.27430450929993527620908220893075554,

10510.24330971182260890021436749299395174,

10510.33113809472365802239474533022147286,

10510.21646359964877690034093489239714209,

10510.18546892767567365813448374780641658,

10510.21130922467616185732613607588081288,

10510.10279600724394380471276705093947948,

10510.18546846180073306841260750697802765,

10510.27329709803826586967642853979680895,



```
10510.15447580460734762105091195474193578, none,  
10510.15962933459559753629439868067034152,  
10510.15447547822172875194075435755578698, none,  
10510.21130888333652133061171130910282331,  
10510.20615520683932324155373741096345514, none, none, none]
```

```
1 --> 2 target = [25795896.38402523067483074087277410546211,  
1.856299066338193589775327311327804340868,  
10510.26915308875887989631979082506670442]  
one interval r = 24835338.05090069689887353986876751975138 ..  
497005345829595149611601949518525726267/2000000000000000000000000000000000  
0  
Time Approximations 0.036.
```

```
hint used Hint := [24850265.39735153163244735697452997613798, 3, 1, 1,  
24743509.60268659785926677493114233979369,  
24850237.69689665459305925255508686501973 ..  
1242513364573987488725142465429694354713/5000000000000000000000000000000000  
00, 298960418182500/22468879468420441 .. 25089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.239829) | S ---> P  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=  
2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850265.39735153163244735697452997613798,  
rm=24743509.60268659785926677493114233979369}, {r =  
24850237.69689665459305925255508686501973 ..  
24850267.29147974977450284930859388709426, rm =  
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.32167e-30, 0., -.79116278674e-34]  
Solution in 0.281s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 0.933r=2.48503e+07 in  
[24850237.69689665459305925255508686501973 ..  
1242513364573987488725142465429694354713/5000000000000000000000000000000000  
00]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 25089000]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678944242001671184086773476800,  
10510.36930058692371237634766420582120505,  
10510.39514076165434656325405307424286284,  
10510.28046624356371105066247575283169983,  
10510.36313884729203957386907803459205855,  
10510.33729983825909943170848255074907818,  
10510.33214580458397924448755880606587243,  
10510.30529788725177007207568075770432126,  
10510.21747048872573259267125122487784566,  
10510.27430497004874370166927236627703638,  
10510.19163019582321647064300607748399615,  
10510.26914997035212898869267993991496724,  
10510.27430450929993527620908220893075554,
```



















```
hint used Hint := [25795892.97947239655457473800077316699564, 3, 1, 1,
22860366.33050119628977322107538139299992,
25795867.25698799409451243993168295190626 ..
1289794834371977270055988053158890743643/500000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888989.06493588745880515057802801897934, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=
4.31731e+14 scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795892.97947239655457473800077316699564,
rm=22860366.33050119628977322107538139299992}, {r =
25795867.25698799409451243993168295190626 ..
25795896.68743954540111976106317781487286, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06493588745880515057802801897934}, avoid={});
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1.8e-31
Equations at solution: [-.18e-30, -.18e-30, .50474700143e-34]
Solution in 0.224s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.789r=2.57959e+07 in
[25795867.25698799409451243993168295190626 ..
1289794834371977270055988053158890743643/500000000000000000000000000000
00]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23888989.06493588745880515057802801897934]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678944242001671184086773476800,
10510.36930058692371237634766420582120505,
10510.39514076165434656325405307424286284,
10510.28046624356371105066247575283169983,
10510.36313884729203957386907803459205855,
10510.33729983825909943170848255074907818,
10510.33214580458397924448755880606587243,
10510.30529788725177007207568075770432126,
10510.21747048872573259267125122487784566,
10510.27430497004874370166927236627703638,
10510.19163019582321647064300607748399615,
10510.26914997035212898869267993991496724,
10510.27430450929993527620908220893075554,
10510.24330971182260890021436749299395174,
10510.33113809472365802239474533022147286,
10510.21646359964877690034093489239714209,
10510.18546892767567365813448374780641658,
10510.21130922467616185732613607588081288,
10510.10279600724394380471276705093947948,
10510.18546846180073306841260750697802765,
10510.27329709803826586967642853979680895,
10510.15447580460734762105091195474193578,
10510.23714797616856965313372581358434462,
10510.15962933459559753629439868067034152,
```



```
Cascade time 46.733
counts: 28, 28
```

```
1 --> 0 target = [23888999.9999999998120924569179055286563,  
2.632556434722355239463044349216850203813,  
10510.45814019397464533071640743390949062]  
one interval r = 24835349.43352072287057128313519631678990 ..  
621256961929631416754757956084258127247/250000000000000000000000000000  
0
```

```
hint used Hint := [24850273.21919080441727298772802245902434, 3, 1, 1,  
20396435.39659592643000634328381896732011,  
24850248.88301387793152980010731189259064 ..  
621256961929631488519759813441433737669/250000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 23889000., 1]
```

```
Exiting SolveHard() after 1.913r=2.48503e+07 in
[24850248.88301387793152980010731189259064 ..
621256961929631488519759813441433737669/250000000000000000000000000000
0]
```

```
Tau [10510.45813678933081241966144032763648064,
10510.36930058673522444580574188482627010, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]
```



```
2 --> 0 target = [23888999.99999999998120924569179055286563,
2.632556434722355239463044349216850203813,
10510.45814019397464533071640743390949062]
one interval r = 25781113.99083725307395805900958882469624 ..
20636726006246775098839576385438623333/80000000000000000000000000000000
Time Approximations 0.028.
```

```
hint used Hint := [25795903.79985511561316781642113695093491, 3, 1, 1,
22860361.79220184767639985483105161990097,
25795878.07759835662287683941521694813384 ..
161224421923802901739910205661944662513/62500000000000000000000000000000
, 298960418182500/22468879468420441 .. 23889000., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.63282) | P <-- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=
4.31733e+14 scos=4.39724e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795903.79985511561316781642113695093491,
rm=22860361.79220184767639985483105161990097}, {r =
25795878.07759835662287683941521694813384 ..
25795907.50780846427838563290591114600208, rm =
.1330553304194287328500223794129351168576e-1 .. 23889000.}, avoid={});
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=5e-32
Equations at solution: [.5e-31, .5e-31, -.14e-34]
Solution in 0.284s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.635r=2.57959e+07 in
[25795878.07759835662287683941521694813384 ..
161224421923802901739910205661944662513/62500000000000000000000000000000
]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23889000.]: target and source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678933081241966144032763648064,
10510.36930058673522444580574188482627010,
10510.39514076188132199387311352346771550, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]
```

```
Start Generation 2
2 --> 1 target = [24850273.21919080107500596272783138461959,
1.476968812057095584191303005247280614908,
10510.36930388284198816179841884198430223]
one interval r = 25781108.70735774325388516951360296614305 ..
1289795113945692104050507073410626603027/50000000000000000000000000000000
00
Time Approximations 0.034.
```

```
hint used Hint := [25795900.39533807920941978281062023785801, 3, -1, 1,
24743535.44822274617462265333263319921312,
25795872.84858705064741864171344489129222 ..
2579590227891383795486680697763551821623/100000000000000000000000000000000
000, 298960418182500/22468879468420441 ..
```



















none, none, none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none]

1 --> 2 target = [25795900.39533808354246762934891234188036,  
1.856309428639576188784971889852280098888,  
10510.33730295667508861133905883779988707]  
one interval r = 24835342.15555717022354529661425637881347 ..  
2485027132512925511433083377734912376991/1000000000000000000000000000000  
000  
Time Approximations 0.037.

hint used Hint := [24850269.43101309861241733616879334665430, 3, 1, 1,  
24743514.60014700975744022125022662330031,  
24850241.73069462631526237165506015616915 ..  
1242513566256462912427735010225393921567/500000000000000000000000000000  
00, 298960418182500/22468879468420441 .. 25089000, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.239829) | S --> P  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=  
2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850269.43101309861241733616879334665430,  
rm=24743514.60014700975744022125022662330031}, {r=  
24850241.73069462631526237165506015616915 ..  
24850271.32512925824855470020450787843134, rm =  
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.29692e-30, 0., -.72e-34]  
Solution in 0.305s

Time Plot 0 s.  
Exiting SolveHard() after 1.754r=2.48503e+07 in  
[24850241.73069462631526237165506015616915 ..  
1242513566256462912427735010225393921567/500000000000000000000000000000  
00]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 25089000]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678933081241966144032763648064,  
10510.36930058673522444580574188482627010,  
10510.39514076188132199387311352346771550,  
10510.28046624345600938597212098046407499,  
10510.36313884710504351100073654361210698,  
10510.33729983851758788818714046428573600,  
10510.33214580444625633198399927081512587,  
10510.30529788709628705220189094075918872, none,  
10510.27430496994253376579750709730888899, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none]

1 --> 0 target = [23888989.43146328547494698146835000840470,  
2.632526185978468910175964722176218307274,  
10510.28046964805050266086914947147220084]  
one interval r = 24835338.73249666198338251427461288830033 ..







```

hint used Hint := [25795893.34215400624221088590056918107762, 3, 1, 1,
22860366.17838899487248578273534549876757,
25795867.61967723367271871049090258979880 ..
1289794852506034629127592665641586312049/500000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888989.43146328644175807501347600997649, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=
4.31731e+14 scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795893.34215400624221088590056918107762,
rm=22860366.17838899487248578273534549876757}, {r =
25795867.61967723367271871049090258979880 ..
25795897.05012069258255185331283172624098, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.43146328644175807501347600997649}, avoid={}));
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=6e-32
Equations at solution: [.6e-31, .6e-31, -.42e-34]
Solution in 0.239s

```

```

Time Plot 0 s.
Exiting SolveHard() after 0.781r=2.57959e+07 in
[25795867.61967723367271871049090258979880 ..
1289794852506034629127592665641586312049/500000000000000000000000000000
00]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23888989.43146328644175807501347600997649]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

```

```

Tau [10510.45813678933081241966144032763648064,
10510.36930058673522444580574188482627010,
10510.39514076188132199387311352346771550,
10510.28046624345600938597212098046407499,
10510.36313884710504351100073654361210698,
10510.33729983851758788818714046428573600,
10510.33214580444625633198399927081512587,
10510.30529788709628705220189094075918872,
10510.21747048895661394714581524021423684,
10510.27430496994253376579750709730888899,
10510.19163019563863446929644660166321474, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none]

```

```

1 --> 0 target = [23888992.50558472531172541413312509655619,
2.632534984523479702269790277107423529622,
10510.33214920905510117685174533633358020]
one interval r = 24835341.84514835757320692388799824957633 ..
621256775502257926199517275312952911127/250000000000000000000000000000
0
Time Approximations 0.034.

```

```

hint used Hint := [24850265.76205359199870736703593497152527, 3, 1, 1,
20396437.16957127706091945345483725838621,

```



```
24850241.42564446195929857082855634229261 ..
1242513551004516068014443586172817582043/50000000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888992.50558472677941562199764206716244, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.533028) | P <--- S
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=
5.43203e+14 scos=2.39475e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850265.76205359199870736703593497152527,
rm=20396437.16957127706091945345483725838621}, {r =
24850241.42564446195929857082855634229261 ..
24850271.02009032136028887172345635164086, rm =
.1330553304194287328500223794129351168576e-1 ..
23888992.50558472677941562199764206716244}, avoid={}));
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=1.1e-31
Equations at solution: [.20e-30, .11e-30, -.72e-34]
Solution in 0.361s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.767r=2.48503e+07 in
[24850241.42564446195929857082855634229261 ..
1242513551004516068014443586172817582043/50000000000000000000000000000000
00]
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .
. 23888992.50558472677941562199764206716244]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678933081241966144032763648064,
10510.36930058673522444580574188482627010,
10510.39514076188132199387311352346771550,
10510.28046624345600938597212098046407499,
10510.36313884710504351100073654361210698,
10510.33729983851758788818714046428573600,
10510.33214580444625633198399927081512587,
10510.30529788709628705220189094075918872,
10510.21747048895661394714581524021423684,
10510.27430496994253376579750709730888899,
10510.19163019563863446929644660166321474, none, none,
10510.24330971160800567713850806172232578, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0 target = [23888992.50558472531172541413312509655619,
2.632534984523479702269790277107423529622,
10510.33214920905510117685174533633358020]
one interval r = 25781106.49760329950233501605963054463378 ..
322448751149850583766048200013925549841/12500000000000000000000000000000
0
Time Approximations 0.036.
```

```
hint used Hint := [25795896.38402523646204355452114396262515, 3, 1, 1,
22860364.90258708217946452135489353704915,
25795870.66161245800881604810046494170563 ..
```



























```
-.1286762695354506538357536802500935137354e-14
one interval r = 23873935.37540036425536621845487429204610 ..
1194449545427563571681935226178076814569/500000000000000000000000000000
00
Time Approximations 0.048.
```

```
hint used Hint := [23888985.62428976346864819081258538077918, 3, -1, 1,
20396632.00572718336484608918971225516342,
23888961.16565299817948948119449178210506 ..
2388899090855127150824991682392263419777/1000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S --> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=
-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888985.62428976346864819081258538077918,
rm=20396632.00572718336484608918971225516342}, {r =
23888961.16565299817948948119449178210506 ..
23888990.90855127150824991682392263419777, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={});
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1.6e-31
Equations at solution: [.28e-30, .16e-30, -.101e-33]
Solution in 1.082s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.969r=2.38890e+07 in
[23888961.16565299817948948119449178210506 ..
2388899090855127150824991682392263419777/1000000000000000000000000000000
000]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678933081241966144032763648064,
10510.36930058673522444580574188482627010,
10510.39514076188132199387311352346771550,
10510.28046624345600938597212098046407499,
10510.36313884710504351100073654361210698,
10510.33729983851758788818714046428573600,
10510.33214580444625633198399927081512587,
10510.30529788709628705220189094075918872,
10510.21747048895661394714581524021423684,
10510.27430496994253376579750709730888899,
10510.19163019563863446929644660166321474,
10510.26914997055298914220315913588134434,
10510.27430450919372547790089388273269588,
10510.24330971160800567713850806172232578,
10510.33113809498363834719068051958324598,
10510.21646359957408011765160684539768666,
10510.18546892749258338431523877996467885,
10510.21130922490853493824075611915195054, none, none,
10510.27329709832975925460199729614509822, none, none, none, none,
none, none, none, none, none, none]
```



```
2 --> 1 target = [24850262.70323078216364658570639838932353,
1.476940274035382038373747613092308847795,
10510.19163349169793688490634062915410482]
one interval r = 25781098.14047726156997542996633208900764 ..
25795891821193715368926840938016418829/10000000000000000000000000000000000
Time Approximations 0.029.
```

```
hint used Hint := [25795889.93759639200267360170740297062535, 3, -1, 1,
24743522.41979937237431350459260520845964,
25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/20000000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
24850262.70323078644925875505002433594091, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.76017) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=
-2.37384e+14 scos=5.95461e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25795889.93759639200267360170740297062535,
rm=24743522.41979937237431350459260520845964}, {r =
25795862.39063358446748128489577933480278 ..
25795891.82119371218082845151233599574945, rm =
.1330553304194287328500223794129351168576e-1 ..
24850262.70323078644925875505002433594091}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=1e-32
Equations at solution: [.14846e-30, -.1e-31, -.71e-34]
Solution in 0.309s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.814r=2.57959e+07 in
[25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/20000000000000000000000000000000000
0]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 24850262.70323078644925875505002433594091]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678933081241966144032763648064,
10510.36930058673522444580574188482627010,
10510.39514076188132199387311352346771550,
10510.28046624345600938597212098046407499,
10510.36313884710504351100073654361210698,
10510.33729983851758788818714046428573600,
10510.33214580444625633198399927081512587,
10510.30529788709628705220189094075918872,
10510.21747048895661394714581524021423684,
10510.27430496994253376579750709730888899,
10510.19163019563863446929644660166321474,
10510.26914997055298914220315913588134434,
10510.27430450919372547790089388273269588,
10510.24330971160800567713850806172232578,
10510.33113809498363834719068051958324598,
10510.21646359957408011765160684539768666,
10510.18546892749258338431523877996467885,
```



```

10510.21130922490853493824075611915195054, none, none,
10510.27329709832975925460199729614509822, none, none,
10510.15962933485799191575007696258798772, none, none, none, none,
none, none, none]

0 --> 1 target = [24850262.70323078216364658570639838932353,
1.476940274035382038373747613092308847795,
10510.19163349169793688490634062915410482]
Imaginary part neglected:
-.1286762695354506538357536802500935137354e-14
one interval r = 23873928.44821858601326525618560642901329 ..
2388898414707082196624544899683665118351/1000000000000000000000000000000
000
Time Approximations 0.039.

hint used Hint := [23888978.86275416794827744619407470166172, 3, -1, 1,
20396633.60486737240849086583158368073964,
23888954.40382902066202638611670198664112 ..
298612301838385292588525592212900916387/1250000000000000000000000000000
0, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=
-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888978.86275416794827744619407470166172,
rm=20396633.60486737240849086583158368073964}, {r =
23888954.40382902066202638611670198664112 ..
23888984.14707082340708204737703207331096, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1.3e-31
Equations at solution: [-.23e-30, -.13e-30, .95e-34]
Solution in 0.329s

Time Plot 0 s.
Exiting SolveHard() after 1.961r=2.38890e+07 in
[23888954.40382902066202638611670198664112 ..
298612301838385292588525592212900916387/1250000000000000000000000000000
0]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678933081241966144032763648064,
10510.36930058673522444580574188482627010,
10510.39514076188132199387311352346771550,
10510.28046624345600938597212098046407499,
10510.36313884710504351100073654361210698,
10510.33729983851758788818714046428573600,
10510.33214580444625633198399927081512587,
10510.30529788709628705220189094075918872,
10510.21747048895661394714581524021423684,
10510.27430496994253376579750709730888899,
10510.19163019563863446929644660166321474,
10510.26914997055298914220315913588134434,

```



```

10510.27430450919372547790089388273269588,
10510.24330971160800567713850806172232578,
10510.33113809498363834719068051958324598,
10510.21646359957408011765160684539768666,
10510.18546892749258338431523877996467885,
10510.21130922490853493824075611915195054,
10510.10279600714014806605292071423390155, none,
10510.27329709832975925460199729614509822, none, none,
10510.15962933485799191575007696258798772, none, none, none, none,
none, none, none]

0 --> 2 target = [25795893.34215400918248578581317037515996,
1.856291208189616047976570301468282909171,
10510.21747360708325352495946368222697513]
Imaginary part neglected:
-.1286762695354506538357536802500935137354e-14
one interval r = 23873930.02298357309446615514111659036285 ..
2388898568416735648951335000620688025949/1000000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [23888981.93694517027845377679197924121306, 3, -1, 1,
22860455.76745222490936139362415353022295,
23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/4000000000000000000000000000000000,
298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.367179) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31730e+14 scos=4.39725e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888981.93694517027845377679197924121306,
rm=22860455.76745222490936139362415353022295}, {r =
23888955.94100364913487175812205942139954 ..
23888985.68416735818079539166696781319750, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}));
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.8e-31
Equations at solution: [-.18e-30, -.18e-30, .73e-34]
Solution in 0.402s

Time Plot 0 s.
Exiting SolveHard() after 2.005r=2.38890e+07 in
[23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/400000000000000000000000000000000]
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 ..
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678933081241966144032763648064,
10510.36930058673522444580574188482627010,
10510.39514076188132199387311352346771550,
10510.28046624345600938597212098046407499,
10510.36313884710504351100073654361210698,
10510.33729983851758788818714046428573600,
10510.33214580444625633198399927081512587,
```



```

10510.30529788709628705220189094075918872,
10510.21747048895661394714581524021423684,
10510.27430496994253376579750709730888899,
10510.19163019563863446929644660166321474,
10510.26914997055298914220315913588134434,
10510.27430450919372547790089388273269588,
10510.24330971160800567713850806172232578,
10510.33113809498363834719068051958324598,
10510.21646359957408011765160684539768666,
10510.18546892749258338431523877996467885,
10510.21130922490853493824075611915195054,
10510.10279600714014806605292071423390155, none,
10510.27329709832975925460199729614509822,
10510.15447580447353062389924730157237058, none,
10510.15962933485799191575007696258798772, none, none, none, none,
none, none, none]

1 --> 2 target = [25795893.34215400918248578581317037515996,
1.856291208189616047976570301468282909171,
10510.21747360708325352495946368222697513]
one interval r = 24835334.93824082382273250767058746120234 ..
1242513211633239642850610559509580611961/500000000000000000000000000000000000
00
Time Approximations 0.039.

hint used Hint := [24850262.33852743516297758053367205548154, 3, 1, 1,
24743505.81299238129247202614818714422035,
24850234.63796912173141920432208117660707 ..
2485026423266479739181704602953513976831/1000000000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.23983) | S ---> P
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=
2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850262.33852743516297758053367205548154,
rm=24743505.81299238129247202614818714422035}, {r =
24850234.63796912173141920432208117660707 ..
24850264.23266479739181704602953513976831, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.27218e-30, 0., -.53e-34]
Solution in 1.027s

Time Plot 0 s.
Exiting SolveHard() after 1.728r=2.48503e+07 in
[24850234.63796912173141920432208117660707 ..
2485026423266479739181704602953513976831/1000000000000000000000000000000000000
000]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 25089000]: target and source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678933081241966144032763648064,
10510.36930058673522444580574188482627010,

```



[illegible]



Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 24850265.76205359199870736703593497152527]: target and source on the  
different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678933081241966144032763648064,  
10510.36930058673522444580574188482627010,  
10510.39514076188132199387311352346771550,  
10510.28046624345600938597212098046407499,  
10510.36313884710504351100073654361210698,  
10510.33729983851758788818714046428573600,  
10510.33214580444625633198399927081512587,  
10510.30529788709628705220189094075918872,  
10510.21747048895661394714581524021423684,  
10510.27430496994253376579750709730888899,  
10510.19163019563863446929644660166321474,  
10510.26914997055298914220315913588134434,  
10510.27430450919372547790089388273269588,  
10510.24330971160800567713850806172232578,  
10510.33113809498363834719068051958324598,  
10510.21646359957408011765160684539768666,  
10510.18546892749258338431523877996467885,  
10510.21130922490853493824075611915195054,  
10510.10279600714014806605292071423390155,  
10510.18546846161764292318118495836104556,  
10510.27329709832975925460199729614509822,  
10510.15447580447353062389924730157237058, none,  
10510.15962933485799191575007696258798772, none, none,  
10510.21130888356889447184671086777842895, none, none, none, none]

0 --> 1 target = [24850265.76205358721471964806727971512813,  
1.476948574977268581330437546185732433515,  
10510.24331300768111330535407815365135976]  
Imaginary part neglected:  
-.1286762695354506538357536802500935137354e-14  
one interval r = 23873931.59770124994330755281219936095342 ..  
1194449361060866796303476081166628187087/500000000000000000000000000000  
00  
Time Approximations 0.044.

hint used Hint := [23888981.93692575516559285951162594064654, 3, -1, 1,  
20396632.87781872303409295730403849988110,  
23888957.47813172092635353311983299926098 ..  
597224680530433446694670341991304966889/250000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.466972) | S ---> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=  
-5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888981.93692575516559285951162594064654,  
rm=20396632.87781872303409295730403849988110}, {r =  
23888957.47813172092635353311983299926098 ..  
23888987.22121733786778681367965219867556, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));



Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=3e-32  
Equations at solution: [.5e-31, .3e-31, -.20e-34]  
Solution in 0.372s

Time Plot 0 s.

Exiting SolveHard() after 2.054r=2.38890e+07 in

[23888957.47813172092635353311983299926098 ..  
597224680530433446694670341991304966889/2500000000000000000000000000000000  
0]

Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.

Clockwise ray.

Ray outgoing at target.

Solve Side.

Tau [10510.45813678933081241966144032763648064,  
10510.36930058673522444580574188482627010,  
10510.39514076188132199387311352346771550,  
10510.28046624345600938597212098046407499,  
10510.36313884710504351100073654361210698,  
10510.33729983851758788818714046428573600,  
10510.33214580444625633198399927081512587,  
10510.30529788709628705220189094075918872,  
10510.21747048895661394714581524021423684,  
10510.27430496994253376579750709730888899,  
10510.19163019563863446929644660166321474,  
10510.26914997055298914220315913588134434,  
10510.27430450919372547790089388273269588,  
10510.24330971160800567713850806172232578,  
10510.33113809498363834719068051958324598,  
10510.21646359957408011765160684539768666,  
10510.18546892749258338431523877996467885,  
10510.21130922490853493824075611915195054,  
10510.10279600714014806605292071423390155,  
10510.18546846161764292318118495836104556,  
10510.27329709832975925460199729614509822,  
10510.15447580447353062389924730157237058, none,  
10510.15962933485799191575007696258798772,  
10510.15447547808791181763597173123619868, none,  
10510.21130888356889447184671086777842895, none, none, none, none]

0 --> 2 target = [25795896.38402523890668232756652133307735,  
1.856299066227386153066014303680425368824,  
10510.26915308869293838216522243742296510]

Imaginary part neglected:

-.1286762695354506538357536802500935137354e-14

one interval r = 23873933.17245664360020005600123277162126 ..

1194449437915187672302587845089621799897/5000000000000000000000000000000000  
00

Time Approximations 0.043.

hint used Hint := [23888985.01109652402578398653593278001387, 3, -1, 1,  
22860454.49191911041368999760273437756943,  
23888959.01529623342321627788824672003212 ..  
59722471895759389095528762418719523933/2500000000000000000000000000000000,  
298960418182500/22468879468420441 .. 24089000, 1]

I search for an scattering ray on opposite branches with 0<sv<1











```
0 --> 2 target = [25795900.03265731056317535071700672718752,
1.856308491726282239653497027718527052273,
10510.33114121313955216003632281281337005]
Imaginary part neglected:
-.1286762695354506538357536802500935137354e-14
one interval r = 23873936.95014365178121034159522129962199 ..
119444962228124845688747268186675236631/50000000000000000000000000000000
Time Approximations 0.044.
```

```
hint used Hint := [23888988.69843567795653101185231580305160, 3, -1, 1,
22860452.96192886545581288218987601576608,
23888962.70280479206299436081506076835447 ..
2388899244562496946280763877837409340903/100000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.36718) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31731e+14 scos=4.39724e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888988.69843567795653101185231580305160,
rm=22860452.96192886545581288218987601576608}, {r =
23888962.70280479206299436081506076835447 ..
23888992.44562496946280763877837409340903, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={{}};
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=5.4e-31
Equations at solution: [.56e-30, .54e-30, -.203e-33]
Solution in 0.388s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.077r=2.38890e+07 in
[23888962.70280479206299436081506076835447 ..
2388899244562496946280763877837409340903/100000000000000000000000000000000
000]
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678933081241966144032763648064,
10510.36930058673522444580574188482627010,
10510.39514076188132199387311352346771550,
10510.28046624345600938597212098046407499,
10510.36313884710504351100073654361210698,
10510.33729983851758788818714046428573600,
10510.33214580444625633198399927081512587,
10510.30529788709628705220189094075918872,
10510.21747048895661394714581524021423684,
10510.27430496994253376579750709730888899,
10510.19163019563863446929644660166321474,
10510.26914997055298914220315913588134434,
10510.27430450919372547790089388273269588,
10510.24330971160800567713850806172232578,
10510.33113809498363834719068051958324598,
10510.21646359957408011765160684539768666,
10510.18546892749258338431523877996467885,
10510.21130922490853493824075611915195054,
```



[illegible]

```
Time Plot 0 s.  
Exiting SolveHard() after 1.816r=2.48503e+07 in  
[24850241.36598119958086914129896830053765 ..  
2485027096042925490424856544845137371367/1000000000000000000000000000000  
000]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 25089000]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

Tau [10510.45813678933081241966144032763648064,  
10510.36930058673522444580574188482627010,  
10510.39514076188132199387311352346771550,  
10510.28046624345600938597212098046407499,  
10510.36313884710504351100073654361210698,  
10510.33729983851758788818714046428573600,  
10510.33214580444625633198399927081512587,  
10510.30529788709628705220189094075918872,



```
1 --> 0   target = [23888989.06493588645064419063883736978768,  
2.632525136931166248672478965308023490870,  
10510.27430791378650762044256641986283257]  
one interval r = 24835338.36137568388911382385556979236584 ..  
31062834495729838816753651782765519247/1250000000000000000000000000000  
Time Approximations 0.04.
```

```
hint used Hint := [24850262.33852773859450921827957797170818, 3, 1, 1,  
20396437.98350304935080318674410988058300,  
24850238.00201200929744389905914961384924 ..  
1242513379829193745424075414664369797603/500000000000000000000000000000  
00, 298960418182500/22468879468420441 ..  
23888989.06493588745880515057802801897934, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.533028) | P <--- S  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=  
5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850262.33852773859450921827957797170818,  
rm=20396437.98350304935080318674410988058300}, {r =  
24850238.00201200929744389905914961384924 ..  
24850267.59658387490848150829328739595206, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.06493588745880515057802801897934}, avoid={{}});  
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=1.0e-31  
Equations at solution: [.17e-30, .10e-30, -.67e-34]  
Solution in 1.116s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 1.81r=2.48503e+07 in  
[24850238.00201200929744389905914961384924 ..  
1242513379829193745424075414664369797603/500000000000000000000000000000  
00]  
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888989.06493588745880515057802801897934]: target and source on the
```



different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

```
Tau [10510.45813678933081241966144032763648064,
10510.36930058673522444580574188482627010,
10510.39514076188132199387311352346771550,
10510.28046624345600938597212098046407499,
10510.36313884710504351100073654361210698,
10510.33729983851758788818714046428573600,
10510.33214580444625633198399927081512587,
10510.30529788709628705220189094075918872,
10510.21747048895661394714581524021423684,
10510.27430496994253376579750709730888899,
10510.19163019563863446929644660166321474,
10510.26914997055298914220315913588134434,
10510.27430450919372547790089388273269588,
10510.24330971160800567713850806172232578,
10510.33113809498363834719068051958324598,
10510.21646359957408011765160684539768666,
10510.18546892749258338431523877996467885,
10510.21130922490853493824075611915195054,
10510.10279600714014806605292071423390155,
10510.18546846161764292318118495836104556,
10510.27329709832975925460199729614509822,
10510.15447580447353062389924730157237058,
10510.23714797595545829044696107285800440,
10510.15962933485799191575007696258798772,
10510.15447547808791181763597173123619868,
10510.29913613965857531387707561944567292,
10510.21130888356889447184671086777842895,
10510.20615520667548509028714318634048704,
10510.18546846674417648010275728180291861, none,
10510.26814323587476829470404153017654360]
```

[illegible]

```
hint used Hint := [25795892.97947239655457473800077316699564, 3, 1, 1,  
22860366.33050119628977322107538139299992,  
25795867.25698799409451243993168295190626 ..  
1289794834371977270055988053158890743643/500000000000000000000000000000  
00, 298960418182500/22468879468420441 ..  
23888989.06493588745880515057802801897934, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.632821) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31731e+14 scos=4.39725e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795892.97947239655457473800077316699564,  
rm=22860366.33050119628977322107538139299992}, {r =
```















```
24850273.21919080441727298772802245902434}, avoid={});
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.14846e-30, 0., .52e-34]
Solution in 0.325s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 0.895r=2.57959e+07 in  
[25795872.84858705064741864171344489129222 ..  
2579590227891383795486680697763551821623/1000000000000000000000000000000000]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 24850273.21919080441727298772802245902434]: target and source on the  
different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678936277210271162349695104919,  
10510.36930058669187131635190383176941460,  
10510.39514076178734552660970468805579276, none, none,  
10510.33729983841125882117464390586214021, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none]
```

```
0 --> 1 target = [24850273.21919080459349870962524691822360,
1.476968812115461624456953491913823190758,
10510.36930388290466688945828848765890746]
one interval r = 23873939.27584648045341324738567842046658 ..
1194449735784687094754357753768704044901/500000000000000000000000000000
00
Time Approximations 0.046.
```

```
hint used Hint := [23888989.43146328644175807501347600997649, 3, -1, 1,  
20396631.10527514558221865082982151479995,  
23888964.97298890387184335817087833215779 ..  
597224867892343544545306555277700499657/250000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.466972) | S ---> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=  
-5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888989.43146328644175807501347600997649,  
rm=20396631.10527514558221865082982151479995}, {r =  
23888964.97298890387184335817087833215779 ..  
23888994.71569374178181226221110801998628, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}));  
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=4e-32  
Equations at solution: [.7e-31, .4e-31, -.27197380161e-34]  
Solution in 0.385s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 2.147r=2.38890e+07 in  
[23888964.97298890387184335817087833215779 ..  
597224867892343544545306555277700499657/25000000000000000000000000000000  
0]  
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
```



. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678936277210271162349695104919,  
10510.36930058669187131635190383176941460,  
10510.39514076178734552660970468805579276,  
10510.28046624350840324023402147460059791, none,  
10510.33729983841125882117464390586214021, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none]

0 --> 2 target = [25795903.79985511843730434389427801992523,  
1.856318223668597603444893978455217529158,  
10510.39514388012297152245601968270375105]  
one interval r = 23873940.85057848602238064630419450099093 ..  
2388899625275549489073819733085403734653/1000000000000000000000000000000  
000  
Time Approximations 0.043.

hint used Hint := [23888992.50558472677941562199764206716244, 3, -1, 1,  
22860451.38218918783573792125602312359251,  
23888966.51012875395781144697926248654402 ..  
597224906318887365922168983423648563467/2500000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.36718) | S ---> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=  
-4.31732e+14 scos=4.39724e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888992.50558472677941562199764206716244,  
rm=22860451.38218918783573792125602312359251}, {r =  
23888966.51012875395781144697926248654402 ..  
23888996.25275549463688675933694594253868, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={});  
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.8e-31  
Equations at solution: [-.18e-30, -.18e-30, .75144984178e-34]  
Solution in 0.38s

Time Plot 0 s.  
Exiting SolveHard() after 2.117r=2.38890e+07 in  
[23888966.51012875395781144697926248654402 ..  
597224906318887365922168983423648563467/2500000000000000000000000000000  
0]  
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678936277210271162349695104919,  
10510.36930058669187131635190383176941460,  
10510.39514076178734552660970468805579276,  
10510.28046624350840324023402147460059791, none,  
10510.33729983841125882117464390586214021,  
10510.33214580451179713126030149560056969, none, none, none, none,







```

000
Time Approximations 0.048.

hint used Hint := [23888989.06496329485872911197089488868350, 3, -1, 1,
22860452.80984339454018821965990898965218,
23888963.06934924828335479031706673264809 ..
597224820303770076400571671810948623397/25000000000000000000000000000000
0, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.36718) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31731e+14 scos=4.39724e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888989.06496329485872911197089488868350,
rm=22860452.80984339454018821965990898965218}, {r =
23888963.06934924828335479031706673264809 ..
23888992.81215080305602286687243794493588, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={});
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=0
Equations at solution: [0., 0., -.6209384257e-35]
Solution in 0.425s

Time Plot 0 s.
Exiting SolveHard() after 2.171r=2.38890e+07 in
[23888963.06934924828335479031706673264809 ..
597224820303770076400571671810948623397/25000000000000000000000000000000
0]
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678936277210271162349695104919,
10510.36930058669187131635190383176941460,
10510.39514076178734552660970468805579276,
10510.28046624350840324023402147460059791,
10510.36313884706248481682747886984740132,
10510.33729983841125882117464390586214021,
10510.33214580451179713126030149560056969, none, none,
10510.27430496999572198535019175400701962, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2 target = [25795900.39533808176120809347666252994451,
1.856309428767960689726537815915000616062,
10510.33730295673198832741936122275236480]
one interval r = 24835342.15555717345298116968611608037791 ..
124251356625646291453923074047215621939/50000000000000000000000000000000
Time Approximations 0.043.

hint used Hint := [24850269.43101309861241733616879334665430, 3, 1, 1,
24743514.60014700975744022125022662330031,
24850241.73069462631526237165506015616915 ..
1242513566256462912427735010225393921567/50000000000000000000000000000000
00, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1

```















Time Plot 0 s.  
Exiting SolveHard() after 1.862r=2.48503e+07 in  
[24850241.42564446195929857082855634229261 ..  
1242513551004516068014443586172817582043/500000000000000000000000000000  
00]  
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888992.50558472677941562199764206716244]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678936277210271162349695104919,  
10510.36930058669187131635190383176941460,  
10510.39514076178734552660970468805579276,  
10510.28046624350840324023402147460059791,  
10510.36313884706248481682747886984740132,  
10510.33729983841125882117464390586214021,  
10510.33214580451179713126030149560056969,  
10510.30529788704137575144450785994595153,  
10510.21747048888307162292316848735550131,  
10510.27430496999572198535019175400701962,  
10510.19163019561571549450591291687516553, none, none,  
10510.24330971159823363554495781487122968, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none]

2 --> 0 target = [23888992.50558472730758651289405520936217,  
2.632534984431676772014060964779699559552,  
10510.33214920914114386475423592840554768]  
one interval r = 25781106.49760329944366564069784134364840 ..  
1289795004599402331769107533507532489883/500000000000000000000000000000  
00  
Time Approximations 0.031.

hint used Hint := [25795896.38402523646204355452114396262515, 3, 1, 1,  
22860364.90258708217946452135489353704915,  
25795870.66161245800881604810046494170563 ..  
2579590009198804353979547140796504033263/100000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
23888992.50558472677941562199764206716244, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.63282) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31731e+14 scos=4.39724e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795896.38402523646204355452114396262515,  
rm=22860364.90258708217946452135489353704915}, {r =  
25795870.66161245800881604810046494170563 ..  
25795900.09198804353979547140796504033263, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888992.50558472677941562199764206716244}, avoid={}));  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=5e-32  
Equations at solution: [-.5e-31, -.5e-31, .28e-34]  
Solution in 0.238s

Time Plot 0 s.































```
.1330553304194287328500223794129351168576e-1 ..  
24850262.70323078644925875505002433594091}, avoid={}));  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.24744e-30, 0., -.57e-34]  
Solution in 0.29s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.779r=2.57959e+07 in
[25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/2000000000000000000000000000000000
0]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 24850262.70323078644925875505002433594091]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678936277210271162349695104919,
10510.36930058669187131635190383176941460,
10510.39514076178734552660970468805579276,
10510.28046624350840324023402147460059791,
10510.36313884706248481682747886984740132,
10510.33729983841125882117464390586214021,
10510.33214580451179713126030149560056969,
10510.30529788704137575144450785994595153,
10510.21747048888307162292316848735550131,
10510.27430496999572198535019175400701962,
10510.19163019561571549450591291687516553,
10510.26914997049259374186062316656308809,
10510.27430450924691376679472211041741953,
10510.24330971159823363554495781487122968,
10510.33113809487810371606763850429239840,
10510.21646359961491581113019441809010224,
10510.18546892747045877416691836921871364,
10510.21130922483578697820024207469177798, none, none,
10510.27329709821187201005974519201030586, none, none,
10510.15962933477209701939071089144820286, none, none, none, none,
none, none, none]
```

```
0 --> 1   target = [24850262.70323078689164841321321833776014,
1.476940274093751360852125025019498934321,
10510.19163349178104976723497325328756448]
one interval r = 23873928.44821858787506899598101358734104 ..
1194449207353541189392653735775907166539/500000000000000000000000000000
00
```

Time Approximations 0.039.

```
hint used Hint := [23888978.86275416794827744619407470166172, 3, -1, 1,  
20396633.60486737240849086583158368073964,  
23888954.40382902066202638611670198664112 ..  
298612301838385292588525592212900916387/1250000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.466972) | S ---> P  
rGuessMin=2.38739e+07    rGuessMax=2.38890e+07    rmGuess=2.03966e+07    k=  
-5.43203e+14    scos=2.39475e+14
```































```

Tau [10510.45813678936277210271162349695104919,
10510.36930058669187131635190383176941460,
10510.39514076178734552660970468805579276,
10510.28046624350840324023402147460059791,
10510.36313884706248481682747886984740132,
10510.33729983841125882117464390586214021,
10510.33214580451179713126030149560056969,
10510.30529788704137575144450785994595153,
10510.21747048888307162292316848735550131,
10510.27430496999572198535019175400701962,
10510.19163019561571549450591291687516553,
10510.26914997049259374186062316656308809,
10510.27430450924691376679472211041741953,
10510.24330971159823363554495781487122968,
10510.33113809487810371606763850429239840,
10510.21646359961491581113019441809010224,
10510.18546892747045877416691836921871364,
10510.21130922483578697820024207469177798,
10510.10279600721297605842881569826114257,
10510.18546846159551838803856206170755407,
10510.27329709821187201005974519201030586,
10510.15447580455950553804388557312750920,
10510.23714797594648068662828047909251898,
10510.15962933477209701939071089144820286,
10510.15447547817388673139142244936653785, none,
10510.21130888349614651612820698233184261,
10510.20615520677460690717782631193733386, none, none, none]

0 --> 2 target = [25795900.03265730882867609705782618862550,
1.856308491854666861391518585160216066589,
10510.33114121319724631200628434169503715]
one interval r = 23873936.95014365209392590054367902242961 ..
2388899244562496944732276518074647606399/1000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [23888988.69843567795653101185231580305160, 3, -1, 1,
22860452.96192886545581288218987601576608,
23888962.70280479206299436081506076835447 ..
2388899244562496946280763877837409340903/1000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.36718) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31731e+14 scos=4.39724e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888988.69843567795653101185231580305160,
rm=22860452.96192886545581288218987601576608}, {r =
23888962.70280479206299436081506076835447 ..
23888992.44562496946280763877837409340903, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={});
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.9e-31
Equations at solution: [.20e-30, .19e-30, -.95390651312e-34]
Solution in 0.382s

Time Plot 0 s.
Exiting SolveHard() after 2.086r=2.38890e+07 in

```











```
hint used Hint := [24850262.33852773859450921827957797170818, 3, 1, 1,
20396437.98350304935080318674410988058300,
24850238.00201200929744389905914961384924 ..
1242513379829193745424075414664369797603/50000000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888989.06493588745880515057802801897934, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.533028) | P <--- S
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=
5.43203e+14 scos=2.39475e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850262.33852773859450921827957797170818,
rm=20396437.98350304935080318674410988058300}, {r =
24850238.00201200929744389905914961384924 ..
24850267.59658387490848150829328739595206, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06493588745880515057802801897934}, avoid={});
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=1e-32
Equations at solution: [-.2e-31, -.1e-31, .15e-34]
Solution in 1.124s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.768r=2.48503e+07 in
[24850238.00201200929744389905914961384924 ..
1242513379829193745424075414664369797603/50000000000000000000000000000000
00]
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .
. 23888989.06493588745880515057802801897934]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678936277210271162349695104919,
10510.36930058669187131635190383176941460,
10510.39514076178734552660970468805579276,
10510.28046624350840324023402147460059791,
10510.36313884706248481682747886984740132,
10510.33729983841125882117464390586214021,
10510.33214580451179713126030149560056969,
10510.30529788704137575144450785994595153,
10510.21747048888307162292316848735550131,
10510.27430496999572198535019175400701962,
10510.19163019561571549450591291687516553,
10510.26914997049259374186062316656308809,
10510.27430450924691376679472211041741953,
10510.24330971159823363554495781487122968,
10510.33113809487810371606763850429239840,
10510.21646359961491581113019441809010224,
10510.18546892747045877416691836921871364,
10510.21130922483578697820024207469177798,
10510.10279600721297605842881569826114257,
10510.18546846159551838803856206170755407,
10510.27329709821187201005974519201030586,
10510.15447580455950553804388557312750920,
10510.23714797594648068662828047909251898,
10510.15962933477209701939071089144820286,
```



```
10510.15447547817388673139142244936653785,  
10510.29913613960445844961802396874157850,  
10510.21130888349614651612820698233184261,  
10510.20615520677460690717782631193733386,  
10510.18546846672205193929552328398115400, none,  
10510.26814323592875094903738533902009972]
```

```
2 --> 0 target = [23888989.06493588771173339644884542451834,  
2.632525136839361215379296098226410515891,  
10510.27430791386019779795915268683887458]  
one interval r = 25781103.05751677065665549973697998355876 ..  
2579589668743954822435907953690453399209/1000000000000000000000000000000  
000
```

Time Approximations 0.032.

```
hint used Hint := [25795892.97947239655457473800077316699564, 3, 1, 1,  
22860366.33050119628977322107538139299992,  
25795867.25698799409451243993168295190626 ..  
1289794834371977270055988053158890743643/5000000000000000000000000000000  
00, 298960418182500/22468879468420441 ..  
23888989.06493588745880515057802801897934, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1  
(0.632821) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31731e+14 scos=4.39725e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795892.97947239655457473800077316699564,  
rm=22860366.33050119628977322107538139299992}, {r =  
25795867.25698799409451243993168295190626 ..  
25795896.68743954540111976106317781487286, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.06493588745880515057802801897934}, avoid={});  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1.4e-31  
Equations at solution: [-.15e-30, -.14e-30, .43e-34]  
Solution in 0.301s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 0.835r=2.57959e+07 in  
[25795867.25698799409451243993168295190626 ..  
1289794834371977270055988053158890743643/5000000000000000000000000000000  
00]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23888989.06493588745880515057802801897934]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678936277210271162349695104919,  
10510.36930058669187131635190383176941460,  
10510.39514076178734552660970468805579276,  
10510.28046624350840324023402147460059791,  
10510.36313884706248481682747886984740132,  
10510.33729983841125882117464390586214021,  
10510.33214580451179713126030149560056969,  
10510.30529788704137575144450785994595153,  
10510.21747048888307162292316848735550131,
```







Time Plot 0 s.  
Exiting SolveHard() after 1.869r=2.48503e+07 in  
[24850248.88301387793152980010731189259064 ..  
621256961929631488519759813441433737669/2500000000000000000000000000000000  
0]  
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23889000.]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678925322818606134583815945501,  
10510.36930058668129353201123910284808332, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none]

2 --> 0 target = [23889000.00000000000102999489633015888692,  
2.632556434371823080062332803943576500051,  
10510.45814019389391773254936284216952111]  
one interval r = 25781113.99083724873676500088992032213252 ..  
2579590750780846458364137327026536129969/1000000000000000000000000000000000  
000  
Time Approximations 0.033.

hint used Hint := [25795903.79985511561316781642113695093491, 3, 1, 1,  
22860361.79220184767639985483105161990097,  
25795878.07759835662287683941521694813384 ..  
161224421923802901739910205661944662513/6250000000000000000000000000000000  
, 298960418182500/22468879468420441 .. 23889000., 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.63282) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31733e+14 scos=4.39724e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795903.79985511561316781642113695093491,  
rm=22860361.79220184767639985483105161990097}, {r =  
25795878.07759835662287683941521694813384 ..  
25795907.50780846427838563290591114600208, rm =  
.1330553304194287328500223794129351168576e-1 .. 23889000.}, avoid={});  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=2.0e-31  
Equations at solution: [-.21e-30, -.20e-30, .95e-34]  
Solution in 1.123s

Time Plot 0 s.  
Exiting SolveHard() after 1.712r=2.57959e+07 in  
[25795878.07759835662287683941521694813384 ..  
161224421923802901739910205661944662513/6250000000000000000000000000000000  
]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23889000.]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678925322818606134583815945501,  
10510.36930058668129353201123910284808332,



```
10510.39514076171935929900237070393559133, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none]
```

Start Generation 2

```
2 --> 1 target = [24850273.21919080489595851491921835475879,
1.476968812080919893659633542798301739393,
10510.36930388278672996800633310908564384]
one interval r = 25781108.70735774043145456211932565441280 ..
1289795113945691964510661989519072880417/500000000000000000000000000000
00
```

Time Approximations 0.03.

```
hint used Hint := [25795900.39533807920941978281062023785801, 3, -1, 1,
24743535.44822274617462265333263319921312,
25795872.84858705064741864171344489129222 ..
2579590227891383795486680697763551821623/1000000000000000000000000000000
000, 298960418182500/22468879468420441 ..
24850273.21919080441727298772802245902434, 1]
```

I search for an scattering ray on opposite branches with  $0 < s_v < 1$

```
(0.760172) | P <--- S
rGuessMin=2.57811e+07    rGuessMax=2.57959e+07    rmGuess=2.47435e+07    k=
-2.37383e+14    scos=5.95462e+14
```

branch outgoing at target, Clockwise

```
(Scattering) fsolve(eqs, {r=25795900.39533807920941978281062023785801,
rm=24743535.4482227461746226533263319921312}, {r =
25795872.84858705064741864171344489129222 ..
25795902.27891383795486680697763551821623, rm =
.1330553304194287328500223794129351168576e-1 ..
```

```
24850273.21919080441727298772802245902434}, avoid={}));
```

Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0

Equations at solution: [.12372e-30, 0., .5e-35]

Solution in 0.309s

Time Plot 0 s.

Exiting SolveHard() after 0.841r=2.57959e+07 in

```
[25795872.84858705064741864171344489129222 ..  
2579590227891383795486680697763551821623/1000000000000000000000000000000  
000]
```

```
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 24850273.21919080441727298772802245902434]: target and source on the
different branches.
```

Clockwise ray.

Ray outgoing at target.

Solve Side.

```
Tau [10510.45813678925322818606134583815945501,  
10510.36930058668129353201123910284808332,  
10510.39514076171935929900237070393559133, none, none,  
10510.33729983836226253676391229027615267, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none]
```

```
0 --> 1 target = [24850273.21919080489595851491921835475879,
1.476968812080919893659633542798301739393,
10510.36930388278672996800633310908564384]
```

Imaginary part neglected:



-.5253186706111498631583399819990125295017e-15  
one interval r = 23873939.27584648141024332068423190886771 ..  
59722486789234357059632613243417091637/2500000000000000000000000000000000  
Time Approximations 0.045.

hint used Hint := [23888989.43146328644175807501347600997649, 3, -1, 1,  
20396631.10527514558221865082982151479995,  
23888964.97298890387184335817087833215779 ..  
597224867892343544545306555277700499657/2500000000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.466972) | S --> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=  
-5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888989.43146328644175807501347600997649,  
rm=20396631.10527514558221865082982151479995}, {r =  
23888964.97298890387184335817087833215779 ..  
23888994.71569374178181226221110801998628, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={});  
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=6e-32  
Equations at solution: [-.11e-30, -.6e-31, .26e-34]  
Solution in 0.392s

Time Plot 0 s.  
Exiting SolveHard() after 2.11r=2.38890e+07 in  
[23888964.97298890387184335817087833215779 ..  
597224867892343544545306555277700499657/2500000000000000000000000000000000  
0]  
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678925322818606134583815945501,  
10510.36930058668129353201123910284808332,  
10510.39514076171935929900237070393559133,  
10510.28046624342936361019930770340868976, none,  
10510.33729983836226253676391229027615267, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none]

0 --> 2 target = [25795903.79985511660721390853159521027593,  
1.856318223502859020772884450582716707364,  
10510.39514387999687282440884009404747370]  
Imaginary part neglected:  
-.5253186706111498631583399819990125295017e-15  
one interval r = 23873940.85057848648181639680254190378058 ..  
2388899625275549533400807317257624900417/1000000000000000000000000000000000  
000  
Time Approximations 0.04.

hint used Hint := [23888992.50558472677941562199764206716244, 3, -1, 1,  
22860451.38218918783573792125602312359251,  
23888966.51012875395781144697926248654402 ..  
597224906318887365922168983423648563467/2500000000000000000000000000000000















. 25089000]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678925322818606134583815945501,  
10510.36930058668129353201123910284808332,  
10510.39514076171935929900237070393559133,  
10510.28046624342936361019930770340868976,  
10510.36313884705457368990712050570285365,  
10510.33729983836226253676391229027615267,  
10510.33214580441643400104963210420665049,  
10510.30529788705245457789572428826882783, none,  
10510.27430496991934876960483486929755874, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none]

1 --> 0 target = [23888989.43146328852481409930957941299334,  
2.632526185627945423147178579073664062941,  
10510.28046964802071352054353185680470188]  
Imaginary part neglected:  
-.1477780416846654559580033374526473400447e-14  
one interval r = 24835338.73249666740013227144658539652425 ..  
497005359225697138220438235623995705407/2000000000000000000000000000000000  
0  
Time Approximations 0.882.

hint used Hint := [24850262.70323078644925875505002433594091, 3, 1, 1,  
20396437.89679703248883954261208042438711,  
24850238.36672641289161845524403523557614 ..  
2485026796128485539651696969285982263711/1000000000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
23888989.43146328644175807501347600997649, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.533028) | P <--- S  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=  
5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850262.70323078644925875505002433594091,  
rm=20396437.89679703248883954261208042438711}, {r =  
24850238.36672641289161845524403523557614 ..  
24850267.96128485539651696969285982263711, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.43146328644175807501347600997649}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=8e-32  
Equations at solution: [.14e-30, .8e-31, -.54e-34]  
Solution in 0.368s

Time Plot 0 s.  
Exiting SolveHard() after 1.886r=2.48503e+07 in  
[24850238.36672641289161845524403523557614 ..  
2485026796128485539651696969285982263711/1000000000000000000000000000000000  
000]  
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888989.43146328644175807501347600997649]: target and source on the  
different branches.  
Counterclockwise ray.



Ray outgoing at target.  
Solve Side.

Tau [10510.45813678925322818606134583815945501,  
10510.36930058668129353201123910284808332,  
10510.39514076171935929900237070393559133,  
10510.28046624342936361019930770340868976,  
10510.36313884705457368990712050570285365,  
10510.33729983836226253676391229027615267,  
10510.33214580441643400104963210420665049,  
10510.30529788705245457789572428826882783, none,  
10510.27430496991934876960483486929755874,  
10510.19163019563564197207652302246685312, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none]

2 --> 0 target = [23888989.43146328852481409930957941299334,  
2.632526185627945423147178579073664062941,  
10510.28046964802071352054353185680470188]  
one interval r = 25781103.42398366889101190698468646063829 ..  
1289794852506034747399147729862888083939/500000000000000000000000000000  
00  
Time Approximations 0.035.

hint used Hint := [25795893.34215400624221088590056918107762, 3, 1, 1,  
22860366.17838899487248578273534549876757,  
25795867.61967723367271871049090258979880 ..  
1289794852506034629127592665641586312049/500000000000000000000000000000  
00, 298960418182500/22468879468420441 ..  
23888989.43146328644175807501347600997649, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.632821) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31731e+14 scos=4.39725e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795893.34215400624221088590056918107762,  
rm=22860366.17838899487248578273534549876757}, {r =  
25795867.61967723367271871049090258979880 ..  
25795897.05012069258255185331283172624098, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.43146328644175807501347600997649}, avoid={});  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1.8e-31  
Equations at solution: [.18e-30, .18e-30, -.62e-34]  
Solution in 0.255s

Time Plot 0 s.  
Exiting SolveHard() after 0.83r=2.57959e+07 in  
[25795867.61967723367271871049090258979880 ..  
1289794852506034629127592665641586312049/500000000000000000000000000000  
00]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23888989.43146328644175807501347600997649]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.







```
Tau [10510.45813678925322818606134583815945501,
10510.36930058668129353201123910284808332,
10510.39514076171935929900237070393559133,
10510.28046624342936361019930770340868976,
10510.36313884705457368990712050570285365,
10510.33729983836226253676391229027615267,
10510.33214580441643400104963210420665049,
10510.30529788705245457789572428826882783,
10510.21747048884558963642866965835747745,
10510.27430496991934876960483486929755874,
10510.19163019563564197207652302246685312, none, none,
10510.24330971160183662661610206978386352, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0 target = [23888992.50558472817262174131652463960891,
2.632534984172955674424181693800343134688,
10510.33214920902213548136369169079131108]
one interval r = 25781106.49760329800573540964019484105243 ..
2579590009198804522262052436569180216921/1000000000000000000000000000000
000
Time Approximations 0.034.
```

```
hint used Hint := [25795896.38402523646204355452114396262515, 3, 1, 1,
22860364.90258708217946452135489353704915,
25795870.66161245800881604810046494170563 ..
2579590009198804353979547140796504033263/1000000000000000000000000000000
000, 298960418182500/22468879468420441 ..
23888992.50558472677941562199764206716244, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.63282) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=
4.31731e+14 scos=4.39724e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795896.38402523646204355452114396262515,
rm=22860364.90258708217946452135489353704915}, {r =
25795870.66161245800881604810046494170563 ..
25795900.09198804353979547140796504033263, rm =
.1330553304194287328500223794129351168576e-1 ..
23888992.50558472677941562199764206716244}, avoid={}));
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=2.2e-31
Equations at solution: [-.23e-30, -.22e-30, .91e-34]
Solution in 1.051s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.63r=2.57959e+07 in
[25795870.66161245800881604810046494170563 ..
2579590009198804353979547140796504033263/1000000000000000000000000000000
000]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23888992.50558472677941562199764206716244]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678925322818606134583815945501,
```







[illegible]

```
Time Plot 0 s.  
Exiting SolveHard() after 2.149r=2.38890e+07 in  
[23888964.60644587167575168019887114912430 ..  
1194449717458466618404658405105548818961/500000000000000000000000000000  
00]  
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.
```

Tau [10510.45813678925322818606134583815945501,  
10510.36930058668129353201123910284808332,







```
Tau [10510.45813678925322818606134583815945501,  
10510.36930058668129353201123910284808332,  
10510.39514076171935929900237070393559133,  
10510.28046624342936361019930770340868976,  
10510.36313884705457368990712050570285365,  
10510.33729983836226253676391229027615267,  
10510.33214580441643400104963210420665049,  
10510.30529788705245457789572428826882783,  
10510.21747048884558963642866965835747745,  
10510.27430496991934876960483486929755874,  
10510.19163019563564197207652302246685312,  
10510.26914997043878827987189961234110660,  
10510.27430450917054079192202572830997020,  
10510.24330971160183662661610206978386352,  
10510.33113809483177409056895333509028460, none,  
10510.18546892749305166376885518470432254, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none]  
  
2 --> 0 target = [23888989.06496329710630348268248640997677,  
2.632525136659086737301021094613488234469,  
10510.27430837450898767554601161272319441]  
one interval r = 25781103.05754417321876963322474340285547 ..  
322448708593333346471681985134529133603/1250000000000000000000000000  
0  
Time Approximations 0.029.  
  
hint used Hint := [25795892.97949951637755150436992208534955, 3, 1, 1,  
22860366.33048982198263378361971969356546,  
25795867.25701511448801980095492062593890 ..  
644897417186666629737809369304875630137/2500000000000000000000000000  
0, 298960418182500/22468879468420441 ..  
23888989.06496329485872911197089488868350, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.632821) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31731e+14 scos=4.39725e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795892.97949951637755150436992208534955,  
rm=22860366.33048982198263378361971969356546}, {r =  
25795867.25701511448801980095492062593890 ..  
25795896.68746666518951237477219502520548, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.06496329485872911197089488868350}, avoid={}));  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1.3e-31  
Equations at solution: [-.13e-30, -.13e-30, .61e-34]  
Solution in 0.22s  
  
Time Plot 0 s.  
Exiting SolveHard() after 0.714r=2.57959e+07 in  
[25795867.25701511448801980095492062593890 ..  
644897417186666629737809369304875630137/2500000000000000000000000000  
0]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23888989.06496329485872911197089488868350]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.
```



Solve Side.

```
Tau [10510.45813678925322818606134583815945501,
10510.36930058668129353201123910284808332,
10510.39514076171935929900237070393559133,
10510.28046624342936361019930770340868976,
10510.36313884705457368990712050570285365,
10510.33729983836226253676391229027615267,
10510.33214580441643400104963210420665049,
10510.30529788705245457789572428826882783,
10510.21747048884558963642866965835747745,
10510.27430496991934876960483486929755874,
10510.19163019563564197207652302246685312,
10510.26914997043878827987189961234110660,
10510.27430450917054079192202572830997020,
10510.24330971160183662661610206978386352,
10510.33113809483177409056895333509028460, none,
10510.18546892749305166376885518470432254,
10510.21130922480097140194592369127918077, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
2 --> 1 target = [24850269.43101310010491768474374401227019,  
1.476958531752689819495282703855785504779,  
10510.30530118314079390061162234971338123]  
one interval r = 25781104.90082928623664044695278660523013 ..  
257958985117097589952094339701959171361/1000000000000000000000000000  
0
```

Time Approximations 0.028.

```
hint used Hint := [25795896.62812623062600675830146784169746, 3, -1, 1,  
24743530.75497470617154487139175541092326,  
25795869.08129891002682354003571513818579 ..  
2579589851170975665166085604166659164517/1000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
24850269.43101309861241733616879334665430, 1]
```

```
I search for an scattering ray on opposite branches with 0<sv<1
(0.760171) | P <--- S
rGuessMin=2.57811e+07    rGuessMax=2.57959e+07    rmGuess=2.47435e+07    k=
-2.37384e+14    scos=5.95462e+14
```

```
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25795896.62812623062600675830146784169746,
rm=24743530.75497470617154487139175541092326}, {r =
25795869.08129891002682354003571513818579 ..
25795898.51170975665166085604166659164517, rm =
.1330553304194287328500223794129351168576e-1 ..
24850269.43101309861241733616879334665430}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [0., 0., .6e-35]
Solution in 0.284s
```

Time Plot 0 s.

```
Exiting SolveHard() after 1.709r=2.57959e+07 in  
[25795869.08129891002682354003571513818579 ..  
2579589851170975665166085604166659164517/10000000000000000000000000000000  
000]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 24850269.43101309861241733616879334665430]: target and source on the
```



different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

```
Tau [10510.45813678925322818606134583815945501,
10510.36930058668129353201123910284808332,
10510.39514076171935929900237070393559133,
10510.28046624342936361019930770340868976,
10510.36313884705457368990712050570285365,
10510.33729983836226253676391229027615267,
10510.33214580441643400104963210420665049,
10510.30529788705245457789572428826882783,
10510.21747048884558963642866965835747745,
10510.27430496991934876960483486929755874,
10510.19163019563564197207652302246685312,
10510.26914997043878827987189961234110660,
10510.27430450917054079192202572830997020,
10510.24330971160183662661610206978386352,
10510.33113809483177409056895333509028460, none,
10510.18546892749305166376885518470432254,
10510.21130922480097140194592369127918077, none, none,
10510.27329709818453234810768809811652638, none, none, none, none,
none, none, none, none, none, none]
```

[illegible]

```
hint used Hint := [23888985.62428976346864819081258538077918, 3, -1, 1,  
20396632.00572718336484608918971225516342,  
23888961.16565299817948948119449178210506 ..  
2388899090855127150824991682392263419777/100000000000000000000000000  
000, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.466972) | S ---> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=  
-5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888985.62428976346864819081258538077918,  
rm=20396632.00572718336484608918971225516342}, {r =  
23888961.16565299817948948119449178210506 ..  
23888990.90855127150824991682392263419777, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));  
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=7e-32  
Equations at solution: [.12e-30, .7e-31, -.41e-34]  
Solution in 1.16s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 1.952r=2.38890e+07 in  
[23888961.16565299817948948119449178210506 ..
```







Time Plot 0 s.  
Exiting SolveHard() after 0.895r=2.57959e+07 in  
[25795862.39063358446748128489577933480278 ..  
515917836423874243616569030246719914989/2000000000000000000000000000000000  
0]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 24850262.70323078644925875505002433594091]: target and source on the  
different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678925322818606134583815945501,  
10510.36930058668129353201123910284808332,  
10510.39514076171935929900237070393559133,  
10510.28046624342936361019930770340868976,  
10510.36313884705457368990712050570285365,  
10510.33729983836226253676391229027615267,  
10510.33214580441643400104963210420665049,  
10510.30529788705245457789572428826882783,  
10510.21747048884558963642866965835747745,  
10510.27430496991934876960483486929755874,  
10510.19163019563564197207652302246685312,  
10510.26914997043878827987189961234110660,  
10510.27430450917054079192202572830997020,  
10510.24330971160183662661610206978386352,  
10510.33113809483177409056895333509028460,  
10510.21646359955753277369403545570735710,  
10510.18546892749305166376885518470432254,  
10510.21130922480097140194592369127918077, none, none,  
10510.27329709818453234810768809811652638, none, none,  
10510.15962933475360501051603164586404453, none, none, none, none,  
none, none, none]

0 --> 1 target = [24850262.70323078899960925380052838157496,  
1.476940274059214529755724311742204940403,  
10510.19163349169361710770244135162422226]  
Imaginary part neglected:  
-.5253186706111498631583399819990125295017e-15  
one interval r = 23873928.44821859069093095969592069835588 ..  
2388898414707082653118476515043211006911/1000000000000000000000000000000000  
000  
Time Approximations 0.039.

hint used Hint := [23888978.86275416794827744619407470166172, 3, -1, 1,  
20396633.60486737240849086583158368073964,  
23888954.40382902066202638611670198664112 ..  
298612301838385292588525592212900916387/1250000000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.466972) | S --> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=  
-5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888978.86275416794827744619407470166172,  
rm=20396633.60486737240849086583158368073964}, {r =  
23888954.40382902066202638611670198664112 ..















one interval r = 25781101.21410471461537275799816830637262 ..  
515917897261409527372851898952851590157/2000000000000000000000000000000000  
0  
Time Approximations 0.029.

hint used Hint := [25795892.97947942498571214559240875822627, 3, -1, 1,  
24743526.20943107487634918939497374544011,  
25795865.43257821616570179673528437159660 ..  
644897371576761841392917646201273823237/2500000000000000000000000000000000  
0, 298960418182500/22468879468420441 ..  
24850265.76205359199870736703593497152527, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.76017) | P <-- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=  
-2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=25795892.97947942498571214559240875822627,  
rm=24743526.20943107487634918939497374544011}, {r =  
25795865.43257821616570179673528437159660 ..  
25795894.86307047365571670584805095292948, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850265.76205359199870736703593497152527}, avoid={}));  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.7423e-31, 0., -.4e-35]  
Solution in 0.29s

Time Plot 0 s.  
Exiting SolveHard() after 0.788r=2.57959e+07 in  
[25795865.43257821616570179673528437159660 ..  
644897371576761841392917646201273823237/2500000000000000000000000000000000  
0]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 24850265.76205359199870736703593497152527]: target and source on the  
different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678925322818606134583815945501,  
10510.36930058668129353201123910284808332,  
10510.39514076171935929900237070393559133,  
10510.28046624342936361019930770340868976,  
10510.36313884705457368990712050570285365,  
10510.33729983836226253676391229027615267,  
10510.33214580441643400104963210420665049,  
10510.30529788705245457789572428826882783,  
10510.21747048884558963642866965835747745,  
10510.27430496991934876960483486929755874,  
10510.19163019563564197207652302246685312,  
10510.26914997043878827987189961234110660,  
10510.27430450917054079192202572830997020,  
10510.24330971160183662661610206978386352,  
10510.33113809483177409056895333509028460,  
10510.21646359955753277369403545570735710,  
10510.18546892749305166376885518470432254,  
10510.21130922480097140194592369127918077,  
10510.10279600716444066559974875731644243,



```

10510.18546846161811151585560850058018054,
10510.27329709818453234810768809811652638,
10510.15447580449464660344246913372789687, none,
10510.15962933475360501051603164586404453, none, none,
10510.21130888346133101207065340085673354, none, none, none, none]

0 --> 1 target = [24850265.76205359386264302984536726444897,
1.476948575001100562479000632560181837575,
10510.24331300767361697484684790181579458]
Imaginary part neglected:
-.5253186706111498631583399819990125295017e-15
one interval r = 23873931.59770125442736393731650794825103 ..
2388898722121734030202849581452204854219/1000000000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [23888981.93692575516559285951162594064654, 3, -1, 1,
20396632.87781872303409295730403849988110,
23888957.47813172092635353311983299926098 ..
597224680530433446694670341991304966889/25000000000000000000000000000000000000
0, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=
-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888981.93692575516559285951162594064654,
rm=20396632.87781872303409295730403849988110}, {r =
23888957.47813172092635353311983299926098 ..
23888987.22121733786778681367965219867556, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1.2e-31
Equations at solution: [.22e-30, .12e-30, -.84e-34]
Solution in 0.399s

Time Plot 0 s.
Exiting SolveHard() after 2.062r=2.38890e+07 in
[23888957.47813172092635353311983299926098 ..
597224680530433446694670341991304966889/25000000000000000000000000000000000000
0]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678925322818606134583815945501,
10510.36930058668129353201123910284808332,
10510.39514076171935929900237070393559133,
10510.28046624342936361019930770340868976,
10510.36313884705457368990712050570285365,
10510.33729983836226253676391229027615267,
10510.33214580441643400104963210420665049,
10510.30529788705245457789572428826882783,
10510.21747048884558963642866965835747745,
10510.27430496991934876960483486929755874,
10510.19163019563564197207652302246685312,

```



```

10510.26914997043878827987189961234110660,  

10510.27430450917054079192202572830997020,  

10510.24330971160183662661610206978386352,  

10510.33113809483177409056895333509028460,  

10510.21646359955753277369403545570735710,  

10510.18546892749305166376885518470432254,  

10510.21130922480097140194592369127918077,  

10510.10279600716444066559974875731644243,  

10510.18546846161811151585560850058018054,  

10510.27329709818453234810768809811652638,  

10510.15447580449464660344246913372789687, none,  

10510.15962933475360501051603164586404453,  

10510.15447547810902786573987438251388024, none,  

10510.21130888346133101207065340085673354, none, none, none, none]  

0 --> 2 target = [25795896.38402523883366468301317006532146,  

1.856299066190041211930554028653060743203,  

10510.26915308868385383250239826714603417]  

Imaginary part neglected:  

-.5253186706111498631583399819990125295017e-15  

one interval r = 23873933.17245664798745597604524245662855 ..  

597224718957593943188118105615058535417/2500000000000000000000000000000000  

0  

Time Approximations 0.039.  

hint used Hint := [23888985.01109652402578398653593278001387, 3, -1, 1,  

22860454.49191911041368999760273437756943,  

23888959.01529623342321627788824672003212 ..  

59722471895759389095528762418719523933/2500000000000000000000000000000000,  

298960418182500/22468879468420441 .. 24089000, 1]  

I search for an scattering ray on opposite branches with 0<sv<1  

(0.367179) | S ---> P  

rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=  

-4.31731e+14 scos=4.39725e+14  

branch outgoing at target, Clockwise  

(Scattering) fsolve(eqs, {r=23888985.01109652402578398653593278001387,  

rm=22860454.49191911041368999760273437756943}, {r =  

23888959.01529623342321627788824672003212 ..  

23888988.75830375563821150496748780957320, rm =  

.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}));  

Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.1e-31  

Equations at solution: [-.12e-30, -.11e-30, .37e-34]  

Solution in 0.343s  

Time Plot 0 s.  

Exiting SolveHard() after 2.087r=2.38890e+07 in  

[23888959.01529623342321627788824672003212 ..  

59722471895759389095528762418719523933/2500000000000000000000000000000000]  

Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 ..  

. 24089000]: target and source on the different branches.  

Clockwise ray.  

Ray outgoing at target.  

Solve Side.  

Tau [10510.45813678925322818606134583815945501,  

10510.36930058668129353201123910284808332,  

10510.39514076171935929900237070393559133,
```



























Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678925322818606134583815945501,  
10510.36930058668129353201123910284808332,  
10510.39514076171935929900237070393559133,  
10510.28046624342936361019930770340868976,  
10510.36313884705457368990712050570285365,  
10510.33729983836226253676391229027615267,  
10510.33214580441643400104963210420665049,  
10510.30529788705245457789572428826882783,  
10510.21747048884558963642866965835747745,  
10510.27430496991934876960483486929755874,  
10510.19163019563564197207652302246685312,  
10510.26914997043878827987189961234110660,  
10510.27430450917054079192202572830997020,  
10510.24330971160183662661610206978386352,  
10510.33113809483177409056895333509028460,  
10510.21646359955753277369403545570735710,  
10510.18546892749305166376885518470432254,  
10510.21130922480097140194592369127918077,  
10510.10279600716444066559974875731644243,  
10510.18546846161811151585560850058018054,  
10510.27329709818453234810768809811652638,  
10510.15447580449464660344246913372789687,  
10510.23714797595275033026776732159886323,  
10510.15962933475360501051603164586404453,  
10510.15447547810902786573987438251388024,  
10510.29913613961820393647301750097163196,  
10510.21130888346133101207065340085673354,  
10510.20615520669342452176424343901347893,  
10510.18546846674464506976991561268462549,  
10510.21130876405287126013006169139318814,  
10510.26814323585504438815448265081177238]

```
Cascade time 48.147
counts: 28, 28
```

## Iteration 5

```
Start Generation 1  
1 --> 0   target = [23889000.00000000015249914394173842228344,  
2.632556434680728693764692937115665254874,  
10510.45814019403260272106689752860471625]  
one interval r = 24835349.43352072274321240269846032264326 ..  
621256961929631413776474608542854469743/250000000000000000000000000000  
0
```

Time Approximations 0.037.

```

hint used Hint := [24850273.21919080441727298772802245902434, 3, 1, 1,
20396435.39659592643000634328381896732011,
24850248.88301387793152980010731189259064 ..
621256961929631488519759813441433737669/25000000000000000000000000000000
0, 298960418182500/22468879468420441 .. 23889000., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.533028) |      P  <--- S

```



```
rGuessMin=2.48353e+07    rGuessMax=2.48503e+07    rmGuess=2.03964e+07    k=
5.43204e+14    scos=2.39474e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850273.21919080441727298772802245902434,
rm=20396435.39659592643000634328381896732011}, {r =
24850248.88301387793152980010731189259064 ..
24850278.47718525954079039253765734950676, rm =
.1330553304194287328500223794129351168576e-1 .. 23889000.}), avoid={}));
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=6e-32
Equations at solution: [-.10e-30, -.6e-31, .10996433726e-34]
Solution in 0.39s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.083r=2.48503e+07 in
[24850248.88301387793152980010731189259064 ..
621256961929631488519759813441433737669/2500000000000000000000000000000000
0]
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .
. 23889000.]: target and source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678918192452608718760264360075,
10510.36930058688480355785706375052275675, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 0 target = [23889000.00000000015249914394173842228344,
2.632556434680728693764692937115665254874,
10510.45814019403260272106689752860471625]
Imaginary part neglected:
-.5924510391192590374527634437975863507978e-15
one interval r = 25781113.99083725804534328465549944725246 ..
2579590750780847379462342617042850581171/1000000000000000000000000000000000
000
Time Approximations 0.031.
```

```
hint used Hint := [25795903.79985511561316781642113695093491, 3, 1, 1,
22860361.79220184767639985483105161990097,
25795878.07759835662287683941521694813384 ..
161224421923802901739910205661944662513/6250000000000000000000000000000000
, 298960418182500/22468879468420441 .. 23889000., 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.63282) | P <--- S
rGuessMin=2.57811e+07    rGuessMax=2.57959e+07    rmGuess=2.28604e+07    k=
4.31733e+14    scos=4.39724e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795903.79985511561316781642113695093491,
rm=22860361.79220184767639985483105161990097}, {r =
25795878.07759835662287683941521694813384 ..
25795907.50780846427838563290591114600208, rm =
.1330553304194287328500223794129351168576e-1 .. 23889000.}), avoid={}));
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=2.0e-31
Equations at solution: [-.22e-30, -.20e-30, .8e-34]
Solution in 0.292s
```







Clockwise ray.  
Ray outgoing at target.  
Solve Side.

```
Tau [10510.45813678918192452608718760264360075,  
10510.36930058688480355785706375052275675,  
10510.39514076190213764159560681497762278, none, none,  
10510.33729983854857721313045262806652442, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none]
```

```
0 --> 1 target = [24850273.21919080084340316299852942141956,  
1.476968811987539337548895852844488931969,  
10510.36930388289804534991660059860698046]  
one interval r = 23873939.27584647989402208324489801657867 ..  
2388899471569374134727141762489515843899/1000000000000000000000000000000000000  
000  
Time Approximations 0.044.
```

```
hint used Hint := [23888989.43146328644175807501347600997649, 3, -1, 1,  
20396631.10527514558221865082982151479995,  
23888964.97298890387184335817087833215779 ..  
597224867892343544545306555277700499657/2500000000000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.466972) | S ---> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=  
-5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888989.43146328644175807501347600997649,  
rm=20396631.10527514558221865082982151479995}, {r =  
23888964.97298890387184335817087833215779 ..  
23888994.71569374178181226221110801998628, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={});  
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=2e-32  
Equations at solution: [.3e-31, .2e-31, -.31e-34]  
Solution in 0.366s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 2.148r=2.38890e+07 in  
[23888964.97298890387184335817087833215779 ..  
597224867892343544545306555277700499657/2500000000000000000000000000000000000  
0]  
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678918192452608718760264360075,  
10510.36930058688480355785706375052275675,  
10510.39514076190213764159560681497762278,  
10510.28046624330332111411460936353971213, none,  
10510.33729983854857721313045262806652442, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none]
```











```

(Scattering) fsolve(eqs, {r=23888989.06496329485872911197089488868350,
rm=22860452.80984339454018821965990898965218}, {r =
23888963.06934924828335479031706673264809 ..
23888992.81215080305602286687243794493588, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}));
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=3.5e-31
Equations at solution: [.36e-30, .35e-30, -.145e-33]
Solution in 0.443s

Time Plot 0 s.
Exiting SolveHard() after 2.198r=2.38890e+07 in
[23888963.06934924828335479031706673264809 ..
597224820303770076400571671810948623397/2500000000000000000000000000000000
0]
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678918192452608718760264360075,
10510.36930058688480355785706375052275675,
10510.39514076190213764159560681497762278,
10510.28046624330332111411460936353971213,
10510.36313884725478644946357276183466111,
10510.33729983854857721313045262806652442,
10510.33214580428355820921906585999872256, none, none,
10510.27430496979000930419279020638252749, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]

1 --> 2 target = [25795900.39533808865595098688523665790576,
1.856309428507396102023070450568965497941,
10510.33730295673631448780354580427698008]
one interval r = 24835342.15555717029304250928909848570652 ..
497005426502585103772997690992264946617/2000000000000000000000000000000000
0
Time Approximations 0.038.

hint used Hint := [24850269.43101309861241733616879334665430, 3, 1, 1,
24743514.60014700975744022125022662330031,
24850241.73069462631526237165506015616915 ..
1242513566256462912427735010225393921567/5000000000000000000000000000000000
00, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.239829) | S --> P
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=
2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850269.43101309861241733616879334665430,
rm=24743514.60014700975744022125022662330031}, {r =
24850241.73069462631526237165506015616915 ..
24850271.32512925824855470020450787843134, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}), avoid={}));
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.2474e-31, 0., -.23962107620e-34]
Solution in 0.3s

```











. 23888989.43146328644175807501347600997649]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678918192452608718760264360075,  
10510.36930058688480355785706375052275675,  
10510.39514076190213764159560681497762278,  
10510.28046624330332111411460936353971213,  
10510.36313884725478644946357276183466111,  
10510.33729983854857721313045262806652442,  
10510.33214580428355820921906585999872256,  
10510.30529788725620367344474057250350901,  
10510.21747048897362922109921724343201929,  
10510.27430496979000930419279020638252749,  
10510.19163019578441320614224430724738289, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none]

1 --> 0 target = [23888992.50558472466152679573183151493242,  
2.632534984481850805353844360687384907406,  
10510.33214920909924833800771960353635018]  
one interval r = 24835341.84514835661405750009352405379759 ..  
62125677550225790278620037049432190141/25000000000000000000000000000000  
Time Approximations 0.034.

hint used Hint := [24850265.76205359199870736703593497152527, 3, 1, 1,  
20396437.16957127706091945345483725838621,  
24850241.42564446195929857082855634229261 ..  
1242513551004516068014443586172817582043/50000000000000000000000000000000  
00, 298960418182500/22468879468420441 ..  
23888992.50558472677941562199764206716244, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.533028) | P <--- S  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=  
5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850265.76205359199870736703593497152527,  
rm=20396437.16957127706091945345483725838621}, {r =  
24850241.42564446195929857082855634229261 ..  
24850271.02009032136028887172345635164086, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888992.50558472677941562199764206716244}, avoid={});  
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=1.1e-31  
Equations at solution: [.19e-30, .11e-30, -.70017046881e-34]  
Solution in 0.355s

Time Plot 0 s.  
Exiting SolveHard() after 1.952r=2.48503e+07 in  
[24850241.42564446195929857082855634229261 ..  
1242513551004516068014443586172817582043/50000000000000000000000000000000  
00]  
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888992.50558472677941562199764206716244]: target and source on the  
different branches.  
Counterclockwise ray.



Ray outgoing at target.  
Solve Side.

```
Tau [10510.45813678918192452608718760264360075,
10510.36930058688480355785706375052275675,
10510.39514076190213764159560681497762278,
10510.28046624330332111411460936353971213,
10510.36313884725478644946357276183466111,
10510.33729983854857721313045262806652442,
10510.33214580428355820921906585999872256,
10510.30529788725620367344474057250350901,
10510.21747048897362922109921724343201929,
10510.27430496979000930419279020638252749,
10510.19163019578441320614224430724738289, none, none,
10510.24330971174377457186476606275847920, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]
```

[illegible]

```
hint used Hint := [25795896.38402523646204355452114396262515, 3, 1, 1,  
22860364.90258708217946452135489353704915,  
25795870.66161245800881604810046494170563 ..  
2579590009198804353979547140796504033263/100000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
23888992.50558472677941562199764206716244, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.63282) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31731e+14 scos=4.39724e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795896.38402523646204355452114396262515,  
rm=22860364.90258708217946452135489353704915}, {r =  
25795870.66161245800881604810046494170563 ..  
25795900.09198804353979547140796504033263, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888992.50558472677941562199764206716244}, avoid={{}});  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1.9e-31  
Equations at solution: [-.20e-30, -.19e-30, .6e-34]  
Solution in 0.242s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 0.799r=2.57959e+07 in  
[25795870.66161245800881604810046494170563 ..  
2579590009198804353979547140796504033263/1000000000000000000000000000000000]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23888992.50558472677941562199764206716244]: target and source on the  
different branches.
```



Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

```
Tau [10510.45813678918192452608718760264360075,  
10510.36930058688480355785706375052275675,  
10510.39514076190213764159560681497762278,  
10510.28046624330332111411460936353971213,  
10510.36313884725478644946357276183466111,  
10510.33729983854857721313045262806652442,  
10510.33214580428355820921906585999872256,  
10510.30529788725620367344474057250350901,  
10510.21747048897362922109921724343201929,  
10510.27430496979000930419279020638252749,  
10510.19163019578441320614224430724738289,  
10510.26914997055999458101216780248035780, none,  
10510.24330971174377457186476606275847920, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none]
```

[illegible]

```
hint used Hint := [25795900.03265730627104171495990540839944, 3, -1, 1,  
24743534.99638995296918328323923452726440,  
25795872.48589893272346549352544011957533 ..  
644897547905845320097763426011396162913/250000000000000000000000000000  
0, 298960418182500/22468879468420441 ..  
24850272.85449189132526402658550193546657, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.760172) | P <--- S  
rGuessMin=2.57811e+07    rGuessMax=2.57959e+07    rmGuess=2.47435e+07    k=  
-2.37383e+14    scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=25795900.03265730627104171495990540839944,  
rm=24743534.99638995296918328323923452726440}, {r =  
25795872.48589893272346549352544011957533 ..  
25795901.91623381280391053704045584651652, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850272.85449189132526402658550193546657}, avoid={{}});  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.9898e-31, 0., -.5e-34]  
Solution in 0.293s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 1.758r=2.57959e+07 in  
[25795872.48589893272346549352544011957533 ..  
644897547905845320097763426011396162913/2500000000000000000000000000000000  
0]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 ..
```



. 24850272.85449189132526402658550193546657]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678918192452608718760264360075,  
10510.36930058688480355785706375052275675,  
10510.39514076190213764159560681497762278,  
10510.28046624330332111411460936353971213,  
10510.36313884725478644946357276183466111,  
10510.33729983854857721313045262806652442,  
10510.33214580428355820921906585999872256,  
10510.30529788725620367344474057250350901,  
10510.21747048897362922109921724343201929,  
10510.27430496979000930419279020638252749,  
10510.19163019578441320614224430724738289,  
10510.26914997055999458101216780248035780, none,  
10510.24330971174377457186476606275847920,  
10510.33113809501479149855488296623598052, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none]

0 --> 1 target = [24850272.85449188771994595522792240092675,  
1.476967822268394589757102209584908836422,  
10510.36314214326638224922332096398783675]  
one interval r = 23873938.90033936074455755263743807271462 ..  
2388899434916933190194810404573096232867/1000000000000000000000000000000000000  
000

Time Approximations 0.045.

hint used Hint := [23888989.06493588745880515057802801897934, 3, -1, 1,  
20396631.19196530471597662319541189563767,  
23888964.60644587167575168019887114912430 ..  
1194449717458466618404658405105548818961/500000000000000000000000000000000000  
00, 298960418182500/22468879468420441 .. 24089000, 1]

I search for an scattering ray on opposite branches with  $0 < sv < 1$

(0.466972) | S --> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=  
-5.43203e+14 scos=2.39475e+14

branch outgoing at target, Clockwise

(Scattering) fsolve(eqs, {r=23888989.06493588745880515057802801897934,  
rm=20396631.19196530471597662319541189563767}, {r =

23888964.60644587167575168019887114912430 ..

23888994.34916933236809316810211097637922, rm =

.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));

Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=3e-32

Equations at solution: [-.6e-31, -.3e-31, .6e-35]

Solution in 1.166s

Time Plot 0 s.

Exiting SolveHard() after 2.079r=2.38890e+07 in

[23888964.60644587167575168019887114912430 ..

1194449717458466618404658405105548818961/500000000000000000000000000000000000  
00]

Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.

Clockwise ray.







Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

```
Tau [10510.45813678918192452608718760264360075,
10510.36930058688480355785706375052275675,
10510.39514076190213764159560681497762278,
10510.28046624330332111411460936353971213,
10510.36313884725478644946357276183466111,
10510.33729983854857721313045262806652442,
10510.33214580428355820921906585999872256,
10510.30529788725620367344474057250350901,
10510.21747048897362922109921724343201929,
10510.27430496979000930419279020638252749,
10510.19163019578441320614224430724738289,
10510.26914997055999458101216780248035780,
10510.27430450904120103230410027018028030,
10510.24330971174377457186476606275847920,
10510.33113809501479149855488296623598052, none,
10510.18546892763852593126304843357867968, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

```
2 --> 0 target = [23888989.06496329380554360184548475643147,
2.632525136967982470294273058432369073793,
10510.27430837458963685860954315221041663]
Imaginary part neglected:
-.5924510391192590374527634437975863507978e-15
one interval r = 25781103.05754417907575604840008469125413 ..
161224354296666709454946206725924637757/62500000000000000000000000000000
Time Approximations 0.03.
```

```

hint used Hint := [25795892.97949951637755150436992208534955, 3, 1, 1,
22860366.33048982198263378361971969356546,
25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/2500000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
23888989.06496329485872911197089488868350, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=
4.31731e+14 scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795892.97949951637755150436992208534955,
rm=22860366.33048982198263378361971969356546}, {r =
25795867.25701511448801980095492062593890 ..
25795896.68746666518951237477219502520548, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06496329485872911197089488868350}, avoid={{}});
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1.9e-31
Equations at solution: [.19e-30, .19e-30, -.6e-34]
Solution in 0.279s

```

```
Time Plot 0 s.
Exiting SolveHard() after 1.77r=2.57959e+07 in
[25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/2500000000000000000000000000000000
0]
```



Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23888989.06496329485872911197089488868350]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678918192452608718760264360075,  
10510.36930058688480355785706375052275675,  
10510.39514076190213764159560681497762278,  
10510.28046624330332111411460936353971213,  
10510.36313884725478644946357276183466111,  
10510.33729983854857721313045262806652442,  
10510.33214580428355820921906585999872256,  
10510.30529788725620367344474057250350901,  
10510.21747048897362922109921724343201929,  
10510.27430496979000930419279020638252749,  
10510.19163019578441320614224430724738289,  
10510.26914997055999458101216780248035780,  
10510.27430450904120103230410027018028030,  
10510.24330971174377457186476606275847920,  
10510.33113809501479149855488296623598052, none,  
10510.18546892763852593126304843357867968,  
10510.21130922492571402214937475932521987, none, none, none, none,  
none, none, none, none, none, none, none, none]

2 --> 1 target = [24850269.43101309606649312721734589799840,  
1.476958531659309301781390901840192976433,  
10510.30530118325234835222514533505196445]  
Imaginary part neglected:  
-.5924510391192590374527634437975863507978e-15  
one interval r = 25781104.90082929393169730204602666659873 ..  
1289794925585488330467364356516623468417/500000000000000000000000000000  
00  
Time Approximations 0.031.

hint used Hint := [25795896.62812623062600675830146784169746, 3, -1, 1,  
24743530.75497470617154487139175541092326,  
25795869.08129891002682354003571513818579 ..  
2579589851170975665166085604166659164517/100000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
24850269.43101309861241733616879334665430, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.760171) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=  
-2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=25795896.62812623062600675830146784169746,  
rm=24743530.75497470617154487139175541092326}, {r =  
25795869.08129891002682354003571513818579 ..  
25795898.51170975665166085604166659164517, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850269.43101309861241733616879334665430}, avoid={}));  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.32166e-30, 0., -.8e-34]  
Solution in 0.277s







Solution in 0.351s

Time Plot 0 s.

Exiting SolveHard() after 2.1r=2.38890e+07 in

[23888961.16565299817948948119449178210506 ..

```
2388899090855127150824991682392263419777/1000000000000000000000000000000  
000]
```

Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 . . 24089000]: target and source on the different branches.

Clockwise ray.

Ray outgoing at target.

Solve Side.

```
Tau [10510.45813678918192452608718760264360075,  
10510.36930058688480355785706375052275675,  
10510.39514076190213764159560681497762278,  
10510.28046624330332111411460936353971213,  
10510.36313884725478644946357276183466111,  
10510.33729983854857721313045262806652442,  
10510.33214580428355820921906585999872256,  
10510.30529788725620367344474057250350901,  
10510.21747048897362922109921724343201929,  
10510.27430496979000930419279020638252749,  
10510.19163019578441320614224430724738289,  
10510.26914997055999458101216780248035780,  
10510.27430450904120103230410027018028030,  
10510.24330971174377457186476606275847920,  
10510.33113809501479149855488296623598052,  
10510.21646359943172934589603422102085162,  
10510.18546892763852593126304843357867968,  
10510.21130922492571402214937475932521987, none, none,  
10510.27329709837108609430544821610893206, none, none, none, none,  
none, none, none, none, none, none]
```

```
2 --> 1 target = [24850262.70323078170709817304659874223335,
```

1.476940273965825181301333700278326255008,

10510.19163349175019369781798302604181388]

Imaginary part neglected:

$-.5924510391192590374527634437975863507978e-15$

```
one interval r = 25781098.14047726620236957430576323928142 ..
```

257958918211937199545196052394100264899/1000000000000000000000000000000  
0

Time Approximations 0.034.

```
hint used Hint := [25795889.93759639200267360170740297062535, 3, -1, 1,
```

24743522.41979937237431350459260520845964,

25795862.39063358446748128489577933480278 ..

515917836423874243616569030246719914989/2000000000000000000000000000000

0, 298960418182500/22468879468420441 ..

24850262.70323078644925875505002433594091, 11

```

I search for an scattering ray on opposite branches with  $0 < s_v < 1$ 

```

(0.76017) | P <--- S

```
rGuessMin=2.57811e+07    rGuessMax=2.57959e+07    rmGuess=2.47435e+07    k=
```

```

-2.37384e+14      scos=5.95461e+14

```

branch outgoing at target, Clockwise

```
(Scattering) fsolve(eqs, {r=25795889.93759639200267360170740297062535,
```

```
rm=24743522.41979937237431350459260520845964}, {r =
```



















10510.24331300772336027614350690513857333]  
Imaginary part neglected:  
-.5924510391192590374527634437975863507978e-15  
one interval r = 25781101.21410471863424788072863216633333 ..  
1289794743153524017228864201316762620003/500000000000000000000000000000  
00  
Time Approximations 0.034.

hint used Hint := [25795892.97947942498571214559240875822627, 3, -1, 1,  
24743526.20943107487634918939497374544011,  
25795865.43257821616570179673528437159660 ..  
644897371576761841392917646201273823237/250000000000000000000000000000  
0, 298960418182500/22468879468420441 ..  
24850265.76205359199870736703593497152527, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.76017) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=  
-2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=25795892.97947942498571214559240875822627,  
rm=24743526.20943107487634918939497374544011}, {r =  
25795865.43257821616570179673528437159660 ..  
25795894.86307047365571670584805095292948, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850265.76205359199870736703593497152527}, avoid={}));  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.22269e-30, 0., -.5e-34]  
Solution in 0.305s

Time Plot 0 s.  
Exiting SolveHard() after 0.883r=2.57959e+07 in  
[25795865.43257821616570179673528437159660 ..  
644897371576761841392917646201273823237/250000000000000000000000000000  
0]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 24850265.76205359199870736703593497152527]: target and source on the  
different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678918192452608718760264360075,  
10510.36930058688480355785706375052275675,  
10510.39514076190213764159560681497762278,  
10510.28046624330332111411460936353971213,  
10510.36313884725478644946357276183466111,  
10510.33729983854857721313045262806652442,  
10510.33214580428355820921906585999872256,  
10510.30529788725620367344474057250350901,  
10510.21747048897362922109921724343201929,  
10510.27430496979000930419279020638252749,  
10510.19163019578441320614224430724738289,  
10510.26914997055999458101216780248035780,  
10510.27430450904120103230410027018028030,  
10510.24330971174377457186476606275847920,  
10510.33113809501479149855488296623598052,  
10510.21646359943172934589603422102085162,







```

10510.19163019578441320614224430724738289,
10510.26914997055999458101216780248035780,
10510.27430450904120103230410027018028030,
10510.24330971174377457186476606275847920,
10510.33113809501479149855488296623598052,
10510.21646359943172934589603422102085162,
10510.18546892763852593126304843357867968,
10510.21130922492571402214937475932521987,
10510.10279600698365942206778648066986864,
10510.18546846176358548274468123956474821,
10510.27329709837108609430544821610893206,
10510.15447580430703213187994617720403932, none,
10510.15962933488518086035768335434487057,
10510.15447547792141334031928874023850863, none,
10510.21130888358607356878493843384248873, none, none, none, none]

0 --> 2 target = [25795896.38402524260848318535606155665309,
1.856299066095202419478675059591534865238,
10510.26915308873018037248922904220922807]
one interval r = 23873933.17245664251065104582254458860516 ..
2388898875830375238509764647296413098839/1000000000000000000000000000000000000
000
Time Approximations 0.039.

hint used Hint := [23888985.01109652402578398653593278001387, 3, -1, 1,
22860454.49191911041368999760273437756943,
23888959.01529623342321627788824672003212 ..
59722471895759389095528762418719523933/250000000000000000000000000000000000,
298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.367179) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31731e+14 scos=4.39725e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888985.01109652402578398653593278001387,
rm=22860454.49191911041368999760273437756943}, {r =
23888959.01529623342321627788824672003212 ..
23888988.75830375563821150496748780957320, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={{});
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=2.2e-31
Equations at solution: [.23e-30, .22e-30, -.99e-34]
Solution in 0.406s

Time Plot 0 s.
Exiting SolveHard() after 2.157r=2.38890e+07 in
[23888959.01529623342321627788824672003212 ..
59722471895759389095528762418719523933/250000000000000000000000000000000000]
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678918192452608718760264360075,
10510.36930058688480355785706375052275675,
10510.39514076190213764159560681497762278,
10510.28046624330332111411460936353971213,

```



```

10510.36313884725478644946357276183466111,
10510.33729983854857721313045262806652442,
10510.33214580428355820921906585999872256,
10510.30529788725620367344474057250350901,
10510.21747048897362922109921724343201929,
10510.27430496979000930419279020638252749,
10510.19163019578441320614224430724738289,
10510.26914997055999458101216780248035780,
10510.27430450904120103230410027018028030,
10510.24330971174377457186476606275847920,
10510.33113809501479149855488296623598052,
10510.21646359943172934589603422102085162,
10510.18546892763852593126304843357867968,
10510.21130922492571402214937475932521987,
10510.10279600698365942206778648066986864,
10510.18546846176358548274468123956474821,
10510.27329709837108609430544821610893206,
10510.15447580430703213187994617720403932, none,
10510.15962933488518086035768335434487057,
10510.15447547792141334031928874023850863, none,
10510.21130888358607356878493843384248873,
10510.206155206498976777888643354068186847, none, none, none]

1 --> 2 target = [25795896.38402524260848318535606155665309,
1.856299066095202419478675059591534865238,
10510.26915308873018037248922904220922807]
one interval r = 24835338.05090068256086463434726047183699 ..
62125668228699358490205885002478488309/2500000000000000000000000000000000
Time Approximations 0.04.

hint used Hint := [24850265.39735153163244735697452997613798, 3, 1, 1,
24743509.60268659785926677493114233979369,
24850237.69689665459305925255508686501973 ..
1242513364573987488725142465429694354713/5000000000000000000000000000000000
00, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.239829) | S ---> P
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=
2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850265.39735153163244735697452997613798,
rm=24743509.60268659785926677493114233979369}, {r =
24850237.69689665459305925255508686501973 ..
24850267.29147974977450284930859388709426, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={});
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.4949e-31, 0., -.4774165166e-35]
Solution in 1.234s

Time Plot 0 s.
Exiting SolveHard() after 1.931r=2.48503e+07 in
[24850237.69689665459305925255508686501973 ..
1242513364573987488725142465429694354713/5000000000000000000000000000000000
00]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 25089000]: target and source on the different branches.
Counterclockwise ray.

```



Ray outgoing at target.  
Solve Side.

```
Tau [10510.45813678918192452608718760264360075,
10510.36930058688480355785706375052275675,
10510.39514076190213764159560681497762278,
10510.28046624330332111411460936353971213,
10510.36313884725478644946357276183466111,
10510.33729983854857721313045262806652442,
10510.33214580428355820921906585999872256,
10510.30529788725620367344474057250350901,
10510.21747048897362922109921724343201929,
10510.27430496979000930419279020638252749,
10510.19163019578441320614224430724738289,
10510.26914997055999458101216780248035780,
10510.27430450904120103230410027018028030,
10510.24330971174377457186476606275847920,
10510.33113809501479149855488296623598052,
10510.21646359943172934589603422102085162,
10510.18546892763852593126304843357867968,
10510.21130922492571402214937475932521987,
10510.10279600698365942206778648066986864,
10510.18546846176358548274468123956474821,
10510.27329709837108609430544821610893206,
10510.15447580430703213187994617720403932,
10510.23714797609139100931626400575662678,
10510.15962933488518086035768335434487057,
10510.15447547792141334031928874023850863, none,
10510.21130888358607356878493843384248873,
10510.20615520649897677888643354068186847, none, none, none]
```

[illegible]

```
hint used Hint := [23888988.69843567795653101185231580305160, 3, -1, 1,  
22860452.96192886545581288218987601576608,  
23888962.70280479206299436081506076835447 ..  
2388899244562496946280763877837409340903/1000000000000000000000000000  
000, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.36718) | S ---> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=  
-4.31731e+14 scos=4.39724e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888988.69843567795653101185231580305160,  
rm=22860452.96192886545581288218987601576608}, {r =  
23888962.70280479206299436081506076835447 ..  
23888992.44562496946280763877837409340903, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}));  
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=6.2e-31  
Equations at solution: [.64e-30, .62e-30, -.242e-33]  
Solution in 0.392s
```











Time Approximations 0.038.

```
hint used Hint := [24850262.33852773859450921827957797170818, 3, 1, 1,
20396437.98350304935080318674410988058300,
24850238.00201200929744389905914961384924 ..
1242513379829193745424075414664369797603/500000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888989.06493588745880515057802801897934, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.533028) | P <--- S
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=
5.43203e+14 scos=2.39475e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850262.33852773859450921827957797170818,
rm=20396437.98350304935080318674410988058300}, {r =
24850238.00201200929744389905914961384924 ..
24850267.59658387490848150829328739595206, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06493588745880515057802801897934}, avoid={});
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=9e-32
Equations at solution: [-.14e-30, -.9e-31, .49879512711e-34]
Solution in 0.361s
```

Time Plot 0 s.

```
Exiting SolveHard() after 1.956r=2.48503e+07 in
[24850238.00201200929744389905914961384924 ..
1242513379829193745424075414664369797603/500000000000000000000000000000
00]
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .
. 23888989.06493588745880515057802801897934]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678918192452608718760264360075,
10510.36930058688480355785706375052275675,
10510.39514076190213764159560681497762278,
10510.28046624330332111411460936353971213,
10510.36313884725478644946357276183466111,
10510.33729983854857721313045262806652442,
10510.33214580428355820921906585999872256,
10510.30529788725620367344474057250350901,
10510.21747048897362922109921724343201929,
10510.27430496979000930419279020638252749,
10510.19163019578441320614224430724738289,
10510.26914997055999458101216780248035780,
10510.27430450904120103230410027018028030,
10510.24330971174377457186476606275847920,
10510.33113809501479149855488296623598052,
10510.21646359943172934589603422102085162,
10510.18546892763852593126304843357867968,
10510.21130922492571402214937475932521987,
10510.10279600698365942206778648066986864,
10510.18546846176358548274468123956474821,
10510.27329709837108609430544821610893206,
10510.15447580430703213187994617720403932,
```



```

10510.23714797609139100931626400575662678,
10510.15962933488518086035768335434487057,
10510.15447547792141334031928874023850863,
10510.29913613981865576155018811945082147,
10510.21130888358607356878493843384248873,
10510.20615520649897677888643354068186847,
10510.18546846689011904305847566427337344, none,
10510.26814323572240765922262132165573144]

2 --> 0 target = [23888989.06493588640561695891085266203304,
2.632525136889539083844435179240750408931,
10510.27430791384082845876950574586854090]
Imaginary part neglected:
-.5924510391192590374527634437975863507978e-15
one interval r = 25781103.05751677620512130790420979021127 ..
1289794834371977686219806309041615080903/50000000000000000000000000000000
00
Time Approximations 0.033.

hint used Hint := [25795892.97947239655457473800077316699564, 3, 1, 1,
22860366.33050119628977322107538139299992,
25795867.25698799409451243993168295190626 ..
1289794834371977270055988053158890743643/50000000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888989.06493588745880515057802801897934, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=
4.31731e+14 scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795892.97947239655457473800077316699564,
rm=22860366.33050119628977322107538139299992}, {r =
25795867.25698799409451243993168295190626 ..
25795896.68743954540111976106317781487286, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06493588745880515057802801897934}, avoid={});
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=5e-32
Equations at solution: [.5e-31, .5e-31, -.3e-34]
Solution in 0.252s

Time Plot 0 s.
Exiting SolveHard() after 0.824r=2.57959e+07 in
[25795867.25698799409451243993168295190626 ..
1289794834371977270055988053158890743643/50000000000000000000000000000000
00]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23888989.06493588745880515057802801897934]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678918192452608718760264360075,
10510.36930058688480355785706375052275675,
10510.39514076190213764159560681497762278,
10510.28046624330332111411460936353971213,
10510.36313884725478644946357276183466111,

```



10510.33729983854857721313045262806652442,  
10510.33214580428355820921906585999872256,  
10510.30529788725620367344474057250350901,  
10510.21747048897362922109921724343201929,  
10510.27430496979000930419279020638252749,  
10510.19163019578441320614224430724738289,  
10510.26914997055999458101216780248035780,  
10510.27430450904120103230410027018028030,  
10510.24330971174377457186476606275847920,  
10510.33113809501479149855488296623598052,  
10510.21646359943172934589603422102085162,  
10510.18546892763852593126304843357867968,  
10510.21130922492571402214937475932521987,  
10510.10279600698365942206778648066986864,  
10510.18546846176358548274468123956474821,  
10510.27329709837108609430544821610893206,  
10510.15447580430703213187994617720403932,  
10510.23714797609139100931626400575662678,  
10510.15962933488518086035768335434487057,  
10510.15447547792141334031928874023850863,  
10510.29913613981865576155018811945082147,  
10510.21130888358607356878493843384248873,  
10510.20615520649897677888643354068186847,  
10510.18546846689011904305847566427337344,  
10510.21130876417761358612806862993613324,  
10510.268143235722407659222621321655731441

```
Cascade time 48.395
counts: 28, 28
```

## Iteration 6

Start Generation 1

```
1 --> 0   target = [23888999.9999999977209507742520460216516,  
2.632556434476914983066077228762126551528,  
10510.45814019367394768141294231981521135]  
one interval r = 24835349.43352070193412834741845243312108 ..  
248502784771852361021823495183424196061/1000000000000000000000000000000000  
0
```

Time Approximations 0.043.

```
hint used Hint := [24850273.21919080441727298772802245902434, 3, 1, 1,  
20396435.39659592643000634328381896732011,  
24850248.88301387793152980010731189259064 ..  
621256961929631488519759813441433737669/250000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 23889000., 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.533028) | P <--- S  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=  
5.43204e+14 scos=2.39474e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850273.21919080441727298772802245902434,  
rm=20396435.39659592643000634328381896732011}, {r =  
24850248.88301387793152980010731189259064 ..  
24850278.47718525954079039253765734950676, rm =  
.1330553304194287328500223794129351168576e-1 .. 23889000.}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=1.3e-31
```



```
Equations at solution: [-.23e-30, -.13e-30, .7e-34]
Solution in 0.383s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.012r=2.48503e+07 in
[24850248.88301387793152980010731189259064 ..
621256961929631488519759813441433737669/2500000000000000000000000000000000
0]
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .
. 23889000.]: target and source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678938351358937901247904160323,  
10510.36930058635828704106387853035225328, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none]
```

[illegible]

```
hint used Hint := [25795903.79985511561316781642113695093491, 3, 1, 1,  
22860361.79220184767639985483105161990097,  
25795878.07759835662287683941521694813384 ..  
161224421923802901739910205661944662513/625000000000000000000000000000  
, 298960418182500/22468879468420441 .. 23889000., 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.63282) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31733e+14 scos=4.39724e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795903.79985511561316781642113695093491,  
rm=22860361.79220184767639985483105161990097}, {r =  
25795878.07759835662287683941521694813384 ..  
25795907.50780846427838563290591114600208, rm =  
.1330553304194287328500223794129351168576e-1 .. 23889000.}, avoid={}));  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1.5e-31  
Equations at solution: [-.17e-30, -.15e-30, .65e-34]  
Solution in 1.179s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 1.743r=2.57959e+07 in  
[25795878.07759835662287683941521694813384 ..  
161224421923802901739910205661944662513/6250000000000000000000000000000000  
]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23889000.]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```







one interval r = 23873939.27584648070720491912171852092668 ..  
1194449735784687106936227105617021585431/50000000000000000000000000000000  
00

Time Approximations 0.039.

hint used Hint := [23888989.43146328644175807501347600997649, 3, -1, 1,  
20396631.10527514558221865082982151479995,  
23888964.97298890387184335817087833215779 ..  
597224867892343544545306555277700499657/25000000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]

I search for an scattering ray on opposite branches with  $0 < sv < 1$

(0.466972) | S --> P

rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=  
-5.43203e+14 scos=2.39475e+14

branch outgoing at target, Clockwise

(Scattering) fsolve(eqs, {r=23888989.43146328644175807501347600997649,  
rm=20396631.10527514558221865082982151479995}, {r =

23888964.97298890387184335817087833215779 ..

23888994.71569374178181226221110801998628, rm =

.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));

Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1.2e-31

Equations at solution: [-.21e-30, -.12e-30, .86697997784e-34]

Solution in 0.36s

Time Plot 0 s.

Exiting SolveHard() after 2.139r=2.38890e+07 in

[23888964.97298890387184335817087833215779 ..

597224867892343544545306555277700499657/25000000000000000000000000000000  
0]

Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.

Clockwise ray.

Ray outgoing at target.

Solve Side.

Tau [10510.45813678938351358937901247904160323,

10510.36930058635828704106387853035225328,

10510.39514076157030087526317110738268058,

10510.28046624354431081238608621091555154, none,

10510.33729983821289451480090182116482612, none, none, none, none,

none, none, none, none, none, none, none, none, none, none, none, none,

none, none, none, none, none, none, none, none, none]

0 --> 2 target = [25795903.79985510515410180296683100620734,

1.856318223503011443293710646620394790981,

10510.39514387976890074035287061220972481]

one interval r = 23873940.85057848575849358590165103340110 ..

2388899625275549462908055906832453221081/10000000000000000000000000000000  
000

Time Approximations 0.936.

hint used Hint := [23888992.50558472677941562199764206716244, 3, -1, 1,  
22860451.38218918783573792125602312359251,

23888966.51012875395781144697926248654402 ..

597224906318887365922168983423648563467/25000000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]

I search for an scattering ray on opposite branches with  $0 < sv < 1$







Equations at solution: [0., 0., 0.]  
Solution in 0.263s

```
Time Plot 0 s.  
Exiting SolveHard() after 0.966r=2.48503e+07 in  
[24850245.15428919258543501803067780064530 ..  
1242513737429890818728656221501545064373/50000000000000000000000000000000  
00]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 25089000]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678938351358937901247904160323,  
10510.36930058635828704106387853035225328,  
10510.39514076157030087526317110738268058,  
10510.28046624354431081238608621091555154,  
10510.36313884673059182530949692437624322,  
10510.33729983821289451480090182116482612,  
10510.33214580453071551586344205697997057, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none]
```

```
Start Generation 3
0 --> 2 target = [25795900.39533806957749672681645956557312,
1.856309428602377369973884330510731997002,
10510.33730295639659789029381461905324385]
one interval r = 23873937.32565211724213502600907357943550 ..
1194449640607540194207803406947848004181/500000000000000000000000000000
00
Time Approximations 0.049.
```

```
hint used Hint := [23888989.06496329485872911197089488868350, 3, -1, 1,  
22860452.80984339454018821965990898965218,  
23888963.06934924828335479031706673264809 ..  
597224820303770076400571671810948623397/2500000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.36718) | S ---> P  
rGuessMin=2.38739e+07    rGuessMax=2.38890e+07    rmGuess=2.28605e+07    k=  
-4.31731e+14    scos=4.39724e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888989.06496329485872911197089488868350,  
rm=22860452.80984339454018821965990898965218}, {r =  
23888963.06934924828335479031706673264809 ..  
23888992.81215080305602286687243794493588, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));  
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.7e-31  
Equations at solution: [.18e-30, .17e-30, -.49832679641e-34]  
Solution in 0.393s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.263r=2.38890e+07 in
[23888963.06934924828335479031706673264809 ..
597224820303770076400571671810948623397/2500000000000000000000000000000000
0]
```



Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678938351358937901247904160323,  
10510.36930058635828704106387853035225328,  
10510.39514076157030087526317110738268058,  
10510.28046624354431081238608621091555154,  
10510.36313884673059182530949692437624322,  
10510.33729983821289451480090182116482612,  
10510.33214580453071551586344205697997057, none, none,  
10510.27430497003332068617025862749989369, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none]

1 --> 2 target = [25795900.39533806957749672681645956557312,  
1.856309428602377369973884330510731997002,  
10510.33730295639659789029381461905324385]  
one interval r = 24835342.15555715062444936005289065116349 ..  
124251356625646179302577504170686131573/500000000000000000000000000000  
00  
Time Approximations 0.035.

hint used Hint := [24850269.43101309861241733616879334665430, 3, 1, 1,  
24743514.60014700975744022125022662330031,  
24850241.73069462631526237165506015616915 ..  
1242513566256462912427735010225393921567/500000000000000000000000000000  
00, 298960418182500/22468879468420441 .. 25089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.239829) | S ---> P  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=  
2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850269.43101309861241733616879334665430,  
rm=24743514.60014700975744022125022662330031}, {r =  
24850241.73069462631526237165506015616915 ..  
24850271.32512925824855470020450787843134, rm =  
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [-.7423e-31, 0., .1e-34]  
Solution in 0.275s

Time Plot 0 s.  
Exiting SolveHard() after 1.886r=2.48503e+07 in  
[24850241.73069462631526237165506015616915 ..  
1242513566256462912427735010225393921567/500000000000000000000000000000  
00]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 25089000]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678938351358937901247904160323,  
10510.36930058635828704106387853035225328,







```
2 --> 0 target = [23888989.43146328738349005900795592770472,  
2.632526185733034714791700079698387730233,  
10510.28046964778540526827196373540112813]  
one interval r = 25781103.42398365688757113592723209774148 ..  
161224356563254269143788730177141600777/625000000000000000000000000  
Time Approximations 0.03.
```

```
Time Plot 0 s.  
Exiting SolveHard() after 1.726r=2.57959e+07 in  
[25795867.61967723367271871049090258979880 ..  
1289794852506034629127592665641586312049/50000000000000000000000000000000  
00]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23888989.43146328644175807501347600997649]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

Tau [10510.45813678938351358937901247904160323,  
10510.36930058635828704106387853035225328,  
10510.39514076157030087526317110738268058,  
10510.28046624354431081238608621091555154,  
10510.36313884673059182530949692437624322,  
10510.33729983821289451480090182116482612,  
10510.33214580453071551586344205697997057,  
10510.30529788672816311498757758966151614,  
10510.21747048868119303358518567917422204,  
10510.27430497003332068617025862749989369,



10510.19163019529729729241705276245683970, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none]

1 --> 0 target = [23888992.50558472699170608454620236240503,  
2.632534984278044852734986320457223215001,  
10510.33214920878616154171897015218262927]  
one interval r = 24835341.84514833854936358781620091965915 ..  
1242513551004514917973287752483055080511/500000000000000000000000000000  
00  
Time Approximations 0.042.

hint used Hint := [24850265.76205359199870736703593497152527, 3, 1, 1,  
20396437.16957127706091945345483725838621,  
24850241.42564446195929857082855634229261 ..  
1242513551004516068014443586172817582043/500000000000000000000000000000  
00, 298960418182500/22468879468420441 ..  
23888992.50558472677941562199764206716244, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.533028) | P <--- S  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=  
5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850265.76205359199870736703593497152527,  
rm=20396437.16957127706091945345483725838621}, {r =  
24850241.42564446195929857082855634229261 ..  
24850271.02009032136028887172345635164086, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888992.50558472677941562199764206716244}, avoid={});  
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=8e-32  
Equations at solution: [.15e-30, .8e-31, -.8e-34]  
Solution in 1.255s

Time Plot 0 s.  
Exiting SolveHard() after 1.975r=2.48503e+07 in  
[24850241.42564446195929857082855634229261 ..  
1242513551004516068014443586172817582043/500000000000000000000000000000  
00]  
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888992.50558472677941562199764206716244]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678938351358937901247904160323,  
10510.36930058635828704106387853035225328,  
10510.39514076157030087526317110738268058,  
10510.28046624354431081238608621091555154,  
10510.36313884673059182530949692437624322,  
10510.33729983821289451480090182116482612,  
10510.33214580453071551586344205697997057,  
10510.30529788672816311498757758966151614,  
10510.21747048868119303358518567917422204,  
10510.27430497003332068617025862749989369,  
10510.19163019529729729241705276245683970, none, none,  
10510.24330971126282626043789836557696032, none, none, none, none,



none, none, none, none, none, none, none, none, none, none, none, none,  
none]

2 --> 0 target = [23888992.50558472699170608454620236240503,  
2.632534984278044852734986320457223215001,  
10510.33214920878616154171897015218262927]  
one interval r = 25781106.49760328596272589959034926274876 ..  
1289795004599401664924381184108046113659/500000000000000000000000000000  
00  
Time Approximations 0.033.

hint used Hint := [25795896.38402523646204355452114396262515, 3, 1, 1,  
22860364.90258708217946452135489353704915,  
25795870.66161245800881604810046494170563 ..  
2579590009198804353979547140796504033263/100000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
23888992.50558472677941562199764206716244, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.63282) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31731e+14 scos=4.39724e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795896.38402523646204355452114396262515,  
rm=22860364.90258708217946452135489353704915}, {r =  
25795870.66161245800881604810046494170563 ..  
25795900.09198804353979547140796504033263, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888992.50558472677941562199764206716244}, avoid={}));  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=2.1e-31  
Equations at solution: [.22e-30, .21e-30, -.80e-34]  
Solution in 0.296s

Time Plot 0 s.  
Exiting SolveHard() after 0.857r=2.57959e+07 in  
[25795870.66161245800881604810046494170563 ..  
2579590009198804353979547140796504033263/100000000000000000000000000000  
000]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23888992.50558472677941562199764206716244]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678938351358937901247904160323,  
10510.36930058635828704106387853035225328,  
10510.39514076157030087526317110738268058,  
10510.28046624354431081238608621091555154,  
10510.36313884673059182530949692437624322,  
10510.33729983821289451480090182116482612,  
10510.33214580453071551586344205697997057,  
10510.30529788672816311498757758966151614,  
10510.21747048868119303358518567917422204,  
10510.27430497003332068617025862749989369,  
10510.19163019529729729241705276245683970,  
10510.26914997027372599112518853241827321, none,  
10510.24330971126282626043789836557696032, none, none, none, none,



```
none, none, none, none, none, none, none, none, none, none, none, none, none,
none]
```

```
2 --> 1 target = [24850272.85449186857437748233975170672978,  
1.476967822354143421157902173430323567516,  
10510.36314214292974955541794710612119236]  
one interval r = 25781108.34089176312537209145656445398896 ..  
161224386976461267569480937890655294449/6250000000000000000000000000  
Time Approximations 0.028.
```

```
hint used Hint := [25795900.03265730627104171495990540839944, 3, -1, 1,  
24743534.99638995296918328323923452726440,  
25795872.48589893272346549352544011957533 ..  
644897547905845320097763426011396162913/250000000000000000000000000000  
0, 298960418182500/22468879468420441 ..  
24850272.85449189132526402658550193546657, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.760172) | P <--- S  
rGuessMin=2.57811e+07    rGuessMax=2.57959e+07    rmGuess=2.47435e+07    k=  
-2.37383e+14    scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=25795900.03265730627104171495990540839944,  
rm=24743534.99638995296918328323923452726440}, {r =  
25795872.48589893272346549352544011957533 ..  
25795901.91623381280391053704045584651652, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850272.85449189132526402658550193546657}}, avoid={{}});  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [-.7424e-31, 0., .51e-34]  
Solution in 0.311s
```

```

Time Plot 0 s.
Exiting SolveHard() after 1.827r=2.57959e+07 in
[25795872.48589893272346549352544011957533 ..
644897547905845320097763426011396162913/2500000000000000000000000000000000
0]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 24850272.85449189132526402658550193546657]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

```

```
Tau [10510.45813678938351358937901247904160323,  
10510.36930058635828704106387853035225328,  
10510.39514076157030087526317110738268058,  
10510.28046624354431081238608621091555154,  
10510.36313884673059182530949692437624322,  
10510.33729983821289451480090182116482612,  
10510.33214580453071551586344205697997057,  
10510.30529788672816311498757758966151614,  
10510.21747048868119303358518567917422204,  
10510.27430497003332068617025862749989369,  
10510.19163019529729729241705276245683970,  
10510.26914997027372599112518853241827321, none,  
10510.24330971126282626043789836557696032,  
10510.33113809468143069434938959241915420, none, none, none, none,
```











```

none, none, none, none, none, none, none, none, none, none, none]

2 --> 0 target = [23888989.06496329590696435069015296323164,
2.632525136764175862901157796398979700690,
10510.27430837427270413765281790211961283]
one interval r = 25781103.05754416115732139330549996807758 ..
2579589668746665577534911611219111255051/1000000000000000000000000000000
000
Time Approximations 0.029.

hint used Hint := [25795892.97949951637755150436992208534955, 3, 1, 1,
22860366.33048982198263378361971969356546,
25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/25000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
23888989.06496329485872911197089488868350, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=
4.31731e+14 scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795892.97949951637755150436992208534955,
rm=22860366.33048982198263378361971969356546}, {r =
25795867.25701511448801980095492062593890 ..
25795896.68746666518951237477219502520548, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06496329485872911197089488868350}, avoid={}));
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=2e-32
Equations at solution: [.2e-31, .2e-31, -.22e-34]
Solution in 0.28s

Time Plot 0 s.
Exiting SolveHard() after 1.73r=2.57959e+07 in
[25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/25000000000000000000000000000000
0]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23888989.06496329485872911197089488868350]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678938351358937901247904160323,
10510.36930058635828704106387853035225328,
10510.39514076157030087526317110738268058,
10510.28046624354431081238608621091555154,
10510.36313884673059182530949692437624322,
10510.33729983821289451480090182116482612,
10510.33214580453071551586344205697997057,
10510.30529788672816311498757758966151614,
10510.21747048868119303358518567917422204,
10510.27430497003332068617025862749989369,
10510.19163019529729729241705276245683970,
10510.26914997027372599112518853241827321,
10510.27430450928451262127555846252411868,
10510.24330971126282626043789836557696032,

```



```

10510.33113809468143069434938959241915420, none,
10510.18546892715373169930495104290417209,
10510.21130922463559951492609077752434181, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1 target = [24850269.43101307669321021710788288185114,
1.476958531745057515420903993218129936680,
10510.30530118291186972411565694701141878]
one interval r = 25781104.90082927461290075029582298790387 ..
2579589851170974748601305625930602070271/1000000000000000000000000000000
000
Time Approximations 0.034.

hint used Hint := [25795896.62812623062600675830146784169746, 3, -1, 1,
24743530.75497470617154487139175541092326,
25795869.08129891002682354003571513818579 ..
2579589851170975665166085604166659164517/1000000000000000000000000000000
000, 298960418182500/22468879468420441 ..
24850269.43101309861241733616879334665430, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.760171) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=
-2.37384e+14 scos=5.95462e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25795896.62812623062600675830146784169746,
rm=24743530.75497470617154487139175541092326}, {r =
25795869.08129891002682354003571513818579 ..
25795898.51170975665166085604166659164517, rm =
.1330553304194287328500223794129351168576e-1 ..
24850269.43101309861241733616879334665430}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.19795e-30, 0., .36e-34]
Solution in 0.285s

Time Plot 0 s.
Exiting SolveHard() after 0.839r=2.57959e+07 in
[25795869.08129891002682354003571513818579 ..
2579589851170975665166085604166659164517/1000000000000000000000000000000
000]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 24850269.43101309861241733616879334665430]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678938351358937901247904160323,
10510.36930058635828704106387853035225328,
10510.39514076157030087526317110738268058,
10510.28046624354431081238608621091555154,
10510.36313884673059182530949692437624322,
10510.33729983821289451480090182116482612,
10510.33214580453071551586344205697997057,
10510.30529788672816311498757758966151614,
10510.21747048868119303358518567917422204,
10510.27430497003332068617025862749989369,
10510.19163019529729729241705276245683970,

```



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10510.26914997027372599112518853241827321,
10510.27430450928451262127555846252411868,
10510.24330971126282626043789836557696032,
10510.33113809468143069434938959241915420, none,
10510.18546892715373169930495104290417209,
10510.21130922463559951492609077752434181, none, none,
10510.27329709803387935349088560719681378, none, none, none, none,
none, none, none, none, none, none]

0 --> 1 target = [24850269.43101307669321021710788288185114,
1.476958531745057515420903993218129936680,
10510.30530118291186972411565694701141878]
one interval r = 23873935.37540036566275770431936100791382 ..
59722477271378182019047960859487234871/2500000000000000000000000000000000
Time Approximations 0.04.

hint used Hint := [23888985.62428976346864819081258538077918, 3, -1, 1,
20396632.00572718336484608918971225516342,
23888961.16565299817948948119449178210506 ..
2388899090855127150824991682392263419777/10000000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S --> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=
-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888985.62428976346864819081258538077918,
rm=20396632.00572718336484608918971225516342}, {r =
23888961.16565299817948948119449178210506 ..
23888990.90855127150824991682392263419777, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}));
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1.0e-31
Equations at solution: [-.18e-30, -.10e-30, .61351833327e-34]
Solution in 0.346s

Time Plot 0 s.
Exiting SolveHard() after 2.167r=2.38890e+07 in
[23888961.16565299817948948119449178210506 ..
2388899090855127150824991682392263419777/10000000000000000000000000000000000
000]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678938351358937901247904160323,
10510.36930058635828704106387853035225328,
10510.39514076157030087526317110738268058,
10510.28046624354431081238608621091555154,
10510.36313884673059182530949692437624322,
10510.33729983821289451480090182116482612,
10510.33214580453071551586344205697997057,
10510.30529788672816311498757758966151614,
10510.21747048868119303358518567917422204,
10510.27430497003332068617025862749989369,
10510.19163019529729729241705276245683970,

```



```

10510.26914997027372599112518853241827321,
10510.27430450928451262127555846252411868,
10510.24330971126282626043789836557696032,
10510.33113809468143069434938959241915420,
10510.21646359967119500469984951047125095,
10510.18546892715373169930495104290417209,
10510.21130922463559951492609077752434181, none, none,
10510.27329709803387935349088560719681378, none, none, none, none,
none, none, none, none, none, none]

2 --> 1 target = [24850262.70323076475591787356918648545477,
1.476940274051579968370581697129705901649,
10510.19163349145063971445139829592824925]
one interval r = 25781098.14047724931747681218894455995558 ..
2579589182119370323993379093691592766989/1000000000000000000000000000000000000
000
Time Approximations 0.033.

hint used Hint := [25795889.93759639200267360170740297062535, 3, -1, 1,
24743522.41979937237431350459260520845964,
25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/2000000000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
24850262.70323078644925875505002433594091, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.76017) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=
-2.37384e+14 scos=5.95461e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25795889.93759639200267360170740297062535,
rm=24743522.41979937237431350459260520845964}, {r =
25795862.39063358446748128489577933480278 ..
25795891.82119371218082845151233599574945, rm =
.1330553304194287328500223794129351168576e-1 ..
24850262.70323078644925875505002433594091}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.17321e-30, 0., -.45e-34]
Solution in 1.244s

Time Plot 0 s.
Exiting SolveHard() after 1.825r=2.57959e+07 in
[25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/2000000000000000000000000000000000000
0]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 24850262.70323078644925875505002433594091]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678938351358937901247904160323,
10510.36930058635828704106387853035225328,
10510.39514076157030087526317110738268058,
10510.28046624354431081238608621091555154,
10510.36313884673059182530949692437624322,
10510.33729983821289451480090182116482612,

```



```

10510.33214580453071551586344205697997057,
10510.30529788672816311498757758966151614,
10510.21747048868119303358518567917422204,
10510.27430497003332068617025862749989369,
10510.19163019529729729241705276245683970,
10510.26914997027372599112518853241827321,
10510.27430450928451262127555846252411868,
10510.24330971126282626043789836557696032,
10510.33113809468143069434938959241915420,
10510.21646359967119500469984951047125095,
10510.18546892715373169930495104290417209,
10510.21130922463559951492609077752434181, none, none,
10510.27329709803387935349088560719681378, none, none,
10510.15962933458889879045954883317037182, none, none, none, none,
none, none, none]

0 --> 1 target = [24850262.70323076475591787356918648545477,
1.476940274051579968370581697129705901649,
10510.19163349145063971445139829592824925]
one interval r = 23873928.44821858905312932258473918280149 ..
1194449207353541246682570230289986779239/500000000000000000000000000000
00
Time Approximations 0.049.

hint used Hint := [23888978.86275416794827744619407470166172, 3, -1, 1,
20396633.60486737240849086583158368073964,
23888954.40382902066202638611670198664112 ..
298612301838385292588525592212900916387/1250000000000000000000000000000
0, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=
-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888978.86275416794827744619407470166172,
rm=20396633.60486737240849086583158368073964}, {r =
23888954.40382902066202638611670198664112 ..
23888984.14707082340708204737703207331096, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1e-32
Equations at solution: [.2e-31, .1e-31, .13681914649e-34]
Solution in 0.403s

Time Plot 0 s.
Exiting SolveHard() after 1.287r=2.38890e+07 in
[23888954.40382902066202638611670198664112 ..
298612301838385292588525592212900916387/1250000000000000000000000000000
0]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678938351358937901247904160323,
10510.36930058635828704106387853035225328,
10510.39514076157030087526317110738268058,

```



```

10510.28046624354431081238608621091555154,
10510.36313884673059182530949692437624322,
10510.33729983821289451480090182116482612,
10510.33214580453071551586344205697997057,
10510.30529788672816311498757758966151614,
10510.21747048868119303358518567917422204,
10510.27430497003332068617025862749989369,
10510.19163019529729729241705276245683970,
10510.26914997027372599112518853241827321,
10510.27430450928451262127555846252411868,
10510.24330971126282626043789836557696032,
10510.33113809468143069434938959241915420,
10510.21646359967119500469984951047125095,
10510.18546892715373169930495104290417209,
10510.21130922463559951492609077752434181,
10510.10279600726404969149386918685200487, none,
10510.27329709803387935349088560719681378, none, none,
10510.15962933458889879045954883317037182, none, none, none, none,
none, none, none]

0 --> 2 target = [25795893.34215399694043024268117589151567,
1.856291208152421680121550737131754612179,
10510.21747360683403526374736742590644633]
one interval r = 23873930.02298357601724680208281496733312 ..
1194449284208367967131790247921500508167/500000000000000000000000000000
00
Time Approximations 0.045.

hint used Hint := [23888981.93694517027845377679197924121306, 3, -1, 1,
22860455.76745222490936139362415353022295,
23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/400000000000000000000000000000,
298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.367179) | S --> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31730e+14 scos=4.39725e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888981.93694517027845377679197924121306,
rm=22860455.76745222490936139362415353022295}, {r =
23888955.94100364913487175812205942139954 ..
23888985.68416735818079539166696781319750, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=6e-32
Equations at solution: [.7e-31, .6e-31, -.18380216836e-34]
Solution in 0.402s

Time Plot 0 s.
Exiting SolveHard() after 2.233r=2.38890e+07 in
[23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/400000000000000000000000000000]
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

```



```

Tau [10510.45813678938351358937901247904160323,
10510.36930058635828704106387853035225328,
10510.39514076157030087526317110738268058,
10510.28046624354431081238608621091555154,
10510.36313884673059182530949692437624322,
10510.33729983821289451480090182116482612,
10510.33214580453071551586344205697997057,
10510.30529788672816311498757758966151614,
10510.21747048868119303358518567917422204,
10510.27430497003332068617025862749989369,
10510.19163019529729729241705276245683970,
10510.26914997027372599112518853241827321,
10510.27430450928451262127555846252411868,
10510.24330971126282626043789836557696032,
10510.33113809468143069434938959241915420,
10510.21646359967119500469984951047125095,
10510.18546892715373169930495104290417209,
10510.21130922463559951492609077752434181,
10510.10279600726404969149386918685200487, none,
10510.27329709803387935349088560719681378,
10510.15447580459358996117935886618458331, none,
10510.15962933458889879045954883317037182, none, none, none, none,
none, none, none]

1 --> 2 target = [25795893.34215399694043024268117589151567,
1.856291208152421680121550737131754612179,
10510.21747360683403526374736742590644633]
one interval r = 24835334.93824080598656449317600024521256 ..
248502642326647753356111797088543119427/1000000000000000000000000000000000
0
Time Approximations 0.039.

hint used Hint := [24850262.33852743516297758053367205548154, 3, 1, 1,
24743505.81299238129247202614818714422035,
24850234.63796912173141920432208117660707 ..
2485026423266479739181704602953513976831/1000000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.23983) | S ---> P
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=
2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850262.33852743516297758053367205548154,
rm=24743505.81299238129247202614818714422035}, {r =
24850234.63796912173141920432208117660707 ..
24850264.23266479739181704602953513976831, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.29693e-30, 0., .7e-34]
Solution in 0.303s

Time Plot 0 s.
Exiting SolveHard() after 1.994r=2.48503e+07 in
[24850234.63796912173141920432208117660707 ..
2485026423266479739181704602953513976831/1000000000000000000000000000000000
000]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441

```











```
(Scattering) fsolve(eqs, {r=23888981.93692575516559285951162594064654,
rm=20396632.87781872303409295730403849988110}, {r =
23888957.47813172092635353311983299926098 ..
23888987.22121733786778681367965219867556, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}));
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=5e-32
Equations at solution: [-.9e-31, -.5e-31, .24502696998e-34]
Solution in 0.376s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.19r=2.38890e+07 in
[23888957.47813172092635353311983299926098 ..
597224680530433446694670341991304966889/2500000000000000000000000000000000
0]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678938351358937901247904160323,
10510.36930058635828704106387853035225328,
10510.39514076157030087526317110738268058,
10510.28046624354431081238608621091555154,
10510.36313884673059182530949692437624322,
10510.33729983821289451480090182116482612,
10510.33214580453071551586344205697997057,
10510.30529788672816311498757758966151614,
10510.21747048868119303358518567917422204,
10510.27430497003332068617025862749989369,
10510.19163019529729729241705276245683970,
10510.26914997027372599112518853241827321,
10510.27430450928451262127555846252411868,
10510.24330971126282626043789836557696032,
10510.33113809468143069434938959241915420,
10510.21646359967119500469984951047125095,
10510.18546892715373169930495104290417209,
10510.21130922463559951492609077752434181,
10510.10279600726404969149386918685200487,
10510.18546846127879146277237394793356713,
10510.27329709803387935349088560719681378,
10510.15447580459358996117935886618458331, none,
10510.15962933458889879045954883317037182,
10510.15447547820797120596937326804725898, none,
10510.21130888329595910497591642911463739, none, none, none, none]
```

```
0 --> 2 target = [25795896.38402522643849928434216719852051,
1.856299066190191200993478435576010823827,
10510.26915308843987788343479560779280410]
one interval r = 23873933.17245664628881212351473479366001 ..
2388898875830375607060567964253154097199/1000000000000000000000000000000000
000
```

```
Time Approximations 0.044.
```

```
hint used Hint := [23888985.01109652402578398653593278001387, 3, -1, 1,
22860454.49191911041368999760273437756943,
23888959.01529623342321627788824672003212 ..
```



```
59722471895759389095528762418719523933/2500000000000000000000000000000000,
298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.367179) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31731e+14 scos=4.39725e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888985.01109652402578398653593278001387,
rm=22860454.49191911041368999760273437756943}, {r =
23888959.01529623342321627788824672003212 ..
23888988.75830375563821150496748780957320, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.3e-31
Equations at solution: [.13e-30, .13e-30, -.60989999311e-34]
Solution in 0.389s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.243r=2.38890e+07 in
[23888959.01529623342321627788824672003212 ..
59722471895759389095528762418719523933/2500000000000000000000000000000000]
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678938351358937901247904160323,
10510.36930058635828704106387853035225328,
10510.39514076157030087526317110738268058,
10510.28046624354431081238608621091555154,
10510.36313884673059182530949692437624322,
10510.33729983821289451480090182116482612,
10510.33214580453071551586344205697997057,
10510.30529788672816311498757758966151614,
10510.21747048868119303358518567917422204,
10510.27430497003332068617025862749989369,
10510.19163019529729729241705276245683970,
10510.26914997027372599112518853241827321,
10510.27430450928451262127555846252411868,
10510.24330971126282626043789836557696032,
10510.33113809468143069434938959241915420,
10510.21646359967119500469984951047125095,
10510.18546892715373169930495104290417209,
10510.21130922463559951492609077752434181,
10510.10279600726404969149386918685200487,
10510.18546846127879146277237394793356713,
10510.27329709803387935349088560719681378,
10510.15447580459358996117935886618458331, none,
10510.15962933458889879045954883317037182,
10510.15447547820797120596937326804725898, none,
10510.21130888329595910497591642911463739,
10510.20615520679170219506748080659190470, none, none, none]
```

```
1 --> 2 target = [25795896.38402522643849928434216719852051,
1.856299066190191200993478435576010823827,
10510.26915308843987788343479560779280410]
one interval r = 24835338.05090066586839409510968486701333 ..
```







```

10510.15962933458889879045954883317037182,
10510.15447547820797120596937326804725898, none,
10510.21130888329595910497591642911463739,
10510.20615520679170219506748080659190470, none, none, none]

0 --> 2 target = [25795900.03265729674451043624528404697911,
1.856308491689083798803353619237267630605,
10510.33114121286354715953666648735269973]
one interval r = 23873936.95014365307152480387784838632141 ..
47777984891249940794906378307395066481/20000000000000000000000000000000
Time Approximations 0.047.

hint used Hint := [23888988.69843567795653101185231580305160, 3, -1, 1,
22860452.96192886545581288218987601576608,
23888962.70280479206299436081506076835447 ..
2388899244562496946280763877837409340903/10000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.36718) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31731e+14 scos=4.39724e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888988.69843567795653101185231580305160,
rm=22860452.96192886545581288218987601576608}, {r =
23888962.70280479206299436081506076835447 ..
23888992.44562496946280763877837409340903, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.7e-31
Equations at solution: [.18e-30, .17e-30, -.54517638569e-34]
Solution in 0.415s

Time Plot 0 s.
Exiting SolveHard() after 1.318r=2.38890e+07 in
[23888962.70280479206299436081506076835447 ..
2388899244562496946280763877837409340903/10000000000000000000000000000000
000]
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678938351358937901247904160323,
10510.36930058635828704106387853035225328,
10510.39514076157030087526317110738268058,
10510.28046624354431081238608621091555154,
10510.36313884673059182530949692437624322,
10510.33729983821289451480090182116482612,
10510.33214580453071551586344205697997057,
10510.30529788672816311498757758966151614,
10510.21747048868119303358518567917422204,
10510.27430497003332068617025862749989369,
10510.19163019529729729241705276245683970,
10510.26914997027372599112518853241827321,
10510.27430450928451262127555846252411868,
10510.24330971126282626043789836557696032,
10510.33113809468143069434938959241915420,

```



```

10510.21646359967119500469984951047125095,
10510.18546892715373169930495104290417209,
10510.21130922463559951492609077752434181,
10510.10279600726404969149386918685200487,
10510.18546846127879146277237394793356713,
10510.27329709803387935349088560719681378,
10510.15447580459358996117935886618458331,
10510.23714797561276459076178350784478705,
10510.15962933458889879045954883317037182,
10510.15447547820797120596937326804725898, none,
10510.21130888329595910497591642911463739,
10510.20615520679170219506748080659190470, none, none,
10510.26814323596804093190851842602129984]

```

```

1 --> 2 target = [25795900.03265729674451043624528404697911,
1.856308491689083798803353619237267630605,
10510.33114121286354715953666648735269973]
one interval r = 24835341.78443700687109372609387210031866 ..
2485027096042923262218034515118561690139/1000000000000000000000000000000
000

```

Time Approximations 0.04.

```

hint used Hint := [24850269.06631200499486567135274096895840, 3, 1, 1,
24743514.14830449237551739504443492551457,
24850241.36598119958086914129896830053765 ..
2485027096042925490424856544845137371367/1000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 25089000, 1]

```

```

I search for an scattering ray on opposite branches with 0<sv<1
(0.239829) | S --> P
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=
2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850269.06631200499486567135274096895840,
rm=24743514.14830449237551739504443492551457}, {r =
24850241.36598119958086914129896830053765 ..
24850270.96042925490424856544845137371367, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={});
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.39589e-30, 0., -.8e-34]
Solution in 0.321s

```

Time Plot 0 s.

```

Exiting SolveHard() after 2.009r=2.48503e+07 in
[24850241.36598119958086914129896830053765 ..
2485027096042925490424856544845137371367/1000000000000000000000000000000
000]

```

```

Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 25089000]: target and source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

```

```

Tau [10510.45813678938351358937901247904160323,
10510.36930058635828704106387853035225328,
10510.39514076157030087526317110738268058,
10510.28046624354431081238608621091555154,
10510.36313884673059182530949692437624322,

```



```

10510.33729983821289451480090182116482612,
10510.33214580453071551586344205697997057,
10510.30529788672816311498757758966151614,
10510.21747048868119303358518567917422204,
10510.27430497003332068617025862749989369,
10510.19163019529729729241705276245683970,
10510.26914997027372599112518853241827321,
10510.27430450928451262127555846252411868,
10510.24330971126282626043789836557696032,
10510.33113809468143069434938959241915420,
10510.21646359967119500469984951047125095,
10510.18546892715373169930495104290417209,
10510.21130922463559951492609077752434181,
10510.10279600726404969149386918685200487,
10510.18546846127879146277237394793356713,
10510.27329709803387935349088560719681378,
10510.15447580459358996117935886618458331,
10510.23714797561276459076178350784478705,
10510.15962933458889879045954883317037182,
10510.15447547820797120596937326804725898,
10510.29913613929293709870202867285753397,
10510.21130888329595910497591642911463739,
10510.20615520679170219506748080659190470, none, none,
10510.26814323596804093190851842602129984]

```

```

1 --> 0 target = [23888989.06493588850705002078675162330143,
2.632525136685732476486561063115216067361,
10510.27430791352389594480677032448708599]
one interval r = 24835338.36137566524632921463638756928912 ..
1242513379829192636967326187901122261861/500000000000000000000000000000
00
Time Approximations 0.048.

```

```

hint used Hint := [24850262.33852773859450921827957797170818, 3, 1, 1,
20396437.98350304935080318674410988058300,
24850238.00201200929744389905914961384924 ..
1242513379829193745424075414664369797603/500000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888989.06493588745880515057802801897934, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.533028) | P <--- S
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=
5.43203e+14 scos=2.39475e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850262.33852773859450921827957797170818,
rm=20396437.98350304935080318674410988058300}, {r =
24850238.00201200929744389905914961384924 ..
24850267.59658387490848150829328739595206, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06493588745880515057802801897934}, avoid={});
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=1.0e-31
Equations at solution: [-.18e-30, -.10e-30, .6e-34]
Solution in 0.391s

```

```

Time Plot 0 s.
Exiting SolveHard() after 2.199r=2.48503e+07 in
[24850238.00201200929744389905914961384924 ..

```



```

1242513379829193745424075414664369797603/500000000000000000000000000000000000
00]
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .
. 23888989.06493588745880515057802801897934]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678938351358937901247904160323,
10510.36930058635828704106387853035225328,
10510.39514076157030087526317110738268058,
10510.28046624354431081238608621091555154,
10510.36313884673059182530949692437624322,
10510.33729983821289451480090182116482612,
10510.33214580453071551586344205697997057,
10510.30529788672816311498757758966151614,
10510.21747048868119303358518567917422204,
10510.27430497003332068617025862749989369,
10510.19163019529729729241705276245683970,
10510.26914997027372599112518853241827321,
10510.27430450928451262127555846252411868,
10510.24330971126282626043789836557696032,
10510.33113809468143069434938959241915420,
10510.21646359967119500469984951047125095,
10510.18546892715373169930495104290417209,
10510.21130922463559951492609077752434181,
10510.10279600726404969149386918685200487,
10510.18546846127879146277237394793356713,
10510.27329709803387935349088560719681378,
10510.15447580459358996117935886618458331,
10510.23714797561276459076178350784478705,
10510.15962933458889879045954883317037182,
10510.15447547820797120596937326804725898,
10510.29913613929293709870202867285753397,
10510.21130888329595910497591642911463739,
10510.20615520679170219506748080659190470,
10510.18546846640532501809419397632837238, none,
10510.26814323596804093190851842602129984]

2 --> 0 target = [23888989.06493588850705002078675162330143,
2.632525136685732476486561063115216067361,
10510.27430791352389594480677032448708599]
one interval r = 25781103.05751675828669896339330286502203 ..
2579589668743953598696603257694781037287/1000000000000000000000000000000000000
000
Time Approximations 0.034.

hint used Hint := [25795892.97947239655457473800077316699564, 3, 1, 1,
22860366.33050119628977322107538139299992,
25795867.25698799409451243993168295190626 ..
1289794834371977270055988053158890743643/500000000000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888989.06493588745880515057802801897934, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=

```



```
4.31731e+14    scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795892.97947239655457473800077316699564,
rm=22860366.33050119628977322107538139299992}, {r =
25795867.25698799409451243993168295190626 ..
25795896.68743954540111976106317781487286, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06493588745880515057802801897934}, avoid={}));
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=3e-32
Equations at solution: [.3e-31, .3e-31, .5e-35]
Solution in 0.263s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.863r=2.57959e+07 in
[25795867.25698799409451243993168295190626 ..
1289794834371977270055988053158890743643/500000000000000000000000000000
00]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23888989.06493588745880515057802801897934]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678938351358937901247904160323,
10510.36930058635828704106387853035225328,
10510.39514076157030087526317110738268058,
10510.28046624354431081238608621091555154,
10510.36313884673059182530949692437624322,
10510.33729983821289451480090182116482612,
10510.33214580453071551586344205697997057,
10510.30529788672816311498757758966151614,
10510.21747048868119303358518567917422204,
10510.27430497003332068617025862749989369,
10510.19163019529729729241705276245683970,
10510.26914997027372599112518853241827321,
10510.27430450928451262127555846252411868,
10510.24330971126282626043789836557696032,
10510.33113809468143069434938959241915420,
10510.21646359967119500469984951047125095,
10510.18546892715373169930495104290417209,
10510.21130922463559951492609077752434181,
10510.10279600726404969149386918685200487,
10510.18546846127879146277237394793356713,
10510.27329709803387935349088560719681378,
10510.15447580459358996117935886618458331,
10510.23714797561276459076178350784478705,
10510.15962933458889879045954883317037182,
10510.15447547820797120596937326804725898,
10510.29913613929293709870202867285753397,
10510.21130888329595910497591642911463739,
10510.20615520679170219506748080659190470,
10510.18546846640532501809419397632837238,
10510.21130876388749928589846774914656056,
10510.26814323596804093190851842602129984]
```

```
Cascade time 50.397
```



[illegible]























```
24743514.60014700975744022125022662330031,
24850241.73069462631526237165506015616915 ..
1242513566256462912427735010225393921567/50000000000000000000000000000000
00, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.239829) | S --> P
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=
2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850269.43101309861241733616879334665430,
rm=24743514.60014700975744022125022662330031}, {r =
24850241.73069462631526237165506015616915 ..
24850271.32512925824855470020450787843134, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.19794e-30, 0., -.78e-34]
Solution in 0.269s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.98r=2.48503e+07 in
[24850241.73069462631526237165506015616915 ..
1242513566256462912427735010225393921567/50000000000000000000000000000000
00]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 25089000]: target and source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678919554814622893210528524039,
10510.36930058679635290810545398076470715,
10510.39514076206256012794571996982881491,
10510.28046624332455204936034548209098100,
10510.36313884716609711097874014799364018,
10510.33729983869089110286014688393010861,
10510.33214580432265904772480155814019098,
10510.30529788714940572843960125133856867, none,
10510.27430496981100157469317161964690000, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]
```

```
1 --> 0 target = [23888989.43146328557835419259512766974703,
2.632526185899077968327572419818158674543,
10510.28046964806267188131032298908977684]
one interval r = 24835338.73249665917388887214003050701691 ..
497005359225696976506521620407471627083/20000000000000000000000000000000
0
Time Approximations 0.043.
```

```
hint used Hint := [24850262.70323078644925875505002433594091, 3, 1, 1,
20396437.89679703248883954261208042438711,
24850238.36672641289161845524403523557614 ..
2485026796128485539651696969285982263711/10000000000000000000000000000000
000, 298960418182500/22468879468420441 ..
23888989.43146328644175807501347600997649, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.533028) | P <--- S
```























Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=0  
Equations at solution: [0., 0., -.1e-34]  
Solution in 0.372s

Time Plot 0 s.

Exiting SolveHard() after 2.381r=2.38890e+07 in  
[23888964.60644587167575168019887114912430 ..  
1194449717458466618404658405105548818961/500000000000000000000000000000  
00]

Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.

Clockwise ray.

Ray outgoing at target.

Solve Side.

Tau [10510.45813678919554814622893210528524039,  
10510.36930058679635290810545398076470715,  
10510.39514076206256012794571996982881491,  
10510.28046624332455204936034548209098100,  
10510.36313884716609711097874014799364018,  
10510.33729983869089110286014688393010861,  
10510.33214580432265904772480155814019098,  
10510.30529788714940572843960125133856867,  
10510.21747048914165901330972513637924528,  
10510.27430496981100157469317161964690000,  
10510.19163019570356986533345516430023915,  
10510.26914997074589424831897502598040405,  
10510.27430450906219327904070416811660165,  
10510.24330971168080111861301397506512564,  
10510.33113809515686669956767541801888818, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none]

Start Generation 4

1 --> 0 target = [23888989.06496329394952609057529687864997,  
2.632525136930218680532530824723288224680,  
10510.27430837454741040223911986003725455]  
one interval r = 24835338.36140343196080089179215412905611 ..  
2485026759661113911728280278283690695739/100000000000000000000000000000  
000

Time Approximations 0.04.

hint used Hint := [24850262.33855500957466342744981057733590, 3, 1, 1,  
20396437.98349656584284330028372043025521,  
24850238.00203928112673210949390165384854 ..  
99401070386444582936187413117559617381/400000000000000000000000000000,  
298960418182500/22468879468420441 ..  
23888989.06496329485872911197089488868350, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.533028) | P <--- S  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=  
5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850262.33855500957466342744981057733590,  
rm=20396437.98349656584284330028372043025521}, {r =  
24850238.00203928112673210949390165384854 ..  
24850267.59661114573404685327938990434525, rm =  
.1330553304194287328500223794129351168576e-1 ..



```
23888989.06496329485872911197089488868350}, avoid={}));
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=1e-32
Equations at solution: [-.2e-31, -.1e-31, -.12e-34]
Solution in 0.393s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 1.131r=2.48503e+07 in  
[24850238.00203928112673210949390165384854 ..  
99401070386444582936187413117559617381/4000000000000000000000000000000]  
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888989.06496329485872911197089488868350]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678919554814622893210528524039,
10510.36930058679635290810545398076470715,
10510.39514076206256012794571996982881491,
10510.28046624332455204936034548209098100,
10510.36313884716609711097874014799364018,
10510.33729983869089110286014688393010861,
10510.33214580432265904772480155814019098,
10510.30529788714940572843960125133856867,
10510.21747048914165901330972513637924528,
10510.27430496981100157469317161964690000,
10510.19163019570356986533345516430023915,
10510.26914997074589424831897502598040405,
10510.27430450906219327904070416811660165,
10510.24330971168080111861301397506512564,
10510.33113809515686669956767541801888818, none,
10510.18546892755744392593297628891275469, none, none, none, none,
none, none, none, none, none, none, none, none, none]
```

[illegible]

```

hint used Hint := [25795892.97949951637755150436992208534955, 3, 1, 1,
22860366.33048982198263378361971969356546,
25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/2500000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
23888989.06496329485872911197089488868350, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) |      P  <--- S
rGuessMin=2.57811e+07      rGuessMax=2.57959e+07      rmGuess=2.28604e+07      k=
4.31731e+14      scos=4.39725e+14
branch   outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795892.97949951637755150436992208534955,
rm=22860366.33048982198263378361971969356546}, {r =
25795867.25701511448801980095492062593890 ..
25795896.68746666518951237477219502520548, rm =

```























```

000
Time Approximations 0.047.

hint used Hint := [23888981.93694517027845377679197924121306, 3, -1, 1,
22860455.76745222490936139362415353022295,
23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/40000000000000000000000000000000,
298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.367179) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31730e+14 scos=4.39725e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888981.93694517027845377679197924121306,
rm=22860455.76745222490936139362415353022295}, {r =
23888955.94100364913487175812205942139954 ..
23888985.68416735818079539166696781319750, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={});
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=2e-32
Equations at solution: [.1e-31, .2e-31, -.1e-34]
Solution in 0.396s

Time Plot 0 s.
Exiting SolveHard() after 2.265r=2.38890e+07 in
[23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/40000000000000000000000000000000]
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678919554814622893210528524039,
10510.36930058679635290810545398076470715,
10510.39514076206256012794571996982881491,
10510.28046624332455204936034548209098100,
10510.36313884716609711097874014799364018,
10510.33729983869089110286014688393010861,
10510.33214580432265904772480155814019098,
10510.30529788714940572843960125133856867,
10510.21747048914165901330972513637924528,
10510.27430496981100157469317161964690000,
10510.19163019570356986533345516430023915,
10510.26914997074589424831897502598040405,
10510.27430450906219327904070416811660165,
10510.24330971168080111861301397506512564,
10510.33113809515686669956767541801888818,
10510.21646359943461300203958642268709693,
10510.18546892755744392593297628891275469,
10510.21130922509350515006590852752884631,
10510.10279600701249766009483831343723558, none,
10510.27329709849505267889797414715290499,
10510.15447580435374026700144774421538664, none,
10510.15962933503510206868081977023563836, none, none, none, none,
none, none, none]

1 --> 2 target = [25795893.34215400762409308209675369309939,

```







```

10510.27329709849505267889797414715290499,
10510.15447580435374026700144774421538664, none,
10510.15962933503510206868081977023563836, none, none, none, none,
none, none, none]

2 --> 1 target = [24850265.76205358503533959855587505305733,
1.476948574925856680572969041966222357689,
10510.24331300770304604967508457407049917]
one interval r = 25781101.21410471325620950907723481107840 ..
2579589486307047502110585783620135487639/1000000000000000000000000000000
000
Time Approximations 0.032.

hint used Hint := [25795892.97947942498571214559240875822627, 3, -1, 1,
24743526.20943107487634918939497374544011,
25795865.43257821616570179673528437159660 ..
644897371576761841392917646201273823237/25000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
24850265.76205359199870736703593497152527, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.76017) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=
-2.37384e+14 scos=5.95462e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25795892.97947942498571214559240875822627,
rm=24743526.20943107487634918939497374544011}, {r =
25795865.43257821616570179673528437159660 ..
25795894.86307047365571670584805095292948, rm =
.1330553304194287328500223794129351168576e-1 ..
24850265.76205359199870736703593497152527}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.4949e-31, 0., .4e-35]
Solution in 0.253s

Time Plot 0 s.
Exiting SolveHard() after 0.774r=2.57959e+07 in
[25795865.43257821616570179673528437159660 ..
644897371576761841392917646201273823237/25000000000000000000000000000000
0]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 24850265.76205359199870736703593497152527]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678919554814622893210528524039,
10510.36930058679635290810545398076470715,
10510.39514076206256012794571996982881491,
10510.28046624332455204936034548209098100,
10510.36313884716609711097874014799364018,
10510.33729983869089110286014688393010861,
10510.33214580432265904772480155814019098,
10510.30529788714940572843960125133856867,
10510.21747048914165901330972513637924528,
10510.27430496981100157469317161964690000,
10510.19163019570356986533345516430023915,

```















```

1242513364573987488725142465429694354713/50000000000000000000000000000000
00]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 25089000]: target and source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678919554814622893210528524039,
10510.36930058679635290810545398076470715,
10510.39514076206256012794571996982881491,
10510.28046624332455204936034548209098100,
10510.36313884716609711097874014799364018,
10510.33729983869089110286014688393010861,
10510.33214580432265904772480155814019098,
10510.30529788714940572843960125133856867,
10510.21747048914165901330972513637924528,
10510.27430496981100157469317161964690000,
10510.19163019570356986533345516430023915,
10510.26914997074589424831897502598040405,
10510.27430450906219327904070416811660165,
10510.24330971168080111861301397506512564,
10510.33113809515686669956767541801888818,
10510.21646359943461300203958642268709693,
10510.18546892755744392593297628891275469,
10510.21130922509350515006590852752884631,
10510.10279600701249766009483831343723558,
10510.18546846168250345977098236774028140,
10510.27329709849505267889797414715290499,
10510.15447580435374026700144774421538664,
10510.23714797602817887135503713448105101,
10510.15962933503510206868081977023563836,
10510.15447547796812145020022788718401390, none,
10510.21130888375386467458135417510849341,
10510.20615520656355476094062103409590260, none, none, none]

0 --> 2 target = [25795900.03265730830925765697578468041734,
1.856308491723391116783272033610498780026,
10510.33114121314573822118436109365656370]
one interval r = 23873936.95014365151089923920892033284293 ..
2388899244562496888525550581095322805007/10000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [23888988.69843567795653101185231580305160, 3, -1, 1,
22860452.96192886545581288218987601576608,
23888962.70280479206299436081506076835447 ..
2388899244562496946280763877837409340903/10000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.36718) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31731e+14 scos=4.39724e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888988.69843567795653101185231580305160,
rm=22860452.96192886545581288218987601576608}, {r =
23888962.70280479206299436081506076835447 ..

```















```
2 --> 0   target = [23888989.06493588654959803405475247961310,  
2.632525136851775294078647116623349467978,  
10510.27430791379860197863530493176805504]  
one interval r = 25781103.05751676952386576924166409908840 ..  
322448708592994338889641166277683766317/12500000000000000000000000  
0  
Time Approximations 0.03.
```

```
Time Plot 0 s.  
Exiting SolveHard() after 1.953r=2.57959e+07 in  
[25795867.25698799409451243993168295190626 ..  
1289794834371977270055988053158890743643/50000000000000000000000000000000  
00]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23888989.06493588745880515057802801897934]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

Tau [10510.45813678919554814622893210528524039,



10510.36930058679635290810545398076470715,  
10510.39514076206256012794571996982881491,  
10510.28046624332455204936034548209098100,  
10510.36313884716609711097874014799364018,  
10510.33729983869089110286014688393010861,  
10510.33214580432265904772480155814019098,  
10510.30529788714940572843960125133856867,  
10510.21747048914165901330972513637924528,  
10510.27430496981100157469317161964690000,  
10510.19163019570356986533345516430023915,  
10510.26914997074589424831897502598040405,  
10510.27430450906219327904070416811660165,  
10510.24330971168080111861301397506512564,  
10510.33113809515686669956767541801888818,  
10510.21646359943461300203958642268709693,  
10510.18546892755744392593297628891275469,  
10510.21130922509350515006590852752884631,  
10510.10279600701249766009483831343723558,  
10510.18546846168250345977098236774028140,  
10510.27329709849505267889797414715290499,  
10510.15447580435374026700144774421538664,  
10510.23714797602817887135503713448105101,  
10510.15962933503510206868081977023563836,  
10510.15447547796812145020022788718401390,  
10510.29913613971161912784447391670387170,  
10510.21130888375386467458135417510849341,  
10510.20615520656355476094062103409590260,  
10510.18546846680903701396464791451215711,  
10510.21130876434540469028086772641642732,  
10510.268143235743161241457500450274031681

```
Cascade time 51.74
counts: 28, 28
```

Iteration 8

Start Generation 1

```
1 --> 0   target = [23889000.00000000016250608146246437617650,
2.632556434672781343330486851629856645742,
10510.45814019398008722970263023232563935]
one interval r = 24835349.43352072251286216660644188407598 ..
1242513923859262816019712757860537437981/500000000000000000000000000000
00
```

Time Approximations 0.039.

```
hint used Hint := [24850273.21919080441727298772802245902434, 3, 1, 1,  
20396435.39659592643000634328381896732011,  
24850248.88301387793152980010731189259064 ..  
621256961929631488519759813441433737669/250000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 23889000., 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.533028) | P <--- S  
rGuessMin=2.48353e+07    rGuessMax=2.48503e+07    rmGuess=2.03964e+07    k=  
5.43204e+14    scos=2.39474e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850273.21919080441727298772802245902434,  
rm=20396435.39659592643000634328381896732011}, {r=
```







. 23889000.]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678926518707127196888424121963,  
10510.36930058676122375611578715846972933,  
10510.39514076201519009600797947282923323, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none]

Start Generation 2  
2 --> 1 target = [24850273.21919080075959117667919642663199,  
1.476968812016060435279337651588686048639,  
10510.36930388284801099631932735000276593]  
one interval r = 25781108.70735775458818361277456089184102 ..  
1289795113945692664813015212188713172819/500000000000000000000000000000  
00  
Time Approximations 0.029.

hint used Hint := [25795900.39533807920941978281062023785801, 3, -1, 1,  
24743535.44822274617462265333263319921312,  
25795872.84858705064741864171344489129222 ..  
2579590227891383795486680697763551821623/100000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
24850273.21919080441727298772802245902434, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.760172) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=  
-2.37383e+14 scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=25795900.39533807920941978281062023785801,  
rm=24743535.44822274617462265333263319921312}, {r =  
25795872.84858705064741864171344489129222 ..  
25795902.27891383795486680697763551821623, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850273.21919080441727298772802245902434}, avoid={}));  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.9898e-31, 0., -.33e-34]  
Solution in 0.274s

Time Plot 0 s.  
Exiting SolveHard() after 0.777r=2.57959e+07 in  
[25795872.84858705064741864171344489129222 ..  
2579590227891383795486680697763551821623/100000000000000000000000000000  
000]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 24850273.21919080441727298772802245902434]: target and source on the  
different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678926518707127196888424121963,  
10510.36930058676122375611578715846972933,  
10510.39514076201519009600797947282923323, none, none,  
10510.33729983862872857532591939569402997, none, none, none, none,



[illegible]







```
(Scattering) fsolve(eqs, {r=24850272.85449189132526402658550193546657,
rm=24743518.84163067386800243860950511726701}, {r =
24850245.15428919258543501803067780064530 ..
24850274.74859781637457312443003090128746, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}), avoid={});
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.27218e-30, 0., -.45504612886e-34]
Solution in 0.307s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.956r=2.48503e+07 in
[24850245.15428919258543501803067780064530 ..
1242513737429890818728656221501545064373/500000000000000000000000000000
00]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 25089000]: target and source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678926518707127196888424121963,
10510.36930058676122375611578715846972933,
10510.39514076201519009600797947282923323,
10510.28046624339154590064771993823712869,
10510.36313884713021665188144463272333517,
10510.33729983862872857532591939569402997,
10510.33214580440369413419785660105277125, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]
```

```
Start Generation 3
0 --> 2 target = [25795900.39533809406455964217601124817138,
1.856309428741226677870841293295704832789,
10510.33730295666933479840367973639762395]
one interval r = 23873937.32565211561104433853890165995510 ..
1194449640607540114382415971506659344879/500000000000000000000000000000
00
Time Approximations 0.047.
```

```
hint used Hint := [23888989.06496329485872911197089488868350, 3, -1, 1,
22860452.80984339454018821965990898965218,
23888963.06934924828335479031706673264809 ..
597224820303770076400571671810948623397/250000000000000000000000000000
0, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.36718) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31731e+14 scos=4.39724e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888989.06496329485872911197089488868350,
rm=22860452.80984339454018821965990898965218}, {r =
23888963.06934924828335479031706673264809 ..
23888992.81215080305602286687243794493588, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={});
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=4e-32
Equations at solution: [.5e-31, .4e-31, .11e-34]
Solution in 0.392s
```



Time Plot 0 s.  
Exiting SolveHard() after 2.345r=2.38890e+07 in  
[23888963.06934924828335479031706673264809 ..  
597224820303770076400571671810948623397/2500000000000000000000000000000000  
0]  
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678926518707127196888424121963,  
10510.36930058676122375611578715846972933,  
10510.39514076201519009600797947282923323,  
10510.28046624339154590064771993823712869,  
10510.36313884713021665188144463272333517,  
10510.33729983862872857532591939569402997,  
10510.33214580440369413419785660105277125, none, none,  
10510.27430496987724418865164365643260621, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none]

1 --> 2 target = [25795900.39533809406455964217601124817138,  
1.856309428741226677870841293295704832789,  
10510.33730295666933479840367973639762395]  
one interval r = 24835342.15555716919151795375633108447494 ..  
1242513566256462705094072800377701092657/5000000000000000000000000000000000  
00

Time Approximations 0.042.

hint used Hint := [24850269.43101309861241733616879334665430, 3, 1, 1,  
24743514.60014700975744022125022662330031,  
24850241.73069462631526237165506015616915 ..  
1242513566256462912427735010225393921567/5000000000000000000000000000000000  
00, 298960418182500/22468879468420441 .. 25089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.239829) | S ---> P  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=  
2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850269.43101309861241733616879334665430,  
rm=24743514.60014700975744022125022662330031}, {r =  
24850241.73069462631526237165506015616915 ..  
24850271.32512925824855470020450787843134, rm =  
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.34641e-30, 0., -.91288734911e-34]  
Solution in 0.322s

Time Plot 0 s.  
Exiting SolveHard() after 1.048r=2.48503e+07 in  
[24850241.73069462631526237165506015616915 ..  
1242513566256462912427735010225393921567/5000000000000000000000000000000000  
00]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 25089000]: target and source on the different branches.  
Counterclockwise ray.



Ray outgoing at target.  
Solve Side.

Tau [10510.45813678926518707127196888424121963,  
10510.36930058676122375611578715846972933,  
10510.39514076201519009600797947282923323,  
10510.28046624339154590064771993823712869,  
10510.36313884713021665188144463272333517,  
10510.33729983862872857532591939569402997,  
10510.33214580440369413419785660105277125,  
10510.30529788709873276570567574657469706, none,  
10510.27430496987724418865164365643260621, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none]

1 --> 0 target = [23888989.43146328572536290105672924473362,  
2.632526185928895211853318931904504437411,  
10510.28046964805710642292076532446233873]  
one interval r = 24835338.73249666169564695150841126971551 ..  
497005359225697026030995824619125809119/2000000000000000000000000000000000  
0

Time Approximations 0.035.

hint used Hint := [24850262.70323078644925875505002433594091, 3, 1, 1,  
20396437.89679703248883954261208042438711,  
24850238.36672641289161845524403523557614 ..  
2485026796128485539651696969285982263711/1000000000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
23888989.43146328644175807501347600997649, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.533028) | P <--- S  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=  
5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850262.70323078644925875505002433594091,  
rm=20396437.89679703248883954261208042438711}, {r =  
24850238.36672641289161845524403523557614 ..  
24850267.96128485539651696969285982263711, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.43146328644175807501347600997649}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=0  
Equations at solution: [0., 0., -.15289962289e-34]  
Solution in 0.365s

Time Plot 0 s.

Exiting SolveHard() after 2.167r=2.48503e+07 in  
[24850238.36672641289161845524403523557614 ..  
2485026796128485539651696969285982263711/1000000000000000000000000000000000  
000]

Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888989.43146328644175807501347600997649]: target and source on the  
different branches.

Counterclockwise ray.

Ray outgoing at target.

Solve Side.

Tau [10510.45813678926518707127196888424121963,







[illegible]

```
Time Plot 0 s.  
Exiting SolveHard() after 1.124r=2.48503e+07 in  
[24850241.42564446195929857082855634229261 ..  
1242513551004516068014443586172817582043/50000000000000000000000000000000  
00]  
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888992.50558472677941562199764206716244]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

Tau [10510.45813678926518707127196888424121963,  
10510.36930058676122375611578715846972933,  
10510.39514076201519009600797947282923323,  
10510.28046624339154590064771993823712869,  
10510.36313884713021665188144463272333517,  
10510.33729983862872857532591939569402997,







```
10510.30529788709873276570567574657469706,  
10510.21747048909164391277516194664050661,  
10510.27430496987724418865164365643260621,  
10510.19163019566579564173052818862079559,  
10510.26914997070992036108035443708359666, none,  
10510.24330971165706811801077111654645684, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none]
```

```
2 --> 1 target = [24850272.85449188757753849461645403701232,  
1.476967822296915528471105686065463406828,  
10510.36314214321535789978493171963555508]  
one interval r = 25781108.34089178863242405560719116205299 ..  
1289795095811691402788232230995304818381/500000000000000000000000000000  
00
```

Time Approximations 0.03.

```
hint used Hint := [25795900.03265730627104171495990540839944, 3, -1, 1,  
24743534.99638995296918328323923452726440,  
25795872.48589893272346549352544011957533 ..  
644897547905845320097763426011396162913/250000000000000000000000000000  
0, 298960418182500/22468879468420441 ..  
24850272.85449189132526402658550193546657, 1]
```

I search for an scattering ray on opposite branches with  $0 < sv < 1$

```
(0.760172) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=  
-2.37383e+14 scos=5.95462e+14
```

branch outgoing at target, Clockwise

```
(Scattering) fsolve(eqs, {r=25795900.03265730627104171495990540839944,  
rm=24743534.99638995296918328323923452726440}, {r =  
25795872.48589893272346549352544011957533 ..  
25795901.91623381280391053704045584651652, rm =  
.1330553304194287328500223794129351168576e-1 ..
```

```
24850272.85449189132526402658550193546657}, avoid={});
```

Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0

Equations at solution: [.9898e-31, 0., -.28e-34]

Solution in 0.265s

Time Plot 0 s.

Exiting SolveHard() after 0.787r=2.57959e+07 in

```
[25795872.48589893272346549352544011957533 ..  
644897547905845320097763426011396162913/250000000000000000000000000000  
0]
```

Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 24850272.85449189132526402658550193546657]: target and source on the  
different branches.

Clockwise ray.

Ray outgoing at target.

Solve Side.

```
Tau [10510.45813678926518707127196888424121963,  
10510.36930058676122375611578715846972933,  
10510.39514076201519009600797947282923323,  
10510.28046624339154590064771993823712869,  
10510.36313884713021665188144463272333517,  
10510.33729983862872857532591939569402997,  
10510.33214580440369413419785660105277125,
```



```
10510.30529788709873276570567574657469706,  
10510.21747048909164391277516194664050661,  
10510.27430496987724418865164365643260621,  
10510.19163019566579564173052818862079559,  
10510.26914997070992036108035443708359666, none,  
10510.24330971165706811801077111654645684,  
10510.33113809509395286459168065131751488, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none]
```

```
0 --> 1 target = [24850272.85449188757753849461645403701232,  
1.476967822296915528471105686065463406828,  
10510.36314214321535789978493171963555508]  
one interval r = 23873938.90033936085256176013504641566687 ..  
4777798869833866400131352588611537037/20000000000000000000000000000000  
Time Approximations 0.043.
```

```
hint used Hint := [23888989.06493588745880515057802801897934, 3, -1, 1,  
20396631.19196530471597662319541189563767,  
23888964.60644587167575168019887114912430 ..  
1194449717458466618404658405105548818961/50000000000000000000000000000000  
00, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.466972) | S --> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=  
-5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888989.06493588745880515057802801897934,  
rm=20396631.19196530471597662319541189563767}, {r =  
23888964.60644587167575168019887114912430 ..  
23888994.34916933236809316810211097637922, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={});  
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=2e-32  
Equations at solution: [.3e-31, .2e-31, -.21e-34]  
Solution in 0.351s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 2.28r=2.38890e+07 in  
[23888964.60644587167575168019887114912430 ..  
1194449717458466618404658405105548818961/50000000000000000000000000000000  
00]  
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678926518707127196888424121963,  
10510.36930058676122375611578715846972933,  
10510.39514076201519009600797947282923323,  
10510.28046624339154590064771993823712869,  
10510.36313884713021665188144463272333517,  
10510.33729983862872857532591939569402997,  
10510.33214580440369413419785660105277125,  
10510.30529788709873276570567574657469706,  
10510.21747048909164391277516194664050661,  
10510.27430496987724418865164365643260621,  
10510.19163019566579564173052818862079559,
```











```

10510.21747048909164391277516194664050661,
10510.27430496987724418865164365643260621,
10510.19163019566579564173052818862079559,
10510.26914997070992036108035443708359666,
10510.27430450912843582386935196277626326,
10510.24330971165706811801077111654645684,
10510.33113809509395286459168065131751488, none,
10510.18546892751891846564997845450320358,
10510.21130922504273881339453018069691595, none, none, none, none,
none, none, none, none, none, none, none, none, none]

2 --> 1 target = [24850269.43101309397673781930025830936021,
1.476958531687824955807847892835703869068,
10510.30530118316842289262103048223989770]
one interval r = 25781104.90082929839196506233863778710498 ..
2579589851170977102053322268485659722047/1000000000000000000000000000000
000
Time Approximations 0.032.

hint used Hint := [25795896.62812623062600675830146784169746, 3, -1, 1,
24743530.75497470617154487139175541092326,
25795869.08129891002682354003571513818579 ..
2579589851170975665166085604166659164517/1000000000000000000000000000000
000, 298960418182500/22468879468420441 ..
24850269.43101309861241733616879334665430, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.760171) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=
-2.37384e+14 scos=5.95462e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25795896.62812623062600675830146784169746,
rm=24743530.75497470617154487139175541092326}, {r =
25795869.08129891002682354003571513818579 ..
25795898.51170975665166085604166659164517, rm =
.1330553304194287328500223794129351168576e-1 ..
24850269.43101309861241733616879334665430}, avoid={{}});
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.7423e-31, 0., .31e-34]
Solution in 0.28s

Time Plot 0 s.
Exiting SolveHard() after 0.847r=2.57959e+07 in
[25795869.08129891002682354003571513818579 ..
2579589851170975665166085604166659164517/1000000000000000000000000000000
000]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 24850269.43101309861241733616879334665430]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678926518707127196888424121963,
10510.36930058676122375611578715846972933,
10510.39514076201519009600797947282923323,
10510.28046624339154590064771993823712869,
10510.36313884713021665188144463272333517,

```



```

10510.33729983862872857532591939569402997,
10510.33214580440369413419785660105277125,
10510.30529788709873276570567574657469706,
10510.21747048909164391277516194664050661,
10510.27430496987724418865164365643260621,
10510.19163019566579564173052818862079559,
10510.26914997070992036108035443708359666,
10510.27430450912843582386935196277626326,
10510.24330971165706811801077111654645684,
10510.33113809509395286459168065131751488, none,
10510.18546892751891846564997845450320358,
10510.21130922504273881339453018069691595, none, none,
10510.27329709841734633224514633984300021, none, none, none, none,
none, none, none, none, none, none]

0 --> 1 target = [24850269.43101309397673781930025830936021,
1.476958531687824955807847892835703869068,
10510.30530118316842289262103048223989770]
one interval r = 23873935.37540036304539203446029523847069 ..
1194449545427563512421431683801687361999/500000000000000000000000000000
00
Time Approximations 0.047.

hint used Hint := [23888985.62428976346864819081258538077918, 3, -1, 1,
20396632.00572718336484608918971225516342,
23888961.16565299817948948119449178210506 ..
2388899090855127150824991682392263419777/1000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=
-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888985.62428976346864819081258538077918,
rm=20396632.00572718336484608918971225516342}, {r =
23888961.16565299817948948119449178210506 ..
23888990.90855127150824991682392263419777, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=2e-32
Equations at solution: [-.3e-31, -.2e-31, .21e-34]
Solution in 0.383s

Time Plot 0 s.
Exiting SolveHard() after 2.311r=2.38890e+07 in
[23888961.16565299817948948119449178210506 ..
2388899090855127150824991682392263419777/1000000000000000000000000000000
000]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678926518707127196888424121963,
10510.36930058676122375611578715846972933,
10510.39514076201519009600797947282923323,
10510.28046624339154590064771993823712869,
```



```

10510.36313884713021665188144463272333517,  

10510.33729983862872857532591939569402997,  

10510.33214580440369413419785660105277125,  

10510.30529788709873276570567574657469706,  

10510.21747048909164391277516194664050661,  

10510.27430496987724418865164365643260621,  

10510.19163019566579564173052818862079559,  

10510.26914997070992036108035443708359666,  

10510.27430450912843582386935196277626326,  

10510.24330971165706811801077111654645684,  

10510.33113809509395286459168065131751488,  

10510.21646359948606305606567843379303172,  

10510.18546892751891846564997845450320358,  

10510.21130922504273881339453018069691595, none, none,  

10510.27329709841734633224514633984300021, none, none, none, none,  

none, none, none, none, none, none]  
  

2 --> 1 target = [24850262.70323078191699517382577698456839,  

1.476940273994347076083347603219314115509,  

10510.19163349170512158155159581933014403]  

one interval r = 25781098.14047727297345426208664892995628 ..  

515917836423874533053101760983181764861/20000000000000000000000000  

0  

Time Approximations 0.032.  
  

hint used Hint := [25795889.93759639200267360170740297062535, 3, -1, 1,  

24743522.41979937237431350459260520845964,  

25795862.39063358446748128489577933480278 ..  

515917836423874243616569030246719914989/20000000000000000000000000  

0, 298960418182500/22468879468420441 ..  

24850262.70323078644925875505002433594091, 1]  

I search for an scattering ray on opposite branches with 0<sv<1  

(0.76017) | P <--- S  

rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=  

-2.37384e+14 scos=5.95461e+14  

branch outgoing at target, Clockwise  

(Scattering) fsolve(eqs, {r=25795889.93759639200267360170740297062535,  

rm=24743522.41979937237431350459260520845964}, {r =  

25795862.39063358446748128489577933480278 ..  

25795891.82119371218082845151233599574945, rm =  

.1330553304194287328500223794129351168576e-1 ..  

24850262.70323078644925875505002433594091}, avoid={}));  

Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  

Equations at solution: [.9897e-31, 0., -.23e-34]  

Solution in 0.338s  
  

Time Plot 0 s.  

Exiting SolveHard() after 2.132r=2.57959e+07 in  

[25795862.39063358446748128489577933480278 ..  

515917836423874243616569030246719914989/20000000000000000000000000  

0]  

Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  

. 24850262.70323078644925875505002433594091]: target and source on the  

different branches.  

Clockwise ray.  

Ray outgoing at target.  

Solve Side.
```



```

Tau [10510.45813678926518707127196888424121963,
10510.36930058676122375611578715846972933,
10510.39514076201519009600797947282923323,
10510.28046624339154590064771993823712869,
10510.36313884713021665188144463272333517,
10510.33729983862872857532591939569402997,
10510.33214580440369413419785660105277125,
10510.30529788709873276570567574657469706,
10510.21747048909164391277516194664050661,
10510.27430496987724418865164365643260621,
10510.19163019566579564173052818862079559,
10510.26914997070992036108035443708359666,
10510.27430450912843582386935196277626326,
10510.24330971165706811801077111654645684,
10510.33113809509395286459168065131751488,
10510.21646359948606305606567843379303172,
10510.18546892751891846564997845450320358,
10510.21130922504273881339453018069691595, none, none,
10510.27329709841734633224514633984300021, none, none,
10510.15962933497029447194779129608263675, none, none, none, none,
none, none, none]

0 --> 1 target = [24850262.70323078191699517382577698456839,
1.476940273994347076083347603219314115509,
10510.19163349170512158155159581933014403]
one interval r = 23873928.44821858630950754311535739841178 ..
1194449207353541112561101421458588060507/500000000000000000000000000000
00
Time Approximations 0.049.

hint used Hint := [23888978.86275416794827744619407470166172, 3, -1, 1,
20396633.60486737240849086583158368073964,
23888954.40382902066202638611670198664112 ..
298612301838385292588525592212900916387/1250000000000000000000000000000
0, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S --> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=
-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888978.86275416794827744619407470166172,
rm=20396633.60486737240849086583158368073964}, {r =
23888954.40382902066202638611670198664112 ..
23888984.14707082340708204737703207331096, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=2e-32
Equations at solution: [-.3e-31, -.2e-31, .36e-34]
Solution in 0.341s

Time Plot 0 s.
Exiting SolveHard() after 1.282r=2.38890e+07 in
[23888954.40382902066202638611670198664112 ..
298612301838385292588525592212900916387/1250000000000000000000000000000
0]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]:target and source on the different branches.
Clockwise ray.

```







Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678926518707127196888424121963,  
10510.36930058676122375611578715846972933,  
10510.39514076201519009600797947282923323,  
10510.28046624339154590064771993823712869,  
10510.36313884713021665188144463272333517,  
10510.33729983862872857532591939569402997,  
10510.33214580440369413419785660105277125,  
10510.30529788709873276570567574657469706,  
10510.21747048909164391277516194664050661,  
10510.27430496987724418865164365643260621,  
10510.19163019566579564173052818862079559,  
10510.26914997070992036108035443708359666,  
10510.27430450912843582386935196277626326,  
10510.24330971165706811801077111654645684,  
10510.33113809509395286459168065131751488,  
10510.21646359948606305606567843379303172,  
10510.18546892751891846564997845450320358,  
10510.21130922504273881339453018069691595,  
10510.10279600707684644191147089660043549, none,  
10510.27329709841734633224514633984300021,  
10510.15447580443213029003612237275817750, none,  
10510.15962933497029447194779129608263675, none, none, none, none,  
none, none, none]

1 --> 2 target = [25795893.34215402111076740723668408846545,  
1.856291208291270169522312413530010867971,  
10510.21747360710138899052080487730185946]  
one interval r = 24835334.93824082422955205635746440613141 ..  
1242513211633239662925943662415393485687/50000000000000000000000000000000  
00  
Time Approximations 0.036.

hint used Hint := [24850262.33852743516297758053367205548154, 3, 1, 1,  
24743505.81299238129247202614818714422035,  
24850234.63796912173141920432208117660707 ..  
2485026423266479739181704602953513976831/10000000000000000000000000000000  
000, 298960418182500/22468879468420441 .. 25089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.23983) | S ---> P  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=  
2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850262.33852743516297758053367205548154,  
rm=24743505.81299238129247202614818714422035}, {r =  
24850234.63796912173141920432208117660707 ..  
24850264.23266479739181704602953513976831, rm =  
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [-.24743e-30, 0., .49753153582e-34]  
Solution in 0.302s



















```

10510.20615520665598597538147487283497495, none, none, none]

1 --> 2 target = [25795896.38402525212404948708884134724596,
1.856299066329043604762794900601166738915,
10510.26915308873297510098005272181140872]
one interval r = 24835338.05090068566184863230606932482307 ..
1242513364573987321956164229510076420379/50000000000000000000000000000000
00
Time Approximations 0.04.

hint used Hint := [24850265.39735153163244735697452997613798, 3, 1, 1,
24743509.60268659785926677493114233979369,
24850237.69689665459305925255508686501973 ..
1242513364573987488725142465429694354713/50000000000000000000000000000000
00, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.239829) | S --> P
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=
2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850265.39735153163244735697452997613798,
rm=24743509.60268659785926677493114233979369}, {r =
24850237.69689665459305925255508686501973 ..
24850267.29147974977450284930859388709426, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.7423e-31, 0., .45308297969e-34]
Solution in 0.308s

Time Plot 0 s.
Exiting SolveHard() after 1.029r=2.48503e+07 in
[24850237.69689665459305925255508686501973 ..
1242513364573987488725142465429694354713/50000000000000000000000000000000
00]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 25089000]: target and source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678926518707127196888424121963,
10510.36930058676122375611578715846972933,
10510.39514076201519009600797947282923323,
10510.28046624339154590064771993823712869,
10510.36313884713021665188144463272333517,
10510.33729983862872857532591939569402997,
10510.33214580440369413419785660105277125,
10510.30529788709873276570567574657469706,
10510.21747048909164391277516194664050661,
10510.27430496987724418865164365643260621,
10510.19163019566579564173052818862079559,
10510.26914997070992036108035443708359666,
10510.27430450912843582386935196277626326,
10510.24330971165706811801077111654645684,
10510.33113809509395286459168065131751488,
10510.21646359948606305606567843379303172,
10510.18546892751891846564997845450320358,

```







```
1 --> 2   target = [25795900.03265732103664187508783430136736,  
1.856308491827932603118313482158562150383,  
10510.33114121313297217736295213885062566]  
one interval r = 24835341.78443702523869568768603310314442 ..  
621256774010731266688277956960122640789/2500000000000000000000000000  
0
```

```
hint used Hint := [24850269.06631200499486567135274096895840, 3, 1, 1,  
24743514.14830449237551739504443492551457,  
24850241.36598119958086914129896830053765 ..  
2485027096042925490424856544845137371367/1000000000000000000000000000000  
000, 298960418182500/22468879468420441 .. 25089000, 1]
```

```
Time Plot 0 s.  
Exiting SolveHard() after 2.135r=2.48503e+07 in  
[24850241.36598119958086914129896830053765 ..  
2485027096042925490424856544845137371367/1000000000000000000000000000000  
000]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 25089000]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.
```



Solve Side.

```
Tau [10510.45813678926518707127196888424121963,
10510.36930058676122375611578715846972933,
10510.39514076201519009600797947282923323,
10510.28046624339154590064771993823712869,
10510.36313884713021665188144463272333517,
10510.33729983862872857532591939569402997,
10510.33214580440369413419785660105277125,
10510.30529788709873276570567574657469706,
10510.21747048909164391277516194664050661,
10510.27430496987724418865164365643260621,
10510.19163019566579564173052818862079559,
10510.26914997070992036108035443708359666,
10510.27430450912843582386935196277626326,
10510.24330971165706811801077111654645684,
10510.33113809509395286459168065131751488,
10510.21646359948606305606567843379303172,
10510.18546892751891846564997845450320358,
10510.21130922504273881339453018069691595,
10510.10279600707684644191147089660043549,
10510.18546846164397793449143879199780960,
10510.27329709841734633224514633984300021,
10510.15447580443213029003612237275817750,
10510.23714797600369456707744336184815409,
10510.15962933497029447194779129608263675,
10510.15447547804651144283232643136775466,
10510.29913613966019485733464810272497579,
10510.21130888370309830855507402107126701,
10510.20615520665598597538147487283497495, none, none,
10510.26814323580865254916639910578633149]
```

```
1 --> 0 target = [23888989.06493588665191616257674587638457,  
2.632525136881592409693015604031449515540,  
10510.27430791379228521378681196558101636]  
one interval r = 24835338.36137568355161798828049343456341 ..  
621256689914596768086747458200177379087/25000000000000000000000000  
0  
Time Approximations 0.04.
```

```
hint used Hint := [24850262.33852773859450921827957797170818, 3, 1, 1,  
20396437.98350304935080318674410988058300,  
24850238.00201200929744389905914961384924 ..  
1242513379829193745424075414664369797603/500000000000000000000000000000  
00, 298960418182500/22468879468420441 ..  
23888989.06493588745880515057802801897934, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.533028) | P <--- S  
rGuessMin=2.48353e+07    rGuessMax=2.48503e+07    rmGuess=2.03964e+07    k=  
5.43203e+14    scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850262.33852773859450921827957797170818,  
rm=20396437.98350304935080318674410988058300}, {r =  
24850238.00201200929744389905914961384924 ..  
24850267.59658387490848150829328739595206, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.06493588745880515057802801897934}, avoid={{}});
```



Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=0  
Equations at solution: [0., 0., .3332588973e-35]  
Solution in 0.366s

Time Plot 0 s.  
Exiting SolveHard() after 1.061r=2.48503e+07 in  
[24850238.00201200929744389905914961384924 ..  
1242513379829193745424075414664369797603/500000000000000000000000000000  
00]  
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888989.06493588745880515057802801897934]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678926518707127196888424121963,  
10510.36930058676122375611578715846972933,  
10510.39514076201519009600797947282923323,  
10510.28046624339154590064771993823712869,  
10510.36313884713021665188144463272333517,  
10510.33729983862872857532591939569402997,  
10510.33214580440369413419785660105277125,  
10510.30529788709873276570567574657469706,  
10510.21747048909164391277516194664050661,  
10510.27430496987724418865164365643260621,  
10510.19163019566579564173052818862079559,  
10510.26914997070992036108035443708359666,  
10510.27430450912843582386935196277626326,  
10510.24330971165706811801077111654645684,  
10510.33113809509395286459168065131751488,  
10510.21646359948606305606567843379303172,  
10510.18546892751891846564997845450320358,  
10510.21130922504273881339453018069691595,  
10510.10279600707684644191147089660043549,  
10510.18546846164397793449143879199780960,  
10510.27329709841734633224514633984300021,  
10510.15447580443213029003612237275817750,  
10510.23714797600369456707744336184815409,  
10510.15962933497029447194779129608263675,  
10510.15447547804651144283232643136775466,  
10510.29913613966019485733464810272497579,  
10510.21130888370309830855507402107126701,  
10510.20615520665598597538147487283497495,  
10510.18546846677051148455188564106084332, none,  
10510.26814323580865254916639910578633149]

2 --> 0 target = [23888989.06493588665191616257674587638457,  
2.632525136881592409693015604031449515540,  
10510.27430791379228521378681196558101636]  
one interval r = 25781103.05751678276973932430334689833532 ..  
1289794834371978010909720391265708973499/500000000000000000000000000000  
00  
Time Approximations 0.03.

hint used Hint := [25795892.97947239655457473800077316699564, 3, 1, 1,  
22860366.33050119628977322107538139299992,



```
25795867.25698799409451243993168295190626 ..
1289794834371977270055988053158890743643/50000000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888989.06493588745880515057802801897934, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=
4.31731e+14 scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795892.97947239655457473800077316699564,
rm=22860366.33050119628977322107538139299992}, {r =
25795867.25698799409451243993168295190626 ..
25795896.68743954540111976106317781487286, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06493588745880515057802801897934}, avoid={}));
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=2.2e-31
Equations at solution: [-.23e-30, -.22e-30, .105e-33]
Solution in 0.262s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.928r=2.57959e+07 in
[25795867.25698799409451243993168295190626 ..
1289794834371977270055988053158890743643/50000000000000000000000000000000
00]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23888989.06493588745880515057802801897934]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678926518707127196888424121963,
10510.36930058676122375611578715846972933,
10510.39514076201519009600797947282923323,
10510.28046624339154590064771993823712869,
10510.36313884713021665188144463272333517,
10510.33729983862872857532591939569402997,
10510.33214580440369413419785660105277125,
10510.30529788709873276570567574657469706,
10510.21747048909164391277516194664050661,
10510.27430496987724418865164365643260621,
10510.19163019566579564173052818862079559,
10510.26914997070992036108035443708359666,
10510.27430450912843582386935196277626326,
10510.24330971165706811801077111654645684,
10510.33113809509395286459168065131751488,
10510.21646359948606305606567843379303172,
10510.18546892751891846564997845450320358,
10510.21130922504273881339453018069691595,
10510.10279600707684644191147089660043549,
10510.18546846164397793449143879199780960,
10510.27329709841734633224514633984300021,
10510.15447580443213029003612237275817750,
10510.23714797600369456707744336184815409,
10510.15962933497029447194779129608263675,
10510.15447547804651144283232643136775466,
10510.29913613966019485733464810272497579,
```



















. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678944374453044521541735838287,  
10510.36930058675752987385980790398847213,  
10510.39514076169947859885886603113460896,  
10510.28046624358022892604977046691205390, none,  
10510.33729983835493911580361103561920255,  
10510.33214580455274926090447258928235121, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none]

1 --> 2 target = [25795903.79985511086896954007804104979244,  
1.856318223503530852224880903839965517298,  
10510.39514387999521298105901805716377864]  
Imaginary part neglected:  
-.5585484964768019272613486624672638217295e-15  
one interval r = 24835345.63929295565691189118925503676032 ..  
124251373742989067274317501235529776783/50000000000000000000000000000000  
Time Approximations 0.043.

hint used Hint := [24850272.85449189132526402658550193546657, 3, 1, 1,  
24743518.84163067386800243860950511726701,  
24850245.15428919258543501803067780064530 ..  
1242513737429890818728656221501545064373/50000000000000000000000000000000  
00, 298960418182500/22468879468420441 .. 25089000, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.239828) | S --> P  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=  
2.37383e+14 scos=5.95462e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850272.85449189132526402658550193546657,  
rm=24743518.84163067386800243860950511726701}, {r =  
24850245.15428919258543501803067780064530 ..  
24850274.74859781637457312443003090128746, rm =  
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [-.19795e-30, 0., .3e-34]  
Solution in 1.445s

Time Plot 0 s.  
Exiting SolveHard() after 2.161r=2.48503e+07 in  
[24850245.15428919258543501803067780064530 ..  
1242513737429890818728656221501545064373/50000000000000000000000000000000  
00]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 25089000]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678944374453044521541735838287,  
10510.36930058675752987385980790398847213,  
10510.39514076169947859885886603113460896,  
10510.28046624358022892604977046691205390,







```
1.856309428602898735359820281561157794947,  
10510.33730295663577700841029012578768238]  
Imaginary part neglected:  
-.5585484964768019272613486624672638217295e-15  
one interval r = 24835342.15555717205846668041635107380078 ..  
497005426502585138443376432995228064621/2000000000000000000000000000000000  
0
```

Time Approximations 0.042.

```
hint used Hint := [24850269.43101309861241733616879334665430, 3, 1, 1,  
24743514.60014700975744022125022662330031,  
24850241.73069462631526237165506015616915 ..  
1242513566256462912427735010225393921567/5000000000000000000000000000000000  
00, 298960418182500/22468879468420441 .. 25089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.239829) | S ---> P  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=  
2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850269.43101309861241733616879334665430,  
rm=24743514.60014700975744022125022662330031}, {r =  
24850241.73069462631526237165506015616915 ..  
24850271.32512925824855470020450787843134, rm =  
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [-.39590e-30, 0., .6e-34]  
Solution in 0.345s
```

Time Plot 0 s.

```
Exiting SolveHard() after 2.251r=2.48503e+07 in  
[24850241.73069462631526237165506015616915 ..  
1242513566256462912427735010225393921567/5000000000000000000000000000000000  
00]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 25089000]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678944374453044521541735838287,  
10510.36930058675752987385980790398847213,  
10510.39514076169947859885886603113460896,  
10510.28046624358022892604977046691205390,  
10510.36313884712881705587711784745261010,  
10510.33729983835493911580361103561920255,  
10510.33214580455274926090447258928235121,  
10510.30529788713925523009926644841391280, none,  
10510.27430497006822128765885790927044806, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none]
```

```
1 --> 0 target = [23888989.43146328619827032860770676833783,  
2.632526185984013052954627707409538101101,  
10510.28046964800650349120882917245530085]  
Imaginary part neglected:  
-.5585484964768019272613486624672638217295e-15  
one interval r = 24835338.73249666353597787168573143801922 ..
```



2485026796128485311297994455704919717231/1000000000000000000000000000000000000  
000

Time Approximations 0.041.

hint used Hint := [24850262.70323078644925875505002433594091, 3, 1, 1,  
20396437.89679703248883954261208042438711,  
24850238.36672641289161845524403523557614 ..  
2485026796128485539651696969285982263711/1000000000000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
23888989.43146328644175807501347600997649, 1]  
I search for an scattering ray on opposite branches with  $0 < \text{sv} < 1$   
(0.533028) | P <--- S  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=  
5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850262.70323078644925875505002433594091,  
rm=20396437.89679703248883954261208042438711}, {r =  
24850238.36672641289161845524403523557614 ..  
24850267.96128485539651696969285982263711, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.43146328644175807501347600997649}, avoid={});  
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=6e-32  
Equations at solution: [-.11e-30, -.6e-31, .4e-34]  
Solution in 1.462s

Time Plot 0 s.

Exiting SolveHard() after 2.188r=2.48503e+07 in  
[24850238.36672641289161845524403523557614 ..  
2485026796128485539651696969285982263711/1000000000000000000000000000000000000  
000]  
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888989.43146328644175807501347600997649]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678944374453044521541735838287,  
10510.36930058675752987385980790398847213,  
10510.39514076169947859885886603113460896,  
10510.28046624358022892604977046691205390,  
10510.36313884712881705587711784745261010,  
10510.33729983835493911580361103561920255,  
10510.33214580455274926090447258928235121,  
10510.30529788713925523009926644841391280, none,  
10510.27430497006822128765885790927044806,  
10510.19163019567222731750326208892232669, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none,  
none, none, none, none]

2 --> 0 target = [23888989.43146328619827032860770676833783,  
2.632526185984013052954627707409538101101,  
10510.28046964800650349120882917245530085]  
one interval r = 25781103.42398366235178390231547081965581 ..  
2579589705012068847100484171110160138381/1000000000000000000000000000000000000  
000

Time Approximations 0.034.







```

hint used Hint := [24850265.76205359199870736703593497152527, 3, 1, 1,
20396437.16957127706091945345483725838621,
24850241.42564446195929857082855634229261 ..
1242513551004516068014443586172817582043/50000000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888992.50558472677941562199764206716244, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.533028) | P <--- S
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=
5.43203e+14 scos=2.39475e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850265.76205359199870736703593497152527,
rm=20396437.16957127706091945345483725838621}, {r =
24850241.42564446195929857082855634229261 ..
24850271.02009032136028887172345635164086, rm =
.1330553304194287328500223794129351168576e-1 ..
23888992.50558472677941562199764206716244}, avoid={}));
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=1.2e-31
Equations at solution: [-.21e-30, -.12e-30, .8e-34]
Solution in 1.478s

```

```

Time Plot 0 s.
Exiting SolveHard() after 2.198r=2.48503e+07 in
[24850241.42564446195929857082855634229261 ..
1242513551004516068014443586172817582043/50000000000000000000000000000000
00]
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .
. 23888992.50558472677941562199764206716244]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

```

```

Tau [10510.45813678944374453044521541735838287,
10510.36930058675752987385980790398847213,
10510.39514076169947859885886603113460896,
10510.28046624358022892604977046691205390,
10510.36313884712881705587711784745261010,
10510.33729983835493911580361103561920255,
10510.33214580455274926090447258928235121,
10510.30529788713925523009926644841391280,
10510.21747048878605796346581018313544152,
10510.27430497006822128765885790927044806,
10510.19163019567222731750326208892232669, none, none,
10510.24330971162387192942949247759505618, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]

```

```

2 --> 0 target = [23888992.50558472498059389927314996991724,
2.632534984529020827058537770935070288839,
10510.33214920899337539602932613578496613]
one interval r = 25781106.49760329060116185442860284507188 ..
2579590009198803788923857108647323525691/10000000000000000000000000000000
000
Time Approximations 0.036.

```

```

hint used Hint := [25795896.38402523646204355452114396262515, 3, 1, 1,

```



```

22860364.90258708217946452135489353704915,
25795870.66161245800881604810046494170563 ..
2579590009198804353979547140796504033263/1000000000000000000000000000000
000, 298960418182500/22468879468420441 ..
23888992.50558472677941562199764206716244, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.63282) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=
4.31731e+14 scos=4.39724e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795896.38402523646204355452114396262515,
rm=22860364.90258708217946452135489353704915}, {r =
25795870.66161245800881604810046494170563 ..
25795900.09198804353979547140796504033263, rm =
.1330553304194287328500223794129351168576e-1 ..
23888992.50558472677941562199764206716244}, avoid={}));
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=2e-32
Equations at solution: [-.2e-31, -.2e-31, .22e-34]
Solution in 0.293s

```

```

Time Plot 0 s.
Exiting SolveHard() after 0.872r=2.57959e+07 in
[25795870.66161245800881604810046494170563 ..
2579590009198804353979547140796504033263/1000000000000000000000000000000
000]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23888992.50558472677941562199764206716244]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

```

```

Tau [10510.45813678944374453044521541735838287,
10510.36930058675752987385980790398847213,
10510.39514076169947859885886603113460896,
10510.28046624358022892604977046691205390,
10510.36313884712881705587711784745261010,
10510.33729983835493911580361103561920255,
10510.33214580455274926090447258928235121,
10510.30529788713925523009926644841391280,
10510.21747048878605796346581018313544152,
10510.27430497006822128765885790927044806,
10510.19163019567222731750326208892232669,
10510.26914997036470657477894550062699048, none,
10510.24330971162387192942949247759505618, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]

```

```

2 --> 1 target = [24850272.85449188922509982477469451222955,
1.476967822426446754652120160403357593323,
10510.36314214316198650608628035850568916]
one interval r = 25781108.34089176925203718406476611948329 ..
32244877395292261093401675067742743047/1250000000000000000000000000000
Time Approximations 0.033.

```

```

hint used Hint := [25795900.03265730627104171495990540839944, 3, -1, 1,
24743534.99638995296918328323923452726440,

```



```
25795872.48589893272346549352544011957533 ..
644897547905845320097763426011396162913/2500000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
24850272.85449189132526402658550193546657, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.760172) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=
-2.37383e+14 scos=5.95462e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25795900.03265730627104171495990540839944,
rm=24743534.99638995296918328323923452726440}, {r =
25795872.48589893272346549352544011957533 ..
25795901.91623381280391053704045584651652, rm =
.1330553304194287328500223794129351168576e-1 ..
24850272.85449189132526402658550193546657}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.12372e-30, 0., .59e-34]
Solution in 1.414s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.003r=2.57959e+07 in
[25795872.48589893272346549352544011957533 ..
644897547905845320097763426011396162913/2500000000000000000000000000000000
0]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 24850272.85449189132526402658550193546657]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678944374453044521541735838287,
10510.36930058675752987385980790398847213,
10510.39514076169947859885886603113460896,
10510.28046624358022892604977046691205390,
10510.36313884712881705587711784745261010,
10510.33729983835493911580361103561920255,
10510.33214580455274926090447258928235121,
10510.30529788713925523009926644841391280,
10510.21747048878605796346581018313544152,
10510.27430497006822128765885790927044806,
10510.19163019567222731750326208892232669,
10510.26914997036470657477894550062699048, none,
10510.24330971162387192942949247759505618,
10510.33113809482245769237099311008012022, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
0 --> 1 target = [24850272.85449188922509982477469451222955,
1.476967822426446754652120160403357593323,
10510.36314214316198650608628035850568916]
one interval r = 23873938.90033936115988965730142683574505 ..
2388899434916933230888398864644393303599/1000000000000000000000000000000000
000
Time Approximations 0.05.
```

```
hint used Hint := [23888989.06493588745880515057802801897934, 3, -1, 1,
20396631.19196530471597662319541189563767,
```



```
23888964.60644587167575168019887114912430 ..
1194449717458466618404658405105548818961/50000000000000000000000000000000
00, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S --> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=
-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888989.06493588745880515057802801897934,
rm=20396631.19196530471597662319541189563767}, {r =
23888964.60644587167575168019887114912430 ..
23888994.34916933236809316810211097637922, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=2.4e-31
Equations at solution: [.43e-30, .24e-30, -.116e-33]
Solution in 0.4s
```

```
Time Plot 0 s.
Exiting SolveHard() after 1.317r=2.38890e+07 in
[23888964.60644587167575168019887114912430 ..
1194449717458466618404658405105548818961/50000000000000000000000000000000
00]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678944374453044521541735838287,
10510.36930058675752987385980790398847213,
10510.39514076169947859885886603113460896,
10510.28046624358022892604977046691205390,
10510.36313884712881705587711784745261010,
10510.33729983835493911580361103561920255,
10510.33214580455274926090447258928235121,
10510.30529788713925523009926644841391280,
10510.21747048878605796346581018313544152,
10510.27430497006822128765885790927044806,
10510.19163019567222731750326208892232669,
10510.26914997036470657477894550062699048,
10510.27430450931941313354587261104687761,
10510.24330971162387192942949247759505618,
10510.33113809482245769237099311008012022, none, none, none, none,
none, none, none, none, none, none, none, none, none, none]
```

```
Start Generation 4
1 --> 0 target = [23888989.06496329466121760605347299066554,
2.632525137015154027830541403106823593093,
10510.27430837449278484841431579860341504]
Imaginary part neglected:
-.5585484964768019272613486624672638217295e-15
one interval r = 24835338.36140343641581661907146267869442 ..
1242513379830557174812803153660447681/50000000000000000000000000000000
Time Approximations 0.042.
```

```
hint used Hint := [24850262.33855500957466342744981057733590, 3, 1, 1,
20396437.98349656584284330028372043025521,
```







```
22860366.33048982198263378361971969356546,  
25795867.25701511448801980095492062593890 ..  
644897417186666629737809369304875630137/2500000000000000000000000000000000  
0, 298960418182500/22468879468420441 ..  
23888989.06496329485872911197089488868350, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.632821) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31731e+14 scos=4.39725e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795892.97949951637755150436992208534955,  
rm=22860366.33048982198263378361971969356546}, {r =  
25795867.25701511448801980095492062593890 ..  
25795896.68746666518951237477219502520548, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.06496329485872911197089488868350}, avoid={});  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=7e-32  
Equations at solution: [.7e-31, .7e-31, -.46e-34]  
Solution in 0.273s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 0.888r=2.57959e+07 in  
[25795867.25701511448801980095492062593890 ..  
644897417186666629737809369304875630137/2500000000000000000000000000000000  
0]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23888989.06496329485872911197089488868350]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678944374453044521541735838287,  
10510.36930058675752987385980790398847213,  
10510.39514076169947859885886603113460896,  
10510.28046624358022892604977046691205390,  
10510.36313884712881705587711784745261010,  
10510.33729983835493911580361103561920255,  
10510.33214580455274926090447258928235121,  
10510.30529788713925523009926644841391280,  
10510.21747048878605796346581018313544152,  
10510.27430497006822128765885790927044806,  
10510.19163019567222731750326208892232669,  
10510.26914997036470657477894550062699048,  
10510.27430450931941313354587261104687761,  
10510.24330971162387192942949247759505618,  
10510.33113809482245769237099311008012022, none,  
10510.18546892752764421305090976822909877,  
10510.21130922473944693406776312921882517, none, none, none, none,  
none, none, none, none, none, none, none, none, none]
```

```
2 --> 1 target = [24850269.43101309810558499278718845079822,  
1.476958531817362915653021440706832616568,  
10510.30530118315697355933149529795053583]  
one interval r = 25781104.90082928150483183775114584719817 ..  
2579589851170975430698001917391257843379/1000000000000000000000000000000000  
000
```



Time Approximations 0.033.

```
hint used Hint := [25795896.62812623062600675830146784169746, 3, -1, 1,  
24743530.75497470617154487139175541092326,  
25795869.08129891002682354003571513818579 ..  
2579589851170975665166085604166659164517/100000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
24850269.43101309861241733616879334665430, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.760171) | P <--- S  
rGuessMin=2.57811e+07    rGuessMax=2.57959e+07    rmGuess=2.47435e+07    k=  
-2.37384e+14    scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=25795896.62812623062600675830146784169746,  
rm=24743530.75497470617154487139175541092326}, {r =  
25795869.08129891002682354003571513818579 ..  
25795898.51170975665166085604166659164517, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850269.43101309861241733616879334665430}, avoid={{}});  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.4949e-31, 0., -.14e-34]  
Solution in 0.347s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 2.019r=2.57959e+07 in  
[25795869.08129891002682354003571513818579 ..  
2579589851170975665166085604166659164517/100000000000000000000000000000000000000]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 24850269.43101309861241733616879334665430]: target and source on the  
different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678944374453044521541735838287,
10510.36930058675752987385980790398847213,
10510.39514076169947859885886603113460896,
10510.28046624358022892604977046691205390,
10510.36313884712881705587711784745261010,
10510.33729983835493911580361103561920255,
10510.33214580455274926090447258928235121,
10510.30529788713925523009926644841391280,
10510.21747048878605796346581018313544152,
10510.27430497006822128765885790927044806,
10510.19163019567222731750326208892232669,
10510.26914997036470657477894550062699048,
10510.27430450931941313354587261104687761,
10510.24330971162387192942949247759505618,
10510.33113809482245769237099311008012022, none,
10510.18546892752764421305090976822909877,
10510.21130922473944693406776312921882517, none, none,
10510.27329709818777324319365924603605742, none, none, none, none,
none, none, none, none, none, none]
```

```
0 --> 1 target = [24850269.43101309810558499278718845079822,
1.476958531817362915653021440706832616568,
```



```
10510.30530118315697355933149529795053583]
one interval r = 23873935.37540036590753719531457397698259 ..
1194449545427563652517976335619715171693/500000000000000000000000000000
00
Time Approximations 0.053.
```

```
hint used Hint := [23888985.62428976346864819081258538077918, 3, -1, 1,
20396632.00572718336484608918971225516342,
23888961.16565299817948948119449178210506 ..
2388899090855127150824991682392263419777/1000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S --> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=
-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888985.62428976346864819081258538077918,
rm=20396632.00572718336484608918971225516342}, {r =
23888961.16565299817948948119449178210506 ..
23888990.90855127150824991682392263419777, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={});
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=9e-32
Equations at solution: [.16e-30, .9e-31, -.83e-34]
Solution in 0.383s
```

```
Time Plot 0 s.
Exiting SolveHard() after 2.496r=2.38890e+07 in
[23888961.16565299817948948119449178210506 ..
2388899090855127150824991682392263419777/1000000000000000000000000000000
000]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678944374453044521541735838287,
10510.36930058675752987385980790398847213,
10510.39514076169947859885886603113460896,
10510.28046624358022892604977046691205390,
10510.36313884712881705587711784745261010,
10510.33729983835493911580361103561920255,
10510.33214580455274926090447258928235121,
10510.30529788713925523009926644841391280,
10510.21747048878605796346581018313544152,
10510.27430497006822128765885790927044806,
10510.19163019567222731750326208892232669,
10510.26914997036470657477894550062699048,
10510.27430450931941313354587261104687761,
10510.24330971162387192942949247759505618,
10510.33113809482245769237099311008012022,
10510.21646359971896238982101106450545018,
10510.18546892752764421305090976822909877,
10510.21130922473944693406776312921882517, none, none,
10510.2732970981877324319365924603605742, none, none, none, none,
none, none, none, none, none, none]
```







```

10510.18546892752764421305090976822909877,
10510.21130922473944693406776312921882517, none, none,
10510.27329709818777324319365924603605742, none, none,
10510.15962933470663056523227688712368931, none, none, none, none,
none, none, none]

0 --> 1 target = [24850262.70323078402809577503064072665331,
1.476940274123879560153977103187927487369,
10510.19163349165958145963209712520700171]
one interval r = 23873928.44821858709409360050141556749567 ..
298612301838385287816147906506913900783/1250000000000000000000000000000
0
Time Approximations 0.042.

hint used Hint := [23888978.86275416794827744619407470166172, 3, -1, 1,
20396633.60486737240849086583158368073964,
23888954.40382902066202638611670198664112 ..
298612301838385292588525592212900916387/1250000000000000000000000000000
0, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=
-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888978.86275416794827744619407470166172,
rm=20396633.60486737240849086583158368073964}, {r =
23888954.40382902066202638611670198664112 ..
23888984.14707082340708204737703207331096, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}));
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=2.4e-31
Equations at solution: [.42e-30, .24e-30, -.155e-33]
Solution in 0.36s

Time Plot 0 s.
Exiting SolveHard() after 2.443r=2.38890e+07 in
[23888954.40382902066202638611670198664112 ..
298612301838385292588525592212900916387/1250000000000000000000000000000
0]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678944374453044521541735838287,
10510.36930058675752987385980790398847213,
10510.39514076169947859885886603113460896,
10510.28046624358022892604977046691205390,
10510.36313884712881705587711784745261010,
10510.33729983835493911580361103561920255,
10510.33214580455274926090447258928235121,
10510.30529788713925523009926644841391280,
10510.21747048878605796346581018313544152,
10510.27430497006822128765885790927044806,
10510.19163019567222731750326208892232669,
10510.26914997036470657477894550062699048,
10510.27430450931941313354587261104687761,
```



```

10510.24330971162387192942949247759505618,
10510.33113809482245769237099311008012022,
10510.21646359971896238982101106450545018,
10510.18546892752764421305090976822909877,
10510.21130922473944693406776312921882517,
10510.10279600727565501714169929528476998, none,
10510.27329709818777324319365924603605742, none, none,
10510.15962933470663056523227688712368931, none, none, none, none,
none, none, none]

0 --> 2 target = [25795893.34215400122428255150645638052943,
1.856291208152937392222856205122142600690,
10510.21747360703603471073218288687796328]
one interval r = 23873930.02298357363513462357438158714897 ..
1194449284208367851066015878545150531807/500000000000000000000000000000
00
Time Approximations 0.048.

hint used Hint := [23888981.93694517027845377679197924121306, 3, -1, 1,
22860455.76745222490936139362415353022295,
23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/4000000000000000000000000000000,
298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.367179) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31730e+14 scos=4.39725e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888981.93694517027845377679197924121306,
rm=22860455.76745222490936139362415353022295}, {r =
23888955.94100364913487175812205942139954 ..
23888985.68416735818079539166696781319750, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=4e-32
Equations at solution: [-.3e-31, -.4e-31, .19e-34]
Solution in 0.4s

Time Plot 0 s.
Exiting SolveHard() after 2.479r=2.38890e+07 in
[23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/4000000000000000000000000000000]
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678944374453044521541735838287,
10510.36930058675752987385980790398847213,
10510.39514076169947859885886603113460896,
10510.28046624358022892604977046691205390,
10510.36313884712881705587711784745261010,
10510.33729983835493911580361103561920255,
10510.33214580455274926090447258928235121,
10510.30529788713925523009926644841391280,
10510.21747048878605796346581018313544152,
10510.27430497006822128765885790927044806,

```



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10510.19163019567222731750326208892232669,
10510.26914997036470657477894550062699048,
10510.27430450931941313354587261104687761,
10510.24330971162387192942949247759505618,
10510.33113809482245769237099311008012022,
10510.21646359971896238982101106450545018,
10510.18546892752764421305090976822909877,
10510.21130922473944693406776312921882517,
10510.10279600727565501714169929528476998, none,
10510.27329709818777324319365924603605742,
10510.15447580459131094619445158130004285, none,
10510.15962933470663056523227688712368931, none, none, none, none,
none, none, none]

1 --> 2 target = [25795893.34215400122428255150645638052943,
1.856291208152937392222856205122142600690,
10510.21747360703603471073218288687796328]
Imaginary part neglected:
-.5585484964768019272613486624672638217295e-15
one interval r = 24835334.93824082518141336010664191516846 ..
497005284653295883936988238930003592591/200000000000000000000000000000
0
Time Approximations 0.041.

hint used Hint := [24850262.33852743516297758053367205548154, 3, 1, 1,
24743505.81299238129247202614818714422035,
24850234.63796912173141920432208117660707 ..
2485026423266479739181704602953513976831/100000000000000000000000000000
000, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.23983) | S ---> P
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=
2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850262.33852743516297758053367205548154,
rm=24743505.81299238129247202614818714422035}, {r =
24850234.63796912173141920432208117660707 ..
24850264.23266479739181704602953513976831, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.27218e-30, 0., .5e-34]
Solution in 0.308s

Time Plot 0 s.
Exiting SolveHard() after 1.045r=2.48503e+07 in
[24850234.63796912173141920432208117660707 ..
2485026423266479739181704602953513976831/100000000000000000000000000000
000]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 25089000]: target and source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678944374453044521541735838287,
10510.36930058675752987385980790398847213,
10510.39514076169947859885886603113460896,
```



```

10510.28046624358022892604977046691205390,
10510.36313884712881705587711784745261010,
10510.33729983835493911580361103561920255,
10510.33214580455274926090447258928235121,
10510.30529788713925523009926644841391280,
10510.21747048878605796346581018313544152,
10510.27430497006822128765885790927044806,
10510.19163019567222731750326208892232669,
10510.26914997036470657477894550062699048,
10510.27430450931941313354587261104687761,
10510.24330971162387192942949247759505618,
10510.33113809482245769237099311008012022,
10510.21646359971896238982101106450545018,
10510.18546892752764421305090976822909877,
10510.21130922473944693406776312921882517,
10510.10279600727565501714169929528476998,
10510.18546846165270388110462146887098032,
10510.27329709818777324319365924603605742,
10510.15447580459131094619445158130004285, none,
10510.15962933470663056523227688712368931, none, none, none, none,
none, none, none]

2 --> 1 target = [24850265.76205358802995762752967178396284,
1.476948575065763255802891182446941648344,
10510.24331300762503128415926824999830236]
one interval r = 25781101.21410470603165359115269226084415 ..
1289794743153523393414592099929794800427/500000000000000000000000000000000000
00
Time Approximations 0.034.

hint used Hint := [25795892.97947942498571214559240875822627, 3, -1, 1,
24743526.20943107487634918939497374544011,
25795865.43257821616570179673528437159660 ..
644897371576761841392917646201273823237/250000000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
24850265.76205359199870736703593497152527, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.76017) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=
-2.37384e+14 scos=5.95462e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25795892.97947942498571214559240875822627,
rm=24743526.20943107487634918939497374544011}, {r =
25795865.43257821616570179673528437159660 ..
25795894.86307047365571670584805095292948, rm =
.1330553304194287328500223794129351168576e-1 ..
24850265.76205359199870736703593497152527}, avoid={});
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.4949e-31, 0., .21e-34]
Solution in 0.346s

Time Plot 0 s.
Exiting SolveHard() after 2.09r=2.57959e+07 in
[25795865.43257821616570179673528437159660 ..
644897371576761841392917646201273823237/250000000000000000000000000000000000
0]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .

```



. 24850265.76205359199870736703593497152527]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678944374453044521541735838287,  
10510.36930058675752987385980790398847213,  
10510.39514076169947859885886603113460896,  
10510.28046624358022892604977046691205390,  
10510.36313884712881705587711784745261010,  
10510.33729983835493911580361103561920255,  
10510.33214580455274926090447258928235121,  
10510.30529788713925523009926644841391280,  
10510.21747048878605796346581018313544152,  
10510.27430497006822128765885790927044806,  
10510.19163019567222731750326208892232669,  
10510.26914997036470657477894550062699048,  
10510.27430450931941313354587261104687761,  
10510.24330971162387192942949247759505618,  
10510.33113809482245769237099311008012022,  
10510.21646359971896238982101106450545018,  
10510.18546892752764421305090976822909877,  
10510.21130922473944693406776312921882517,  
10510.10279600727565501714169929528476998,  
10510.18546846165270388110462146887098032,  
10510.27329709818777324319365924603605742,  
10510.15447580459131094619445158130004285, none,  
10510.15962933470663056523227688712368931, none, none,  
10510.21130888339980651576422393208094934, none, none, none, none]

0 --> 1 target = [24850265.76205358802995762752967178396284,  
1.476948575065763255802891182446941648344,  
10510.24331300762503128415926824999830236]  
one interval r = 23873931.59770124994382980809929682805409 ..  
2388898722121733593065066745624197280641/1000000000000000000000000000000000000  
000  
Time Approximations 0.053.

hint used Hint := [23888981.93692575516559285951162594064654, 3, -1, 1,  
20396632.87781872303409295730403849988110,  
23888957.47813172092635353311983299926098 ..  
597224680530433446694670341991304966889/2500000000000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.466972) | S --> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=  
-5.43203e+14 scos=2.39475e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888981.93692575516559285951162594064654,  
rm=20396632.87781872303409295730403849988110}, {r =  
23888957.47813172092635353311983299926098 ..  
23888987.22121733786778681367965219867556, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));  
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1.4e-31  
Equations at solution: [.25e-30, .14e-30, -.91e-34]  
Solution in 1.448s



Time Plot 0 s.  
Exiting SolveHard() after 2.41r=2.38890e+07 in  
[23888957.47813172092635353311983299926098 ..  
597224680530433446694670341991304966889/2500000000000000000000000000000000  
0]  
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678944374453044521541735838287,  
10510.36930058675752987385980790398847213,  
10510.39514076169947859885886603113460896,  
10510.28046624358022892604977046691205390,  
10510.36313884712881705587711784745261010,  
10510.33729983835493911580361103561920255,  
10510.33214580455274926090447258928235121,  
10510.30529788713925523009926644841391280,  
10510.21747048878605796346581018313544152,  
10510.27430497006822128765885790927044806,  
10510.19163019567222731750326208892232669,  
10510.26914997036470657477894550062699048,  
10510.27430450931941313354587261104687761,  
10510.24330971162387192942949247759505618,  
10510.33113809482245769237099311008012022,  
10510.21646359971896238982101106450545018,  
10510.18546892752764421305090976822909877,  
10510.21130922473944693406776312921882517,  
10510.10279600727565501714169929528476998,  
10510.18546846165270388110462146887098032,  
10510.27329709818777324319365924603605742,  
10510.15447580459131094619445158130004285, none,  
10510.15962933470663056523227688712368931,  
10510.15447547820569218805061294935646403, none,  
10510.21130888339980651576422393208094934, none, none, none, none]

0 --> 2 target = [25795896.38402522990510550944145188641557,  
1.856299066190704801931185722764250045345,  
10510.26915308862799298418916772127598114]  
one interval r = 23873933.17245664306056832136806821576199 ..  
2388898875830375292339974855499409791701/1000000000000000000000000000000000  
000  
Time Approximations 0.048.

hint used Hint := [23888985.01109652402578398653593278001387, 3, -1, 1,  
22860454.49191911041368999760273437756943,  
23888959.01529623342321627788824672003212 ..  
59722471895759389095528762418719523933/2500000000000000000000000000000000,  
298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.367179) | S --> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=  
-4.31731e+14 scos=4.39725e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888985.01109652402578398653593278001387,  
rm=22860454.49191911041368999760273437756943}, {r =











10510.33114121310170867467177422189751175]  
one interval r = 23873936.95014365289322568883033612041005 ..  
2388899244562497022723543607532223529249/1000000000000000000000000000000  
000  
Time Approximations 1.167.

hint used Hint := [23888988.69843567795653101185231580305160, 3, -1, 1,  
22860452.96192886545581288218987601576608,  
23888962.70280479206299436081506076835447 ..  
2388899244562496946280763877837409340903/1000000000000000000000000000000  
000, 298960418182500/22468879468420441 .. 24089000, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.36718) | S ---> P  
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=  
-4.31731e+14 scos=4.39724e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=23888988.69843567795653101185231580305160,  
rm=22860452.96192886545581288218987601576608}, {r =  
23888962.70280479206299436081506076835447 ..  
23888992.44562496946280763877837409340903, rm =  
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={});  
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=0  
Equations at solution: [0., 0., .6e-35]  
Solution in 0.401s

Time Plot 0 s.  
Exiting SolveHard() after 2.466r=2.38890e+07 in  
[23888962.70280479206299436081506076835447 ..  
2388899244562496946280763877837409340903/1000000000000000000000000000000  
000]  
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678944374453044521541735838287,  
10510.36930058675752987385980790398847213,  
10510.39514076169947859885886603113460896,  
10510.28046624358022892604977046691205390,  
10510.36313884712881705587711784745261010,  
10510.33729983835493911580361103561920255,  
10510.33214580455274926090447258928235121,  
10510.30529788713925523009926644841391280,  
10510.21747048878605796346581018313544152,  
10510.27430497006822128765885790927044806,  
10510.19163019567222731750326208892232669,  
10510.26914997036470657477894550062699048,  
10510.27430450931941313354587261104687761,  
10510.24330971162387192942949247759505618,  
10510.33113809482245769237099311008012022,  
10510.21646359971896238982101106450545018,  
10510.18546892752764421305090976822909877,  
10510.21130922473944693406776312921882517,  
10510.10279600727565501714169929528476998,  
10510.18546846165270388110462146887098032,  
10510.27329709818777324319365924603605742,



```

10510.15447580459131094619445158130004285,
10510.23714797597279265497740730730738621,
10510.15962933470663056523227688712368931,
10510.15447547820569218805061294935646403, none,
10510.21130888339980651576422393208094934,
10510.20615520677553885625122485952046430, none, none,
10510.26814323600192393185232591542074846]

1 --> 2 target = [25795900.03265730315684391107415583173566,
1.856308491689605009459892842409162795402,
10510.33114121310170867467177422189751175]
Imaginary part neglected:
-.5585484964768019272613486624672638217295e-15
one interval r = 24835341.78443702824382984072680890555906 ..
2485027096042925362361352172817951418501/1000000000000000000000000000000
000
Time Approximations 0.041.

hint used Hint := [24850269.06631200499486567135274096895840, 3, 1, 1,
24743514.14830449237551739504443492551457,
24850241.36598119958086914129896830053765 ..
2485027096042925490424856544845137371367/1000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.239829) | S ---> P
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=
2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850269.06631200499486567135274096895840,
rm=24743514.14830449237551739504443492551457}, {r =
24850241.36598119958086914129896830053765 ..
24850270.96042925490424856544845137371367, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={});
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.29692e-30, 0., .8e-34]
Solution in 0.31s

Time Plot 0 s.
Exiting SolveHard() after 1.037r=2.48503e+07 in
[24850241.36598119958086914129896830053765 ..
2485027096042925490424856544845137371367/1000000000000000000000000000000
000]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 25089000]: target and source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678944374453044521541735838287,
10510.36930058675752987385980790398847213,
10510.39514076169947859885886603113460896,
10510.28046624358022892604977046691205390,
10510.36313884712881705587711784745261010,
10510.33729983835493911580361103561920255,
10510.33214580455274926090447258928235121,
10510.30529788713925523009926644841391280,
10510.21747048878605796346581018313544152,

```



[illegible]

```
Time Plot 0 s.  
Exiting SolveHard() after 2.263r=2.48503e+07 in  
[24850238.00201200929744389905914961384924 ..  
1242513379829193745424075414664369797603/50000000000000000000000000000000  
00]
```



Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .  
. 23888989.06493588745880515057802801897934]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678944374453044521541735838287,  
10510.36930058675752987385980790398847213,  
10510.39514076169947859885886603113460896,  
10510.28046624358022892604977046691205390,  
10510.36313884712881705587711784745261010,  
10510.33729983835493911580361103561920255,  
10510.33214580455274926090447258928235121,  
10510.30529788713925523009926644841391280,  
10510.21747048878605796346581018313544152,  
10510.27430497006822128765885790927044806,  
10510.19163019567222731750326208892232669,  
10510.26914997036470657477894550062699048,  
10510.27430450931941313354587261104687761,  
10510.24330971162387192942949247759505618,  
10510.33113809482245769237099311008012022,  
10510.21646359971896238982101106450545018,  
10510.18546892752764421305090976822909877,  
10510.21130922473944693406776312921882517,  
10510.10279600727565501714169929528476998,  
10510.18546846165270388110462146887098032,  
10510.27329709818777324319365924603605742,  
10510.15447580459131094619445158130004285,  
10510.23714797597279265497740730730738621,  
10510.15962933470663056523227688712368931,  
10510.15447547820569218805061294935646403,  
10510.29913613970301161007981200851078976,  
10510.21130888339980651576422393208094934,  
10510.20615520677553885625122485952046430,  
10510.18546846677923744262194143441707954, none,  
10510.26814323600192393185232591542074846]

2 --> 0 target = [23888989.06493588726129796898184631991038,  
2.632525136936710641400755073929290004760,  
10510.27430791374397656634998306294689424]  
one interval r = 25781103.05751676369039153708791099022400 ..  
1289794834371977066753494645171235115651/500000000000000000000000000000  
00  
Time Approximations 0.036.

hint used Hint := [25795892.97947239655457473800077316699564, 3, 1, 1,  
22860366.33050119628977322107538139299992,  
25795867.25698799409451243993168295190626 ..  
1289794834371977270055988053158890743643/500000000000000000000000000000  
00, 298960418182500/22468879468420441 ..  
23888989.06493588745880515057802801897934, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.632821) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31731e+14 scos=4.39725e+14  
branch outgoing at target, Counterclockwise



```
(Scattering) fsolve(eqs, {r=25795892.97947239655457473800077316699564,
rm=22860366.33050119628977322107538139299992}, {r =
25795867.25698799409451243993168295190626 ..
25795896.68743954540111976106317781487286, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06493588745880515057802801897934}, avoid={});
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=2e-32
Equations at solution: [.2e-31, .2e-31, 0.]
Solution in 0.267s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.88r=2.57959e+07 in
[25795867.25698799409451243993168295190626 ..
1289794834371977270055988053158890743643/500000000000000000000000000000
00]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23888989.06493588745880515057802801897934]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678944374453044521541735838287,
10510.36930058675752987385980790398847213,
10510.39514076169947859885886603113460896,
10510.28046624358022892604977046691205390,
10510.36313884712881705587711784745261010,
10510.33729983835493911580361103561920255,
10510.33214580455274926090447258928235121,
10510.30529788713925523009926644841391280,
10510.21747048878605796346581018313544152,
10510.27430497006822128765885790927044806,
10510.19163019567222731750326208892232669,
10510.26914997036470657477894550062699048,
10510.27430450931941313354587261104687761,
10510.24330971162387192942949247759505618,
10510.33113809482245769237099311008012022,
10510.21646359971896238982101106450545018,
10510.18546892752764421305090976822909877,
10510.21130922473944693406776312921882517,
10510.10279600727565501714169929528476998,
10510.18546846165270388110462146887098032,
10510.27329709818777324319365924603605742,
10510.15447580459131094619445158130004285,
10510.23714797597279265497740730730738621,
10510.15962933470663056523227688712368931,
10510.15447547820569218805061294935646403,
10510.29913613970301161007981200851078976,
10510.21130888339980651576422393208094934,
10510.20615520677553885625122485952046430,
10510.18546846677923744262194143441707954,
10510.21130876399134661582198253074460619,
10510.26814323600192393185232591542074846]
```

```
Cascade time 53.408
counts: 28, 28
```



Iteration 10

Start Generation 1

```
1 --> 0 target = [23888999.9999999996915660022500580537673,
2.632556434697108084182444910151929585500,
10510.45814019382948665891485933139799865]
```

Imaginary part neglected:

$- .7899068589605941812732624932066412544169e-15$

one interval  $r = 24835349.43352071915453734324251658134389 \dots$

2485027847718525302182703521084968427751/1000000000000000000000000000000  
000

Time Approximations 0.038.

```
hint used Hint := [24850273.21919080441727298772802245902434, 3, 1, 1,
```

20396435.39659592643000634328381896732011,

24850248.88301387793152980010731189259064 ..

621256961929631488519759813441433737669/2500000000000000000000000000

```
0, 298960418182500/22468879468420441 .. 23889000., 1]
```

I search for an scattering ray on opposite branches with  $0 < s_v < 1$

```
(0.533028) | P <--- S
```

```
rGuessMin=2.48353e+07    rGuessMax=2.48503e+07    rmGuess=2.03964e+07    k=
```

5.43204e+14      scos=2.39474e+14

branch outgoing at target, Counterclockwise

```
(Scattering) fsolve(egs, {r=24850273.21919080441727298772802245902434,
```

```
rm=20396435.39659592643000634328381896732011}, {r =
```

24850248.88301387793152980010731189259064 ..

24850278.47718525954079039253765734950676, rm =

```
.1330553304194287328500223794129351168576e-1 .. 23889000.}, avoid={}));
```

Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=1.4e-31

Equations at solution: [.25e-30, .14e-30, -.93e-34]

Solution in 0.434s

Time Plot 0 s.

Exiting SolveHard() after 2.351r=2.48503e+07 in

[24850248.88301387793152980010731189259064 ..

621256961929631488519759813441433737669/250000000000000000000000000000

01

Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .

```

.23889000.]: target and source on the different branches.

```

Counterclockwise ray.

Ray outgoing at target.

Solve Side.

Tau [10510.45813678926798302660673913220334360,

```
10510.36930058663607015748106989090970339, none, none, none, none,
```

none, none, none, none, none, none, none, none, none, none, none, none, none,

none, none, none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none, none, none,

```

none,
none]

```

```
2 --> 0 target = [23888999.99999999996915660022500580537673,
```

2.632556434697108084182444910151929585500,

10510.458140193829486658914859331397998651

Imaginary part neglected:

imaginary part: -0.5924510391192590343506277762390899878035e-15

```
one interval r = 25781113.99083723551608735316641121687965 ..
```

one received 1 25761119.99089/25551000/5551001112100/500 ...  
1289795375390422574860379830982969364413/500000000000000000000000000000

00



Time Approximations 0.035.

```
hint used Hint := [25795903.79985511561316781642113695093491, 3, 1, 1,  
22860361.79220184767639985483105161990097,  
25795878.07759835662287683941521694813384 ..  
161224421923802901739910205661944662513/625000000000000000000000000000  
, 298960418182500/22468879468420441 .. 23889000., 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.63282) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31733e+14 scos=4.39724e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795903.79985511561316781642113695093491,  
rm=22860361.79220184767639985483105161990097}, {r =  
25795878.07759835662287683941521694813384 ..  
25795907.50780846427838563290591114600208, rm =  
.1330553304194287328500223794129351168576e-1 .. 23889000.}), avoid={}));  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1.3e-31  
Equations at solution: [-.13e-30, -.13e-30, .55e-34]  
Solution in 1.412s
```

Time Plot 0 s.

```
Exiting SolveHard() after 2.019r=2.57959e+07 in  
[25795878.07759835662287683941521694813384 ..  
161224421923802901739910205661944662513/62500000000000000000000000000000  
]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23889000.]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678926798302660673913220334360,
10510.36930058663607015748106989090970339,
10510.39514076158689472017956295031490963, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none]
```

Start Generation 2

[illegible]

```
hint used Hint := [25795900.39533807920941978281062023785801, 3, -1, 1,  
24743535.44822274617462265333263319921312,  
25795872.84858705064741864171344489129222 ..  
2579590227891383795486680697763551821623/1000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
24850273.21919080441727298772802245902434, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.760172) |      P    <--- S
```







Solution in 0.419s

Time Plot 0 s.

Exiting SolveHard() after 2.536r=2.38890e+07 in

[23888964.97298890387184335817087833215779 ..  
597224867892343544545306555277700499657/2500000000000000000000000000000000  
0]

Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.

Clockwise ray.

Ray outgoing at target.

Solve Side.

Tau [10510.45813678926798302660673913220334360,  
10510.36930058663607015748106989090970339,  
10510.39514076158689472017956295031490963,  
10510.28046624337562620006676431241263327, none,  
10510.33729983817593982960513996663102281, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none, none]

0 --> 2 target = [25795903.79985510290079615915653501825769,  
1.856318223690018529163326701919528804171,  
10510.39514387992190915610069578340764056]  
one interval r = 23873940.85057848580495996771750509741563 ..  
2388899625275549467561572171199593577943/1000000000000000000000000000000000  
000

Time Approximations 0.051.

hint used Hint := [23888992.50558472677941562199764206716244, 3, -1, 1,  
22860451.38218918783573792125602312359251,  
23888966.51012875395781144697926248654402 ..  
597224906318887365922168983423648563467/2500000000000000000000000000000000  
0, 298960418182500/22468879468420441 .. 24089000, 1]

I search for an scattering ray on opposite branches with  $0 < sv < 1$

(0.36718) | S ---> P

rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=  
-4.31732e+14 scos=4.39724e+14

branch outgoing at target, Clockwise

(Scattering) fsolve(eqs, {r=23888992.50558472677941562199764206716244,  
rm=22860451.38218918783573792125602312359251}, {r =

23888966.51012875395781144697926248654402 ..

23888996.25275549463688675933694594253868, rm =

.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={});

Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.0e-31

Equations at solution: [.10e-30, .10e-30, -.33e-34]

Solution in 0.381s

Time Plot 0 s.

Exiting SolveHard() after 1.325r=2.38890e+07 in

[23888966.51012875395781144697926248654402 ..  
597224906318887365922168983423648563467/2500000000000000000000000000000000  
0]

Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.

Clockwise ray.

Ray outgoing at target.



Solve Side.

```
Tau [10510.45813678926798302660673913220334360,  
10510.36930058663607015748106989090970339,  
10510.39514076158689472017956295031490963,  
10510.28046624337562620006676431241263327, none,  
10510.33729983817593982960513996663102281,  
10510.33214580441012388566347419144552288, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none, none, none, none, none]
```

```
1 --> 2 target = [25795903.79985510290079615915653501825769,
1.856318223690018529163326701919528804171,
10510.39514387992190915610069578340764056]
Imaginary part neglected:
-.7899068589605941812732624932066412544169e-15
one interval r = 24835345.63929295206600911739634220955282 ..
1242513737429890496240257525353377596817/500000000000000000000000000000
00
Time Approximations 0.039.
```

```
hint used Hint := [24850272.85449189132526402658550193546657, 3, 1, 1,  
24743518.84163067386800243860950511726701,  
24850245.15428919258543501803067780064530 ..  
1242513737429890818728656221501545064373/5000000000000000000000000000  
00, 298960418182500/22468879468420441 .. 25089000, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.239828) | S ---> P  
rGuessMin=2.48353e+07    rGuessMax=2.48503e+07    rmGuess=2.47435e+07    k=  
2.37383e+14    scos=5.95462e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850272.85449189132526402658550193546657,  
rm=24743518.84163067386800243860950511726701}, {r =  
24850245.15428919258543501803067780064530 ..  
24850274.74859781637457312443003090128746, rm =  
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=1e-32  
Equations at solution: [-.34641e-30, .1e-31, .60e-34]  
Solution in 0.345s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 2.321r=2.48503e+07 in  
[24850245.15428919258543501803067780064530 ..  
1242513737429890818728656221501545064373/50000000000000000000000000000000  
00]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 25089000]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

Tau [10510.45813678926798302660673913220334360,  
10510.36930058663607015748106989090970339,  
10510.39514076158689472017956295031490963,  
10510.28046624337562620006676431241263327,  
10510.36313884700291897640215338659606339,  
10510.33729983817593982960513996663102281,



```
10510.33214580441012388566347419144552288, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none, none, none, none]
```

Start Generation 3

```
0 --> 2 target = [25795900.39533806417231372911127617952495,
1.856309428789376313612643628604179195851,
10510.33730295649605777591569507347689587]
one interval r = 23873937.32565211402525225089403315523871 ..
119444964060754003727027525266627801797/50000000000000000000000000000000
Time Approximations 0.044.
```

```
hint used Hint := [23888989.06496329485872911197089488868350, 3, -1, 1,
22860452.80984339454018821965990898965218,
23888963.06934924828335479031706673264809 ..
597224820303770076400571671810948623397/25000000000000000000000000000000
0, 298960418182500/22468879468420441 .. 24089000, 1]
```

I search for an scattering ray on opposite branches with  $0 < sv < 1$

```
(0.36718) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31731e+14 scos=4.39724e+14
```

branch outgoing at target, Clockwise

```
(Scattering) fsolve(eqs, {r=23888989.06496329485872911197089488868350,
rm=22860452.80984339454018821965990898965218}, {r =
23888963.06934924828335479031706673264809 ..
```

```
23888992.81215080305602286687243794493588, rm =
```

```
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={{});
```

Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.7e-31

Equations at solution: [-.18e-30, -.17e-30, .66e-34]

Solution in 0.383s

Time Plot 0 s.

Exiting SolveHard() after 2.551r=2.38890e+07 in

```
[23888963.06934924828335479031706673264809 ..
597224820303770076400571671810948623397/25000000000000000000000000000000
0]
```

Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .  
. 24089000]: target and source on the different branches.

Clockwise ray.

Ray outgoing at target.

Solve Side.

```
Tau [10510.45813678926798302660673913220334360,
10510.36930058663607015748106989090970339,
10510.39514076158689472017956295031490963,
10510.28046624337562620006676431241263327,
10510.36313884700291897640215338659606339,
10510.33729983817593982960513996663102281,
10510.33214580441012388566347419144552288, none, none,
10510.27430496985918060796572129880762572, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none, none, none, none, none]
```

```
1 --> 2 target = [25795900.39533806417231372911127617952495,
1.856309428789376313612643628604179195851,
10510.33730295649605777591569507347689587]
```

Imaginary part neglected:











```
22860366.17838899487248578273534549876757,  
25795867.61967723367271871049090258979880 ..  
1289794852506034629127592665641586312049/50000000000000000000000000000000  
00, 298960418182500/22468879468420441 ..  
23888989.43146328644175807501347600997649, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.632821) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=  
4.31731e+14 scos=4.39725e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=25795893.34215400624221088590056918107762,  
rm=22860366.17838899487248578273534549876757}, {r =  
25795867.61967723367271871049090258979880 ..  
25795897.05012069258255185331283172624098, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.43146328644175807501347600997649}, avoid={});  
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1.1e-31  
Equations at solution: [.12e-30, .11e-30, -.17e-34]  
Solution in 0.249s
```

```
Time Plot 0 s.  
Exiting SolveHard() after 0.852r=2.57959e+07 in  
[25795867.61967723367271871049090258979880 ..  
1289794852506034629127592665641586312049/50000000000000000000000000000000  
00]  
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .  
. 23888989.43146328644175807501347600997649]: target and source on the  
different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.
```

```
Tau [10510.45813678926798302660673913220334360,  
10510.36930058663607015748106989090970339,  
10510.39514076158689472017956295031490963,  
10510.28046624337562620006676431241263327,  
10510.36313884700291897640215338659606339,  
10510.33729983817593982960513996663102281,  
10510.33214580441012388566347419144552288,  
10510.30529788694694170710086473195705585,  
10510.21747048864463291012773950665775737,  
10510.27430496985918060796572129880762572,  
10510.19163019552192640233675056758828625, none, none, none, none,  
none, none, none, none, none, none, none, none, none, none, none,  
none, none, none, none]
```

```
1 --> 0 target = [23888992.50558472688771629550727030707137,  
2.632534984498237092195253909623143536923,  
10510.33214920893663945179178717181320184]  
Imaginary part neglected:  
-.7899068589605941812732624932066412544169e-15  
one interval r = 24835341.84514835546509050599594426146890 ..  
2485027102009031497971539063356418241973/10000000000000000000000000000000  
000  
Time Approximations 0.043.
```

```
hint used Hint := [24850265.76205359199870736703593497152527, 3, 1, 1,
```



```

20396437.16957127706091945345483725838621,
24850241.42564446195929857082855634229261 ..
1242513551004516068014443586172817582043/50000000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888992.50558472677941562199764206716244, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.533028) | P <--- S
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.03964e+07 k=
5.43203e+14 scos=2.39475e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850265.76205359199870736703593497152527,
rm=20396437.16957127706091945345483725838621}, {r =
24850241.42564446195929857082855634229261 ..
24850271.02009032136028887172345635164086, rm =
.1330553304194287328500223794129351168576e-1 ..
23888992.50558472677941562199764206716244}, avoid={}));
Accepted {r=2.48503e+07, rm=2.03964e+07} with Delta=6e-32
Equations at solution: [-.11e-30, -.6e-31, .20e-34]
Solution in 0.407s

```

```

Time Plot 0 s.
Exiting SolveHard() after 2.419r=2.48503e+07 in
[24850241.42564446195929857082855634229261 ..
1242513551004516068014443586172817582043/50000000000000000000000000000000
00]
Scattering ray (rm=2.03964e+07) in [298960418182500/22468879468420441 .
. 23888992.50558472677941562199764206716244]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

```

```

Tau [10510.45813678926798302660673913220334360,
10510.36930058663607015748106989090970339,
10510.39514076158689472017956295031490963,
10510.28046624337562620006676431241263327,
10510.36313884700291897640215338659606339,
10510.33729983817593982960513996663102281,
10510.33214580441012388566347419144552288,
10510.30529788694694170710086473195705585,
10510.21747048864463291012773950665775737,
10510.27430496985918060796572129880762572,
10510.19163019552192640233675056758828625, none, none,
10510.24330971153554831152752518946757588, none, none, none, none,
none, none, none, none, none, none, none, none, none, none, none,
none]

```

```

2 --> 0 target = [23888992.50558472688771629550727030707137,
2.632534984498237092195253909623143536923,
10510.33214920893663945179178717181320184]
Imaginary part neglected:
-.5924510391192590343506277762390899878035e-15
one interval r = 25781106.49760328353216693956424525433949 ..
161224375574925193101431604743511239809/62500000000000000000000000000000
Time Approximations 0.036.

```

```

hint used Hint := [25795896.38402523646204355452114396262515, 3, 1, 1,

```



















```

00
Time Approximations 0.031.

hint used Hint := [25795892.97949951637755150436992208534955, 3, 1, 1,
22860366.33048982198263378361971969356546,
25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/2500000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
23888989.06496329485872911197089488868350, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <-- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=
4.31731e+14 scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795892.97949951637755150436992208534955,
rm=22860366.33048982198263378361971969356546}, {r =
25795867.25701511448801980095492062593890 ..
25795896.68746666518951237477219502520548, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06496329485872911197089488868350}, avoid={});
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1e-32
Equations at solution: [-.2e-31, -.1e-31, .33e-34]
Solution in 0.266s

```

```

Time Plot 0 s.
Exiting SolveHard() after 2.006r=2.57959e+07 in
[25795867.25701511448801980095492062593890 ..
644897417186666629737809369304875630137/2500000000000000000000000000000000
0]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23888989.06496329485872911197089488868350]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

```

```

Tau [10510.45813678926798302660673913220334360,
10510.36930058663607015748106989090970339,
10510.39514076158689472017956295031490963,
10510.28046624337562620006676431241263327,
10510.36313884700291897640215338659606339,
10510.33729983817593982960513996663102281,
10510.33214580441012388566347419144552288,
10510.30529788694694170710086473195705585,
10510.21747048864463291012773950665775737,
10510.27430496985918060796572129880762572,
10510.19163019552192640233675056758828625,
10510.26914997028525877604449021463367879,
10510.27430450911037204827188851759675626,
10510.24330971153554831152752518946757588,
10510.33113809463902004091131457972100071, none,
10510.18546892737290534797909456177900118,
10510.21130922459358393394766976468289780, none, none, none, none,
none, none, none, none, none, none, none, none]

```

```

2 --> 1 target = [24850269.43101308854764239836296897374814,
1.476958531574602409909525957899024353916,

```



10510.30530118298182696943076424024156572]  
Imaginary part neglected:  
-.5924510391192590343506277762390899878035e-15  
one interval r = 25781104.90082926739340688855149572839990 ..  
2579589851170974034429166362079366095003/1000000000000000000000000000000  
000  
Time Approximations 0.035.

hint used Hint := [25795896.62812623062600675830146784169746, 3, -1, 1,  
24743530.75497470617154487139175541092326,  
25795869.08129891002682354003571513818579 ..  
2579589851170975665166085604166659164517/1000000000000000000000000000000  
000, 298960418182500/22468879468420441 ..  
24850269.43101309861241733616879334665430, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.760171) | P <--- S  
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=  
-2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Clockwise  
(Scattering) fsolve(eqs, {r=25795896.62812623062600675830146784169746,  
rm=24743530.75497470617154487139175541092326}, {r =  
25795869.08129891002682354003571513818579 ..  
25795898.51170975665166085604166659164517, rm =  
.1330553304194287328500223794129351168576e-1 ..  
24850269.43101309861241733616879334665430}, avoid={}));  
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.22267e-30, 0., -.83e-34]  
Solution in 0.32s

Time Plot 0 s.  
Exiting SolveHard() after 0.908r=2.57959e+07 in  
[25795869.08129891002682354003571513818579 ..  
2579589851170975665166085604166659164517/1000000000000000000000000000000  
000]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 24850269.43101309861241733616879334665430]: target and source on the  
different branches.  
Clockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678926798302660673913220334360,  
10510.36930058663607015748106989090970339,  
10510.39514076158689472017956295031490963,  
10510.28046624337562620006676431241263327,  
10510.36313884700291897640215338659606339,  
10510.33729983817593982960513996663102281,  
10510.33214580441012388566347419144552288,  
10510.30529788694694170710086473195705585,  
10510.21747048864463291012773950665775737,  
10510.27430496985918060796572129880762572,  
10510.19163019552192640233675056758828625,  
10510.26914997028525877604449021463367879,  
10510.27430450911037204827188851759675626,  
10510.24330971153554831152752518946757588,  
10510.33113809463902004091131457972100071, none,  
10510.18546892737290534797909456177900118,



```

10510.21130922459358393394766976468289780, none, none,
10510.27329709793792011220600221767168779, none, none, none, none,
none, none, none, none, none, none]

0 --> 1 target = [24850269.43101308854764239836296897374814,
1.476958531574602409909525957899024353916,
10510.30530118298182696943076424024156572]
one interval r = 23873935.37540036064791305551861135791756 ..
2388899090855126791391476629985024479263/1000000000000000000000000000000
000
Time Approximations 0.045.

hint used Hint := [23888985.62428976346864819081258538077918, 3, -1, 1,
20396632.00572718336484608918971225516342,
23888961.16565299817948948119449178210506 ..
2388899090855127150824991682392263419777/1000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=
-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888985.62428976346864819081258538077918,
rm=20396632.00572718336484608918971225516342}, {r =
23888961.16565299817948948119449178210506 ..
23888990.90855127150824991682392263419777, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=7e-32
Equations at solution: [-.12e-30, -.7e-31, .54e-34]
Solution in 0.409s

Time Plot 0 s.
Exiting SolveHard() after 2.591r=2.38890e+07 in
[23888961.16565299817948948119449178210506 ..
2388899090855127150824991682392263419777/1000000000000000000000000000000
000]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678926798302660673913220334360,
10510.36930058663607015748106989090970339,
10510.39514076158689472017956295031490963,
10510.28046624337562620006676431241263327,
10510.36313884700291897640215338659606339,
10510.33729983817593982960513996663102281,
10510.33214580441012388566347419144552288,
10510.30529788694694170710086473195705585,
10510.21747048864463291012773950665775737,
10510.27430496985918060796572129880762572,
10510.19163019552192640233675056758828625,
10510.26914997028525877604449021463367879,
10510.27430450911037204827188851759675626,
10510.24330971153554831152752518946757588,
10510.33113809463902004091131457972100071,

```



```

10510.21646359944350591830711131808208214,
10510.18546892737290534797909456177900118,
10510.21130922459358393394766976468289780, none, none,
10510.27329709793792011220600221767168779, none, none, none, none,
none, none, none, none, none, none]

2 --> 1 target = [24850262.70323077695673344987323117002626,
1.476940273881125802607069795262377037191,
10510.19163349152644747757447912749718052]
Imaginary part neglected:
-.5924510391192590343506277762390899878035e-15
one interval r = 25781098.14047724244590844892912656543810 ..
2579589182119369644253894279358247712211/1000000000000000000000000000000
000
Time Approximations 0.035.

hint used Hint := [25795889.93759639200267360170740297062535, 3, -1, 1,
24743522.41979937237431350459260520845964,
25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/2000000000000000000000000000000
0, 298960418182500/22468879468420441 ..
24850262.70323078644925875505002433594091, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.76017) | P <-- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.47435e+07 k=
-2.37384e+14 scos=5.95461e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=25795889.93759639200267360170740297062535,
rm=24743522.41979937237431350459260520845964}, {r =
25795862.39063358446748128489577933480278 ..
25795891.82119371218082845151233599574945, rm =
.1330553304194287328500223794129351168576e-1 ..
24850262.70323078644925875505002433594091}, avoid={}));
Accepted {r=2.57959e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [-.17321e-30, 0., .36e-34]
Solution in 0.321s

Time Plot 0 s.
Exiting SolveHard() after 0.924r=2.57959e+07 in
[25795862.39063358446748128489577933480278 ..
515917836423874243616569030246719914989/2000000000000000000000000000000
0]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 24850262.70323078644925875505002433594091]: target and source on the
different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678926798302660673913220334360,
10510.36930058663607015748106989090970339,
10510.39514076158689472017956295031490963,
10510.28046624337562620006676431241263327,
10510.36313884700291897640215338659606339,
10510.33729983817593982960513996663102281,
10510.33214580441012388566347419144552288,
10510.30529788694694170710086473195705585,

```



```

10510.21747048864463291012773950665775737,
10510.27430496985918060796572129880762572,
10510.19163019552192640233675056758828625,
10510.26914997028525877604449021463367879,
10510.27430450911037204827188851759675626,
10510.24330971153554831152752518946757588,
10510.33113809463902004091131457972100071,
10510.21646359944350591830711131808208214,
10510.18546892737290534797909456177900118,
10510.21130922459358393394766976468289780, none, none,
10510.27329709793792011220600221767168779, none, none,
10510.15962933449879008062805122167217164, none, none, none, none,
none, none, none]

0 --> 1 target = [24850262.70323077695673344987323117002626,
1.476940273881125802607069795262377037191,
10510.19163349152644747757447912749718052]
one interval r = 23873928.44821858439477813133950176751712 ..
2388898414707082038790773900443344856531/1000000000000000000000000000000000000
000
Time Approximations 0.047.

hint used Hint := [23888978.86275416794827744619407470166172, 3, -1, 1,
20396633.60486737240849086583158368073964,
23888954.40382902066202638611670198664112 ..
298612301838385292588525592212900916387/1250000000000000000000000000000000000
0, 298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.466972) | S --> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.03966e+07 k=
-5.43203e+14 scos=2.39475e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888978.86275416794827744619407470166172,
rm=20396633.60486737240849086583158368073964}, {r =
23888954.40382902066202638611670198664112 ..
23888984.14707082340708204737703207331096, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}, avoid={}));
Accepted {r=2.38890e+07, rm=2.03966e+07} with Delta=1.0e-31
Equations at solution: [.17e-30, .10e-30, -.60e-34]
Solution in 0.41s

Time Plot 0 s.
Exiting SolveHard() after 2.555r=2.38890e+07 in
[23888954.40382902066202638611670198664112 ..
298612301838385292588525592212900916387/1250000000000000000000000000000000000
0]
Scattering ray (rm=2.03966e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678926798302660673913220334360,
10510.36930058663607015748106989090970339,
10510.39514076158689472017956295031490963,
10510.28046624337562620006676431241263327,
10510.36313884700291897640215338659606339,

```



```

10510.33729983817593982960513996663102281,
10510.33214580441012388566347419144552288,
10510.30529788694694170710086473195705585,
10510.21747048864463291012773950665775737,
10510.27430496985918060796572129880762572,
10510.19163019552192640233675056758828625,
10510.26914997028525877604449021463367879,
10510.27430450911037204827188851759675626,
10510.24330971153554831152752518946757588,
10510.33113809463902004091131457972100071,
10510.21646359944350591830711131808208214,
10510.18546892737290534797909456177900118,
10510.21130922459358393394766976468289780,
10510.10279600704221111572894192638996838, none,
10510.27329709793792011220600221767168779, none, none,
10510.15962933449879008062805122167217164, none, none, none, none,
none, none, none]

0 --> 2 target = [25795893.34215399155844090503555102922141,
1.856291208339420683747165904322496709035,
10510.21747360693388971110766517019259064]
one interval r = 23873930.02298357282437361412116545710148 ..
1194449284208367811365840435280386662713/500000000000000000000000000000
00
Time Approximations 0.044.

hint used Hint := [23888981.93694517027845377679197924121306, 3, -1, 1,
22860455.76745222490936139362415353022295,
23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/400000000000000000000000000000,
298960418182500/22468879468420441 .. 24089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.367179) | S ---> P
rGuessMin=2.38739e+07 rGuessMax=2.38890e+07 rmGuess=2.28605e+07 k=
-4.31730e+14 scos=4.39725e+14
branch outgoing at target, Clockwise
(Scattering) fsolve(eqs, {r=23888981.93694517027845377679197924121306,
rm=22860455.76745222490936139362415353022295}, {r =
23888955.94100364913487175812205942139954 ..
23888985.68416735818079539166696781319750, rm =
.1330553304194287328500223794129351168576e-1 .. 24089000.}), avoid={}));
Accepted {r=2.38890e+07, rm=2.28605e+07} with Delta=1.9e-31
Equations at solution: [-.20e-30, -.19e-30, .54e-34]
Solution in 0.384s

Time Plot 0 s.
Exiting SolveHard() after 2.475r=2.38890e+07 in
[23888955.94100364913487175812205942139954 ..
9555594273666943272318156666787125279/400000000000000000000000000000]
Scattering ray (rm=2.28605e+07) in [298960418182500/22468879468420441 .
. 24089000]: target and source on the different branches.
Clockwise ray.
Ray outgoing at target.
Solve Side.

Tau [10510.45813678926798302660673913220334360,
10510.36930058663607015748106989090970339,
```



[illegible]







```
Equations at solution: [.12372e-30, 0., -.36e-34]
Solution in 0.343s
```

Time Plot 0 s.

Exiting SolveHard() after 2.119r=2.57959e+07 in

[25795865.43257821616570179673528437159660 ..

```
644897371576761841392917646201273823237/2500000000000000000000000000000000  
0]
```

Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441.  
. 24850265.76205359199870736703593497152527]: target and source on the  
different branches.

Clockwise ray.

Ray outgoing at target.

Solve Side.

Tau [10510.45813678926798302660673913220334360,

10510.36930058663607015748106989090970339,

10510.39514076158689472017956295031490963,

10510.28046624337562620006676431241263327,

10510.36313884700291897640215338659606339,

10510.33729983817593982960513996663102281,

10510.33214580441012388566347419144552288.

10510.30529788694694170710086473195705585,

10510.21747048864463291012773950665775737

10510.27430496985918060796572129880762572

10510.19163019552192640233675056758828625

10510.19105019552192040255075050750820025,  
10510.26914997028525877604449021463367879

10510.28914997028523877804449021483387879,  
10510.27430450811037204827188851759675626

10510.27430450911057204827188851759075828,  
10510.24330971153554831152752518946757588

10510.24330971153554831152752518946757588,  
10510.33113808463803004081131457872100071

10510.33115809483902004091131437972100071,  
10510.31646350044350501830711131808208214

10510.21646359944350591830711131808208214,  
10510.18546882737300534787808456177800118

10510.18546892737290534797909456177900118,  
10510.31130032450358303204766076468300780

10510.21130922459358393394/669/6468289/80,  
10510.10070600704001111570004100600006000

10510.102796007042211115728941926389996838,  
10510.10516046140706460735346405303554134

10510.18546846149796462735346405303554104,  
10510.073200700700700700011000600001767160770

```
10510.27329709793792011220600221767168779,  
10510.15447580441984444377951710915202133, none,
```

10510.15962933449879008062805122167217164, none, none,

```
10510.21130888325394336226416137799270874, none, none, none, none]
```

```
0 --> 1 target = [24850265.76205358462694595264625700010704,
```

1.476948574823019453266474794715753677437,

10510.243313007553874599382750560958564801

```
one interval r = 23873931.59770125102157358131511297869418 ..
```

1194449361060866848998917568985020541901/500000000000000000000000000000  
00

Time Approximations 0.048.

```
hint used Hint := [23888981.93692575516559285951162594064654, 3, -1, 1,
```

20396632.87781872303409295730403849988110,

23888957.47813172092635353311983299926098 ..

597224680530433446694670341991304966889/2500000000000000000000000000000

0, 298960418182500/22468879468420441 . . 24089000, 11

```
I search for an scattering ray on opposite branches with 0<sv<1
```

(0.466972) | S ---> P

```
rGuessMin=2.38739e+07    rGuessMax=2.38890e+07    rmGuess=2.03966e+07    k=
```

```

1E+05BIN 2:55/55E+07 1E+05BIN
-5.43203e+14      scos=2.39475e+14

```











Imaginary part neglected:  
-.7899068589605941812732624932066412544169e-15  
one interval r = 24835338.05090068263177682946998943014247 ..  
2485026729147974346312583386798196867589/1000000000000000000000000000000  
000  
Time Approximations 0.041.

hint used Hint := [24850265.39735153163244735697452997613798, 3, 1, 1,  
24743509.60268659785926677493114233979369,  
24850237.69689665459305925255508686501973 ..  
1242513364573987488725142465429694354713/500000000000000000000000000000  
00, 298960418182500/22468879468420441 .. 25089000, 1]  
I search for an scattering ray on opposite branches with  $0 < sv < 1$   
(0.239829) | S --> P  
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=  
2.37384e+14 scos=5.95462e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850265.39735153163244735697452997613798,  
rm=24743509.60268659785926677493114233979369}, {r =  
24850237.69689665459305925255508686501973 ..  
24850267.29147974977450284930859388709426, rm =  
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));  
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0  
Equations at solution: [.22270e-30, 0., -.71e-34]  
Solution in 0.333s

Time Plot 0 s.  
Exiting SolveHard() after 2.264r=2.48503e+07 in  
[24850237.69689665459305925255508686501973 ..  
1242513364573987488725142465429694354713/500000000000000000000000000000  
00]  
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .  
. 25089000]: target and source on the different branches.  
Counterclockwise ray.  
Ray outgoing at target.  
Solve Side.

Tau [10510.45813678926798302660673913220334360,  
10510.36930058663607015748106989090970339,  
10510.39514076158689472017956295031490963,  
10510.28046624337562620006676431241263327,  
10510.36313884700291897640215338659606339,  
10510.33729983817593982960513996663102281,  
10510.33214580441012388566347419144552288,  
10510.30529788694694170710086473195705585,  
10510.21747048864463291012773950665775737,  
10510.27430496985918060796572129880762572,  
10510.19163019552192640233675056758828625,  
10510.26914997028525877604449021463367879,  
10510.27430450911037204827188851759675626,  
10510.24330971153554831152752518946757588,  
10510.33113809463902004091131457972100071,  
10510.21646359944350591830711131808208214,  
10510.18546892737290534797909456177900118,  
10510.21130922459358393394766976468289780,  
10510.10279600704221111572894192638996838,  
10510.18546846149796462735346405303554104,







```

10510.26914997028525877604449021463367879,
10510.27430450911037204827188851759675626,
10510.24330971153554831152752518946757588,
10510.33113809463902004091131457972100071,
10510.21646359944350591830711131808208214,
10510.18546892737290534797909456177900118,
10510.21130922459358393394766976468289780,
10510.10279600704221111572894192638996838,
10510.18546846149796462735346405303554104,
10510.27329709793792011220600221767168779,
10510.15447580441984444377951710915202133,
10510.23714797588003068966922796948222748,
10510.15962933449879008062805122167217164,
10510.15447547803422553042380932260978082, none,
10510.21130888325394336226416137799270874,
10510.20615520666604951230121488875092120, none, none,
10510.26814323578844489382674290375339482]

```

```

1 --> 2 target = [25795900.03265729101818705466183737745248,
1.856308491876081912843863112822074595523,
10510.33114121295755107691482863992426023]
Imaginary part neglected:
-.7899068589605941812732624932066412544169e-15
one interval r = 24835341.78443702038540780864073685632336 ..
2485027096042924589985945542728479554639/1000000000000000000000000000000
000
Time Approximations 0.042.

```

```

hint used Hint := [24850269.06631200499486567135274096895840, 3, 1, 1,
24743514.14830449237551739504443492551457,
24850241.36598119958086914129896830053765 ..
2485027096042925490424856544845137371367/1000000000000000000000000000000
000, 298960418182500/22468879468420441 .. 25089000, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.239829) | S --> P
rGuessMin=2.48353e+07 rGuessMax=2.48503e+07 rmGuess=2.47435e+07 k=
2.37384e+14 scos=5.95462e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=24850269.06631200499486567135274096895840,
rm=24743514.14830449237551739504443492551457}, {r =
24850241.36598119958086914129896830053765 ..
24850270.96042925490424856544845137371367, rm =
.1330553304194287328500223794129351168576e-1 .. 25089000.}, avoid={}));
Accepted {r=2.48503e+07, rm=2.47435e+07} with Delta=0
Equations at solution: [.12372e-30, 0., -.40e-34]
Solution in 0.347s

```

```

Time Plot 0 s.
Exiting SolveHard() after 1.075r=2.48503e+07 in
[24850241.36598119958086914129896830053765 ..
2485027096042925490424856544845137371367/1000000000000000000000000000000
000]
Scattering ray (rm=2.47435e+07) in [298960418182500/22468879468420441 .
. 25089000]: target and source on the different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.

```



[illegible]

```
hint used Hint := [24850262.33852773859450921827957797170818, 3, 1, 1,  
20396437.98350304935080318674410988058300,  
24850238.00201200929744389905914961384924 ..  
1242513379829193745424075414664369797603/500000000000000000000000000000  
00, 298960418182500/22468879468420441 ..  
23888989.06493588745880515057802801897934, 1]  
I search for an scattering ray on opposite branches with 0<sv<1  
(0.533028) | P <--- S  
rGuessMin=2.48353e+07    rGuessMax=2.48503e+07    rmGuess=2.03964e+07    k=  
5.43203e+14    scos=2.39475e+14  
branch outgoing at target, Counterclockwise  
(Scattering) fsolve(eqs, {r=24850262.33852773859450921827957797170818,  
rm=20396437.98350304935080318674410988058300}, {r =  
24850238.00201200929744389905914961384924 ..  
24850267.59658387490848150829328739595206, rm =  
.1330553304194287328500223794129351168576e-1 ..  
23888989.06493588745880515057802801897934}}, avoid={{}});
```







```
hint used Hint := [25795892.97947239655457473800077316699564, 3, 1, 1,
22860366.33050119628977322107538139299992,
25795867.25698799409451243993168295190626 ..
1289794834371977270055988053158890743643/500000000000000000000000000000
00, 298960418182500/22468879468420441 ..
23888989.06493588745880515057802801897934, 1]
I search for an scattering ray on opposite branches with 0<sv<1
(0.632821) | P <--- S
rGuessMin=2.57811e+07 rGuessMax=2.57959e+07 rmGuess=2.28604e+07 k=
4.31731e+14 scos=4.39725e+14
branch outgoing at target, Counterclockwise
(Scattering) fsolve(eqs, {r=25795892.97947239655457473800077316699564,
rm=22860366.33050119628977322107538139299992}, {r =
25795867.25698799409451243993168295190626 ..
25795896.68743954540111976106317781487286, rm =
.1330553304194287328500223794129351168576e-1 ..
23888989.06493588745880515057802801897934}, avoid={}));
Accepted {r=2.57959e+07, rm=2.28604e+07} with Delta=1.5e-31
Equations at solution: [.16e-30, .15e-30, -.71e-34]
Solution in 0.233s
```

```
Time Plot 0 s.
Exiting SolveHard() after 0.828r=2.57959e+07 in
[25795867.25698799409451243993168295190626 ..
1289794834371977270055988053158890743643/500000000000000000000000000000
00]
Scattering ray (rm=2.28604e+07) in [298960418182500/22468879468420441 .
. 23888989.06493588745880515057802801897934]: target and source on the
different branches.
Counterclockwise ray.
Ray outgoing at target.
Solve Side.
```

```
Tau [10510.45813678926798302660673913220334360,
10510.36930058663607015748106989090970339,
10510.39514076158689472017956295031490963,
10510.28046624337562620006676431241263327,
10510.36313884700291897640215338659606339,
10510.33729983817593982960513996663102281,
10510.33214580441012388566347419144552288,
10510.30529788694694170710086473195705585,
10510.21747048864463291012773950665775737,
10510.27430496985918060796572129880762572,
10510.19163019552192640233675056758828625,
10510.26914997028525877604449021463367879,
10510.27430450911037204827188851759675626,
10510.24330971153554831152752518946757588,
10510.33113809463902004091131457972100071,
10510.21646359944350591830711131808208214,
10510.18546892737290534797909456177900118,
10510.21130922459358393394766976468289780,
10510.10279600704221111572894192638996838,
10510.18546846149796462735346405303554104,
10510.27329709793792011220600221767168779,
10510.15447580441984444377951710915202133,
10510.23714797588003068966922796948222748,
10510.15962933449879008062805122167217164,
```



```
10510.15447547803422553042380932260978082,  
10510.29913613950625971965524819180988248,  
10510.21130888325394336226416137799270874,  
10510.20615520666604951230121488875092120,  
10510.18546846662449817196962748331741883,  
10510.21130876384548321012167300700774508,  
10510.26814323578844489382674290375339482]
```

```
Cascade time 54.852  
counts: 28, 28
```

```
> H;
```

```
[1.97167243179101363486669684351828446301 × 10-23,  
1.642101504730337457050345345544979517220 × 10-20,  
8.406791603895718368569938945812840643246 × 10-21,  
6.173263318963566975692769942448037254403 × 10-22,  
4.600146220681093677612984302624911774703 × 10-20,  
1.211421189330445547748226322655058293042 × 10-20,  
1.512788537997035554342519511653683073070 × 10-20,  
5.232550055286191799690376005305233486399 × 10-21,  
8.889377513365852976839803145270685126528 × 10-21,  
2.697717751527673576950617738788003746142 × 10-22,  
4.665909388295331167028344201053197639577 × 10-20]
```

(108)

```
> nops (H) ;
```

11

(109)

```
> Digits;
```

40

(110)

```
> Histogram(H) ;
```



